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Abstract

This paper presents a theoretical investigation into why losing weight is so difficult even in the absence of rational addiction, time-inconsistent preferences or bounded rationality. We add to the existing literature by focusing on the role that individual metabolism and physical activity have on weight loss. The results from the theoretical model provides multiple steady states and a threshold revealing a situation of "obesity traps" that the individual must surpass in order to successfully lose weight. Any weight-loss efforts that the individual undertakes have to surpass this threshold in order to result in permanent weight loss, otherwise the individual will gradually regain weight and converge to his or her previous body weight. In addition to this we study how price policies affect individual behavior. We show that food taxes, in the long run, increases body weight, even if food consumption decreases; while price policies aimed at promoting physical activity are able to sustain healthier lifestyle, fitness accumulation and a decreases in body weight.

Keywords: Obesity, Dieting, Optimal Control, Multiple Equilibria.
JEL-Classification: D91, I12, I18

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1 Introduction

Over the last decade the incidence of obesity has increased dramatically (Ogden and Carroll, 2010, von Ruesten et al., 2011), leading to greater concerns about the promotion of efficient ways to invert this trend by means of dieting (Baradel et al., 2009, Bish et al., 2012) or physical activity (Mozaffarian et al., 2011). However, although attempts at losing weight may have increased, more than 50% of obese individuals still struggle to lose weight permanently (Kruger et al., 2004). This poses the questions of why weight loss is so difficult to achieve and what factors influence it. Current literature relates difficulties in losing weight to several possible explanations such as: time-inconsistent preferences (Dodd, 2008, Ikeda et al., 2010); willpower depletion (Ozdenoren et al., 2012); the interaction between two conflicting systems driving human behavior (cognitive vs. affective) (Loewenstein and O'Donoghue, 2004, Ruhm, 2012); or, more generally, to the failure of individuals to rationally balance current benefits and future costs related to food consumption and body weight (O'Donoghue and Rabin, 1999).

In this paper we add to existing literature by proposing another, complementary, explanation for why dieting is so difficult, even in the absence of rational addiction, time-inconsistent preferences or bounded rationality. To do so we develop an original assumption regarding individual metabolism. Current economic literature considers individual metabolism to be linearly related to body weight, such that the more an individual weighs the higher his or her caloric expenditure will be (Dragone, 2009, Levy, 2002, Harris and Benedict, 1918). The key assumption is that calories expended by the metabolism in order to keep the basic functions of the organism running (i.e. basal metabolic rate), is a linear and increasing function of body weight; meaning that of two individuals of a similar age, the one with a higher body weight (i.e. the one with the greater "weight stock") will burn a higher fraction of it to keep his organism running. This assumption entails that the marginal effect of an additional kg on an individual caloric expenditure is always positive and that the caloric expenditure of obese/overweight individuals is therefore higher than that of leaner individuals, simply because they weigh more.

This notion of body weight accumulation and energy expenditure contradicts recent insights from medical literature, according to which metabolism actively and non-monotonically reacts to changes in body weight (Catenacci et al., 2011, Ebbeling et al., 2012, Gale et al., 2004, Katan and Ludwig, 2010, Leibel and Hirsch, 1984, Leibel et al., 1995), such that obese/overweight individuals do not burn more calories simply because they weigh more (Johnstone et al., 2005, Cunningham, 1991, Astrup et al., 1999). For the same level of physical activity and food intake, the higher adipose content in the body composition of obese/overweight individuals contributes in reducing their caloric expenditure rather than increasing it, since body fat negatively affects individual's caloric expenditure. Mifflin et al. (1990), Cunningham (1991), Wang et al. (2000) report that the main predictor for total energy expenditure is the level of non-adipose mass that the individual possesses and that fat mass is not one of its main predictors. Moreover, Johnstone et al. (2005) quantify that fat mass accounts for less than 6% of basal metabolic rate variability, while fat-free mass accounts for almost 70%. Based on this evidence we can conclude that given the minimal incidence that fat mass has on an individual's caloric expenditure and the fact that obese/overweight individuals have a positive imbalance of fat mass over fat-free mass, the relationship between body weight and caloric expenditure is non-linear (Thomas et al., 2009, 2011). This means that of two individuals of the same age, the one who weighs more (i.e. the one with a higher adipose content) is predicted to burn fewer calories with respect to the leaner individual. In order to capture this dynamic we will assume that there exists an individual specific level of body weight over which any additional weight gain will decrease caloric expenditure; conversely any weight gain lower than this threshold will increase caloric expenditure.

In addition to this, we also investigate the role of physical activity on body weight accumulation, expanding the current literature which treats body weight outcomes of an individual as resulting only from food consumption (Dragone, 2009, Levy, 2002). Instead, we consider food consumption and physical activity not as two isolated choices, but as two simultaneous decisions. Such instantaneous interconnection is crucial because evidence shows that dietary and lifestyle choices are not only independently associated with long-term weight changes, but also that there exists a substantial aggregate effect resulting from simultaneous decision making between the two (Mozaffarian et al., 2011, Wadden et al., 2011). When dietary and exercise recommendation are coupled together weight-loss attempts are proven to be more pronounced and long-term stable (Wadden et al., 2011), proving that examining body weight accumulation decision neglecting the role of physical activity could lead to partial results. In spite of this, there are only few theoretical approaches that treat the interconnection between eating- and physical-activity choices, and explore how these interact with external economic factors (Yaniv et al., 2009, Dragone et al., 2015).

This paper presents a theoretical investigation of why losing weight is so difficult, focusing on metabolism and physical activity, and which price policy is apt to sustain persistent weight loss. The results from the theoretical model using both the novel metabolic assumption and physical activity as a choice variable are threefold. Firstly, we extend the rational eating literature to show the impact that metabolism and physical activity can have on the steady state decision of a rational individual. Unlike current literature, in which the individual exhibits only a rational and stable outcome of overweight (Dragone, 2009, Levy, 2002), we allow for multiple equilibria and report one characterized by a healthier body weight. Second, this multiplicity of steady states provides the evidence of an "obesity trap" with an associated threshold dividing the two possible outcomes. Intuitively, any weight-loss efforts that the individual undertakes have to surpass this threshold in order to result in permanent weight loss, otherwise the individual will gradually regain weight and converge to his or her previous body weight. This threshold provides the rationale for any unsuccessful dieting efforts even in the absence of rational addiction, time-inconsistent preferences or bounded rationality. Therefore the presence of such a threshold does indeed have an influence on the possible persistence of dieting efforts. Lastly we analyze the impact of price policies on individual behavior, showing that food price policies even if they are able to decrease food consumption lead to an increases in body weight due to a reduction in physical activity. Conversely thin subsidies, in the form of a reduction in the price of physical activity, are able to sustain long-run weight losses and healthier lifestyle (i.e. an increase in physical activity). Hence the policy maker should try to stimulate individual to exercises more, rather than taxing food, in order to sustain persistence long-run weight losses.

2 The Model

2.1 The Utility Function

Consider an individual whose intertemporal utility depends on the current consumption c, physical activity e and the consumption of a composite good q, as well as on body weight w and stock of physical fitness s related to physical exercises. We will consider the following quasi-linear specification for the utility function[1]:

$$U = U(c(t), e(t), w(t), s(t)) + q(t) \tag{1}$$

The utility function is assumed to be jointly concave (i.e. $U_{cc} < 0$, $U_{ee} < 0$, $U_{ww} < 0$ and $U_{ss} < 0$) and separable in all its elements (i.e. $U_{ce} = U_{cw} = ... = U_{se} = 0$).

Consuming too little or too much amount of food could creates health problems lead to health problems (e.g. anorexia, bulimia), in particular in combination with too little physical exercise. This holds also for physical exercise, a small value of e could harm the individual by increasing body atrophies and that the risks of several diseases[2]; on the other hand, much or too intensive physical exercise (high level of e) increases the risk of injuries and body decay (Morton et al., 2009, Howatson and Van Someren, 2008). For these reasons, we assume that there exists a given level c^* and e^* such that $U_c = 0$ and $U_e = 0$, and the individual is consuming both c and e in optimal level, with no disutility whatsoever. Conversely, if $U_c \neq 0$ $U_e \neq 0$ the individual is underconsuming (overconsuming) the two goods with respect to such optimal level. Following Dragone and Savorelli (2012) we say the agent is underconsuming if $U_c > 0$ (utility would be increased by increasing food consumption) and is over-consuming if $U_c < 0$ (utility would be increased by decreasing food consumption). With respect to physical exercise we say that the individual is overexercising (underexercising) is $U_e < 0$ ($U_e > 0$), with respect to e^* which is the level of physical exercise such that the individual is maximizing his instantaneous utility without suffering from any immediate physical discomfort.

Regarding body weight we assume that there exists a BMI that maximizes each agent's health condition; denote the corresponding body weight as w^H, such that $\partial U(w^H)/\partial w = 0$. We say the agent is overweight ($w > w^H$) if $U_w < 0$ (utility would be increased by decreasing body weight) and conversely underweight ($w < w^H$) if $U_w > 0$ (utility would be increased by increasing body weight). It is possible to distinguish between three possible cases. If $w^{sat} > w^H$, satiation induces being overweight; likewise we say that satiation induces one to be underweight if $w^H > w^{sat}$. Lastly if $w^{sat} = w^H$ satiation corresponds to healthy weight.

Physical fitness, s, is a different concepts with respect to physical activity and it is associated with the ability to perform daily tasks without excessive fatigue (Caspersen et al., 1985) and such fitness is supposed to be crated via physical activity consumption. We will assume that there is no maximum point of physical fitness beyond which individual will suffer from any disutility, since physical fitness accumulation is always supposed to produce positive utility. Therefore, since the consumption of e is suppose to generate positive

[1] This assumption is made for expositional convenience, as it implies that all income effects are captured by changes in the demand for the composite good q and that the law of demand holds both in the short run and in the long run.

[2] Including coronary heart disease, hypertension, stroke, diabetes, depression, osteoporosis, and cancers of the breast and colon (Garrett et al., 2004).

physical fitness, s, we assume that $U_s > 0$ and $U_{ss} < 0$.

Finally, we assume that the individual receives a fixed income, M, at each point in time and that there are no financial markets. Thus, the individual's spending equals M at each point in time. Formally, we have

$$M = p_c c + p_e e + q$$

where p_c and p_e are the market prices of food and exercise, while the price of the composite good is normalized to one.

2.2 The Dynamics of Individuals Body Weight and Physical Fitness

The determinants of individual's body weight are given by food consumption, $c(t)$, physical activity $e(t)$, and current body weight, $w(t)$ which will influence the accumulation of body weight in time as follows

$$\dot{w}(t) = c(t) - g(w(t)) - \alpha e(t) \tag{2}$$

Energy intake is linearly an positively related to the amount of food consumption, $c(t)$ while we assume that energy expenses, via physical activity, equals fixed shares, α, of total physical exercises (Westerterp, 2008). Additionally energy expenditure is also related to body weight, and especially to basal metabolic rate, via the $g(.)$ function, which is assumed to be twice continuously-differentiable, positive and concave with a maximum in $g(\bar{w})$.

$g(w(t))$ is a concave function determining the amount of energy expended per unit of body weight, which in medical terms is defined as the basal metabolic rate (Harris and Benedict, 1918), that is, the amount of energy expended by individuals at rest. We assume that the contribution of food consumption to body weight is positive and linear. Additionally, the contribution of body weight to the rate of caloric expenditure is assumed to be increasing (i.e. $g_w \geq 0$) for $w \in [0, \bar{w}]$ and decreasing afterwards, with $g_w < 0$. The idea is the following: the more an individual eats the more weight he will gain; the more weight he gains, the more calories he will burn until he reaches an individual specific body weight, $\bar{w}$, where the effect of additional body weight will actually decrease this basal metabolic rate. This explanation is related to the fact that the main predictor of basal metabolic rate is individual's lean mass (Johnstone et al., 2005, Mifflin et al., 1990, Wang et al., 2000), which is supposed to be lower (in proportion) in heavier individual. Therefore $\bar{w}$ can be thought of as the maximum sustainable level of body weight above which the organism will store additional calories in fat tissue and not in lean mass, resulting then in a net decrease in caloric expenditure since fat mass burns at a lower rate with respect to lean mass.

We will assume that as long as any additional weight gain is resulting in an increases in caloric expenditure, such that $g_w \geq 0$, the individual can be considered has having a positive imbalance between fat free mass and fat mass. Conversely as soon as the individual surpasses $\bar{w}$, resulting in a decreases in caloric expenditure associated with an increases in body weight, $g_w < 0$, the individual can most certainly be considered overweight or obese, since the percentage of fat mass will be strictly higher than the percentage of fat-free mass.

Regarding fitness we have the following equation describing fitness accumulation

$$\dot{s}(t) = e(t) - \delta s(t) \tag{3}$$

Physical exercises is supposed to increased fitness via several components. Caspersen et al. (1985) divide these components in two groups: one related to health and the other related to skills that pertain more to athletic ability [3]. Physical fitness is also assumed to depreciate with time (i.e. aging) at a constant rate $\sigma > 0$.

3 Optimality conditions and stability

The individual's goal is to maximize his intertemporal utility by choosing the amount of food consumption and physical activity, which in turn will affect her body weight and physical fitness. Given an infinite time horizon and a positive discount rate, ρ, the individual's problem can be written as follows (See Appendix A.1):

$$\max_{c(t),e(t)} E\left[\int_0^T e^{-\tilde{\rho}t}[U(c(t), e(t), w(t), s(t))]dt\right] \tag{4}$$

$$\text{subject to } \dot{w}(t) = c(t) - g(w(t)) - \alpha s(t) \tag{5}$$

$$\dot{s}(t) = e(t) - \delta s(t) \tag{6}$$

$$M = p_c c + p_e e + q \tag{7}$$

$$w(t), c(t), e(t), s(t) \geq 0 \tag{8}$$

$$w(0) = w_0, \text{ given} \tag{9}$$

$$s(0) = s_0, \text{ given} \tag{10}$$

Here we have introduced the non-negativity constraint on c, e, s and w since they represent, respectively, food consumption, physical activity and fitness and body weight which, by definition, cannot become negative. The associated current value Hamiltonian (dropping the time indexes for convenience and substituting the budget constraint) is

$$H(c, e, w, s, \lambda, \mu) = U(c, e, w, s) + M - p_c c - p_e e + \lambda(c - g(w) - \alpha e) + \mu(e - \delta s) \tag{11}$$

3.1 Obesity Traps

Given joint concavity, the first-order conditions for this problem are

[3] Caspersen et al. (1985), moreover, collected such components in the following categories: (a) cardio-respiratory endurance, (b) muscular endurance, (c) muscular strength, (d) body composition, and (e) flexibility.

$$
\begin{aligned}
&H_c = 0 \qquad U_c - p_c + \lambda = 0 && \text{(12a)}\\
&H_e = 0 \qquad U_e - p_c + \alpha\lambda + \mu = 0 && \text{(12b)}\\
&\dot{\lambda} = \lambda(\rho + g_w) - U_w && \text{(12c)}\\
&\dot{\mu} = \mu(\rho + \delta) - U_w && \text{(12d)}\\
&\dot{w} = c - g(w) - \alpha e && \text{(12e)}\\
&\dot{s} = e - \delta s && \text{(12f)}
\end{aligned}
$$

If we time-differentiate equation (12a)-(12b) and substitute it in (12c)-(12d) along with (12a)-(12b) we derive an expressions for $\dot{c}$ and $\dot{w}$ which, along with (12e)-(12f) results in the following system of differential equations characterizing the equilibrium

$$
\begin{aligned}
\dot{c} &= \frac{(U_c - p_c)(g_w + \rho) + U_w}{U_{cc}}\\
\dot{e} &= \frac{(\delta + \rho)(\alpha(U_c - p_c) + (U_e - p_e)) - \alpha((U_c - p_c)(g_w + \rho) + U_w) + U_s}{U_{ee}}\\
\dot{w} &= c - g(w) - \alpha e\\
\dot{s} &= e - \delta s
\end{aligned}
\tag{13}
$$

For an internal steady state the following conditions must be satisfied

$$
\begin{aligned}
\dot{c} &= 0 \qquad U_w = -(U_c - p_c)(\rho + g_w) && (14)\\
\dot{w} &= 0 \qquad c^{ss} = g(w^{ss}) && (15)\\
\dot{e} &= 0 \qquad \alpha(U_c - p_c) = -(U_e - p_e) && (16)\\
\dot{s} &= 0 \qquad e^{ss} = \delta s^{ss} && (17)
\end{aligned}
$$

It can be showed that, under special conditions[4], the outcome of the steady state can be associated with an individual with optimal weight. More generally we derive the following trade-offs between food consumption, exercise and body-weight (according to equations (14), (17) and (15))

(a) Being overweight, underconsuming, overexercising;

(b) Being underweight, overconsuming, overexercising ($U_s < U_w$);

(c) Being overweight, underconsuming, underexercising (p_e high);

[4] In this special steady state, body weight is at its optimal ($w^{ss} = w^H$), so $U_w = 0$, and the individual is exercising and consuming food an optimal level to keep w^H constant, according to $\delta w^H = c^H - \alpha e^H$, which will result in a corresponding (optimal) level of fitness accumulation, $\delta s^H = e^H$.

(d) Being underweight, underconsuming, overexercising (p_c high and $U_s < U_w$).

Overexercising is optimal for for overweight individuals (case (a)), such that he is already underconsuming food but try to maintain his body weight under control by overexercising. Overexercising is also optimal for underweight individuals, if the marginal utility of an additional unit of fitness is higher than an additional unit of body weight (i.e. fitness concerns), so they overexercise in order to be even fitter that their current situation. Outcome (c) is associated with a very high price for physical activity so that individual might restrain to consume physical exercise. This creates a decreases in food consumption for already overweight individuals in order to contain their body weight by only dieting. Outcome (d) assume a very high price of food and a very high valuation of own physical fitness over body weight, due to the high price of food the individual cannot overexercise and overconsumes at the same time (like outcome (b)) so he will just overexercise due to a his his fitness concerns (i.e. addicting fitness).

In order to establish the multiplicity of steady states which will lead to the possibility of "obesity trap", we need to prove that our system of differential equations (13) has more than one possible solutions. Such multiplicity boils down to the following instability condition, based upon the analysis on the Jacobian matrix (Appendix B.1)

Proposition 3.1. *The intertemporal problem (4)-(10) is associated with multiplicity of steady states characterized by the sufficient condition for an unstable threshold*

$$g_w(g_w + \rho)U_{cc}(\delta(\delta + \rho)U_{ee} + U_{ss}) < -\delta(\delta + \rho)(\alpha^2 U_{cc} + U_{ee})(H_{ww}) + U_{ss}(1 + U_{ww})] \tag{18}$$

(See Appendix B.1)

From condition (18) we can also derive an additional (sufficient) condition for multiple steady states. In addition to the unstable threshold in (18) we can also see that the equation

$$g_w(g_w + \rho)U_{cc}(\delta(\delta + \rho)U_{ee} + U_{ss}) + \delta(\delta + \rho)(\alpha^2 U_{cc} + U_{ee})(H_{ww}) + U_{ss}(1 + U_{ww})]$$

which is the determinant of the Jacobian associated with the intertemporal problem (4)-(10) (See Appendix B.1) can be positive[5] in two different cases, thus allowing two possible stable steady states. The firs one is when $g_w > 0$, while the second one is for $g_w < 0$. This characterization of steady states based on the sign of the derivative of the $g(.)$ function allows us to say that our model permits two stable steady states, one healthier than the other, since $g_w > 0$ is associated, by definition, to a condition of healthy weight, while $g_w < 0$ is associated with overweight/obesity. More generally we will assume that there will be at most three steady states, divided by the (overweight) threshold defined in equation (18).

Corollary 3.1 (Obesity Traps)**.** *Of the two possible steady states associated with the intertemporal problem*

[5]In an optimal control problem with two state and two control variables a sufficient condition for a steady state to be stable is $|J| > 0$ (Dockner, 1985)

(4)-(10), one is characterized by a healthy body weight (i.e. $g_w > 0$) while the other one is characterized by a condition of overweight/obesity (i.e. $g_w < 0$).

Proposition 3.1 allows us to state that if there are indeed multiple steady states, and condition (18) is therefore satisfied, instability becomes an issue and there exists a threshold such that the individual is trapped in an excessive weight steady state. This result characterize a situation in which there is body resistance to weight-loss that the individual has to surpass in order to permanently lose weight, which can be reconciled with the concept of "homeostasis" reported in medical evidence (Ebbeling et al., 2012, Gale et al., 2004, Katan and Ludwig, 2010, Leibel and Hirsch, 1984, Leibel et al., 1995). Such that the organism contrast dieting effort since losing weight results in a decrease in energy available and in order to keep the organism in a stable energy state metabolism contrast weight-losses.

The multiplicity of steady states gives us a rationale for why, in general, dieting efforts are so difficult and why weight loss is not always guaranteed after a diet is finished. The reason behind this is the presence of a weight threshold above which any weight loss will be permanent. This means that if dieting does not enable the individual to surpass this threshold he or she will start to regain weight once the diet is finished and will then slowly converge back to the previous weight, nullifying all dieting efforts.

4 The Effect of Prices Policies on Individual Behaviour

4.1 Increasing the price of food

The change in the steady state level of physical activity due to a change in the price of food is given by the following expression (See Appendix C.1).

$$\frac{\partial e^{ss}}{\partial p_c} = \frac{\Phi\alpha\delta(\delta+\rho)(H_{ww})}{|J|} < 0 \tag{19}$$

with $\Phi = 1/U_{ww}U_{cc} > 0$.

Similarly we can compute the change in steady state food consumption and body weight (See Appendix C.1)

$$\frac{\partial c^{ss}}{\partial p_c} = \frac{\Phi[\alpha^2\delta(\delta+\rho)(H_{ww}) + g_w[\delta(\delta+\rho)U_{ee} + U_{ss}](g_w+\rho)]}{|J|} < 0 \tag{20}$$

$$\frac{\partial w^{ss}}{\partial p_c} = \frac{\Phi(g_w+\rho)(\delta(\delta+\rho)U_{ee} + U_{ss})}{|J|} > 0 \tag{21}$$

Proposition 4.1. *When the price of food increases, physical activity and food consumption decreases, while body weight increases.*

After an increases in the price of food, after an introduction of food-taxes, there is a decrease in physical activity, since food consumption is more costly individuals will tend to consume less calories decreasing their necessity to engage in physical activity. However decreased food consumption has a downside, the net result between reducing food intake and physical activity is an increases in individual body weight. In fact the marginal effect of p_c on e (equation 19) is higher than the marginal effect on c (equation 20). This means that price policies aimed at increasing the price of food will be able to reduce food consumption although they will also decreases physical activity resulting in a net increases in body weight.

Corollary 4.1. *In the long-run food price policies are unable to sustain body weight reduction.*

Food price policies will indeed be able to decreases food consumption but my making so they will also result in a net increases in body weight resulting from decreasing physical activity. Policy concerns, when taxing food, are aimed at not only reducing food consumption but also at reducing body-weight by possibility increases healthier lifestyle choices, however the introduction of food taxes are unable to obtain such desired outcome.

4.2 Increasing the price of physical activity

The change in the steady state level of physical activity due to a change in the price of food is given by the following expression (See Appendix C.2).

$$\frac{\partial e^{ss}}{\partial p_e} = \frac{\Phi\delta(\delta+\rho)(H_{ww} + g_w(g_w+\rho)U_{cc})}{|J|} < 0 \tag{22}$$

Proposition 4.2. *When the price of physical activity increases, physical activity decreases*

Similarly we can compute the change in steady state food consumption and body weight (See Appendix C.2)

$$\frac{\partial c^{ss}}{\partial p_e} = \frac{\Phi\alpha\delta(\delta+\rho)(H_{ww})}{|J|} < 0 \tag{23}$$

$$\frac{\partial w^{ss}}{\partial p_e} = -\frac{\Phi\alpha\delta(\delta+\rho)(g_w+\rho)}{|J|} > 0 \tag{24}$$

Proposition 4.3. *An increases in the price of physical activity, will decrease food consumption and increase body weight.*

Food consumption decreases after p_e increases since the individual is consuming less calories due to a decreases in his physical exercise. However, a decreases in physical activity associated with and increases in its price will also increases individual's body weight due to a decreases incidence of physical activity in reducing body weight accumulation. In fact the marginal effect of p_e on e (equation 22) is higher than the marginal effect on c (equation 23).

Corollary 4.2. *In the long run thin subsidies, in the form of a decrease in the price of physical exercise, are always able to increases physical activity and decrease body weight*

Price policies aimed at promoting physical activity, via this subsidies, are shown to be more efficient than food-taxes in order to sustain healthier lifestyle, fitness accumulation and a decreases in body weight. In fact. differently from food taxes individual will not be sustained in their dieting effort alone (i.e. a decrease in c) but they will also being motivated in engaging in more physical activity, resulting in a more than proportional result for a single price policy. Thus policy concerns should be devoted in making individual physical activity effort less expensive and possibly more salient so to maximize the results of a single price policy; or when pricing food coupling such tool with a thin subsidies in order to avoid the unexpected result of a decreased physical effort due to the introduction of food taxes.

5 Conclusion

Given the increasing incidence of obesity, the promotion of efficient ways to invert this trend is a major health concern. A reduction in caloric intake or an increase in caloric expenditure via physical activity, are the two main strategies normally prescribed. Although there has been an increase in weight-loss attempts using both of these recommended strategies, their beneficial effects are yet to be seen. Due to unsuccessful attempts and in order to facilitate weight-loss efforts, recent insights from behavioral economics have been used to promote the reduction of excessive weight by motivating individuals to follow a type of behavior that they would not naturally follow by means of monetary and non-monetary incentives (Cawley and Price, 2013, John et al., 2011, Volpp et al., 2008). Although the results seem to support the ability of the scheme to promote weight loss during the time of the experimentation, few of the experiments were able to sustain a long-lasting effect and almost all weight was regained post-experiment. This poses the question of understanding why weight loss is so difficult to achieve and what factors influence weight loss.

In this paper we have presented a theoretical model of rational eating (Dragone, 2009, Levy, 2002) with an original assumption regarding individual metabolism and including physical activity as a choice variable, in order to investigate why weight-loss attempts are so difficult to achieve. The novelty of the model lies in the presence of a non-monotonic relationship between caloric expenditure and body weight, resulting in lower caloric expenditure for overweight/obese individuals. The theoretical analysis helps us to understand why losing weight is so difficult without relying on rational addiction, time-inconsistent preferences or bounded rationality and, most importantly, the conditions under which an incentive for weight loss should be designed such that the individual is able to permanently lose weight. Multiple steady states and a threshold characterizing a situation of "obesity traps", which the individual must try to surpass by means of dieting, are derived. In addition to this we showed that food price policies will actually increases individual body weight, even if food consumption decreases, due to an associated decrease in physical activity. On the contrary, thin subsidies, in the form of a decrease in the price of physical activity, will sustain weight-losses via an increased in physical exercise, in the long-run.

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A Appendix A.

A.1 The intertemporal problem (4) - (10)

In this appendix we will prove that the intertemporal problem (4) - (10), with an infinite time horizon and constant discount rate, is an approximation of a similar model with stochastic time of death.

A.1.1 The Model with stochastic time of death

In order to take into account that individual's longevity is stochastic and impossible to known before time of death, we assume that the agent's life is finite but with an uncertain terminal date, T, (Yaari, 1965). $F(T)$ represents the probability o dying at time T and $f(T)$ is the associated density function. Given initial body weight w_0, and physical fitness s_0 the agent must choose the path of food consumption satisfying the following intertemporal problem

$$\max_{c(t),e(t)} E\left[\int_0^T e^{-\tilde{\rho}t}[U(c(t), e(t), w(t), s(t))]dt\right] \tag{25}$$

$$\text{subject to } \dot{w}(t) = c(t) - g(w(t)) - \alpha s(t) \tag{26}$$

$$\dot{s}(t) = e(t) - \delta s(t) \tag{27}$$

$$M = p_c c + p_e e + q \tag{28}$$

$$w(t), c(t), e(t), s(t) \geq 0 \tag{29}$$

$$w(0) = w_0, \text{ given} \tag{30}$$

$$s(0) = s_0, \text{ given} \tag{31}$$

where $\tilde{\rho} > 0$ is the intertemporal discount rate representing the agent's impatience. The objective function (25) , differently from (4) represents the expected lifetime utility function of an agent with stochastic terminal time. Following Dragone et al. (2015), Yaari (1965) we can exploit a very useful result in order to prove the equivalency of the two. Equation (25) can be equivalently represented in terms of the following objective function

$$\int_0^T [1 - F(t)]e^{-\tilde{\rho}t}U(.)dt$$

where $1-F(t)$ is the probability of living beyond t (Yaari, 1965, Levy, 2002). In order to prove the equivalency of (25) and (4) we will focus on the special case where $f(T) = \hat{\rho}e^{-\hat{\rho}t}$ so that the density function is exponential. Under this assumption the expected intertemporal utility of the agent can be equivalently written as follows

$$E\left[\int_0^T e^{-\tilde{\rho}t}U(.)dt\right] = \int_0^T e^{-\hat{\rho}t}e^{-\tilde{\rho}t}U(.)dt = \int_0^T e^{-\rho t}U(.)dt \tag{32}$$

where $\rho = \hat{\rho} + \tilde{\rho}$ and it is the overall discount rate depending on impatience $(\tilde{\rho})$ and on the hazard rate $(\hat{\rho})$. Equation (32) represents the discounted stream of utility of an infinitely-lived agent, however, an alternative interpretation is possible, whereby the objective function (32) represents the expected intertemporal utility of an agent with stochastic life and whose hazard rate is constant. Thus, providing a bridge between finite and infinite horizon models.

B Appendix B.

B.1 The Derivation of Condition (18)

The Jacobian associated with the intertemporal problem (4)-(10) is the following

$$J = \begin{pmatrix} \rho + g_w & 0 & \frac{U_{ww}+g_{ww}(U_c-p_c)}{U_{cc}} & 0 \\ \frac{\alpha(\delta+\rho)U_{cc}-\alpha(\rho+g_w)U_{cc}}{U_{ee}} & \delta+\rho & -\frac{\alpha(U_{ww}+g'_{ww}(U_c-p_c))}{U_{ee}} & \frac{U_{ss}}{U_{ee}} \\ 1 & -\alpha & -g_w & 0 \\ 0 & 1 & 0 & -\delta \end{pmatrix} \tag{33}$$

The trace is positive and equals to $tr(J) = 2\rho > 0$ while the determinant is harder to sign

$$\begin{aligned} \det(J) = \Phi[\delta(\delta+\rho)(\alpha^2 U_{cc} + U_{ee})(H_{ww}) + U_{ss}(1+U_{ww})]] \\ + \Phi[g_w(g_w+\rho)U_{cc}(\delta(\delta+\rho)U_{ee} + U_{ss})] \end{aligned} \tag{34}$$

with $\Phi = 1/U_{ww}U_{cc} > 0$ and $H_{ww} = (U_{ww} - g_{ww}(p_c - U_c) < 0$.

Following the formula from Dockner (1985) the eigenvalues of the Jacobian, at the steady state, are given by the following expression

$$e_i = \frac{\rho}{2} \pm \sqrt{\left(\frac{\rho}{2}\right)^2 - \frac{K}{2} \pm \frac{1}{2}\sqrt{K^2 - 4|J|}} \tag{35}$$

with $i = 1, 2, 3, 4$. $|K|$ is the determinant of the Jacobian matrix and K is the sum of the principal minors of dimension 2 of J:

$$K = \begin{vmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial w} \\ \frac{\partial \dot{w}}{\partial c} & \frac{\partial \dot{w}}{\partial w} \end{vmatrix} + \begin{vmatrix} \frac{\partial \dot{e}}{\partial e} & \frac{\partial \dot{e}}{\partial s} \\ \frac{\partial \dot{s}}{\partial e} & \frac{\partial \dot{s}}{\partial s} \end{vmatrix} + 2 \begin{vmatrix} \frac{\partial \dot{c}}{\partial e} & \frac{\partial \dot{c}}{\partial s} \\ \frac{\partial \dot{w}}{\partial e} & \frac{\partial \dot{w}}{\partial s} \end{vmatrix} = \psi - g_w(\rho + g_w) - \frac{H_{ww}}{U_{cc}} - \frac{U_{ss}}{U_{ee}} < 0$$

With $\psi = -\delta(\delta+\rho) < 0$. Dockner (1985) shows that, if K is negative and $K^2/4 \geq \det(J) > 0$, the steady state is a saddle point with real eigenvalues, two being positive and two being negative, and therefore the optimal trajectories that converge to the steady state are locally monotonic. Such conditions is the analog of the one-dimensional case when $|J| > 0$. Therefore, the fact that the determinant of the Jacobian is positive and that K is negative is sufficient to show that it is always possible to converge to the steady state and that the steady state has a saddle point stability[6]

After some manipulations, it can be shown that $K^2 > 4|J|$, thus the steady state is a saddle point with real eigenvalues, two being positive and two being negative, and therefore the optimal trajectories that converge to the steady state are locally monotonic (Dockner, 1985), therefore the conditions $|J| > 0$ and $|K| < 0$ are sufficient for saddle point stability. The condition $|J| > 0$ requires that

$$g_w(g_w + \rho)U_{cc}(\delta(\delta + \rho)U_{ee} + U_{ss}) + \delta(\delta + \rho)(\alpha^2 U_{cc} + U_{ee})(H_{ww}) + U_{ss}(1 + U_{ww})] > 0 \tag{36}$$

Since the second term in (36) is always positive, such condition is verified both for $g_W > 0$ and $g_w < 0$ as long as $g_w + \rho < 0$, proving the possibility of multiple steady states.

To derive the instability conditions (18) we have to characterize the cases in which $|J| < 0$. therefore we can rewrite the determinant as

$$\begin{aligned} &g_w(g_w + \rho)U_{cc}(\delta(\delta + \rho)U_{ee} + U_{ss}) < \\ &- \delta(\delta + \rho)(\alpha^2 U_{cc} + U_{ee})(H_{ww}) + U_{ss}(1 + U_{ww})] \end{aligned} \tag{37}$$

which is exactly condition (18). Given the assumption on $U(.)$, $g(.)$ and more importantly the concavity assumption on H (i.e. $H_{ww} < 0$), condition (37) is verified when, $g_w < 0$, and especially when $g_w + \rho > 0$.

C Appendix C.

C.1 The derivation of expressions (19), (20) and (21)

The change in the steady state level of food consumption and physical activity due to a change in the price of food are given by the following expression

$$\frac{\partial c^{ss}}{\partial p_c} = -\frac{|P|}{|J|} \qquad \frac{\partial e^{ss}}{\partial p_c} = -\frac{|K|}{|J|} \qquad \frac{\partial w^{ss}}{\partial p_c} = -\frac{|W|}{|J|} \tag{38}$$

where P and K are, respectively

$$P = \begin{bmatrix} \frac{\partial \dot{c}}{\partial p_c} & \frac{\partial \dot{c}}{\partial e} & \frac{\partial \dot{c}}{\partial w} & \frac{\partial \dot{c}}{\partial s} \\ \frac{\partial \dot{e}}{\partial p_c} & \frac{\partial \dot{e}}{\partial e} & \frac{\partial \dot{e}}{\partial w} & \frac{\partial \dot{e}}{\partial s} \\ \frac{\partial \dot{w}}{\partial p_c} & \frac{\partial \dot{w}}{\partial e} & \frac{\partial \dot{w}}{\partial w} & \frac{\partial \dot{w}}{\partial s} \\ \frac{\partial \dot{s}}{\partial p_c} & \frac{\partial \dot{s}}{\partial e} & \frac{\partial \dot{s}}{\partial w} & \frac{\partial \dot{s}}{\partial s} \end{bmatrix} \qquad K = \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial p_c} & \frac{\partial \dot{c}}{\partial w} & \frac{\partial \dot{c}}{\partial s} \\ \frac{\partial \dot{e}}{\partial c} & \frac{\partial \dot{e}}{\partial p_c} & \frac{\partial \dot{e}}{\partial w} & \frac{\partial \dot{e}}{\partial s} \\ \frac{\partial \dot{w}}{\partial c} & \frac{\partial \dot{w}}{\partial p_c} & \frac{\partial \dot{w}}{\partial w} & \frac{\partial \dot{w}}{\partial s} \\ \frac{\partial \dot{s}}{\partial c} & \frac{\partial \dot{s}}{\partial p_c} & \frac{\partial \dot{s}}{\partial w} & \frac{\partial \dot{s}}{\partial s} \end{bmatrix} \qquad W = \begin{bmatrix} \frac{\partial \dot{c}}{\partial c} & \frac{\partial \dot{c}}{\partial e} & \frac{\partial \dot{c}}{\partial p_c} & \frac{\partial \dot{c}}{\partial s} \\ \frac{\partial \dot{e}}{\partial c} & \frac{\partial \dot{e}}{\partial e} & \frac{\partial \dot{e}}{\partial p_c} & \frac{\partial \dot{e}}{\partial s} \\ \frac{\partial \dot{w}}{\partial c} & \frac{\partial \dot{w}}{\partial e} & \frac{\partial \dot{w}}{\partial p_c} & \frac{\partial \dot{w}}{\partial s} \\ \frac{\partial \dot{s}}{\partial c} & \frac{\partial \dot{s}}{\partial e} & \frac{\partial \dot{s}}{\partial p_c} & \frac{\partial \dot{s}}{\partial s} \end{bmatrix}$$

Since the necessary condition for a stable steady states implies that $|J| > 0$ (and also $g_w + \rho < 0$) (See Appendix B.1) then the sign of the response to a price change is given by the sign of the determinants of P,

[6]Dockner (1985) shows also that f K is negative but $\det(J) > K^2/4 > 0$, the steady state is a stable focus, the eigenvalues are imaginary (one pair with positive real part and one with negative real part) and the optimal trajectories imply dampened oscillations toward the steady state. Thus the steady state is still stable with the main difference being whether the optimal paths of consumption are monotonic or oscillating.

K and W which are given by the following expressions

—K—=-$\Phi\alpha\delta(\delta+\rho)(H_{ww})$

—P—=-$\Phi[\alpha^2\delta(\delta+\rho)(H_{ww})+g_w[\delta(\delta+\rho)U_{ee}+U_{ss}](g_w+\rho)]$

—W—=-$\Phi(g_w+\rho)(\delta(\delta+\rho)U_{ee}+U_{ss})$

(39)

Since $|J|$ is the determinant of the Jacobian matrix expressions (19), (20) and (21) are immediately derived.

C.2 The derivation of expressions (22), (23) and (24)

Similarly to the response to price change or food the change in the steady state level of food consumption, physical activity and body weight due to a change in the price of physical activity are given by the following expression

$$\frac{\partial c^{ss}}{\partial p_e}=-\frac{|P_2|}{|J|} \qquad \frac{\partial e^{ss}}{\partial p_e}=-\frac{|K_2|}{|J|} \qquad \frac{\partial w^{ss}}{\partial p_e}=-\frac{|W_2|}{|J|} \tag{40}$$

where P and K are, respectively

$$P_2=\begin{bmatrix} \frac{\partial\dot{c}}{\partial p_e} & \frac{\partial\dot{c}}{\partial e} & \frac{\partial\dot{c}}{\partial w} & \frac{\partial\dot{c}}{\partial s} \\ \frac{\partial\dot{e}}{\partial p_e} & \frac{\partial\dot{e}}{\partial e} & \frac{\partial\dot{e}}{\partial w} & \frac{\partial\dot{e}}{\partial s} \\ \frac{\partial\dot{w}}{\partial p_e} & \frac{\partial\dot{w}}{\partial e} & \frac{\partial\dot{w}}{\partial w} & \frac{\partial\dot{w}}{\partial s} \\ \frac{\partial\dot{s}}{\partial p_e} & \frac{\partial\dot{s}}{\partial e} & \frac{\partial\dot{s}}{\partial w} & \frac{\partial\dot{s}}{\partial s} \end{bmatrix} \qquad K_2=\begin{bmatrix} \frac{\partial\dot{c}}{\partial c} & \frac{\partial\dot{c}}{\partial p_e} & \frac{\partial\dot{c}}{\partial w} & \frac{\partial\dot{c}}{\partial s} \\ \frac{\partial\dot{e}}{\partial c} & \frac{\partial\dot{e}}{\partial p_e} & \frac{\partial\dot{e}}{\partial w} & \frac{\partial\dot{e}}{\partial s} \\ \frac{\partial\dot{w}}{\partial c} & \frac{\partial\dot{w}}{\partial p_e} & \frac{\partial\dot{w}}{\partial w} & \frac{\partial\dot{w}}{\partial s} \\ \frac{\partial\dot{s}}{\partial c} & \frac{\partial\dot{s}}{\partial p_e} & \frac{\partial\dot{s}}{\partial w} & \frac{\partial\dot{s}}{\partial s} \end{bmatrix} \qquad W_2=\begin{bmatrix} \frac{\partial\dot{c}}{\partial c} & \frac{\partial\dot{c}}{\partial e} & \frac{\partial\dot{c}}{\partial p_e} & \frac{\partial\dot{c}}{\partial s} \\ \frac{\partial\dot{e}}{\partial c} & \frac{\partial\dot{e}}{\partial e} & \frac{\partial\dot{e}}{\partial p_e} & \frac{\partial\dot{e}}{\partial s} \\ \frac{\partial\dot{w}}{\partial c} & \frac{\partial\dot{w}}{\partial e} & \frac{\partial\dot{w}}{\partial p_e} & \frac{\partial\dot{w}}{\partial s} \\ \frac{\partial\dot{s}}{\partial c} & \frac{\partial\dot{s}}{\partial e} & \frac{\partial\dot{s}}{\partial p_e} & \frac{\partial\dot{s}}{\partial s} \end{bmatrix}$$

It holds true that since the necessary condition for a stable steady states implies that $|J|>0$ (and also $g_w+\rho<0$) (See Appendix B.1) then the sign of the response to a price change is given by the sign of the determinants of P_2, K_2 and W_2 which are given by the following expressions

—K$_2| = -\Phi\delta(\delta+\rho)(H_{ww}+g_w(g_w+\rho)U_{cc})$

—P$_2| = -\Phi\alpha\delta(\delta + \rho)(H_{ww})$

—W$_2| = -\Phi\alpha\delta(\delta + \rho)(g_w + \rho)$

(41)

Since $|J|$ is the determinant of the Jacobian matrix expressions (22), (23) and (24) are immediately derived.