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Testing for Long Memory in ISE Using ARFIMA-FIGARCH Model and Structural Break Test

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Abstract

This study examines long memory in Istanbul Stock Exchange (ISE) by using the structural break test in variance and ARFIMA-FIGARCH model. Our findings indicate that long memory does not exist in the equity return; however, it exits in volatility. Consequently, ISE is found as a weak form inefficient market due to volatility as it has a predictable component.

I. Introduction

According to the efficient market hypothesis that Fama (1970) suggested all the information related with the financial assets affect their price and in regard to that information, price movements occur accordingly in the market. In an efficient market, the stock returns show a random walk causing it impossible to make a prediction from the past patterns. For this reason, many studies on the financial asset returns focus on if the returns have long memory features or not. If the stock returns have long-memory, then the previous returns will lead to make predictions on future returns making a weak form efficient market hypothesis to lose its validity.

In literature, there are a huge number of studies that investigates stock market efficiency in developed and developing countries, using long memory models.[1]

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There are some studies on ISE showing the same evidence that the index return indicates a long memory forming a weak form inefficient market.[2]

In this study, existence of long memory in ISE is examined using ARFIMA-GARCH model. Unlike other studies on ISE, presence of structural break in unconditional variance of index return series is examined with Iterated Cumulative Sums of Squares (ICSS) method.

II. Methodology

Multiple Structural Break Test in Unconditional Variance

Inclan and Tiao (1994) proposed a method that based on ICSS to detect structural breaks in the unconditional variance of a stochastic process. In order to test null hypothesis of constant unconditional variance against the alternative hypothesis of a break in the unconditional variance, Inclan and Tiao (1994) proposed to use the statistic given by

$$IT = \sqrt{T/2}D_k \qquad (1)$$

where $D_k = (C_k / C_T) - (k/T)$ and $C_k = \sum_{t=1}^{k} r_t^2$ be the cumulative sum of squares of a series of uncorrelated random variables with mean 0 and variance σ_t^2, $t = 1, 2, ..., T$. The value of k $(k = 1,\ldots, T)$ maximizes $\left|\sqrt{T/2}D_k\right|$ is the estimate of the break date. Under the variance homogeneity IT statistic behaves like a Brownian bridge asymptotically. At the %5 significance level, the critical value computed by Inclan and Tiao (1994) is $C_{0.05} = 1.358$.

The most serious drawback of the IT test statistic is designed for independently and identically distributed random variables. This is very strong assumption for

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[1] Examples for the study are Barkoulas et al. (2000), Resende and Teixeira (2002), Tolvi (2003), Caporale and Gil-Alana (2004), Cajueiro and Tabak (2006), Lux and Kaizoji (2007), McMillan and Thupayagale (2008), Blasco and Santamaria (1996), Vougas (2004), Gil-Alana (2006) .

[2] Balaban (1995 and 1996), Kılıç (2004), Christodoulou-Volos and Siokis (2006), Assaf (2007), DiSario et al. (2008).

financial data as most return series include conditional heteroskedasticity. For this purpose, Sanso et al. (2004) modified IT test statistic for conditional heteroskedasticity. Modified IT test statistic given by

$$\kappa_2 = \sup_k \left| T^{-1/2} G_k \right| \tag{2}$$

where $G_k = \hat{\omega}_4^{-1/2}\left(C_k - \frac{k}{T} C_T \right)$ and $\hat{\omega}_4$ is a consistent estimator of ω_4. Non-parametric estimator of ω_4,

$$\hat{\omega}_4 = \frac{1}{T}\sum_{t=1}^{T}\left(r_t^2 - \hat{\sigma}^2\right)^2 + \frac{2}{T}\sum_{l=1}^{m}\omega(l,m)\sum_{t=l+1}^{T}\left(r_t^2 - \hat{\sigma}^2\right)\left(r_{t-1}^2 - \hat{\sigma}^2\right) \tag{3}$$

where $\omega(l,m)$ is a lag window, such as the Barlett, defined as $\omega(l,m) = 1 - l/(m+1)$, or the quadratic spectral.

In the test procedure, if we are looking for only with the possible of single point change, then the D_k function would provide a satisfactory procedure. But when we are interested in finding multiple change points on an observed series the usefulness of the D_k function becomes questionable because of the masking effect. A solution is an iterative scheme based on successive application of D_k to pieces of the series, dividing consecutively after a possible change point is found[3] (Inclan and Tiao, 1994).

Long Memory Models

Fractional integration first appears in literature in the studies of Granger and Joyeux (1980) and Hosking (1981). The model, known as Autoregressive Fractionally Integrated Moving Average (ARFIMA), allows for increased flexibility in modeling low-frequency dynamics. ARFIMA model is written as follows;

$$\Phi(L)(1-L)^d Y_t = \Theta(L)\varepsilon_t,\ \varepsilon_t \sim (0, \sigma_\varepsilon^2) \tag{4}$$

[3] See Inclan and Tiao (1994) for ICSS procedure details.

where d is fractional integration parameter as a real number, L is lag operator and ε_t is white noise residual. Polynomial structures of Equation (4) lie outside the unit circle, satisfying the stationarity and invariability conditions. The fractional differencing lag operator $(1-L)^d$ is defined by the binomial expansion as follows.

$$(1-L)^d = 1 - dL + \frac{d(d-1)}{2!}L^2 - \frac{d(d-1)(d-2)}{3!}L^3 + ... \tag{5}$$

ARFIMA process is nonstationary when d ≥ 0.5. For 0 < d < 0.5, ARFIMA process is said to exhibit long memory. The process exhibits short memory for d = 0 and intermediate memory for d < 0.

Baillie et al. (1996) proposed fractional integrated GARCH (FIGARCH) model to determine long memory in return volatility. GARCH (p, q) model in ARMA for squared error form given by

$$[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{6}$$

where $v_t \equiv \varepsilon_t^2 - \sigma_t^2$. To satisfy the covariance stationary conditions, roots $[1-\alpha(L)-\beta(L)]$ and $[1-\beta(L)]$ lie outside the unit circle. FIGARCH model is derived from standard GARCH model with fractional difference operator, $(1-L)^d$. FIGARCH model is written as follows:

$$\phi(L)(1-L)^{\bar{d}}\varepsilon_t^2 = \omega + [1-\beta(L)]v_t \tag{7}$$

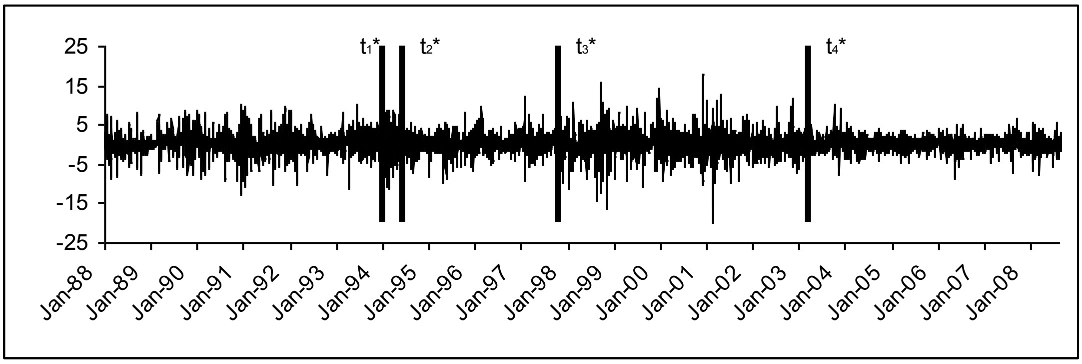
FIGARCH ($p, \bar{d}, q$) model is transformed standard GARCH when $\bar{d} = 0$ and IGARCH model when $\bar{d} = 1$.

III. Data and Empirical Results

In this study, the data set which is comprised of daily closing prices of ISE with respective transaction series from 1988 to 2008, totaling 5150 observations is used. The closing prices of ISE 100 index is collected from The Central Bank of the Republic of Turkey. The return series is adjusted by taking logarithmic difference as

$r_t = 100 \times \ln(p_t / p_{t-1})$. Figure 1 shows the ISE 100 index return series. The economic crisis in 1997 in Asia, in 1998 in Russia, in 2000 and 2001 in Turkey affect ISE causing its volatility to rise. After 2001 economic crisis, a stability programme has been applied and Turkey managed to achieve economic growth from 2003 through 2007. During that period ISE index return showed lower volatility.

Figure 1: ISE 100 Index Return



The summary statistics is given in Table 1. The daily average index return is positive and it is 0.168. Normality test statistics indicates that distribution of return series is significantly different from normality and in the form of leptokurtic. According to the ARCH test result, conditional variance of return series exhibit conditional heteroscedasticity properties.[4]

Table 1: Summary Statistics

Series	Mean	Std. Dev.	Skewness	Kurtosis	J-B Tests	ARCH (5)	Q (10)
Return	0.168	2.880	-0.053	6.252	2272 [0.000]	142.9 [0.000]	90.655 [0.000]

1) J-B indicates Jarque-Bera normality test, ARCH indicates LM conditional variance test, Q(.) indicates Box-Pierce serial correlation test.

To prove the existence of structural breaks in unconditional variance, κ_2 tests are used. We found 4 break points in variance of return series and the break point dates are determined as 31/12/1993, 17/06/1994, 24/10/1997 and 25/03/2003. While investigating the existence of the long memory as a priori, we assume that return series has constant unconditional variance. For this reason, investigating long memory with unstable unconditional variance will give deviated results. In this

[4] Phillips-Perron and KPSS unit root test provides that return series are stationary.

study, in order to make unconditional variance stable the method those Noira et al. (2004) is applied.

$\mathrm{var}(r_t) = \sigma_t^2$ will be estimated over each of the intervals limited by the regime-shift points of the unconditional variance as given below:

$$\hat{\sigma}_t^2 = \sum_{i=0}^{2} \hat{\sigma}_i^2 \left[I\left(t_i^* \le t \le t_{i+1}^*\right) \right] \quad (\text{t} = 1,\ldots,\text{n and } \text{i} = 0, 1,\ldots,4) \tag{8}$$

where $\hat{\sigma}_i^2 = \left(1/\left(t_{i+1}^* - t_i^*\right)\right) \sum_{t=t^*}^{\left(t_{i+1}^*\right)-1} r_t^2$, $t_0^* = 1$, $t_1^* = 1498$, $t_2^* = 1612$, $t_3^* = 2454$, $t_4^* = 3784$, $t_5^* = 5150$ and $I(.)$ is the indicator function that takes the value 1 if the argument is true, and 0 otherwise. To enable unconditional variance stable, return series are filtered as $r_t^f = r_t / \hat{\sigma}_t \quad (\text{t} = 1,\ldots,\text{n})$. As a result, the return series is filtered and the existence of long memory is calculated according to the filtered series.[5]

In Table 2, the GARCH model results are given. The ARMA model is determined with Box-Jenkins approach and Akaike information criteria. The sum of the GARCH parameters' for non-filtered return series is 0.983, and for filtered return series is 0.928 indicating the existence of the long memory in volatility. Structural breaks cause volatility to increase more than it is to be expected.

Table 2: GARCH Model Results

	r	***r^f***
Constant (mean)	0.159 [0.000]	0.060 [0.000]
AR(1)	0.100 [0.000]	0.100 [0.000]
Constant (variance)	0.213 [0.001]	0.077 [0.000]
$\boldsymbol{\alpha}$	0.150 [0.000]	0.159 [0.000]
$\boldsymbol{\beta}$	0.832 [0.000]	0.769 [0.000]
df	7.407 [0.000]	8.345 [0.000]
$\boldsymbol{\alpha + \beta}$	0.983	0.928
$\boldsymbol{Q_1(5)}$	23.987	15.743
$\boldsymbol{Q_2(5)}$	7.448	5.221

1) r and r^f indicates non-filtered and filtered return series respectively, *df* indicates student t distribution parameter, $Q_1(.)$ indicates Box-Pierce test for standardized residuals, $Q_2(.)$ indicates Box-Pierce test for squared standardized residuals, [.] indicates p values.

The presence of long memory in volatility is investigated by the FIGARCH model and the results are given in Table 3. Fractional integration parameter, d, for

[5] The existence of the structural break is re-investigated for filtered return series and break is not determined.

non-filtered return series is 0.394, for filtered series is 0.345 and it is found significantly different from zero. According to these results, volatility of index return series has long memory and it is a predictable structure. Also fractional integration parameter is obtained smaller for filtered return series supports that long-range dependence may be due to the presence of structural breaks.

Table 3: FIGARCH Model Results

	r	r^f
Constant (mean)	0.162 [0.000]	0.060 [0.000]
AR(1)	0.101 [0.000]	0.102 [0.000]
Constant (variance)	0.389 [0.001]	0.093 [0.004]
$\tilde{d}$	0.394 [0.000]	0.345 [0.000]
α	0.076 [0.530]	0.009 [0.953]
β	0.291 [0.037]	0.185 [0.326]
df	8.114 [0.000]	8.450 [0.000]
$Q_1(5)$	20.848	16.647
$Q_2(5)$	5.080	4.751

1) r and r^f indicates non-filtered and filtered return series respectively, *df* indicates student t distribution parameter, $Q_1(.)$ indicates Box-Pierce test for standardized residuals, $Q_2(.)$ indicates Box-Pierce test for squared standardized residuals, [.] indicates p values.

Finally, the existence of long memory in the mean of process is investigated by ARFIMA-FIGARCH model and the results can be seen on Table 4. Fractional integration parameter is obtained 0.035 and 0.037 respectively for non-filtered and filtered return series and is not significantly different from zero at %5 level. This result indicates return exhibit short memory properties and cannot be estimated in the long run. In volatility equation, long memory properties are also significantly detected.

Table 4: ARFIMA-FIGARCH Model Results

	r	r^f
Constant (mean)	0.164 [0.000]	0.059 [0.000]
d	0.035 [0.069]	0.037 [0.050]
AR(1)	0.064 [0.004]	0.063 [0.005]
Constant (variance)	0.395 [0.001]	0.094 [0.003]
$\tilde{d}$	0.393 [0.000]	0.346 [0.000]
α	0.069 [0.566]	0.006 [0.969]
β	0.284 [0.043]	0.181 [0.327]
df	8.092 [0.000]	8.396 [0.000]
$Q_1(5)$	11.946	8.269
$Q_2(5)$	5.016	4.688

1) r and r^f indicates non-filtered and filtered return series respectively, *df* indicates student t distribution parameter, $Q_1(.)$ indicates Box-Pierce test for standardized residuals, $Q_2(.)$ indicates Box-Pierce test for squared standardized residuals, [.] indicates p values.

IV. Conclusion

In this study, the weak form market efficiency of ISE 100 index is examined by using the structural break test in variance and ARFIMA-FIGARCH model. Structural break test provides 4 break points on the return series. Related to the break dates, the return series is filtered and investigated whether the return and volatility carry long memory features or not. The results indicate that on the return, the existence of the long memory is not perceived but on volatility, there is long memory existence. According to this result, volatility has a predictable structure and it indicates that ISE is not a weak form efficient market. Furthermore, smaller number is generated from fractional integration parameter on the filtered series (for volatility equation) supporting the reason of spurious long memory. Finally, structural breaks cause volatility to increase more than it is to be expected.

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