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ABSTRACT

We study how the strategic interaction of liquid-asset suppliers depends on the financial market conditions that determine asset liquidity. In our model, two asset suppliers try to profit from the liquidity services their assets confer. Asset liquidity is indirect in the sense that assets can be sold for money in over-the-counter (OTC) secondary markets. These secondary markets are segmented and customers will be drawn to the market where they expect to find the best terms. Understanding this, asset-suppliers play a differentiated Cournot game, where product differentiation here stems from differences in OTC microstructure. We find that small differences in OTC microstructure can induce very large differences in the relative liquidity of two assets. Asset demand curves can slope upward for even modest degrees of increasing returns in the matching technology. And if one asset supplier has an exogenous advantage over another, the favored agent may want to strategically increase asset supply for the purpose of driving competitors out of the secondary market altogether.

JEL Classification: E31, E43, E52, G12

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1 Introduction

A branch of the recent literature often referred to as "New Monetarist Economics" (see Lagos, Rocheteau, and Wright, 2015) has highlighted the importance of asset liquidity for the determination of asset prices.[1] The main message of this strand of the literature is that if assets can help agents facilitate transactions in markets characterized by certain frictions, such as anonymity or imperfect commitment, then the price of these assets may reflect not only their role as stores of value, as is standard in finance, but also their role as facilitators of transactions, i.e., their *liquidity role*, as is standard in monetary theory. An alternative way of phrasing this statement is that asset prices will not be equal to the present value of the stream of dividends, but they may also include *liquidity premia*. This result is of first-order importance as it highlights that the baseline asset pricing model (e.g., Lucas, 1978) may be missing a crucial component of asset price determination.[2] However, all the studies that predict the existence of liquidity premia assume that asset supply is fixed (and exogenous), usually at a level such that the marginal unit of the asset can help the agent purchase consumption in a decentralized/frictional goods market. Since asset supply is evidently crucial for the validity of this result, one can argue that it should not be treated as an exogenous parameter of the model.

The goal of this paper it to develop a theory of endogenous determination of the supply of (potentially) liquid assets, and to study its implications on asset prices. Our model has three key ingredients. The first is strategic interaction among asset issuers. More precisely, the agencies that issue assets realize that equilibrium asset prices, and, hence, the rate at which they can borrow, depend not only on their own decisions but also on those made by issuers of similar (and, hence, competing) assets. The second ingredient of our model is an empirically relevant concept of asset liquidity. In our framework, agents can liquidate assets for money in Over-the-Counter (OTC) secondary markets characterized by search and bargaining frictions, as in Duffie, Gârleanu, and Pedersen (2005), implying that assets are imperfect substitutes for money and will have downward sloping demand curves.[3] The third ingredient is partial market segmentation: agents can choose to visit the secondary market for any asset, but they must choose

[1] See for example Geromichalos, Licari, and Suarez-Lledo (2007), Lagos and Rocheteau (2008), Lagos (2011), Lester, Postlewaite, and Wright (2012), Andolfatto and Martin (2013), and Hu and Rocheteau (2015).

[2] For instance, some recent papers have shown that adopting models where assets are priced both for their role as stores of value and for their liquidity services, may be the key to rationalizing some long-standing asset pricing-related puzzles. Examples of such papers include Lagos (2010) (equity premium and risk-free rate puzzles), and Geromichalos, Herrenbrueck, and Salyer (2013) and Williamson (2015) (term premium puzzle).

[3] The empirical relevance of our liquidity mechanism is reflected in the words of Brian Roseboro, the Assistant Secretary of the US Treasury for the period 2001-2004, who states that secondary market liquidity is important because it encourages "more aggressive bidding in the primary market" ("A Review of Treasury's Debt Management Policy", June 3, 2002, available at http://www.treas.gov/press/releases/po3149.htm). The official's words indicate that the Treasury benefits from having its assets trade in liquid secondary markets, not because it obtains a direct benefit from secondary trade, but because agents who expect to be able to sell their assets easily in the secondary market, are more willing to pay higher prices in the primary market.

one at a time.[4] Within this framework, the various asset issuers play a *differentiated Cournot-Nash game*, where, crucially, the product (asset) differentiation stems from differences in the *microstructure* of the secondary OTC market where each asset is traded.

In the model, we focus on the case of a duopoly. Assuming that the matching technology in the OTC markets exhibits constant returns to scale (CRS) tends to make asset supplies strategic substitutes. In this case, the strategic interaction resembles a Cournot game, in the sense that equilibrium issue sizes are low, and the prices of both assets include liquidity premia. We also explore the possibility of increasing returns to scale (IRS) in secondary asset market trade, an assumption that seems realistic for financial market search because it implies that both buyers and sellers can find trading partners more easily in larger markets.[5] We find that IRS in OTC market matching tends to make asset supplies strategic complements. In this case, the strategic interaction resembles a Bertrand game, in the sense that equilibrium issue sizes can be large, and that equilibria tend to be in a corner in which only one of the two OTC markets operates.

Studying the endogenous determination of OTC market participation provides a number of new insights. Even for modest degrees of increasing returns, asset demand curves can be upward sloping (because an asset in large supply is likely to be more liquid). There are multiple equilibria in every period's 'stage game' as trade could be concentrated in the market for either asset, as well as mixed between both. Small differences in the microstructure of an OTC market can be magnified into a big *endogenous* liquidity advantage for one asset. Therefore, our paper does not only endogenize the supply of (potentially) liquid assets, but also their degree of liquidity: this is precisely why we have been careful about reminding the reader that assets are 'potentially' liquid. In fact, small exogenous liquidity differences may not only represent differences in the microstructure of the two markets, but they could also reflect the outcome of play in previous periods. This means that the dynamic game between the issuers can in principle become very complex: folk theorems, habits, threats, and learning are in play.

We are particularly interested in the case in which one of the assets has an exogenous liquidity advantage. More precisely, we fix the matching efficiency in one OTC market (say, for asset A) and study the effect of changes in the matching efficiency of the market for asset B on equilibrium variables. As the matching efficiency in market B becomes worse than that in market A, issuer A increases her asset supply and issuer B decreases it, but the strategic pattern of a Cournot game is maintained. The exogenous liquidity advantage of asset A is magnified by the entry choices of agents, which, in turn, feeds back into a rising (falling) liquidity premium on asset A (B). As matching efficiency in market B declines further, there comes a point at which issuer A has an incentive to boost up her supply and drive B out of the secondary market altogether. At that point asset B becomes fully illiquid, in the sense that agents only hold it as a store of value, and its equilibrium price equals its fundamental value. As the matching

[4] Perfectly integrated markets are equivalent to one special case of our model.
[5] Vayanos and Wang (2007) provide a careful justification for this assumption.

efficiency in market B falls even further, the threat of competition by asset B becomes so small so that, eventually, issuer A becomes a monopolist in the supply of liquid assets.

It should be noted that the aforementioned results refer to the CRS case. Even with a small degree of IRS in matching, the incentive to drive out the other issuer and 'capture' all secondary market trade becomes very strong, because traders would prefer to be in the thick market, and through their own entry help making it even thicker.

Finally, our model delivers some important results regarding welfare. Unlike output, social welfare tends to be maximized for small-to-intermediate quantities of liquid asset, featuring positive liquidity premia. This does not tell us whether a monopoly or a Cournot duopoly of asset issuers would be better; in fact, either is possible, depending on parameters. However, it does tell us that aggressive competition for secondary market liquidity, where issuers issue large amounts, is suboptimal. Consequently, market segmentation and exogenous liquidity differences can be good for social welfare because they tend to discourage such aggressive competition.

The model has a number of fruitful applications. The first one is the process of private money creation, because privately issued forms of "money" are really just forms of debt that obtain liquidity endogenously in competition with other assets, and are chosen by profit maximizing firms. The second major application is the superior liquidity of US federal debt over state and municipal debt. Certainly, the US treasury claims not to manage its borrowing strategically. (However, this is exactly what a competitor attempting to be a Stackelberg leader would do.) At any rate, our model still explains how small exogenous advantages of Treasury debt can be magnified into big liquidity differences. This case is even stronger once we consider that liquidity differences that were originally endogenous can become calcified over time, either through habit or through the long-term formation of specialized dealer networks.

The present paper is related to a growing literature that studies the liquidity properties of assets other than fiat money, initiated by Geromichalos et al. (2007) (who study the case of a real financial assets) and Lagos and Rocheteau (2008) (who study the case of physical capital), and continued by Nosal and Rocheteau (2012), Jacquet and Tan (2012), Andolfatto, Berentsen, and Waller (2013), Rocheteau and Wright (2013), and others. A common feature of the aforementioned papers is that assets possess liquidity properties because they serve directly as a medium of exchange (MOE) in frictional decentralized markets.[6] Some other papers show that assets can carry liquidity premia even if they do not serve directly as means of payment, because they can serve as collateral (Ferraris and Watanabe, 2011; Venkateswaran and Wright, 2013; Andolfatto, Martin, and Zhang, 2015; Geromichalos, Lee, Lee, and Oikawa, 2015).

As opposed to these papers, where assets are *liquid* because they help agents bypass certain

[6] Consequently, in most of these papers, assets compete with money as MOE. In recent work, Fernández-Villaverde and Sanches (2016) extend the Lagos and Wright (2005) framework to study the interesting question of competition among privately issued electronic currencies, such as Bitcoin and Ethereum.

frictions, such as anonymity and imperfect commitment, in decentralized markets (by serving directly as means of payment or as collateral) in our paper liquidity is *indirect*, as in Geromichalos and Herrenbrueck (2012): Assets are liquid not because agents can use them directly to purchase consumption goods, but because they can sell them in a secondary asset market in order to acquire more money (and in our case this market is OTC). This idea is exploited in a number of recent papers, including Berentsen, Huber, and Marchesiani (2014, 2015), Herrenbrueck (2014), Mattesini and Nosal (2015), Herrenbrueck and Geromichalos (2015) and Han (2015). This strand of the literature is also related to the work of Lagos and Zhang (2015); however, in that paper agents need money in order to purchase assets (rather than goods) in an OTC financial market. Also, a central idea in our paper is that a secondary asset market allows agents who have an idiosyncratic consumption shock to rebalance their portfolios after that shock has occurred. This idea draws upon the pioneering work of Berentsen, Camera, and Waller (2007), although in their analysis the channeling of liquidity towards the agents who need it most takes place through a competitive banking sector.

Our paper is somewhat related to an older Industrial Organization literature that studies the effect of the existence of secondary asset markets for durable goods on the pricing decisions of the producers of these goods. Examples of these papers include Rust (1985), Rust (1986), Swan (1972) and Manski (1982). In these papers, the existence of a secondary market, where buyers could sell the durable good in the *future*, affects the pricing decisions of sellers *now* because it affects the buyers' valuation for the good and, hence, their demand for it.[7] In our model, if assets could not be traded in secondary markets (i.e., if they had to be held to maturity), agents would be only willing to buy them at their fundamental value. The fact that secondary markets exist endows assets with (indirect) liquidity properties, which, in turn, allows issuers to borrow funds at low rates (or, equivalently, sell bonds at a price which includes a liquidity premium). Two differences (among many) between our model and the aforementioned ones is that here the sellers of assets are oligopolists rather than monopolists, and that the secondary markets are characterized by search and bargaining frictions.

Naturally, our work is also related to a growing literature, initiated by the pioneering work of Duffie et al. (2005), which studies how frictions in OTC financial markets can affect asset prices and trade. A non-exhaustive list of such papers includes Weill (2007), Vayanos and Weill (2008), Lagos and Rocheteau (2009), Lagos, Rocheteau, and Weill (2011), Chiu and Koeppl (2011), Afonso and Lagos (2015), Üslü (2015), and Chang and Zhang (2015).

[7] Within the context of financial, rather than commodity, markets, this idea is also exploited by Geromichalos et al. (2013) and Arseneau, Rappoport, and Vardoulakis (2015).

2 The economy with exogenous asset supply

2.1 Environment

Time is discrete and the horizon is infinite. Each period consists of three sub-periods where different economic activities take place. We start with an intuitive description of these markets; a detailed characterization will follow. In the first sub-period, two distinct OTC financial markets open. We refer to them as OTC$_j$, $j = \{A, B\}$. OTC$_j$ is the market where agents who hold assets of type j can sell them for money. For instance, one can think of asset A as T-Bills and asset B as municipal bonds or corporate AAA bonds. In the second sub-period, agents visit a decentralized good market characterized by bilateral trade, anonymity, and imperfect commitment. We refer to this market as the DM. Due to the aforementioned frictions, a need for a MOE arises in this market, and in our model only fiat money can play this role. During the third sub-period, economic activity takes place in a centralized market, which is similar in spirit to the settlement market of Lagos and Wright (2005) (henceforth, LW). We refer to this market as the CM. There are two types of agents, buyers and sellers, depending on their role in the DM. Agents live forever and their types (i.e., buyers or sellers) are permanent. The measure of both types of agents is normalized to the unit. There are also two agencies, $j = \{A, B\}$, that issue asset j in its respective *primary market* which opens within the third sub-period.

All agents discount the future between periods (but not sub-periods) at rate $\beta \in (0, 1)$. Buyers consume in the first and the third sub-periods and supply labor in the first sub-period. Their preferences for consumption and labor within a period are given by $\mathcal{U}(X, H, q)$, where X, H represent consumption and labor in the CM, respectively, and q consumption in the DM. Sellers consume only in the CM, and they produce in both the CM and the DM. Their preferences are given by $\mathcal{V}(X, H, q)$, where X, H are as above, and q stands for units of production in the DM. Interpreting the CM as a pure liquidity market, we adopt the functional forms:

$$\mathcal{U}(X, H, q) = X - H + u(q),$$
$$\mathcal{V}(X, H, h) = X - H - q.$$

We assume that u is twice continuously differentiable with $u(0) = 0$, $u' > 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Let q^* denote the optimal level of production in a bilateral meeting in the DM, i.e., $q^* \equiv \{q : u'(q^*) = 1\}$. The issuers of assets are only present in the CM. Their preferences are given by $\mathcal{Y}(X, H) = X - H$, where X, H are as above. The issuers also discount the future at rate β. What makes them special is that they can issue assets that potentially carry liquidity premia, thus allowing them to obtain net profits out of this operation.[8]

[8] Alternatively, one could assume that the issuers have to finance certain expenditures and, hence, have to borrow at least a certain amount, but can choose to borrow more if doing so is profitable. As long as that lower

We now provide a detailed description of the various sub-periods. In the third sub-period, all agents consume and produce a general good or fruit. All agents (including the issuers) have access to a technology that transforms one unit of labor into one unit of the fruit. Agents can choose to hold any amount of money which they can purchase at the ongoing price φ_t (in real terms). They can also purchase any amount of asset j at price p_j, $j = \{A, B\}$ (in nominal terms). These assets are one-period nominal bonds: each unit of (either) asset purchased in period t's CM pays one dollar in the CM of $t+1$. The supply of the assets, (A_t, B_t), is chosen strategically by the issuers, i.e., each issuer chooses the supply of her asset as a best response to her *rival's* action in order to maximize profits, realizing that both her and her rival's assets provide indirect liquidity services to the asset purchaser (described in detail below). The supply of money is controlled by the monetary authority, and it evolves according to $M_{t+1} = (1+\mu)M_t$, with $\mu > \beta - 1$. New money is introduced, or withdrawn if $\mu < 0$, via lump-sum transfers to buyers in the CM. Money has no intrinsic value, but it is portable, storable, divisible, and recognizable by all agents. In short, it possesses all the properties that make it an acceptable MOE in the DM.

After making their portfolio decisions in the CM, buyers receive an idiosyncratic consumption shock. More precisely, a measure $\ell < 1$ of buyers find out that they have a desire to consume in the forthcoming DM. We refer to these buyers as the C-types, and to the remaining $1-\ell$ buyers as the N-types. Since buyers did not know whether they would be C or N-type when they were making their portfolio choices, N-types will typically hold some cash that they do not want to use in the current period, while C-types may find themselves short of cash (since carrying money is costly). The OTC round of trade is placed strategically after the idiosyncratic uncertainty has been resolved, but before the DM opens, in order to allow agents to allocate the liquid asset into the hands of those who have a better use for it, i.e., the C-types. Hence, C-types can enter OTC$_j$ in order to sell assets of type $j = \{A, B\}$ for money. To make things interesting and realistic, we assume that the OTC financial markets are segmented: an agent who wants to sell or purchase assets is free to enter either OTC$_A$ or OTC$_B$, but she must choose only one of these markets. Hence, coordination is extremely important, and agents will pick the market where they expect to find better trading conditions.

Once C-types and N-types have decided which market they wish to enter, a matching function, $f_j(C_j, N_j)$, brings sellers (C-types) and buyers (N-types) of assets within OTC$_j$ together in bilateral matches. Throughout the paper we use the specific functional form:

$$f_j(x,y) = \alpha_j \left(\frac{xy}{x+y}\right)^{1-\rho} (xy)^{\rho},$$

with $\alpha_j \in [0,1]$ and $\rho \in [0,1]$. Notice that $f_j(x,y) \leq \min\{x,y\}$. The term α_j captures exogenous liquidity factors in OTC$_j$, such as the density of the dealer network. Also, notice that this

bound is not too large, our results would remain valid under the alternative specification.

specific functional form allows us to study both the case of CRS, i.e., when $\rho = 0$, and IRS, i.e., when $\rho > 0$. Within any match in either of the OTC markets the C-type makes a take-it-or-leave-it (TIOLI) offer with probability $\lambda \in [0, 1]$, otherwise the N-type does.

The second sub-period is the standard decentralized goods market of the LW model. C-type buyers meet bilaterally with sellers and negotiate over the terms of trade. Due to anonymity and lack of commitment exchange has to be *quid pro quo* and, as we have already mentioned, only money can serve as a MOE. [9] Since all the interesting insights of the paper follow from agents' interaction in the OTC round of trade, we wish to keep the DM as simple as possible. To that end, we assume that all C-type buyers match with a seller, and that in any DM meeting the buyer makes a TIOLI offer to the seller.

The timing of the model is summarized in Figure 1. It is important to highlight that the secondary OTC markets are completely unrelated to the primary markets where assets are first issued. Nevertheless, the microstructure of the OTC markets, summarized by the parameters α_j, ρ, and λ, will critically affect the liquidity properties of the assets and, consequently, their selling price in the primary market (or, equivalently, the rate at which the issuers can borrow funds by selling new bonds). It is quite clear that issuer $j = \{A, B\}$ will always benefit by a higher value of α_j (i.e., by having her assets trade in a more liquid secondary market). However, it is understood that here issuers have no control over the microstructure of the secondary market where their assets trade.[10]

2.2 Value functions

We begin with the description of the value functions in the CM. Consider first a buyer who enters this market with m units of fiat money and d_j units of asset $j = \{A, B\}$. The Bellman equation of the buyer is given by:

$$W(m, d_A, d_B) = \max_{\substack{X,H,\hat{m},\\ \hat{d}_A,\hat{d}_B,\eta_i}} \left\{ X - H + \mathbb{E}_i \left\{ \eta_i \Omega^i_A\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big) + (1-\eta_i)\Omega^i_B\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big) \right\} \right\}$$

$$\text{s.t. } X + \varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) = H + \varphi(m + \mu M + d_A + d_B),$$

$$0 \leq \eta_i \leq 1, \text{ for } i = \{C, N\},$$

[9] Here we shall make this an assumption of the model. However, a number of recent papers in the monetary-search literature, such as Rocheteau (2011) and Lester et al. (2012) do not place any restrictions on which objects can serve as MOE and show that, under asymmetric information, fiat money will endogenously arise as a superior MOE, thus, providing a micro-founded justification for our assumption.

[10] We believe that this is a fairly realistic description of secondary OTC financial markets. For instance, the US Treasury (i.e., the issuer of US government debt) has no control over the organization of the network of Interdealer Brokers who carry out the vast majority of secondary trade of US treasuries.

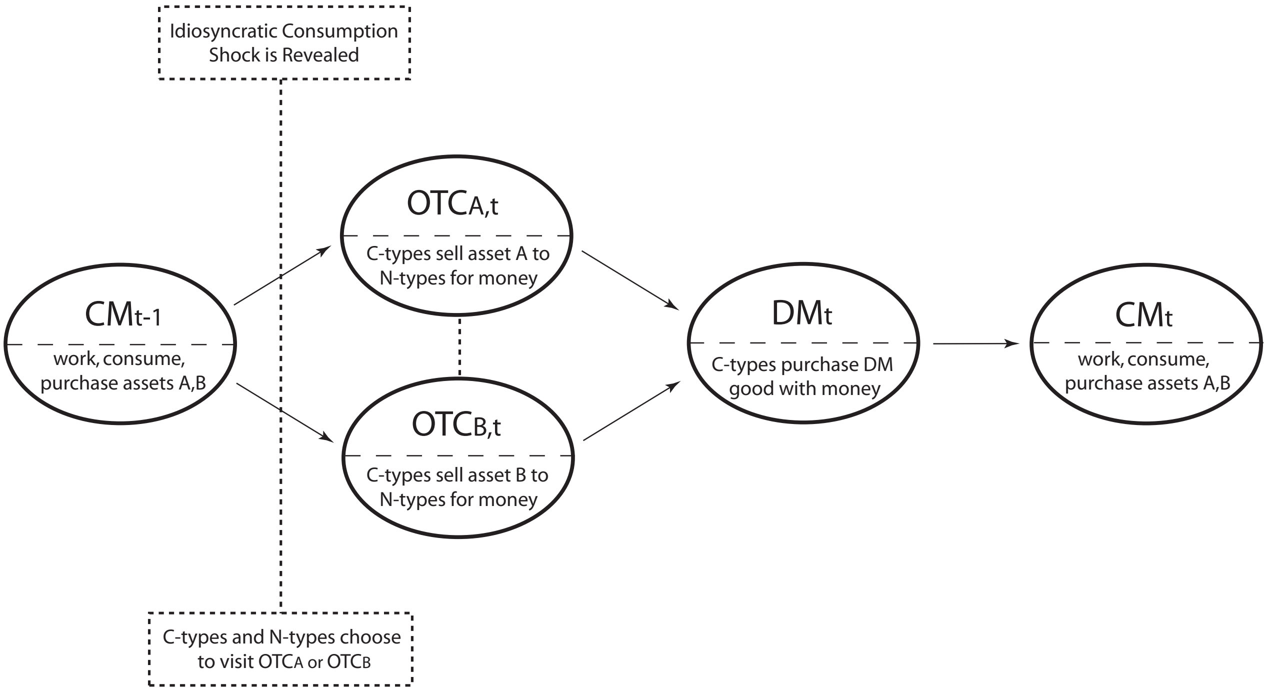


Figure 1: Timing of events.

where variables with hats denote portfolio choices for the next period, and $\mathbb{E}$ denotes the expectations operator. The price of money is expressed in terms of the general good but the price of bonds is expressed in nominal terms. The function Ω_j^i represents the value function in the OTC market for asset $j \in \{A, B\}$ for a buyer of type $i \in \{C, N\}$, to be described in more detail below, and η_i represents the probability with which the buyer chooses to enter the OTC$_A$, conditional on being of type i. At the optimum, X and H are indeterminate but their difference is not. Using this fact and substituting $X - H$ from the budget constraint into W yields:

$$\begin{aligned} W(m, d_A, d_B) &= \varphi(m + \mu M + d_A + d_B) \\ &\quad + \max_{\substack{\hat{m},\hat{d}_A,\hat{d}_B,\\ \eta_C,\eta_N}} \Big\{ -\varphi(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B) + \ell\left[\eta_C\Omega_A^C\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big) + (1-\eta_C)\Omega_B^C\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\right] \\ &\quad + (1-\ell)\left[\eta_N\Omega_A^N\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big) + (1-\eta_N)\Omega_B^N\Big(\hat{m}, \hat{d}_A, \hat{d}_B\Big)\right]\Big\}. \end{aligned} \tag{1}$$

In the last expression, we have also used the fact that the representative buyer will be a C-type with probability ℓ in order replace the expectations operator. As is standard in models that build on LW, the optimal choice of the agent does not depend on the current state (due to the quasi-linearity of $\mathcal{U}$), and the CM value function is linear. We collect all the terms in (1) that do not depend on the state variables m, d_A, d_B and write:

$$W(m, d_A, d_B) = \varphi(m + d_A + d_B) + \Upsilon, \tag{2}$$

where the constant Υ collects the remaining terms.

As is well-known, a seller will not wish to leave the CM with positive amounts of money and bond holdings.[11] Therefore, when entering the CM a seller can only hold money that she received as payment in the preceding DM, and her CM value function is given by:

$$W^S(m) = \max_{X,H} \left\{ X - H + V^S \right\}$$
$$\text{s.t. } X = H + \varphi m,$$

where V^S denotes the seller's value function in the forthcoming DM. We can again use the budget constraint to substitute $X - H$ and show that W^S will be linear:

$$W^S(m) = \varphi m + V^S \equiv \Upsilon^S + \varphi m. \tag{3}$$

We now turn to the description of the OTC value functions. First, let $H_C \in [0,1]$ and $H_N \in [0,1]$ denote the fraction of C-types and N-types, respectively, who enter OTC$_A$. Then, the measure of asset sellers and buyers in OTC$_A$ is given by $H_C\ell$ and $H_N(1-\ell)$, respectively, and the measure of asset sellers and buyers in OTC$_B$ is given by $(1-H_C)\ell$ and $(1-H_N)(1-\ell)$, respectively. Given that matching is random, the matching probabilities (or arrival rates) for an agent of type $i = \{C, N\}$ in OTC$_j$, $j = \{A, B\}$, denoted by α_{ij}, are as follows:

$$\alpha_{CA} \equiv \frac{f_A}{H_C\ell}, \tag{4}$$
$$\alpha_{CB} \equiv \frac{f_B}{(1-H_C)\ell}, \tag{5}$$
$$\alpha_{NA} \equiv \frac{f_A}{H_N(1-\ell)}, \tag{6}$$
$$\alpha_{NB} \equiv \frac{f_B}{(1-H_N)(1-\ell)}. \tag{7}$$

Having described the various arrival rates, α_{ij}, we can now define the value function for an agent of type $i = \{C, N\}$ who decides to enter OTC$_j$, $j = \{A, B\}$. Let ζ_j, denote the amount of money that gets transferred to the C-type, and χ_j the amount of assets (of type j) that gets transferred to the N-type in a typical match in OTC$_j$, $j = \{A, B\}$. These terms are described in detail in Lemma 2 below. We have:

$$\Omega_A^C(m, d_A, d_B) = \alpha_{CA} V\left(m + \zeta_A, d_A - \chi_A, d_B\right) + \left(1 - \alpha_{CA}\right) V(m, d_A, d_B), \tag{8}$$
$$\Omega_B^C(m, d_A, d_B) = \alpha_{CB} V\left(m + \zeta_B, d_A, d_B - \chi_B\right) + \left(1 - \alpha_{CB}\right) V(m, d_A, d_B), \tag{9}$$
$$\Omega_A^N(m, d_A, d_B) = \alpha_{NA} W\left(m - \zeta_A, d_A + \chi_A, d_B\right) + \left(1 - \alpha_{NA}\right) W(m, d_A, d_B), \tag{10}$$

[11] For a careful proof of this result, see Rocheteau and Wright (2005).

$$\Omega_B^N(m,d_A,d_B) = \alpha_{NB} W\left(m-\zeta_B, d_A, d_B+\chi_B\right) + \left(1-\alpha_{NB}\right) W(m,d_A,d_B), \tag{11}$$

where V denotes a buyer's value function in the DM. Notice that N-type buyers proceed directly to next period's CM.

Lastly, consider the value functions in the DM. Let q denote the quantity of goods traded, and τ the total payment in units of fiat money. These terms are described in detail in Lemma 1 below. The DM value function for a buyer who enters that market with portfolio (m, d_A, d_B) is given by:

$$V(m,d_A,d_B) = u(q) + W(m-\tau, d_A, d_B), \tag{12}$$

and the DM value function for a seller (who enters with no money or assets) is given by:

$$V^S = -q + \beta W^S(\tau).$$

2.3 The terms of trade in the OTC markets and the DM

Consider a meeting between a C-type buyer with portfolio (m, d_A, d_B) and a seller who, in the beginning of the DM sub-period, holds no money or assets. The two parties bargain over a quantity q to be produced by the seller and a cash payment τ, to be made by the buyer. The buyer makes a TIOLI offer maximizing her surplus subject to the seller's participation constraint and the cash constraint. The bargaining problem can be described by:

$$\max_{\tau,q} \{u(q) + W(m-\tau, d_A, d_B) - W(m, d_A, d_B)\}$$
$$\text{s.t. } -q + W^S(\tau) - W^S(0) = 0,$$

and the cash constraint $\tau \leq m$. Substituting the value functions W, W^S from (2) and (3) into the expressions above, allows us to simplify this problem to:

$$\max_{\tau,q} \{u(q) - \varphi\tau\}$$
$$\text{s.t. } q = \varphi\tau,$$

and $\tau \leq m$. The solution to the bargaining problem is described in the following lemma.

Lemma 1. *Let m^* denote the amount of money that, given the CM value of money, φ, allows the buyer to purchase the first-best quantity q^*, i.e., let $m^* = q^*/\varphi$. Then, the solution to the bargaining problem is given by $\tau(m) = \min\{m, m^*\}$ and $q(m) = \varphi \min\{m, m^*\}$.*

Proof. The proof is standard and it is, therefore, omitted. □

The solution to the bargaining problem is straightforward. The only variable that affects the solution is the buyer's money holdings. As long as the buyer carries m^* or more, the first-best quantity q^* will always be produced. If, on the other hand, $m < m^*$, the buyer does not have enough cash to induce the seller to produce q^*. The cash constrained buyer will give up all her money, $\tau(m) = m$, and the seller will produce the quantity of good that satisfies her participation constraint under $\tau(m) = m$, namely, $q = \varphi m$.

While Lemma 1 describes the bargaining solution for all possible money holdings by the C-type buyer, we know that, since $\mu > \beta - 1$, the cost of carrying money is strictly positive, and a buyer will never choose to hold $m > m^*$.[12] Hence, from now on we will focus on the binding branch of the bargaining solution, i.e., we will set $\tau(m) = m$ and $q(m) = \varphi m$.

We now describe the terms of trade in the OTC round of trade. Consider a meeting in OTC$_j$, $j = \{A, B\}$, between a C-type carrying a portfolio (m, d_A, d_B) and an N-type with portfolio $(\tilde{m}, \tilde{d}_A, \tilde{d}_B)$. These agents negotiate over an amount of money, ζ_j, to be transferred to the C-type, and an amount of type-j assets, χ_j, to be transferred to the N-type. Recall that the C-type (N-type) makes a TIOLI offer to the other party with probability λ $(1 - \lambda)$. In the match under consideration, the surpluses for the C-type and the N-type agents, respectively, are given by:

$$\begin{aligned} S_{Cj} &= V(m + \zeta_j, d_A - \mathbb{I}\{j = A\}\, \chi_A, d_B - \mathbb{I}\{j = B\}\, \chi_B) - V(m, d_A, d_B) \\ &= u(\varphi(m + \zeta_j)) - u(\varphi m) - \varphi \chi_j, \end{aligned} \tag{13}$$

$$S_{Nj} = W(m - \zeta_j, d_A + \mathbb{I}\{j = A\}\, \chi_A, d_B + \mathbb{I}\{j = B\}\, \chi_B) - W(m, d_A, d_B) = \varphi(\chi_j - \zeta_j), \tag{14}$$

where $\mathbb{I}$ denotes the identity function, and the second equalities in the equations above exploit the definitions of the functions V, W (i.e., equations (12) and (2), respectively).

Consider first the case in which the C-type makes the TIOLI offer. Then, the bargaining problem is equivalent to maximizing S_{Cj} (with respect to ζ_j, χ_j), subject to $S_{Nj} = 0$ and $\chi_j \leq d_j$. On the other hand, if it is the N-type who makes the offer, the problem is equivalent to maximizing S_{Nj}, subject to $S_{Cj} = 0$ and $\chi_j \leq d_j$.[13] The solution to the bargaining problem is described in the following lemma.

Lemma 2. *a) Suppose that the C-type is making the TIOLI offer. Define* $\bar{d}^C \equiv m^* - m$. *Then, the bargaining solution is given by* $\chi_j(m, d_j) = \zeta_j(m, d_j) = \min\{d_j, \bar{d}^C\}$.

b) Suppose that the N-type is making the TIOLI offer. Define $\bar{d}^N \equiv [u(q^*) - u(\varphi m)]/\varphi$. *Then, the*

[12] Even if the buyer in question matches with an N-type in the preceding OTC round and acquires some extra liquidity, she will never choose to adjust her post-OTC money balances in a way that these exceed m^*. This would be unnecessary since carrying m^* is already enough to buy her the first-best quantity in the forthcoming DM.

[13] Here, we implicitly assume that $m + \tilde{m} \geq m^*$. This assumption implies that the maximum amount of money that the C-type will ever want to acquire is $m^* - m$ (i.e., the amount she is missing in order to reach the first-best m^*), or, equivalently, that the N-type's money holdings, $\tilde{m}$, do not affect the bargaining solution, which would generate even more subcases that one needs to worry about. Later, when we study equilibrium, we will restrict attention to low inflation rates, thus making sure that agents will never find themselves in the region where $m + \tilde{m} < m^*$.

bargaining solution is given by $\chi_j(m, d_j) = \min\{d_j, \bar{d}^N\}$ *and:*

$$\zeta_j(m, d_j) = \begin{cases} \tilde{\zeta}_j(m, d_j), & \text{if } d_j < \bar{d}^N, \\ m^* - m, & \text{if } d_j \geq \bar{d}^N, \end{cases}$$

where we have defined:

$$\tilde{\zeta}_j(m, d_j) \equiv \{\zeta : u(\varphi(m + \zeta)) - u(\varphi m) - \varphi d_j = 0\}.$$

Proof. It is straightforward to check that the suggested answer satisfies the necessary and sufficient conditions for maximization in each case. □

The OTC bargaining solution is intuitive. Regardless of which agent makes the TIOLI offer, her objective is to maximize the available surplus of the match. This surplus is generated by transferring more money to the C-type, and it is maximized when the C-type's post-OTC money holdings are $m + \zeta_j = m^*$. However, in order to "afford" this transfer of liquidity, the C-type needs to have enough assets, and the critical level of asset holdings that allows her to acquire $\zeta_j = m^* - m$ *does depend* on who makes the offer. In particular, if the i-type makes the offer that critical level is given by $\bar{d}^i$, $i = \{C, N\}$, where, clearly, $\bar{d}^N > \bar{d}^C$ since if the N-type makes the offer she will require a higher amount of assets in order to transfer $m^* - m$ units of money.

Summing up, if the C-type carries a sufficient amount of assets (defined as $\bar{d}^i$ when the i-type makes the offer), then the money transfer will be optimal, i.e., $\zeta_j = m^* - m$, regardless of who makes the offer, and the asset transfer will satisfy $\chi_j = \bar{d}^i$, where i is the type of agent who makes the offer. On the other hand, if the C-type is constrained by her asset holdings (i.e., if $d_j < \bar{d}^i$ when the i-type makes the offer), then the C-type will give up all her assets, $\chi_j = d_j$, and she will receive a money transfer which is smaller than $m^* - m$ and depends on who makes the offer. More precisely, it satisfies $\zeta_j = d_j$, if the C-type makes the offer, and $\zeta_j = \tilde{\zeta}_j$, if the N-type makes the offer. It is easy to verify that $\tilde{\zeta}_j < d_j$, for all $d_j < \bar{d}^N$, since if the N-type makes the offer she will transfer a lower amount of money to the C-type (for any given amount of assets $d_j < \bar{d}^N$ that she receives).

2.4 Optimal behavior

In this sub-section, we describe the optimal behavior of the representative buyer. As is standard in models that build on LW, all buyers will choose the same optimal portfolio holdings regardless of their trading histories in the preceding DM. This result follows from the "no-wealth effects" property, which, in turn, stems from the quasi linear preferences. What is new here is that agents do not only choose an optimal portfolio, $(\hat{m}, \hat{d}_A, \hat{d}_B)$, but also the OTC market which they expect to enter in order to sell or buy assets, once their type has been revealed. The typical buyer's choice is reflected by her objective function, denoted by $J(\hat{m}, \hat{d}_A, \hat{d}_B)$. This function summarizes the buyer's cost and benefit from choosing any particular portfolio $(\hat{m}, \hat{d}_A, \hat{d}_B)$. To obtain J, substitute the expressions Ω^i_j, $i = \{C, N\}$ and $j = \{A, B\}$, from equations (8)-(11) into the maximization operator in equation (1). After replacing the value functions W and V from equations (12) and (2), respectively, and focusing only on the terms that contain the control variables $(\hat{m}, \hat{d}_A, \hat{d}_B)$, we obtain:

$$\begin{aligned} J(\hat{m}, \hat{d}_A, \hat{d}_B) = & -\varphi\left(\hat{m} + p_A\hat{d}_A + p_B\hat{d}_B\right) + \beta\hat{\varphi}\left(\hat{m} + \hat{d}_A + \hat{d}_B\right) \\ & + \beta\ell\left[u\left(\hat{\varphi}\hat{m}\right) - \hat{\varphi}\hat{m} + \max\left\{\underbrace{\lambda\alpha_{CA}S_{CA}}_{\text{enter A}}, \underbrace{\lambda\alpha_{CB}S_{CB}}_{\text{enter B}}\right\}\right]. \end{aligned} \tag{15}$$

The interpretation of the objective function is quite intuitive. The first term represents the cost that the buyer needs to pay in order to purchase the portfolio $(\hat{m}, \hat{d}_A, \hat{d}_B)$ in the CM, and the second term represents the benefit from selling these assets in the CM of the next period. Notice that if one was to shut down the DM market (and, hence, all liquidity considerations), the buyer's objective function would consist only of these two terms. However, as indicated by the third term in (15), the agent may turn out to be a C-type (with probability ℓ), in which case she can use her money, $\hat{m}$, to purchase consumption in the DM (thus, generating a net surplus equal to $u\left(\hat{\varphi}\hat{m}\right) - \hat{\varphi}\hat{m}$), and she can enter OTC$_j$, $j = A$ or B, in order to acquire more money by selling her assets, $\hat{d}_A$ or $\hat{d}_B$, depending on her entry choice. In the last expression, the terms S_{Cj} represent the surplus for the C-type in OTC$_j$, but notice that the agent will actually enjoy this surplus only if she gets to match in that market *and* make the TIOLI offer, an event that occurs with probability $\lambda\alpha_{Cj}$. Exploiting the OTC bargaining solution (i.e., Lemma 2) and equation (13), one can verify that, for $j = \{A, B\}$,

$$S_{Cj} = \begin{cases} u(q^*) - u(\hat{\varphi}\hat{m}) - q^* + \hat{\varphi}\hat{m}, & \text{if } \hat{d}_j > m^* - \hat{m}, \\ u(\hat{\varphi}\hat{m} + \hat{\varphi}\hat{d}_j) - u(\hat{\varphi}\hat{m}) - \hat{\varphi}\hat{d}_j, & \text{otherwise,} \end{cases} \tag{16}$$

where the condition $\hat{d}_j > m^* - \hat{m}$ states that in this case the agent's asset holdings are "abundant", i.e., they allow her to reach the first-best amount of money, m^*, through OTC trade.

Having established the representative buyer's objective function, two important observations are in order. First, while we have only imposed an exogenous segmentation assumption on the OTC markets, an *endogenous* segmentation will also arise in the primary markets, i.e., buyers will typically choose to purchase only asset A or asset B in the CM. This is true because, in equilibrium, assets will trade at a premium (Geromichalos and Herrenbrueck, 2012), and buyers will only pay this premium if they expect to trade the asset in the OTC. Since they can only enter one OTC (and anticipate having to choose eventually), they will choose *ex ante* (i.e. in the CM) a bang-bang solution $\eta_C \in \{0,1\}$, and, accordingly, they will "specialize" in asset A or B.[14] This, in turn, implies that the representative buyer's choice of asset holdings (including fiat money) is complementary to the choice of which asset market to trade in. That is, buyers who choose to specialize in asset A will typically choose to hold a different portfolio than those who specialize in asset B. For instance, we shall see in what follows that agents who choose to trade in a less liquid OTC market will insure themselves against the liquidity shock by carrying a higher amount of money.

The second important observation is that the agent's choice of which OTC market to enter if she turns out to be an N-type is completely unrelated with her portfolio choice in the CM.[15] This follows directly from Lemma 2, which specifies that only the asset and money holdings of the C-type buyer matter for the bargaining solution in OTC trade. As a result, regardless of her portfolio choice, which by the time the N-type makes her OTC entry choice is sunk, this agent will enter OTC$_A$ only if:

$$(1-\lambda)\alpha_{NA}S_{NA} \geq (1-\lambda)\alpha_{NB}S_{NB}.$$

In the last expression, the terms S_{Nj} represent the surplus for the N-type in OTC$_j$. Exploiting Lemma 2 and equation (14), one can verify that, for $j = \{A, B\}$,

$$S_{Nj} \quad = \quad \begin{cases} u(q^*) - u(\hat{\varphi}\tilde{m}) - q^* + \hat{\varphi}\tilde{m}, & \text{if } \tilde{d}_j > [u(q^*) - u(\hat{\varphi}\tilde{m})]/\hat{\varphi}\,, \\[2ex] \hat{\varphi}\tilde{m} + \hat{\varphi}\tilde{d}_j - u^{-1}\left[u(\hat{\varphi}\tilde{m}) + \hat{\varphi}\tilde{d}_j\right], & \text{otherwise,} \end{cases} \tag{17}$$

where $\tilde{m}, \tilde{d}_j$ stand for the representative buyer's expectation about the money and asset-j holdings, respectively, that her trading partner, a C-type, will carry into OTC$_j$ (recall that here we consider the representative buyer who turned out to be an N-type, thus, her own portfolio does not affect the surplus of the match). The condition $\tilde{d}_j > [u(q^*) - u(\hat{\varphi}\tilde{m})]/\hat{\varphi}$ states that the asset holdings of the C-type are large enough to allow her post-OTC money balances to reach the first-best amount, m^*.

2.5 Equilibrium

In this section we define a steady-state equilibrium for the model, treating the supplies of assets, A, B as given. Of course, endogenizing the asset supplies by studying the game played between the two issuers is one of the most important tasks of this paper, and it is carried out

[14] Buyers will only hold the other asset if indifferent, i.e. if that asset is abundant or illiquid.

[15] The careful reader may have already noticed this point: None of the terms that appear in equation (15) (i.e., the definition of the objective function) involves the event in which the buyer turns out to be an N-type.

in Section 3.1. In steady state, the cost of holding money can be summarized by the parameter $i \equiv (1+\mu-\beta)/\beta$; exploiting the Fisher equation, this parameter represents the nominal interest rate on an *illiquid* asset. The restriction $\mu > \beta - 1$ translates into $i > 0$. Notice that in any equilibrium it must be true that $p_j \geq 1/(1+i)$, $j = \{A, B\}$, since violation of these conditions would generate an infinite demand for the assets.

We have thirteen endogenous variables.[16] First, we have the equilibrium real balances $\{z_A, z_B\}$ held by the buyer who chooses to specialize in asset A, choosing $\eta_C = 1$, or B, choosing $\eta_C = 0$ (recall from the discussion in Section 2.4 that an agent who chooses to trade in OTC$_A$ will typically make different portfolio choices than the one who chooses o trade in OTC$_B$).

Next, we have the equilibrium quantities $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, \tilde{q}_{1A}, \tilde{q}_{1B}\}$. The first four represent the quantity of DM good purchased by a C-type buyer who either did not trade in the OTC market (indexed by 0), or who traded *and* made the TIOLI offer (indexed by 1), depending on whether they chose to specialize in asset A or asset B. The last two terms (i.e., the $\tilde{q}$'s) represent the quantity of DM good purchased by a buyer who traded in her chosen OTC, A or B, but did not get to make the TIOLI offer (because the N-type did). It is clear from Lemma 2 that the purchasing power of the C-type in the DM will depend on whether she got to make the offer or not, and, naturally, we have $q_{1j} \geq \tilde{q}_{1j}$, for all j.[17]

Next, we have the prices of the three assets $\{\varphi, p_A, p_B\}$. And, finally, we have the market entry choices $\{H_C, H_N\}$, i.e., the fractions of C-types and N-types, respectively, who choose to enter OTC$_A$.

We now showcase how seven out the thirteen endogenous variables can be derived directly from the following six variables, $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, H_C, H_N\}$. First, it is clear that $z_j = q_{0j}$, for $j = \{A, B\}$, since the C-type who does not trade in the OTC can only purchase the amount of DM goods that her own real money holdings, z_j, allow her to afford. Second, the price of money solves:

$$\varphi = \frac{H_C q_{0A} + (1 - H_C) q_{0B}}{M}. \tag{18}$$

This equation follows directly from the market clearing condition in the market for money. Third, the equilibrium asset prices must satisfy the demand equations:[18]

$$p_j = \frac{1}{1+i}\Bigg(1 + \ell\, \alpha_{Cj}\lambda \cdot [u'(q_{1j}) - 1]\Bigg), \text{ for } j = \{A, B\}. \tag{19}$$

[16] This count actually excludes the terms of trade in the OTC markets, since these follow directly from the main endogenous variables described in this section and Lemma 2.

[17] More precisely, we have $q_{1j} > \tilde{q}_{1j}$, unless the C-type's asset holdings satisfy $d_j \geq [u(q^*) - u(\varphi m)]/\varphi$. Then, even if the N-type makes the offer the C-type can afford a money transfer of $m^* - m$, and we have $q_{1j} = \tilde{q}_{1j} = q^*$.

[18] These follow directly from obtaining the first-order conditions in the buyer's objective function, i.e., equation (15), and imposing equilibrium quantities. Notice that the asset prices do not only depend on the variables q_{1j}, but also on the equilibrium values of H_C, H_N which affect the arrival rates α_{Cj}; see equations (4) and (5).

For future reference, notice that as long as $q_{1j} < q^*$, the marginal unit of the asset allows the agent to acquire additional money which she can use in order to boost her consumption in the DM. In this case, the agent is willing to pay a *liquidity premium* in order to hold the asset. On the other hand, if $q_{1j} = q^*$, the term inside the square bracket becomes zero, and $p_j = 1/(1+i)$, which is simply the *fundamental price* of a one-period nominal bond.

Finally, the quantities consumed in the DM by buyers who did not make the TIOLI offer in the preceding OTC market satisfy:

$$\tilde{q}_{1A} = \min\left\{q^*,\ u^{-1}\left(u(q_{0A}) + \varphi\,\frac{A}{H_C}\right)\right\}, \tag{20}$$

$$\tilde{q}_{1B} = \min\left\{q^*,\ u^{-1}\left(u(q_{0B}) + \varphi\,\frac{B}{1-H_C}\right)\right\}, \tag{21}$$

where φ has been explicitly defined as a function of the variables q_{0j} in (18). (These equations are derived from substituting equilibrium variables into part (b) of Lemma 2.)

The analysis so far establishes that if one had solved for $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, H_C, H_N\}$, then the remaining seven variables could also be immediately determined. Hence, hereafter we refer to these six variables as the "core" variables of the model. We now turn to the description of the equilibrium conditions that determine the core variables. Throughout this discussion, recall that the terms H_C, H_N are also implicitly affecting the arrival rates α_{Cj}.

First, the money demand for those specializing in asset j satisfies:

$$i = \ell\,(1-\alpha_{Cj}\lambda)\cdot[u'(q_{0j}) - 1] + \ell\,\alpha_{Cj}\lambda\cdot[u'(q_{1j}) - 1]\,, \quad \text{for}\ \ j = \{A, B\}. \tag{22}$$

Note that we have defined $\alpha_{ij} = 0$ if there is no entry at all into market j. If that is the case, q_{0j} and q_{1j} are still defined even though nobody actually trades at those quantities.

Next, the OTC trading protocol links q_{0j} and q_{1j}. Consider for instance market A. The bargaining solution, evaluated at equilibrium quantities, becomes:

$$q_{1A} = \min\left\{q^*,\ q_{0A} + \frac{\varphi A}{H_C}\right\},$$

where $\varphi A/H_C$ stands for the real value of assets that the C-type brings into OTC$_A$.[19] Even though the real aggregate supply of asset A is φA, the buyer under consideration holds more than the average because some buyers choose not to hold asset A at all (i.e., the buyers who specialize on asset B). Throughout the paper, when we talk about "asset concentration" or

[19] If the C-type's asset holdings are plentiful in the OTC, then we know that this agent will be able to purchase the first-best amount of money in the DM, hence, $q_{1A} = q^*$. On the other hand, if the asset is scarce in OTC trade, the C-type gives away all of her assets, $\varphi A/H_C$. Moreover, since here we are in the case where the C-type makes the offer, she will swap assets for money at a one-to-one ratio. As a result, in equilibrium it must be that $q_{1A} = q_{0A} + \varphi A/H_C$, which explains the last expression.

"asset dilution", we refer to this effect. After substituting the value of money from (18) in the last expression, we obtain two equations, one for each market:

$$q_{1A} = \min\left\{q^*,\ q_{0A} + \frac{A}{M} \cdot \frac{H_C q_{0A} + (1 - H_C) q_{0B}}{H_C}\right\}, \tag{23}$$

$$q_{1B} = \min\left\{q^*,\ q_{0B} + \frac{B}{M} \cdot \frac{H_C q_{0A} + (1 - H_C) q_{0B}}{1 - H_C}\right\}. \tag{24}$$

If it happens that $H_C = 1$ (no C-types enter the B-market) and $B > 0$, then we define $q_{1B} = q^*$, because a C-type of infinitesimal size who decided to deviate and hold asset B could hold the entire stock of it, which would certainly satiate them in an OTC trade – in the hypothetical case that there was an N-type in the B-market willing to trade with them. Similarly, if $H_C = 0$ and $A > 0$, then we define $q_{1A} = q^*$.

How large can the aggregate supply of an asset be for the asset to remain scarce in OTC trades? Clearly, the asset is more likely to be scarce if its ownership is diluted, i.e. if many buyers choose to hold that asset at the end of a period. So for example, asset A is most likely to be scarce if $H_C = 1$. But in this special case, Equation (23) tells us that the asset is scarce ($q_{1A} < q^*$) only if the condition $1 + A/M < q^*/q_{0A}$ is satisfied. On the boundary, $q_{1A} = q^*$, so we can use the money demand equation (22) to obtain the bounds:

$$\bar{A} \equiv M\left(q^*/\bar{q}_{0A} - 1\right), \text{ where } \bar{q}_{0A} \text{ solves } i = [\ell - \lambda\, f_A(\ell, 1-\ell)]\ [u'(\bar{q}_{0A}) - 1]\,,$$
$$\bar{B} \equiv M\left(q^*/\bar{q}_{0B} - 1\right), \text{ where } \bar{q}_{0B} \text{ solves } i = [\ell - \lambda\, f_B(\ell, 1-\ell)]\ [u'(\bar{q}_{0B}) - 1]\,.$$

There are three things to notice here. First, if $A > \bar{A}$, then asset A is certain to be abundant but the reverse is not always true, because asset ownership can be concentrated in the hands of a few buyers. Second, if we fix $H_C = H_N = 1$ so that OTC trade is completely concentrated in the A-market, then asset A is indeed abundant if and only if $A > \bar{A}$, and conversely for asset B. Third, if the market for asset A has an exogenous liquidity advantage ($\alpha_A > \alpha_B$), then $\bar{A} > \bar{B}$, and vice versa. In order to have a term for the maximal upper bound on asset supply beyond which either asset is certain to be abundant, we define:

$$\bar{D} \equiv \max\{\bar{A}, \bar{B}\}.$$

The remaining task is to characterize the OTC market entry choices. Consider first a C-type. As we have already discussed, this type at the beginning of the period has already made the choice to hold either asset A or asset B, so the choice of which market to enter has also been made at that time. The critical choice takes place in the preceding CM, where the agent chooses which asset to specialize in. Evaluating equation (16) at equilibrium quantities, we find that if the C-type makes the TIOLI offer, her surplus of trading in market $j \in \{A, B\}$ equals:[20]

$$S_{Cj} = u(q_{1j}) - u(q_{0j}) - q_{1j} + q_{0j}. \tag{25}$$

But since the C-type's entry choice is made concurrently with her portfolio choice, this surplus has to be balanced not only against the probability of matching, α_{Cj}, and the probability of making the offer (vs getting zero surplus), λ, but also against the cost of carrying the asset in the first place. Hence, we define the "net" surplus that the buyer obtains if she enters in OTC$_j$, conditional on being a C-type, as:

$$\tilde{S}_{Cj} \equiv -iq_{0j} - [(1+i)p_j - 1](q_{1j} - q_{0j}) + \alpha_{Cj}\lambda S_{Cj}.$$

Furthermore, we can use the money and asset demand (i.e., equations (19) and (22)) to substitute for i and p_j in the last expression. After some algebra, we obtain:

$$\tilde{S}_{Cj} = (1 - \alpha_{Cj}\lambda) \cdot [u(q_{0j}) - u'(q_{0j})q_{0j}] + \alpha_{Cj}\lambda \cdot [u(q_{1j}) - u'(q_{1j})q_{1j}]\,. \tag{26}$$

It is now straightforward to characterize the optimal entry of C-type buyers. In particular, the typical C-type will set:

$$\eta_C \;=\; \begin{cases} 1, & \text{if } \tilde{S}_{CA} > \tilde{S}_{CB}, \\ 0, & \text{if } \tilde{S}_{CA} < \tilde{S}_{CB}, \\ \in [0,1], & \text{if } \tilde{S}_{CA} = \tilde{S}_{CB}, \end{cases} \tag{27}$$

where the terms $\tilde{S}_{Cj}$, $j = \{A, B\}$, are defined in (26). Notice that a necessary condition for type-C buyers to enter both OTC markets is that $\tilde{S}_{CA} = \tilde{S}_{CB}$.

Finally, we want to characterize the market choice of the N-type buyers. Since these agents are asset buyers, their own asset holdings do not matter, so they can enter the market for either asset independently of which asset they chose to hold in the preceding CM. Then, the typical N-type will simply enter the market in which she expects a greater surplus (accounting for the probability of trading and making the TIOLI offer). Evaluating equation (17) at equilibrium quantities implies that the surplus for the N-type who chooses to enter OTC$_A$ is given by:

[20] This equality holds regardless of whether the asset is plentiful in the OTC meeting or not. Consider first the case of plentiful assets. For this case evaluating the relevant (i.e., the "abundant") branch of equation (16) at equilibrium quantities yields $S_{Cj} = u(q^*) - u(q_{0j}) - q^* + q_{0j}$, which is exactly what one would obtain if $q_{1j} = q^*$ was imposed on equation (25). Next, consider the case of scarce assets and for simplicity focus on OTC$_A$. In this case, evaluating (16) at equilibrium quantities yields $S_{Cj} = u(q_{1j}) - u(q_{0j}) - \varphi A/H_C$, where $\varphi A/H_C$ is the real value of assets that the C-type brings into OTC$_A$. But as we know from the discussion that leads to equation (23), here $q_{1A} = q_{0A} + \varphi A/H_C$. Hence, the validity of equation (25) is once again verified.

$$S_{NA} = \begin{cases} u(q^*) - u(q_{0A}) - q^* + q_{0A}, & \text{if } \frac{A}{H_C} > \frac{u(q^*)-u(q_{0A})}{\varphi}, \\ q_{0A} + \varphi\frac{A}{H_C} - u^{-1}\left(\varphi\frac{A}{H_C} + u(q_{0A})\right), & \text{otherwise,} \end{cases} \tag{28}$$

and the surplus for the N-type who chooses to enter OTC$_B$ is given by:

$$S_{NB} = \begin{cases} u(q^*) - u(q_{0B}) - q^* + q_{0B}, & \text{if } \frac{B}{1-H_C} > \frac{u(q^*)-u(q_{0B})}{\varphi}, \\ q_{0B} + \varphi\frac{B}{1-H_C} - u^{-1}\left(\varphi\frac{B}{1-H_C} + u(q_{0B})\right), & \text{otherwise.} \end{cases} \tag{29}$$

In (28) and (29) we have used the term φ to keep these expressions relatively short, but it is understood that φ is itself a function of the core variables, as defined in (18).

It is now straightforward to characterize the optimal entry of N-type buyers. In particular, the typical N-type will set:

$$\eta_N = \begin{cases} 1, & \text{if } \alpha_{NA}S_{NA} > \alpha_{NA}S_{NB}, \\ 0, & \text{if } \alpha_{NA}S_{NA} < \alpha_{NA}S_{NB}, \\ \in [0,1], & \text{if } \alpha_{NA}S_{NA} = \alpha_{NB}S_{NB}, \end{cases} \tag{30}$$

where the terms S_{Nj}, $j = \{A, B\}$, are defined in(28) and (29). Clearly, a necessary condition for type-N buyers to enter both OTC markets is that $\alpha_{NA}S_{NA} = \alpha_{NB}S_{NB}$.

The following definition summarizes a steady state equilibrium in the model with fixed asset supplies.

Definition 1. Assume (for now) that asset supplies are fixed and equal to $(A, B) \in \mathbb{R}^+ \times \mathbb{R}^+$. A steady state equilibrium for the core variables of the model is a list $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, H_C, H_N\}$ such that equations (22), for $j = \{A, B\}$, (23), and (24) are satisfied, and the typical buyer's entry choice satisfies (27) and (30) with $\eta_C = H_C$ and $\eta_N = H_N$.

2.6 Characterization of equilibrium

We are now ready to characterize the equilibria of the economy, summarized by the core variables $\{q_{0A}, q_{1A}, q_{0B}, q_{1B}, H_C, H_N\}$, conditional on the asset supplies $A, B \geq 0$. As we show below, the system admits a closed form solution in one special case, which we call "balanced CRS": there are CRS to matching in the OTC markets ($\rho = 0$) and neither asset has an exogenous liquidity advantage ($\alpha_A = \alpha_B$).[21] However, a general analytical characterization is technically

[21] We use the word "balance" to describe the situation where $\alpha_A = \alpha_B$. We could also use "symmetry", but we reserve that word for equilibria where *all* variables indexed by A equal their B-counterparts (e.g., $p_A = p_B$). Notice that even in the balanced environment, there may not be any symmetric equilibria if, say, $A > B$.

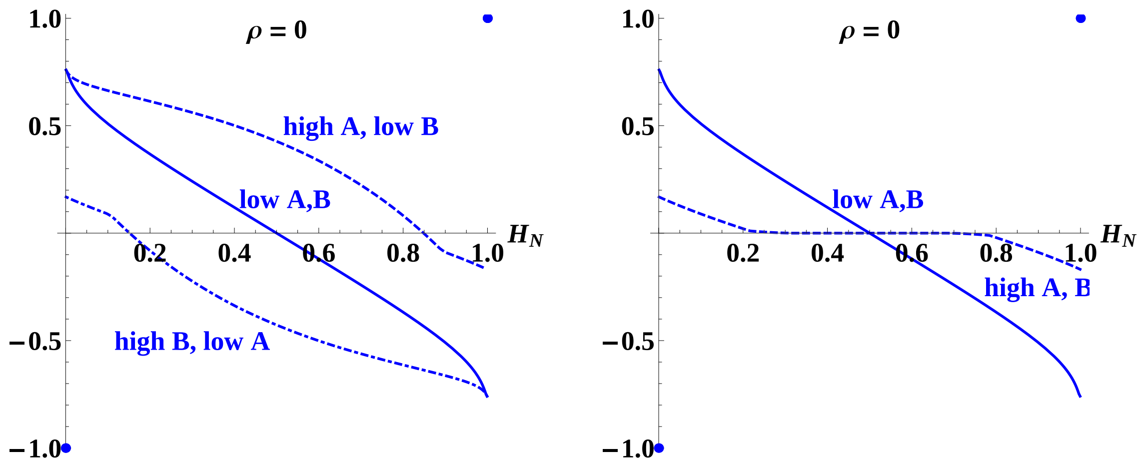


Figure 2: The function $G(H_N)$ for $\rho = 0$ and varying asset supplies.

impossible, and the rest of the analysis will therefore be numerical.[22] To understand how the equilibria are constructed, it is instructive to begin with the following exercise. Fixing a level of H_N (the proportion of N-types who enter the A-market), we compute the optimal portfolio choices through Equations (22)-(24) and (27), and finally, we compute the function:

$$G(H_N) = \frac{\alpha_{NA} S_{NA} - \alpha_{NB} S_{NB}}{\alpha_{NA} S_{NA} + \alpha_{NB} S_{NB}},$$

where all the surplus terms have the optimal portfolios substituted. So the function $G(H_N)$ measures the relative return to an *individual* N-type buyer from choosing the A-market over the B-market, assuming a proportion H_N of all *other* N-type buyers enters the A-market, and scaled to lie between -1 and +1 because that makes the visualization and interpretation much easier.

Figures 2 and 3 show how this relative return depends on H_N, and how this dependence changes with different values of ρ. In all cases, an exogenously high H_N causes a high value of

[22] We have a core system of six equations, and most of the endogenous variables show up in multiple equations (for instance, all terms but q_{1B}, H_N appear in Equation (23), and all terms but $q_{0i}, q_{1i}, i \neq j$, appear in Equation (22), and so on). Moreover, the equations are non-linear and include corners, due to the presence of the $\min\{\}$ terms and the various branches that characterize the agents' market entry decision. One may wonder whether it would be worthwhile to impose some simplifying assumptions that would allow us to achieve an analytical characterization of equilibrium. We believe that the model presented here constitutes the most parsimonious framework that can capture all the salient features of the economic problem we are studying. In that sense, any further simplification would eliminate insights from our model that we think are essential. A few examples may make our point more clear. A simplifying assumption adopted often in these types of models is that the bargaining power of agents is equal to either 0 or 1. (Recall that this is precisely what we have assumed for the DM, because not many interesting things happen in that market.) But imposing a similar assumption in the OTC would be a bad idea: it would imply that either the C-types or the N-types get no surplus from OTC trade, which would render their entry decision indeterminate. But as we have explained, the agent's decision which market to visit is one of the most important economic forces in our model. As another example, some related papers (Mattesini and Nosal, 2015; Geromichalos and Jung, 2016) gain tractability by assuming that asset trade takes place *only* in OTC markets, and the original asset holdings are given to agents in the CM as endowments, i.e. there is no primary asset market. Clearly, such an assumption here would deprive the model of its most important ingredient: the endogenous determination of asset supply.

H_C, too; when there are many buyers in an asset market, sellers would like to go to the same market. Of course, nobody would try to trade in a ghost town, so (for any matching function in fact) it must be the case that $H_C = 0$ if and only if $H_N = 0$, and $H_C = 1$ if and only if $H_N = 1$. Therefore, the corners are always equilibria. If $H_N = H_C = 0$, then $G(H_N) = -1$, so indeed all N-types prefer the B-market; and if $H_N = H_C = 1$, then $G(H_N) = +1$, so all N-types prefer the A-market, and nobody would want to deviate.

However, the interior equilibria (i.e., the ones where $H_N \in (0, 1)$), are more interesting than the corners, so let us analyze $G(H_N)$ for $H_N \in (0, 1)$. Our claim is that with CRS ($\rho = 0$) and low enough asset supplies, as H_N becomes relatively large an individual N-type prefers to deviate and enter the B-market. This may seem puzzling – the matching function exhibits CRS, so why are there not many equilibria with constant market tightness? In words, if H_N is large then there are many buyers competing for sellers, but as long as H_C increases along with H_N, there would be many sellers, too. When asset supplies are small enough, however, then there is an additional effect: concentration and dilution of the asset portfolios as H_C changes. If many C-types hold the A-asset, and the supply of A is not too large, then each of them will hold only a small amount, which implies a small trading surplus in the A-market and a larger trading surplus in the B-market. For this reason, the curves in the left panel of Figure 2 slope down in the interior. If many N-types were to enter the A-market, many C-types would follow; but consequently, each of these C-types would hold a small quantity of the A-asset, trading returns would be poor in that market, and therefore an individual N-type would be better off switching to the other. In fact, with $\rho = 0$, there is only one interior equilibrium and it is stable in the sense that if a small proportion of N-type agents makes a mistake and enters the 'wrong' market, everybody else's best response would be to switch until the equilibrium proportions are restored. This is not true for the corners: if only one market is open, then C- and N-types would jointly prefer to open up the other market, and they will quickly do so once someone makes the first move.

Everything else equal, N-types are more likely to enter the A-market if: (i) $\alpha_A > \alpha_B$, because then the A-market has an exogenous matching advantage; (ii) $A > B$, because then there is a larger potential surplus when trading asset A; (iii) H_C is large, because the more C-types enter the A-market, the easier it becomes for N-types to match; and (iv), if H_N is small, which reduces the congestion between N-types in the A-market.

Analogous considerations apply for the C-types (considerations (i) and (ii) are identical and (iii) and (iv) are reversed). Consequently, a larger supply of an asset shifts everybody's entry choices towards the market for that asset. In the right panel of Figure 2, however, we can see that this process has its limits: when the asset supplies are large enough, then the dilution effect has no power, and all buyers (C or N) will be indifferent between a range of possible entry choices. However, this range may be less than the full set $[0, 1]$. For example, say that $A = B = 0.75\bar{D}$. If $H_C = H_N \in [.25, .75]$, then each individual C-type is carrying enough

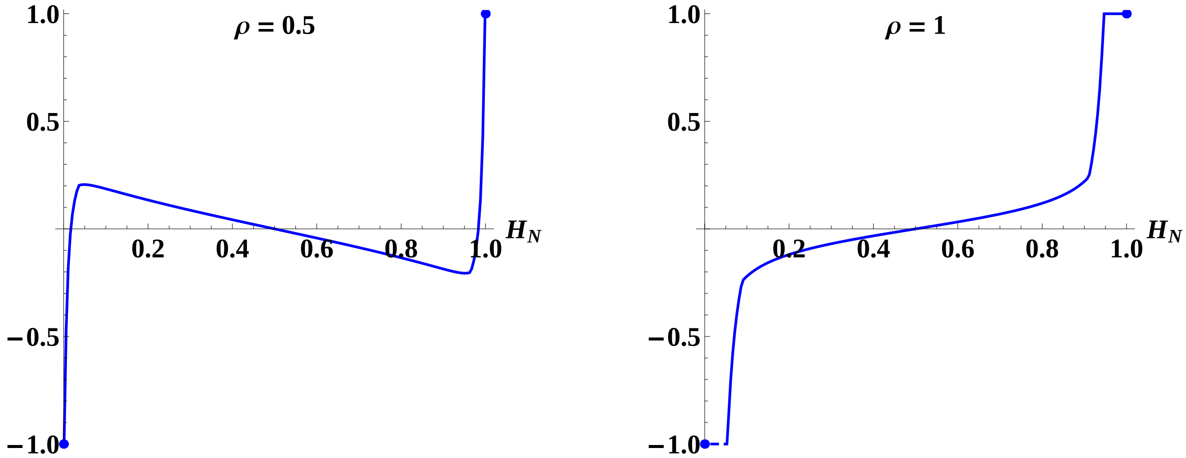


Figure 3: The function $G(H_N)$ given different degrees of IRS in matching; in both figures we have $A + B < \bar{D}$ so that there is an overall scarcity of liquidity.

assets to obtain the first-best quantity of money in case she is matched and makes the offer. As $H_C = H_N$, traders have the same matching probability in each market, and maximal surplus in either one of them, so they are indifferent between the markets. But if H_C were smaller than $.25$, then a C-type who chooses to hold the B-asset would no longer carry enough of it to obtain the first-best after OTC trade; consequently, N-types will find higher surplus in the A-market, and $G(H_N) > 0$ for $H_N < .25$.

The entry choice is more complex when there are increasing returns to matching in the OTC markets. For example, the left panel of Figure 3 shows a moderate amount of increasing returns, $\rho = 0.5$. In this case, both C- and N-types would prefer to trade in a thicker market with more overall entry. We can see that in the example, there are now five equilibria: the two corners (which are both stable now in the sense of being robust to errors by a small number of agents), the stable interior equilibrium, and two unstable asymmetric equilibria. With strongly increasing returns, $\rho = 1$, we can see that the picture is completely reversed: agents so strongly prefer to be in the thicker market that this completely dominates the effect of dilution of asset portfolios. The corners are now the only robust equilibria; there exists an interior equilibrium by continuity, but if it was ever played, a small variation in any parameter of the model would quickly drive the buyers into one of the corner equilibria.

Now that we understand the structure of the possible equilibria, we want to compare asset prices in these equilibria, and interpret their comparative statics with respect to asset supplies as the aggregate demand for these assets. These comparative statics are shown in Figure 4. In all graphs, the supply of asset A is on the horizontal axis and the supply of B is held fixed and indicated by a gray vertical line. We show three cases: first, the simplest case of balanced CRS ($\rho = 0$ and $\alpha_A = \alpha_B$); second, giving an exogenous advantage to asset A ($\alpha_A > \alpha_B$); and third, without an advantage for either asset but with IRS in matching. In all three examples, the

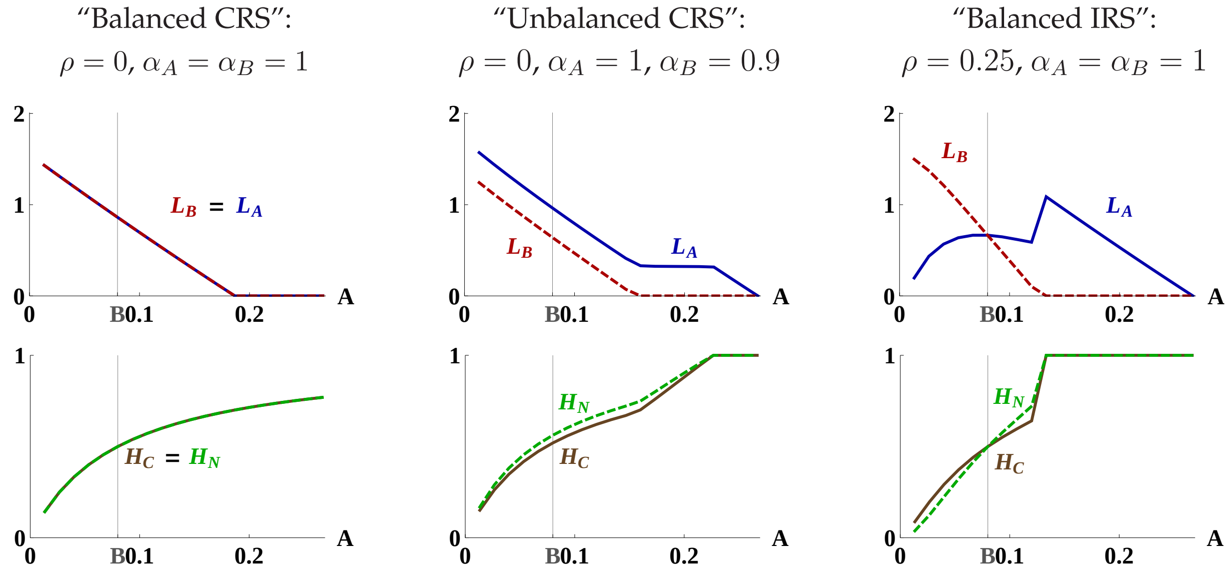


Figure 4: Net liquidity premia L_j (in %) and entry choices, varying A and holding B fixed.

graphs in the top row show the *net liquidity premia* of assets A and B, defined as:

$$L_j \equiv (1+i)p_j - 1 = \ell\, \alpha_{Cj} \lambda \left[u'(q_{1j}) - 1 \right].$$

The graphs in the bottom row of the figure show the market entry choices H_C and H_N.

Notice first that some standard results are replicated in our model. First, the liquidity premium of an asset is zero if that asset is in very large supply, no matter how liquid the market for that asset is. The reason is that as the asset supply becomes large enough, $q_{1j} \to 1$. (One should be careful with the terms here: the asset does not *lose* its liquidity properties, but it fulfills its liquidity role inframarginally; it still contributes to the overall supply of liquidity in the sense that money demand will be lower than it would be if that asset did not exist.) Furthermore, real balances decrease with inflation so the need to liquidate assets in the OTC markets becomes stronger with inflation; if the asset supplies are small enough, the liquidity premium of any liquid asset will rise with inflation, too.

In addition to these standard results, our model also delivers new insights into asset pricing in this environment of segmented OTC markets. Three results stand out. The first is that when matching in the markets exhibits "balanced CRS" (that is, CRS and neither market has an exogenous liquidity advantage), there exists a unique interior equilibrium when the asset supplies are not too large. In this equilibrium, $H_C = H_N = A/(A+B)$, so the ratio of buyers to sellers is 1 in each market, and the assets turn out to be perfect substitutes: we have $p_A = p_B$ and all the equilibrium quantities and prices only depend on the *sum* of the asset supplies, $A + B$.[23]

[23] This outcome is reminiscent of a celebrated result in the competitive search literature that under CRS in matching, one market being open is isomorphic to many markets being open, in equilibrium. However, this is not

The leftmost column of Figure 4 illustrates this.

The second result is that exogenous liquidity differences are *amplified* by the market entry process, even with CRS. Consider a case where $\alpha_A > \alpha_B$, so that OTC$_A$ has an exogenous liquidity advantage. As a consequence, and as illustrated in the middle column of Figure 4, both H_C and H_N increase, but the latter increases more. Intuitively, the N-types only consider the potential trading surplus in the OTC market when deciding which market to enter while the C-types also consider the ex-ante cost of carrying either asset, and therefore the N-types are more sensitive to liquidity differences when choosing their market. The end result is that market tightness from the point of view of asset sellers rises in the more liquid market and falls in the less liquid one: formally, we observe that the elasticity of the endogenous ratio α_{CA}/α_{CB} with respect to the exogenous ratio α_A/α_B is more than 1. Crucially, it is the point of view of OTC asset sellers that matters for asset pricing at the issue stage; people who buy a newly issued asset are concerned about the conditions at which they can sell it down the road, but people who plan to buy the asset later in the secondary market have no influence on the issue price. As a consequence, even a small divergence of α_A and α_B will drive a wedge between the liquidity premia on the two assets.

The third result is that IRS in matching encourage market concentration, i.e. corner equilibria. This is illustrated in the rightmost column of Figure 4. Near the origin, we have a case of $A \ll B$, so asset A is barely traded in OTC markets (though not entirely absent due to the fact that ownership of asset B is much more diluted). As the supply of A increases, more agents are willing to trade it in the OTC market because of the increase in potential trading surplus; and crucially, N-types are more sensitive to this increase, so the ratio H_N/H_C rises as A increases. This is important because again, it means that asset A becomes rapidly more attractive to C-types through *two* channels (market tightness and IRS).[24] As asset demand in the CM by future C-types determines the issue price, the resulting increase in liquidity is so strong that it makes the price of asset A upward sloping in its supply – at least, until that supply is so large that the force of diminishing marginal utility takes over. But we are not done. When the supply of A becomes even larger, all OTC trade becomes concentrated in the market for A and B ceases to be liquid at all. As this happens, the price of asset A jumps upward discontinuously; later, we will see that this effect of increasing returns provides a powerful incentive to the issuer of an asset to issue up to the point where competing assets are driven out of secondary markets.[25]

quite the case here: as long as the asset supplies are small enough, the asset concentration/dilution effect ensures that there is a unique equilibrium. If the sum of the asset supplies becomes large enough so that $A + B > \bar{D}$, then we have a continuum of interior equilibria; but all of them satisfy $H_C = H_N$, $q_{0A} = q_{0B}$, $q_{1A} = q_{1B} = q^*$, and $p_A = p_B = 1/(1+i)$.

[24] To be precise: with IRS and $\alpha_A = \alpha_B$, we observe $H_C < H_N$ in the interior if and only if $A < B$. The more plentiful asset is more liquid.

[25] There is a question of equilibrium selection with IRS because there may not be a stable interior equilibrium. If there are two corners, which one do we select? We have to make a particular assumption and what we used in our analysis is that the corner of the more liquid asset, or the one in larger supply, has an advantage in being

To summarize our results: we find that liquidity premia are always zero if asset supply is large but may be positive if asset supply is small enough. The liquidity premium on a particular asset is always decreasing in that *same* asset's supply with CRS; but with IRS, liquidity depends positively on issue size and asset demand curves can therefore have *upward sloping* segments. However, the liquidity premium on an asset is always decreasing in the supply of *other* assets, which opens the door to strategic interaction.

3 The economy with strategically chosen asset supply

3.1 The game between the asset issuers

We look at the non-cooperative game between two issuers who seek to maximize their utility, described above: they live only in the CM, where they can work, consume, and issue assets. Their utility within the period is $\mathcal{Y}(X, H) = X - H$, where X, H denote consumption and work effort, and they discount the future by the same factor β as the other agents. They take into account that the real price at which they can sell their asset, φp_j, depends on the supplies of both assets. For example, the problem of issuer A who has issued A^- assets in the previous period can be described by the following Bellman equation:

$$W^A(A^-) = \max_{X,H,A} \left\{X - H + \beta W^A(A)\right\}$$
$$\text{s.t. } X + \varphi A^- = H + \varphi p_A A,$$

which we can simplify to yield:

$$W^A(A^-) = -\varphi A^- + \max_{A} \left\{\varphi p_A A + \beta W^A(A)\right\}.$$

Just like for private agents, the issuer's choice of A does not depend on their previous choices. We can use this, plus the fact that in steady state $\varphi/\hat{\varphi} = (1 + \mu)$, to solve for issuer A's objective:

$$\begin{aligned} J^A &= \frac{\varphi}{1+i}\left[(1+i)p_A - 1\right] A \\ &= \frac{\varphi}{1+i}\Big(\ell\, \alpha_{CA}\lambda \left[u'(q_{1A}) - 1\right]\Big) A. \end{aligned} \tag{31}$$

selected. Specifically, we computed the function G at the starting point $\alpha_A A/(\alpha_A A + \alpha_B B)$ and then iterated in the direction of the sign of $G(H_N)$ until convergence or until reaching a corner. If the corner is not stable, e.g. as is the case with balanced CRS, we will always reach an interior equilibrium. On the other hand, a stable interior equilibrium may exist but not be found if the corner is stable and the starting point is close to it. There may be more realistic assumptions: e.g. if trade was concentrated at a corner in recent periods, this corner will likely be selected again. However, while realistic, this assumption does not really provide a path for our numerical analysis.

With an analogous derivation, issuer B's objective is:

$$J^B = \frac{\varphi}{1+i}\big(\ell\,\alpha_{CB}\lambda\,[u'(q_{1B}) - 1]\big)B. \tag{32}$$

Simply put, each issuer seeks to maximize the product of the net liquidity premium L_j and the supply of their asset, taking into account that their choice of asset supply affects the general equilibrium choices of the buyers.

The next step is to choose a solution concept for the game between the issuers. This is not a trivial question because there are very many options. The simplest approach is to model the interaction between the issuers as a static game, ignoring the fact that these issuers interact in every period, which suggests a static equilibrium concept such as Nash Equilibrium. But even within the static framework, there are alternatives that might be relevant in our context. For example, if one of the issuers has the ability to commit to an issue size before the other, then the appropriate equilibrium concept would be Stackelberg rather than Nash.

And if we take the repeated interaction seriously, there are even more possibilities. For one, folk theorems would support many more outcomes that are not equilibria of the static game, including collusion. To complicate things even further, we have multiple equilibria in the stage game, but is it *realistic* to think that all of them could be played independently of previous outcomes? For example, say that the OTC traders played the corner equilibrium of $H_C = H_N = 1$ in the last twenty periods, and say that the A-issuer issued a positive quantity but less than $\bar{D}$. If the B-issuer were to issue a larger quantity $B > A$, and assuming $\alpha_B \geq \alpha_A$, then the OTC traders might be collectively better off to switch to an interior equilibrium or even to the B-corner. But could the traders realistically coordinate in this way given the recent history of A being the only asset with a liquid market? If the answer is "no", then asset issuers in a given period would be competing not only for current profits, but also for future monopolies.[26]

Given the difficulty of arguing which of the many possibilities is the most relevant one for our theory, the analysis in this section will proceed in two parts that should be distinguished. One, we will describe the payoff structure facing the asset issuers in the stage game and analyze how this structure depends on parameters such as whether one of the assets has an exogenous liquidity advantage (α_A, α_B), and whether the matching function exhibits CRS or IRS (ρ). We believe this will allow our readers to extrapolate what kind of outcomes one might obtain with their preferred solution concept, whether static Stackelberg or dynamic triggers. Two, for the sake of concreteness, we will solve for static Nash equilibria of the stage game and analyze how these equilibria, as well as resulting macroeconomic outcomes such as DM production, depend on the parameters of the economy.

[26] Choosing a solution concept for the game between the issuers is *in addition to* choosing a solution concept for the entry choices (discussed in Footnote 25). There are a lot of doors one could open here.

3.2 Strategic structure of the game

In this subsection, we analyze the strategic structure of the game and the incentives asset issuers face; in the next subsection, we will analyze how static Nash equilibria of the game depend on the parameters of the economy, including the ones which govern the strategic structure.

First, we analyze an economy with "balanced CRS" in financial markets ($\rho = 0$ and $\alpha_A = \alpha_B \equiv \alpha$). As we saw in Section 2.6 above, the two corner equilibria are not robust to small errors: if an arbitrarily small measure of C-types happens to enter a market with no N-types, then N-types can profitably deviate by entering that market. As illustrated in Figure 2, more and more buyers (C and N) will enter that market until the interior equilibrium is reached; consequently, in the case of balanced CRS, the interior equilibrium is the interesting one to study.

Using the guess-and-verify method, it is easy to show that $H_C = H_N = A/(A+B)$ together with $q_{0A} = q_{0B}$ and $q_{1A} = q_{1B}$ satisfies all of the equilibrium equations, and that this is the unique solution when the sum $A + B$ is small enough. In that case, we also have $q_{1j} < q^*$, and the net liquidity premia $L_A = L_B > 0$ depend only on the sum $A + B$: the assets are perfect substitutes and are priced along a common demand curve. This case of balanced CRS is therefore isomorphic to a version of the model where the assets could be traded in the *same* OTC market rather than in segmented markets as we assume here. And because the assets are perfect substitutes (and as long as the inverse demand curve $L_j(A + B)$ is not too convex), the only Nash equilibrium of the game between the issuers is the symmetric Cournot equilibrium where both assets are issued in the same quantity, each approximately one-third of the quantity $\bar{D}$ that would drive the liquidity premium to zero.

Second, we analyze how an economy with CRS in financial markets is affected by exogenous liquidity differences. Specifically, we set α_A equal to 1 and let α_B vary. The results are illustrated in Figure 5; the leftmost column illustrates the balanced CRS case, and the rightmost column illustrates how if asset B has too much of a disadvantage, then the interior equilibrium of OTC market entry ceases to exist. Issuer A issues the monopoly quantity, approximately one-half of $\bar{D}$, and issuer B issues an arbitrary amount because asset B is illiquid in any case.

The intermediate values of α_B are the most interesting. If α_B is close to but below α_A, then the demand curve for asset A has a kink as shown in the middle column of Figure 4. As long as α_B is large enough, the Cournot equilibrium survives, though it shifts very slightly to be biased in favor of A issuing more and receiving a larger payoff. When α_B becomes smaller, however, then A may prefer to issue a very large quantity that would concentrate OTC trade in the A-market and drive B's liquidity premium to zero. In this equilibrium, despite receiving a zero payoff, B does not issue an arbitrary amount: if B issues too little, A would have an incentive to issue the (smaller) monopoly quantity, in which case B has an incentive to issue more, so this is not an equilibrium. We will analyze the consequences for the economy in more detail later, but we can already see that the total supply of liquid assets is largest if B is somewhat illiquid,

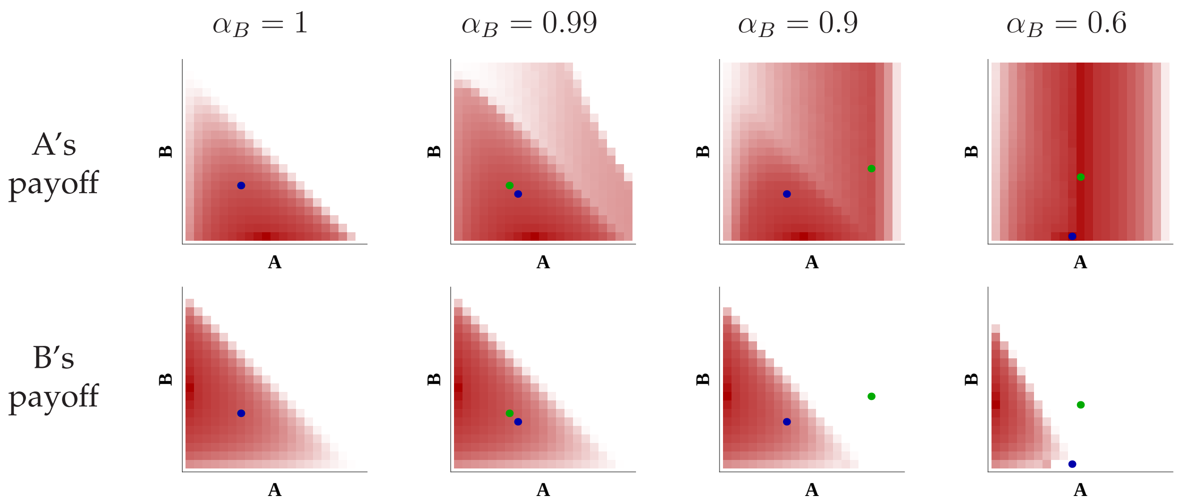


Figure 5: Payoffs as functions of asset supplies, with CRS ($\rho = 0$) and asset A having an exogenous liquidity advantage over asset B ($\alpha_B \leq \alpha_A = 1$). Darker shades of red indicate larger payoffs, white indicates zero. The blue and green points indicate particular Nash equilibria.

smallest if B is very illiquid, and in between if both A and B are very liquid.

To summarize: with CRS in financial markets, the structure of the game resembles Cournot competition, in varying flavors. If not too unbalanced, CRS supports the interior equilibrium in OTC markets where every asset is somewhat liquid. We see relatively low issue sizes, and we can extrapolate that they would be larger with Stackelberg replacing Nash, and smaller if the issuers collude.

As the third case, we analyze how an economy without exogenous liquidity differences is affected by IRS in financial markets. Specifically, we set $\alpha_A = \alpha_B$ equal to 1 and let ρ vary. The results are illustrated in Figure 6; the leftmost column shows the balanced CRS case, and the rightmost column illustrates how a strong degree of IRS makes the symmetric interior equilibrium so unstable that it effectively ceases to exist. In the latter case, the returns to issuing *more* than one's competitor become enormous because buyers strongly prefer to only trade in one market.[27] Consequently, there is a Nash equilibrium where quantity A is so close to $\bar{D}$ that issuer B does not find it profitable to issue any more, because in either case their asset would trade at a zero liquidity premium, either due to being illiquid or due to being plentiful. And there is another Nash equilibrium with A and B's roles reversed.

For intermediate values of ρ, we see a smooth transformation of the playing field. For low ρ, assets tend to be strategic substitutes where players prefer to issuer neither too little nor too much, but for high ρ, assets become strategic complements where players strongly prefer to issue *more* than the other. We also see a transition from Cournot-type equilibria of low issue sizes and both assets being liquid to Bertrand-type equilibria of high issue sizes and only one asset

[27] And in computing equilibria, we made the assumption that buyers are more likely to pick the corner of the asset of which there is a larger quantity. See 25 for a discussion.

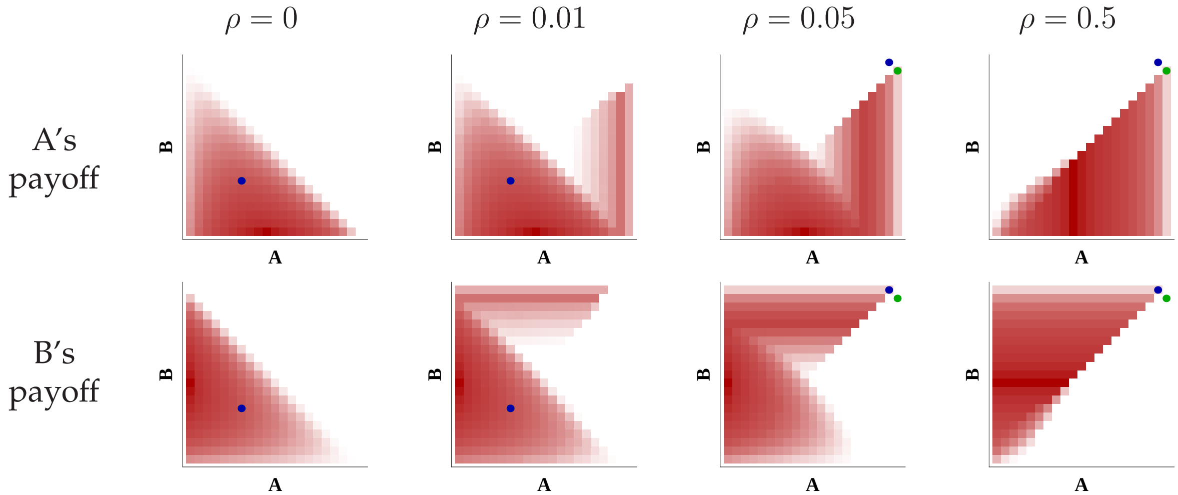


Figure 6: Payoffs as functions of asset supplies, with IRS but no exogenous liquidity advantage ($\alpha_A = \alpha_B = 1$). Darker shades of red indicate larger payoffs, white indicates zero. The green and blue dots indicate particular Nash equilibria.

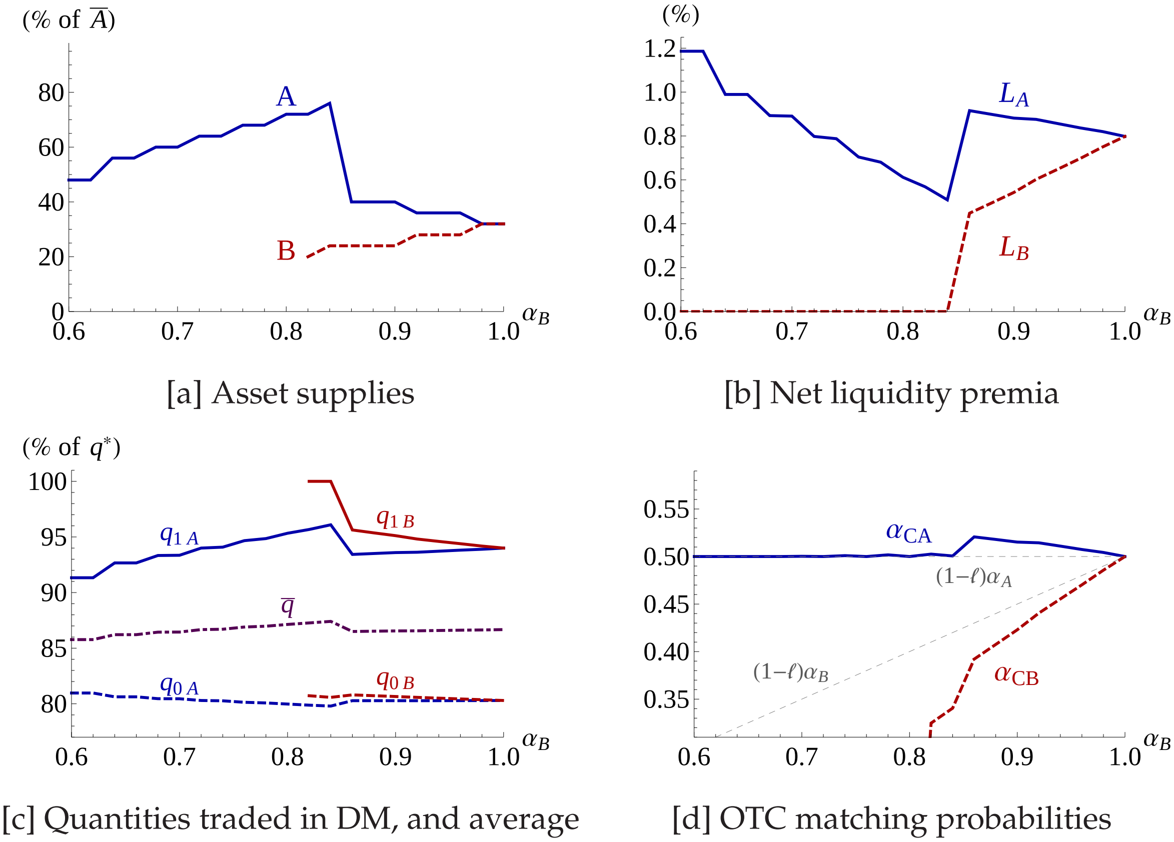
being liquid – and this transition is anything but smooth, but happens all of a sudden around a critical value of ρ.

To summarize: with a large enough degree of IRS in financial markets, the structure of the game resembles Bertrand competition rather than Cournot. This supports corner equilibria, where one asset ends up being very liquid and the other one not liquid at all. As one would expect of a static Bertrand game, issue sizes become very large; however, this might not be the case for a richer model of dynamic competition. One could imagine that if issuer A succeeds in concentrating trade in the A-corner for enough periods, buyers might eventually 'learn' that A is more liquid than B and form a habit of selecting the A-corner no matter how much B issues. This would permit A to issue the monopoly quantity from that point on.

3.3 Comparative statics

In this section, we analyze the comparative statics of the strategic equilibria described with respect to α_B, considering both CRS in matching and small amounts of increasing returns. Our goal is to understand what happens if one of the assets (A for concreteness) has an exogenous liquidity advantage, and how the answer to this question interacts with the degree of increasing returns in matching. Throughout this section, we hold $\alpha_A = 1$ fixed.

One thing to be aware of is how we compute the Nash Equilibria of the game between the issuers. We iterate best responses of the two issuers on a finite grid of possible asset supplies which excludes asset supplies which we know can never give positive payoffs: zero and supplies exceeding $\bar{D}$. The starting point is the smallest positive asset supply on the grid (e.g., the



[a] Asset supplies

[b] Net liquidity premia

[c] Quantities traded in DM, and average

[d] OTC matching probabilities

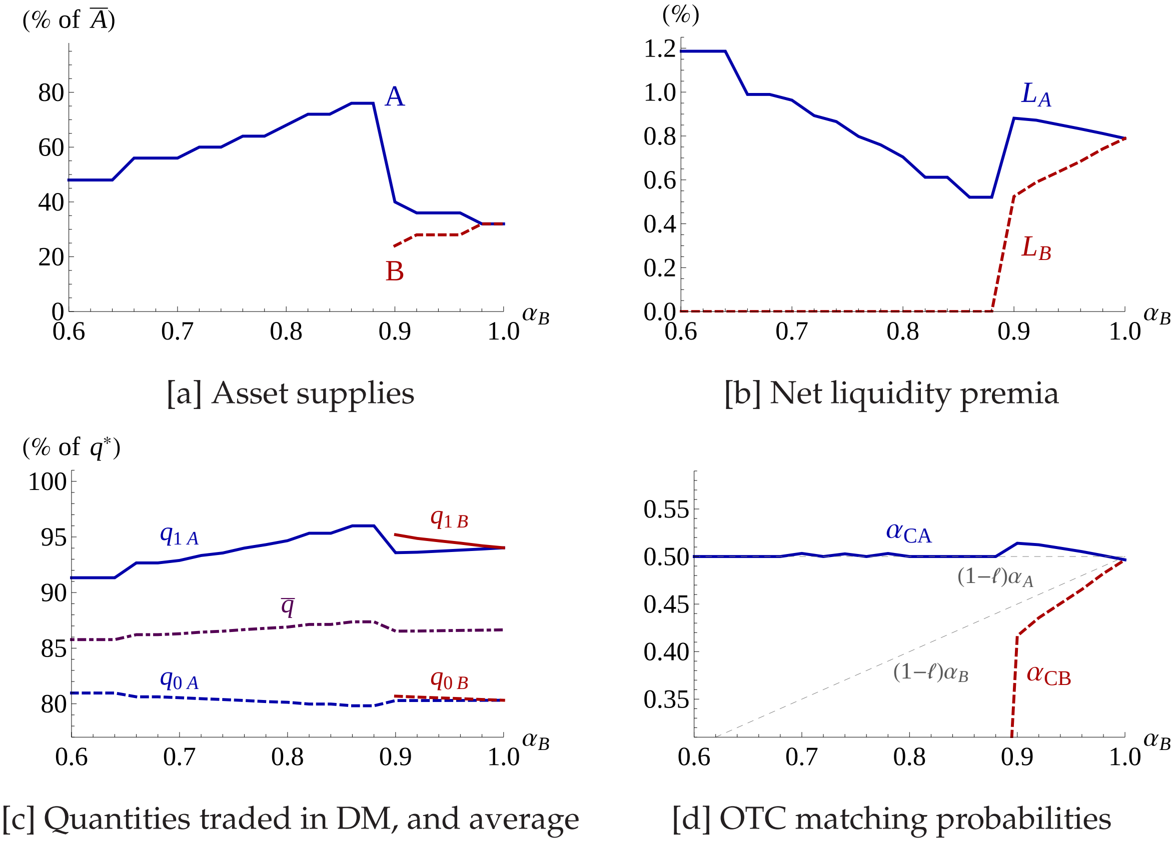
Figure 7: Comparative statics of the strategic equilibria with respect to α_B, with CRS ($\rho = 0$).

point $(0.05\bar{D}, 0.05\bar{D})$ on a 20×20-grid). The remaining choice is whether we let A or B move first; in this section, all equilibria are computed with A moving first.[28]

The comparative statics of the strategic equilibria with respect to α_B, with $\alpha_A = 1$ and CRS ($\rho = 0$), are illustrated in Figure 7. As α_B declines slightly from 1 (the balanced case), A begins to issue more and B begins to issue less (panel [a]), but the strategic pattern of a Cournot game is maintained. The exogenous liquidity advantage of asset A is magnified by the entry choices of agents (panel [d]), which feeds back into a rising liquidity premium on asset A and a falling liquidity premium on asset B (panel [b]). Outputs diverge: C-types who hold asset A end up purchasing smaller quantities q_{0A} and q_{1A}, but the probability that they will obtain the larger one of the two, q_{1A}, increases. Conversely, C-types who still hold asset B despite its liquidity disadvantage are compensated with higher quantities q_{0B} and q_{1B} (panel [c]).

As α_B declines further, we observe a discontinuity. At some point, the benefit to A from ramping up the issue size all the way to drive out B from the financial markets becomes too strong, so this is exactly what happens. Asset B becomes fully illiquid, and therefore its issue

[28] The equilibria where B moves first are usually identical or payoff-identical, the main difference being that the algorithm is slightly more likely to find the A-corner in this case, for intermediate values of α_B. In Figures 5 and 6, Nash Equilibria where A moves first are indicated with a blue dot, and those where B moves first are indicated with a green dot. Our approach to finding the asset demand choices of private agents is described in Footnote 25.



[a] Asset supplies

[b] Net liquidity premia

[c] Quantities traded in DM, and average

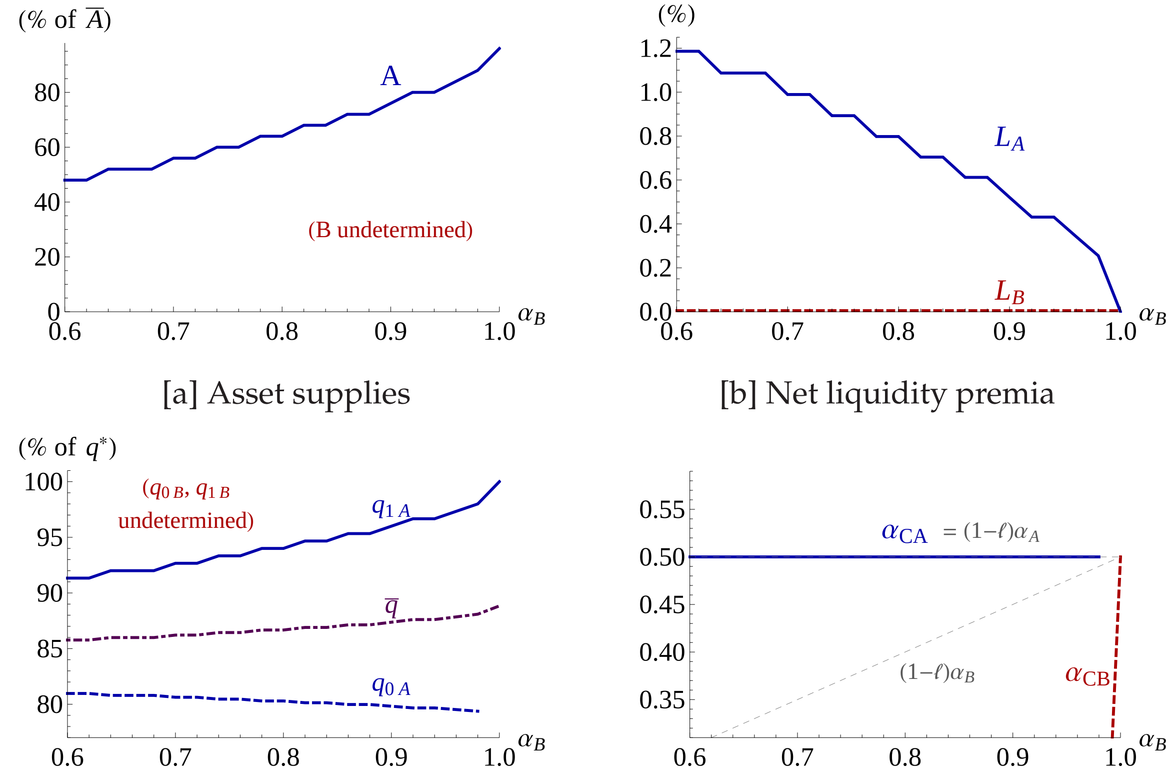
[d] OTC matching probabilities

Figure 8: Comparative statics with respect to α_B, with just the tiniest bit of IRS ($\rho = 0.01$).

size and the quantities q_{0B} and q_{1B} become indeterminate. As a result of this aggressive competition, total output of DM goods is highest at the discontinuity. If α_B declines even more, the threat of trading asset B gradually diminishes; eventually, A becomes a monopolist who issues an intermediate quantity of asset A and total output declines to its lowest value. (Welfare is a more complicated story, as we explain in Section 3.4 below.)

When we allow for a very small degree of increasing returns in matching, $\rho = 0.01$ (illustrated in Figure 8), the results are almost identical to those with CRS, as one might expect given that ρ is so close to zero. Even so, we can see that the transition from the interior equilibrium to the A-corner where asset B is illiquid (and its supply is indeterminate) happens 'sooner', i.e., for a higher value of α_B than under CRS. Increasing returns make it slightly easier for A to drive B out of the market: in the example, A will do so for $\alpha_B = 0.87$ under $\rho = 0.01$ but not under CRS.

It would be a natural guess that when increasing returns are strong enough, the Cournot-style equilibrium is completely eliminated in favor of aggressive competition for secondary market liquidity. But *how* strong do they need to be? Our perhaps surprising answer is: "not very". In fact, as Figure 9 illustrates, the transition occurs somewhere between $\rho = 0.01$ and $\rho = 0.02$; in the latter case, even with a relatively tiny degree of IRS, competition for liquidity



[a] Asset supplies

[b] Net liquidity premia

[c] Quantities traded in DM, and average

[d] OTC matching probabilities

Figure 9: Comparative statics with respect to α_B, with a little bit more IRS ($\rho = 0.02$).

is fierce and only one market is open in all strategic equilibria. However, this does not mean that the exogenous liquidity parameter α_B stops mattering. When $\alpha_B = \alpha_A = 1$, an issuer who wishes to capture the secondary market must issue the quantity $\bar{D}$ which eliminates any payoff, akin to Bertrand competition. But as α_B declines, so does the threat of B's competition. As a consequence, as α_B declines, the equilibrium asset supply A falls. Consequently, the quantity q_{1A} of DM trade for C-types who traded at full surplus in the OTC market also falls but the quantity q_{0A} of DM trade for C-types who did not trade rises (because agents self-insure against the lower trading surplus by holding more money). On balance, and in our particular example, the effect on q_{1A} dominates so that if α_B declines, total DM production falls along with the supply of liquid assets.

3.4 The relationship between asset supplies, output, and welfare

It is well known that in monetary-liquidity models of this kind, average output and welfare do not necessarily move in the same direction. And neither of them obeys a general relationship with the supply of liquid assets. For example, consider the corner equilibrium where only the OTC market for asset A is open (or assume for a moment that asset A is the only asset).

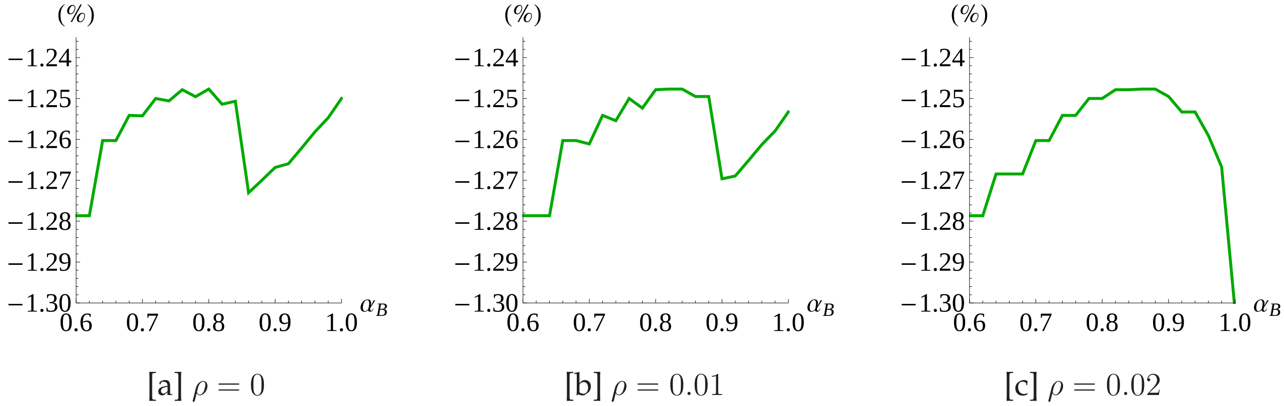


Figure 10: Welfare as a function of α_B, measured as equivalent CM consumption, in percent deviations from the first-best.

According to a recent result by Herrenbrueck and Geromichalos (2015) and Huber and Kim (2015), it can be shown that welfare is a *decreasing* function of the asset supply in a neighborhood $(\bar{A} - \epsilon, \bar{A})$. Why? First, note that as A increases (but is still below $\bar{A}$), q_{0A} falls and q_{1A} rises, so the effect on average output is ambiguous and depends on parameters. However, the welfare effects of these changes are weighted by the marginal utility term $u'(q) - 1$. So if A is close to $\bar{A}$, then $u'(q_{1A})$ is close to $u'(q^*) = 1$: the welfare gain to successful traders vanishes but the welfare loss to unsuccessful traders does not, and the overall welfare effect must be negative. This is visualized by taking together panels [a] and [c] of Figure 9, showing increasing asset supply and output near $\bar{A}$, with panel [c] of Figure 10, which shows the welfare drop.

However, no result currently exists that would tell us what happens for low levels of asset supply. In our numerical example, we can see that when α_B varies in the range $[0.6, 0.8]$, then the supply of liquid assets (only A in this case), average output, and welfare move in the same direction. However, we know that other outcomes are possible: for example, in Herrenbrueck and Geromichalos (2015) we showed that if u is quadratic and OTC trade is seller-take-all, then output is constant and welfare is monotonically decreasing in the supply of liquid assets.

What does this imply for the relationship between market microstructure and welfare? First, using the fact that an asset supply close to $\bar{D}$ is always 'too much' from a welfare perspective, we would argue that any condition that leads to aggressive competition among the asset issuers is best avoided. In particular, the general intuition that Bertrand competition tends to be good for social welfare is not valid when it comes to liquid assets. The same reasoning would apply when matching is CRS and we compare a Cournot oligopoly of few versus many competitors.

Second, there is much less to say when we are far from the aggressive "everyone issues $\bar{D}$" case. In our figures, for example, a duopoly is better for welfare than a monopoly, but we have already explained that this is not robust to other parameters.

Third, and perhaps surprisingly, the effect of the exogenous 'market quality' parameter α_B on welfare is not monotonic. In fact, for IRS and $\alpha_B \approx \alpha_A$, the effect is negative. For CRS, we have shown that $\alpha_B \ll \alpha_A$ promotes a monopoly and $\alpha_B \approx \alpha_A$ promotes a Cournot duopoly, but it is *intermediate* values of α_B that promote the most aggressive competition, the largest supply of liquid assets, and low welfare. It is also important to recognize that very little of the pattern in Figure 10 is due to the direct effect of α_B on trading frequency, as the [d]-panels of Figures 7-9 show: asset B is endogenously illiquid for $\alpha_B < 0.9$, no OTC trade in that asset actually takes place, but the threat that it might still affects the equilibrium.

3.5 Closed-form solutions in the special case of balanced CRS

As we showed in Section 3.2 above, in the special case of $\rho = 0$ and $\alpha_A = \alpha_B \equiv \alpha$ there is an equilibrium of the economy with $H_C = H_N = A/(A+B)$, and a symmetric solution of the variables $q_{0A} = q_{0B}$, $q_{1A} = q_{1B}$, and $p_A = p_B$. So we can drop the asset subscript for the rest of this section. If we further assume that $u(q) \equiv \log(q)$, which normalizes the first-best level of DM production to $q^* = 1$, there is a closed-form solution both for the subgame and for the Cournot-Nash equilibrium of the issuers.

First, define the parameter $\kappa \equiv (1-\ell)\alpha\lambda$, which summarizes financial market liquidity from a C-type's point of view. (The $(1-\ell)$-term is the measure of N-types in the economy, and it enters here through the CRS matching function.) The upper bound on the overall asset supply where assets become abundant in OTC trade is $\bar{D} \equiv i/[\ell(1-\kappa)]$, and for fixed asset supplies which satisfy $A + B < \bar{D}$, we obtain the equilibrium:

$$
\begin{aligned}
q_0 &= \frac{1 - \kappa + \kappa \frac{M}{M+A+B}}{1 + i/\ell} \\
q_1 &= \frac{1 + (1-\kappa)\frac{A+B}{M}}{1 + i/\ell} \\
p &= \frac{1}{1+i} \times \left(1 + \ell\kappa \frac{i/\ell - (1-\kappa)\frac{A+B}{M}}{(1-\kappa)\frac{M+A+B}{M} + \kappa} \right)
\end{aligned}
$$

For large enough asset supplies, i.e. $A + B \geq \bar{D}$, it is easy to solve for $q_0 = [1+\bar{D}]^{-1}$, $q_1 = 1$, and $p = 1/(1+i)$.

This formula for the asset price is not quite linear in the asset supplies, but it is not too badly behaved. So simple calculus gives us the asset supplies and prices in a static Cournot-Nash equilibrium:

$$
A = B = \frac{M}{8(1-\kappa)} \left(i/\ell - 3 + \sqrt{(1+i/\ell)(9+i/\ell)} \right)
$$

$$
p = \frac{1}{1+i} \times \left(1 + \ell\kappa \, \frac{3(1+i/\ell) - \sqrt{(1+i/\ell)(9+i/\ell)}}{1 + i/\ell + \sqrt{(1+i/\ell)(9+i/\ell)}} \right)
$$

And using L'Hospital's rule, the net liquidity premium $L \equiv (1+i)p - 1$ can be approximated by $\kappa i/3$ for small values of i. This reflects the fact that in a Cournot equilibrium with a linear demand curve and two competitors, each will issue one-third of the maximal profitable quantity and the resulting profit margin will be one-third of the maximal profit margin. Here, the demand curve is not linear, but close enough for low i. The maximal 'profit margin' is κi; i measures the inflation wedge and therefore the need for liquidity, and κ measures the extent to which a financial asset which cannot be used as money, but can be traded for money in an OTC market, can satisfy this need for liquidity.

In fact, we can extend this special case of the model to $N > 2$ issuers in the natural way. As long as we maintain balanced CRS in financial markets, the symmetric Cournot-Nash equilibrium exists and is tractable. In that equilibrium, a fraction $1/N$ of both C-types and N-types trades each particular asset, each issuer issues approximately $1/(N+1)$ of the maximal profitable quantity $\bar{D}$, and the resulting net liquidity premium will be approximately equal to $\kappa i/(N+1)$. Total output and welfare can be computed and analyzed as needed.

4 Conclusion

We develop a model of endogenous determination of the supply of assets whose liquidity properties and, hence, equilibrium prices depend both on the exogenous characteristics (or the microstructure) of the secondary markets where these assets trade, and on the endogenous entry decisions of buyers and sellers of assets. We study the game played between two issuers of assets, allowing for asset differentiation, which reflects differences in the microstructure of the secondary OTC market where each asset is traded, and which the respective issuer cannot control. Assuming CRS in the matching technology tends to make asset supplies strategic substitutes. In this case, the outcome of the issue game resembles a Cournot game, in the sense that asset supplies are low and the prices of both assets include liquidity premia. We also explore the possibility of IRS in the matching technology (an assumption considered plausible in the theoretical finance literature). With IRS the outcome of the game resembles a Bertrand game, in the sense that asset supplies are large, and the severity of competition can lead to situations where only one OTC market operates, and only the issuer of that asset enjoys liquidity rents.

Studying the endogenous and strategic determination of asset supply offers a number of new insights. We show that even for modest degrees of increasing returns, asset demand curves can be upward sloping because IRS encourages market concentration and agents are more likely to concentrate in market of an asset with plentiful supply. We also show that small differences in the microstructure of an OTC market can be magnified into a big endogenous liquidity advantage for one asset, because traders would prefer to be in the thick market, and through their own entry help make it even thicker. The model has a number of fruitful applications, including the superior liquidity of US federal debt over municipal and corporate debt. For instance, our model can explain how small exogenous advantages of Treasury debt can be magnified into big liquidity differences. In fact, from a welfare perspective, market segmentation and big liquidity differentials may be good: aggressive competition for secondary market liquidity tends to produce asset supplies that we know are too large to be optimal.

References

Afonso, G. and R. Lagos (2015). Trade dynamics in the market for federal funds. *Econometrica 83*(1), 263–313.

Andolfatto, D., A. Berentsen, and C. Waller (2013). Optimal disclosure policy and undue diligence. *Journal of Economic Theory*.

Andolfatto, D. and F. M. Martin (2013). Information disclosure and exchange media. *Review of Economic Dynamics 16*(3), 527–539.

Andolfatto, D., F. M. Martin, and S. Zhang (2015). Rehypothecation and liquidity. *FRB St Louis Paper No. FEDLWP2015-003*.

Arseneau, D. M., D. Rappoport, and A. Vardoulakis (2015). Secondary market liquidity and the optimal capital structure. *Available at SSRN 2594558*.

Berentsen, A., G. Camera, and C. Waller (2007). Money, credit and banking. *Journal of Economic Theory 135*(1), 171–195.

Berentsen, A., S. Huber, and A. Marchesiani (2014). Degreasing the wheels of finance. *International economic review 55*(3), 735–763.

Berentsen, A., S. Huber, and A. Marchesiani (2015). The societal benefit of a financial transaction tax. *University of Zurich, Department of Economics, Working Paper* (176).

Chang, B. and S. Zhang (2015). Endogenous market making and network formation. *Available at SSRN 2600242*.

Chiu, J. and T. Koeppl (2011). Trading dynamics with adverse selection and search: Market freeze, intervention and recovery.

Duffie, D., N. Gârleanu, and L. H. Pedersen (2005, November). Over-the-counter markets. *Econometrica 73*(6), 1815–1847.

Fernández-Villaverde, J. and D. Sanches (2016). Can currency competition work? Technical report, National Bureau of Economic Research.

Ferraris, L. and M. Watanabe (2011). Collateral fluctuations in a monetary economy. *Journal of Economic Theory 146*(5), 1915–1940.

Geromichalos, A. and L. Herrenbrueck (2012). Monetary policy, asset prices, and liquidity in over-the-counter markets. Technical report, Working Papers, University of California, Department of Economics.

Geromichalos, A., L. Herrenbrueck, and K. Salyer (2013). A search-theoretic model of the term premium. Technical report, Working Papers, University of California, Department of Economics.

Geromichalos, A. and K. M. Jung (2016). Monetary policy and efficiency in over-the-counter financial trade. Technical report, Working Papers, University of California, Department of Economics.

Geromichalos, A., J. Lee, S. Lee, and K. Oikawa (2015). Over-the-counter trade and the value of assets as collateral. *Economic Theory 1–33. DOI:10.1007/s00199-015-0904-9*.

Geromichalos, A., J. M. Licari, and J. Suarez-Lledo (2007, October). Monetary policy and asset prices. *Review of Economic Dynamics 10*(4), 761–779.

Han, H. (2015). Over-the-counter markets, intermediation, and monetary policy.

Herrenbrueck, L. (2014). Quantitative easing and the liquidity channel of monetary policy. Technical report.

Herrenbrueck, L. and A. Geromichalos (2015). A tractable model of indirect asset liquidity. Technical report.

Hu, T.-W. and G. Rocheteau (2015). Monetary policy and asset prices: A mechanism design approach. *Journal of Money, Credit and Banking 47*(S2), 39–76.

Huber, S. and J. Kim (2015). Central bank purchases of government bonds. *University of Zurich, Department of Economics, Working Paper* (193).

Jacquet, N. L. and S. Tan (2012). Money and asset prices with uninsurable risks. *Journal of Monetary Economics 59*(8), 784–797.

Lagos, R. (2010, November). Asset prices and liquidity in an exchange economy. *Journal of Monetary Economics 57*(8), 913–930.

Lagos, R. (2011, October). Asset prices, liquidity, and monetary policy in an exchange economy. *Journal of Money, Credit and Banking 43*, 521–552.

Lagos, R. and G. Rocheteau (2008, September). Money and capital as competing media of exchange. *Journal of Economic Theory 142*(1), 247–258.

Lagos, R. and G. Rocheteau (2009). Liquidity in asset markets with search frictions. *Econometrica 77*(2), 403–426.

Lagos, R., G. Rocheteau, and P.-O. Weill (2011). Crises and liquidity in over-the-counter markets. *Journal of Economic Theory 146*(6), 2169–2205.

Lagos, R., G. Rocheteau, and R. Wright (2015). Liquidity: A new monetarist perspective. *Journal of Economic Literature, forthcoming*.

Lagos, R. and R. Wright (2005, June). A unified framework for monetary theory and policy analysis. *Journal of Political Economy 113*(3), 463–484.

Lagos, R. and S. Zhang (2015). Monetary exchange in over-the-counter markets: A theory of speculative bubbles, the fed model, and self-fulfilling liquidity crises. Technical report, National Bureau of Economic Research.

Lester, B., A. Postlewaite, and R. Wright (2012). Information, liquidity, asset prices, and monetary policy. *The Review of Economic Studies 79*(3), 1209–1238.

Lucas, Robert E, J. (1978, November). Asset prices in an exchange economy. *Econometrica 46*(6), 1429–45.

Manski, C. (1982). Equilibrium in second hand markets. Technical report, Mimeo.

Mattesini, F. and E. Nosal (2015). Liquidity and asset prices in a monetary model with otc asset markets. *Journal of Economic Theory*.

Nosal, E. and G. Rocheteau (2012). Pairwise trade, asset prices, and monetary policy. *Journal of Economic Dynamics and Control*.

Rocheteau, G. (2011). Payments and liquidity under adverse selection. *Journal of Monetary Economics 58*(3), 191–205.

Rocheteau, G. and R. Wright (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica 73*(1), 175–202.

Rocheteau, G. and R. Wright (2013). Liquidity and asset-market dynamics. *Journal of Monetary Economics 60*(2), 275–294.

Rust, J. (1985). Stationary equilibrium in a market for durable assets. *Econometrica: Journal of the Econometric Society*, 783–805.

Rust, J. (1986). When is it optimal to kill off the market for used durable goods? *Econometrica: Journal of the Econometric Society*, 65–86.

Swan, P. L. (1972). Optimum durability, second-hand markets, and planned obsolescence. *Journal of Political Economy 80*(3), 575–585.

Üslü, S. (2015). Pricing and liquidity in decentralized asset markets. Technical report, Technical report, UCLA.

Vayanos, D. and T. Wang (2007). Search and endogenous concentration of liquidity in asset markets. *Journal of Economic Theory 136*(1), 66–104.

Vayanos, D. and P.-o. Weill (2008). A search-based theory of the on-the-run phenomenon. *The Journal of Finance 63*(3), 1361–1398.

Venkateswaran, V. and R. Wright (2013). Pledgability and liquidity: a new monetarist model of financial and macroeconomic activity. Technical report, National Bureau of Economic Research.

Weill, P.-O. (2007). Leaning against the wind. *The Review of Economic Studies 74*(4), 1329–1354.

Williamson, S. D. (2015). Scarce collateral, the term premium, and quantitative easing. *Journal of Economic Theory doi:10.1016/j.jet.2015.07.010*.