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Baron and Myerson (BM) (1982) propose an incentive-compatible, individually rational and ex-ante socially optimal direct-revelation mechanism to regulate a monopolistic firm with unknown costs. We show that their mechanism is not ex-post Pareto dominated by any other feasible direct-revelation mechanism. However, there also exist an uncountable number of feasible direct-revelation mechanisms that are not ex-post Pareto dominated by the BM mechanism. To investigate whether the BM mechanism remains in the set of ex-post undominated mechanisms when the Pareto axiom is slightly weakened, we introduce the ϵ-Pareto dominance. This concept requires the relevant dominance relationships to hold in the support of the regulator's beliefs everywhere but at a set of points of measure ϵ, which can be arbitrarily small. We show that a modification of the BM mechanism which always equates the price to the marginal cost can ϵ-Pareto dominate the BM mechanism at uncountably many regulatory environments, while it is never ϵ-Pareto dominated by the BM mechanism at any regulatory environment.

Keywords: Monopoly; Regulation; Asymmetric Information; Pareto Efficiency

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1 Introduction

The seminal paper of Baron and Myerson (1982) (BM) was the first study in the economics literature to introduce a general social welfare function -a weighted sum of the producer and consumer welfares- to deal with the problem of regulating a monopolistic firm with unknown costs. This piece of work, along with an earlier study of Loeb and Magat (1979), also pioneered in characterizing an incentive-compatible solution to the regulation problem. The regulatory solution of BM was based upon the well-known Revelation Principle (Dasgupta, Hammond and Maskin, 1979; Myerson, 1979; Harris and Townsend, 1981), allowing the regulator to restrict herself to incentive-compatible revelation mechanisms that require the monopolistic firm to report its unknown cost information and guarantee that it has no incentive for misreporting.[1] Mechanisms considered by BM include four functions (schedules) defined over the set of possible cost reports: price and quantity functions which must agree on a given inverse demand curve, a probability function specifying the set of cost reports at which the monopolistic firm will be permitted to operate, and a subsidy function specifying the money transfer from consumers to the monopolistic firm. Demanding these four functions to be incentive-compatible requires that the marginal welfare of the monopolistic firm -which turns out to be affinely linear in the regulated quantity of the output under the cost structure assumed by BM- is nonincreasing in its cost report. In cases the social welfare function puts a lower weight on the producer welfare than on the consumer welfare, the regulator has an incentive to

[1] Unlike the mechanism of BM, the incentive scheme offered by Loeb and Magat (1979) does not use a direct-revelation mechanism that asks the monopolistic firm to report its private cost information. Instead, it delegates the output decision to the monopolistic firm that is also offered the right to the whole social surplus. However, the outcome of this scheme -that is optimal only if the social welfare treats consumers and the producer equally- can be obtained as a special case of the outcome of the BM mechanism, which is optimal under a general social welfare function admitting unequal treatments of consumers and the producer as well.

contract/shrink the quantity schedule in order to limit the producer welfare, consisting of informational rents. However, this contraction would also suppress the consumer welfare; hence a tradeoff. The regulator optimally balances this tradeoff by choosing the quantity schedule (and the other three schedules) to maximize the expected value of the social welfare under her beliefs - about the monopolistic firm's unknown cost parameter - over some known support $[\theta_0, \theta_1]$.

Since the expected (ex-ante) and the actual (ex-post) values of the social welfare need not be the same, one may wonder why the regulatory model of BM did not choose to maximize the actual, instead of the expected, social welfare. The reason is simply that this is indeed impossible, since at any value θ of the unknown cost parameter, the informational rents of the regulated firm and consequently the ex-post social welfare depend not only on the quantity to be produced at the cost level θ, but also on the part of the quantity schedule over the possible cost reports higher than θ. To put it in a different way, the value of the quantity at any cost report θ (marginally) affects not only the ex-post social welfare calculated at θ but also the ex-post social welfare calculated at any cost report lower than θ, i.e., the interval $[\theta_0, \theta)$. BM optimally balances such integral effects on the social welfare caused by the regulator's choice of the quantity schedule by using her prior beliefs about the possible cost values, i.e., by choosing a feasible direct-revelation mechanism that maximizes the expected social welfare (Proposition 1, borrowed from BM 1982).

At this point, we may ask whether there exists a feasible direct-revelation mechanism that is ex-post more efficient than the BM mechanism at all values of the cost information. The answer (we provide in Proposition 2) is 'no', since the ex-ante social efficiency of the BM mechanism implies that it must also be ex-post Pareto undominated. However, we also show that there exist an uncountable number of feasible direct-revelation mechanisms that are not ex-post Pareto dominated by the BM mechanism (Proposition 3). Propositions 2 and 3 altogether reveal that one can find an uncountable set of feasible direct-revelation mechanisms that are ex-post Pareto non-comparable to the BM mechanism.

The main objective of this paper is to study whether the BM mechanism would remain to be an ex-post Pareto undominated mechanism if the Pareto concept were relaxed slightly. To that aim, we introduce a new concept called ϵ-Pareto dominance that requires the relevant dominance relationships (regarding the welfares of the producer and consumers) to hold in the support of the regulator's beliefs everywhere but at some points with measure ϵ. Using this concept, we show that a modification of the BM mechanism which always requires marginal cost pricing, irrespective of the weight of the producer welfare in the social welfare, can dominate the BM mechanism at uncountably many regulatory environments (Proposition 4). While this result is not universally valid for all environments (Proposition 5), the modified BM mechanism may be argued to be ex-post superior to the original mechanism since the former is never dominated by the latter at any regulatory environment (Proposition 6).

The rest of the paper is organized as follows: Section 2 presents the regulatory model borrowed from BM (1982) and Section 3 presents the regulatory mechanism of BM. Section 4 presents our results and Section 5 concludes.

2 Model

We consider the BM's (1982) model of regulation involving a monopolistic firm with unknown costs. The firm faces the cost function

$$C(q,\theta) = (c_0 + c_1\theta)q + (k_0 + k_1\theta) \quad \text{if } q > 0, \quad \text{and} \quad C(0,\theta) = 0, \tag{1}$$

where c_0, c_1, k_0, k_1 are known constants satisfying $c_1 \geq 0$ and $k_1 \geq 0$. The parameter θ is restricted to a known interval $[\theta_0, \theta_1]$, where $\theta_1 > \theta_0 \geq 0$.

The monopolistic firm also faces an inverse demand function which is denoted by $P(.)$. So, the price at the output level q is equal to $P(q)$. Then, the total value to consumers of an output quantity $q \geq 0$ can be calculated as

$$V(q) = \int_0^q P(\tilde{q})d\tilde{q}, \tag{2}$$

and the consumer surplus as $V(q) - P(q)q$.

The demand function as well as the form of the cost function and all of its parameters other than θ are known to the regulator. While the regulator does not know the actual value of the cost parameter θ (before the implementation of the regulatory mechanism), she has (known) prior beliefs about it. These beliefs are represented by the probability density function $f(.)$, which is positive and continuous over the known support $[\theta_0, \theta_1]$. Let $F(.)$ denote the corresponding cumulative distribution function.

3 Baron and Myerson's (1982) Regulatory Mechanism

The regulatory mechanism considered by BM involves the outcome functions (r, p, q, s) that will be characterized below. After the regulator announces these functions, the monopolistic firm is asked to report a cost value in $[\theta_0, \theta_1]$. When the reported cost is $\tilde{\theta}$, $r(\tilde{\theta})$ is the probability that the monopolistic firm is allowed to operate, $p(\tilde{\theta})$ and $q(\tilde{\theta})$ are the regulated price and quantity respectively, and $s(\tilde{\theta})$ is the expected value of the subsidy received by the monopolistic firm, conditional on the probability that it is allowed to operate.

If the monopolistic firm with the cost parameter θ submits the cost report $\tilde{\theta}$, its expected profits (i.e., the producer welfare) would become

$$\pi(\tilde{\theta}, \theta) = \left[p(\tilde{\theta})q(\tilde{\theta}) - C(q(\tilde{\theta}), \theta)\right] r(\tilde{\theta}) + s(\tilde{\theta}). \tag{3}$$

A regulatory policy $\langle r, p, q, s \rangle$ is called *feasible* if it satisfies the following conditions for all $\theta \in [\theta_0, \theta_1]$:

(i) $r(\theta)$ is a probability function, i.e.,

$$0 \leq r(\theta) \leq 1, \tag{4}$$

(ii) $p(\theta)$ and $q(\theta)$ agree on the inverse demand curve, i.e.,

$$p(\theta) = P(q(\theta)), \tag{5}$$

(iii) the regulatory policy is *incentive-compatible*, i.e.,

$$\pi(\theta, \theta) \geq \pi(\tilde{\theta}, \theta), \quad \text{for all } \tilde{\theta} \in [\theta_0, \theta_1], \tag{6}$$

(iv) the regulatory policy is *individually rational* under truthful revelation, i.e.,

$$\pi(\theta, \theta) \geq 0. \tag{7}$$

Given a feasible regulatory policy (r, p, q, s), the consumer welfare (the consumer surplus net of the subsidy paid to the monopolistic firm) and the producer welfare at any cost level $\theta \in [\theta_0, \theta_1]$ become

$$CW(\theta) = [V(q(\theta)) - p(\theta)q(\theta)]\, r(\theta) - s(\theta), \tag{8}$$

and

$$\pi(\theta) \equiv \pi(\theta, \theta) = [p(\theta)q(\theta) - C(q(\theta), \theta)]\, r(\theta) + s(\theta), \tag{9}$$

respectively. The social welfare $SW(\theta)$ is defined as the sum of the consumer welfare $CW(\theta)$ and a fraction of the producer welfare $\pi(\theta)$. Formally,

$$SW(\theta) \;=\; CW(\theta) + \alpha\, \pi(\theta) = [V(q(\theta)) - C(q(\theta), \theta))]\, r(\theta) - (1 - \alpha)\, \pi(\theta), \tag{10}$$

where $\alpha \in [0,1]$ is the (relative) weight of the producer welfare. The objective of the regulator is to choose a feasible regulatory policy that will maximize the expected value of $SW(\theta)$ in (10), conditional on her prior beliefs $f(.)$ about the parameter θ. That is, the regulator aims to solve

$$\max_{r(.),p(.),q(.),s(.)} \int_{\theta_0}^{\theta_1} SW(\theta) f(\theta) d\theta \ \text{ subject to } \ (4)-(7). \tag{11}$$

To present the solution to the above problem, the following definitions will be necessary. Let

$$z_\alpha(\theta) = \theta + (1-\alpha)\frac{F(\theta)}{f(\theta)}, \tag{12}$$

for any $\theta \in [\theta_0, \theta_1]$, and

$$h_\alpha(\phi) = z_\alpha(F^{-1}(\phi)) \tag{13}$$

for any $\phi \in [0,1]$. Define

$$H_\alpha(\phi) = \int_0^\phi h_\alpha(\tilde{\phi}) d\tilde{\phi} \tag{14}$$

and

$$\bar{H}_\alpha(\phi) = \operatorname{conv} H_\alpha(\phi), \tag{15}$$

i.e., $\bar{H}_\alpha(.)$ is the highest convex function on the interval $[0,1]$ satisfying $\bar{H}_\alpha(\phi) \leq H_\alpha(\phi)$ for all $\phi \in [0,1]$. Then, let

$$\bar{h}_\alpha(\phi) = \bar{H}'_\alpha(\phi), \tag{16}$$

for any $\phi \in [0,1]$, and finally define

$$\bar{z}_\alpha(\theta) = \bar{h}_\alpha(F(\theta)) \tag{17}$$

for any $\theta \in [\theta_0, \theta_1]$.

Proposition 1. **(Baron and Myerson, 1982)** *The solution to the regulator's problem in (11) is given by the mechanism* $(\bar{r}, \bar{p}, \bar{q}, \bar{s})$ *satisfying equations (18)-(21) for all* $\theta \in [\theta_0, \theta_1]$*:*

$$\bar{p}(\theta) = c_0 + c_1 \bar{z}_\alpha(\theta) \tag{18}$$

$$P(\bar{q}(\theta)) = \bar{p}(\theta) \tag{19}$$

$$\bar{r}(\theta) = \begin{cases} 1 & \text{if } V(\bar{q}(\theta)) - \bar{p}(\theta)\bar{q}(\theta) \geq k_0 + k_1 \bar{z}_\alpha(\theta) \\ 0 & \text{otherwise} \end{cases} \tag{20}$$

$$\bar{s}(\theta) = \left[(c_0 + c_1\theta)\bar{q}(\theta) + k_0 + k_1\theta - \bar{p}(\theta)\bar{q}(\theta)\right]\bar{r}(\theta) + \int_\theta^{\theta_1} \bar{r}(\tilde{\theta})(c_1\bar{q}(\tilde{\theta}) + k_1)d\tilde{\theta} \tag{21}$$

Proof. See pages 920-921 of Baron and Myerson (1982). *Q.E.D.*

Note that inserting the optimal subsidy (21) into the producer welfare in (9) at the optimal regulatory mechanism yields

$$\pi^{BM}(\theta) = \int_\theta^{\theta_1} \bar{r}(\tilde{\theta})(c_1\bar{q}(\tilde{\theta}) + k_1)d\tilde{\theta}, \tag{22}$$

implying that the BM mechanism yields purely informational rents to the regulated firm due to its private information. Also note that at the optimal regulatory policy the actual social welfare is given by

$$SW^{BM}(\theta) = V(\bar{q}(\theta)) - C(\bar{q}(\theta), \theta) - (1-\alpha)\pi^{BM}(\theta), \tag{23}$$

implying that the informational rents of the monopolistic firm yields a positive dead-weight loss in welfare, of the magnitude $(1-\alpha)\pi^{BM}(\theta)$, unless $\alpha = 1$.

4 Results

Let $\Theta = [\theta_0, \theta_1]$. Given any regulatory mechanism Ψ and any $\theta \in \Theta$, let $SW^{\Psi}(\theta)$, $CW^{\Psi}(\theta)$, and $\pi^{\Psi}(\theta)$ respectively denote the corresponding social welfare, consumer welfare, and producer welfare at θ.

Definition 1. A regulatory mechanism Ψ (ex-post) *Pareto dominates* another regulatory mechanism $\hat{\Psi}$ if $CW^{\Psi}(\theta) > CW^{\hat{\Psi}}(\theta)$ and $\pi^{\Psi}(\theta) > \pi^{\hat{\Psi}}(\theta)$ for all $\theta \in \Theta$.

Since, we do not consider ex-ante Pareto domination, hereafter we will simply call ex-post Pareto domination as Pareto domination.

Definition 2. A direct-revelation mechanism of the class (r, p, q, s) is called feasible if it satisfies the feasibility conditions (4)-(7) of BM.

Let $\mathcal{M}^{\mathcal{F}}$ denote the set of all feasible direct-revelation mechanisms of the class (r, p, q, s), studied by BM.

Proposition 2. *No mechanism in $\mathcal{M}^{\mathcal{F}}$ Pareto dominates the BM mechanism.*

Proof. The proof follows from the fact that the BM mechanism maximizes

$$\int_{\theta \in \Theta} SW(\theta) f(\theta) d\theta = \int_{\theta \in \Theta} \left[CW(\theta) + \alpha \pi(\theta) \right] f(\theta) d\theta. \tag{24}$$

Suppose there exists a mechanism $\Psi \in \mathcal{M}^{\mathcal{F}}$ such that $CW^{\Psi}(\theta) > CW^{BM}(\theta)$ and $\pi^{\Psi}(\theta) > \pi^{BM}(\theta)$ for all $\theta \in \Theta$. Then, we would have

$$\int_{\theta \in \Theta} SW^{\Psi}(\theta) f(\theta) d\theta > \int_{\theta \in \Theta} SW^{BM}(\theta) f(\theta) d\theta, \tag{25}$$

a contradiction. *Q.E.D.*

The above result reveals that there is a ground for defending the Bayesian approach employed by the BM in regulatory mechanism design, because under the incentive-compatibility and other feasibility conditions, no direct-revelation mechanism, whether it is Bayesian or non-Bayesian, can Pareto dominate the BM mechanism. However, as shown by the next result it is also true that the BM mechanism is not a Pareto dominant mechanism; i.e., it does not Pareto dominate all other feasible direct-revelation mechanisms.

Proposition 3. *There exists an uncountable number of mechanisms in $\mathcal{M}^{\mathcal{F}}$ that are not Pareto dominated by the BM mechanism.*

Proof. First, let $\alpha \in [0, 1)$. Consider a mechanism $\Psi = (r^{\Psi}, p^{\Psi}, q^{\Psi}, s^{\Psi}) \in \mathcal{M}^{\mathcal{F}}$ that modifies the BM mechanism by changing $\bar{p}(\theta) = c_0 + c_1 \bar{z}_{\alpha}(\theta)$ in (18) with

$$p^{\Psi}(\theta) = c_0 + c_1[\gamma \bar{z}_{\alpha}(\theta) + (1 - \gamma)\theta], \tag{26}$$

where $\gamma \in (0, 1)$. Thus, Ψ is characterized by (26) and (19)-(21) subject to the change of the variables $\bar{r}, \bar{p}, \bar{q}, \bar{s}$ by $r^{\Psi}, p^{\Psi}, q^{\Psi}, s^{\Psi}$, respectively. Clearly, $p^{\Psi}(\theta) < \bar{p}(\theta)$, hence $q^{\Psi}(\theta) > \bar{q}(\theta)$ and $r^{\Psi}(\theta) > \bar{r}(\theta)$ for all $\theta \in (\theta_0, \theta_1]$. We have $\pi^{\Psi}(\theta_1) = \pi^{BM}(\theta_1) = 0$, but $\pi^{\Psi}(\theta) > \pi^{BM}(\theta)$ for all $\theta \in [\theta_0, \theta_1)$, implying that Ψ is not Pareto dominated by the BM mechanism.

Now, let $\alpha = 1$. Consider a mechanism $\Psi = (r^{\Psi}, p^{\Psi}, q^{\Psi}, s^{\Psi}) \in \mathcal{M}^{\mathcal{F}}$ that modifies the BM mechanism by changing $\bar{p}(\theta) = c_0 + c_1 \bar{z}_1(\theta) = c_0 + c_1\theta$ in (18) with

$$p^{\Psi}(\theta) = c_0 + c_1[\gamma \bar{z}_0(\theta) + (1 - \gamma)\theta], \tag{27}$$

where $\gamma \in (0, 1)$. Thus, Ψ is characterized by (27) and (19)-(21) subject to the change of the variables $\bar{r}, \bar{p}, \bar{q}, \bar{s}$ by $r^{\Psi}, p^{\Psi}, q^{\Psi}, s^{\Psi}$, respectively. Clearly, for all $\theta \in (\theta_0, \theta_1]$ we have $p^{\Psi}(\theta) > \bar{p}(\theta)$, implying $q^{\Psi}(\theta) < \bar{q}(\theta)$ and $r^{\Psi}(\theta) < \bar{r}(\theta)$. This further implies $\pi^{\Psi}(\theta_0) < \pi^{BM}(\theta_0)$. On the other hand, we have $p^{\Psi}(\theta_0) = \bar{p}(\theta_0)$, and therefore $q^{\Psi}(\theta_0) = \bar{q}(\theta_0)$ and $r^{\Psi}(\theta_0) = \bar{r}(\theta_0)$. So, $CW^{\Psi}(\theta_0) = [V(q^{\Psi}(\theta_0)) - C(q^{\Psi}(\theta_0), \theta_0)]r^{\Psi}(\theta_0) - \pi^{\Psi}(\theta_0) > [V(\bar{q}(\theta_0)) - C(\bar{q}(\theta_0), \theta_0)]\bar{r}(\theta_0) - \pi^{BM}(\theta_0) = CW^{BM}(\theta_0)$, implying that Ψ is not Pareto dominated by the BM mechanism. Finally, note that this conclusion can be reached by uncountably many mechanisms in $\mathcal{M}^{\mathcal{F}}$ since $\gamma \in (0, 1)$ and each γ corresponds to a distinct mechanism Ψ. *Q.E.D.*

The above proposition rests upon two observations about the BM mechanism. The first is that when $\alpha \neq 1$, the optimal regulatory price implied by the BM mechanism is always higher than the marginal cost of the regulated firm to limit the marginal informational rents (the optimal allowed quantity) of the monopolistic firm. Evidently, a simple modification to the BM mechanism where the price policy becomes any weighted average of the BM price policy and the marginal cost pricing policy, while satisfying incentive-compatibility and other feasibility conditions, would lead to higher marginal informational rents, hence a higher level of producer welfare at all values of θ except for θ_1. The second observation is that when $\alpha = 1$, the optimal regulatory price implied by the BM mechanism is always equal to the marginal cost of the regulated firm, and therefore the informational rents of the regulated firm are at the highest possible level that can be attained by a feasible direct-revelation mechanism. However, these maximal rents for the regulated firm imply that the subsidies that must be paid by consumers are ex-post suboptimally high at low values of θ. Now, a modification to the BM mechanism could shift the price schedule, without violating feasibility conditions, to any weighted average of the BM price policy at $\alpha = 1$ and the BM price policy at $\alpha = 0$, to reduce the informational rents at any cost value. Evidently, this modification would not change the quantity of output q or the induced gross surplus $V(q) - C(q, \theta)$ at the lowest cost value θ_0, since irrespective of α, the BM mechanism always implements marginal cost pricing at $\theta = \theta_0$. Therefore, the consumer welfare, which is the gross surplus net of the informational rents, would be higher, when $\theta = \theta_0$, under the modified mechanism than under the BM mechanism. So, for both when $\alpha \neq 1$ and when $\alpha = 1$, one can find an uncountable number of feasible direct-revelation mechanisms under which the welfare of either consumers or the producer would be higher, at some values of cost information, compared to the welfare implied by the BM mechanism. Proposition 2 and 3 altogether establish that there are uncountably many feasible direct-revelation mechanisms that are Pareto non-comparable to the BM mechanism.

Below, we will investigate whether the BM mechanism would remain to be a Pareto undominated mechanism if the Pareto axiom were relaxed slightly.

Definition 3. Let $\lambda(.)$ be the Lebesque measure on $\Re$ and let ϵ be any real number such that $0 \leq \epsilon < \lambda(\Theta)$. A regulatory mechanism Ψ is said to *ϵ-Pareto dominate* another regulatory mechanism Ψ' if $\lambda(\Theta) - \lambda(\Theta^*) \leq \epsilon$ where $\Theta^* = \{\theta \in \Theta : CW^{\Psi}(\theta) > CW^{\Psi'}(\theta)$ and $\pi^{\Psi}(\theta) > \pi^{\Psi'}(\theta)\}$.

The strongest form of domination in Definition 3 is 0-Pareto dominance, where the measure of the cost values at which the dominance relationships for the consumer and the producer welfare hold true is equal to the measure of Θ. Note also that 0-Pareto dominance does not imply, while implied by, the standard Pareto dominance in Definition 1. Below, we will describe a mechanism that we will Pareto compare to the BM mechanism under our relaxed domination concept.

Definition 4. Let (r^M, p^M, q^M, s^M) denote the modified BM mechanism, which satisfies equations (28)-(31) for all $\theta \in [\theta_0, \theta_1]$:

$$p^M(\theta) = c_0 + c_1\theta \tag{28}$$

$$P(q^M(\theta)) = p^M(\theta) \tag{29}$$

$$r^M(\theta) = \begin{cases} 1 & \text{if } V(q^M(\theta)) - p^M(\theta)q^M(\theta) \geq k_0 + k_1\bar{z}_\alpha(\theta) \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

$$\begin{aligned} s^M(\theta) \;&=\; \left[(c_0 + c_1\theta)q^M(\theta) + k_0 + k_1\theta - p^M(\theta)q^M(\theta)\right] r^M(\theta) \\ &\quad + \int_\theta^{\theta_1} r^M(\tilde{\theta})(c_1 q^M(\tilde{\theta}) + k_1)d\tilde{\theta} \end{aligned} \tag{31}$$

The essential feature of the above mechanism is that the price is always (for all values of α) equal to the marginal cost of production, while this would be true under the BM mechanism only if $\alpha = 1$. Clearly, the above mechanism is incentive-compatible since $p^M(\theta)$ is increasing, and therefore $q^M(\theta)$ and $r^M(\theta)$ are decreasing, in θ over $[\theta_0, \theta_1]$. It is also evident by Proposition 1 that the modified mechanism cannot maximize the expected (ex-ante) social welfare, which is maximized by the BM mechanism. The next question we will deal with is whether the modified BM mechanism can ϵ-Pareto dominate the BM mechanism at any regulatory environment.

Definition 5. We describe a *regulatory environment* by the list $\langle P(.), C(.,.), f(.)\rangle$, involving the inverse demand and cost functions faced by the monopolistic firm, and the beliefs of the regulator about the private cost parameter of the regulated firm.

Since the model of BM does not specify the inverse demand and belief functions, their regulatory model presented in Section 2 actually corresponds to a wide class of regulatory environments according to our definition above. Here, we will continue to focus on the same class of environments, by keeping all of the structures in Section 2. Before presenting our results, we should finally note that in the case where the welfares of consumers and the producer are equally weighted in the social objective ($\alpha = 1$), the original and the modified BM mechanism lead to the same welfare allocation at all possible costs. Thus, for the following results we will restrict ourselves to the case where $\alpha \in [0, 1)$.

Proposition 4. *Let $\alpha \in [0,1)$. For any $\epsilon \in (0, \theta_1 - \theta_0)$, there exist an uncountable number of regulatory environments at which the modified BM mechanism ϵ-Pareto dominates the BM mechanism.*

Proof. Let $\alpha \in [0,1)$. Pick any $\epsilon \in (0, \theta_1 - \theta_0)$. Consider a regulatory environment $\langle P(.), C(.,.), f(.)\rangle$, where

$$P(q) = a - q, \quad \text{for all} \quad q \in [0, a], \quad \text{with}$$

$$a \geq \min\left\{2\theta_1 - \theta_0, \theta_0 + \frac{(\theta_1 - \theta_0)^2}{\epsilon}\right\} \quad \text{(Assumption-a)}$$

$$C(q, \theta) = \theta q, \quad \text{if } q > 0, \quad \text{and} \quad C(0, \theta) = 0, \quad \text{for all } \theta \in [\theta_0, \theta_1],$$

$$f(\theta) = \frac{1}{\theta_1 - \theta_0}, \quad \text{for all } \theta \in [\theta_0, \theta_1].$$

We can calculate $F(\theta) = (\theta - \theta_0)/(\theta_1 - \theta_0)$, and $F(\theta)/f(\theta) = \theta - \theta_0$ for all $\theta \in [\theta_0, \theta_1]$. Pick any θ and first consider the modified BM mechanism. The producer welfare and consumer welfare are given by

$$\pi^M(\theta) \;=\; \int_\theta^{\theta_1} r^M(\tilde{\theta}) q^M(\tilde{\theta}) d\tilde{\theta}, \tag{32}$$

and

$$CW^M(\theta) \;=\; \left[V(q^M(\theta)) - C(q^M(\theta), \theta))\right] r^M(\theta) - \pi^M(\theta), \tag{33}$$

correspondingly. Since the parameters k_0 and k_1 in equation (1) both become zero under the assumed cost function, (30) would imply $r^M(\theta) = 1$. Then, from (28) and (29) and using the fact that $c_0 = 0$ and $c_1 = 1$, we have $q^M(\tilde{\theta}) = a - \tilde{\theta}$ for all $\tilde{\theta} \in [\theta_0, \theta_1]$. Note that Assumption-a implies $a > \theta_1$, thus $q^M(\tilde{\theta}) \geq 0$ for all $\tilde{\theta} \in [\theta_0, \theta_1]$. Then, equations (32) and (33) respectively become

$$\pi^M(\theta) = \int_\theta^{\theta_1} (a - \tilde{\theta}) d\tilde{\theta}, \tag{34}$$

and

$$CW^M(\theta) = \frac{(a-\theta)^2}{2} - \int_\theta^{\theta_1} \left(a - \tilde{\theta}\right) d\tilde{\theta}. \tag{35}$$

Now, we consider the BM mechanism (18)-(21). We have $z_\alpha(\theta) = \theta + (1-\alpha)(\theta - \theta_0)$, which is is increasing in θ, hence $\bar{z}_\alpha(.) = z_\alpha(.)$ from (12)-(17). Moreover, $\bar{q}(\tilde{\theta}) = a - \tilde{\theta} - (1-\alpha)(\tilde{\theta} - \theta_0)$ for all $\tilde{\theta} \in [\theta_0, \theta_1]$, from (18) and (19). Assumption-a implies $a \geq \theta_1 + (1-\alpha)(\theta_1 - \theta_0)$, thus $\bar{q}(\tilde{\theta}) \geq 0$ for all $\tilde{\theta} \in [\theta_0, \theta_1]$. Moreover, since $k_0 = k_1 = 0$, equation (20) implies $\bar{r}(\tilde{\theta}) = 1$ for all $\tilde{\theta} \in [\theta_0, \theta_1]$. Thus we have

$$\pi^{BM}(\theta) = \int_\theta^{\theta_1} (a - \tilde{\theta} - (1-\alpha)(\tilde{\theta} - \theta_0)) d\tilde{\theta}, \tag{36}$$

and

$$\begin{aligned} CW^{BM}(\theta) = {} & \frac{(a-\theta)}{2} \left(a - \theta - (1-\alpha)(\theta - \theta_0)\right) \\ & - \int_\theta^{\theta_1} \left(a - \tilde{\theta} - (1-\alpha)(\tilde{\theta} - \theta_0)\right) d\tilde{\theta}. \end{aligned} \tag{37}$$

Subtracting (36) and (37) correspondingly from (34) and (35), we obtain the welfare differentials

$$\pi^M(\theta) - \pi^{BM}(\theta) = (1-\alpha) \int_\theta^{\theta_1} (\tilde{\theta} - \theta_0) d\tilde{\theta}, \tag{38}$$

and

$$CW^M(\theta) - CW^{BM}(\theta) = (1-\alpha)\frac{(a-\theta)}{2}(\theta - \theta_0) - (1-\alpha) \int_\theta^{\theta_1} (\tilde{\theta} - \theta_0) d\tilde{\theta}. \tag{39}$$

Clearly, $\pi^M(\theta) - \pi^{BM}(\theta) > 0$ for all $\theta \in [\theta_0, \theta_1)$. Also, one can easily check that $CW^M(\theta) - CW^{BM}(\theta) > 0$ if and only if $\theta \in (\theta^*, \theta_1]$ where

$$\theta^* = \theta_0 + \frac{(\theta_1 - \theta_0)^2}{a - \theta_0}. \tag{40}$$

Assumption-a implies that $\theta^* > \theta_0$ and $\theta^* < \theta_1$. Therefore, for all $\theta \in (\theta^*, \theta_1)$, it is true that $CW^M(\theta) - CW^{BM}(\theta) > 0$ and $\pi^M(\theta) - \pi^{BM}(\theta) > 0$. Let $\Theta^* = (\theta^*, \theta_1)$, and let $\lambda(.)$ be the Lebesque measure on $\Re$. Note that

$$\lambda(\Theta) - \lambda(\Theta^*) = \theta^* - \theta_0 = \frac{(\theta_1 - \theta_0)^2}{a - \theta_0} \leq \epsilon$$

by equation (40) and Assumption-a. Thus, the modified BM mechanism ϵ-Pareto dominates the BM mechanism. Finally, note that the set of values for the parameter a satisfying Assumption-a is uncountable. *Q.E.D.*

Proposition 4 suggests that in situations where the outcomes of the modified and the original BM mechanism are not identical (i.e., $\alpha \neq 1$), even for arbitrarily small values of ϵ one can find many regulatory environments at which the modified BM mechanism ϵ-Pareto dominates the BM mechanism. However, the set of these regulatory environments is not the universal set, as illustrated by the next proposition.

Proposition 5. *Let $\alpha \in [0, 1)$. There exist an uncountable number of regulatory environments at which the BM mechanism is not ϵ-Pareto dominated by the modified BM mechanism for any $\epsilon \in (0, \theta_1 - \theta_0)$.*

Proof. Let $\alpha \in [0, 1)$. Pick any $\epsilon \in (0, \theta_1 - \theta_0)$. Consider the regulatory environments in the proof of Proposition 4 with Assumption-a being changed to

$$a \in \left[2\theta_1 - \theta_0,\, \theta_0 + \frac{(\theta_1 - \theta_0)^2}{\epsilon}\right) \quad \text{(Assumption-b)}.$$

That proof showed that for the considered environments the modified BM mechanism ϵ-Pareto dominates the BM mechanism only if $a \geq \theta_0 + (\theta_1 - \theta_0)^2/\epsilon$. Then, Assumption-b ensures that the BM mechanism is not ϵ-Pareto dominated. Finally, note that the set of values for the parameter a satisfying Assumption-b is uncountable. *Q.E.D.*

While the modified mechanism can ϵ-Pareto dominate the BM mechanism at some regulatory environments, it is unable to do so at some others. Despite this, the modified mechanism can be argued to be superior to the BM mechanism, since the modified mechanism is not ϵ-Pareto dominated by the BM mechanism for any $\epsilon \in [0, \theta_1 - \theta_0)$.

Proposition 6. *Let $\alpha \in [0,1)$. There exists no regulatory environment at which the BM mechanism ϵ-Pareto dominates the modified BM mechanism for any $\epsilon \in [0, \theta_1 - \theta_0)$.*

Proof. Let $\alpha \in [0,1)$. Consider any regulatory environment $\langle P(.), C(.,.),\ f(.)\rangle$ satisfying the structures in Section 2, and pick any $\theta \in \Theta$. Note that

$$\pi^{BM}(\theta) - \pi^{M}(\theta) = \int_{\theta}^{\theta_1} \bar{r}(\tilde{\theta})(c_1\bar{q}(\tilde{\theta}) + k_1)d\tilde{\theta} - \int_{\theta}^{\theta_1} r^{M}(\tilde{\theta})(c_1 q^{M}(\tilde{\theta}) + k_1)d\tilde{\theta} \leq 0, \quad (41)$$

since $p^M(\tilde{\theta}) < \bar{p}(\tilde{\theta})$, and consequently $q^M(\tilde{\theta}) > \bar{q}(\tilde{\theta})$ and $r^M(\tilde{\theta}) > \bar{r}(\tilde{\theta})$ for all $\tilde{\theta} \in \Theta$. So, if $\Theta^* = \{\theta \in \Theta : CW^{BM}(\theta) > CW^{M}(\theta)$ and $\pi^{BM}(\theta) > \bar{\pi}^{M}(\theta)\}$ then $\Theta^* = \emptyset$. Let $\lambda(.)$ be the Lebesque measure on $\Re$. Then,

$$\lambda(\Theta) - \lambda(\Theta^*) = \lambda(\Theta) = \theta_1 - \theta_0. \quad (42)$$

It follows from Definition 3 that the BM mechanism cannot ϵ-Pareto dominate the modified BM mechanism for any $\epsilon \in [0, \theta_1 - \theta_0)$. *Q.E.D.*

5 Conclusion

In this paper, we have studied the ex-post efficiency of the BM (1982) mechanism that regulates a monopolistic firm with unknown costs. We have showed that the regulatory mechanism of BM, which maximizes the expected social welfare subject to some feasibility conditions involving individual rationality and incentive-compatibility (Proposition 1, borrowed from BM, 1982), is not ex-post Pareto dominated by any other feasible direct-revelation mechanism (Proposition 2), however it is not an ex-post Pareto dominant mechanism either , for there exists (many) feasible mechanisms that are not Pareto dominated by the BM mechanism (Proposition 3).

We have next investigated whether the BM mechanism would survive as an ex-post undominated mechanism if the Pareto dominance notion were to be slightly relaxed. To that aim, we have introduced the ϵ-Pareto dominance requiring the relevant dominance relationships to hold in the support of the regulator's beliefs everywhere with a possible exception of some points of measure ϵ. With regard to this weakened Pareto concept, a modified version of the BM mechanism which requires marginal cost pricing at all possible costs is found to dominate the BM mechanism at an uncountable set of regulatory environments (Proposition 4), though this set is not as large as the universal set (Proposition 5). Nevertheless, the modified mechanism may be argued to be superior to the original one for it is never ϵ-Pareto dominated by the original BM mechanism at any regulatory environment (Proposition 6).

Given the problem of selecting a desirable mechanism among infinitely many feasible direct-revelation mechanisms that are ex-post Pareto non-comparable, the solution devised by BM, which employed a Bayesian regulator endowed with the objective of maximizing the expected social welfare, is definitely very attractive, since their approach optimally balances the Pareto tension between the producer and consumer welfare at different values of cost using the regulator's beliefs about the unknown cost information. Nevertheless, the Bayesian approach in monopoly regulation, or in mechanism design at large, has been criticized by many economists, including Crew and Kleindorfer (1986), Vogelsang (1988), Koray and Sertel (1990), and Laffont (1990), on the grounds of the (un)accountability of the regulator's prior beliefs and a related moral hazard problem. Some of these criticisms were formally investigated by Koray and Saglam (2005), who show how a non-benevolent regulator in the BM model of regulation can extract rents from consumers or the regulated firm by manipulating her beliefs to the benefit of either of the two parties. Despite all criticisms, the BM mechanism is still the best mechanism at hand to deal with the problem of regulating a single-period monopoly under asymmetric information. However, as we show in this paper, there exist some regulatory environments where the use of the BM mechanism might no longer be argued to be indispensable, since at these environments some other mechanisms may lead to ex-post superior outcomes for all parties in the society at almost all values of the cost information.

We should finally note that although the special focus of this study has been the problem of monopoly regulation, all of our results can directly be extended to the class of principal-agent models considered by Guesnerie and Laffont (1984).

References

Baron, D. and R. B. Myerson (1982), "Regulating a monopolist with unknown costs." *Econometrica*, 50, 911-930.

Crew, M.A. and P. R. Kleindorfer (1986), *The Economics of Public Utility Regulation.* Cambridge, MA: MIT Press.

Dasgupta, P. S., P. J. Hammond, and E. S. Maskin (1979), "The implementation of social choice rules: Some results on incentive compatibility." *Review of Economic Studies*, 46, 185-216.

Guesnerie, R. and J. J. Laffont (1984), "A complete solution to a class of principal-agent problems with an application to the control of a self-managed firm." *Journal of Public Economics*, 25, 329-369.

Harris, M. and R. M. Townsend (1981), "Resource allocation under asymmetric information." *Econometrica*, 49, 33-64.

Koray, S. and I. Saglam (2005), "The need for regulating a Bayesian regulator." *Journal of Regulatory Economics*, 28, 5-21.

Koray, S. and M. R. Sertel (1990), "Pretend-but-perform regulation and limit pricing." *European Journal of Political Economy*, 6, 451-72.

Laffont, J. J. (1994), "The new Economics of regulation ten years after." *Econometrica*, 62, 507-37.

Loeb, M. and W. A. Magat (1979), "A decentralized method for utility regulation." *Journal of Law and Economics*, 22, 399-404.

Myerson, R. B. (1979), "Incentive compatibility and the bargaining problem." *Econometrica*, 47, 61-74.

Vogelsang, I. (1988), "A little paradox in the design of regulatory mechanisms." *International Economic Review*, 29, 467-76.