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1 May 2016

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Abstract

This paper compares the outcomes of two three-stage games of two firms competing for quantity with managerial delegation. In fact, we prove that simultaneous choice of managers by the proprietors of the firms followed by Stackelberg-type competition is equivalent to sequential choice of managers followed by Cournot-type competition. We prove equivalence in a general setting, namely, when the duopolistic model is characterised by a non-linear inverse demand function of the form $p_i = a - (q_i)^n - \gamma(q_j)^n$, $i, j = 1, 2$ and $n \geq 1$.

Keywords: Strategic delegation; Cournot competition; Stackelberg competition

JEL Classification: D43, L13, L21

1 Introduction

In modern day corporation practices the adoption of a managing scheme where owners delegate decision powers to managers is widely adopted and considered as standard. Traditionally, the owner of a firm strives to stimulate the aggressiveness of his manager by committing to an incentive contract rewarding the manager's performance based on a combination of factors such as market share, output and/or profits.

The role of strategic delegation in oligopoly was first investigated by the works of Fershtman and Judd (1985), Vickers (1985) and Sklivas (1987). They based their investigation on the study of two-stage complete information games. In the first stage, the owners of each firm publicly announce the rewarding schemes put forward to their managers, while in the second stage, firms' managers compete for quantities or prices, according to delegated objective functions.

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The effects of strategic delegation on the standard competition models of industrial economics such as the Cournot and Stackelberg models is a topic of increased interest to both scholars and practitioners and the results have the potential to guide regulation.

In their 2008 paper Kopel and Löffler (2008) consider a duopoly game where the two firms are in a Stackelberg-leader-follower-type situation. Amongst other things, they examine whether the market leader sustains her advantage if at a stage prior to quantity decisions the owners of the firms are given the ability to choose their delegation schemes.

The current paper elaborates further on the ideas first developed in the above paper and Stamatopoulos's (2013) pre-print that was privately communicated to us. We examine and compare the outcomes of two three-stage complete information games that both involve managerial delegation by means of a weighted contract rewarding profits and quantity prior to quantity competition. In the first stage of the first game, the G-game, the owners of the first firm commit on the incentive rate of their managerial contract. In the second stage, the owners of the second firm react making up their mind with regards to the incentive rate of their manager. In the third stage of the G-game, the managers of the two firms compete for quantity in a Cournot fashion. In the first stage of the H-game the two owners make simultaneous offers to their managers. In the second stage, the manager of the first firm acts as a leader setting a quantity level and in the third stage of the H-game the manager of the second company reacts accordingly gauging her output level. We prove that equilibrium prices and quantities are the same for the two games. Further, we show that the first mover in the sequential delegation game, the G-game, earns the same profit as the second mover in the sequential quantity game, the H-game. Similarly, the second mover of the delegation game achieves the same profit as the first mover in the quantity game.

We, further, contribute to the literature by considering a duopoly market of a differentiated commodity characterized by a non-linear inverse demand function. To overcome the potential restrictiveness of linear scale curves while, at the same time, exploring the limits of our considerations we consider an inverse demand function of the form $p_i = a-(q_i)^n-\gamma(q_j)^n$ where p_i is i's market price and q_i and q_j with $i, j = 1, 2$ are the quantities produced by the two firms.

The rest of the paper is organized as follows: the model and the results are found in section 2, section 3 sheds light into the heuristics of the result, section 4 derives Stamatopoulos' (2013) linear demand case followed by a brief conclusion in section 5. All technical details are collected in the Appendix.

2 The model

Consider a duopoly market with differentiated commodities. Firm i faces the inverse demand function $p_i = a - f(q_i) - \gamma f(q_j)$ where p_i is i's market price, q_i and q_j

are the quantities produced by firms i and j, with $i, j = 1, 2$, $i \neq j$, $f : \mathbf{R}_+ \rightarrow \mathbf{R}_+$ a homogeneous function of degree $n \in \mathcal{N}$ and $\gamma \in (0, 1]$. The cost function is assumed to be linear, the same for both firms. Let, further, c with $c < a$ denote the marginal production cost. Firms follow a corporate structure characterized by ownership-management separation. Following Vickers (1985), we assume that the managers are offered objective functions that are combinations of profit and quantity sold. The manager of firm i is thus delegated the objective function

$$v_i(q_1, q_2) = (p_i - c)q_i + a_i q_i, \;\; a_i \geq 0, \;\; i = 1, 2$$

The total payoff of manager i is given by $\lambda_i v_i + t_i$. Parameters λ_i and t_i do not affect the choice of quantities and are chosen by the firm's owners to just give the manager his reservation utility. The incentive rate a_i is chosen by the owners of firm i so that they maximize the profit function

$$\pi_i(q_1, q_2) = (p_i - c)q_i, \;\; i = 1, 2$$

The G-game is structured as follows: in the first stage, the owners of firm 1 decide on the incentive rate of their manager, a_1. The owners of firm 2 observe the offer of the first firm and in stage two they react by choosing a_2. Finally, in stage three the managers of the two firms compete selecting quantities q_1 and q_2 in a Cournot fashion.

The structure of the H-game is as follows: in the first stage the owners of the two firms simultaneously choose a_1 and a_2 making their choices publicly known. In the second stage, the manager of the first firm sets a level of quantity and in the third stage, the manager of the second firm reacts by selecting an optimum level of quantity for her firm.

As usually the two games are solved by backward induction. Let $q_1^G(a_1, a_2), q_2^G(a_1, a_2)$ denote the quantities solving the Cournot sub-game played at stage 3 of the G-game. If we denote by $\pi_i(a_1, a_2) = \pi_i(q_1^G(a_1, a_2), q_2^G(a_1, a_2))$, $i = 1, 2$ the profit functions of the two owners respectively then, in stage two, the owners of the second firm select a_2 by solving $\max_{a_2 \geq 0} \pi_2(a_1, a_2)$. Let $a_2(a_1)$ denote their choice. Then, in the first stage, the owners of the first firm solve the problem $\max_{a_1 \geq 0} \pi_1(a_1, a_2(a_1))$. Let, further, a_i^G and p_i^G denote the equilibrium incentive rate and product price of firm i in the G-game, $i = 1, 2$. Then,

$$a_1^G = \frac{n(n+1)\gamma^2\Omega_1}{(n+1)\Omega}(a - c),$$

$$a_2^G = \frac{n(n+1)(n+1-\gamma)\gamma^2\Omega - n^2(n+1)\gamma^5\Omega_1}{(n+1)^3(n+1-\gamma^2)\Omega}(a - c),$$

$$p_1^G = \frac{n(n+1)^3(n+1-\gamma)(n+1-\gamma^2)\Omega}{[(n+1)^2 - \gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a - c)$$

$$+\frac{-n(n+1)^3(n+1-\gamma^2)^2\gamma^2\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c)$$
$$+\frac{-n^2(n+1)(n+1-\gamma)\gamma^3\Omega+n^3(n+1)\gamma^6\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,,$$

$$\begin{aligned} p_2^G &= \frac{n(n+1)^3(n+1-\gamma)(n+1-\gamma^2)\Omega}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\ &\quad +\frac{-n^2(n+1)^3(n+1-\gamma^2)\gamma^3\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\ &\quad +\frac{-n(n+1)(n+1-\gamma)(n+1-\gamma^2)\gamma^2\Omega+n^2(n+1)(n+1-\gamma^2)\gamma^5\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,, \end{aligned}$$

where

$$\Omega_1=(n+1-\gamma)[(n+1)^2-\gamma^2][(n+1)^2(n+1-\gamma^2)-n\gamma^3]\,,$$

and

$$\Omega=[(n+1)^3(n+1-\gamma^2)+n\gamma^4][(n+1)^2(n+1-\gamma^2)^2-n^2\gamma^4]\,.$$

The precise calculations for the above formulae can be found in the Appendix.

Similarly, with respect to the H-game, at stage three the manager of firm 2 responds to the quantity level $q_1(a_1,a_2)$ set by the manager of firm 1 at stage two, by solving the maximization problem $\max_{q_2\geq 0} u_2(q_1,q_2)$. If we denote by $r_2(q_1)$ the solution to this problem, at stage two the manager of firm 1 solves the problem $\max_{q_1} u_1(q_1,r_2(q_1))$ where

$$u_1(q_1,r_2(q_1))=(a-f(q_1)-\gamma f(r_2(q_1))-c)q_1+a_1q_1\,.$$

If $q_1^H(a_1,a_2)$ denotes the solution to this problem and $q_2^H(a_1,a_2)=r_2(q_1^H(a_1,a_2))$ then, at stage one the owners of firms 1 and 2 simultaneously solve the respective problems $\max_{a_1}\pi_1(q_1^H(a_1,a_2),q_2^H(a_1,a_2))$ and $\max_{a_2}\pi_2(q_1^H(a_1,a_2),q_2^H(a_1,a_2))$. Let a_i^H and p_i^H denote the equilibrium incentive rate and product price of firm 2 in H-game. Again, the precise calculations can be found in the Appendix,

$$a_1^H=0\,,\quad a_2^H=\frac{[(n+1)^2-(n+1)\gamma-n\gamma^2]n(n+1)\gamma^2}{(n+1)[(n+1)^2-(2n+1)\gamma^2][(n+1)^2-n\gamma^2]}(a-c)\,,$$

$$\begin{aligned} p_1^H &= \frac{(n+1)[n(n+1)^2-n(n+1)\gamma(\gamma+1)+n\gamma^3]\Omega}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\ &\quad +\frac{-n^2(n+1)(n+1-\gamma^2)\gamma^3\Omega_1}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,, \end{aligned}$$

$$\begin{aligned} p_2^H &= \frac{n(n+1)[(n+1)^2-(n+1)\gamma-n\gamma^2]\Omega}{(n+1)^2(n+1-\gamma^2)\Omega}(a-c) \\ &\quad +\frac{n(n+1)[(2n+1)\gamma^2-(n+1)^2]\gamma^2\Omega_1}{(n+1)^2(n+1-\gamma^2)\Omega}(a-c)\,. \end{aligned}$$

Putting everything together, we get

Theorem 2.1 *For games G and H the following hold true:*

(i) $a_1^G = a_2^H > a_2^G > a_1^H = 0$.

(ii) $p_1^G = p_2^H$, $p_2^G = p_1^H$ *and* $q_1^G = q_2^H$, $q_2^G = q_1^H$.

(iii) $\pi_1^G = \pi_2^H > \pi_2^G = \pi_1^H$.

Proof. The details of the proof are included in the Appendix.

3 The intuition behind the result

In analyzing the main result, Theorem 2.1, we follow the reasoning presented first in [Kopel and Löffler (2008)] and subsequently in [Stamatopoulos (2014)].

Parts (ii) and (iii) of Theorem 2.1 follow readily from part (i) of the same theorem. The H-game follows similar lines to those of the corresponding game in [Kopel and Löffler (2008)]. It is, therefore, not surprising that the leader, the owner of firm 1 in this case, has no incentive to delegate, i.e. $a_1^H = 0$ while the follower, firm 2, does have an incentive to delegate decision power to a manager. This result is directly derived in the Appendix.

The rest of the main theorem follows from the fact that $a_1^G = a_2^H$; i.e., the incentive rate of the first mover in the delegation sub-game of G is equal to the incentive rate of the second mover in the quantity sub-game of H. In consistency with the notation used in section 2, let $q_1^G(a_1) = q_1^G(a_1, a_2(a_1))$ denote the equilibrium quantity of firm 1 in the G-game as a function of a_1, i.e. the incentive rate delegated to the manager of firm 1 being the first mover in the corresponding delegation sub-game. A direct application of the chain rule allows for the splitting of the rate of change of $q_1^G(a_1)$ into two summands as follows:

$$\frac{dq_1^G}{da_1} = \underbrace{\frac{\partial q_1^G}{\partial a_1}}_{\text{direct effect}} + \underbrace{\frac{\partial q_1^G}{\partial a_2}\frac{da_2(a_1)}{da_1}}_{\text{indirect effect}} . \tag{3.0.1}$$

In a similar fashion, the rate of change of $q_2^H(a_2) = q_2^H(a_2, q_1^H(a_1, a_2))$ as a function of a_2 can be split

$$\frac{dq_2^H}{da_2} = \underbrace{\frac{\partial q_2^H}{\partial a_2}}_{\text{direct effect}} + \underbrace{\frac{\partial q_2^H}{\partial q_1}\frac{\partial q_1^H}{\partial a_2}}_{\text{indirect effect}} \tag{3.0.2}$$

Equations (3.0.1) and (3.0.2) decompose the effect of own delegation incentive on own performance into the direct effect measuring the immediate impact of a_1 on q_1^G and a_2 on q_2^H respectively and the indirect effect measuring the corresponding impact via the opponent's response. Given our framework, it turns out that both direct as well as indirect effects are mutually equal. Further, if we let $q_2^G(a_1) = q_2^G(a_1, a_2(a_1))$ and $q_1^H(a_2) = q_1^H(a_1, a_2)$

$$\frac{dq_2^G(a_1, a_2(a_1))}{da_1} = \frac{dq_1^H(a_1, a_2)}{da_2} \tag{3.0.3}$$

is also true.

4 A "linear" example

Consider the case where the inverse demand function is given by

$$p_i(q_1, q_2) = a - q_i - \gamma q_j$$

with $i \neq j, i, j \in \{1, 2\}$ and marginal cost $c < a$.

We, first, look at the G-game. The quantities chosen in the Cournot stage are

$$q_1^G(a_1, a_2) = \frac{(2-\gamma)(a-c) + 2a_1 - \gamma a_2}{4-\gamma^2}, \;\; q_2^G(a_1, a_2) = \frac{(2-\gamma)(a-c) + 2a_2 - \gamma a_1}{4-\gamma^2}.$$

At stage 2, the reaction function of the follower, i.e. the owner of firm 2, is

$$a_2(a_1) = \frac{\gamma^2(2-\gamma)(a-c) - \gamma^3 a_1}{4(2-\gamma^2)}.$$

In equilibrium,

$$a_1^G = \frac{\gamma^2\Omega_1}{\Omega}(a-c), \;\; a_2^G = \frac{(2-\gamma)\gamma^2\Omega - \gamma^5\Omega_1}{4(2-\gamma^2)\Omega}(a-c), \;\; a_1^G > a_2^G$$

where $\Omega_1 = (2-\gamma)(4-\gamma^2)[4(2-\gamma^2) - \gamma^3]$ and $\Omega = [8(2-\gamma^2) + \gamma^4][4(2-\gamma^2)^2 - \gamma^4]$.
Equilibrium prices and quantities in terms of Ω and Ω_1 for firm 1 are given by

$$p_1^G = \left[\frac{2(2-\gamma)[4(2-\gamma^2) - \gamma^3]\Omega}{8(2-\gamma^2)(4-\gamma^2)\Omega} + \frac{2\gamma^2[\gamma^4 - 4(2-\gamma^2)^2]\Omega_1}{8(2-\gamma^2)(4-\gamma^2)\Omega}\right](a-c)$$

$$q_1^G = \left[\frac{(2-\gamma)[4(2-\gamma^2) - \gamma^3]\Omega}{4(2-\gamma^2)(4-\gamma^2)\Omega} + \frac{\gamma^2[8(2-\gamma^2) + \gamma^4]\Omega_1}{4(2-\gamma^2)(4-\gamma^2)\Omega}\right](a-c),$$

while for firm 2 are given by

$$p_2^G = \left[\frac{2(2-\gamma)(2-\gamma^2)(4-\gamma^2)\Omega}{8(2-\gamma^2)(4-\gamma^2)\Omega} + \frac{2(2-\gamma^2)\gamma^3(\gamma^2-4)\Omega_1}{8(2-\gamma^2)(4-\gamma^2)\Omega}\right](a-c)$$

$$q_2^G = \left[\frac{2(2-\gamma)(4-\gamma^2)\Omega}{4(2-\gamma^2)(4-\gamma^2)\Omega} + \frac{2\gamma^3(\gamma^2-4)\Omega_1}{4(2-\gamma^2)(4-\gamma^2)\Omega}\right](a-c).$$

We, then, consider the H-game. Quantities in the last two stages are given by

$$q_1^H(a_1,a_2)=\frac{(2-\gamma)(a-c)-\gamma a_2+2a_1}{2(2-\gamma^2)}$$

and

$$q_2^H(a_1,a_2)=\frac{a-c+a_2-\gamma q_1^H(a_1,a_2)}{2}=\frac{(4-2\gamma-\gamma^2)(a-c)-2\gamma a_1+(4-\gamma^2)a_2}{4(2-\gamma^2)}.$$

In the delegation stage we have

$$a_1^H=0\,,\quad a_2^H=\frac{\gamma^2\Omega_1}{\Omega}(a-c)=a_1^G$$

In addition, it is straightforward to show that $p_2^H=p_1^G$ and $p_1^H=p_2^G$ showing further, that $\pi_2^H=\pi_1^G$ and $\pi_1^H=\pi_2^G$.

5 Conclusion

This paper focuses on a comparative study of the classical competition models of Cournot and Stackelberg when policy decisions are taken by managers. We have proved that sequential delegation under Cournot quantity competition and simultaneous delegation under Stackelberg quantity competition produce the same market outcomes. In this given framework, prices and quantities are found to be identical for these two modes of competition. Further, being the first mover in the delegation game under the first framework is equivalent -in terms of profits- to being the second mover in the quantity game under the second. A similar equality holds between the profits of the second mover in the delegation game and the profits of the first mover in the quantity game. The result could be proved in an even more general setting that of an inverse demand function of the form $p_i=a-(q_i)^n-\gamma(q_j)^m$ where p_i is i's market price and q_i and q_j with $i,j=1,2$ are the quantities produced by the two firms.

Whether a similar result can be derived for the case of more than two firms is a question for future research.

6 Appendix

We first prove a lemma

Lemma 6.1 *Let $f(q)$ be a non-negative real, differentiable function, $f:\mathbf{R}_+\to\mathbf{R}_+$. The function f is homogeneous if and only if $qf'(q)=nf(q)$, for some $n\in\mathbf{N}$.*

Proof. It is a known exercise to show that if f is homogeneous $f(q) = Cq^n$ for some $C > 0$ and $n \in \mathbf{N}$. A straightforward check proves $qf'(q) = nf(q)$.
To prove the converse,

$$qf'(q) = nf(q) \;\Rightarrow\; \frac{f'(q)}{f(q)} = \frac{n}{q} \;\Rightarrow\; \int \frac{f'(q)}{f(q)}\,dq = \int \frac{n}{q}\,dq \;\Rightarrow\; f(q) = Cq^n$$

and f is homogeneous.

Proof of Theorem 2.1: We focus first at the G-game. At the third stage of the game the managers of the two firms compete for quantities à la Cournot. The solution of the system

$$\frac{\vartheta u_1}{\vartheta q_1} = 0\,, \quad \frac{\vartheta u_2}{\vartheta q_2} = 0$$

gives

$$f(q_1) = \frac{a - c + a_1 - \gamma f(q_2)}{n+1}\,, \quad f(q_2) = \frac{a - c + a_2 - \gamma f(q_1)}{n+1}$$

which implies that the quantities chosen at stage 3 as functions of the incentive coefficients a_1, a_2 are given by

$$q_1^G(a_1, a_2) = f^{-1}\left(\frac{(n+1-\gamma)(a-c) + (n+1)a_1 - \gamma a_2}{(n+1)^2 - \gamma^2}\right) \tag{6.1.1}$$

and

$$q_2^G(a_1, a_2) = f^{-1}\left(\frac{(n+1-\gamma)(a-c) + (n+1)a_2 - \gamma a_1}{(n+1)^2 - \gamma^2}\right)\,. \tag{6.1.2}$$

At stage 2, the owners of firm 2 choose a_2 so that to maximise their profit

$$\pi_2(a_1, a_2) = (a - \gamma f(q_1^G(a_1, a_2)) - f(q_2^G(a_1, a_2)) - c)q_2^G(a_1, a_2)\,.$$

We know by Lemma 6.1 that for $i, j = 1, 2$

$$\frac{\vartheta q_i}{\vartheta a_j} = \frac{q_i}{nf(q_i)}\frac{\vartheta}{\vartheta a_j}f(q_i) = \frac{q_i}{nf(q_i)}\frac{n+1}{(n+1)^2 - \gamma^2}\,. \tag{6.1.3}$$

In solving $\frac{\vartheta \pi_2}{\vartheta a_2} = 0$ we take into account (6.1.3) for $i = j = 2$ to get

$$\left(\frac{\gamma^2}{(n+1)^2 - \gamma^2} - \frac{n+1}{(n+1)^2 - \gamma^2}\right)f(q_2) + (a - \gamma f(q_1) - f(q_2) - c)\frac{1}{n}\frac{n+1}{(n+1)^2 - \gamma^2} = 0$$

which yields a_2 as a function of a_1

$$a_2(a_1) = \frac{n(n+1-\gamma)\gamma^2(a-c) - n\gamma^3 a_1}{(n+1)^2(n+1-\gamma^2)}\,. \tag{6.1.4}$$

At stage 1, the owners of firm 1 maximise their profit

$$\pi_1(a_1) = (a - f(q_1^G(a_1)) - \gamma f(q_2^G(a_1))q_1^G(a_1)\,.$$

where $q_1^G(a_1) = q_1^G(a_1, a_2(a_1))$ and $q_2^G(a_1) = q_2^G(a_1, a_2(a_1))$. Plugging the value of a_2 of equation (6.1.4) into equations (6.1.1) and (6.1.2) respectively we get

$$\begin{aligned} f(q_1^G(a_1)) &= \frac{(n+1-\gamma)[(n+1)^2(n+1-\gamma^2) - n\gamma^3](a-c)}{[(n+1)^2-\gamma^2](n+1)^2(n+1-\gamma^2)} \\ &\quad + \frac{[(n+1)^3(n+1-\gamma^2) + n\gamma^4]a_1}{[(n+1)^2-\gamma^2](n+1)^2(n+1-\gamma^2)} \end{aligned}$$

and

$$\begin{aligned} f(q_2^G(a_1)) &= \frac{(n+1)(n+1-\gamma)[(n+1)(n+1-\gamma^2) + n\gamma^2](a-c)}{[(n+1)^2-\gamma^2](n+1)^2(n+1-\gamma^2)} \\ &\quad + \frac{-(n+1)[(n+1)(n+1-\gamma^2)\gamma + n\gamma^3]a_1}{[(n+1)^2-\gamma^2](n+1)^2(n+1-\gamma^2)} . \end{aligned}$$

Solving $\frac{d\pi_1(a_1)}{da_1} = 0$ gives

$$a_1^G = \frac{n(n+1)\gamma^2\Omega_1}{(n+1)\Omega}(a-c)\,, \tag{6.1.5}$$

$$a_2^G = \frac{n(n+1)(n+1-\gamma)\gamma^2\Omega - n^2(n+1)\gamma^5\Omega_1}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,, \tag{6.1.6}$$

with

$$\Omega_1 = (n+1-\gamma)[(n+1)^2-\gamma^2][(n+1)^2(n+1-\gamma^2) - n\gamma^3]\,,$$

and

$$\Omega = [(n+1)^3(n+1-\gamma^2) + n\gamma^4][(n+1)^2(n+1-\gamma^2)^2 - n^2\gamma^4]\,.$$

We, now, come to the H-game. At stage 3, the manager of firm 2 solves the problem

$$\max_{q_2 \geq 0} u_2(q_1.q_2) = \max_{q_2 \geq 0}[(a - \gamma f(q_1) - f(q_2) - c)q_2 + a_2 q_2]$$

to find the response function

$$f(q_2(q_1)) = \frac{a - c + a_2 - \gamma f(q_1)}{n+1} \tag{6.1.7}$$

or equivalently, since by Lemma 6.1 f is 1–1,

$$q_2(q_1) = f^{-1}\left(\frac{a - c + a_2 - \gamma f(q_1)}{n+1}\right) .$$

Then, at stage 2 the manager of firm 1, having taken into account the response of the manager of firm 2 at stage 3, would seek to maximize his objective function

$$u_1(q_1) = (a - f(q_1) - \gamma f(q_2(q_1)) - c)q_1 + a_1 q_1 .$$

A straightforward calculation gives

$$q_1^H(a_1,a_2) = f^{-1}\left(\frac{(n+1-\gamma)(a-c)-\gamma a_2+(n+1)a_1}{(n+1)(n+1-\gamma^2)}\right)$$

and consequently

$$\begin{aligned} q_2^H(a_1,a_2) &= f^{-1}\left(\frac{[(n+1)^2-(n+1)\gamma-n\gamma^2](a-c)-(n+1)\gamma a_1}{(n+1)^2(n+1-\gamma^2)}\right. \\ &\quad \left. +\frac{[(n+1)^2-n\gamma^2]a_2}{(n+1)^2(n+1-\gamma^2)}\right). \end{aligned}$$

At stage 3 the owners of the two firms simultaneously solve their respective maximization problems

$$\max_{a_1\geq 0}\pi_1(a_1,a_2), \quad \max_{a_2\geq 0}\pi_2(a_1,a_2)$$

where

$$\begin{aligned} \pi_1(a_1,a_2) &= (a-f(q_1(a_1,a_2))-\gamma f(q_2(a_1,a_2))-c)q_1(a_1,a_2) \\ &= (a-f(q_1(a_1,a_2))-\gamma r_2(f(q_1(a_1,a_2)))-c)q_1(a_1,a_2) \end{aligned}$$

with

$$r_2(t)=\frac{a-c+a_2-\gamma t}{n+1}$$

from equation (6.1.7) and

$$\pi_2(a_1,a_2)=(a-f(q_2(a_1,a_2))-\gamma f(q_1(a_1,a_2))-c)q_2(a_1,a_2).$$

Because of (6.1.3) applied for $i=j=1$

$$\begin{aligned} \frac{\vartheta\pi_1}{\vartheta a_1} &= \left(-\frac{\vartheta f(q_1(a_1,a_2))}{\vartheta a_1}-\gamma\frac{dr_2}{df(q_1)}\frac{\vartheta f(q_1(a_1,a_2))}{\vartheta a_1}\right)q_1(a_1,a_2) \\ &\quad +(a-f(q_1(a_1,a_2))-\gamma r_2(f(q_1(a_1,a_2)))-c)\frac{\vartheta q_1(a_1,a_2)}{\vartheta a_1} \\ &= \left(-\frac{df(q_1)}{dq_1}-\gamma\frac{dr_2}{df(q_1)}\frac{df(q_1)}{dq_1}\right)\frac{\vartheta q_1(a_1,a_2)}{\vartheta a_1}q_1(a_1,a_2) \\ &\quad +(a-f(q_1(a_1,a_2))-\gamma r_2(f(q_1(a_1,a_2)))-c)\frac{\vartheta q_1(a_1,a_2)}{\vartheta a_1} \\ &= \left(\left(-\frac{df(q_1)}{dq_1}-\gamma\frac{df(q_2(q_1))}{dq_1}\right)q_1+(a-f(q_1)-\gamma f(q_2(q_1))-c)\right)\frac{\vartheta q_1(a_1,a_2)}{\vartheta a_1} \\ &= \frac{d\pi_1(q_1)}{dq_1}\frac{\vartheta q_1(a_1,a_2)}{\vartheta a_1}=-a_1\frac{\vartheta q_1(a_1,a_2)}{\vartheta a_1}<0 \end{aligned}$$

therefore, $\pi_1(a_1, a_2)$ is a decreasing function of a_1 taking its maximum value at $a_1^H = 0$. Taking this into account $\pi_2(a_1, a_2)$ becomes

$$\begin{aligned}\pi_2(a_2) &= \Bigg(\frac{(n+1)^2(n+1-\gamma^2)(a-c)}{(n+1)^2(n+1-\gamma^2)}\\ &\quad + \frac{-\gamma(n+1)(n+1-\gamma)(a-c)+\gamma^2(n+1)a_2}{(n+1)^2(n+1-\gamma^2)}\\ &\quad + \frac{-[(n+1)^2-(n+1)\gamma-n\gamma^2](a-c)-[(n+1)^2-n\gamma^2]a_2}{(n+1)^2(n+1-\gamma^2)}\Bigg)\, q_2(0, a_2)\,.\end{aligned}$$

The first order condition following a lengthy, albeit straightforward, calculation gives

$$a_2^H = \frac{[(n+1)^2-(n+1)\gamma-n\gamma^2]n(n+1)\gamma^2}{(n+1)[(n+1)^2-(2n+1)\gamma^2][(n+1)^2-n\gamma^2]}(a-c)\,. \tag{6.1.8}$$

One verifies directly that

$$a_1^G = a_2^H > a_2^G > a_1^H = 0$$

proving the first part of the Theorem 2.1. For the second part we calculate $p_i^G(a_1^G, a_2^G)$ and $p_i^H(a_1^H, a_2^H)$ for $i = 1, 2$.

$$\begin{aligned}p_1^G(a_1^G, a_2^G) &= a - f(q_1(a_1^G, a_2^G)) - \gamma f(q_2(a_1^G, a_2^G)) - c\\ &= \frac{[(n+1)^2-\gamma^2](a-c)-(n+1-\gamma)(a-c)-(n+1)a_1^G+\gamma a_2^G}{(n+1)^2-\gamma^2}\\ &\quad -\gamma\frac{(n+1-\gamma)(a-c)+(n+1)a_2^G-\gamma a_1^G}{(n+1)^2-\gamma^2}\\ &= \frac{n(n+1-\gamma)(a-c)+(\gamma^2-n-1)a_1^G-n\gamma a_2^G}{(n+1)^2-\gamma^2}\,,\end{aligned}$$

$$\begin{aligned}p_2^G(a_1^G, a_2^G) &= a - \gamma f(q_1(a_1^G, a_2^G)) - f(q_2(a_1^G, a_2^G)) - c\\ &= \frac{[(n+1)^2-\gamma^2](a-c)-\gamma(n+1-\gamma)(a-c)-\gamma(n+1)a_1^G+\gamma^2 a_2^G}{(n+1)^2-\gamma^2}\\ &\quad -\frac{(n+1-\gamma)(a-c)+(n+1)a_2^G-\gamma a_1^G}{(n+1)^2-\gamma^2}\\ &= \frac{n(n+1-\gamma)(a-c)-\gamma n a_1^G+(\gamma^2-n-1)a_2^G}{(n+1)^2-\gamma^2}\,.\end{aligned}$$

Also,

$$\begin{aligned}p_1^H(a_1^H, a_2^H) &= a - f(q_1(a_1^H, a_2^H)) - \gamma f(q_2(a_1^H, a_2^H)) - c\\ &= \frac{(n+1)^2(n+1-\gamma^2)(a-c)}{(n+1)^2(n+1-\gamma^2)}\\ &\quad -\frac{(n+1)(n+1-\gamma)(a-c)-(n+1)\gamma a_2^H+(n+1)^2 a_1^H}{(n+1)^2(n+1-\gamma^2)}\end{aligned}$$

$$
\begin{aligned}
& -\frac{\gamma[(n+1)^2-(n+1)\gamma-n\gamma^2](a-c)-(n+1)\gamma^2 a_1^H+\gamma[(n+1)^2-n\gamma^2]a_2^H}{(n+1)^2(n+1-\gamma^2)} \\
= & \frac{[n(n+1)^2-n(n+1)\gamma(\gamma+1)+n\gamma^3](a-c)}{(n+1)^2(n+1-\gamma^2)} \\
& +\frac{(n+1)(\gamma^2-n-1)a_1^H}{(n+1)^2(n+1-\gamma^2)}+\frac{n\gamma(\gamma^2-n-1)a_2^H}{(n+1)^2(n+1-\gamma^2)},
\end{aligned}
$$

and

$$
\begin{aligned}
p_2^H(a_1^H,a_2^H) &= a-\gamma f(q_1(a_1^H,a_2^H))-f(q_2(a_1^H,a_2^H))-c \\
&= \frac{(n+1)^2(n+1-\gamma^2)(a-c)}{(n+1)^2(n+1-\gamma^2)} \\
& \quad -\frac{\gamma(n+1)(n+1-\gamma)(a-c)-\gamma^2(n+1)a_2^H+\gamma(n+1)^2 a_1^H}{(n+1)^2(n+1-\gamma^2)} \\
& \quad -\frac{[(n+1)^2-(n+1)\gamma-n\gamma^2](a-c)-(n+1)\gamma a_1^H+[(n+1)^2-n\gamma^2]a_2^H}{(n+1)^2(n+1-\gamma^2)} \\
&= \frac{n[(n+1)^2-(n+1)\gamma-n\gamma^2](a-c)}{(n+1)^2(n+1-\gamma^2)} \\
& \quad -\frac{n(n+1)\gamma a_1^H}{(n+1)^2(n+1-\gamma^2)}+\frac{[(2n+1)\gamma^2-(n+1)^2]a_2^H}{(n+1)^2(n+1-\gamma^2)}.
\end{aligned}
$$

The first part of Theorem 2.1, i.e.

$$
a_1^H=0\,, \quad ,a_2^H=a_1^G=\frac{n(n+1)\gamma^2\Omega_1(a-c)}{(n+1)\Omega}\,,
$$

implies that

$$
\begin{aligned}
p_1^G &= \frac{n(n+1)^3(n+1-\gamma^2)(n+1-\gamma)\Omega}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\
& \quad +\frac{-n(n+1)^3(n+1-\gamma^2)^2\gamma^2\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\
& \quad +\frac{-n^2(n+1)(n+1-\gamma)\gamma^3\Omega+n^3(n+1)\gamma^6\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,,
\end{aligned}
$$

$$
\begin{aligned}
p_2^G &= \frac{n(n+1)^3(n+1-\gamma)(n+1-\gamma^2)\Omega}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\
& \quad +\frac{-n^2(n+1)^3(n+1-\gamma^2)\gamma^3\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c) \\
& \quad +\frac{-n(n+1)(n+1-\gamma)(n+1-\gamma^2)\gamma^2\Omega+n^2(n+1)(n+1-\gamma^2)\gamma^5\Omega_1}{[(n+1)^2-\gamma^2](n+1)^3(n+1-\gamma^2)\Omega}(a-c)
\end{aligned}
$$

and

$$p_1^H = \frac{(n+1)[n(n+1)^2 - n(n+1)\gamma(\gamma+1) + n\gamma^3]\Omega}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c)$$
$$+\frac{-n^2(n+1)(n+1-\gamma^2)\gamma^3\Omega_1}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,,$$

$$p_2^H = \frac{n(n+1)[(n+1)^2 - (n+1)\gamma - n\gamma^2]\Omega}{(n+1)^2(n+1-\gamma^2)}(a-c)$$
$$+\frac{n(n+1)[(2n+1)\gamma^2 - (n+1)^2]\gamma^2\Omega_1}{(n+1)^2(n+1-\gamma^2)}(a-c)\,.$$

By comparing the coefficients of Ω and Ω_1 one directly establishes

$$p_1^G = p_2^H \quad \text{and} \quad p_2^G = p_1^H$$

proving half of Theorem 2.1(ii).
To the end of proving the remaining half of Theorem 2.1(ii), we observe that

$$f(q_1^G) = \frac{(n+1)^2(n+1-\gamma)(n+1-\gamma^2)\Omega + n(n+1)^3(n+1-\gamma^2)\gamma^2\Omega_1}{[(n+1)^2-\gamma^2](n+1-\gamma^2)(n+1)^2\Omega}(a-c)$$
$$-\gamma\frac{n(n+1-\gamma)\gamma^2\Omega - n^2\gamma^5\Omega_1}{[(n+1)^2-\gamma^2](n+1-\gamma^2)(n+1)^2\Omega}(a-c)\,,$$

$$f(q_2^G) = \frac{(n+1)^2(n+1-\gamma)(n+1-\gamma^2)\Omega}{[(n+1)^2-\gamma^2](n+1-\gamma^2)(n+1)^2\Omega}(a-c)$$
$$+\frac{n(n+1)(n+1-\gamma)\gamma^2\Omega - n^2(n+1)\gamma^5\Omega_1}{[(n+1)^2-\gamma^2](n+1-\gamma^2)(n+1)^2\Omega}(a-c)$$
$$-\gamma\frac{n(n+1)^2(n+1-\gamma^2)\gamma^2\Omega_1}{[(n+1)^2-\gamma^2](n+1-\gamma^2)(n+1)^2\Omega}(a-c)\,.$$

For $q_i^H, 1 = 1, 2$, we get

$$f(q_1^H) = \left[\frac{(n+1)(n+1-\gamma)\Omega}{(n+1)^2(n+1-\gamma^2)\Omega} - \gamma\frac{n(n+1)\gamma^2\Omega_1}{(n+1)^2(n+1-\gamma^2)\Omega}\right](a-c)\,,$$

$$f(q_2^H) = \frac{(n+1)[(n+1)(n+1-\gamma^2) - \gamma(n+1-\gamma)]\Omega}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c)$$
$$+\frac{n(n+1)[(n+1)(n+1-\gamma^2)+\gamma^2]\gamma^2\Omega_1}{(n+1)^3(n+1-\gamma^2)\Omega}(a-c)\,.$$

By comparing, once more, the coefficients of Ω and Ω_1 in the above formulae and because f is 1–1 one directly establishes

$$q_1^G = q_2^H \quad \text{and} \quad q_2^G = q_1^H$$

which completes the proof of Theorem 2.1(ii).
The proof of Theorem 2.1(iii) follows, now, immediately.

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