Tax burden optimization on economic agents by modeling interaction in the taxation system

Sokolovskyi, Dmytro and Sokolovska, Olena

Institute of Industrial Economics of National Academy of Sciences of Ukraine, Institute of Industrial Economics of National Academy of Sciences of Ukraine

5 May 2016

Introduction

In the current context the developing and transition countries, including Ukraine, face the problems of insufficient revenues, collected in different level budgets. The economic reforms need rather more costs (i.e. revenues), than stable situation. For this reason, any arrears in developing and transition countries create more problems than in developed ones. The budget arrears due to direct nonpayment, concealment of income, transferring the economic activity into illegal sector leads to budget shortfall. In this situation the threats come from insufficiently clear planning of tax base (number of taxpayers, their revenues), from insufficiently clear behavior rating of economic agents in case of changes in state tax policy and also from insufficiently organized inspection of tax collection. All foresaid and-or can lead to default by economic agents from their obligations.

Our research provides the development of a game-theoretical model of interaction between economic agent and tax authorities.

This approach differs from existing methods (which are widespread in practice) for control of tax rates based on political decisions, the objective of which is to increase the budget revenues, to ease the financial burden on some industry, to bring the domestic legislation in line with the international one, etc. But in such a case the sufficiently rigorous substantiation of optimality of adopted decision is absent.

In theory, one of the main aspects of budget nonfulfillment problem is the issue of tax evasion, which is well known in world literature as «tax evasion problem». For purpose of our research the studies of tax evasion by means of game-theoretic modeling are of interest.

Literature review

The application of game-theoretic tools to solve the problem of tax evasion first was made by M. Alligham, A. Sandmo (1972), in which the simple basic model of interaction between taxpayer and tax inspector was proposed. In 2004 the literature review of the tax evasion theory for last 30 years was prepared by A. Sandmo; in this context we can mention the article of J. Slemrod and S. Yitzhaki (2000), in which they overviewed the main aspects of tax avoidance, evasion and administration.

Hereafter, T. Srinivasan (1973), T. McCaleb (1973), B. Singh (1976), V. Christiansen (1980), J. Baldry (1984) analyzed problems of optimization of correlation between identifying tax violation and amounts of penalties, and also application of those penalties to struggle with tax evasion.

The relationship between tax rate levels and tax evasion in a context where the utility of a taxpayer depends on both his own consumption and his relative position with respect to the average declared income of the economy is analyzed in study of J. Panadés i Martí (2002).

Generally, such theoretical studies concern the possibility determination of application of various fiscal instruments (rates of taxes and penalties, probability of tax declaration audit) to counterwork of tax evasion.

Among studies in applied economics, concerned with evaluation of specific figures of tax evasion for certain countries, for purposes of our research we examined the following ones. E. Engel, J. Hines Jr. (1999), N. Gemmell, M. Ratto (2012) proposed the model of behavior of rational taxpayer, where the current taxpayer' evasion is the decreasing function of his prior evasion. The model was estimated using the data of aggregate behavior of American taxpayers over the 1947-1993 period. Despite the feasibility of proposed approach, it can't be used for our study because it characterizes the individuals' behavior (moreover with American mentality and in conditions of American tax system), but not the enterprises' behavior.

A rational taxpayer also appears in study of G. Coricelli et al. (2007). This article states that the main requirement for individuals to evade taxes will be the positive expected benefit of cheating (positive value of utility function), i.e. the expected benefits are greater than expected costs. But as former study, this one analyses the actions of individuals (the features of their psychological behavior under tax evasion and their emotion), but not the actions of firms; this complicates the possibility of using the results of concerned article for our study.

J. de Melo et al. (1992) in their study compared the tax systems of low-income countries (which is more close to Ukrainian realias); however they analyzed Madagascar's tax system as a comparative base, which characterizes by various distortionary taxes. Using a 10-sectors model and general-equilibrium calculations, the authors estimated revenue losses from exemptions, tax evasion, and smuggling for three types of taxes: import duties, value-added taxes, excise taxes, moreover to simplify the model they united all these taxes into specific aggregate tax. The application of given model to Ukraine is complicated by reason of presence of less distorting tax system and by reason of model's incompleteness, because in Ukrainian tax system besides taxes there are direct ones, the impact of which differs from that of indirect taxes.

We should note that mentioned investigations, concerning the tax evasion problem in the context of applied economics, unfortunately rarely use the game-theoretic tools. From this perspective we can mention the research of J. Alm and M. McKee (2004), V. Lipatov (2003) and V. Bilotkach (2006).

Alm and McKee (2004) using game-theoretic tools investigated compliance behavior when returns are selected for audit based upon the deviation of each individual's tax report from the average report of all other taxpayers. The V. Lipatov's model assumes the minimal information about auditing probability and the possibility of payoffs for tax inspectors from taxpayers. V. Lipatov does not optimize the actions of tax authority (government), and resolves itself into adopting of agents' decisions, which are satisfactory but not optimal. Bilotkach (2006) examined the tax evasion by Ukrainian enterprises through underreporting activity. In the theoretic game the author calculated the equilibrium points for businessman (taxpayer, who can hide part of his profit and offer bribe to official in the case of detecting) and tax inspector, who can take a bribe in exchange for concealment of tax violation. But we should note, that such a model will be feasible only after introducing into the model the third person – a customer (a principal), because if the principal and the tax inspector are all in one, than taking bribes on his part will be absolutely irrational behavior.

Congdon and Kling (2010, pp. 375-386) consider the theoretical implications of behavioral economics for tax policy and the welfare consequences of taxation.

But the autonomous analysis of tax evasion problem does not give the exhaustive information about budget nonfulfillment. Insufficiently fulfilled budget often takes place not because the agents do not pay taxes and do not declare their revenues (or another tax base), but because of complexity of identifying the unfairness of their behavior. Such situation can lead to certain problems for economy (shifting the agents' activity into shadow sector, capital flight, reducing of foreign investment etc).

Thus, the important factor, which determines the actions of all interested parties in tax evasion model, is the detection probability of unfair acts of taxpayer, and also the possibility of erroneously identified tax violation (in conditions of absence of tax evasion). It is our opinion that when we take into account this parameter, that was not considered in research articles mentioned above, that can influence essentially on contractors' behavior.

In this context our model differs from existing ones properly by the fact that, besides of traditional unsecured detection of taxpayers' tax evasion by tax inspector, we also introduce the possibility of recognition the "fair" taxpayer as a tax evading person.

Basing on analysis of research in the area of tax evasion it can be concluded that one of the currently important problems, which now is not solved, is the problem of optimization of tax burden, which is grounded on analysis of dependence between economic agents' behavior and changes of tax pressure. Indispensably it should be taken into account the possibility of opportunistic behavior of economic agents. Solving of mentioned problem can allow increasing of budgetary performance on different levels.

Consequently, the target of given research is the current tax system and also the set of industries; the scope of research is the influence of changes of tax burden on economic agents' behavior. The goal of research is to define the real tax burden which depends on the nominal tax burden falling on enterprises of different industries; also the response of enterprises to possible tax rates changes will be analyzed.

The goal of research determines the choice of following research tasks:

- ✓ to classify a number of taxes and industries with relation to easiness of tax evasion with the purpose of calculation the values of general behavioral function parameters, which are specific to given taxes and industries;
- ✓ to simulate the behavior of economic agents in various industries using aggregate tax – to define the general function of behavior and to do its parameterization;
- ✓ to define the real tax burden according to its nominal value for enterprises in various industries;
- ✓ to define the points of optimal tax burden and points of enterprises' market out in various industries.

For analysis and empirical estimation we've used a game theory, clustering, statistical analysis.

In our model we assumed the following:

1) the perfect demand elasticity for firms' production (i.e. the prices are set endogenously);

2) the taxpayers practice bounded rationality in their actions, i.e. they try to maximize their revenues, irrespective to means of objective achievement, taking the decisions according to their awareness.
3) the motives of behavior and principles of environment valuation are invariable for all economic agents in all industries.

Accordingly to this, we conclude that behavior models for firms in various industries must to coincide by structure but to differ by specific values of parameters; the last ones are peculiar for each industry.

Classification of taxes and industries

Basing on economic and statistical analysis for further research we've decided to limit the number of taxes to six:

1) profit tax;
2) payroll tax (as burden on production cost, which the economic agent can manipulate);
3) land tax;
4) excise tax;
5) customs duty.

One of the main taxes – the VAT, did not consider in the model, because it has a specific procedure of assessment and refund.

Further it is reasonable to specialize some taxes, notably:

- ✓ customs duties – on export and import duty, since the export duty is levied only for some raw materials, while import tariff is levied on most of products, imported in Ukraine by economic agents. The income tariff can levy at ad valorem, specific and combined rates, which can have a zero value;
- ✓ excise tax – on excise tax on import and domestic excise tax. The differentiation is made according to procedure of paying and procedure of control of those taxes.

So, the taxes are the following:

1) profit tax;
2) payroll taxes;
3) land tax;
4) domestic excise tax;
5) excise tax on import;
6) export duty;
7) import tariff.

Principles of identification of some taxes in terms of possibility and methods of tax evasion

Objects of taxation are legal facts, which cause the engagement of subject of taxation to pay taxes. The concealment (full or partial) of tax object and change of tax object can be regarded as illegal act related to tax object.

The tax base can be considered as a part of income or assets (net of tax allowances) of taxpayer secured in legislation and which comes into account at

calculation of tax amount. The tax evasion through improper use of allowance, through underdeclaration of income can be regarded as illegal act related to tax base.

Taxes related to external trade differ by procedure of taxpaying, notably, they are paid by event (i.e. the basis for taxpaying is only the fact of product's border crossing) in contrast to taxes non related to external trade (in this case the basis for taxpaying is the occurrence of determined interim).

The same is for procedure of control over taxpayers: for taxes related to external trade, the control exercises directly at the moment of product's border crossing; for taxes non-related to external trade the control exercises at specified time.

The evasion from different taxes involves different cost. The main rule is the following: it is easier to evade taxes, which need technically complicated calculation of financial results, and it is more complicate to evade taxes, which are based on simple physical or cost indicators.

The ranking of taxes by easiness of tax evasion was made as follows.

The easiest is the evasion from paying profit tax and payroll taxes. The easiness of evasion from paying profit tax is caused by possibility of ambiguous profit calculation in accounting documents. Moreover, large enterprises, notably in metallurgical industry and machinery, often shift the significant part of their profit into offshore zones.

Basing on analysis of statistical data the taxes were ranked by easiness of tax evasion, namely:

1) profit tax, payroll taxes;
2) domestic excise tax;
3) excise tax on import; export duty; import tariff;
4) land tax.

Such ranking by easiness of tax evasion is confirmed by tax debt profile. Particularly, one of the higher debts is the debt related to non-payment of profit tax, and one of the smaller debts is the debt related to land tax.

Factors affecting easiness of tax evasion are classified in Tables 1-2.

Table 1. Factors affecting easiness of evasion of taxes non-related to external trade

Taxes	Profit tax	Payroll taxes	Land tax	Domestic excise tax
Method of tax evasion				
I. Illegal acts related to tax object			+	+
II. Illegal acts related to tax base	+	+	+	+
III. Non-commodity transactions	+	+		
Procedure of taxpaying (by event or by time)	T	T	T	
Procedure of control of taxpaying				
Procedure of control (by event or by time)	T	T	T	
Penalty charging rules				

Industries are grouped according to:

✓ share of different taxes;

✓ procedure of paying of each tax in given industry;
✓ general characteristics of firms operating in given industry, which influence on behavior of separate firm.

Basing on analysis of Ukrainian legislation for each industry we established a list of taxes, paying by firms operating in this industry. Industries and appropriate taxes are presented in Table 3.

Table 2. Factors affecting easiness of evasion of taxes related to external trade

Taxes	Excise tax on import	Import tariff	Export duty
Method of tax evasion			
I. Illegal acts related to tax object	+	+	+
II. Illegal acts related to tax base	+	+	+
III. Non-commodity transactions			+
Procedure of taxpaying (by event or by time)	E	E	E
Procedure of control of taxpaying			
Procedure of control (by event or by time)	E	E	E
Penalty charging rules			

Table 3. Taxes paid by industry

Taxes	**Industry**							
	CA	**CB**	**DA**	**DJ**	**DK, DL, DM**	**G**	**I**	**J**
Profit tax, t^p	+	+	+	+	+	+	+	+
Personal income tax, t^w	–	–	–	–	–	–	–	–
Payroll taxes, t^l	+	+	+	+	+	+	+	+
Land tax, t^h	+	+	+	+	+	+	+	+
VAT, t^{VAT}	–	–	+	–	–	–	–	–
Domestic excise tax, t^{a1}	+	+	+	+	+	+	+	+
Excise tax on import, t^{a2}	+	+	+	+	+	+	+	+
Export duty, t^{ex}	+	+	+	+	+	+	+	+
Import tariff, t^{im}	+	+	+	+	+	+	+	+

Legend
CA – mining of fuel and energy raw materials;
CB – mining of raw materials (except fuel and energy raw materials);
DA – production of food, beverages and tobacco;
DJ – production of metal and metal products;
DK – machinery and equipment;
DL – electrical, electronic and optical machinery and apparatus;
DM – production of vehicles and transport equipment;
G – commerce; auto service; reparation of household goods and items of personal-use;
I – transport and communications;
J – financial business;

«+» means that the given tax influences on certain industry; «–» means that there is no influence.

Model of taxpayer's behavior

In the case when the nominal tax rate increases (and this tax affects on economics of enterprise), this leads to increase of cost and decrease of profit – regardless the type o tax. This means, that all taxes irrespective of their name and procedure of assessment can be regarded as profit taxes. It is important because of in practice the rates of some taxes can increase, the rates of another ones – decrease. In this situation only the final impact matters – did the total tax burden increase or decrease, i.e. did the share of profit, withdrawing for social needs will be increased or decreased.

If this share increases, the firms will tend to decrease the increment of tax burden due to additional tax evasion, associated with additional cost. And if this cost is not extremely high, this will result to the certain increase of scope of tax evasion and further enlargement of gap between nominal and real tax coefficient[1].

The decrease of share of profit leads to decrease of gain, proceeded from tax evasion. Thus let us assume that the firm will decrease a little the scope of tax evasion, and the gap between nominal and real tax coefficient will be also decreased.

To define how exactly the relation between nominal and real tax coefficient will be changed for each industry, it is important to take in account the value of additional cost, related to increase of scope of tax evasion (or cost-cutting in case where the scope of tax evasion decrease), and also the value of cost, needed to detect the evidence of tax evasion.

Because of the fact that the procedure of taxpaying provides the interaction between agents – taxpayers and principals – the state authorities, which control the tax collection, is expedient to model such process basing on game-theoretic model of interaction between mentioned agents.

To define the value of certain strategies of interaction, we envisage the following parameters:

- ✓ tax base (income, profit, volumes of external trade etc, further – "income") (R);
- ✓ tax rate (τ);
- ✓ value (function, coefficient) of penalties for tax evasion (γ);
- ✓ cost of concealment procedure, related to tax evasion (depends, particularly, on complication of tax evasion in certain industries) (ν);
- ✓ cost needed to agent to maintain the transparence of his activity (depends on industry) (μ);
- ✓ cost of procedure of control for tax treatment (depends, particularly, on complexity of inspections in different industries) (π);
- ✓ value of detecting by principal the real tax evasion of agents (depends on costs of concealment of tax evasion bearing by agents, and inspection costs bearing by principal) (p);

[1] Every tax rate (for example, land tax) can be increased, but tax evasion will not take place to tax with increased rate; it will take place to those taxes, for which the agent has a best practice to evade (for example, the profit tax and VAT).

- ✓ value of detecting by principal the fictitious tax evasion of agents (depends on costs of maintaining the transparence bearing by agents, and inspection costs bearing by principal) (q);
- ✓ cost (complication) of estimation (maid by principal) of predicted value of collected tax;
- ✓ frequency of tax inspections.

The game, modeling the interaction between agents in the process of taxpaying, is presented in (1)-(4).[2]

$$\Gamma = \left(\Pr; Ag; \left(G, H\right)\left(\Pr \times Ag\right)\right), \tag{1}$$

where

$$\Pr = \begin{pmatrix} pr_0 \\ pr_1 \end{pmatrix} \tag{2}$$

– set of tax inspector's strategies, pr_0 – does not inspect; pr_1 – inspects;

$$Ag = \left(ag_0; ag_1\right) \tag{3}$$

– set of taxpayer's strategies, ag_0 – does not pay; ag_1 – pays;

$$\begin{aligned} \left(G, H\right)\left(\Pr \times Ag\right) &= \left(g_{ij}, h_{ij}\right) = \\ &= \begin{pmatrix} \{0; R - v\} & \{\tau R; (1-\tau)R - \mu\} \\ \begin{Bmatrix} p(v,\pi)\tau(1+\gamma)R - \pi; \\ \left(1 - p(v,\pi)\tau(1+\gamma)\right)R - v \end{Bmatrix} & \begin{Bmatrix} \left(1 + q(\mu,\pi)(1+\gamma)\right)\tau R - \pi; \\ \left(1 - \tau\left(1 + q(\mu,\pi)(1+\gamma)\right)\right)R - \mu \end{Bmatrix} \end{pmatrix} \end{aligned} \tag{4}$$

The following additional assumptions of the model are:

1. the taxpayer's incentive to evade taxes grows up with the increase of aggregate tax. As a consequence, when the aggregate tax increases, the expectable share of taxpayers, who come to a decision about tax concealment, also increases.
2. for each taxpayer there is a threshold; before achieving the threshold, he fairly pays taxes, after exceeding this threshold, the taxpayer decides to evade taxes.
3. for each taxpayer there is a "threshold of reproduction on a simple scale" ($\hat{\tau}$). This means that if the profit after aggregate tax is equal to zero, the economic agent withdraws from market (or he passes completely into shadow sector), and aggregate tax he paid is identical to zero[3].

---

[2] A shortened version of the model (1)-(4) is presented in D. Sokolovskyi and O. Sokolovska (2013).

[3] This assumption have something in common with conclusions of D. Hibbs Jr. and V. Piculescu (2005), who proposed a model of how institutional benefits, taxation and government regulations affect the productive activity of private enterprises of more than 10,000 ones in 80 countries covering the main regions of the world for 1998-2000 period. They concluded that incentives to evade taxation and produce underground depend not only on statutory tax rates, but such incentives are related to firm-specific thresholds of tax toleration.

According to 1) the certain agent to determined level fairly pays the aggregate tax (bringing an income τR for principal); after that he changes his strategy for tax evasion strategy with principal income $p\tau(1+\gamma)R$. According to 2) the certain agent pays the aggregate tax (fairly or with partial evasion) until the amount of his aggregate tax is $\tau < \hat{\tau}$, after that the agent abandons the market, zeroing the amount of paid tax (Fig. 1).

To extend this principle for all taxpayers, it should be taken into account that the number of opportunists increases monotonously with the increasing of tax rate. Subsequently, the general income equation can be written as follows:

$$R_{real} = \left(1-\tau^n\right)\tau R + p\tau^{n+1}(1+\gamma)R =$$

$$= \tau R - \tau^{n+1}R + p\tau^{n+1}(1+\gamma)R;$$

$$R_{real} = \left(1-(1-p-p\gamma)\tau^n\right)\tau R.$$

Reducing this equation by R and taking into account the dependences from industry and time, we obtain dependence of real tax rate on its nominal rate:

$$\tilde{t}^{\varepsilon}(j) = \left(1-\left(1-p^{\varepsilon}-p^{\varepsilon}\gamma^{\varepsilon}\right)b^{\varepsilon}\left(t^{\varepsilon}(j)\right)^n\right)t^{\varepsilon}(j), \qquad (5)$$

where $t^{\varepsilon}(j)$ is the nominal tax coefficient for ε-th industry at moment j;

p^{ε} is the coefficient of complication of tax evasion for ε-th industry;

γ^{ε} is the generalized coefficient of penalties for ε-th industry;

b^{ε} is the scale coefficient related to level of replacement for ε-th industry;

n is the coefficient of curvature of graph function.

Figure 1. Changes in taxpayer's behavior with the increase of aggregate tax

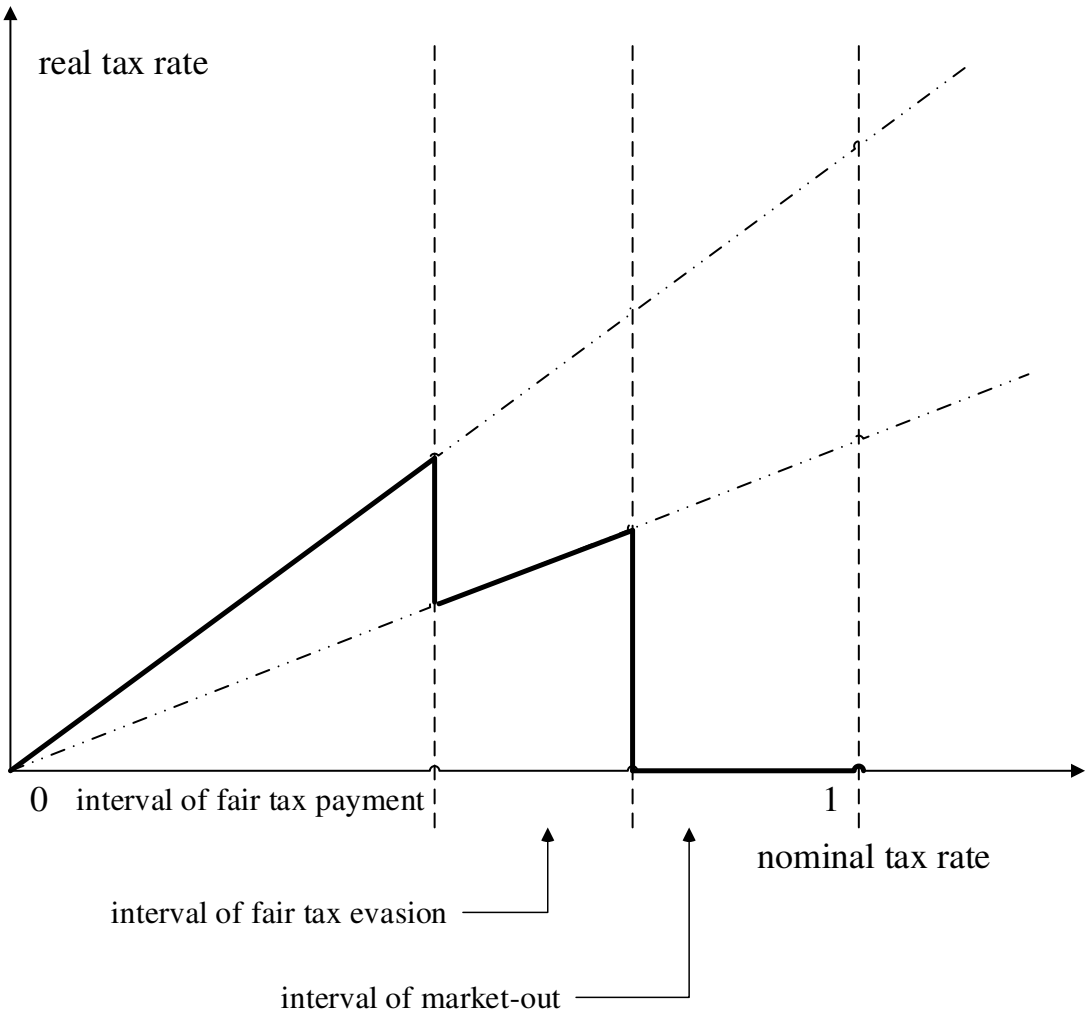
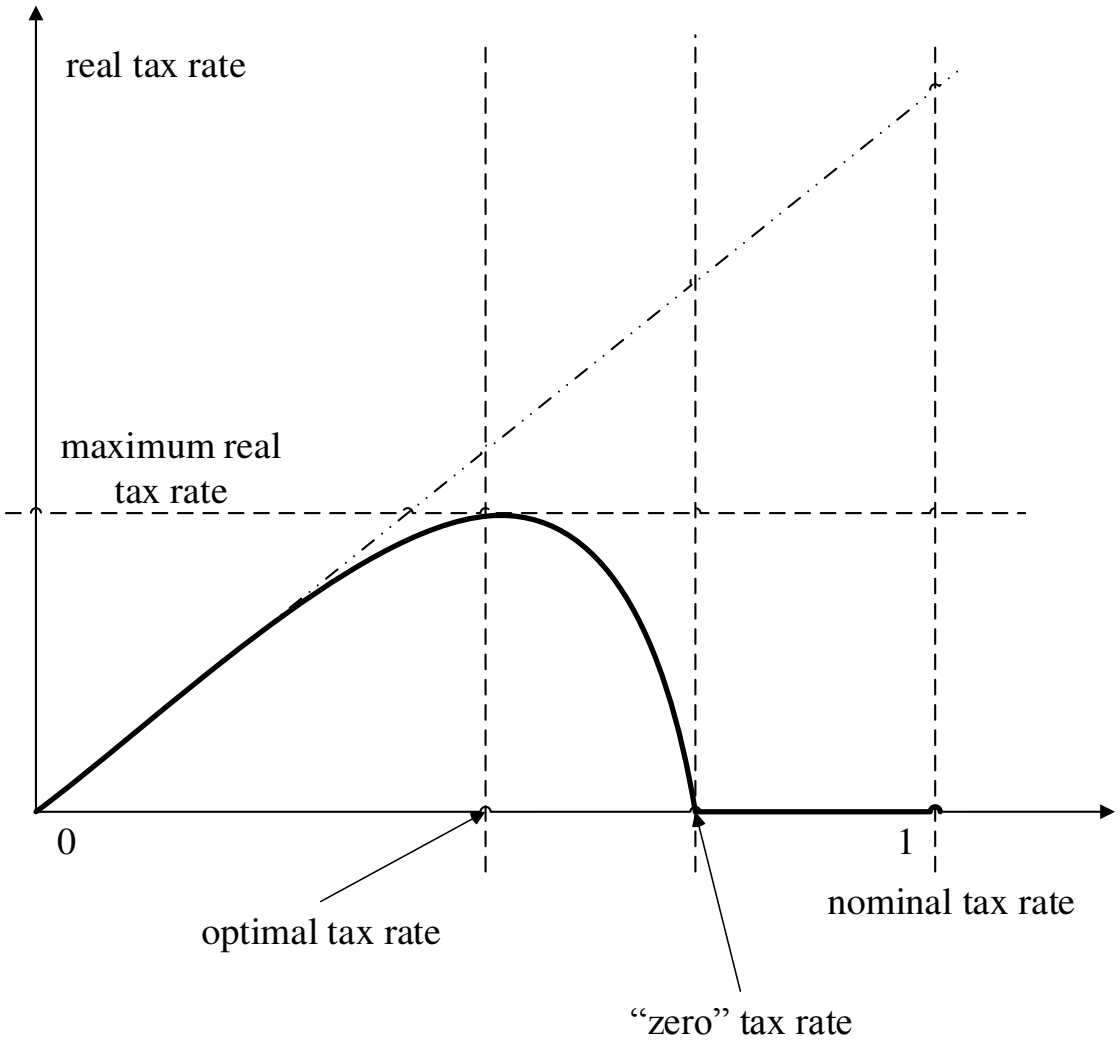


Fig. 2 represents the type of dependence (5)

Figure 2. Dependence of real tax burden on nominal one



For each industry the tax structure differs. So it makes sense to assume that the degree of easiness/complication of tax evasion for each industry is predicated upon the tax structure, paid by firms of given industry, and does not depend directly on other aspects of functioning of those firms.

To estimate the real tax burden after changing of nominal tax rates we introduce the following notation:

$$\left(t_i^{\varepsilon}(j)\right)_{\substack{i=1,2,\ldots N;\\ \varepsilon=1,2,\ldots M}} = \begin{pmatrix} t_1^1(j) & t_1^2(j) & \ldots & t_1^M(j) \\ t_2^1(j) & t_2^2(j) & \ldots & t_2^M(j) \\ \cdot & \cdot & \cdot & \cdot \\ t_N^1(j) & t_N^2(j) & \ldots & t_N^M(j) \end{pmatrix}$$

is the matrix of nominal tax rates at moment j, besides

$t_i^{\varepsilon}(j)$ is the tax rate i for firms of ε-th industry at moment j;

N denotes the number of taxes,

M denotes the number of industries;

$$\left(t^{\varepsilon}(j)\right)_{\varepsilon=1,2,\ldots M}=\left(t^{1}(j)\quad t^{2}(j)\quad\ldots\quad t^{M}(j)\right);$$

is the vector of nominal tax burden at moment *t*;

$$\left(Va^{\varepsilon}(j)\right)_{\varepsilon=1,2,\ldots M}=\left(Va^{1}(j)\quad Va^{2}(j)\quad\ldots\quad Va^{M}(j)\right);$$

is the vector of tax bases at moment *j*;

$S_i^{\varepsilon}(j)$ denotes the statistically defined amount of *i*-th tax paid by firms of *ε*-th industry at moment *j*;

$\overline{S}_i^{\varepsilon}(j)$ denotes the statistically defined amount of *i*-th tax liabilities (tax debt) for firms of *ε*-th industry at moment *j*;

$S^{\varepsilon}(j)=\sum_{i=1}^{N}S_i^{\varepsilon}(j)$ denotes the total tax amount, paid by *ε*-th industry at moment *j*.

$\overline{S}^{\varepsilon}(j)=\sum_{i=1}^{N}\overline{S}_i^{\varepsilon}(j)$ denotes the total tax liabilities (tax debt) for *ε*-th industry at moment *j*;

$P_i^{\varepsilon}(j)$ denotes the level of tax evasion for firms of *ε*-th industry at moment *j*;

$\delta S_i^{\varepsilon}(j)$ is the real share of tax *i* in total tax burden for firms of *ε*th industry at moment *j,*

$$\delta S_i^{\varepsilon}(j)=\frac{S_i^{\varepsilon}(j)}{S^{\varepsilon}(j)}=\frac{S_i^{\varepsilon}(j)}{\sum_{i=1}^{N}S_i^{\varepsilon}(j)},$$

$\delta\tilde{S}_i^{\varepsilon}(j)$ is the nominal share of tax *i* in total tax burden for firms of *ε*th industry at moment *j,*

$$\delta\tilde{S}_i^{\varepsilon}(j)=\frac{S_i^{\varepsilon}(j)+\overline{S}_i^{\varepsilon}(j)}{S^{\varepsilon}j+\overline{S}^{\varepsilon}(j)}=\frac{S_i^{\varepsilon}(j)+\overline{S}_i^{\varepsilon}(j)}{\sum_{i=1}^{N}\left(S^{\varepsilon}(j)+\overline{S}^{\varepsilon}(j)\right)},$$

Proceeding from aforesaid patterns, we propose the following algorithm to estimate the influence of nominal tax rates change on firms' behavior of each industry.

Assume that for the next moment *j+1* the matrix of nominal tax rates change is defined:

$$\left(\Delta t_i^{\varepsilon}(j)\right)_{\substack{i=1,2,\dots N;\\ \varepsilon=1,2,\dots M}} = \begin{pmatrix} \Delta t_1^1(j+1) & \Delta t_1^2(j+1) & \dots & \Delta t_1^M(j+1) \\ \Delta t_2^1(j+1) & \Delta t_2^2(j+1) & \dots & \Delta t_2^M(j+1) \\ \cdot & \cdot & \cdot & \cdot \\ \Delta t_N^1(j+1) & \Delta t_N^2(j+1) & \dots & \Delta t_N^M(j+1) \end{pmatrix},$$

i.e., the matrix of nominal tax rates at moment *j+1* will be the following

$$\left(\Delta t_i^{\varepsilon}(j+1)\right) = \left(t_i^{\varepsilon}(j)\right) + \left(\Delta t_i^{\varepsilon}(j)\right)\Big|_{\substack{i=1,2,\dots N;\\ \varepsilon=1,2,\dots M}};$$

$$\begin{pmatrix} t_1^1(j+1) & t_1^2(j+1) & \dots & t_1^M(j+1) \\ t_2^1(j+1) & t_2^2(j+1) & \dots & t_2^M(j+1) \\ \cdot & \cdot & \cdot & \cdot \\ t_N^1(j+1) & t_N^2(j+1) & \dots & t_N^M(j+1) \end{pmatrix} =$$

$$= \begin{pmatrix} t_1^1(j) & t_1^2(j) & \dots & t_1^M(j) \\ t_2^1(j) & t_2^2(j) & \dots & t_2^M(j) \\ \cdot & \cdot & \cdot & \cdot \\ t_N^1(j) & t_N^2(j) & \dots & t_N^M(j) \end{pmatrix} + \begin{pmatrix} \Delta t_1^1(j+1) & \Delta t_1^2(j+1) & \dots & \Delta t_1^M(j+1) \\ \Delta t_2^1(j+1) & \Delta t_2^2(j+1) & \dots & \Delta t_2^M(j+1) \\ \cdot & \cdot & \cdot & \cdot \\ \Delta t_N^1(j+1) & \Delta t_N^2(j+1) & \dots & \Delta t_N^M(j+1) \end{pmatrix}.$$

In this case

$$\forall i,\varepsilon : i=1,2,\dots N;\varepsilon=1,2,\dots M: \quad S_i^{\varepsilon}(j+1) = S_i^{\varepsilon}(j)\frac{\Delta t_i^{\varepsilon}(j+1)}{t_i^{\varepsilon}(j)}$$

$$\forall \varepsilon : \varepsilon=1,2,\dots M: \quad S^{\varepsilon}(j+1) = S^{\varepsilon}(j) + \sum_{\varepsilon=1}^{M} S_i^{\varepsilon}(j+1) =$$

$$= S^{\varepsilon}(j) + \sum_{\varepsilon=1}^{M} \frac{\Delta t_i^{\varepsilon}(j+1)}{t_i^{\varepsilon}(j)}.$$

The vector of nominal tax burden at moment *j+1* will be:

$$\left(t^{\varepsilon}(j+1)\right)_{\varepsilon=1,2,\dots M} = \left(\frac{S^1(j+1)}{Va^1(j+1)} \quad \frac{S^2(j+1)}{Va^2(j+1)} \quad \dots \quad \frac{S^M(j+1)}{Va^M(j+1)}\right);$$

and for real tax burden (according (5)):

$$\tilde{t}^{\varepsilon}(j+1) = \left(1 - a^{\varepsilon}\left(t^{\varepsilon}(j+1)\right)^{n^{\varepsilon}}\right) t^{\varepsilon}(j+1), \qquad (6)$$

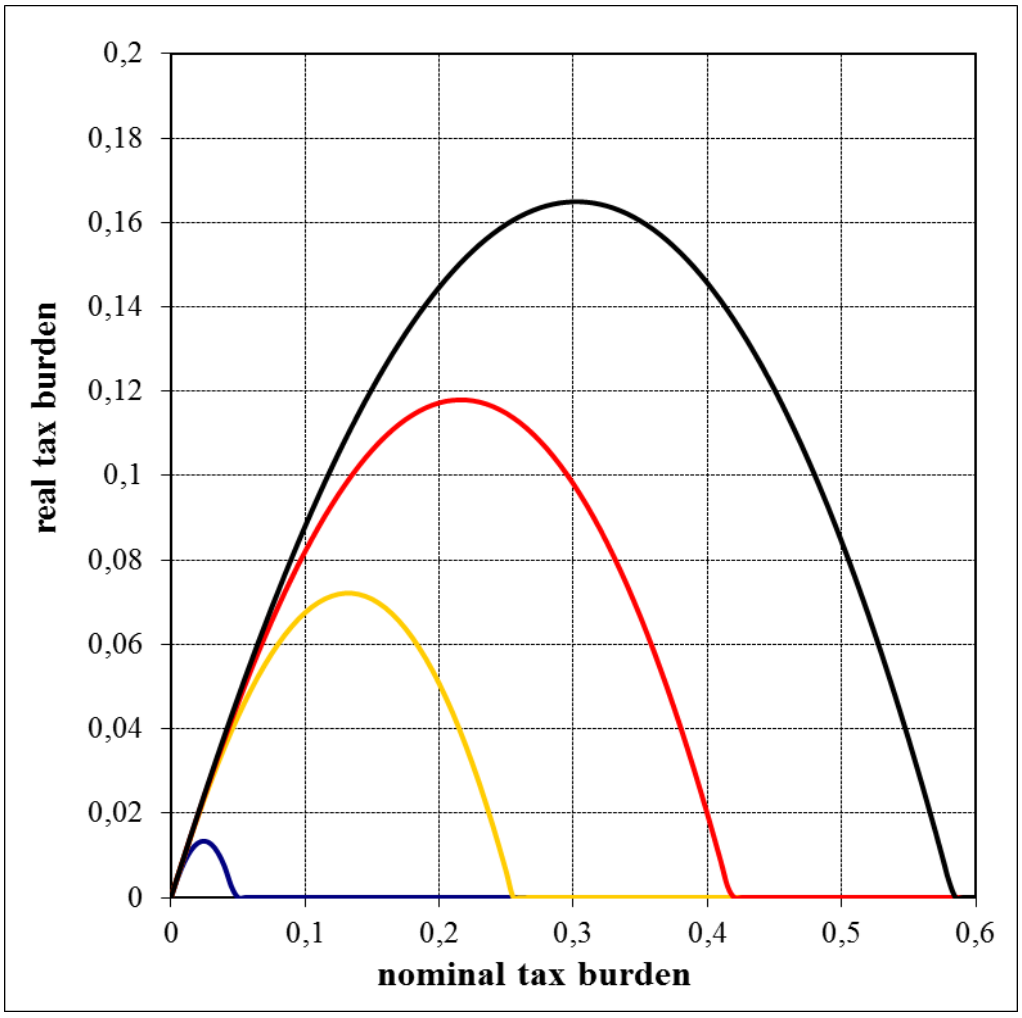
Since the coefficient of equation (5) base on statistical estimation, the possible statistical error and the divergence of obtained numerical values, should be taken into

account; therefore it is expedient to unite the similar (by value) functions of tax burden using the averaging of a^{ε} and b^{ε} values. With consideration of values of parameters, obtained after analysis of statistical data for Donets region for pre-war period (by 2014), it appeared rational to divide all industries of region into four groups[4]:

1) mining of fuel and energy raw materials;
2) production of vehicles and transport equipment; commerce; auto service; reparation of household goods and items of personal-use; transport and communications;
3) production of food, beverages and tobacco; machinery and equipment;
4) mining of raw materials (except fuel and energy raw materials); production of metal and metal products; electrical, electronic and optical machinery and apparatus.

For each of those groups we obtained a value of function of real tax burden with respect to tax base – the added value[5]; fig. 3 represents the graphs of those functions and also the optimal and threshold values of tax burden (which are presented in Table 4).

Figure 3. Dependence of real tax burden on nominal one for groups of industries



[4] Those groups represent the main budget revenue generating industries in Donetsk region.

[5] The added value as a tax base seems more economically feasible than, for example, the sales revenue, because taxes do not influence on firm's material cost. This added value was calculated as firm's gross sales after deduction of material cost.

Table 4. Optimal and threshold values of tax burden for groups of industries

Group number	Tax burden, %	
	optimal	threshold
1	*2,5*	*5,0*
2	*13,0*	*25,5*
3	*21,7*	*42,0*
4	*30,0*	*58,5*

Conclusion

Since the rational economic agents usually try to minimize the tax pressure, in particularly using tax evasion, for effective planning of budget revenues the task of optimization of tax burden depending on industry and also the analysis of firm's reaction for change of tax rates are currently important.

The given study proposes the approach to classify the industries with relation of easiness of tax evasion of agents operating in certain industry. Basing on assumptions of their behavior we define the type of dependence of real tax rates on their nominal ones. It is confirmed that the graph, describing the agent's behavior (depending on change of tax pressure), has two key points: maximum – the optimal tax rate (if this rate increase, the real tax revenues fall), and also the point of simple reproduction (after achieving this level, firms stop to pay taxes at all, i.e. they either shift into informal sector or close down).We calculated the concrete values of those parameters for economic agents operating in different group of industries. Those groups are formed on the base of estimation of easiness of tax evasion with relation to definite taxes and industries.

The successful model realization with practical confirmation of developed hypotheses will allow revealing the interaction between tax rates and revenues, collected by different level budgets, between returns of economic agents and tax evasion level. This will permit to estimate the efficiency of actual tax burden for various industries. According to this one will be able to determine the practical guidelines for Ukrainian tax policy optimization.

We should note that the most of model parameters are not the statistical data and they should be modeled in addition. Moreover, the type of dependence of p and q values on v, μ and π variables is to be defined. To strengthen the model adequacy, the possibility of side payment (here it is a case of briberies, of criminal protection racket etc) is to be considered. Also it is important to take into account the history of interaction, the additional personal ownership of information about tax inspections, the different variants of penalties etc. Also the further specification and extension for classification of taxes and industries with relation to easiness of tax evasion are required

The specified problems are in the scope of our further research.