The perils of first-order conditions of New Keynesian models

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Abstract
This paper presents a case where first-order conditions used for the Calvo sticky-price New Keynesian model is insufficient and in fact results in calculation of a wrong equilibrium.

1 Calvo model

The Calvo sticky-price model [1] presented here has the same household optimization problem as before except that the budget constraint is now

\[ P_t C_t + \frac{B_t}{R_t} \leq W_t N_t + D_t + B_{t-1} \]  

Thus real wage first-order condition is given by

\[ \frac{W_t}{P_t} = C_t^\sigma N_t^{\sigma} \]  

Again assume \( P_0 = 1 \).

In the Calvo model, production function for an individual firm remains the same, but now price dispersion affects the final output. Production function will be changed to:

\[ Y_t = s_t A_t N_t^{1-\alpha} \]  

For the Calvo model, \( 0 < s_t \leq 1 \), and no one value can be pre-determined for \( s_t \) without specified monetary policy.

Decreasing returns to scale is required for what follows - in constant returns to scale, the method presented below would fail, as will be seen.

I will assume \( P_0 = 1 \).

The household has utility function of

\[ U(C, N) = \frac{C^{1-\sigma}}{1 - \sigma} - \frac{N^{1+\varphi}}{1 + \varphi} \]  

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and tries to maximize intertemporal utility of

\[ V = \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \]  (5)

But intertemporal nature will not matter for this paper, and thus focus will be given to \( U \), instead of \( V \).

Here, \( C_u \) is the level that brings the maximum utility to the household given \( C_u = s_t A_t N_u^{1-\alpha} \).

Now for simplification, suppose \( \sigma \to 0, \varphi \to 0 \). Technically, these values are greater than 0, but the equilibrium will not deviate too much from when one assumes \( \sigma = 0, \varphi = 0 \).

In such a case, the limiting utility function is given by

\[ U(C_0, N_0) = C_0 - N_0 \]  (6)

Let \( A_0 = 1 \). Then, \( N_0 = \left( \frac{C_0}{s_0} \right)^{\frac{1}{1-\alpha}} \). Let \( s_0 \to 1 \) for another simplification. Then \( N_0 = C_0^{\frac{1}{1-\alpha}} \). Let us find out \( C_u \) by solving first-order condition of maximizing \( U \) (here, first-order condition will be OK).

\[ C_u = (1 - \alpha)^{\frac{1}{1-\alpha}} \]  (7)

\[ C_u = xW_u N_u = xC_u^{\sigma + \frac{1-\sigma}{1-\alpha}} = xC_u^{\frac{1}{1-\alpha}} \]  (8)

\[ x = \frac{1}{1-\alpha} \]  (9)

\[ N_u = (1 - \alpha)^{\frac{1}{2}} \]  (10)

Let \( C_0 = kC_u \), and let \( k' = k^{1/(1-\alpha)} \). Then,

\[ D_0 = C_0 - W_0 N_0 = C_0 - C_0^\sigma N_0^{1+\sigma} = C_0 - N_0 = kC_u - k'C_u^{\frac{1}{1-\alpha}} \]  (11)

\[ W_0 N_u = N_u = C_u^{\frac{1}{1-\alpha}} \]  (12)

\[ W_0 N_u + D_0 = kC_u - k'C_u^{\frac{1}{1-\alpha}} + C_u^{\frac{1}{1-\alpha}} \]  (13)

Suppose \( k \approx 0 \) (I will simply set it as \( k = 0 \)), and let \( 0 < q < 1 \). It is wished that \( qC_u \leq W_0 N_u + D_0 \). Does this hold?

\[ qC_u \leq kC_u - k'C_u^{\frac{1}{1-\alpha}} + C_u^{\frac{1}{1-\alpha}} \]  (14)

\[ q \leq k - k'C_u^{\frac{1}{1-\alpha}} + C_u^{\frac{1}{1-\alpha}} \]  (15)

\[ q - k \leq C_u^{\frac{1}{1-\alpha}} - k'C_u^{\frac{1}{1-\alpha}} \]  (16)

Substituting the \( C_u \) equation,

\[ q - k \leq (1 - \alpha)(1 - k') \]  (17)
with $k = 0$

$$q \leq 1 - \alpha$$  \hspace{1cm} (18)

Recall utility function:

$$T(C) = C - C^{\frac{1}{1-\alpha}}$$  \hspace{1cm} (19)

Derivative of $T$ at $0 < C < 1$ satisfies $0 < T'(C)$. Thus, Equation 18 proves that there exists a case where the equilibrium thought to be allowed by New Keynesian modelling is not actually an equilibrium.

\section{Conclusion}

This paper suggests that first-order conditions derived from optimization problems are not sufficient to find Calvo New Keynesian model equilibrium.

\section*{References}