# ‘Williamson’s Fallacy’ in Estimation of Inter-Regional Inequality

Gluschenko, Konstantin

Institute of Economics and Industrial Engineering, Siberian Branch of the Russian Academy of Sciences, Novosibirsk State University

20 November 2015

# ‘Williamson’s Fallacy’ in Estimation of Inter-Regional Inequality

KONSTANTIN GLUSCHENKO
Institute of Economics and Industrial Engineering, Siberian Branch of the Russian Academy of Sciences, Russia;
and
Novosibirsk State University, Russia

pr. Lavrentieva 17, 630090 Novosibirsk, Russia.
Tel.: +7 383 330 2548;
fax: +7 383 330 2580.
*E-mail address:* glu@nsu.ru.

**First version: November 2015**

**This version: May 2016**

ABSTRACT *While estimating regional inequality, many economists use inequality indices weighted by regions’ shares in the national population. Despite this approach is widespread, its adequacy has not received attention in the regional science literature. This paper proves that such approach is conceptually inconsistent, yielding an estimate of interpersonal inequality among the whole population of the country rather than an estimate of regional inequality. Moreover, the population-weighted inequality indices do not meet requirements to an adequate inequality measure.*

KEYWORDS: *inequality index; weighting by population; Williamson coefficient of variation; inequality axioms*

JEL CLASSIFICATION: D31; D63; R10

## 1. Introduction

Studying economic inequality in a country, one may consider distribution of, say, income between individuals or between country's regions. Not only does the latter introduce spatial dimension in studies of inequality, but it can also reveal important links remaining overlooked with treating the country as a whole. For example, while the literature on civil war has found little support for a link between individual-level economic inequality and civil war, Deiwiks et al. (2012) find strong evidence that regional inequality affects the risk of secessionist conflict. In both cases, the same statistical methodology and inequality indices (which amount to a few tens) are applied, with the difference that regions rather than individuals are taken as observations while estimating regional inequality. However, there is a modification of the inequality indices that is used to measure regional inequality.

Apparently, Williamson (1965) was the first who put forward the idea of weighting indices that measure inequality between regions of a country by regions' shares in the national population. Since then such an approach became fairly widespread in regional studies. Publications that apply it number in hundreds. Therefore I am able to cite only a small part of them, using a dozen of recent journal articles as a 'sample'. Table 1 tabulates them, reporting inequality indices applied as well as geographical and temporal coverage of respective studies. In this table, $CV$ = coefficient of variation, $G$ = Gini index, $Th$ = Theil index, $MLD$ = mean logarithmic deviation, $\sigma$ = standard deviation of logarithms and $RMD$ = relative mean deviation. Subscript $w$ indicates that the index is a population-weighted one.

Most studies from Table 1 use regional GDP – elsewhere called gross regional product – per capita as a well-being indicator. An exception is Doran & Jordan (2013) who exploit regional gross value added per capita; a few studies consider some additional indicators. The table shows that the application of the population-weighted inequality indices is greatly varied both in geographical terms and time spans (note that if different countries are involved in a study, the case at hand is not international inequality; the study deals with regional inequalities in relevant countries or in a set of countries). Inequality indices used are also manifold. The most popular ones are the coefficient of variation, Gini and Theil indices (many other, 'out of sample', papers confirm this). Therefore only these three indices will be dealt with in what follows. It should be noted that the population-weighted indices are present not only in the literature on economic inequality; they find use in studies of inequality in the areas of health care, education, energy policy, etc.

**Table 1.** Selected recent studies that use population-weighted inequality indices.

| Author(s) | Index(es) used | Geographical coverage | Time span |
|---|---|---|---|
| Doran & Jordan (2013) | $Th_w$ | 14 EU countries (NUTS 2 regions) | 1980–2009 |
| Enflo & Rosés (2015) | $MLD_w$ | Sweden | 1860–2000 |
| Ezcurra & Rodríguez-Pose (2014) | $Th_w$, $CV_w$, $MLD_w$, $\sigma_w$ | 22 emerging countries | 1990–2006 |
| Kyriacou & Roca-Sagalés (2014) | $CV_w$, $MLD_w$, $\sigma_w$ | 22 OECD countries | 1990–2005 |
| Lessmann (2014) | $CV_w$ | 56 countries | 1980–2009 |
| Li & Gibson (2013) | $G_w$, $CV_w$, $Th_w$ | China | 1990–2010 |
| Martínez-Galarraga et al. (2015) | $Th_w$ | Spain | 1860–2000 |
| Mussini (2015) | $G_w$ | 28 EU countries (NUTS 3 regions) | 2003–2011 |
| Petrakos & Psycharis (2016) | $CV_w$ | Greece | 2000–2012 |
| Sacchi & Salotti (2014) | $CV_w$, $\sigma_w$ | 21 OECD countries | 1981–2005 |
| Wijerathna et al. (2014) | $G_w$, $CV_w$, $RMD_w$ | Sri Lanka | 1996–2011 |
| Zubarevich & Safronov (2011) | $G_w$, $CV_w$ | Russia, Ukraine, Kazakhstan | 1998–2009 |

Williamson did not provide a more or less detailed substantiation of his idea, merely noted that an unweighted inequality index ‘will be determined in part by the somewhat arbitrary political definition of regional units’ and ‘[t]he preference for an unweighted index over a weighted one, we think, is indefensible’ (Williamson, 1965, pp. 11, 34). Nor such substantiations appeared in the next 50 years. Even a handbook chapter on measuring regional divides only asserts that the use of unweighted inequality indices ‘may lead to unrealistic results in certain cases, affecting our perception of convergence or divergence trends’ (Ezcurra & Rodríguez-Pose, 2009, p. 332), providing no proof or example. The only attempt to explore properties of the population-weighted indices is due to Portnov & Felsenstein (2010); it will be discussed in Section 4.

It seems that more often than not relevant studies apply Williamson’s idea mechanically, not considering the benefit of weighting and its sense. A number of them (including some of the cited studies) estimate both population-weighted and unweighted indices without discussing distinctions between these. It appears that the authors of such studies believe weighted and unweighted versions of an index to be interchangeable or complementary (like, say, the coefficient of variation and Theil index). Yet even Williamson’s brief notes cited above are open to question.

First, the political division of a country is the reality which regional researchers should deal with, irrespective of whether they believe it to be ‘somewhat arbitrary’ or ‘natural’. Certainly, they may discuss its shortcomings and find ways of improvement, but it is a quite different story unrelated to the issue of regional inequality. Therefore the desire for ‘adjustment’ of existing political division through weighting regional disparities seems strange.

Second, why do we need taking into account differences in regional population at all? But we can estimate inequality among groups in country’s population without regard for sizes of these groups. For instance, while estimating wage inequality between industrial workers, builders, teachers, lawyers and so on, we do not care what shares of these occupational groups in the total population (or employees) are. What is a fundamental difference between this and the case when each population group consists of inhabitants of one region?

Third, on closer inspection results of estimating inequality with the use of population-weighted indices look striking; they may prove to be evidently unrealistic. The next section provides a glowing example.

The purpose of this paper is to show that application of the population-weighted indices for measuring regional inequality is nothing but a fallacy. The main point is that they measure not inequality between regions but something else and therefore yield distorted estimates of regional inequality. Albeit Williamson’s approach has received some criticism in the literature (which will be discussed in Section 4), it has overlooked this point. Moreover, this paper proves that these indices do not meet requirements to an adequate inequality measure.

The rest of the paper is organized as follows. Section 2 reveals the true sense of estimations of inequality obtained with the use of the population-weighted indices. Section 3 analyzes properties of the population-weighted indices, providing proofs that they violate two important axioms. Section 4 discusses arguments against and in favour of the population weighting that are found in the literature. Section 5 summarizes conclusions drawn in the paper.

## 2. What Do Population-Weighted Indices Measure?

Consider cross-region income distribution $y = (y_i)$, $i = 1, \ldots, m$; $y_i$ = per capita income in region $i$ and $\bar{y}$ = the arithmetic average of regional per capita incomes ($\bar{y} = (y_1 + \ldots + y_m)/m$). Then the coefficient of variation measuring regional inequality has the form

$$CV = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y})^2 / m}}{\bar{y}}. \tag{1}$$

Now let $N_i$ = population of region $i$; $N$ = population of the country; $n_i = N_i/N$ = region's share in the national population (region's weight); $n = (n_i)$ will be called population distribution. The weighted average of regional per capita incomes ( $\bar{y}_{(w)} = n_1 y_1 + \ldots + n_m y_m$ ) is denoted by $\bar{y}_{(w)}$. It exactly equals the national per capita income: $\bar{y}_{(w)} = (Y_1 + \ldots + Y_m) / N = Y / N$, where $Y_i$ stands for region's total income ($Y_i = N_i y_i$) and $Y$ represents the national total income. Under this notation, the Williamson coefficient of variation (Williamson, 1965, p. 11) – sometimes called the Williamson index – looks like

$$CV_w = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y}_{(w)})^2 n_i}}{\bar{y}_{(w)}}. \tag{2}$$

The Gini and Theil indices can be respectively written as

$$G = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m}|y_i - y_k|}{2m^2 \bar{y}}; \tag{3}$$

$$Th = \frac{1}{m}\sum_{i=1}^{m}\frac{y_i}{\bar{y}}\ln(\frac{y_i}{\bar{y}}). \tag{4}$$

Their population-weighted counterparts take the forms

$$G_w = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} n_i n_k |y_i - y_k|}{2\bar{y}_{(w)}}; \tag{5}$$

$$Th_w = \sum_{i=1}^{m} n_i \frac{y_i}{\bar{y}_{(w)}}\ln(\frac{y_i}{\bar{y}_{(w)}}). \tag{6}$$

Sometimes, the weighting by population is present in the Theil index implicitly. For example, Doran & Jordan (2013, p. 25–26) construct the index from regions' shares of total income, $Y_i/Y$, and regions' shares of total population, $N_i/N$. Martínez-Galarraga et al. (2015, p. 510) use a similar way. It is easily seen that such index is equivalent to that represented by Formula (6):

$$\sum_{i=1}^{m}\frac{Y_i}{Y}\ln(\frac{Y_i / Y}{N_i / N}) = \sum_{i=1}^{m}\frac{N_i y_i}{N\bar{y}_{(w)}}\ln(\frac{Y_i / N_i}{Y / N}) = \sum_{i=1}^{m} n_i \frac{y_i}{\bar{y}_{(w)}}\ln(\frac{y_i}{\bar{y}_{(w)}}) = Th_w.$$

Let us apply the population-weighted indices to estimate regional inequality in a simple two-region case. Consider two Chinese regions, mainland China as a whole and Macao, the Special Administrative Region of the People's Republic of China (and the richest territory of the world). In hoary antiquity, when the Portuguese occupied as large part of the Chinese territory as they could (or needed), Macao might be deemed a 'somewhat arbitrary' regional unit. Nowadays, it is quite natural, as Macao has its own currency, and citizens of China from other regions need visa to get there. Table 2 reports data on these regions.

**Table 2.** Per capita income and population in mainland China and Macao in 2014.

| Region | PPP-adjusted GDP per capita ($y_i$), current international dollars* | Population ($N_i$), million people** | Region's weight ($n_i$) |
|---|---|---|---|
| Mainland China | 13,217 | 1,376.049 | 0.999573 |
| Macao | 139,767 | 0.588 | 0.000427 |

* World Bank (2015)
** United Nations (2015, p. 13)

Estimating income inequality between mainland China and Macao, we get results listed in Table 3. It reports values of the population-weighted coefficient of variation and Gini and Theil indices defined by Formulae (2), (5) and (6) as well as values of the unweighted indices according to Formulae (1), (3) and (4). For comparability sake, the table also reports these indices standardized so that they take on values in the range of [0, 1]. That is, an index is divided by its maximum corresponding to perfect inequality. For our case of two observations, the maxima of $CV$, $G$ and $Th$ are respectively 1, 0.5 and log(2). The maxima of $CV_w$, $G_w$ and $Th_w$ approximately equal 1, 48.4 and 7.8; the way of computing these maxima will be explained in Section 3 and summarized in its Table 7.

**Table 3.** Estimates of income inequality between mainland China and Macao.

| Index | Population-weighted | | Unweighted | |
|---|---|---|---|---|
| | Raw | Standardized | Raw | Standardized |
| Coefficient of variation | 0.197 | 0.004 | 0.827 | 0.827 |
| Gini index | 0.004 | 0.004 | 0.414 | 0.827 |
| Theil index | 0.007 | 0.001 | 0.399 | 0.576 |
| Average income | $\bar{y}_{(w)} = 13{,}721$ | | $\bar{y} = 76{,}492$ | |

While the unweighted indices indicate a high degree of inequality, the population-weighted ones yield the reverse pattern. The standardized values of $CV_w$ and $G_w$ (coinciding up to the sixth decimal digit) have the value of 0.004, or 0.4% in percentage terms; and standardized $Th_w$ is even less than 0.1%. This suggests that there is (almost) no income inequality between the average mainland Chinese and average inhabitant of Macao. Indeed, our perception of spatial inequality is greatly distorted, but in the sense that is opposite to the view of Ezcurra & Rodríguez-Pose (2009, p. 332): it is the population weighting that gives rise to distortions. In the two-region case, the result evidently contradicts common sense. However, a sufficiently great number of regions in empirical studies masks such absurdities, creating an impression that estimates of inequality with the use of the population weighting are reasonable.

Then what is the reason for that low inequality suggested by the population-weighted inequality indices? What is the sense of the estimates obtained? To understand what the weighted indices measure, let us estimate inequality among all citizens of a country, basing on cross-region income distribution. The 'national' coefficient of variation ($CV_{nat}$) with $y_l$ standing for personal income of $l$-th citizen of the country looks like

$$CV_{nat} = \frac{\sqrt{\sum_{l=1}^{N}(y_l - \bar{y})^2 / N}}{\bar{y}}.$$

Obviously, the population-average income in this formula – national per capita income, $\bar{y} = (y_1 + ... + y_N)/N$ – equals the weighted average of regional per capita incomes $\bar{y}_{(w)}$. Lacking information on intra-regional income distributions, we are forced to assume that all inhabitants of a region have the same income equalling per capita income in this region. Then square deviations $(y_l - \bar{y}_{(w)})^2$ are uniform for all $l$ relating to inhabitants of the same region, say $i$. Hence, their sum over all inhabitants of the region is $(y_i - \bar{y}_{(w)})^2 N_i$. Summing up such sums over all regions, we come to the Williamson coefficient of variation:

$$CV_{nat} = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y}_{(w)})^2 N_i / N}}{\bar{y}_{(w)}} = \frac{\sqrt{\sum_{i=1}^{m}(y_i - \bar{y}_{(w)})^2 n_i}}{\bar{y}_{(w)}} = CV_w.$$

Thus, the population-weighted coefficient of variation is not a measure of inequality between regions; instead, it measures national inequality, i.e. interpersonal inequality in the whole population of the country. In doing so, it does not (and cannot) take into account intra-regional inequalities. Certainly, this relates not only to the coefficient of variation but to any other inequality index (maybe, except for those based on partial information from cross-region income distributions, e.g. the relative range of disparities, $R = \max_i y_i / \min_i y_i$, interquartile range, and the like; however, it seems that the weighting is hardly applicable to them).

This explains the sense of results obtained with the population-weighted indices in Table 3. These measure inequality between inhabitants of the united mainland China and Macao. Provided that inequality within mainland China is zero (as all its inhabitants are supposed to have the same income), adding less than one million people – even with extremely high income – to its 1.4-billion population can increase the degree of the overall inequality only slightly.

It is seen that there is a conceptual distinction between the unweighted and population-weighted estimates of inequality. In the former case, all regions enjoy equal rights in the sense that all $y_i$ are equiprobable (i.e. the probability of finding income $y_i$ in a randomly chosen region is the same for all $i$ and equals $1/m$). Albeit speaking of regions, we actually deal with individuals, representative (or 'average', i.e. having the region-average income) inhabitants of each region. While estimating regional inequality, we compare their incomes without regard for how many people live in respective regions (like we do while comparing wages across occupations). Indeed, the fact that the average inhabitant of Macao is almost 11 times richer than the average mainland Chinese in no way changes because of the fact that the population of Macao is 2,340 times smaller than the population of mainland China.

Introducing regional weights implies that a region is represented by all its inhabitants rather than by one 'average' inhabitant. That is, we consider region $i$ as a group of $N_i$ people, each individual within the group having income $y_i$. Then the probability of $y_i$ differs across regions, becoming proportional to their populations, $n_i$. Thus, $(n_i)$ is in fact a proxy of the personal-income distribution in the country. In other words, it is a grouping of the whole country's population into income classes $(y_i)$ of different sizes $(N_i)$. The regional division matters no more; the impression that the case at hand is inequality between regions is but an illusion owing to that the grouping proceeds from the data by region. Actually we get an estimate of interpersonal inequality in the country. As such it is very crude, since it neglects inequality within regions and – what is much more important – the income classes $y_i$ (constructed from cross-region data) in fact heavily overlap because of overlapping intra-regional income distributions.

It follows herefrom that a population-weighted estimate of inequality is biased with regard to estimates of both regional inequality (as it measures a different value) and interpersonal inequality (as it does not take account of within-region income disparities). In both cases, the result can be misleading as the example of two Chinese regions demonstrates.

The bias can have either direction depending on a particular combination of regional per capita incomes and populations. Williamson (1965, p. 12) reports values of both weighted and unweighted coefficients of variation estimated on regional data from 24 countries. Regional inequality estimated by $CV_w$ proves to be overstated in about a half of countries, and understated in another half. The biases (relative to the unweighted estimates) range from –52.6% (in India) to +37.6% (in Puerto Fico). The case of India is an example of quite misleading result in an actual study (covering 18 regions): the population-weighted index understates the extent of regional inequality there by more than a half.

One more evidence is due to Petrakos & Psycharis (2016). They estimate the evolution of regional inequality in Greece across its NUTS 2 and NUTS 3 regions over 2000–2012, using both population-weighted and unweighted coefficient of variation. The trend of $CV_w$ is upward, while $CV$ has either a downward trend (in the case of NUTS 3 regions) or is stable (for NUTS 2 regions). Thus, if one considered the weighted estimates, the conclusion would be that regional inequality rises, whereas actually it remains unchanged or even decreases.

Mussini (2015) estimates inequality between NUTS 3 regions in the EU-28 over 2003–2011 (applying $G_w$) and decomposes its changes into those caused by population change, re-ranking of regions and growth of regional per capita incomes. In the light of the above considerations the intuitive sense of the first component becomes absolutely obscure. As it has been shown above, the population-weighted index measures national inequality. Imagine that the cross-individual income distribution in a country remains invariant while the cross-region population distribution changes. Then the effect of population change in the decomposition of inequality change reflects nothing but a result of replacing one improper division of the population into income classes by another (also improper) one.

## 3. Some Properties of the Population-Weighted Indices

An adequate inequality index should satisfy a number of axioms, i.e. desirable properties of an inequality measure (see, e.g. Cowell, 2000). Ezcurra & Rodríguez-Pose (2009, pp. 332–333) argue – with no proof – that a number of the population-weighted inequality indices, including the coefficient of variation and Gini and Theil indices, fulfil the basic axioms, namely, scale invariance, population principle, anonymity and principle of transfers (the Pigou-Dalton principle). Some other papers, a few above-cited ones among them, contain similar assertions.

Indeed, these indices are scale-invariant; the check is easy and straightforward. The fulfilment of the population principle (or replication invariance) seems questionable, since it would hold under a replication of not only the income distribution, but the population distribution as well, which is beyond the axiom conditions. As for the anonymity (symmetry) principle and principle of transfers, the population-weighted inequality indices violate them (while their unweighted counterparts do satisfy). This fact will be proved below regarding the population-weighted coefficient of variation. Such proofs for the population-weighted Gini and Theil indices need more cumbersome mathematics; therefore only numerical examples will illustrate violations of these axioms by them.

Adjusting Jenkins & van Kerm's (2009, p. 52) definition to the case of regions, the anonymity principle requires the inequality index to depend only on per capita income values used to construct it and not additional information such as what the region is with a particular per capita income or what regional populations are. In other words, the index must be invariant to any permutation of income observations.

Consider a cross-region income distribution $y = (y_1,\ldots, y_N)$ and its permutation $y_*$, i.e. $y = (\ldots, y_i, \ldots, y_k, \ldots)$ and $y_* = (\ldots, y_k, \ldots, y_i, \ldots)$; the rest elements in $y_*$ remain the same as in $y$; hereafter $y_k > y_i$. One can expect the value of the population-weighted inequality index to change under such a transformation if for no other reason than it changes the weighted average:

$$\Delta\bar{y}_{(w)} = \bar{y}_{*(w)} - \bar{y}_{(w)} = (n_i - n_k)(y_k - y_i)\,. \tag{7}$$

It is seen that the weighted average remains intact only in the trivial case of $n_i = n_k$.

The change in the population-weighted coefficient of variation is characterized by the following equation:

$$\Delta CV_w^2 = CV_w^2(y_*) - CV_w^2(y) = \frac{\Delta\bar{y}_{(w)}}{(\bar{y}_{(w)} + \Delta\bar{y}_{(w)})^2}\left(y_i + y_k - (2\bar{y}_{(w)} + \Delta\bar{y}_{(w)})\frac{\overline{y^2}_{(w)}}{\bar{y}^2_{(w)}}\right), \tag{8}$$

where $\overline{y^2}_{(w)}$ is the weighted average of squared incomes and $\bar{y}^2_{(w)}$ is the square of the weighted average; $\Delta\bar{y}_{(w)}$ is defined by Formula (7). Note that $\overline{y^2}_{(w)} / \bar{y}^2_{(w)} = CV_w^2(y) + 1$; hence, it always (given that $y_k \neq y_i$) exceeds unity. Thus, $\Delta CV_W^2$ depends on six variables: $y_i$, $y_k$, $n_i$, $n_k$, $\bar{y}_{(w)}$, and $\overline{y^2}_{(w)}$. (This number may be reduced by one, replacing the latter two variables with $CV_w(y)$.) The signs of the relationship

$$\frac{y_i + y_k}{2\bar{y}_{(w)} + \Delta\bar{y}_{(w)}} \cdot \frac{\bar{y}^2_{(w)}}{\overline{y^2}_{(w)}} - 1 \equiv F(y_i, y_k, n_i, n_k, \bar{y}_{(w)}, \overline{y^2}_{(w)}) - 1 \tag{9}$$

and $\Delta\bar{y}_{(w)}$ determine the sign of $\Delta CV_W^2$, hence the direction of change in the inequality measure: $\operatorname{sgn}(\Delta CV_w^2) = \operatorname{sgn}(F(\cdot) - 1) \cdot \operatorname{sgn}(\Delta\bar{y}_{(w)})$. Table 4 shows different possible cases.

**Table 4.** Permutation-induced changes in the population-weighted coefficient of variation.

| | $n_i > n_k$ ($\Delta\bar{y}_{(w)} > 0$) | $n_i < n_k$ ($\Delta\bar{y}_{(w)} < 0$) |
|---|---|---|
| $F(\cdot) > 1$ | $CV_w$ increases | $CV_w$ decreases |
| $F(\cdot) < 1$ | $CV_w$ decreases | $CV_w$ increases |

Given too many variables in $F(\cdot)$, its behaviour is not amenable to more or less comprehensive formal analysis. It is possible for some particular cases only. For instance, if both $y_i$ and $y_k$ are less than the weighted average and $n_i > n_k$, then $F(\cdot) < 1$ knowingly holds and $CV_w$ diminishes.

In principle, the case of $\Delta CV_w^2 = 0$ is possible as well. Let all regions except $i$ and $k$ have the same per capita income $y_r$. Then we can aggregate them into a single 'region' $r$ with income $y_r$ and weight $n_r = 1 - (n_i + n_k)$. (Such a 'region' will be used elsewhere below.) In this instance $F(\cdot) = F(y_i, y_k, n_i, n_k; y_r)$. Keeping all variables except $y_r$ constant, we can find the value of $y_r$ such that $F(y_r) = 1$. Equation $F(y_r) = 1$ is a cubic one with respect to $y_r$; its closed-form solution is very cumbersome and therefore is not reported. (In fact, we can dispense with it, solving the equation numerically.) This equation may have a real positive root, albeit not always. However, no significance should be attached to this fact. First, probability of finding an actual cross-region income distribution (along with the population distribution) that satisfies $F(\cdot) = 1$ even for some single pair of $i$ and $k$ seems to be close to zero. Second, particular cases of satisfying the anonymity principle do not matter at all, while the only (non-degenerate) case – even a single numerical example – of its violation would evidence that the inequality index under consideration does have this unpleasant property.

Table 5 provides numerical examples that illustrate four cases listed in Table 4 and the case of no change in the population-weighted coefficient of variation. It tabulates three income distributions and their permutations – (A), (B) and (C), the population distribution $n = (n_j)$ being uniform across these. Therefore, $\Delta\bar{y}_{(w)} > 0$ holds for all three cases of transition from $y$ to $y_*$. However, we can also consider reverse transitions from $y_*$ to $y$, exchanging indices $i$ and $k$; in these transitions, $\Delta\bar{y}_{(w)} < 0$. Along with the coefficient of variation, the table reports the population-weighted Gini and Theil indices. Besides, for comparison sake, it also reports values of unweighted inequality indices.

**Table 5.** Permutation-induced changes in the population-weighted inequality indices.

| Region index | $n$ | (A) | | (B) | | (C) | |
|---|---|---|---|---|---|---|---|
| | | $y$ | $y_*$ | $y$ | $y_*$ | $y$ | $y_*$ |
| $i$ | 0.15 | 150 | 300 | 150 | 300 | 150 | 300 |
| $k$ | 0.05 | 300 | 150 | 300 | 150 | 300 | 150 |
| $r$ | 0.80 | 400 | 400 | 100 | 100 | 218.9 | 218.9 |
| $\bar{y}_{(w)}$ | | 357.5 | 372.5 | 117.5 | 132.5 | 212.62 | 227.62 |
| $CV_w$ | | 0.251 | 0.167 | 0.387 | 0.537 | 0.149 | 0.149 |
| $G_w$ | | 0.098 | 0.062 | 0.129 | 0.205 | 0.059 | 0.060 |
| $Th_w$ | | 0.039 | 0.017 | 0.057 | 0.115 | 0.011 | 0.011 |
| $CV$ | | 0.363 | | 0.464 | | 0.275 | |
| $G$ | | 0.196 | | 0.242 | | 0.149 | |
| $Th$ | | 0.070 | | 0.104 | | 0.038 | |

Case (A) is that of diminishing values of the population-weighted inequality measures caused by the exchange of incomes between two regions; $F(\cdot) < 1$ here. The decrease is fairly sizeable, equalling more than one third for $CV_w$ and more than a half for $Th_w$. Considering the reverse transition, we have $\Delta\bar{y}_{(w)} < 0$ and $F(\cdot) > 1$; the permutation of regional incomes causes the weighted inequality indices to rise. In case (B), the effect of permutation in $y$ is an increase in the weighted indices, as $F(\cdot) > 1$; the reverse permutation has the adverse effect. At last, the weighted coefficient of variation does not change under the permutation in case (C). Interestingly, the weighted Gini and Theil indices are also near-invariant in this case: $\Delta G_w = 3.7\cdot10^{-4}$ and $\Delta Th_w = 5.5\cdot10^{-4}$. Comparing values of the respective weighted and unweighted indices in Table 3, we can see that the weighting leads to significant undervaluation of inequality, except for $Th_w(y_*)$ and $CV_w(y_*)$ in case (B).

Let us turn to the principle of transfers which 'is usually taken to be indispensable in most of the inequality literature' (Cowell, 2000, p. 98). Let cross-region income distribution $y = (\ldots, y_i, \ldots, y_k, \ldots)$ be transformed into $y_* = (\ldots, y_{*i} = y_i + \theta, \ldots, y_{*k} = y_k - \theta, \ldots)$, where $y_{*j} = y_j$ for $j \neq i, k$, and $0 < \theta < \theta_{\max} = (y_k - y_i)/2$, thus keeping region $k$ still richer than $i$. The principle of transfers requires the inequality index to decrease under such a transformation. This requirement for the weighted coefficient of variation (denoting $CV_{w*} \equiv CV_w(y_*)$) can be represented as

$$\frac{dCV_{w*}}{d\theta} = \frac{1}{\bar{y}_{*(w)}CV_{w*}}\left(\frac{n_i y_{*i} - n_k y_{*k}}{\bar{y}_{*(w)}} - (CV_{w*}^2 + 1)(n_i - n_k)\right) < 0. \qquad (10)$$

Condition (10) unambiguously holds only if $n_i y_{*i} < n_k y_{*k}$ and $n_i > n_k$, as both summands in the right-hand side of the equation have negative sign. However, as $\theta$ rises, $y_{*i}$ and $y_{*k}$ become progressively closer to each other, which inevitably causes $n_i y_{*i} - n_k y_{*k}$ to change its sign to positive. When the signs of summands in the right-hand side of Equation (10) are different (in the case of $n_i < n_k$ they always are), the resulting sign of their sum depends on particular combination of $y$, $n$ and the value of $\theta$. Then it is not inconceivable that the derivative of $CV_{w*}$ is positive somewhere in the definitional domain of $\theta$, so violating the principle of transfers.

To show that $dCV_{w*}/d\theta > 0$ is possible, consider the case when the transfer is close to the right bound of its domain, $\theta \approx (y_k - y_i)/2$. Then $y_{*i} \approx y_{*k} \approx (y_k + y_i)/2$. In this instance, provided that $n_i > n_k$, $dCV_{w*}/d\theta > 0$ if $(y_i + y_k)/2 > (CV_{w*}^2 + 1)\bar{y}_{*(w)}$. Let $y_i = (1+\alpha)\bar{y}_{*(w)}$ and $y_k = (1+\beta)\bar{y}_{*(w)}$ (note that $\alpha$ may be negative), then the latter inequality looks like $(\alpha + \beta)/2 > CV_{w*}^2$. Such a relationship is fairly realistic. Usually $CV_{w*} < 1$, therefore $\alpha$ and $\beta$ should not be too great. For example, if $CV_{w*} = 0.7$, the principle of transfers will be violated with, say, $y_i = 1.2\bar{y}_{*(w)}$ and $y_k = 1.8\bar{y}_{*(w)}$ in the neighbourhood of $\theta = 0.3\bar{y}_{*(w)}$, or with $y_i = 0.9\bar{y}_{*(w)}$ and $y_k = 2.1\bar{y}_{*(w)}$ near $\theta = 0.6\bar{y}_{*(w)}$. Note that with $n_i > n_k$, a necessary condition for $dCV_{w*}/d\theta > 0$ is exceedance of the weighted average by $y_k$, $y_k > \bar{y}_{*(w)} = \bar{y}_{(w)} + (n_i - n_k)\theta > \bar{y}_{(w)}$.

Provided that $n_i < n_k$, $dCV_{w*}/d\theta > 0$ if $(y_i + y_k)/2 < (CV_{w*}^2 + 1)\bar{y}_{*(w)}$. This inequality obviously holds when both $y_i$ and $y_k$ are below the weighted average $\bar{y}_{*(w)}$, or when $\alpha \le -\beta$. It also may be true if both variables are above $\bar{y}_{*(w)}$, e.g. with $y_i = 1.1\bar{y}_{*(w)}$ and $y_k = 1.8\bar{y}_{*(w)}$ near $\theta = 0.35\bar{y}_{*(w)}$, given that $CV_{w*} = 0.7$.

Considering $CV_{w*}$ as a function of transfer, $CV_w(y_*) = CV_w(\theta)$ (then $CV_w(y) = CV_w(0)$), we can distinguish four types of its behaviour (depending on particular $y$ and $n$). They are depicted in Figure 1, with $CV_w(\theta)$ normalized to $CV_w(0)$ and $\theta$ normalized to $\theta_{\max}$.

Type 1 is a monotonic rise in the weighted coefficient of variation everywhere in the definitional domain of $\theta$. In type 2, $CV_w(\theta)$ decreases at first and then begins to rise (i.e $dCV_{w*}/d\theta$ changes its sign from negative to positive). Starting with some $\theta$, it reaches the initial value, $CV_w(0)$, and then exceeds it more and more. Type 3 is qualitatively similar to type 2, except for $CV_w(\theta)$ does not reach the initial value by the end of the domain of $\theta$. At last, type 4 is a monotonely decreasing $CV_w(\theta)$.

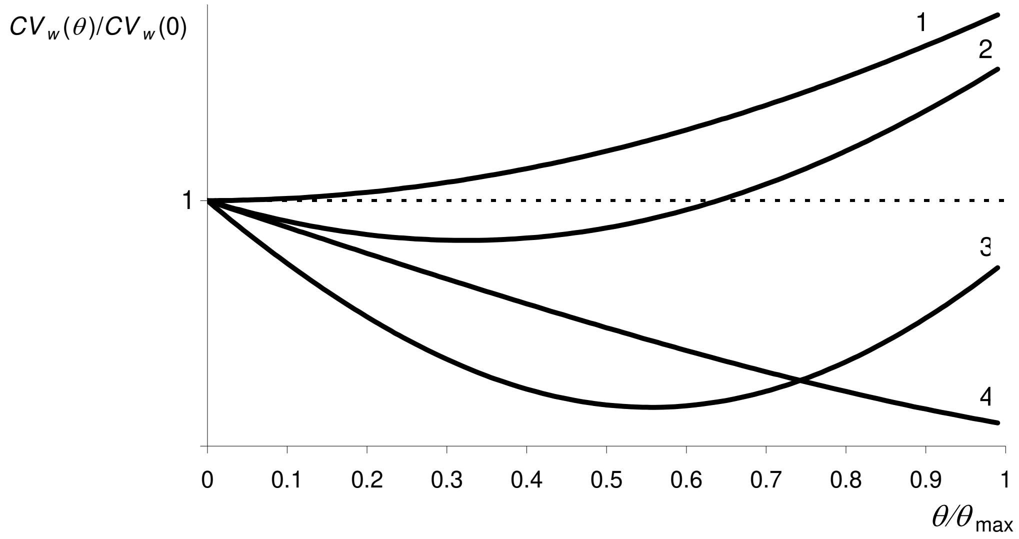


**Figure 1.** Different types of behaviour of $CV_w(\theta)$.
*Note:* for all curves, $n$ = (0.15, 0.05, 0,8), $y_i$ = 100 and $y_k$ = 300; $y_r$ = 420 for curve 1, $y_r$ = 350 for curve 2, $y_r$ = 300 for curve 3, and $y_r$ = 30 for curve 4.

The weighted Gini and Theil indices have the same four types of behaviour. A peculiarity of the Gini index is a break on curve $G_w(\theta)$ in some point (instead of a smooth inflection) in the case of behaviour of types 2 and 3. However, given the same $y$ and $n$, $G_w(\theta)$ and $Th_w(\theta)$ may differ from $CV_w(\theta)$ in the type of behaviour. For instance, curves of the weighted Gini index corresponding to curves 1, 2 and 3 in Figure 1 behave according to type 1; the behaviour is similar only in the case of curve 4. Curves of the weighted Theil index corresponding to curves 2, 3 and 4 in Figure 1 have the same type of behaviour, while behaviour of type 2 corresponds to curve 1of $CV_w$.

The violations of the principle of transfers have serious implications for empirical studies. Let we study the evolution of income inequality in some country (assume that the population distribution remains invariant). Provided that the behaviour of the population-weighted inequality measure is of type 1, we would observe increasing inequality with income gaps between regions of the country becoming progressively smaller over time. In the case of behaviour of types 2 and 3, the results will appear even more striking and unaccountable. At first, inequality falls with decreasing income gaps, as could be expected; but then from some point on, further decrease in the income gaps leads to rise in inequality.

Certainly, the situation is much more involved in actual empirical studies. For example, the population-weighted inequality measure may have varied types of behaviour for different region pairs ($i$, $k$); besides, an increase in per capita income in the poorer region of a pair is not equal, as a rule, to decrease in the richer region. But the above results evidence that in any case these features of the population-weighted inequality measures will produce (unpredictable) distortions in the pattern of the evolution of inequality.

Usually (albeit not always), dynamics of inequality obtained with the use of different unweighted inequality measures, say, the coefficient of variation, Gini and Theil indices, is qualitatively similar, having the same directions of change in inequality and their turning points. Since different population-weighted indices computed on the same data may have different types of behaviour, they can provide quite diverse patterns of the evolution of inequality in a country, depending on a particular index applied.

Table 6 gives numerical examples of violating the transfer principle for cases (A) $n_i < n_k$ and (B) $n_i > n_k$. It tabulates results for the baseline distribution $y$ and its transformations $y_*(\theta)$ with $\theta = 10$ and $\theta = 90$ ($\theta_{\max} = 100$).

**Table 6.** Transfer-induced changes in the population-weighted inequality indices.

| Region index | (A) | | | | (B) | | | |
|---|---|---|---|---|---|---|---|---|
| | $n$ | $y$ | $y_*(10)$ | $y_*(90)$ | $n$ | $y$ | $y_*(10)$ | $y_*(90)$ |
| $i$ | 0.05 | 100 | 110 | 190 | 0.18 | 100 | 110 | 190 |
| $k$ | 0.15 | 300 | 290 | 210 | 0.02 | 300 | 290 | 210 |
| $r$ | 0.80 | 370 | 370 | 370 | 0.80 | 110 | 110 | 110 |
| $\bar{y}_{(w)}$ | | 346.0 | 345.0 | 337.0 | | 112.0 | 120.0 | 126.4 |
| $CV_w$ | | 0.178 | 0.177 | 0.196 | | 0.242 | 0.222 | 0.260 |
| $G_w$ | | 0.060 | 0.062 | 0.079 | | 0.046 | 0.031 | 0.104 |
| $Th_w$ | | 0.021 | 0.020 | 0.022 | | 0.020 | 0.017 | 0.030 |
| $CV$ | | 0.446 | 0.424 | 0.314 | | 0.541 | 0.499 | 0.254 |
| $G$ | | 0.234 | 0.225 | 0.156 | | 0.261 | 0.235 | 0.131 |
| $Th$ | | 0.114 | 0.101 | 0.047 | | 0.136 | 0.116 | 0.035 |

In case (A), the population-weighted coefficient of variation and Theil index have behaviour of type 2. Their values decrease with the small transfer $\theta = 10$ and increase with the greater transfer $\theta = 90$. The weighted Gini index behaves according to type 1, its value rising with both transfers. In case (B), all three weighted indices have behaviour of type 2, falling with $\theta = 10$ and rising with $\theta = 90$. Figure 2 illustrates this case graphically for the whole domain of $\theta$.

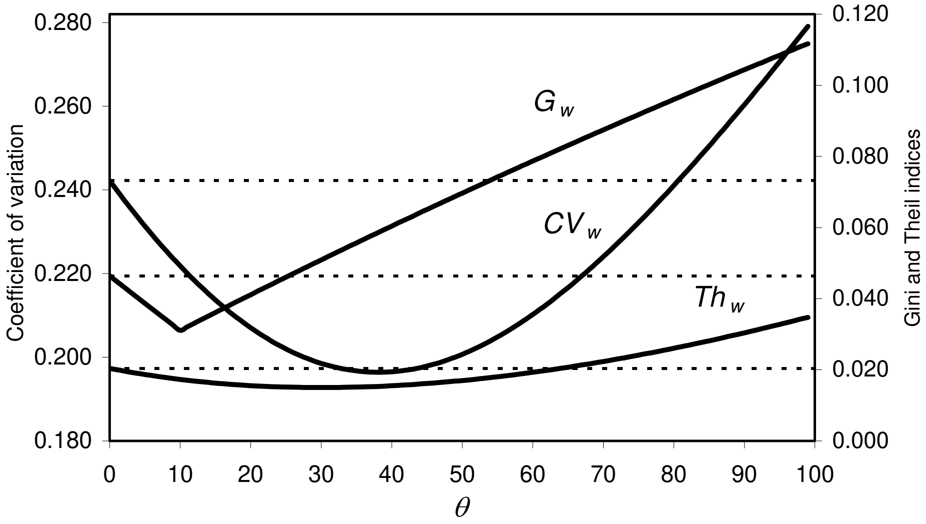


**Figure 2.** Population-weighted indices as functions of transfer.
*Note:* the dashed lines correspond to initial levels (with $\theta = 0$) of the indices.

An inequality index has a maximum in the case of perfect inequality, when the only region has a nonzero income. While the Gini coefficient has the upper bound of this maximum, some other inequality indices (the coefficient of variation and Theil index among them) have not. To judge how great inequality is from an estimate obtained, we should know how far it is from perfect inequality. Therefore it would be desirable to normalize inequality indices to their maxima (the Gini coefficient needs such normalization only in cases of a small number of regions, say, less than 20). In fact, Williamson's (1965) results are not comparable across countries, as the number of regions varies in his sample from 6 to 75; thus, the perfect-inequality values of the unweighted coefficient of variation differ between these extreme cases by the factor of more than 3.8. If the unweighted Theil index were applied, this ratio would equal 2.4. However, Theil (1967, p. 92) objects to normalization, giving an example of two situations. The first society consists of two individuals, only one of them having nonzero income; in the second society, all income belongs to the only of two million persons. The second society is evidently much more unequal. Nonetheless, considerations of cross-country comparability and uniform 'benchmark' of perfect inequality seem more important than Theil's argument (the more so as the number of regions does not differ that dramatically across countries).

A consequence of violating the anonymity principle by the population-weighted inequality indices is a striking and unpleasant feature: they have no unambiguous maxima. Now the value taken on by an index in the case of perfect inequality depends on which particular region possesses all country's income (or to which region the only person having nonzero income is placed). Denote such region by $k$. Table 7 summarizes differences between the maxima of inequality indices without and with the population weighting.

**Table 7.** Maxima of unweighted and weighted inequality indices.

| Index | Unweighted | Population-weighted |
|---|---|---|
| Coefficient of variation | $\sqrt{m-1}$ | $\sqrt{1/n_k-1}$ |
| Gini index | $(m-1)/m$ | $1-n_k$ |
| Theil index | $\log(m)$ | $\log(1/n_k)$ |

We could take the 'maximum of maxima', assigning $k$ to the least populated region. (It is such maxima that have been used to compute standardized values of the population-weighted indices in Table 3.) All the same, this 'global maximum' would depend on the cross-region distribution of country's population. Then the values of a weighted inequality index are not comparable even between countries with an equal number of regions. Moreover, such 'benchmark' of perfect inequality may vary over time in the same country with varying $n_k$ (or even $k$, if some other region becomes the least populated one).

## 4. Contras and Pros

Williamson's approach to measuring regional inequality did receive some criticism in the literature. Metwally & Jensen (1973) point out:

> Williamson's coefficient […] fails to take into account either the dispersion of incomes nationally, or what is more important in a spatial context, the dispersion of incomes within regions. […] It is possible for this coefficient to decrease over time, suggesting a convergence in regional mean incomes, while dispersion in actual incomes could show an opposite trend. (Metwally & Jensen, 1973, p. 135)

As it is seen, the authors mean measuring national (interpersonal) inequality; therefore their criticism is beside the point. But Williamson (1965) in no way intended to estimate inequality among countries' populations. There is not a grain of evidence of such purpose in his paper; quite the contrary, he highlights throughout the paper that he deals with regional inequality.

Fisch (1984) raises a similar objection:

> Williamson's coefficients of variations ignore a […] critical issue in relation to spatial inequality: the unequal regional distribution of population by income class. (Fisch, 1984, p. 91)

Again, the case in point is inability of the population-weighted coefficient of variation to adequately approximate interpersonal income inequality in the whole country.

In fact, objections due to Metwally & Jensen (1973) and Fisch (1984) are not those to the population weighting. The essence is in that they believe the national inequality rather than regional one to be more proper for Williamson's (1965) research.

Parr (1974) considers a different aspect; he notes:

> [T]he value of the [Williamson] index is likely to be influenced by the regionalization scheme employed, and there will be a tendency for the value of the index to be high when the regionalization involves a relatively large number of regions. (Parr, 1974, p. 84)

This is so indeed concerning the unweighted coefficient of variation with its maximum rising as the square root of the number of regions, but it is not true for the population-weighted index in the general case (as it has been shown in the previous section). The further Parr's note is connected with the weighted index though:

> [T]here is no way of knowing whether the official statistical regions on which the index is based reflect the extent of spatial income differentiation, given the particular number of regions involved. (Parr, 1974, p. 84)

To manage with this problem, the author suggests a bootstrap procedure of placing a number of points, corresponding to the number of official regions, at random over the territory of the country, thus obtaining a standard of spatial income differentiation against which the original index could be compared. It is not entirely clear what is meant, but it seems that this procedure would yield something like an approximation of the maximum of $\sqrt{1/n_i - 1}$.

Thus, the above considerations do not concern the main sin of the population-weighted indices, their failure in providing unbiased estimates of regional inequality (as well as their unpleasant properties as inequality measures at all). It is not inconceivable that such criticism exists somewhere in the literature; however, I failed in finding it.

Let us turn to arguments in favour of weighting inequality indices by population.

Portnov & Felsenstein (2010) explore the sensitivity of four unweighted and four population-weighted inequality measures to changes in the ranking, size and number of regions into which a country is divided, explicitly treating regions as groups of people. One of their tests consists in comparison between two situations that differ in the cross-region population distribution and national per capita income, keeping the cross-region income distribution invariant. Surprisingly, the values of the unweighted indices change across the situations, although they should not, being independent of the population distribution. A closer look shows that this is due to the mistaken use of $\bar{y}_{(w)}$ instead of $\bar{y}$ in calculation of these indices. In one more test, the population distribution randomly changes, the cross-region income distribution and national per capita income being kept constant. As one would expect, the weighted inequality indices react to these changes, while the unweighted ones remain constant. The authors believe the latter to be a shortcoming. They conclude:

> These [unweighted] indices may thus lead to spurious results when used for small countries, which are often characterized by rapid changes in population patterns. (Portnov & Felsenstein, 2010, p. 217)

They also conclude that the population-weighted indices – the Williamson coefficient of variation, Gini index and Coulter coefficient – may be considered as more or less reliable regional inequality measures (Portnov & Felsenstein, 2010, pp. 217–218). Both conclusions are fallacious. Explicitly treating regions as groups of people, the authors implicitly deal with the estimation of interpersonal inequality in the country, misinterpreting it as the estimation of regional inequality. Therefore, their results in no way can be deemed a proof of the use of weighting.

Studies on international inequality also widely use the population-weighted indices. From all appearances, economists engaged in studies of international inequality 'reinvented' Williamson's approach. In contrast to regional researchers, they are aware of the conceptual distinction between unweighted and population-weighted inequality indices, explicitly interpreting the latter as approximate measures of inequality among the world population, and not between nations. The surprising thing is that as if there were a barrier between the literature on regional inequality and that on international inequality. The former almost never references to the latter (Akita et al., 2011, can be mentioned as one of extremely rare examples). The conversance with the literature on international inequality would surely prevent regional researchers from misinterpreting the population-weighted indices as measures of regional inequality.

While the literature on regional inequality does not discuss the need for the population weighting in inequality indices, getting by short notes like those cited in Introduction, the literature on international inequality widely debates the question ‘To weight or not to weight?’. Both viewpoints are considered in detail by e.g. Firebaugh (2003) and Ravallion (2005). Under interpretation of the population-weighted estimates as proxies of inequality among the world population, the arguments in favour of weighting look reasonable; at least, they are seriously substantiated.

However, the results of applying the population-weighted indices for estimation of global inequality are disappointing. As Milanovic (2005, p. 10) notices, population-weighted inequality ‘deals neither only with nations nor individuals but falls somewhere in between’. Moreover, it may be misleading (Milanovic, 2012, p. 8). The reason for the use of these rough and possibly misleading estimates is the lack of data on relevant within-country income distributions needed to estimate world inequality. Milanovic (2012) estimates income inequality in the world as a whole over 1952–2006, applying the Gini coefficient weighted by populations of the countries. He also reports estimates of global inequality for 1988–2005 based on household survey data (i.e. taking into account income distributions within countries). These prove to be, first, much higher that the population-weighted estimates, and, second, sliding upward, while the trend of the population-weighted estimates over the respective time span is downward. Thus, estimates obtained with the use of the weighted Gini index turn out, indeed, quite misleading.

The debate regarding the population weighting in the literature on international inequality focuses on the issue of what an adequate characterization of inequality in the world is, either inter-country inequality or interpersonal inequality among the world population. In my view, this debate is fairly pointless. It must be agreed with Firebaugh (2003), who notes that the answer depends on the goal:

> [T]he issue of unweighted versus weighted between-nation inequality reduces to this question: are we interested in between-nation income inequality because of what it tells us about the average difference between nations’ income ratios, or because of what it tells us about the average difference between individuals’ income ratios? (Firebaugh, 2003, p. 129)

Assume that a regional research correctly interprets the population-weighted inequality indices as approximate estimates of national inequality rather than regional inequality. Is it reasonable to apply them? It is the lack of relevant data that forces to use these indices in studies on international inequality. However, such roundabout way in relation to a single country does not make sense. Nowadays, many (if not most) national statistical agencies report data on personal-income distributions in their countries. Estimating national inequality even on so rough distribution as consisting of quintile income classes, we obtain much more exact results than those based on the cross-region income distribution.

## 5. Conclusions

Following Williamson (1965), many economists estimate regional inequality with the use of indices weighted by regions' shares in the national population. A simple analysis in this paper shows that this approach is conceptually inconsistent. Instead of an estimate of regional inequality, we get an estimate of interpersonal inequality among the whole population of the country. Therefore the population-weighted estimates of inequality are biased with regard to estimates of both regional inequality (as they measure a different value) and interpersonal inequality (as they do not and cannot take account of within-region income disparities). In both cases, the result may be not only distorted, but also quite misleading.

Moreover, the population-weighted inequality indices do not satisfy requirements for an adequate inequality measure. They violate two important axioms, the anonymity principle and principle of transfers. This may lead to estimates of inequality evolution that contradict common sense. One more consequence is the absence of unambiguous maxima of the population-weighted inequality indices. This makes it impossible to standardize estimates of inequality with the aim of cross-time or cross-country comparability.

It is worth noting that even the interpretation of a population-weighted inequality index as an approximate measure of interpersonal inequality of the whole country's population is not always true. It holds only regarding indicators which can be applied to an individual, e.g. personal income, wage, housing, education, etc. Otherwise, the meaning of the population-weighted index is obscure. Estimating regional income inequality, many authors use regional GDP per capita to characterize incomes in regions. However, there is no inequality in the national GDP (as the total of regional GRPs) per capita between country's citizens. There are many other indicators that characterize situation of a region, but cannot be applied to its certain inhabitant, e.g. unemployment rate, crime rate, investment per capita, etc. Zubarevich & Safronov (2011) estimate, in addition to income inequality, regional inequality in investment per capita, unemployment rate and poverty rate. Again, there is no, e.g., unemployment inequality between country's inhabitants; only the national average unemployment rate exists. In such cases, the population-weighted inequality indices have no intuitive interpretation at all; it is totally incomprehensible what they measure.

**References**

Akita, T., Kurniawan, P. A. & Miyata, S. (2011) Structural changes and regional income inequality in Indonesia: a bidimensional decomposition analysis, *Asian Economic Journal* 25, 55–77.

Cowell, F. A. (2000) Measurement of inequality, in: A.B. Atkinson & F. Bourguignon (eds), *Handbook of Income Distribution*, vol. 1, pp. 86–166, Amsterdam, North Holland.

Deiwiks, C., Cederman, L. & Gleditsch, K. S. (2012) Inequality and conflict in federations, *Journal of Peace Research*, 49, 289–304.

Doran, J. & Jordan, D. (2013) Decomposing European NUTS2 regional inequality from 1980 to 2009: national and European policy implications, *Journal of Economic Studies*, 40, 22–38.

Enflo, K. & Rosés, J. R. (2015) Coping with regional inequality in Sweden: structural change, migrations, and policy, 1860–2000, *Economic History Review* 68, 191–217.

Ezcurra, R. & Rodríguez-Pose, A. (2009) Measuring the regional divide, in: R. Capello & P. Nijkamp (eds) *Handbook of Regional Growth and Development Theories*, pp. 329–353, Cheltenham, Edward Elgar.

Ezcurra, R. & Rodríguez-Pose, A. (2014) Trade openness and spatial inequality in emerging countries, *Spatial Economic Analysis*, 9, 162–201.

Fisch, O. (1984) Regional income inequality and economic development, *Regional Science and Urban Economics*, 14, 89–111.

Firebaugh, G. (2003) *The New Geography of Global Income Inequality*, Cambridge, MA, Harvard University Press.

Jenkins, S. P. & van Kerm, P. (2009) The measurement of economic inequality, in: V. Salvedra, B. Nolan & T.M. Smeeding (eds), *Economic Inequality*, pp. 40–67, Oxford, Oxford University Press.

Kyriacou, A. P. & Roca-Sagalés O. (2014) Regional disparities and government quality: redistributive conflict crowds out good government, *Spatial Economic Analysis*, 9, 183–201.

Lessmann, C. (2014) Spatial inequality and development – Is there an inverted-U relationship?

*Journal of Development Economics*, 106, 35–51.
Li, C. & Gibson, J. (2013) Rising regional inequality in China: fact or artifact? *World Development*, 2013, 47, 16–29.
Martínez-Galarraga, J., Rosés, J. R. & Tirado, D. A. (2015) The long-term patterns of regional income inequality in Spain, 1860–2000, *Regional Studies*, 49, 502–517.
Metwally, M. M. & Jensen R. C. (1973) A note on the measurement of regional income dispersion, *Economic Development and Cultural Change*, 22, 135–136.
Milanovic, B. (2005) *Worlds apart: measuring international and global inequality*, Princeton, MA, Princeton University Press.
Milanovic, B. (2012) Global inequality recalculated and updated: the effect of new PPP estimates on global inequality and 2005 estimates, *Journal of Economic Inequality*, 10, 1–18.
Mussini, M. (2015) Decomposing changes in inequality and welfare between EU regions: the roles of population change, re-ranking and income growth, *Social Indicators Research*, DOI 10.1007/s11205-015-1184-2
Parr, J. B. (1974) Welfare differences within a nation: a comment, *Papers of the Regional Science Association*, 32, 83–91.
Petrakos, G. & Psycharis, Y. (2016) The spatial aspects of economic crisis in Greece, *Cambridge Journal of Regions, Economy and Society*, 9,137–152.
Portnov, B. A. & Felsenstein D. (2010) On the suitability of income inequality measures for regional analysis: some evidence from simulation analysis and bootstrapping tests, *Socio-Economic Planning Sciences*, 44, 212–219.
Ravallion, M. (2005) Competing concepts of inequality in the globalization debate, in: S.M .Collins & C.L. Graham (eds), *Brookings Trade Forum: 2004: Globalization, Poverty, and Inequality*, pp. 1–38, Washington, D.C., Brookings Institution Press.
Sacchi, A. & Salotti, S. (2014) How regional inequality affects fiscal decentralisation: accounting for the autonomy of subcentral governments, *Environment and Planning C: Government and Policy* 32, 144–162.
Theil, H. (1967) *Economics and Information Theory*, Amsterdam, North Holland.
United Nations (2015) World population prospects: the 2015 revision, key findings and advance tables, Working Paper No. ESA/P/WP.241, United Nations, Department of Economic and Social Affairs, Population Division, New York.
Wijerathna, D., Bandara, J. S., Smith, C. & Naranpanawa, A. (2014) Regional disparities in

Sri Lanka: an empirical analysis, *Asia-Pacific Development Journal* 21, 77–102.

Williamson, J. G. (1965) Regional inequality and the process of national development: a description of patterns, *Economic Development and Cultural Change*, 13, 1–84.

World Bank (2015) World development indicators. – GDP per capita, PPP (current international $). http://data.worldbank.org/indicator/NY.GDP.PCAP.PP.CD. Accessed Sept. 26, 2015.

Zubarevich, N.V. & Safronov, S.G. (2011) Regional inequality in large post-Soviet countries, *Regional Research of Russia*, 1, 15–26.