# What determines the direction of technological progress?

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# What Determines the Direction of Technological Progress?

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**Abstract**

What are the key determinants of the direction of technological progress is of central importance for many problems in macroeconomics. In the existing literature, the changing relative production factor prices as suggested by Hicks (1932) and the relative market sizes as indicated by Acemoglu (2002) are considered as the two major determinants. However, by allowing for adjustment costs in factor accumulation processes to expand Acemoglu's (2003) model, this paper argues that, at least in the steady-state equilibrium, the direction of technological progress may be due to neither of them, but to the relative size of material factor price elasticities, and is biased towards the factor with the relatively smaller elasticity. **In addition**, contrary to the Uzawa(1961) steady-state theorem, this paper demonstrates that along a steady-state equilibrium path, technological progress can simultaneously include labor- and capital-augmenting elements alongside with unchanged factor income shares. **Furthermore**, this paper identifies more general conditions for the existence of a steady-state equilibrium of which Uzawa's theorem obtains as a special case. **Based on these results**, the paper argues that technological progress may have not included labor-augmentation during the preindustrial era because labor supply was infinitely elastic with respect to wages, and no capital-augmentation after the industrial revolution because of the high capital supply elasticity with respect to the interest rate.

**Key Words:** steady-state, direction of technical change, Uzawa's steady-state theorem, price elasiticities, factor income shares, adjustment cost

**JEL:** E13; O33; O11; Q01

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## 1 Introduction

According to the summary of Kaldor (1961), the stylized characteristics of economic growth in developed countries indicate that while per-capita output and physical capital have grown over time, the ratio of physical capital to output and the income shares of labor and physical capital have remained basically constant since the industrial revolution. These characteristics have been associated with the claim that technological progress is purely labor-augmenting. In contrast, Ashraf and Galor (2011) show that during the preindustrial era, technological progress had resulted in larger populations and higher density, but not in higher per-capita income. These characteristics indicate that technological progress included hardly any labor-augmentation before the industrial revolution. Why was the nature of technological progress so different before and after the industrial revolution? What are the determinants of the direction of technological progress and its change? This paper proposes a model which endogenizes the direction of technological progress in an attempt to provide some answers.

The economy analyzed below uses a standard neoclassical production function with labor and capital. The quantity or quality growth rates of the respective factors are the result of intentional investments by economic agents. Technological progress is measured by the rate at which these inputs improve, and the direction of technological progress is represented by the relative pace of these improvements.

The paper proves that under certain conditions there exists an equilibrium path regardless of whether decisions are made within a decentralized market environment or in a socially centralized manner. It provides very simple and clear conclusions concerning the direction of technological progress. Specifically, it shows that the direction of technological progress is neither determined by the change of relative factor prices as suggested by Hicks (1932) nor by the relative market size as argued by Acemoglu (2002). Rather, that direction depends on the relative supply elasticities of material factors with respect to their respective prices, and is biased towards the factor with the relatively smaller elasticity. That is, whether technological progress tends to be capital- or labor-augmenting is not determined by the change in the relative prices itself, but by the relative sensitivity of any material factor accumulation to its own price. The type of technology that augments the factor which is less sensitive to its price change will progress faster.

The intuition behind this result is the following. A higher factor price encourages not only invention but also factor accumulation. If the supply elasticity of one of the factors is very large, it may not be optimal to develop an invention that economizes the use of that factor when its price is relatively increasing in the short run. Accordingly, technological progress is affected by the relative supply elasticities and not the change in the relative price *per-se*. To obtain balanced growth, it is necessary to invest more resources in the development of factor-saving technologies in the factor that has the smaller supply elasticity. If these elasticities are identical for the two factors, technological progress will be equally economical in both. In extreme cases, when a factor has infinite supply elasticity, it is not necessary to invest resources to develop any economizing technology for that factor. If both factors are supplied with infinite elasticities, there is no need to invest resources in innovation at all.

With this intuition in mind, the paper gives the following answers to the aforementioned questions. In the pre-industrial era technological progress was not labor-augmenting because labor supply was very elastic (as described by Malthus). After the industrial revolution, the demographic transition reduced the supply elasticity of labor. Moreover, land was replaced by reproducible physical capital. As the natural resources needed for the production of capital were almost unlimited, the elasticity of capital accumulation with respect to its price became very large. However, this situation is changing as more developing countries started industrializing, putting ever growing pressure on natural resources and the environment. Consequently, the model predicts that in the future technological progress will include more and more capital-augmenting elements.

The paper also draws some conclusions that differ from those found in the existing literature. First, along a steady-state equilibrium path, technological progress can include both labor- and capital-augmenting elements while factor income shares remain unchanged. This stands in contrast the Uzawa (1961) theorem which says that only purely labor-augmenting elements can be present. Second, as capital augmentation can be consistent with stabile factor income shares, that technical change maybe not be the reason for the worldwide decline in labor shares during last few decades.

The ideas in this paper are closely related to previous literature. As early as in 1932, Hicks (1932) wrote: "A change in the relative prices of the factors of production is itself a spur to invention, and to invention of a particular kind-directed to economizing the use of a factor which has become relatively expensive" (pp. 124-125). However, as noted by Kennedy (1964), innovation faced not only the incentive created by relative factor prices but also the constraints of the "innovation possibility frontier". Based on Kennedy, Samuelson (1965) and Drandakis and Phelps (1966) built growth models to formalize the contribution of the induced innovations idea, whereby firms choose their technologies to maximize the current rate of cost reduction. However, this literature was criticized for its lack of micro-foundations. Consequently, for almost thirty years there was little research on the direction of technological progress. Only the work of Acemoglu (1998, 2002, 2003, 2007, and 2009) which studied the issue using the framework of endogenous technological change (as developed by Romer, 1990, and Aghion and Howitt, 1992) has renewed interest in this question. In contrast to the papers of the 1960s, Acemoglu's models start from a microeconomic model of technical change, where innovations are carried out by profit-maximizing firms. Funk (2002) and Irmen (2015) also study the determinants of technological progress within perfectly competitive environments. However, none of these papers takes into account the impact of adjustment costs. As a result, they are bound by the Uzawa (1961) theorem, whereby the steady-state direction of technological progress must be purely labor augmenting.

Many have noted that the Uzawa theorem lacks economic intuition (Aghion and Howitt, 1998, p16; Acemoglu, 2003, 2009; Jones, 2005; Jones and Scrimgeour, 2008). Schlicht (2006) provides a very simple proof of the theorem, from which it becomes clear that the absence of adjustment costs in the capital accumulation equation is the key to the result. However, the literature (Eisner and Strotz, 1963; Lucas, 1967; Foley and Sidrauski, 1970; Mussa, 1977) on adjustment costs points out that an investment function without such costs is not realistic, and leads to some counterfactual results when used to analyze macroeconomic problems. Adjustment costs for investment have been incorporated in macroeconomics textbooks and economic growth theory (Barro and Sala-i-Martin, 2004; Romer, 2006; Acemoglu, 2009), without addressing the impact of adjustment costs on the direction of technological progress. Sato and Ramachandran (2000) prove that if capital accumulation is a nonlinear function of investment, technological progress is not purely labor-augmenting along a steady-state path. However, they do not point out the relationship between nonlinear investment and adjustment costs. Li and Huang (2012, 2015) and Irmen (2013) note that the use of nonlinear investment functions provides one form of modeling adjustment costs. The former using the Ramsey (1928) framework, and the latter using the Schlicht (2006) method prove that including adjustment costs in the capital accumulation process, technological progress can include both labor- and capital-augmenting elements in steady-state. However, these papers do not consider the role of adjustment costs under a growth model with endogenous technological progress, so they also do not discuss the determinants of the direction of technological progress.

The rest of the paper is organized as follows. The second section describes the economic environment of the benchmark model, and analyses the behavior of households and firms; The third section provides the determinants of the direction of technological progress; The fourth section discusses the direction of technological progress when material factors have infinite supply elasticites; The fifth section derives the direction of technological progress in a social planning equilibrium; The sixth section discusses the direction of technological progress under assumptions that differ from those of the benchmark model; The seventh section concludes.

## 2. Benchmark model

### 2.1 Economic Environment

The economic environment of the model is an extension of Acemoglu (2003). The economy consists of two kinds of material factors, denoted by K and L,[3] and three sectors of production; a final goods sector, an intermediate goods sector and a research and development (R&D) sector. The preference structure and production functions are identical to Acemoglu's. However, the current analysis differs from that of Acemoglu's in the factor accumulation functions and the innovation possibilities frontier.

#### 2.1.1 Final good production

The aggregate production function is given by

$$Y = \left[\gamma Y_L{}^{(\varepsilon-1)/\varepsilon} + (1-\gamma) Y_K{}^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \quad ,0 \le \varepsilon < \infty \tag{1}$$

where Y is an aggregate output produced from inputs produced by labor-intensive and capital-intensive processes, respectively $Y_L$ and $Y_K$, and the factor-elasticity of substitution is given by ε, with $0 < \varepsilon < +\infty$.

The labor-intensive and capital-intensive inputs are produced competitively using identical constant elasticity of substitution (CES) production functions with corresponding intermediate inputs, X(i) and Z(i):

$$Y_L = \left[\int_0^N X(i)^\beta di\right]^{1/\beta} \quad and \quad Y_K = \left[\int_0^M Z(i)^\beta di\right]^{1/\beta} \quad , 0 < \beta < 1 \tag{2}$$

where the elasticity of substitution is given by $v = 1/(1-\beta)$, where β determines the monopoly power of the intermediate product producers: the smaller β is, the greater the monopoly power

[3]According to the context of any application they can be respectively capital and labor, skilled and unskilled labor, physical capital and human capital, etc.

becomes. Here N and M represent the measure of different types of labor- and capital-intensive intermediate inputs, respectively. As will be seen below, an increase in N or in M corresponds to a labor- or capital-augmenting technical change.

**2.1.2 Intermediate input production**

Intermediate inputs are supplied by monopolists who hold the right to use the relevant patent, and are produced linearly from their respective factors:

$$X(i) = L(i) \text{ and } \mathrm{Z(i)} = \mathrm{K(i)} \tag{3}$$

**2.1.3 Accumulation of material factors**

While the above follows precisely the Acemoglu (2003) formulation, the following provides an extension. Specifically, we assume:

$$\begin{cases} \dot{\mathrm{K}} = \mathrm{b}_K I_K^{\alpha_K}, & \mathrm{b}_K > 0, \quad 0 \leq \alpha_K \leq 1 \\ \dot{\mathrm{L}} = \mathrm{b}_L I_L^{\alpha_L}, & \mathrm{b}_L > 0, \quad 0 \leq \alpha_L \leq 1 \end{cases} \tag{4}$$

where $\mathrm{I}_K$ and $\mathrm{I}_L$ are respectively resource investments needed to accumulate K and L.

These material factors accumulation processes are the most important extension of Acemoglu (2003) model in this paper. Acemoglu assumes that $\alpha_K = 1$ which is the usual case of the neoclassical growth model. If $\alpha_L = 1$, then equation (4) implies a Malthusian specification.[4] When $0 < \alpha_{\mathrm{K}} < 1$ and $0 < \alpha_{\mathrm{L}} < 1$, the marginal returns of the investment processes are diminishing. This may reflect, among other things, that investment in either factor is associated with adjustment costs (Li and Huang, 2012, 2015; Irmen, 2013).

**2.1.4 The innovation possibilities frontier**

The technology innovation functions are given by

$$\begin{cases} \dot{\mathrm{M}} = \mathrm{b}_M I_M^{\alpha_M}, & \mathrm{b}_M > 0, \quad 0 \leq \alpha_M \leq \beta/(1-\beta) \\ \dot{\mathrm{N}} = \mathrm{b}_N I_N^{\alpha_N}, & \mathrm{b}_N > 0, \quad 0 \leq \alpha_N \leq \beta/(1-\beta) \end{cases} \tag{5}$$

Where $\mathrm{I_M}$ and $\mathrm{I_N}$ are investments needed to develop new varieties M and N of the respective intermediate inputs. Function (5) describes the experimental equipment ideas put forward by Rivera-Natiz and Romer (1991). When $\alpha_{\mathrm{M}} = 1$ and $\alpha_{\mathrm{N}} = 1$, equation (5) is the same as equation (34) (without depreciation) of knowledge creation in Acemoglu (2003) . While $\alpha_{\mathrm{M}}$ and $\alpha_{\mathrm{N}}$ are not necessary equal to 1, their values are limited by the value of $\beta$.

**2.1.5 The representative household**

The representative household owns material factors such as capital and labor, as well as the indefinite rights over the use of patents of the production of intermediate goods. The household's goal is to maximize the discounted flow of utility, given by:

$$\mathrm{U} = \int_0^\infty \frac{C(t)^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt \tag{6}$$

where $C(t)$ is consumption at time t, $\rho > 0$ is the discount rate, and $\theta > 0$ is a utility curvature coefficient of the household.

**2.1.6 Budget constraint**

The representative household's income can be used either for consumption or for investment. The latter consists of four options: it can be used to increase the material factors, K and L, or the

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[4] Suppose $\alpha_{\mathrm{L}} = 1$ and let $I_{\mathrm{L}} = sY$. Then equation (4) implies $\dot{\mathrm{L}}/\mathrm{L} = \mathrm{sb}_L \mathrm{y}$, where $\mathrm{sb}_L$ is exogenous. Defining $\mathrm{y} = \mathrm{Y/L}$ to represent per capita income, one obtains that the labor growth rate is proportional to per capita income, which corresponds to the famous Malthusian assumption on population growth.

"number" of intermediate goods of either type. While the material factors K and L are rented in competitive factor markets, the representative household is a monopoly producer of the intermediate goods. Accordingly, the household faces the following budget constraint:

$$\mathrm{C} + \mathrm{I}_K + \mathrm{I}_L + \mathrm{I}_N + \mathrm{I}_M = wL + rK + \int_0^N \pi_{X(i)} di + \int_0^M \pi_{Z(i)} di \tag{7}$$

Where $\mathrm{I} = \mathrm{I}_K + \mathrm{I}_L + \mathrm{I}_N + \mathrm{I}_M$ is total investment, $w$ and $r$ are market rental prices of L and K, $\pi_{\mathrm{X}(i)}$ and $\pi_{\mathrm{Z}(i)}$ are monopoly profits of the respective intermediate inputs. For the sake of simplicity, this paper will ignore corner solutions and assume that consumption and all investments are strictly positive, that is, C > 0, $\mathrm{I}_K > 0$, $\mathrm{I}_L > 0$, $\mathrm{I}_N > 0$ and $\mathrm{I}_M > 0$.

**2.2 Enterprise behavior**

The analysis of enterprise behavior is similar to that of Acemoglu (2003), and only the main results are reported here. Through that analysis, one can obtain the prices of the material factors K and L, and the monopoly profits of each intermediate product.

**2.2.1 Demand for intermediate goods.**

The goods Y, $\mathrm{Y_L}$ and $\mathrm{Y_K}$ are traded in perfectly competitive markets. The final good Y serves as the numeraire, and $p_{\mathrm{L}}$ and $p_{\mathrm{K}}$ are respectively market prices of $\mathrm{Y_L}$ and $\mathrm{Y_K}$. The demand for $\mathrm{Y_L}$ and $\mathrm{Y_K}$ are derived from profit maximization of the final good producers.

$$\begin{cases} \mathrm{p}_K = (1-\gamma)[\gamma + (1-\gamma)(\mathrm{Y}_K/\mathrm{Y}_L)^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)}(\mathrm{Y}_K/\mathrm{Y}_L)^{-1/\varepsilon} \\ \mathrm{p}_L = \gamma[\gamma + (1-\gamma)(\mathrm{Y}_K/\mathrm{Y}_L)^{(\varepsilon-1)/\varepsilon}]^{1/(\varepsilon-1)} \end{cases} \tag{8}$$

Taking the prices, $p_{\mathrm{Z}(i)}$ and $p_{\mathrm{X}(i)}$, of the generic inputs, X(i) and Z(i), as given, demand for these inputs is obtained from profit maximization:

$$\begin{cases} \mathrm{Z(i)} = \mathrm{Y}_K\left(\mathrm{p}_K/\mathrm{p}_{Z(i)}\right)^{1/(1-\beta)} \\ \mathrm{X(i)} = \mathrm{Y}_L\left(\mathrm{p}_L/\mathrm{p}_{X(i)}\right)^{1/(1-\beta)} \end{cases} \tag{9}$$

**2.2.2 Factor market clearing.**

Because intermediate goods are supplied by monopolists who hold the relevant patents, and are produced linearly from their respective factors (see equation 3), we can obtain the price of intermediate inputs from the profit maximization conditions of the monopolies:

$$\begin{cases} \mathrm{p}_{Z(i)} = \mathrm{r}/\beta \\ \mathrm{p}_{X(i)} = \mathrm{w}/\beta \end{cases} \tag{10}$$

Equations (10) indicate that each of the intermediate inputs has the same mark-up over marginal cost. Substituting (10) into (9), we find that all capital-intensive and all labor-intensive intermediate goods are produced in equal (respective) quantities.

$$\begin{cases} \mathrm{Z(i)} = \mathrm{Z} = \mathrm{Y}_K(\beta\mathrm{p}_K/\mathrm{r})^{1/(1-\beta)} \\ \mathrm{X(i)} = \mathrm{X} = \mathrm{Y}_L(\beta\mathrm{p}_L/\mathrm{w})^{1/(1-\beta)} \end{cases} \tag{11}$$

By the production functions of the intermediate inputs (3), the monopolists' demand for labor and capital are respectively equal. The material factor market clearing condition implies:

$$\begin{cases} \mathrm{Z(i)} = \mathrm{K/M} \\ \mathrm{X(i)} = \mathrm{L/N} \end{cases} \tag{12}$$

Substituting equations (12) into (2), we obtain the equilibrium quantities of labor-intensive and capital-intensive goods:

$$\begin{cases} Y_L = \left[\int_0^N X(i)^\beta di\right]^{1/\beta} = N^{(1-\beta)/\beta} L \\ Y_K = \left[\int_0^M Z(i)^\beta di\right]^{1/\beta} = M^{(1-\beta)/\beta} K \end{cases} \tag{13}$$

Finally, substituting equations (13) into (1), we obtain the amount of the final good produced:

$$Y = \left[\gamma(N^{(1-\beta)/\beta} L)^{(\varepsilon-1)/\varepsilon} + (1-\gamma)(M^{(1-\beta)/\beta} K)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \tag{14}$$

In order to simplify notation, we follow Acemoglu (2003) by letting $A \equiv N^{(1-\beta)/\beta}$and $B \equiv M^{(1-\beta)/\beta}$, to obtain:

$$Y = \left[\gamma(AL)^{(\varepsilon-1)/\varepsilon} + (1-\gamma)(BK)^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \tag{15}$$

Therefore, increasing the variety of capital-intensive or labor-intensive intermediate goods, M and N, implies progress of the capital-augmenting or labor-augmenting technologies B and A.

Let $k \equiv BK/AL$ be the ratio of effective capital to effective labor, then

$$k = (M^{(1-\beta)/\beta} K)/(N^{(1-\beta)/\beta} L) \tag{16}$$

and (15) can be rewritten as:

$$f(k) \equiv Y/AL = \left[\gamma + (1-\gamma)k^{(\varepsilon-1)/\varepsilon}\right]^{\varepsilon/(\varepsilon-1)} \tag{17}$$

Using equation (17), we transform the market prices of the capital-intensive and labor-intensive goods (8) into the following forms:

$$\begin{cases} p_K = f'(k) \\ p_L = f(k) - kf'(k) \end{cases} \tag{18}$$

Substituting equation (18), (13), and (12) into (11), we have

$$\begin{cases} r = \beta M^{(1-\beta)/\beta} f'(k) \\ w = \beta N^{(1-\beta)/\beta}[f(k) - kf'(k)] \end{cases} \tag{19}$$

Equations (19) indicate that the prices of the material factors are positively related to the respective "number" of the intermediate goods.

By equations (19), (13) and (10), we find the monopoly profits of the intermediate goods producers:

$$\begin{cases} \pi_Z = (p_Z - r)Z = (1-\beta)M^{(1-2\beta)/\beta} K f'(k) \\ \pi_X = (p_X - w)X = (1-\beta)N^{(1-2\beta)/\beta} L[f(k) - kf'(k)] \end{cases} \tag{20}$$

Equations (20) show that there is a positive relationship between the monopoly profits of the intermediate inputs and the quantity of material factors. This implies that developing the technology using abundant factor will generate more monopoly profits. Acemoglu (2002) names it "the market size effect" in innovation.

### 2.3 Consumer behavior

Households maximize their objective (6) subject to the budget constraint (7), taking as given the factor accumulation and technological change processes (4) and (5).

The corresponding Euler conditions are given by equations (21) (see Appendix A):

$$\begin{cases} \dot{C}/C = \left[\alpha_K b_K I_K^{\alpha_K - 1} r - (\alpha_K - 1)\dot{I}_K/I_K - \rho\right]/\theta \\ \dot{C}/C = \left[\alpha_L b_L I_L^{\alpha_L - 1} w - (\alpha_L - 1)\dot{I}_L/I_L - \rho\right]/\theta \\ \dot{C}/C = \left[\alpha_M b_M I_M^{\alpha_M - 1} \pi_Z - (\alpha_M - 1)\dot{I}_M/I_M - \rho\right]/\theta \\ \dot{C}/C = \left[\alpha_N b_N I_N^{\alpha_N - 1} \pi_X - (\alpha_N - 1)\dot{I}_N/I_N - \rho\right]/\theta \end{cases} \tag{21}$$

Equations (21) reflect the conditions of the optimal allocation of income among consumption and the four kinds of investment. The first equation in (21) is the necessary condition for the optimal allocation between physic capital investment and consumption. It is worth noting that when $\alpha_K = b_K = 1$, the equation simplifies to the familiar form $\dot{C}/C = (r - \rho)/\theta$. In that environment a constant value of $\dot{C}/C$ implies that *r* must be constant. However, if $0 < \alpha_K < 1$, the rate *r* cannot be constant when $\dot{C}/C$ and $\dot{I}_K/I_K$ are constant, unless $\dot{r}/r = (1 - \alpha_K)\dot{I}_K/I_K$. Thus steady-state growth does not necessarily imply a constant market rental price of capital. The second equation in (21) is the necessary condition for the optimal allocation between labor investment and consumption. The third equation in (21) is the necessary condition for the optimal allocation between investments in new varieties of capital-intensive intermediates and consumption, and the fourth equation in (21) is the necessary condition for the optimal allocation between investments in new varieties of labor-intensive intermediates and consumption. The optimal allocation is achieved when the four equations hold simultaneously. As long as one equation of the formula (21) is not satisfied, the household can obtain a higher level of utility by reallocating its income among consumption and investments.

Finally, the transversality condition is

$$\lim_{t \to \infty} K(t) \exp\left[-\int_0^t r(\upsilon)d\upsilon\right] = 0 \tag{22}$$

### 2.4. Market Equilibrium and Steady-State Equilibrium

#### 2.4.1. Market Equilibrium

**Definition 1:** A *market equilibrium* is obtained when households maximize life-time utility and producers maximize profits, the factor and product markets clear and households meet the Euler equations.

Substituting (18), (19) into the family of Euler equations (21), we obtain the market equilibrium Euler equations:

$$\begin{cases} \dot{C}/C = \left[\alpha_K b_K I_K^{\alpha_K - 1} \beta M^{(1-\beta)/\beta} f'(k) - (\alpha_K - 1)\dot{I}_K/I_K - \rho\right]/\theta \\ \dot{C}/C = \left[\alpha_L b_L I_L^{\alpha_L - 1} \beta N^{(1-\beta)/\beta} [f(k) - kf'(k)] - (\alpha_L - 1)\dot{I}_L/I_L - \rho\right]/\theta \\ \dot{C}/C = \left[\alpha_M b_M I_M^{\alpha_M - 1} (1-\beta) M^{(1-2\beta)/\beta} K f'(k) - (\alpha_M - 1)\dot{I}_M/I_M - \rho\right]/\theta \\ \dot{C}/C = \left[\alpha_N b_N I_N^{\alpha_N - 1} (1-\beta) N^{(1-2\beta)/\beta} L[f(k) - kf'(k)] - (\alpha_N - 1)\dot{I}_N/I_N - \rho\right]/\theta \end{cases} \tag{23}$$

#### 2.4.2. Definition and Existence of Steady-State Equilibrium

**Definition 2:** A *steady-state growth equilibrium* (hereafter SSGE) is a market equilibrium in which the growth rates of the endogenous variables (Y, C, I, $I_K$, $I_L$, $I_M$, $I_N$, K, L, M, N) are nonnegative constants.

The definition is identical to that of most of existing literature (Barro and Sala-i-Martin, 2004; Schlicht, 2006), but slightly different from the definition of a balanced growth path in Acemoglu (2003). Specifically, Definition 2 does not require that the growth rate of K be equal to that of Y and does not require *r* and K/Y to be constant.

Notice that in this model, not only the material factors K and L but also labor- and capital-augmenting technological progress are endogenous. Therefore the model nests several economic models in existing growth literature. From the point of view of labor growth, it encompasses the Malthusian model. From the capital accumulation point of view, it includes the neoclassical growth model. Finally, from the point of view of technological progress, it incorporates a two dimensional endogenous technological progress a-la Romer (1990).

The conditions required for the existence of a steady-state equilibrium are given in Proposition 1.

**Proposition 1**: An SSGE exists only if

$$\begin{cases} \alpha_K + [(1-\beta)/\beta]\alpha_M = 1 \\ \alpha_L + [(1-\beta)/\beta]\alpha_N = 1 \end{cases} \tag{24}$$

**Proof**: See Appendix B.

Because $\alpha_K$、$\alpha_L$、$\alpha_M$、$\alpha_N$ and β all are exogenous parameters, conditions (24) indicate that the model's steady-state equilibrium is a knife edge path. Specifically, the first equation of (24) jointly constrains the parameters of the physical capital accumulation and the capital-augmentation functions, while the second equation constrains the analogous parameters of the labor factor.[5] Knife-edge conditions are common in the growth

From the proposition we can get the follow 2 Lemmas.

**Lemma 1**: In an SSGE,

$$\dot{B}/B = (1-\alpha_K)\dot{Y}/Y \tag{25}$$

**Proof**: See Appendix C.

Lemma 1 shows that if $\dot{Y}/Y > 0$, along an SSGE, a necessary condition for $\dot{B}/B > 0$ is $\alpha_K < 1$. Otherwise, if $\alpha_K = 1$, then $\dot{B}/B$ must be 0. The Uzawa (1961) theorem is a special case of Lemma 1.[6]

**Lemma 2**: In an SSGE,

$$\dot{A}/A = (1-\alpha_L)\dot{Y}/Y \tag{26}$$

**Proof**: See Appendix C.

Lemma 2 shows that if $\dot{Y}/Y > 0$, in an SSGE, a necessary condition for $\dot{A}/A > 0$ is $\alpha_L < 1$. Otherwise, if $\alpha_L = 1$ then $\dot{A}/A$ must be 0. As the Malthusian model also assumes that $\alpha_L = 1$, there is an analogy to the Uzawa steady-state theorem, implying that in this case a steady-state equilibrium cannot include a labor-augmenting element (Li and Jiuli, 2016).

### 2.4.3. The results of steady-state growth equilibrium

In the sequel we assume that conditions (24) are satisfied. Define $s_N \equiv I_N/Y$, $s_M \equiv I_M/Y$, $s_K \equiv I_K/Y$, $s_L \equiv I_L/Y$, and $s_C \equiv C/Y$. The budget constraint becomes:

$$s_C + s_N + s_M + s_K + s_L = 1 \tag{27}$$

Using (4), (5), (16), (17) and (27), the Euler equations (23) can be re-written as：

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[5] Knife-edge conditions are commonly found in the growth literature. See, e.g., Jones (1995); Christiaans (2004); Growiec (2010).

[6] Under the standard assumption that $\dot{K} = I - \delta K$, we have $\alpha_K = 1$, so that $\dot{B}/B$ must be 0 which is Uzawa's theorem.

$$\begin{cases} \dfrac{\dot{C}}{C} = \dfrac{\rho}{\beta\alpha_K{}^2 kf'(k)/[s_K f(k)] + 1 - \alpha_K - \theta} \\ \dfrac{\dot{C}}{C} = \dfrac{\rho}{\beta\alpha_L{}^2[f(k) - kf'(k)]/[s_L f(k)] + 1 - \alpha_L - \theta} \\ \dfrac{\dot{C}}{C} = \dfrac{\rho}{(1-\beta)\alpha_M{}^2 kf'(k)/[s_M f(k)] + 1 - \alpha_M - \theta} \\ \dfrac{\dot{C}}{C} = \dfrac{\rho}{(1-\beta)\alpha_N{}^2[f(k) - kf'(k)]/[s_N f(k)] + 1 - \alpha_N - \theta} \end{cases} \quad (28)$$

(see Appendix D).

On the other hand, as shown in Appendix E, from equation (4) and (5) we also obtain

$$\begin{cases} \dfrac{\dot{C}}{C} = (b_M s_M{}^{\alpha_M}/\alpha_M)^{1-\beta}(b_K s_K{}^{\alpha_K}/\alpha_K)^{\beta}[f(k)/k]^{\beta} \\ \dfrac{\dot{C}}{C} = (b_N s_N{}^{\alpha_N}/\alpha_N)^{1-\beta}(b_L s_L{}^{\alpha_L}/\alpha_L)^{\beta}[f(k)]^{\beta} \end{cases} \quad (29)$$

The seven equations in (27), (28) and (29) can be solved for the seven steady-state equilibrium variables $(\dot{C}/C)^*, k^*, s_C^*, s_K^*, s_L^*, s_M^*, s_N^*$. These variables are determined by the underlying parameters $\rho, \theta, \beta, \gamma, \eta, \alpha_L, \alpha_K, \alpha_N, \alpha_M, b_L, b_K, b_N, b_M$.

Specifically, we can obtain

$$(\dot{Y}/Y)^* = (\dot{I}/I)^* = (\dot{I}_K/I_K)^* = (\dot{I}_L/I_L)^* = (\dot{I}_M/I_M)^* = (\dot{I}_N/I_N)^* = g \quad (30)$$

where $g = g(\rho, \theta, \beta, \gamma, \varepsilon, \alpha_L, \alpha_K, \alpha_N, \alpha_M, b_L, b_K, b_N, b_M)$.

From (30), (4) and (5) we obtain

$$\begin{cases} (\dot{K}/K)^* = \alpha_K g \\ (\dot{L}/L)^* = \alpha_L g \\ (\dot{M}/M)^* = \alpha_M g \\ (\dot{N}/N)^* = \alpha_N g \end{cases} \quad (31)$$

Finally, equations (31) and the definition of B and A imply:

$$\begin{cases} (\dot{B}/B)^* = [(1-\beta)/\beta]\alpha_M g = (1-\alpha_K)g \\ (\dot{A}/A)^* = [(1-\beta)/\beta]\alpha_N g = (1-\alpha_L)g \end{cases} \quad (32)$$

If $[(1-\beta)/\beta]\alpha_M > 0$ (or $\alpha_K < 1$) and $[(1-\beta)/\beta]\alpha_N > 0$ (or $\alpha_L < 1$), then technological progress will include both labor- and capital-augmenting elements along the steady-state equilibrium path.

#### 2.4.4. Factor shares along the steady-state growth equilibrium path

Let $\varphi_L \equiv wL/Y$, $\varphi_K \equiv rK/Y$, $\varphi_N \equiv \pi_N N/Y$, $\varphi_M \equiv \pi_M M/Y$ respectively represent labor, capital and monopoly profit shares of the labor-intensive and capital-intensive intermediate goods producers in total output. Let $\varphi \equiv \varphi_K/\varphi_L$ denote the ratio of capital to labor share. Using the production function (17) we obtain

$$\begin{cases} \varphi_L = \dfrac{\beta\gamma}{\gamma + (1-\gamma)(k^*)^{(\varepsilon-1)/\varepsilon}} \\ \varphi_K = \dfrac{\beta(1-\gamma)(k^*)^{(\varepsilon-1)/\varepsilon}}{\gamma + (1-\gamma)(k^*)^{(\varepsilon-1)/\varepsilon}} \\ \varphi_N = \dfrac{(1-\beta)\gamma}{\gamma + (1-\gamma)(k^*)^{(\varepsilon-1)/\varepsilon}} \\ \varphi_M = \dfrac{(1-\beta)(1-\gamma)(k^*)^{(\varepsilon-1)/\varepsilon}}{\gamma + (1-\gamma)(k^*)^{(\varepsilon-1)/\varepsilon}} \\ \varphi = \dfrac{1-\gamma}{\gamma}(k^*)^{(\varepsilon-1)/\varepsilon} \end{cases} \tag{33}$$

Notice that because $k^*$ is a constant along the steady state equilibrium path these shares are also constant. This fact implies that the labor share can remain unchanged even if technological progress includes capital-augmentation.

The last observation is related to the extensive discussion that has recently developed over the global decline in labor shares and increased income inequality (e.g., Karabarbounis and Neiman, 2013; Piketty 2014). Some authors have argued that the bias of technological progress towards capital-augmentation is an important cause of these phenomena. However, the result above implies that there is no necessary connection between capital-augmentation and declining labor shares. Nevertheless, as equations (34) imply, the total share of innovation monopoly profits is $(1-\beta)$. That share may continue to increase, thereby exerting an important impact on income distribution. In particular, if workers cannot extract some of these monopoly profits, their share in total income may continue to decline.

**2.4.5. Numerical calculation of steady-state growth equilibrium**

Given the non-linearity of equation (27)-(29), we select 5 sets of parameter values as examples and use Matlab program to solve the system as reported in Table 1. The results show that there indeed exists an optimal allocation of resources in each set of parameters to achieve steady-state equilibrium, reported in table 1.

Table 1: Solutions of steady state equilibria

| | | Set 1 | Set 2 | Set 3 | Set 4 | Set 5 |
|---|---|---|---|---|---|---|
| Variables | $s_C$ | 0.8882 | 0.8778 | 0.8830 | 0.8764 | 0.8830 |
| | $s_K$ | 0.0279 | 0.0201 | 0.0401 | 0.0228 | 0.0287 |
| | $s_L$ | 0.0279 | 0.0410 | 0.0287 | 0.0178 | 0.0401 |
| | $s_M$ | 0.0279 | 0.0410 | 0.0195 | 0.0465 | 0.0287 |
| | $s_N$ | 0.0279 | 0.0201 | 0.0287 | 0.0365 | 0.0195 |
| | $k^*$ | 1.0000 | 1.0000 | 1.0914 | 1.2754 | 0.9162 |
| | $\dot{C}/C$ | 0.0234 | 0.0251 | 0.0242 | 0.0254 | 0.0242 |
| | $\dot{K}/K$ | 0.0117 | 0.0100 | 0.0145 | 0.0102 | 0.0121 |
| | $\dot{L}/L$ | 0.0117 | 0.0151 | 0.0121 | 0.0102 | 0.0145 |
| | $\dot{B}/B$ | 0.0117 | 0.0151 | 0.0097 | 0.0152 | 0.0121 |
| | $\dot{A}/A$ | 0.0117 | 0.0100 | 0.0121 | 0.0152 | 0.0097 |
| Parameters | $\gamma$ | 0.5 | 0.5 | 0.5 | 0.4 | 0.5 |
| | $\varepsilon$ | 1.2 | 1.2 | 1.2 | 0.6 | 1.2 |
| | $\alpha_L$ | 0.50 | 0.60 | 0.50 | 0.4 | 0.60 |
| | $\alpha_K$ | 0.50 | 0.40 | 0.60 | 0.4 | 0.50 |

Except for the parameters γ, ε , $\alpha_L$ and $\alpha_K$ which vary across the equilibria, the rest of the parameters are held constant at ρ=0.05、θ=0.6、β=0.5、$b_L = b_K = b_N = b_M = 0.07$. Technological progress is Hicks neutral in the first and the fourth set. The second set and the fifth set are biased to more capital-augmentation while the third is biased to more labor-augmentation. The numerical results demonstrate not only the existence of equilibria, but also that technological progress can include both labor- and capital-augmenting elements, depending on whether the value of ε is greater or less than one.

## 3. Determinants of the direction of technological progress

**Definition 3:** The *direction of technological progress* is the ratio between the rate of capital-augmentation to that of labor-augmentation, i.e. $\mathrm{DT} \equiv (\dot{B}/B)/(\dot{A}/A)$.

When $\dot{A}/A > 0$ and $\dot{B}/B = 0$ then DT＝0， and technological progress is purely labor-augmenting (i.e. Harrod-neutral)； when $\dot{A}/A = 0$ and $\dot{B}/B > 0$， then DT→+∞， and technological progress is purely capital-augmenting (i.e. Solow-neutral)； when $\dot{A}/A = \dot{B}/B > 0$， DT＝1， and technological progress is Hicks-neutral.

Figure 1 shows different directions of technological progress:

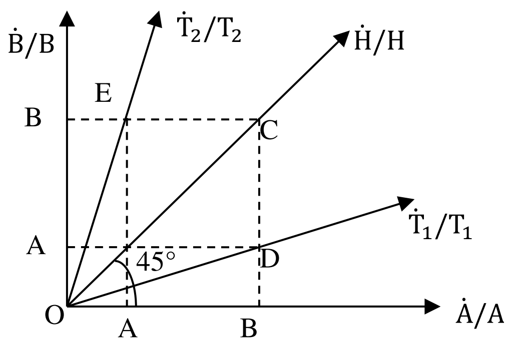


Figure 1: Direction of technological progress

Clearly, the axes represent Harrod-neutral (horizontal) and Solow-neutral (vertical) technical change. The diagonal $\dot{H}/H$ line represents the location of Hicks-neutral technical changes. The ray $\dot{T}_1/T_1$ indicates technical progress which is close to Harrod-neutrality, while $\dot{T}_2/T_2$ is close to Solow-neutrality. Different types of technical changes may be associated with the same growth rates but different directions. They may also have the same direction but different growth rates.

In a steady-state equilibrium, we obtain：

$$\mathrm{DT} \equiv \frac{\dot{B}/B}{\dot{A}/A} = \frac{\alpha_M}{\alpha_N} \tag{34}$$

Using this condition in equation (24) we also get:

$$\mathrm{DT} \equiv \frac{\dot{B}/B}{\dot{A}/A} = \frac{1-\alpha_K}{1-\alpha_L} \tag{35}$$

Equations (34) and (35) show that the direction of technological progress is determined by the exponents of the innovation investment functions, namely $\alpha_M$ and $\alpha_N$, or of the material factors accumulation functions, namely $\alpha_K$ and $\alpha_L$. In order to interpret equation (35), we define next the price elasticities of labor and capital supply, and then discuss the relationship between these elasticities and the direction of technological progress.

**Definition 4**: The *price elasticity* of any variable X is given by

$$\varepsilon_{X,p} \equiv \frac{\dot{X}/X}{\dot{p}_X/p_X} \tag{36}$$

**Lemma 3**: In a SSGE the price elasticity of capital and labor are given by:

$$\begin{cases} \varepsilon_{K,r} = \alpha_K/(1-\alpha_K) \\ \varepsilon_{L,w} = \alpha_L/(1-\alpha_L) \end{cases} \tag{37}$$

*where* $\varepsilon_{K,r}$ and $\varepsilon_{L,w}$ represent the price elasticity of capital and labor respectively.

**Proof**: See Appendix F.

Equations (37) indicate that the price elasticities of capital and labor are determined by the exponents of the material factor accumulation functions, namely, $\alpha_K$ and $\alpha_L$. When $\alpha_K = 1$, the price elasticity of capital is infinite, when $\alpha_L$=1, the price elasticity of labor is infinite. Using formula (32) and (37), we obtain:

$$\begin{cases} \dot{B}/B = g/(1+\varepsilon_{K,r}) \\ \dot{A}/A = g/(1+\varepsilon_{L,w}) \end{cases} \tag{38}$$

Which directly implies:

$$DT = \frac{\dot{B}/B}{\dot{A}/A} = \frac{(1+\varepsilon_{L,w})}{(1+\varepsilon_{K,r})} \tag{39}$$

The interpretation of equation (39) is summarized as Proposition 2.

**Proposition 2:** Along a steady-state growth equilibrium path, the direction of technological progress is determined by the relative price elasticities of the factor accumulation processes and is biased towards the factor with the relatively smaller elasticity.

Proposition 2 shows that the key determinant of the direction of technological progress is neither the change in relative price nor the relative size of markets, but the relative size of the price elasticities of the material factors. The change in the relative price of the material factors does not determine the direction of technological progress *per-se*. This is so because a change in the relative price will not only induce economizing on the factor that became more expensive, but also increased accumulation of it. If the supply elasticity of that factor is bigger than that of the other, the relative price change cannot continue in long run. Therefore, it is not reasonable to invest too many resources in developing new technologies that economize the use of that factor in long run.

## 4. The direction of technological progress under infinite supply elasticities of material factors

Based on the above conclusions, this section turns to a historical perspective on the direction of technological progress.

### 4.1 Why was there no labor-augmentation in the preindustrial era?

According to the empirical work of Ashraf and Galor (2011), in the preindustrial era technological progress brought about only an increase in population and its density while per capita income was nearly unchanged for thousands of years. This indicates that the technological progress did not include labor-augmenting elements. According to the model presented above, this is due to the very high elasticity of population with respect to wages in those times

Specifically, suppose that the population growth follows the Malthusian mechanism, as follows:

$$\dot{L}/L = ay - b \tag{40}$$

where $\dot{L}/L$ represents the rate of population growth, y represents per capita income, and "a" and "b" are positive exogenous parameters.

Since the wage depends on per capita income, namely $w = \alpha y$, with $0 < \alpha \leq 1$, we obtain

$$\varepsilon_{L,w} = \frac{\dot{L}/L}{\dot{w}/w} = \frac{(a/\alpha)w - b}{\dot{w}/w} \tag{41}$$

If $\dot{w}/w \geq 0$, as time progresses we get:

$$\lim_{t\to\infty} \varepsilon_{L,w} = \lim_{t\to\infty} \frac{(a/\alpha)w_0 \exp[\int_{\tau=0}^{t}(\dot{w}_\tau/w_\tau)d\tau] - b}{\dot{w}/w} = \infty \tag{42}$$

Therefore, in a Malthusian world the supply elasticity of labor becomes exceedingly large. Consequently, we obtain from equation (38) that in the preindustrial era $\dot{A}/A = g/(1+\varepsilon_{L,w}) = 0$ i.e. technological progress did not include the labor-augmenting element.

### 4.2. Why is technological progress purely labor-augmenting after the industrial revolution?

According to the summary of Kaldor (1961), since the industrial revolution per-capita capital and income continue to rise, but the productivity of capital has remained nearly remained. These characteristics indicate that technological progress is purely labor-augmenting. Indeed, by assuming that type of technical change the neoclassical growth model gets a steady-state growth path that meet the "Kaldor facts". However, that model cannot explain why technological progress *must* be purely labor-augmenting. The above structure implies that it is because the price elasticity of capital accumulation is infinite.

Taking the standard assumption of neoclassical growth model, the capital accumulation function is given by:

$$\dot{K} = sY - \delta K \tag{43}$$

where $s \equiv (Y - C)/Y$ represents saving rate. Under a constant returns to scale production function, $Y/K = r/\alpha$, where $\alpha \equiv \frac{\partial Y}{\partial K}\frac{K}{Y}$ represents the elasticity of output with respect to capital. From here we get:

$$\varepsilon_{K,r} = \frac{\dot{K}/K}{\dot{r}/r} = \frac{sY/K - \delta}{\dot{r}/r} = \frac{(s/\alpha)r - \delta}{\dot{r}/r} \tag{44}$$

With $\dot{r}/r \geq 0$, in the limit this implies:

$$\lim_{t\to\infty} \varepsilon_{K,r} = \lim_{t\to\infty} \frac{(s/\alpha)r_0 \exp[\int_{\tau=0}^{t}(\dot{r}_\tau/r_\tau)d\tau] - \delta}{\dot{r}/r} = \infty \tag{45}$$

Accordingly, from equation (38) we obtain $\dot{B}/B = g/(1+\varepsilon_{K,r}) = 0$, namely, technological progress must be purely labor-augmenting.

### 4.3. Why was technological progress in the non-agricultural sector slow during the rapid development period of the Chinese economy?

Since the Chinese economy has been reformed and opened, it has rapidly grown for more than 30 years. Empirical research (e.g. Young, 2003) found that the total factor productivity growth in the non-agricultural sector was very slow during this period, a fact that has often been criticized. However, given the above results, when an economy endowed with a large amount of surplus labor, then it is an *optimal* choice within a market economy to opt for a low growth in total factor productivity, because both supply elasticities of capital and labor are very large.

Specifically, when $\varepsilon_{L,w}=\infty$ and $\varepsilon_{K,r}=\infty$, equation (38) shows that $\dot{B}/B$ and $\dot{A}/A$ both are both equal to zero in steady-state. With unlimited supply of labor, the optimal choice should be to make full use of the surplus labor and accumulate capital to induce economic growth. That growth is then mainly driven by increases in factor quantities rather than their qualities.

### 4.4. Will technological progress remain labor-augmenting in the future?

The Uzawa (1961) steady-state theorem says that technological progress must be purely labor-augmenting in the steady-state growth path of the neoclassical growth model. However, this theorem is neither in line with economic intuition, nor can it explain why technological progress did not include labor-augmentation element in the preindustrial era. In addition, it is also unreasonable to expect that technological progress will be purely labor-augmenting in the future and not change with the changing environment.

For a long time after the industrial revolution, the supply elasticity of capital was very large, inducing (as we have seen) labor-augmenting technological progress. Land has been replaced by capital, removing the constraint on economic growth due to limited land. In the neoclassical growth model, physical capital is completely renewable. However, in reality, that creation of such capital requires non-renewable resources. for nearly two hundred years, because only a few countries experienced industrialization, the pressure on these resources was low and their capital supply elasticity could be viewed to have been nearly infinite. However, with many developing countries like China starting o industrialize, the constraint imposed by the nonrenewable natural resources and ecological environment on the accumulation of physical capital become tighter and tighter. Therefore, the elasticity of supply elasticity of capital is likely to decline, changing the mix of technological progress.

## 5. The direction of technological progress in social planning equilibrium

The previous benchmark model gives the equilibrium of decisions in a decentralized market. Due to the externality of innovation, the decentralized equilibrium is not Pareto optimal. Does the direction of technological progress also deviate from the Pareto optimal one? The following will answer the question by solving the social planning program.

Assume that the social planner maximizes the utility of the representative household under the constraints of the production function and social resource constraint. In the social planning program the decision on the production of the intermediate inputs should be based on marginal cost pricing and not on the monopoly pricing. Therefore, investment in innovation depends on the marginal revenue and not the monopoly profits. From the production function (15), the marginal revenue of K, L, M and N are as follows:

$$\begin{cases} \partial Y/\partial \mathrm{K} = M^{(1-\beta)/\beta} f'(k) \\ \partial Y/\partial \mathrm{L} = N^{(1-\beta)/\beta}[f(k) - kf'(k)] \\ \partial Y/\partial \mathrm{M} = M^{(1-2\beta)/\beta} Kf'(k) \\ \partial Y/\partial \mathrm{N} = N^{(1-2\beta)/\beta} L[f(k) - kf'(k)] \end{cases} \quad (46)$$

The social resource constraint is

$$\mathrm{C} + I_K + I_L + I_N + I_M = Y \quad (47)$$

The social welfare function is the utility function of household (6). Using the optimal control technique, we obtain the social Euler equations as follow,

$$\begin{cases} \frac{\dot{C}}{C} = \{\alpha_K b_K I_K{}^{\alpha_K - 1} M^{(1-\beta)/\beta} f'(k) - (\alpha_K - 1)\dot{I}_K/I_K - \rho\}/\theta \\ \frac{\dot{C}}{C} = \{\alpha_L b_L I_L{}^{\alpha_L - 1} N^{(1-\beta)/\beta} [f(k) - kf'(k)] - (\alpha_L - 1)\dot{I}_L/I_L - \rho\}/\theta \\ \frac{\dot{C}}{C} = \{[(1-\beta)/\beta]\alpha_M b_M I_M{}^{\alpha_M - 1} M^{(1-2\beta)/\beta} K f'(k) - (\alpha_M - 1)\dot{I}_M/I_M - \rho\}/\theta \\ \frac{\dot{C}}{C} = \{[(1-\beta)/\beta]\alpha_N b_N I_N{}^{\alpha_N - 1} N^{(1-2\beta)/\beta} L[f(k) - kf'(k)] - (\alpha_N - 1)\dot{I}_N/I_N - \rho\}/\theta \end{cases} \quad (48)$$

By comparing the Euler equations of the social planner to those of the decentralized equilibrium, we observe that the social marginal revenue of the factors equals the price of intermediate varieties multiplied by a factor of $1/\beta$. since $\beta < 1$, if the innovation of intermediates and accumulation of material factors by society will higher than by that of the decentralized agents. Therefore, the decentralized equilibrium is not Pareto optimal.

In order to arrive at a Pareto optimal result in the decentralized environment, it is necessary to subsidize *both* capital and labor, which is different from Romer (1990) where labor-augmentation is admissible. The rate to subsidy is the market prices of capital and labor multiplied by $1/\beta$. Because the intermediate goods are produced by both capital and labor, the Pareto optimal allocation cannot be achieved by subsidizing the monopoly profits of the intermediate goods producers.

Since the rate of output growth in the two environments is different, the rates of both capital- and labor-augmentation are also different. However, the direction of technological progress are the same because the ratio between the capital- and labor-augmentations rates is still $\frac{\dot{B}/B}{\dot{A}/A} = \frac{1-\alpha_K}{1-\alpha_L} = \frac{(1+\varepsilon_{L,w})}{(1+\varepsilon_{K,r})}$. Therefore, although the structure of market affects the resources allocation, it does not affect the direction of technological progress which is determined by the relative size of the elasticities of material factors which reflect the relative scarcity of endowments in the dynamic process.

## 6. Discussion and Extensions

In the benchmark model, the innovation of new varieties of intermediate goods and the accumulation of material factors are both the results of investment. However, Acemoglu (2003) discussed one case in which new varieties of intermediate goods is created by the R&D efforts of scientists, and in Acemoglu (2009) (chapter 15) he presents another case where the growth rates of both capital and labor are exogenous. In addition, in the benchmark model, labor-intensive and capital- intensive products $Y_L$ and $Y_K$ are produced only by labor-intensive intermediates X(i) or capital-intensive intermediates Z(i) respectively, and the intermediary goods X(i) and Z(i) are produced linearly only by L and K respectively. However, Acemoglu (2002, 2003) also discussed the case where the $Y_L$ and $Y_K$ are produced by the intermediary goods and material factors, using the appropriate Cobb-Douglas functions. The effects of these cases on the direction of technological progress of are discussed in following.

### 6.1. Innovation Possibilities Frontier

Acemoglu (2003) discussed the case of the following innovation possibilities frontier:

$$\begin{cases} \dot{N} = d_l M^{\eta} N^{1-\eta} S_l - \delta N \\ \dot{M} = d_k M^{\eta} N^{1-\eta} S_k - \delta M \end{cases} \text{, where } S_l + S_k = S \tag{49}$$

where S represents the total amount of scientists which is given exogenously but suffices to meet the needs for the two intermediate goods departments. $S_l$ and $S_k$ represent, respectlively, the scientists who carry the R&D of the labor-and capital-intensive intermediate goods. Their sum is equal to S. Other assumptions of benchmark model are unchanged.

From equation (49) we can obtain

$$\begin{cases} \dot{N}/N = d_l (M/N)^{\eta} S_l - \delta \\ \dot{M}/M = d_k (M/N)^{\eta} S_k - \delta \end{cases} \tag{50}$$

The rates of technological progress are constant in steady-state equilibrium. Therefore, if such equilibrium exists, there is a $S_l^*$ that satisfies:

$$(M/N)^* = d_k(S - S_l^*)/(d_l S_l^*) \tag{51}$$

Substituting this into equation equations (50), we obtain:

$$\dot{M}/M = \dot{N}/N = d_l [d_k(S - S_l^*)/(d_l S_l^*)]^{\eta} S_l^* - \delta \tag{52}$$

Because $\frac{M^{(1-\beta)/\beta} K}{N^{(1-\beta)/\beta} L}$ is a constant in steady-state, equation (52) requires $\dot{K}/K = \dot{L}/L$, and for a steady- state equilibrium to exist, there is again a knife-edge condition:

$$\alpha_L = \alpha_K \tag{53}$$

Since R&D investment does not use resources, the household budget constraint is modified as:

$$wL + rK + w_S S = C + I_K + I_L \tag{54}$$

Unlike the benchmark model, the income of the household includes the rental revenue of capital and the wages of labor and scientists. The expenditures consist of consumption and investment in the accumulation of material factors. The household allocates income to maximize intertemporal utility.

The budget constraint can be reformulated as:

$$s_C + s_K + s_L = 1 \tag{55}$$

From the Euler equation we can obtain the following equations in the steady-state

$$\begin{cases} \frac{\dot{C}}{C} = \frac{\rho}{\alpha_K{}^2 k f'(k)/[s_K f(k)] + 1 - \alpha_K - \theta} \\ \frac{\dot{C}}{C} = \frac{\rho}{\alpha_L{}^2 [f(k) - k f'(k)]/[s_L f(k)] + 1 - \alpha_L - \theta} \end{cases} \tag{56}$$

When $\alpha_L = \alpha_K$, we obtain from the above:

$$s_K/s_L = k f'(k)/[f(k) - k f'(k)] \tag{57}$$

Because of $\dot{K}/K = \dot{L}/L$ and $\alpha_L = \alpha_K$ in steady-state, the material factors accumulation function (4) imply:[7]

$$\frac{K}{L} = \frac{b_K s_K{}^{\alpha_K}}{b_L s_L{}^{\alpha_L}} \tag{58}$$

Substituting equations (51) and (58) into $k \equiv \frac{M^{(1-\beta)/\beta} K}{N^{(1-\beta)/\beta} L}$ ,we obtain:

[7] See Appendix G.

$$\mathrm{k} = \frac{\mathrm{b}_K \mathrm{s}_K{}^{\alpha_K}}{\mathrm{b}_L \mathrm{s}_L{}^{\alpha_L}} \left[\frac{d_k(S - S_l^*)}{(d_l S_l^*)}\right]^{(1-\beta)/\beta} \tag{59}$$

Finally, from $\mathrm{Y} = \mathrm{N}^{(1-\beta)/\beta}\mathrm{Lf(k)}$ and the steady-state growth rates of Y, C, N and L, we get:

$$\frac{\dot{C}}{\mathrm{C}} = \frac{1}{1-\alpha_L}\frac{1-\beta}{\beta}\left\{d_l S_l^* \left[\frac{d_k(S - S_l^*)}{d_l S_l^*}\right]^{\eta} - \delta\right\} \tag{60}$$

In sum, the model generates six independent equations, (55)- (60), with six independent unknown variables, i.e. $s_C$、$s_L$、$s_K$、$\dot{C}/C$、$k$ and $S_l^*$. In addition we have to impose $\alpha_L = \alpha_K < 1$, which is a specific case of the steady state equilibrium condition in the benchmark model. Equation (52) shows that the technological progress is Hicks neutral. From $\alpha_L = \alpha_K$ we can obtain $\varepsilon_L = \varepsilon_K$, therefore the direction of technological progress is still determined by $\mathrm{DT} = (1+\varepsilon_L)/(1+\varepsilon_K)$. Acemoglu (2003) argues that if the innovation possibilities frontier is specified as in equations (49), the model will have no steady-state equilibrium. The discussion above shows that this is due to Acemoglu's assumption that because he assumes $\alpha_K = 1$ and $\alpha_L \neq \alpha_K$, and not because of equation (49).

**6.2. Exogenous capital and labor growth rates**

Aemoglu (2009, chapter 15) suggests another model with exogenous capital and labor growth rates. He argues that the technological progress will be labor-augmenting. However, it seems that the steady-state technological progress cannot be labor-augmenting.

Keep all assumptions unchanged except that growth rates of material factors are:

$$\begin{cases} \dot{\mathrm{K}}/\mathrm{K} = \mathrm{b}_K > 0 \\ \dot{\mathrm{L}}/\mathrm{L} = \mathrm{b}_L > 0 \end{cases} \tag{61}$$

*where* $\mathrm{b}_K$ and $\mathrm{b}_L$ are exogenously given.

Since the accumulation of material factors does not require resources, the budget constraint equation is modified as follows:

$$\mathrm{wL} + \mathrm{rK} + \int_0^N \pi_{X(i)} di + \int_0^M \pi_{Z(i)} di = C + I_N + I_M \tag{62}$$

Given equations (65), the necessary condition for the existence of a steady-state becomes:

$$\frac{b_L}{1 - [(1-\beta)/\beta]\alpha_N} = \frac{b_K}{1 - [(1-\beta)/\beta]\alpha_M} \tag{63}$$

When equation (66) holds, the consumption growth rate is:

$$\frac{\dot{C}}{C} = \frac{b_L}{1 - [(1-\beta)/\beta]\alpha_N} = \frac{b_K}{1 - [(1-\beta)/\beta]\alpha_M} \tag{64}$$

Moreover, the steady state Euler equations become:[8]

$$\begin{cases} \dfrac{\dot{C}}{\mathrm{C}} = \dfrac{\rho}{(1-\beta)\alpha_N{}^2[f(k) - kf'(k)]/[s_N f(k)] + 1 - \alpha_N - \theta} \\ \dfrac{\dot{C}}{\mathrm{C}} = \dfrac{\rho}{(1-\beta)\alpha_M{}^2 kf'(k)/[s_M f(k)] + 1 - \alpha_M - \theta} \end{cases} \tag{65}$$

The budget constraint can be rewritten as:

$$\mathrm{s_C} + \mathrm{s_N} + \mathrm{s_M} = 1 \tag{66}$$

Equation (5) of the steady-state innovation possibilities frontier yields:[9]

[8]Notice that it is no longer the case that $\dot{C}/C = (r-\rho)/\theta$. Here not only are the accumulation processes of material factors exogenous, but also innovation does not require resource investment since the new intermediate goods are created by special scientists who have no alternative costs. Therefore, the household has no choice but to consume its entire income.

$$\mathrm{f(k)}^{\alpha_N-\alpha_M}=\frac{\alpha_N b_M}{\alpha_M b_N}\frac{{S_M}^{\alpha_M}}{{S_N}^{\alpha_N}}\frac{N_T/M_T}{\left({N_T}^{(1-\beta)/\beta}L_T\right)^{\alpha_N-\alpha_M}} \tag{67}$$

$where\ N_T$、 $M_T$ and $L_T$ are the initial value of the steady-state equilibrium path. These do not affect the growth rates of the endogenous variables and can be set at any positive value. Given these values, there are five independent equations, (63)-(68), and five variables, $\dot{C}/C$、$k$、$s_C$、$s_N$、$s_M$, which yields the equilibrium solution.

The capital- and labor-augmentation rates are:

$$\begin{cases}\dfrac{\dot{B}}{B}=\dfrac{[(1-\beta)/\beta]\alpha_M}{1-[(1-\beta)/\beta]\alpha_M}b_K\\ \dfrac{\dot{A}}{A}=\dfrac{[(1-\beta)/\beta]\alpha_N}{1-[(1-\beta)/\beta]\alpha_N}b_L\end{cases} \tag{68}$$

The supply elasticities of capital and labor are given by:

$$\begin{cases}\varepsilon_K=\dfrac{\dot{K}/K}{\dot{r}/r}=\dfrac{\dot{K}/K}{\dot{B}/B}=\dfrac{1-[(1-\beta)/\beta]\alpha_M}{[(1-\beta)/\beta]\alpha_M}\\ \varepsilon_L=\dfrac{\dot{L}/L}{\dot{w}/w}=\dfrac{\dot{L}/L}{\dot{A}/A}=\dfrac{1-[(1-\beta)/\beta]\alpha_N}{[(1-\beta)/\beta]\alpha_N}\end{cases} \tag{69}$$

From equations (63), (68) and (69) we can obtain the direction of technological progress as follow:

$$\mathrm{DT}=\frac{[(1-\beta)/\beta]\alpha_M}{[(1-\beta)/\beta]\alpha_N}=\frac{1+\varepsilon_L}{1+\varepsilon_K} \tag{70}$$

Equation (68) shows that technological progress will include both labor- and capital-augmenting elements when the accumulation rates of both capital and labor are exogenous. Therefore, though a steady-state equilibrium path exists, balanced growth does not.

**6.3. Different forms of technological progress**

Acemoglu (2002, 2003) suggests yet another form of technological progress in which labor-intensive and capital-intensive products are produced using the following functions:

$$\begin{cases}\mathrm{Y}_L=\dfrac{1}{1-\beta}\left[\displaystyle\int_0^N X(i)^{1-\beta}di\right]L^\beta\\ \mathrm{Y}_K=\dfrac{1}{1-\beta}\left[\displaystyle\int_0^M Z(i)^{1-\beta}di\right]K^\beta\end{cases},0<\beta<1 \tag{71}$$

If we keep the material factor accumulation and innovation functions and the innovation possibilities frontier as in the benchmark model, the necessary conditions for the existence of a steady-state equilibrium become:

$$\begin{cases}\alpha_K+\alpha_M=1\\ \alpha_L+\alpha_N=1\end{cases} \tag{72}$$

If equations (72) are met, there is a unique solution of the steady state equilibrium conditions. The technological progress can include both labor- and capital-augmenting elements, and its direction is still determined by $\mathrm{DT}=(1+\varepsilon_L)/(1+\varepsilon_K)$.

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[9] See Appendix H.

## 7. Conclusions

What determines the direction of technological progress? This is one of the central issues of the theory of economic growth. By developing a growth model with not only endogenous accumulation of material factors (capital and labor) but also endogenous labor- and capital-augmenting technological progress, this paper proves that the determinants of the direction of technological progress are neither the change in the relative prices of the factors of production as suggested by Hicks (1932) nor the relative size of markets as advocated by Acemoglu (2002). Instead, it is the relative size of the supply elasticities of material factors with respect to their respective prices, and is biased towards the factor with the relatively smaller elasticity.

The paper provides new insights concerning the switch in the direction of economic growth between the preindustrial era and the period following the industrial revolution. Specifically, empirical studies by Kaldor (1961) and Ashraf and Galor (2011) show that technological progress before the industrial revolution was nearly completely devoid of labor-augmenting elements, while after the industrial revolution it was almost purely labor-augmenting. The paper argues that these facts may be due to the very high labor supply elasticity in a Malthusian world on the one hand, and a very high renewable capital supply elasticity after the industrial revolution. Concerning things to come, the model predicts that technological progress will include more and more capital-augmenting element when the elasticity of the capital supply starts decreasing because of constraints on the use of non-reproducible resources and the environment become binding.

Finally, this paper also sheds some light on another important issue in current macroeconomics. From the point of view of our model, when innovations are carried out by profit-maximizing firms, the role of innovation in economic growth becomes more and more important. As a result, the share of profits of innovating monopolies may rise. If workers cannot extract a part of these profits, the labor share may be declining, which may explain the global decline in labor shares over the last decades. On the contrary, there is no necessary connection between capital-augmentation and declining labor shares.

**Appendix A： The process of derivation of the Euler equations (21)**

Let the Hamilton associated with the optimization problem be:

$$\mathrm{H} = \mathrm{U(C)e}^{-\rho t} + \lambda_K b_K I_K^{\alpha_K} + \lambda_L b_L I_L^{\alpha_L} + \lambda_M b_M I_M^{\alpha_M} + \lambda_N b_N I_N^{\alpha_N}$$
$$+\mu[wL + rK + \pi_X N + \pi_Z M - C - (I_K + I_L + I_N + I_M) \qquad (A1)$$

The first-order conditions are:

$$\begin{cases} C^{-\theta}e^{-\rho t} = \lambda_M \alpha_M b_M I_M^{\alpha_M - 1} \\ C^{-\theta}e^{-\rho t} = \lambda_K \alpha_K b_K I_K^{\alpha_K - 1} \\ C^{-\theta}e^{-\rho t} = \lambda_L \alpha_L b_L I_L^{\alpha_L - 1} \\ C^{-\theta}e^{-\rho t} = \lambda_N \alpha_N b_N I_N^{\alpha_N - 1} \\ C^{-\theta}e^{-\rho t} = \mu \end{cases} \tag{A2}$$

Taking log-derivatives of both sides of (A2) over time, we obtain

$$\begin{cases} -\theta\frac{\dot{C}}{C} - \rho = \frac{\dot{\lambda}_M}{\lambda_M} + (\alpha_M - 1)\frac{\dot{I}_M}{I_M} \\ -\theta\frac{\dot{C}}{C} - \rho = \frac{\dot{\lambda}_K}{\lambda_K} + (\alpha_K - 1)\frac{\dot{I}_K}{I_K} \\ -\theta\frac{\dot{C}}{C} - \rho = \frac{\dot{\lambda}_L}{\lambda_L} + (\alpha_L - 1)\frac{\dot{I}_L}{I_L} \\ -\theta\frac{\dot{C}}{C} - \rho = \frac{\dot{\lambda}_N}{\lambda_N} + (\alpha_N - 1)\frac{\dot{I}_N}{I_N} \\ -\theta\frac{\dot{C}}{C} - \rho = \frac{\dot{\mu}}{\mu} \end{cases} \tag{A3}$$

The motion equations of λ are:

$$\begin{cases} \dot{\lambda}_M = -\partial H/\partial M = -\mu\pi_Z \\ \dot{\lambda}_K = -\partial H/\partial K = -\mu r \\ \dot{\lambda}_N = -\partial H/\partial N = -\mu\pi_X \\ \dot{\lambda}_L = -\partial H/\partial L = -\mu w \end{cases} \tag{A4}$$

Based on (A2) and (A4),

$$\begin{cases} \dot{\lambda}_M/\lambda_M = -\pi_Z \alpha_M b_M I_M^{\alpha_M - 1} \\ \dot{\lambda}_K/\lambda_K = -r\alpha_K b_K I_K^{\alpha_K - 1} \\ \dot{\lambda}_N/\lambda_N = -\pi_X \alpha_N b_N I_N^{\alpha_N - 1} \\ \dot{\lambda}_L/\lambda_L = -w\alpha_L b_L I_L^{\alpha_L - 1} \end{cases} \tag{A5}$$

Using (A5) in (A3), we obtain the Euler equations (21).

$$\begin{cases} \frac{\dot{C}}{C} = \frac{1}{\theta}\left\{\pi_Z \alpha_M b_M I_M^{\alpha_M - 1} - (\alpha_M - 1)\frac{\dot{I}_M}{I_M} - \rho\right\} \\ \frac{\dot{C}}{C} = \frac{1}{\theta}\left\{r\alpha_K b_K I_K^{\alpha_K - 1} - (\alpha_K - 1)\frac{\dot{I}_K}{I_K} - \rho\right\} \\ \frac{\dot{C}}{C} = \frac{1}{\theta}\left\{w\alpha_L b_L I_L^{\alpha_L - 1} - (\alpha_L - 1)\frac{\dot{I}_L}{I_L} - \rho\right\} \\ \frac{\dot{C}}{C} = \frac{1}{\theta}\left\{\pi_X \alpha_N b_N I_N^{\alpha_N - 1} - (\alpha_N - 1)\frac{\dot{I}_N}{I_N} - \rho\right\} \end{cases} \tag{21}$$

**Appendix B: process of derivation of equation (24)**

Proof: First, from the budget constraint (7) and the definition of a steady-state growth equilibrium, we obtain

$$\frac{\dot{Y}}{Y} = \frac{\dot{I}}{I} = \frac{\dot{I}_M}{I_M} = \frac{\dot{I}_N}{I_N} = \frac{\dot{I}_L}{I_L} = \frac{\dot{I}_K}{I_K} = \frac{\dot{C}}{C} \tag{B1}$$

Then, according to the factor accumulation functions (4) and the innovation possibilities frontier (5), the following must hold in steady-state

$$\begin{cases}\dot{K}/K = \alpha_K \dot{I}_K/I_K \\ \dot{L}/L = \alpha_L \dot{I}_L/I_L\end{cases} \quad (B2)$$

$$\begin{cases}\dot{M}/M = \alpha_M \dot{I}_M/I_M \\ \dot{L}/L = \alpha_N \dot{I}_N/I_N\end{cases} \quad (B3)$$

Using the intensive form of the production function (17), we obtain

$$Y = N^{(1-\beta)/\beta} L f(k) = M^{(1-\beta)/\beta} K f(k)/k \quad (B4)$$

In a steady-state growth equilibrium, due to the fact that *k* is constant, we obtain:

$$\begin{cases}\dot{K}/K + (1-\beta)/\beta \dot{M}/M = \dot{Y}/Y \\ \dot{L}/L + (1-\beta)/\beta \dot{N}/N = \dot{Y}/Y\end{cases} \quad (B5)$$

Substitute (B1), (B2) and (B3) into (B5), if $\dot{Y}/Y > 0$ then we can obtain the necessary condition to exist a steady-state equilibrium equation (24)

$$\begin{cases}\alpha_K + [(1-\beta)/\beta]\alpha_M = 1 \\ \alpha_L + [(1-\beta)/\beta]\alpha_N = 1\end{cases} \quad (24)$$

**Appendix C: the process of derivation of the lemma 1 and lemma 2.**

From the definition of labor- and capital-augmenting technological progress we can obtain

$$\begin{cases}\dot{B}/B = [(1-\beta)/\beta]\dot{M}/M \\ \dot{A}/A = [(1-\beta)/\beta]\dot{N}/N\end{cases} \quad (C1)$$

Insert (B1) and (B3) into (C1) obtain：

$$\begin{cases}\dot{B}/B = [(1-\beta)/\beta]\alpha_M \dot{Y}/Y \\ \dot{A}/A = [(1-\beta)/\beta]\alpha_N \dot{Y}/Y\end{cases} \quad (C2)$$

From equation (24) and (C2) we can obtain：

$$\begin{cases}\dot{B}/B = (1-\alpha_K)\dot{Y}/Y \\ \dot{A}/A = (1-\alpha_L)\dot{Y}/Y\end{cases} \quad (C3)$$

**Appendix D: the process of derivation of equation (28)**

From the Euler equations (23) we can obtain

$$\begin{cases}\frac{\dot{C}}{C} = \left[\alpha_K b_K \frac{I_K^{\alpha_K}}{K}\frac{Y}{I_K}\frac{M^{(1-\beta)/\beta}K}{Y}\beta f'(k) - (\alpha_K - 1)\frac{\dot{I}_K}{I_K} - \rho\right]/\theta \\ \frac{\dot{C}}{C} = \left[\alpha_L b_L \frac{I_L^{\alpha_L}}{L}\frac{Y}{I_L}\frac{N^{(1-\beta)/\beta}L}{Y}\beta[f(k) - kf'(k)] - (\alpha_L - 1)\frac{\dot{I}_L}{I_L} - \rho\right]/\theta \\ \frac{\dot{C}}{C} = \left[\alpha_M b_M \frac{I_M^{\alpha_M}}{M}\frac{Y}{I_M}\frac{M^{(1-\beta)/\beta}K}{Y}(1-\beta)f'(k) - (\alpha_M - 1)\frac{\dot{I}_M}{I_M} - \rho\right]/\theta \\ \frac{\dot{C}}{C} = \left[\alpha_N b_N \frac{I_N^{\alpha_N}}{N}\frac{Y}{I_N}\frac{N^{(1-\beta)/\beta}L}{Y}(1-\beta)[f(k) - kf'(k)] - (\alpha_N - 1)\frac{\dot{I}_N}{I_N} - \rho\right]/\theta\end{cases} \quad (D1)$$

Using the function (4) and (5) obtain

$$\begin{cases}\dot{K}/K = b_K I_K^{\alpha_K}/K \\ \dot{L}/L = b_L I_L^{\alpha_L}/L \\ \dot{M}/M = b_M I_M^{\alpha_M}/M \\ \dot{N}/N = b_N I_N^{\alpha_N}/N\end{cases} \quad (D2)$$

Substitute (D2), $s_N \equiv I_N/Y$, $s_M \equiv I_M/Y$, $s_K \equiv I_K/Y$, $s_L \equiv I_L/Y$, $k = (M^{(1-\beta)/\beta}K)/(N^{(1-\beta)/\beta}L)$ and $f(k) \equiv Y/(N^{(1-\beta)/\beta}L)$ into (D1) obtain

$$\begin{cases}\frac{\dot{C}}{C}=\left[\alpha_K\frac{1}{s_K}\frac{kf'(k)}{f(k)}\beta\frac{\dot{K}}{K}-(\alpha_K-1)\frac{\dot{I}_K}{I_K}-\rho\right]/\theta\\ \frac{\dot{C}}{C}=\left[\alpha_L\frac{\dot{L}}{L}\frac{1}{s_L}\frac{[f(k)-kf'(k)]}{f(k)}\beta-(\alpha_L-1)\frac{\dot{I}_L}{I_L}-\rho\right]/\theta\\ \frac{\dot{C}}{C}=\left[\alpha_M\frac{\dot{M}}{M}\frac{1}{s_M}\frac{kf'(k)}{f(k)}(1-\beta)-(\alpha_M-1)\frac{\dot{I}_M}{I_M}-\rho\right]/\theta\\ \frac{\dot{C}}{C}=\left[\alpha_N\frac{\dot{N}}{N}\frac{1}{s_N}\frac{[f(k)-kf'(k)]}{f(k)}(1-\beta)-(\alpha_N-1)\frac{\dot{I}_N}{I_N}-\rho\right]/\theta\end{cases}\tag{D3}$$

Insert (B1), (B2) and (B3) into (D3) obtain

$$\begin{cases}\frac{\dot{C}}{C}=\left[\alpha_K{}^2\beta\frac{1}{s_K}\frac{kf'(k)}{f(k)}\frac{\dot{C}}{C}-(\alpha_K-1)\frac{\dot{C}}{C}-\rho\right]/\theta\\ \frac{\dot{C}}{C}=\left[\alpha_L{}^2\beta\frac{1}{s_L}\frac{f(k)-kf'(k)}{f(k)}\frac{\dot{C}}{C}-(\alpha_L-1)\frac{\dot{C}}{C}-\rho\right]/\theta\\ \frac{\dot{C}}{C}=\left[\alpha_M{}^2(1-\beta)\frac{1}{s_M}\frac{kf'(k)}{f(k)}\frac{\dot{C}}{C}-(\alpha_M-1)\frac{\dot{C}}{C}-\rho\right]/\theta\\ \frac{\dot{C}}{C}=\left[\alpha_N{}^2(1-\beta)\frac{1}{s_N}\frac{f(k)-kf'(k)}{f(k)}\frac{\dot{C}}{C}-(\alpha_N-1)\frac{\dot{C}}{C}-\rho\right]/\theta\end{cases}\tag{D4}$$

Rearrange (D4) we can obtain equation (28).

**Appendix E: the process of derivation of equation (29).**

Using the definition of investment rate obtain

$$\begin{cases}I_K=s_KY=s_KM^{(1-\beta)/\beta}Kf(k)/k\\ I_L=s_LY=s_LN^{(1-\beta)/\beta}Lf(k)\\ I_M=s_MY=s_MM^{(1-\beta)/\beta}Kf(k)/k\\ I_N=s_NY=s_NN^{(1-\beta)/\beta}Lf(k)\end{cases}\tag{E1}$$

Insert (E1) into (D2) obtain

$$\begin{cases}\frac{\dot{K}}{K}=b_K[s_Kf(k)/k]^{\alpha_K}\frac{M^{\alpha_K(1-\beta)/\beta}}{K^{1-\alpha_K}}\\ \frac{\dot{L}}{L}=b_L[s_Lf(k)]^{\alpha_L}\frac{N^{\alpha_L(1-\beta)/\beta}}{L^{1-\alpha_L}}\\ \frac{\dot{M}}{M}=b_M[s_Mf(k)/k]^{\alpha_M}\frac{K^{\alpha_M}}{M^{1-\alpha_M(1-\beta)/\beta}}\\ \frac{\dot{N}}{N}=b_N[s_Nf(k)]^{\alpha_N}\frac{L^{\alpha_N}}{N^{1-\alpha_N(1-\beta)/\beta}}\end{cases}\tag{E2}$$

Using equation (24) $\begin{cases}\alpha_K+[(1-\beta)/\beta]\alpha_M=1\\ \alpha_L+[(1-\beta)/\beta]\alpha_N=1\end{cases}$ in equation (E2) obtain

$$\begin{cases}\frac{\dot{K}}{K}=b_K[s_Kf(k)/k]^{\alpha_K}\left(\frac{M^{\alpha_K}}{K^{\alpha_M}}\right)^{(1-\beta)/\beta}\\ \frac{\dot{L}}{L}=b_L[s_Lf(k)]^{\alpha_L}\left(\frac{N^{\alpha_L}}{L^{\alpha_N}}\right)^{(1-\beta)/\beta}\\ \frac{\dot{M}}{M}=b_M[s_Mf(k)/k]^{\alpha_M}\frac{K^{\alpha_M}}{M^{\alpha_K}}\\ \frac{\dot{N}}{N}=b_N[s_Nf(k)]^{\alpha_N}\frac{L^{\alpha_N}}{N^{\alpha_L}}\end{cases}\tag{E3}$$

Using (B1), (B2), (B3) and (D3) obtain

$$\begin{cases} \alpha_K \dfrac{\dot{C}}{C} = b_K[s_K f(k)/k]^{\alpha_K} \left(\dfrac{M^{\alpha_K}}{K^{\alpha_M}}\right)^{(1-\beta)/\beta} \\ \alpha_L \dfrac{\dot{C}}{C} = b_L[s_L f(k)]^{\alpha_L} \left(\dfrac{N^{\alpha_L}}{L^{\alpha_N}}\right)^{(1-\beta)/\beta} \\ \alpha_M \dfrac{\dot{C}}{C} = b_M[s_M f(k)/k]^{\alpha_M} \dfrac{K^{\alpha_M}}{M^{\alpha_K}} \\ \alpha_N \dfrac{\dot{C}}{C} = b_N[s_N f(k)]^{\alpha_N} \dfrac{L^{\alpha_N}}{N^{\alpha_L}} \end{cases} \quad (E4)$$

Using the first and the third equation in formula (E4) to remove $M^{\alpha_K}/K^{\alpha_M}$, using the second and fourth equation in (E4) to remove $N^{\alpha_L}/L^{\alpha_N}$ we can obtain equations (29)。

**Appendix F: the process of derivation of equations (37)**

Proof: first, from the equations (19) $\begin{cases} r = \beta M^{(1-\beta)/\beta} f'(k) \\ w = \beta N^{(1-\beta)/\beta}[f(k) - kf'(k)] \end{cases}$ in steady-state to get

$$\begin{cases} \dot{r}/r = [(1-\beta)/\beta]\dot{M}/M \\ \dot{w}/w = [(1-\beta)/\beta]\dot{N}/N \end{cases} \quad (F1)$$

Substitute (F1) into equation (34) obtain:

$$\begin{cases} \varepsilon_{K,r} = (\dot{K}/K)/\{[(1-\beta)/\beta]\dot{M}/M\} \\ \varepsilon_{L,w} = (\dot{L}/L)/\{[(1-\beta)/\beta]\dot{N}/N\} \end{cases} \quad (F2)$$

Substitute (B2) and (B3) into (F2) to obtain:

$$\begin{cases} \varepsilon_{K,r} = \alpha_K/\{[(1-\beta)/\beta]\alpha_M\} \\ \varepsilon_{L,w} = \alpha_L/\{[(1-\beta)/\beta]\alpha_N\} \end{cases} \quad (F3)$$

From the equations (24) we obtain

$$\begin{cases} [(1-\beta)/\beta]\alpha_M = 1 - \alpha_K \\ [(1-\beta)/\beta]\alpha_N = 1 - \alpha_L \end{cases} \quad (F4)$$

Substitute (F4) into (F3) to obtain equations (37)

$$\begin{cases} \varepsilon_{K,r} = \alpha_K/(1-\alpha_K) \\ \varepsilon_{L,w} = \alpha_L/(1-\alpha_L) \end{cases} \quad (37)$$

**Appendix G: the process of derivation of equation (57)**

From the function of accumulation of material factors equations (4) obtain

$$\begin{cases} \dfrac{\dot{K}}{K} = \dfrac{b_K I_K^{\alpha_K}}{K} = b_K \dfrac{[s_K Y]^{\alpha_K}}{K} \\ \dfrac{\dot{L}}{L} = \dfrac{b_L I_L^{\alpha_L}}{L} = b_L \dfrac{[s_L Y]^{\alpha_L}}{L} \end{cases} \quad (G1)$$

From the relationship between growth rate of C, K and L we can obtain

$$\begin{cases} \alpha_K \dfrac{\dot{C}}{C} = b_K \dfrac{[s_K Y]^{\alpha_K}}{K} \\ \alpha_L \dfrac{\dot{C}}{C} = b_L \dfrac{[s_L Y]^{\alpha_L}}{L} \end{cases} \quad (G2)$$

Because $\alpha_K = \alpha_L$ in the case, from (G2) we can obtain

$$b_K \frac{[s_K Y]^{\alpha_K}}{K} = b_L \frac{[s_L Y]^{\alpha_L}}{L} \quad (G3)$$

Rearrange (G3) we can get equation (57).

**Appendix H: the process of derivation of equation (67).**

From innovation possibilities frontier equation (5) we obtain

$$\begin{cases} \frac{\dot{\mathrm{M}}}{\mathrm{M}} = \frac{\mathrm{b}_M I_M^{\alpha_M}}{\mathrm{M}} = \mathrm{b}_M \left[\frac{s_M f(k)}{k}\right]^{\alpha_M} \frac{K^{\alpha_M}}{M^{1-\alpha_M(1-\beta)/\beta}} \\ \frac{\dot{\mathrm{N}}}{\mathrm{N}} = \frac{\mathrm{b}_N I_N^{\alpha_N}}{\mathrm{N}} = \mathrm{b}_N [s_N f(k)]^{\alpha_N} \frac{L^{\alpha_N}}{N^{1-\alpha_N(1-\beta)/\beta}} \end{cases} \qquad (H1)$$

From the growth rates of N, M and C in steady-state we can obtain

$$\begin{cases} \alpha_M \frac{\dot{\mathrm{C}}}{\mathrm{C}} = \frac{\mathrm{b}_M I_M^{\alpha_M}}{\mathrm{M}} = \mathrm{b}_M \left[\frac{s_M f(k)}{k}\right]^{\alpha_M} \frac{K^{\alpha_M}}{M^{1-\alpha_M(1-\beta)/\beta}} \\ \alpha_N \frac{\dot{\mathrm{C}}}{\mathrm{C}} = \frac{\mathrm{b}_N I_N^{\alpha_N}}{\mathrm{N}} = \mathrm{b}_N [s_N f(k)]^{\alpha_N} \frac{L^{\alpha_N}}{N^{1-\alpha_N(1-\beta)/\beta}} \end{cases} \qquad (H2)$$

From equation (H2) we can obtain

$$\frac{\alpha_M}{\alpha_N} = \frac{\mathrm{b}_M}{\mathrm{b}_N} \frac{[s_M]^{\alpha_M} f(k)^{\alpha_M - \alpha_N}}{[s_N]^{\alpha_N}} \frac{1}{(N^{(1-\beta)/\beta} L)^{\alpha_N - \alpha_M}} \frac{N}{M} \qquad (H3)$$

Rearrange equation (H3) to obtain (67)

$$\mathrm{f(k)}^{\alpha_N - \alpha_M} = \frac{\alpha_N b_M}{\alpha_M b_N} \frac{{s_M}^{\alpha_M}}{{s_N}^{\alpha_N}} \frac{N_T / M_T}{\left({N_T}^{(1-\beta)/\beta} L_T\right)^{\alpha_N - \alpha_M}} \qquad (67)$$

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