The perils of first-order conditions of New Keynesian models

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Abstract

This paper presents a case where first-order conditions used for the Calvo sticky-price New Keynesian model is insufficient and in fact results in calculation of a wrong equilibrium.

1 Calvo model

The Calvo sticky-price model [1] presented here has the same household optimization problem as before except that the budget constraint is now

$$P_t C_t + \frac{B_t}{R_t} \leq W_t N_t + D_t + B_{t-1} \tag{1}$$

Thus real wage first-order condition is given by

$$\frac{W_t}{P_t} = {C_t}^{\sigma} {N_t}^{\varphi} \tag{2}$$

Again assume $P_0 = 1$.
In the Calvo model, production function for an individual firm remains the same, but now price dispersion affects the final output. Production function will be changed to:

$$Y_t = s_t A_t {N_t}^{1-\alpha} \tag{3}$$

For the Calvo model, $0 < s_t \leq 1$, and no one value can be pre-determined for s_t without specified monetary policy.
Decreasing returns to scale is required for what follows - in constant returns to scale, the method presented below would fail, as will be seen.
I will assume $P_0 = 1$.
The household has utility function of

$$U(C, N) = \frac{C^{1-\sigma}}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi} \tag{4}$$

and tries to maximize intertemporal utility of

$$V = \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{5}$$

But intertemporal nature will not matter for this paper, and thus focus will be given to U, instead of V.
Here, C_u is the level that brings the maximum utility to the household given $C_u = s_t A_t {N_u}^{1-\alpha}$.
Now for simplification, suppose $\sigma \to 0$, $\varphi \to 0$. Technically, these values are greater than 0, but the equilibrium will not deviate too much from when one assumes $\sigma = 0$, $\varphi = 0$.
In such a case, the limiting utility function is given by

$$U(C_0, N_0) = C_0 - N_0 \tag{6}$$

Let $A_0 = 1$. Then, $N_0 = \left(\frac{C_0}{s_0}\right)^{\frac{1}{1-\alpha}}$. Let $s_0 \to 1$ for another simplification. Then $N_0 = C_0^{\frac{1}{1-\alpha}}$. Let us find out C_u by solving first-order condition of maximizing U (here, first-order condition will be OK).

$$C_u = (1-\alpha)^{\frac{1-\alpha}{\alpha}} \tag{7}$$

$$C_u = xW_u N_u = x{C_u}^{\sigma + \frac{1+\varphi}{1-\alpha}} = x{C_u}^{\frac{1}{1-\alpha}} \tag{8}$$

$$x = \frac{1}{1-\alpha} \tag{9}$$

$$N_u = (1-\alpha)^{\frac{1}{\alpha}} \tag{10}$$

Let $C_0 = kC_u$, and let $k' = k^{1/(1-\alpha)}$ Then,

$$D_0 = C_0 - W_0 N_0 = C_0 - {C_0}^{\sigma} {N_0}^{1+\varphi} = C_0 - N_0 = kC_u - k'{C_u}^{\frac{1}{1-\alpha}} \tag{11}$$

$$W_0 N_u = N_u = {C_u}^{\frac{1}{1-\alpha}} \tag{12}$$

$$W_0 N_u + D_0 = kC_u - k'{C_u}^{\frac{1}{1-\alpha}} + {C_u}^{\frac{1}{1-\alpha}} \tag{13}$$

Suppose $k \approx 0$ (I will simply set it as $k = 0$), and let $0 < q < 1$. It is wished that $qC_u \leq W_0 N_u + D_0$. Does this hold?

$$qC_u \leq kC_u - k'{C_u}^{\frac{1}{1-\alpha}} + {C_u}^{\frac{1}{1-\alpha}} \tag{14}$$

$$q \leq k - k'{C_u}^{\frac{\alpha}{1-\alpha}} + {C_u}^{\frac{\alpha}{1-\alpha}} \tag{15}$$

$$q - k \leq {C_u}^{\frac{\alpha}{1-\alpha}} - k'{C_u}^{\frac{\alpha}{1-\alpha}} \tag{16}$$

Substituting the C_u equation,

$$q - k \leq (1-\alpha)(1-k') \tag{17}$$

with $k = 0$

$$q \leq 1 - \alpha \tag{18}$$

Recall utility function:

$$T(C) = C - C^{\frac{1}{1-\alpha}} \tag{19}$$

Derivative of T at $0 < C < 1$ satisfies $0 < T'(C)$. Thus, Equation 18 proves that there exists a case where the equilibrium thought to be allowed by New Keynesian modelling is not actually an equilibrium.

2 Conclusion

This paper suggests that first-order conditions derived from optimization problems are not sufficient to find Calvo New Keynesian model equilibrium.

References

[1] Calvo, G. A. (1983). "Staggered prices in a utility-maximizing framework", Journal of Monetary Economics 12 (3): 383–398.