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Demographic factors as determinants of external balances

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Abstract

This paper develops, solves and calibrate a dynamic general equilibrium model to examine the effect of demographic factors on the external balance of countries. This is quite an advanced multi-country model with different age structures across country. I find that this different age structure has a large impact on savings, resulting in poorer countries having large external balance surpluses whilst more developed countries will have large deficits.

1 Introduction.

Computing external balances remains a major concern in macroeconomics. The economic literature has identified exchange rate, monetary and fiscal policy as general determinants of external balance in an economy; demographic factors being generally ignored in these analyses. Currently, the world is subjected to demographic transition that is characterized by tremendous changes in population ages structures. This demographic transition is likely to impact people saving and investment decisions, and have therefore considerable impact on international trade chairing. Evaluating such impact is the main purpose of this thesis.

Demographic statistics (2013) [1] suggest that 82.5% of world population live in developing and underdeveloped countries whereas 17.5% live in advanced countries. The population growth of developing countries is expected to reach 86.4% of the worldwide population in 2050 whereas the later will decrease up to 13.6% in advanced economies. The same statistics suggest furthermore that this trend will remain the same until 2100. As consequence, workers are expected to reach a higher number in Southern Economies than in developed economies (Table 1). The observed gap are generally explained by differences between demographic growth (meaning asymmetric distribution of birth rate and life expectancy at birth(Figure1.1, Figure1.2) [2] et Figure 1.3[3] present this situation.

According to the life cycle theory, an economic agent makes consumption decision depending upon the overall of his aggregated incomes at a given time. Owing to the fact that income generally changes through time, it can be expected that aggregated savings will depend upon the demographic structure. Economies that will encompass a higher number of workers will more likely save and less consume whereas those of higher number of retired people will less save and more consume. Would this help to explain the differences in savings and investment among nations? Will saving contraction in advanced economies create deficits or explain surpluses in southern Economies? This thesis will try to shade some light on these issues.

Morever, age structure displayed in Figure 1.3, thereunder suggest high international demographic differences. African countries' populations are younger while those of European countries are older. In fact, countries where people are getting older can be subjected to two opposite cases: If they have accumulated foreign assets, due to regular current account surpluses, they will probably get an additional income to find a greater consumption. If not, they have to decrease international demand or accept borrowing from abroad.

[1] Population Division of the Department of the United Nations Secretariat (2013),World Population Prospects the 2012 Revision Highlights and advance Tables, p.2.

[2] http://www.un.org/en/development/desa/population/publications/pdf/ageing/WorldPopulationAgeing2013.pdf

[3] http://www.ansIrs.com/topic/population-pyramid

Figure 1.1: Distribution and elderly population growth- World Population ageing 2013

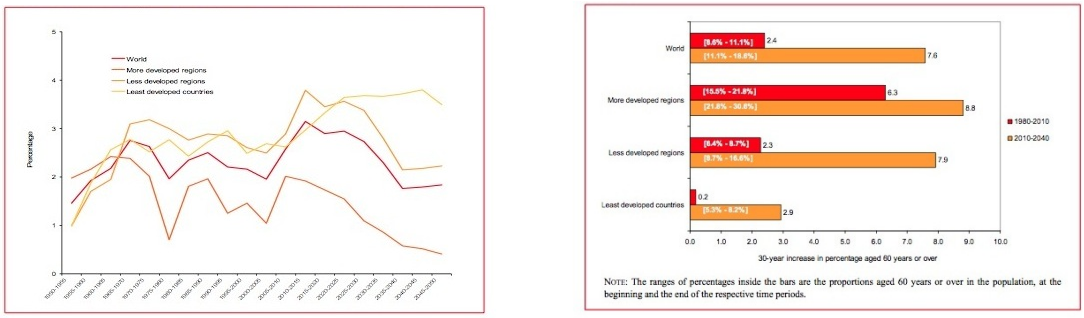
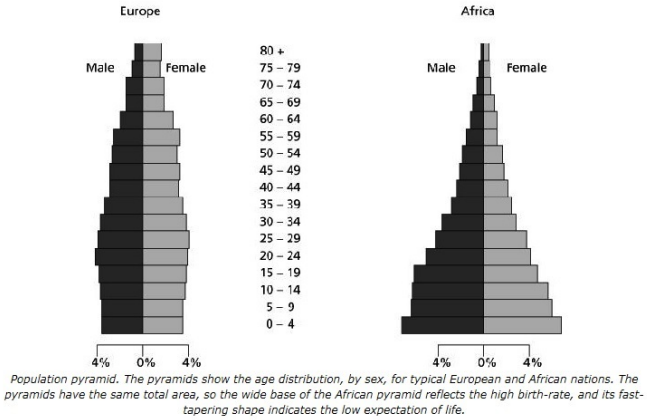


Figure 1.2: population pyramid



Population pyramid. The pyramids show the age distribution, by sex, for typical European and African nations. The pyramids have the same total area, so the wide base of the African pyramid reflects the high birth-rate, and its fast-tapering shape indicates the low expectation of life.

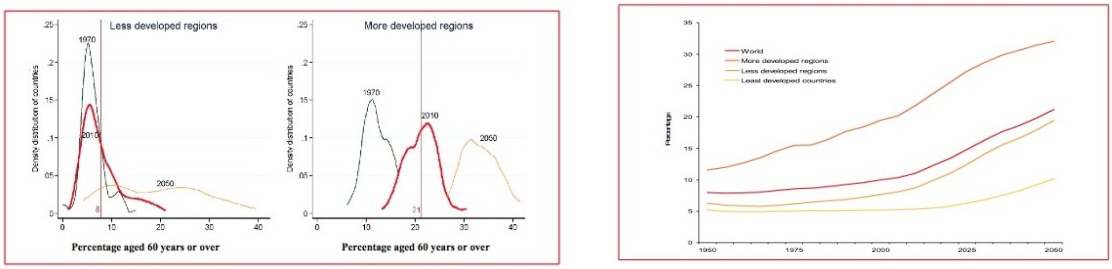
Existing literature pinpoints determinants of external balances in developed economies but mostly focus on technological, capital flows, exchange rate, monetary and fiscal policy effects on external balances. Not considering demographic factors is quite surprising inasmuch as life cycle theory. (Modigliani & Brumberg 1954) links savings differences to demographic variables. These authors suggest a macroeconomic point of view provided by microeconomic thinking of consumer intertemporal choice. The later is supposed to allocate optimally his expenses among periods, by considering the fact that his income fluctuates during the course of his life. Following (Modigliani 1970), (Graham 1987) and (Masson & al 1996) countries with higher number of retired people are likely to have loIr rate of savings. (Blanchard 1985)proposes his overlapping generations model (OLG model). He assumes that, despite their age, economic agents are all subjected to face a constant probability of death. This longevity risk will have an impact on macroeconomic

Table 1: Distribution of world population

Development group or major area	*1950*	*1980*	*2013*	*2050*			
				Low	*Medium*	*High*	*Constant-fertility*
World	100.0	100.0	100.0	100.0	100.0	100.0	100.0
More developed regions	32.2	24.3	17.5	13.8	13.6	13.5	11.4
Less developed regions	67.8	75.7	82.5	86.2	86.4	86.5	88.6
Least developed countries	7.7	8.8	12.5	19.1	19.0	18.8	23.0
Other less developed countries	60.1	66.8	70.0	67.1	67.4	67.7	65.6
Africa	9.1	10.8	15.5	25.4	25.1	24.7	29.0
Asia	55.3	59.2	60.0	53.7	54.1	54.4	52.3
Europe	21.7	15.6	10.4	7.5	7.4	7.4	6.1
Latin America and the Caribbean	6.6	8.2	8.6	8.1	8.2	8.3	8.0
Northern America	6.8	5.7	5.0	4.7	4.7	4.6	4.1
Oceania	0.5	0.5	0.5	0.6	0.6	0.6	0.6

Source: Population Division of the Department of Economic and Social Affairs of the United Nations Secretariat (2013). *World Population Prospect*
United Nations.

Figure 1.3: Distribution of the elderly population- World Population ageing 2013



equilibrium through the marginal propensity of consumption which is positively related to death probability. (Gertler 1999) improves the OLG model by including life cycle of economic agents. Gertler argue that population is divided into two different groups: workers (people in activity) and the retired. At each period, workers are subjected to a constant probability of retirement while retirees are facing a constant probability of dying: these two probabilities vary regardless the economic agent age. Thus, consumption decision of workers, are only explained by expected income on the life horizon.

(Ferrero 2010) proposes an international approach of Gertler model, in order to analyse the determinants of deficits in the USA by considering the G6 countries. Ferrero model, enables to separately study life expectancy and demographic growth in the two regions through their trade balances. The quantitative

analysis highlights the impact of life expectancy on consumption-saving decisions. (Ferrero 2010) eleads to a conclusion that external balances of the USA compared to those of G6 are mainly a result of differences in demographic growth and productivity in the two areas.

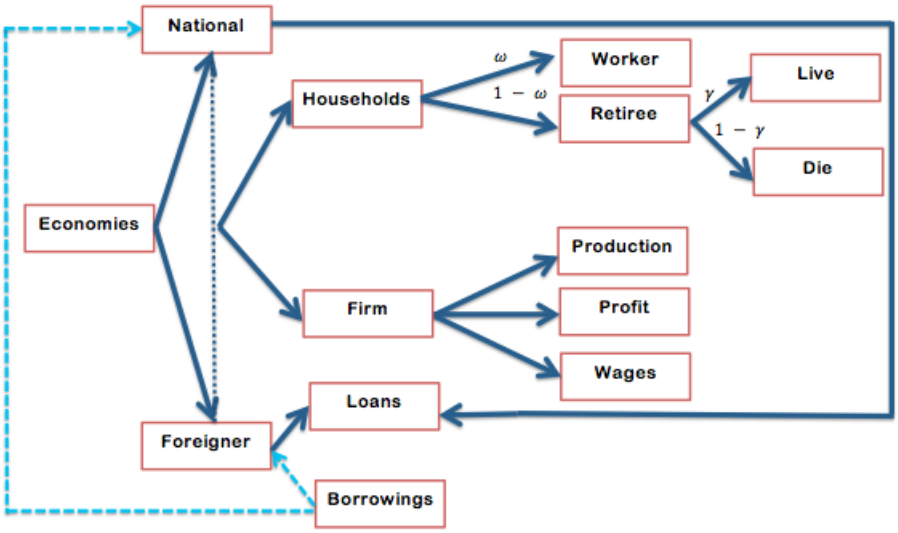
Others studies are interested in the effect of demographic factors on the macrcoeconomics aggretates. (Backus et al. 2013) seek to pintpoint the effects of demography on the macroeconomic fluctuations in capital flows, using an overlapping generations model with an uncertain survival duration. Their findings suggest that a decrease in mortality affects aggregated assets accumulation in twofold :by varying household savings and population structure. The study also finds that in opened economy, demographic factors can impulse persistent capital flows. (Baxter 1995a) develops a two countries international model in which capital accumulations and flows play an important role.

Some authors have tried to check the contribution of demographic factors on other macroeconomic variables. For instance, (Lane & Milesi-Ferretti 2001) argue from USA empirical data and G6 that external debt in industrialized countries depends on output per capita, public debt indicators and demographic indicators. (Borsch-supan et al. 2006) and (Floden 2003) and (Floden 2003) are interested in observed differences in demographic trends to explain the low frequency of international capital mobility during the last 50 years and predict their movements. They find important quantitative effects of demographic variables on external balances. (Chen & al 2009) argue that changes in current accounts in the USA are affected by time difference productivity. (Higgins 1998) focuses his study on analyzing the correlation between age distribution, saving and current account balance.

2 Model

To analyze the impact of demographics on external balances, I develop a two-country version of the (Gertler 1999) model. The model structure is shown in Figure 2.4. Each country is populated by workers, retirees and firms. Workers and retirees make consumption-savings choice and consumption-leisure choice to maximize the utility on their life cycle. Workers face each period at a constant probability ω remaining active in the following period, and thus a constant probability $1 - \omega$ to become retiree. Retired agents make similar choices, taking into account the fact that their work is less productive and therefore less paid than the active agents. The retired agents face a constant probability γ to survive at the end of each period, and thus a probability $1 - \gamma$ of dying in each period. In general, the risks of longevity and retirement transition greatly complicate the calculation of individual decisions, including the consumption function. To work around this

Figure 2.4: Description schématique du modèle international



methodological problem, I adopt the (Gertler 1999) approach. For this reason, the gross return to the period t for financial investments of survivor retirees is equal to $\frac{R_t}{\gamma}$. This assumption is equivalent to the existence of a competitive market for reverse mortgages a la (Yaari 1965) and (Blanchard 1985).

2.1 Intratemporels choices

The retiree's utility is derived from the consumption of goods and leisure, according to $d_t^r = (c_t^r)^{\upsilon} (X_t \ell_t^r)^{1-\upsilon}$ and $d_t^w = (c_t^w)^{\upsilon} (X_t \ell_t^w)^{1-\upsilon}$ for the worker, where υ is the share of goods in the basket d_t^r and d_t^w. In fact, X_t denotes the long run productivity of the labor factor, c_t^r (c_t^w) and ℓ_t^r (ℓ_t^w) denote the consumption of goods and the retiree's leisure (worker)[4].

2.1.1 Consumption - leisure choice of the worker

The worker chooses the quantities of goods and leisure that minimize the cost of his consumption basket d_t^w. The worker's intratemporal problem is:

$$\min_{\{c_t^w, \ell_t^w\}} \quad \phi_t^w = c_t^w + W_t \ell_t^w$$

[4] In the presence of long run economic growth, the marginal utility of leisure must follow the same rate of increase in labor productivity to ensure the existence of a stationary equilibrium for ℓ_t^r et ℓ_t^w.

$$\text{subject to } \; d_t^w = (c_t^w)^{\upsilon} \, (X_t \, \ell_t^w)^{1-\upsilon}$$

where ϕ_t^w is the optimal cost of d_t^w, expressed in terms of goods c_t^w.

The first order conditions for the minimization of the cost of purchasing the basket d_t^w give us the demand functions for c_t^w and ℓ_t^w.

$$c_t^w = \upsilon \, P_t \, d_t^w. \tag{2.1}$$

The worker's leisure demand is directly proportional to the value of the consumer basket and inversely proportional to his salary as follows :

$$\ell_t^w = (1 - \upsilon) \, \frac{P_t \, d_t^w}{W_t}. \tag{2.2}$$

Finally, the price of the worker's consumption basket in terms of consumer goods (the optimum value of ϕ_t^w for $d_t^w = 1$) is:

$$P_t = \tau \left(\frac{W_t}{X_t} \right)^{1-\upsilon} \tag{2.3}$$

where

$$\tau = \frac{1}{\upsilon^{\upsilon} \, (1 - \upsilon)^{1-\upsilon}} \tag{2.4}$$

2.1.2 Consumption - leisure choice of the retiree

The retired agent chooses the amount of goods and leisure that minimize the cost of his consumption basket d_t^r and intratemporal problem of the retiree is as follows :

$$\begin{array}{l} \min\limits_{\{c_t^r, \ell_t^r\}} \; \phi_t^r = c_t^r + W_t^r \, \ell_t^r \\ \text{s.t} \quad d_t^r = (c_t^r)^{\upsilon} \, (X_t \, \ell_t^r)^{1-\upsilon} \end{array}$$

where ϕ_t^r is the optimal cost of the basket d_t^r, expressed in terms of consumer goods. The demand equations of the retired agent are the same as those of the worker. The only difference comes from the fact that the retiree is less productive than the worker. This difference is taken into account by the retiree's real wage.

$$W_t^r = \xi \, W_t \tag{2.5}$$

with ($\xi < 1$). I can infer that the price index of retirees consumption basket is proportional to that of the worker.

$$P_t^r = \xi^{1-\upsilon} \, P_t \tag{2.6}$$

2.2 Intertemporal choices of agents

Each country is populated by a cohort of active households[5] (w) and a cohort of retired households[6] (r). Intertemporal preferences of individuals in the cohort $z = \{w, r\}$ are given by a value function of the utility which takes into account, the characteristics of each agent of the cohort :

$$V_t{}^z = \left\{ (C_t^z)^\rho + \beta^z \left[E_t \left(V_{t+1}^z | \ z \right) \right]^\rho \right\}^{1/\rho} \tag{2.7}$$

The fonction (2.7) is a version of the family of recursive utilities a la (Kreps & Porteus 1991) and (Epstein & Zin 1989). Such preferences between risk aversion from intertemporal substitution, making the individual risk-neutral, while retaining the flexibility regarding the value of the intertemporal elasticity of substitution. The value function and the discount factor differ across cohorts because the future value of the utility depends on the status of the agent. So:

$$E_t \left(V_{t+1} | \ z \right) = \begin{cases} V_{t+1}^r \ si \ z = r \\ \omega V_{t+1}^w + (1 - \omega) V_{t+1}^r \ si \ z = w \end{cases} \quad ; \quad \beta^z = \begin{cases} \beta\gamma \ si \ z = r \\ \beta \quad si \ \ z = w \end{cases}$$

where $E_t \left(V_{t+1}^z | \ z \right)$ is the rational expectation of the function of the conditional future period based on z cohort. β^z is the discount factor for future utilities of the agents of the cohort z and γ is the probability of survival of a retiree. Note that the discount rate of future utilities of a retiree ($\beta^r = \beta\gamma$) is less than that of the worker ($\beta^w = \beta$) because of the probability of survival of the retiree which is less than a unit. The γ et ω probabilities are are, by definition, independent of the age and status(z) of agents. This formulation preferences generate some equivalent decisions in the presence of idiosyncratic risk income. However, the curvature parameter ρ introduced a smoothing between consumption of today and tomorrow.

2.3 Wealth constraint with adjustment costs

The wealth constraint of retiree and financial assets is modeled so that it takes into account the survival probability γ of the retiree and capital adjustment costs.

Consider having a retiree's principal and interest $\left(\frac{1+r_t}{\gamma} \right) b_t^r$ in financial assets and y_t^r as independent income. By assumption, the return on capital r_t is determined for the period $t-1$. Under this assumption,

[5] worker
[6] retirer

r_{t+1} is a known value in period t. The instantaneous budget constraint of the agent retired in period t is:

$$d_t^r + b_{t+1}^r = \left(\frac{1+r_t}{\gamma}\right) b_t^r + y_t^r \tag{2.8}$$

I define $R_{t,\,t+i}$ as follows to simplify the notation.

$$R_{t,\,t+i} = \prod_{k=1}^{i} \left(\frac{\gamma}{1+r_{t+k}}\right) \forall i \geq 1 \Rightarrow R_{t,\,t+k} = \left(\frac{\gamma}{1+r_{t+1}}\right)\left(\frac{\gamma}{1+r_{t+2}}\right)...\left(\frac{\gamma}{1+r_{t+k}}\right) \tag{2.9}$$

$$R_{t,\,t+i} = 1 \; for \; i = 0 \tag{2.10}$$

I get the following intertemporal budget constraint:

$$\sum_{i=0}^{+\infty} R_{t,\,t+i}\, d_{t+i}^r = \left(\frac{1+r_t}{\gamma}\right) b_t^r + \sum_{i=0}^{+\infty} R_{t,\,t+i}\, y_{t+i}^r. \tag{2.11}$$

I define financial (af_t^r), human (h_t^r) and (a_t^r) wealth of the retired agent as follows:

$$af_t^r = \left(\frac{1+r_t}{\gamma}\right) b_t^r \tag{2.12}$$

$$h_t^r = \sum_{i=0}^{+\infty} R_{t,\,t+i}\, y_{t+i}^r \tag{2.13}$$

$$a_t^r = af_t^r + h_t^r \tag{2.14}$$

implying that,

$$a_{t+1}^r = \frac{1}{R_{t,\,t+1}} \left[\left(\frac{1+r_t}{\gamma}\right) b_t^r - d_t^r\right] + \frac{1}{R_{t,\,t+1}} \sum_{i=0}^{+\infty} R_{t,\,t+i}\, y_{t+i}^r. \tag{2.15}$$

so:

$$a_{t+1}^r = \frac{1}{R_{t,\,t+1}} \left(af_t^r + h_t^r - d_t^r\right) = \frac{1}{R_{t,\,t+1}} \left(a_t^r - d_t^r\right), \tag{2.16}$$

The equation (2.16) shows the evolution of the agent's wealth. The financial wealth of the retired agent in an economy with capital corresponds to the capital position and financial assets:

$$b_t^r = q_t\, k_t^r + f_t^r \tag{2.17}$$

where q_t is Tobin's Q and f_t^r other financial assets of the retired agent. The retiree wealth constraint of the model with adjustment costs is given by the expressions (2.16) and (2.17). This gives the wealth constraint of the worker following the same approach while imposing constraint $\gamma = 1$ (the worker has a probability to survive until the next period $\gamma = 1$).

2.3.1 The intertemporal choices of the retiree

The retired agent optimally chooses his consumption basket d_t^r and his total future wealth a_{t+1}^r that maximize (2.18) constrained the evolution of his wealth (2.19) and a terminal condition that requires him to pay all his debts before death.

$$\max_{\left\{d_t^r, a_{t+1}^r, \lambda_t^r\right\}} \quad V_t{}^r = \left\{ (d_t{}^r)^\rho + \beta\, \gamma \left[V_{t+1}^r(a_{t+1}^r) \right]^\rho \right\}^{1/\rho} \tag{2.18}$$

$$s.c \quad a_{t+1}^r = \frac{R_{t+1}}{\gamma} \left(a_t^r - P_t^r\, d_t^r \right) \tag{2.19}$$

Together, the first order conditions of the problem of the retired agent used to deduce the Euler condition of consumption of the retired agent.

$$d_{t+1}^r = \beta^\sigma d_t^r (P_t^r)^\sigma (P_{t+1}^r)^{-\sigma} R_{t+1}^\sigma \tag{2.20}$$

where $\sigma = \frac{1}{1-\rho}$ is the intertemporal elasticity of substitution and R_{t+1} is the real interest rate of assets between periods t and $t+1$. The equation (2.20) shows how the intertemporal consumption path varies with the factor of interest, current and future prices of consumer goods.

I assume that the consumption expenditure of the retired agent (measured in consumer units) is proportional to its total wealth (human and financial wealth):

$$P_t^r\, d_t^r = (\epsilon_t\, \pi_t)\, a_t^r \tag{2.21}$$

where the retirees' marginal propensity to spend wealth $\epsilon_t\, \pi_t$ is written in this way because the similar propensity of workers is π_t. I deduce the dynamic equation of the marginal propensity to spend replacing (2.21) in (2.20) and simplifying.

$$\epsilon_t \pi_t = 1 - \frac{\gamma\, \beta^\sigma\, R_{t+1}^{\sigma-1}\, P_t^{\sigma-1} P_{t+1}^{1-\sigma} (\epsilon_t\, \pi_t)}{(\epsilon_{t+1}\, \pi_{t+1})} \tag{2.22}$$

This relationship shows that the retired agent 's marginal propensity to spend varies with the real interest rate and the path of the price of consumer goods. It should be noted that $\epsilon_t \pi_t$ is a constant and equal to $1 - \beta\gamma$ when the intertemporal elasticity of substitution σ is 1. The human wealth of the retiree is the sum of his wage at the time t and future human wealth h^r_{t+1} updated with the factor $\frac{\gamma}{R_{t+1}}$ which takes into account the probability of survival.

$$h^r_t = \xi\, W_t + \frac{\gamma\, h^r_{t+1}}{R_{t+1}} \tag{2.23}$$

Furthermore, to solve this model and calculate its long run equilibrium, we must know the parametric forms of the value function of retired and active agents. The structure of the model suggests that these functions are proportional to the agent's consumption. In the case of the retired agent, this suggests a form of solution ${V_t}^r = \triangle^r_t\, d^r_t$.

$$(\triangle^r_t)^\rho = \beta^\sigma \gamma\, P_t^{\sigma-1} P_{t+1}^{1-\sigma} R_{t+1}^{\sigma-1} \left(\triangle^r_{t+1}\right)^\rho \tag{2.24}$$

Comparing (2.24) and (2.20) can be deduced that,

$$\triangle^r_t = (\epsilon_t \pi_t)^{\frac{-1}{\rho}} \tag{2.25}$$

2.3.2 The intertemporal choices of the worker

The worker chooses his consumption basket d^w_t and his future total wealth a^w_{t+1} to maximize his utility (2.26) subject to the total wealth constraint.

$$\max_{\{d^w_t\}} \quad {V_t}^w = \left\{ ({d_t}^w)^\rho + \beta \left[\omega V^w_{t+1} + (1-\omega)\, V^r_{t+1}\right]^\rho \right\}^{1/\rho} \tag{2.26}$$

$$s.t \quad a^w_{t+1} = R_{t+1}\left(a^w_t - P_t\, d^w_t\right) \tag{2.27}$$

Together, the first order conditions of the worker's problem can be inferred from the Euler consumption condition of the worker.

$$d^w_t = \beta^{-\sigma} \left(\frac{P_{t+1}}{P_t}\right)^\sigma R^{-\sigma}_{t+1} \left(\omega d^w_{t+1} + (1-\omega) d^r_{t+1} \epsilon^{\frac{\sigma}{1-\sigma}}_{t+1}\right) \Omega^{-\sigma}_{t+1} \tag{2.28}$$

where Ω_{t+1} is the multiplicative factor of real interest rate. This factor takes into account the status of the worker and his retired status :

$$\Omega_{t+1} = \omega + \chi\,(1-\omega)\,\epsilon_{t+1}^{\frac{1}{1-\sigma}} \tag{2.29}$$

with $\chi = \left(\frac{1}{\xi}\right)^{1-\upsilon}$.

The equation (2.28) shows how the consumption path varies with the real interest rate, current and future price of goods and the status of the worker.

I assume that the consumption expenditure of the worker (measured in consumer goods units) is proportional to its total wealth (human wealth and financial wealth). The total wealth of a worker to the period t but becomes retiree at $t+1$ is a_{t+1}^{w}, that is mean a worker at $t+1$ (the worker receives therefore no life annuity on his capital, he only withdraws from period $t+2$). Furthermore spared a unit although the consumer relates to the following period $R_{t+1}\Omega_{t+1}$ baskets. I can therefore deduce that the discount factor is given by $\frac{1}{R_{t+1}\Omega_{t+1}}$.

$$P_t\,d_t^{w} = \pi_t\,a_t^{w} \tag{2.30}$$

I deduce the dynamic equation of the marginal propensity to spend replacing (2.30) in (2.28) and simplifying for:

$$\pi_t = 1 - \frac{\beta^{\sigma} R_{t+1}^{\sigma-1}\,P_t^{\sigma-1} P_{t+1}^{1-\sigma}\Omega_{t+1}^{\sigma-1}\,\pi_t}{\pi_{t+1}} \tag{2.31}$$

This relationship shows that the marginal propensity to consume of the worker varies with the real interest rate and the path of the price of consumer goods. It should be noted that π_t is simply equal to $1-\beta$ when σ, elasticity of substitution is 1. In other words, the worker's marginal propensity to consume is constant in time when the intertemporal elasticity of substitution is unitary. The human wealth of the worker is the sum of his wage, his future wealth which takes into account his activity status $\omega\,h_{t+1}^{w}$ and that taking into account his retired status in the future $\chi(1-\omega)\,\epsilon_{t+1}^{\frac{1}{1-\sigma}}\,h_{t+1}^{r}$.

$$h_t^{w} = W_t + \frac{\chi\,(1-\omega)\,\epsilon_{t+1}^{\frac{1}{1-\sigma}}\,h_{t+1}^{r} + \omega\,h_{t+1}^{w}}{R_{t+1}\,\Omega_{t+1}} \tag{2.32}$$

where $h_t^{w}$is the present human wealth of the worker and $\frac{1}{R_{t+1}\Omega_{t+1}}$ the discount rate of the human wealth. The active agent saves for retirement.

The likelihood of losing his wage after retirement pushes the worker to update the stream of future earnings when faced with high real interest rate. This effect was reflected by the presence of the probability

of remaining in the working population which affects the discount rate. A high discount rate reduces the value of human wealth compared to the case of infinite horizon, thereby reducing current consumption and increasing savings for retirement. The Ω_{t+1} factor varies positively with the ratio of the marginal propensity to consume of the retiree and that of the worker ϵ_{t+1} and the probability of becoming retiree.

If $\epsilon_{t+1} > 1$, then $\Omega_{t+1} > 1$. $\Omega_{t+1} > 1$ implies that the worker discounts the future at a relatively high rate compared to the infinite horizon when Ω_{t+1} is introduced multiplicative way with the real interest rate R_{t+1} in the consumption function. The gain of marginal utility of a unit of wealth for the retired agent is less than that of a worker.

As in the case of the retired agent, I postulate that the active agent value function is proportional to the consumption basket ${V_t}^w = \triangle_t^w d_t^w$, $\triangle_t^w$ the proportion applicable to the choice of the period t. This gives us :

$$\triangle_t^w = (\pi_t)^{\frac{-1}{\rho}} . \tag{2.33}$$

2.4 Firms

Firms maximize the present value of their net profit under constraint of investment by engaging the factors of production, capital and labor. Firms are responsible for the accumulation of capital and households are owners of firms via an implied equity market. Convex adjustment costs are forcing firms decisions in respect of capital. The presence of these adjustment costs significantly alter the behavior of the firm, particularly following an exogenous shock after which it is encouraged to gradually adjust capital spending.

The aggregate production function of the representative firm is given by a Cobb Douglas technology.

$$Y_t = Z_t \, K_t^{1-\alpha} \, (X_t \, H_t)^{\alpha} \tag{2.34}$$

where H_t is the aggregate labor demand, Z_t multifactor productivity, K_t the capital stock, X_t the labor factor productivity and Y_t the production.

The labor factor productivity X_t increases at rate $(1 + x)$. It is exogenous and neutral a la Harod.

$$X_{t+1} = (1 + x) \, X_t. \tag{2.35}$$

Multifactor productivity Z_t is modeled in taking into account the idea that the two countries may have

different levels of productivity in the short term, so that

$$Z_{t+1} = Z^{1-\phi}\, Z_t^{\phi} \tag{2.36}$$

with $0 < \phi < 1$ and Z the long run multifactor productivité [7]. The possibility of transitory shocks to Z_t that fade at a rate ϕ. The value of the representative firm function is given by

$$\begin{aligned} J\left(k_t\right) \quad = \quad & Z_t K_t^{1-\alpha}\left(X_t H_t\right)^{\alpha} - \tfrac{1}{2}\Phi\left(\tfrac{I_t}{K_t} - (x+\delta)\right)^2 - W_t\, H_t - I_t + \frac{J\left[K_{t+1}\right]}{R_{t+1}} + \\ & q_t\left(I_t + (1-\delta)K_t - K_{t+1}\right) \end{aligned} \tag{2.37}$$

where q_t is the relative price of capital in place (Tobin's Q) and $\Phi \geq 0$ is the adjustment costs parameter. Finally δ is the capital depreciation rate.

The first order conditions of the equation (2.37) imply

$$W_t = \alpha \frac{Z_t\, K_t^{1-\alpha}\left(X_t\, H_t\right)^{\alpha}}{H_t} \tag{2.38}$$

The equation (2.38) determines the H_t employment. The firm adjusts to the point where H_t real wage is equal to the marginal productivity of labor.

The investment function is given by the following expression:

$$I_t = \frac{q_t - 1}{\Phi}\, K_t + (x+\delta)\, K_t. \tag{2.39}$$

$$K_{t+1} = I_t + (1-\delta)K_t \tag{2.40}$$

Finally, an implicit capital rental market determines the rent of capital q_t:

$$q_t = \frac{1}{R_{t+1}} \left[\begin{array}{l} -\tfrac{1}{2}\Phi\left(\frac{I_{t+1}}{K_{t+1}} - (x+\delta)\right)^2 + \Phi\frac{I_t+1}{K_{t+1}}\left(\frac{I_t+1}{K_{t+1}} - (x+\delta)\right) + \\ \\ (1-\delta)q_{t+1} + \frac{(1-\alpha)\, Y_{t+1}}{K_{t+1}} \end{array} \right] \tag{2.41}$$

[7] The value of $Z = 1$ will be used at the steady state and the dynamic transition of the model.

An asset market at time t determines the real interest rate R_{t+1} is equal to the capital marginal product.

$$R_{t+1} = (1-\alpha)\frac{Y_t}{K_t} + (1-\delta) \tag{2.42}$$

2.5 The population dynamics

The structure of the life cycle is as follows: at time t, workers (w) and retirees (r) have N_t^w and N_t^r workforce respectively. Workers in period t have a constant probability ω to remain in the working population at time $t+1$ and a probability $(1-\omega)$ to become a retiree. The survival probability of a retiree between the period t and $t+1$ is γ and dying from it is $(1-\gamma)$. The number of births at time $t+1$ is given by $(n_t - \omega)N_t^w$. The average lifetime of an agent in the population of workers is $\frac{1}{1-\omega}$ periods and that remain in the retired population is $\frac{1}{1-\gamma}$ periods.

The number of workers increases at the gross rate n_t[8]:

$$N_{t+1}^w = (n_t - \omega)N_t^w + \omega N_t^w = n_t N_t^w \tag{2.43}$$

The number of retired agents at time $t+1$ is equal to the sum of the number of new retirees at time t and retired survivors.

$$N_{t+1}^r = (1-\omega)N_t^w + \gamma N_t^r \tag{2.44}$$

Dividing equation (2.44) by N_{t+1}^w, I find the dynamic of ψ_t dependency ratio of retirees compared to active agents.

$$\psi_{t+1} = \frac{1-\omega}{n_t} + \gamma\frac{\psi_t}{n_t} \tag{2.45}$$

In the long run: $\psi_t = \psi_{t+1} = \psi$.

$$\psi = \frac{1-\omega}{n-\gamma}\,. \tag{2.46}$$

To take into account the population dynamics in the two-country model, I subdivide the world's population: the world population of workers and retired population. World Population of workers P_t^w is composed of the population of workers of the national economy N_t^w and that of the workers of the foreign economy NE_t^w.

$$P_t^w = N_t^w + NE_t^w \tag{2.47}$$

[8] where $n_t = 1 + n$

The share of the population of workers of the local economy is symbolized by θ_t and the foreign economy $\theta_t^E = 1 - \theta_t$.

The dynamics of the size of the world population of workers at time $t + 1$ is given by the following expression:

$$P_{t+1}^w = n_t N_t^w + n_t^E N E_t^w \tag{2.48}$$

In other words, the equation (2.48) shows that the number of the world population at time $t + 1$ is equal to the sum of the workers of local economy $n_t N_t^w$ and that of the foreign economy $n_t^E N E_t^w$. The dynamics of the workers population share of the national economy, which depends on the population growth rate of the local economy, the foreign economic growth rate, and shares of the populations of workers of each economy at time t is given by:

$$\theta_{t+1} = \frac{\theta_t \, n_t}{\theta_t \, n_t + (1 - \theta_t) \, n_t^E} \tag{2.49}$$

In the steady state, $\theta_{t+1} = \theta_t = \theta$. This share depends on the history of population growth rate in individual countries and in the simulations presented below assumes that $\theta = \frac{1}{2}$ is the starting point.

In the simulations, I assume that the population growth rate n_t follows a first order autoregressive process.

$$n_{t+1} = n^{1-\rho} \, n_t^{\rho}, \tag{2.50}$$

where ρ determines the duration of a shock when it occurs and n the population growth rate of the steady state. It should be noted that both economies population growth rate will be the same in the long run to ensure long run balance, but they can differ in the short run.

2.6 Aggregation of individual choices

After deducting the consumption functions and wealth of individual agents, it is now possible to aggregate individual decisions in order to calculate the overall spending of the economy on consumer goods and leisure. Aggregated variables are identified in capital letters to avoid confusion between individual choices and aggregate values. Moreover, this aggregation takes into account the fact that the population growth rate of two countries may differ in the short and medium term.

The total wealth of retirees is the sum of the aggregate financial wealth of retired agents who survive,

γAF_t^r, and their aggregate human wealth H_t^r:

$$A_t^r = \gamma AF_t^r + H_t^r \tag{2.51}$$

The aggregate human wealth of retired agents is given by

$$\begin{aligned} H_t^r &= \xi\, W_t\, N_t^r + \frac{\gamma\, N_t^r\, H_{t+1}^r}{R_{t+1} N_{t+1}^r} \\ &= \xi\, W_t\, N_t^r + \frac{\gamma\, \psi_t\, H_{t+1}^r}{R_{t+1}((1-\omega) + \gamma\, \psi_t)} \end{aligned} \tag{2.52}$$

The equation (2.52) shows that the aggregate human wealth H_t^r is the sum of the present value of the aggregate wealth of retirees outliving each period. In fact, the wealth of retirees is discounted at $R_{t+1}((1-\omega) + \gamma\, \psi_t)$. This discount rate reflects the number of retirees populating the economy in period t. In this context, an increase factor $((1-\omega) + \gamma\, \psi_t)$ causes a drop in aggregate human wealth of retirees.

The total wealth of workers A_t^w is composed of aggregate human wealth of workers at time t and the aggregate financial wealth of workers at time t :

$$A_t^w = AF_t^w + H_t^w \tag{2.53}$$

The aggregate human wealth of workers H_t^w is composed of human wealth of workers at time t, the updated wealth of workers at time $t+1$ and human wealth of the workers present during retirement. It is given by the following relationship:

$$\begin{aligned} H_t^w &= W_t\, N_t^w + \frac{\frac{\omega\, N_t^w\, H_{t+1}^w}{N_{t+1}^w} + \frac{\chi\,(1-\omega)\,\epsilon_{t+1}^{\frac{1}{1-\sigma}}\, N_t^w\, H_{t+1}^r}{N_{t+1}^r}}{R_{t+1}\Omega_{t+1}} \\ &= W_t\, N_t^w + \frac{\omega\, H_{t+1}^w}{n_t\, R_{t+1}\Omega_{t+1}} + \frac{\chi\,(1-\omega)\,\epsilon_{t+1}^{\frac{1}{1-\sigma}}\, H_{t+1}^r}{((1-\omega) + \gamma\, \psi_t) R_{t+1}\Omega_{t+1}} \end{aligned} \tag{2.54}$$

The equation (2.54) shows that the human wealth H_t^w is a sum of the present value of the aggregate wealth of the economy in each period. The discount rate $n_t\, R_{t+1}\, \Omega_{t+1}$ is the product of workers discount rate and population growth rate. The population growth rate increase the discount rate because as the population increases, the human wealth of workers decline. The decline in demand for leisure of the workers when the

wage W_t increases and the demand for goods falls when the price P_t rises.

$$N_t^w \, \ell_t^w = \frac{(1-\upsilon) \, P_t \, D_t^w}{W_t} \tag{2.55}$$

Investment demand is a function of capital stock and takes into account the adjustment costs :

$$I_t = \frac{q_t - 1}{\Phi} \, K_t + (x + \delta) \, K_t \tag{2.56}$$

Moreover, the financial wealth of the economy Af_t is the difference between total wealth of the economy $A_t^r + A_t^w$ and aggregate human wealth of the economy $H_t^r + H_t^w$:

$$Af_t = A_t^r + A_t^w - (H_t^r + H_t^w) \tag{2.57}$$

The share of financial wealth Af_t^r held by retired employees is given by λ_t et and that of workers Af_t^w is given by $1 - \lambda_t$.

$$\lambda_t = \frac{Af_t^r}{Af_t} \tag{2.58}$$

$$1 - \lambda_t = \frac{Af_t^w}{Af_t} \tag{2.59}$$

The aggregate function of spending in the economy is the sum of consumption of workers and retirees. It is given by

$$B_t = D_t^r + D_t^w = \frac{\epsilon_t \pi_t \, (Af_t^r + H_t^r)}{P_t^r} + \frac{\pi_t \, (Af_t^w + H_t^w)}{P_t} \tag{2.60}$$

Expressing the expression (2.60) in terms of share of financial wealth held by pensioners agents tees and active agents, I obtain:

$$B_t = \pi_t \left((1 - \lambda_t) + \epsilon_t \lambda_t \right) Af_t + \pi_t \left((1 - \varsigma_t) + \epsilon_t \varsigma_t \right) H_t^g \tag{2.61}$$

where

$$\varsigma_t = \frac{H_t^r}{H_t^g} \quad et \quad 1 - \varsigma_t = \frac{H_t^w}{H_t^g} \tag{2.62}$$

and $H_t^g = H_t^w + H_t^r$ is the human wealth of the economy, ς_t the share of this wealth held by retirees and $1 - \varsigma_t$ the share of this wealth held by workers. The dynamics of aggregate financial wealth of retired agents

in period $t+1$ is then modeled as follows:

$$A^r_{t+1} = R_t\, A^r_t + W_t\, \xi\, N^r_t - P^r_t\, C^r_t + (1-\omega)\,(R_t\, A^w_t + W_t\, N^w_t - P_t\, C^w_t) \tag{2.63}$$

The equation (2.63) shows that the aggregate wealth of the retired agents at time $t+1$ is the sum of the wealth from already retired employees, their employment income minus consumption in period t and wealth from new retirees in period t minus their consumption ($(1-\omega)\,(R_t\, A^w_t + W_t\, N^w_t - P_t\, C^w_t)$).

The dynamics of the aggregate financial wealth of workers to the period $t+1$ is:

$$A^w_{t+1} = \omega\,(R_t\, A^w_t + W_t\, N^w_t - P_t\, C^w_t) \Rightarrow R_t\, A^w_t + W_t\, N^w_t - P_t\, C^w_t \tag{2.64}$$

The equation (2.64) shows that the aggregate wealth of workers at time $t+1$ is the amount of wealth from assets of the workers in period t, their human wealth of period t minus consumption.

The dynamics of the share of financial wealth of the economy owned by retirees is obtained by combining (2.58), (2.59), (2.63) et (2.64):

$$\begin{aligned}\lambda_{t+1} = {} & (1-\omega)\,\frac{AF^w_t\, R_{t+1}}{AF_{t+1}} - (1-\omega)\,\frac{D^w_t\, R_{t+1}\, P_t}{AF_{t+1}} + \frac{\xi\, N^r_t\, R_{t+1} W_t}{AF_{t+1}} \\ & + (1-\omega)\,\frac{N^w_t\, R_{t+1}\, W_t}{AF_{t+1}} - \frac{H^r_t\, R_{t+1}\, \epsilon_t\, \pi_t}{AF_{t+1}} + \frac{AF_t\, R_{t+1}\, \lambda_t}{AF_{t+1}} - \frac{AF_t\, R_{t+1}\, \epsilon_t\, \pi_t\, \lambda_t}{AF_{t+1}}\end{aligned} \tag{2.65}$$

International capital flows equalize the real interest rate in both countries, so the interest rate of local R_t and foreign interest rate R^E_t are always equal.

$$R_t = R^E_t \tag{2.66}$$

The sum of financial wealth in bonds of the local country F_t and foreign exchange F^E_t must be zero at equilibrium because the loans of a country is borrowing of another:

$$F_t + F^E_t = 0 \tag{2.67}$$

By incorporating demographic factors in equation (2.67) via the multiplication and division of the equation by the size of the population of workers of local N^w_t and foreign economy NE^w_t, I express the flows of

international capital in the workforce of workers population in each country.

$$\theta_t f_t + (1 - \theta_t) f_t^E = 0 \tag{2.68}$$

The internal balance between supply and demand is given by the following identity which states that GDP is the sum of expenditures (consumption and investment) and external balances related to the accumulation of external assets :

$$Y_t = C_t^r + C_t^w + I_t + XN_t \tag{2.69}$$

In the equilibrium, the external balances of both countries are summed to zero.

$$XN_t + XN_t^E = 0 \tag{2.70}$$

Taking into account the share of the population of workers in each country, the expression of the external balances becomes

$$\theta_t xn_t + (1 - \theta_t) xn_t^E = 0. \tag{2.71}$$

where xn_t and xn_t^E represent the external balances of the local economy and foreign economies taking into account demographic factors.

Considering F_t as net assets of the local economy, the local economy ready abroad at world real interest rate R_t. The dynamics of these assets depends on the market of goods and assets. Net foreign assets represent payments received from the rest of the world in exchange for exports. The constraint of the international economy of resources becomes

$$F_{t+1} = R_t F_t + XN_t. \tag{2.72}$$

The aggregate financial wealth AF_t of the economy is the sum of the return on capital and assets at time t with q_t the capital price, this is the logic of adjustment costs:

$$AF_t = R_t(q_t K_t + F_t). \tag{2.73}$$

The aggregate employment in the economy is composed of the labor supply of workers and retirees

$$H_t = N_t^w L_t^w + N_t^r \xi L_t^r \tag{2.74}$$

where $\xi\, L_t^r$ is the actual amount of work done by retirees and $\xi \in (0,1)$ represents the productivity of a work unit provided by a senior (retiree) compared to a younger person (worker). Each individual has a unit time period which he can use either to work or to enjoy leisure.

The model considers that the two countries are symmetrical (the agent behaviors are similar in both countries and they are modeled in the same way) with the exception of the coefficients. However, all local economic equations are similar to those of foreign economy and the relevant foreign economic variables are indexed by the letter E.

2.7 The steady state

At the steady state, all variables grow at the same exogenous rate equal to the growth rate of the labor productivity X_t. I can therefore represent the variables of the model in relation to labor productivity.

$$h^r = \frac{\alpha(1-\omega)\,\xi\, k^{1-\alpha} z\, R\left(l^w + \frac{\xi\,(1-\omega)\,l^r}{n-\gamma}\right)^{\alpha-1}}{(n-\gamma)\,(R-(1+x)\gamma)} \tag{2.75}$$

where $(n-\gamma)\,(R-(1+x)\gamma)$ is the discount rate of human wealth retiree taking into account the life cycle. The equation (2.76) below shows the human wealth of workers:

$$h^w = \frac{(n-\gamma)(1+x)\,\epsilon^{\frac{1}{1-\sigma}}\left(\frac{1}{\xi}\right)^{1-\upsilon} h^r + \alpha\,\xi\, k^{1-\alpha}\, z\left(l^w + \frac{\xi\,(1-\omega)\,l^r}{n-\gamma}\right)^{\alpha-1}\Omega}{R\,\Omega - \omega(1+x)}, \tag{2.76}$$

where $R\,\Omega - \omega(1+x)$ is the discount rate of the human wealth of the workers. The equation (2.77) below shows the total wealth of retirees. It is the sum of his human and financial wealth.

$$a^r = h^r + (f+k)R\,\gamma\,\lambda \tag{2.77}$$

As a result, the equation (2.78) below shows the total wealth of workers. It is the sum of its human and financial wealth.

$$a^w = h^w + (f+k)(1-\lambda)R \tag{2.78}$$

The equation (2.79) below shows the share of the financial wealth of the retirees in the total financial wealth of the local economy.

$$\lambda = \frac{\begin{array}{l} R(1-\omega)(f+k) - (a^w + \varepsilon h^r)\,\pi + \\ \alpha\, k^{1-\alpha} z \left(l^w + \frac{\xi\,(1-\omega)\,l^r}{n-\gamma}\right)^{\alpha-1} \left((1-\omega) + \frac{\xi\,(1-\omega)}{n-\gamma}\right)^{\alpha-1} \end{array}}{(f+k)\,(n\,(1+x) + R(\varepsilon\,\pi - \omega))} \tag{2.79}$$

The equations (2.80), (2.81), (2.82) and (2.83) give the real interest rate, the marginal propensity to consume of the retired agent, the marginal propensity to consume of the active agent and the factor multiplying the discount rate respectively.

$$R = z\,(1-\alpha)\left(l^w + \frac{\xi\,(1-\omega)\,l^r}{n-\gamma}\right)^{\alpha} k^{-\alpha} + 1 - \delta \tag{2.80}$$

$$\epsilon\pi = 1 - R^{\sigma-1}\beta^{\sigma}\gamma \tag{2.81}$$

$$\pi = 1 - R^{\sigma-1}\beta^{\sigma}\Omega^{\sigma-1} \tag{2.82}$$

$$\Omega = \omega + (1-\omega)\,\epsilon^{\frac{1}{1-\sigma}} \left(\frac{1}{\xi}\right)^{1-\upsilon} \tag{2.83}$$

$$l^r = \frac{n-\gamma}{\xi(1-\omega)} \left[\frac{a^r(n-\gamma)\epsilon\pi \left(\frac{1}{\xi}\right)^{1-\upsilon}(1-\upsilon)}{\alpha \left(\frac{1}{\xi}\right)^{-\upsilon}(1-\omega)k^{1-\alpha} z\,\ell^r}\right]^{\frac{1}{\alpha-1}} - \frac{n-\gamma}{\xi(1-\omega)} l^w \tag{2.84}$$

The expressions (2.84) and (2.85) respectively give the labor supply of the retirees and that of the workers.

$$l^w = \left(\frac{a^w \pi(1-\upsilon)}{\alpha\, z\,\ell^w\, k^{1-\alpha}}\right)^{\frac{1}{\alpha-1}} - \frac{\xi(1-\omega)}{n-\gamma} l^r \tag{2.85}$$

The equations (2.86) et (2.87) respectively give the external balance of the economy and financial assets

(bonds) of the local economy.

$$xn = \left[z \left(l^w + \frac{\xi(1-\omega) l^r}{n-\gamma} \right)^{\alpha} k^{-\alpha} - (x+\delta) \right] k - (a^w + \epsilon a^r)\pi\upsilon \tag{2.86}$$

$$f = \frac{xn}{n(1+x) - R} \tag{2.87}$$

with xn the external balance of the local economy in the steady state. In this context, the equation (2.88) shows that the sum of local and foreign assets, while the equation (2.89) shows that the sum of the external balances of the steady state is zero.

$$\theta f + (1-\theta) f^E = 0 \tag{2.88}$$

$$\theta xn + (1-\theta) xn^E = 0 \tag{2.89}$$

Finally, equation (2.90) shows that real interest rate should be equal in both countries.

$$R = R^E \tag{2.90}$$

3 Calibration and description of the experiment

The Model calibration is enabled by using macroeconomic data, quantitative literature and demographic data got from the "World Population Prospects the 2012 Revision[9] ." I consider two general cases. In the first, I denotes the symmetric case, all demographic parameters are the same in both countries. In the second case, I denotes asymmetric cases, some demographic parameters are not similar.

In the symmetric case, I consider that the two populations grow at the average rate of 1,01%. There probability ω of remaining in the population of workers is 0.977. This implies that the average age to stay in the population of workers $\frac{1}{1-\omega}$ is 45 years in the North as in the South. And therefore the probability of staying in the retired population is equal to $\gamma = 0.9$. The probability of surviving after retirement means that the average number of periods of stay in the retired population $\frac{1}{1-\gamma}$ is 10 years and it is the same in both countries.

[9]Population Division of the Department of the United Nations Secretariat (2013),World Population Prospects the 2012 Revision Highlights and advance Tables.

I firstly consider a situation of a different retirement life expectancy between the two areas (retirement life expectancy assumed to be low in developing countries) and a decrease in the working time in advanced economies without changing life expectancy. Regarding age structure, I consider that retirees in developed countries have longer life expectancy than those of developing countries. This retirement life expectancy is 10 years on average of northern retirees and therefore a probability of surviving after retirement is 0.9. In contrast, retirees from developing country have a retirement life expectancy of 5 years on average and hence a survival probability of 0.8.

I consider, furthermore, the decrease of working time over one year in advanced economies without varying life expectancy. As a consequence, the probability of surviving within the working group decreases at time t and thus the increasing of the retirement life expectancy in developed countries. Scenarios presented in both tables Table 2 and Table 3 take into account differences between the two countries.

Table 2: Parameter values ??for symmetric scenario (benchmark)

	Symmetric scenario (benchmark)													
Parameters	β	υ	δ	σ	α	ω	γ	x	n	ϕ	ξ	Φ	z	ρ
National economy														
Parameter values	0.96	0.4	0.1	0.25	0.667	0.977	0.9	0.01	1.01	0.75	0.55	0.75	1	0.9
Foreign economy														
Parameter values	0.96	0.4	0.1	0.25	0.667	0.977	0.9	0.01	1.01	0.75	0.55	0.75	1	0.9

Notes: The two countries have similar demographic characteristics.

Table 3: Reduced life expectancy at retirement in the South

	Symmetric scenario													
Paramètres	β	υ	δ	σ	α	ω	γ	x	n	ϕ	ξ	Φ	z	ρ
National economy														
Parameter values	0.96	0.4	0.1	0.25	0.667	0.977	**0.9**	0.01	1.01	0.75	0.55	0.75	1	0.9
Foreign economy														
Parameter values	0.96	0.4	0.1	0.25	0.667	0.977	**0.8**	0.01	1.01	0.75	0.55	0.75	1	0.9

Notes: The probability γ to survive South retired agent is less than that of a retired officer of the North.
This shows that life expectancy at retirement is loIr in the South.

Table 4: Reducing the activity period of one year and increase the length of retirement a year without reducing life expectancy in the North

	Symmetric scenario													
Paramètres	β	υ	δ	σ	α	ω	γ	x	n	ϕ	ξ	Φ	z	ρ
National economy														
Parameter values	0.96	0.4	0.1	0.25	0.667	**0.97645**	**0.90909**	0.01	1.01	0.75	0.55	0.75	1	0.9
Foreign economy														
Parameter values	0.96	0.4	0.1	0.25	0.667	0.977	**0.9**	0.01	1.01	0.75	0.55	0.75	1	0.9

Notes: Reducing years spent at work a year to increase the life expectancy of the retired one year without reducing life expectancy in the North increases the probability of working ω and improved the probability of survival during retirement in the North

4 Quantitative results

4.1 Decline in life expectancy in the South

As this thesis analyzes the impacts of demographic factors on the external balances, it is relevant to focus on the demographic variables such as the expectation of life at retirement and number of years at work. In fact, life expectancy can vary and cause differences in the behavior of agents (retired agent and workers). These changes in behavior of a class of economic agents can influence the rest of the economy because of general equilibrium effects.

4.1.1 Effects in the South

The table (Table 5 in the Appendix A) shows the impact of the decline in life expectancy at retirement in the South and the consequences of this change on the model variables. The table reveals that the decline in the life expectancy at the retirement ($\gamma^E = 0.8$) in the Southern economy is reflected increases in the marginal propensity to consume. In fact, this passes of 0.086 to 0.106 for the workers and 0.149 to 0.243 ($\epsilon^E \pi^E$) for the retirees. Reducing the retirement life expectancy also means reductions in the human wealth of retired and active agents that pass from 0.745 to 0.237 and 10.296 to 8.189, respectively. Higher marginal propensities to spend causes a decline in hours worked for both categories of agents. This decline in retired life expectancy encourages the retirees to work less. This leads to a decrease in employment (h^E) and therefore higher wages (W^E) which increased from 0.948 to 0.9485 in the economy of the South. The decrease in working hours of retirees causes a decline in human wealth $(h^r)^E$ and their total wealth as the decline in life expectancy at retirement incentive to save less. The disincentive to savings causes a decline in the share of financial wealth of the retirees of the South.

Moreover, the decline in hours worked causes the increase in working capital compared kh^E in the South. At time of change, the decline in hours worked in the economy of the South leads the decline in South R^E real interest rate relative to that of North R. This encourages South savers to place their assets in foreign. This is what explains the increase of f^E regarded as the South's economy ready for the northern economy. In other words, capital move to the northern economy. It is for this reason that kh capital-labor ratio increases in the North. The increase in the capital-labor ratio in the two regions leads ultimately a lower real interest rate in both zones. This increase also causes an increase in wages.

It should be noted that the mobility of capital and goods cause an equalization of factor prices in both zones (Stolper-Samuelson). The production decline in the economy of the South (y^E) and increases in

the northern economy (y). These production changes are variations in employment and capital in the two regions. The decline in production in the South is not completely transferred to the consumer because of the negative external balance possible by the accumulation of f^E. Moreover, the decline in life expectancy at retirement in the South (γ^E) makes it less attractive for capital investment in the life of retired agents is short.

4.1.2 Effects in the North

The increase in the real interest rate in the economy of the South, following a decline in life expectancy at retirement is expressed at the moment of change, the decline in hours worked in the economy of South resulting in the decline in South R^E real interest rate relative to that of North R. This encourages South savers to invest their assets abroad. In other words, capital move to the northern economy. It is for this reason that the capital labor ratio kh increases in the North. To this end, the northern economy accumulates South foreign assets and capital (k) and therefore, the northern economy accumulates debts ($f < 0$) to the economy of the South.

As the northern economy accumulates debts, it should have a positive long run external balance ($xn > 0$). The increase of capital in the northern economy causes higher wages (W). This increase in wages encourages agents (active agent and retired) to work more at the expense of leisure time that decrease. Rising real wages leads, ceteris paribus, the increase in human wealth of active and retired agents in the northern economy (h^w and h^r). The marginal propensities to spend of workers and retirees of the North ($\epsilon\pi$ and π) decrease. This loIr marginal propensities to consume for agents leads to loIr retiree consumer spending and the workers (d^r et d^w) in the northern economy. Production increases in the northern economy (y). This change comes from the increase in employment and capital in the northern economy.

4.2 Reduction of working time without change in life expectancy in the North

The table (see Table 6 in Appendix A) below present the impact of the decrease in reducing the number of periods spent at work for a year without changing the life expectancy of the agent in the northern economy and its spread on the variables of international model.

4.2.1 Effects on the northern economy

This change in behavior has the effect of reducing the period $\frac{1}{1-\omega}$ during which the northern workers can work at higher wages (activity period) and increase the period $\frac{1}{1-\gamma}$ during which they can work in low-wage ξW (retirement period). This change has implications for reducing the value of time in the northern economy (the Becker "full income" is lower) and therefore wages (W) because the active agents work more to compensate for the decline in period working to increase their retirement period of one year. This wealth effect mostly sustained by workers encourages them to increase working hours (l^w increases in the North) and thus reduce leisure.

The workers of the northern economy must work harder each period to offset the decline in the activity period. The increase of working hours reduced the capital-labor ratio (kh) in the North. This lower capital-labor ratio produces an increase in the marginal productivity of capital and real interest rate R to the northern economy. This means that the capital becomes more productive in the North. Given the rising real interest rate in the North, South savers are interested in investing in the North and therefore the northern economy gets debt in the South ($f < 0$) and South economy becomes lending ($f^E > 0$) . This North borrowings must be offset by a current account surplus ($xn > 0$) in the long run.

Furthermore, the reduction of the period of activity causes a change in the consumption decisions of workers of the northern economy. In fact, the decline in the period of activity causes a decline in the ratio of marginal propensities to spend of the workers and that of the retired agent ϵ in the northern economy. The decrease of the marginal propensity to consume of the workers ϵ for the benefit of the increase in the marginal propensity to spend retired agent. This shows that the worker is declining consumption since the rise of his retirement. He consumes less during his period of activity to offset the rise in the retirement period. This implies decreasing consumption expenditure (d^w) of the worker in favor of the increase of the retirement consumption expenditures (d^r).

The decline in the period of activity of the worker in the North causes an increasing human wealth (wages and human wealth updated) because his human wealth increases more than wages. This increase in human wealth of the worker is explained by the higher discount factor of the human wealth $\frac{1}{R\,\Omega}$ of the worker (the discount factor Ω and R increase) in the North. The human wealth of the northern retiree increases given the rise in productivity in the North. The production of the northern economy increases. This increase is caused by the increase in employment and capital. This economic growth is accompanied by rising current account balance in the North. The share of the financial wealth of northern retirees increases.

4.2.2 Effects on the economy of the South

In fact, the rise in the real interest rate in the northern economy spreads through the economy of the South since the two economies have the same real interest rate that is the global rate. The rise in world real interest rate lowered the capital-labor ratio in the South. This lower capital-labor ratio causes decreasing wages in the South. This change has no direct effect on the Becker "full income" of the South. Southern workers are mainly affected by the substitution effect caused by decreasing wages. Decreasing wages causes a disincentive to work in the South. Hence, active and retired agents of the South reduce their labor supply. This change primarily affects workers who have to work harder and earn more capital to finance retirement. Because they have more investment income, retirees consume more. The rising level of consumption of retirees $(d^r)^E$ (the increase in the demand for leisure and consumer goods $(c^r)^E$). The share of financial wealth held by retirees is increasing in the South as rising real interest rate. However, the share of wealth held by retirees is higher in the North.

The southern economy lends to the North ($f^E > 0$). This accumulation of assets must be compensated by a current account deficit ($xn^E < 0$) in the long run. The decline in the labor supply (h^E) and capital (k^E) in the southern economy cause a decline in production in the South. Decreasing wages in the economy of the South and rising $R^E\,\Omega^E$ causes decline in human wealth of workers in the economy of the South. This shows that the workers discount the future at a higher rate. The decline in human wealth of workers results in a decrease in its consumption of goods $(c^w)^E$. Decreasing wages encourages southern workers to demand more leisure. The decrease in consumption of goods associated with the increase in demand for leisure cause an increase in the expense of the workers of the North $(d^w)^E$ (the marginal propensity to spend π^E increases). The investment in South economy decline since the South savers invest in the North.

5 dynamic path

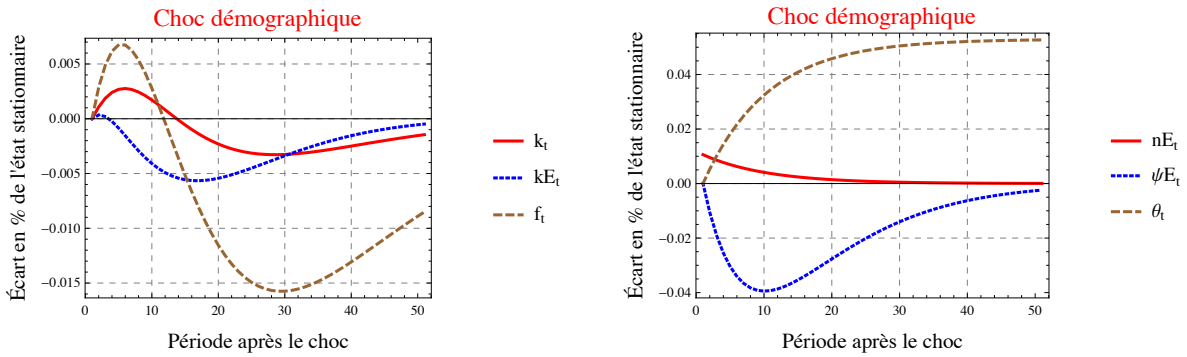
5.1 Demographic shock : baby boom in the South (+ 1,06%)

According to the projections of the Population Division of the United Nations 2012-2013, the southern population will grow by 2.07 % on average during the period 2013-2050. Taking into account these projections, I can see that population growth compared to the steady state varies from 1.06%.

Sustainable increases but not the permanent population growth rate in developing countries represents a positive supply shock in this economy. The probability of remaining worker makes sure that the size of

the southern group of workers increases relative to retirees for a while. Eventually, workers will spend in retirement and the size of retired agents will increase in the South. This shock to the population growth rate (increase nE_t) permanently increases the size of the population of southern workers (θE_t) relatively to that of the North. The baby boom also causes relative abundance of resources (such as labor input in the economy of the South, Figures 5.5) following this durable shock. This is what explains the decline in the real interest rate in the South (RE_t in Figure 5.7).

Figure 5.5: Effects on capital, wealth and demographics



The baby boom in the South (increase in labor supply) initially causes a temporary decrease in real wages (Figure 5.6) in the South. Subsequently, the increase in capital stock (Figure 5.5) promotes an increase in labor productivity and thus higher real wages. The temporary decline in wages has a depressive effect on human wealth of retirees (see hr_t Figure 5.8). The effect is reversed for workers (see hw_t, Figure 5.8).

The marginal propensity to spend of workers in the South decrease because of a large share of income going to workers who naturally have a lower marginal propensity to spend (see κE_t value that represents the marginal propensity to spend of workers in the South π_t^E, Figure 5.8). The temporary decline of ϵE_t ratio (see Figure 5.6) causes a decreasing in the marginal propensity to spend of retired agents from the South ($\epsilon_t \pi_t$). So, saving increases in the South due to lower marginal propensity to spend. Moreover, the capital can not instantly adjust because of adjustment costs, asset accumulation will also decrease.

The savings of the southern economy is relatively superior to that of the North ($\theta E_t f E_t$>0, see Figure 5.7) which borrows. The increase in the labor supply hE_t of the South associated with inflexible capital

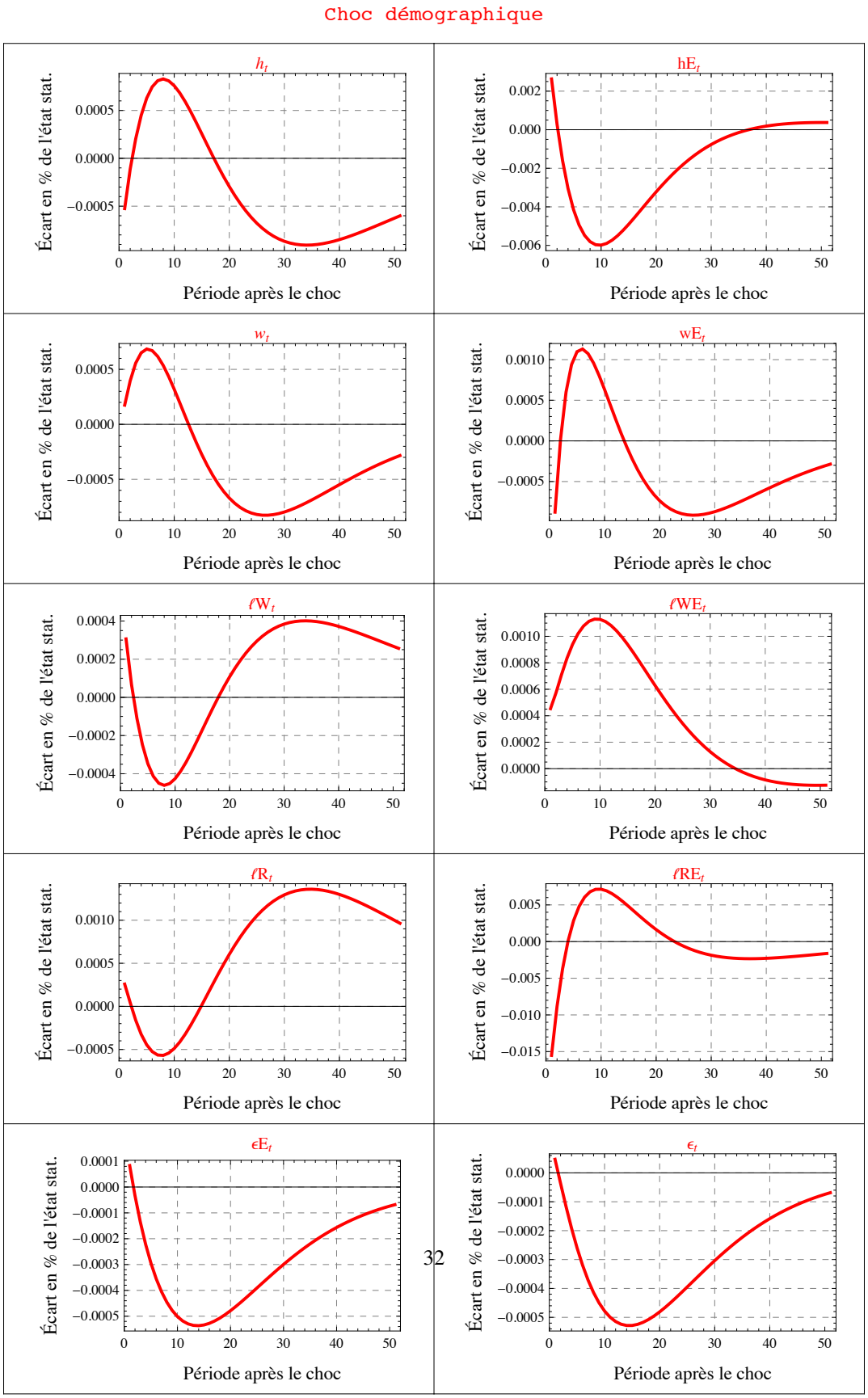


Figure 5.6: Employment and leisure

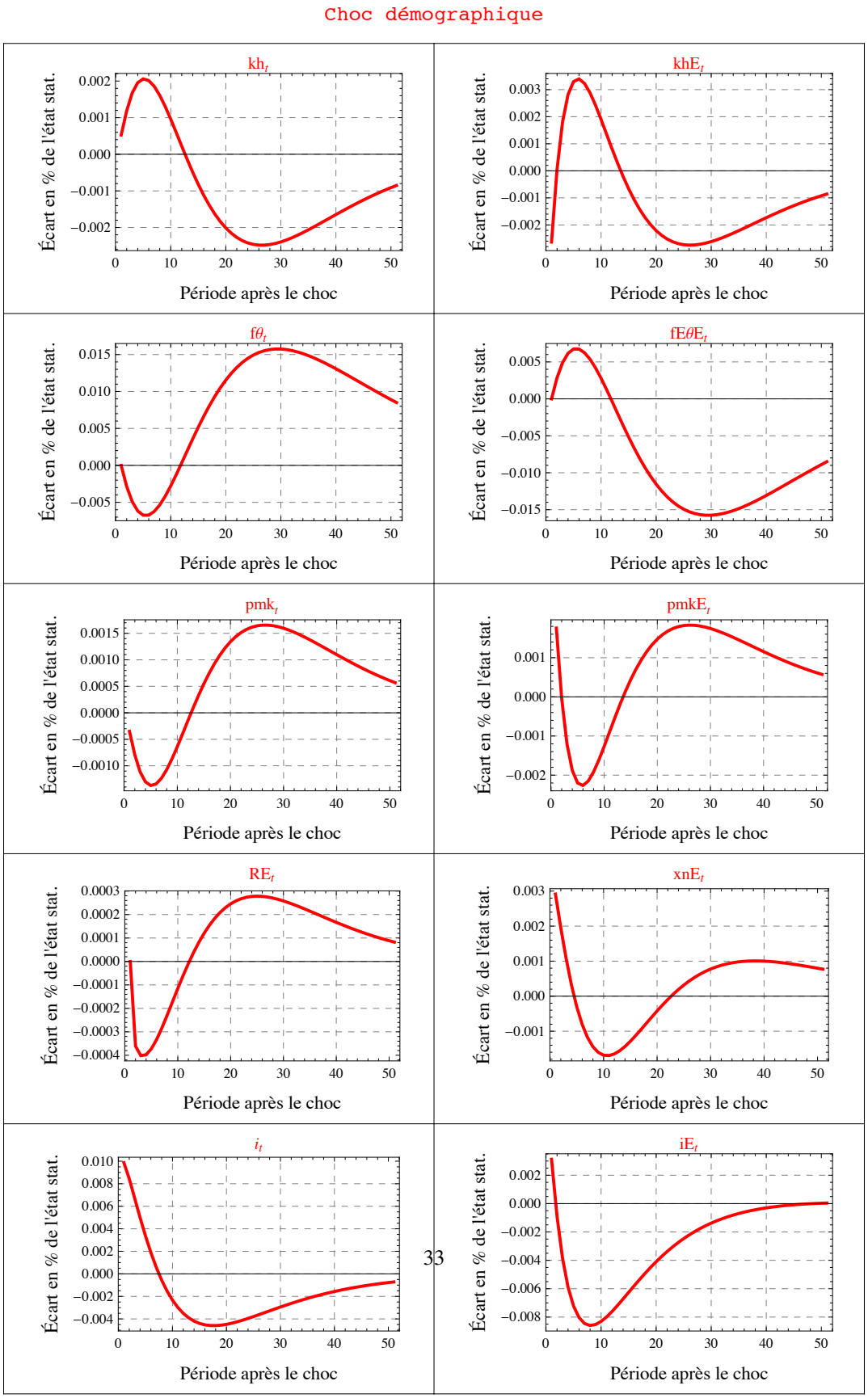


Figure 5.7: External balances, investment and financial wealth

stock generate a lower capital-labor ratio (khE_t see Figure 5.7) and thus an increase in the marginal product of capital ($pmkE_t$). This also causes a rise in Tobin's Q as the decline of labor - capital ratio that generate rent increase of capital (qE_t) (Figure 5.8).

The initial decline in the real interest rate (RE_t) in the South is transmitted to the northern economy given equal real interest rate in both countries. The Tobin's Q (qE_t) of the northern economy is rising, the northern economy has an incentive to accumulate capital (k_t) (see Figure 5.8 and Figure 5.5). Declining Northern real interest rate causes a rise in the marginal propensity to spend of northern retirees (rising $\epsilon_t \pi_t$ given the decline $\epsilon_t \pi_t$ coupled to that of π_t see Figure 5.6 and Figure 5.8 with $\pi_t = \kappa_t$). This increase in expenditures associated with the higher investment is an alternative way to explain the negative external balance in the North (see $xnE_t > 0$ and therefore $xn_t < 0$ 5.6). The increase in capital expenditures is due to the increasing population of workers in the South to meet the future demand of this population.

The decline in local demand in the economy of the South causes a decline in prices of local goods in the South to maintain the balance between supply and demand (Figure 5.9). The consumption expenditure of retirees from the North is varied to increase due to the positive change in the total wealth of the retirees from the South as shown in Figure 5.9. Hence the retiree consumption expenditures increases in the North and so they dissave to consume more. Production increases due to the increase in employment (Figure 5.15 of Annex B.1). However, The northern local demand on the rise, prices will have to rise in order to ensure balance between supply and demand on the local market, ceteris paribus (Figure 5.9). This increase in local prices will boost imports since the decreasing of prices abroad in order to meet the drop in local production caused by the decreasing in employment in the North. This increase in northern economy imports leads to a deficit in the external balance of the country. The northern economy borrows to finance its investment given its low savings rate ($\theta_t f_t$<0, see Figure 5.7). Rising foreign prices, that mean, prices in the northern economy, stimulate exports from South economy. This increase in exports causes the South external balance surplus as presented in Figure 5.7. So, population growth has a positive effect on the external balance of the South. The baby boom is also reflected in a capital accumulation in the South.

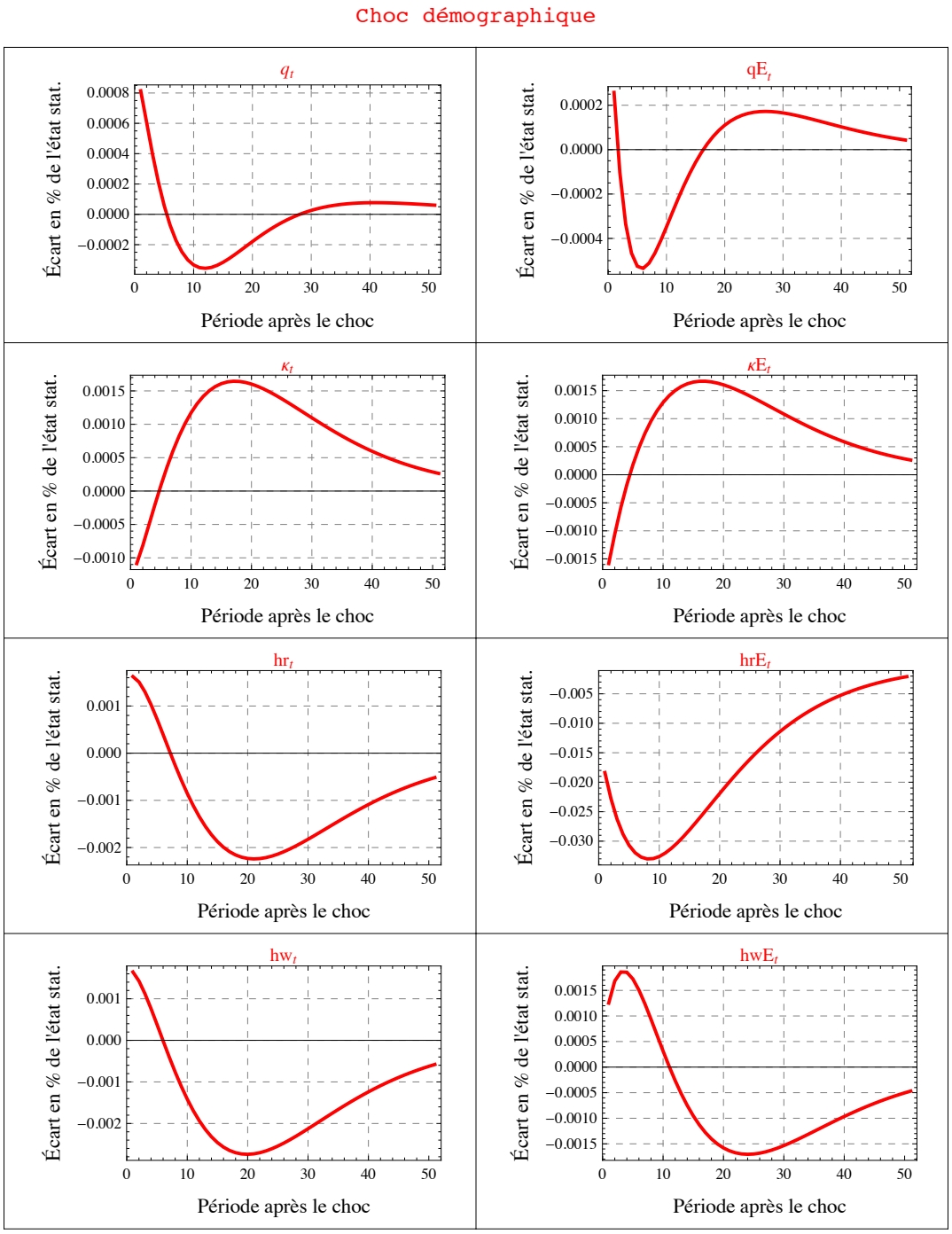


Figure 5.8: Tobin's Q and human resources

Choc démographique

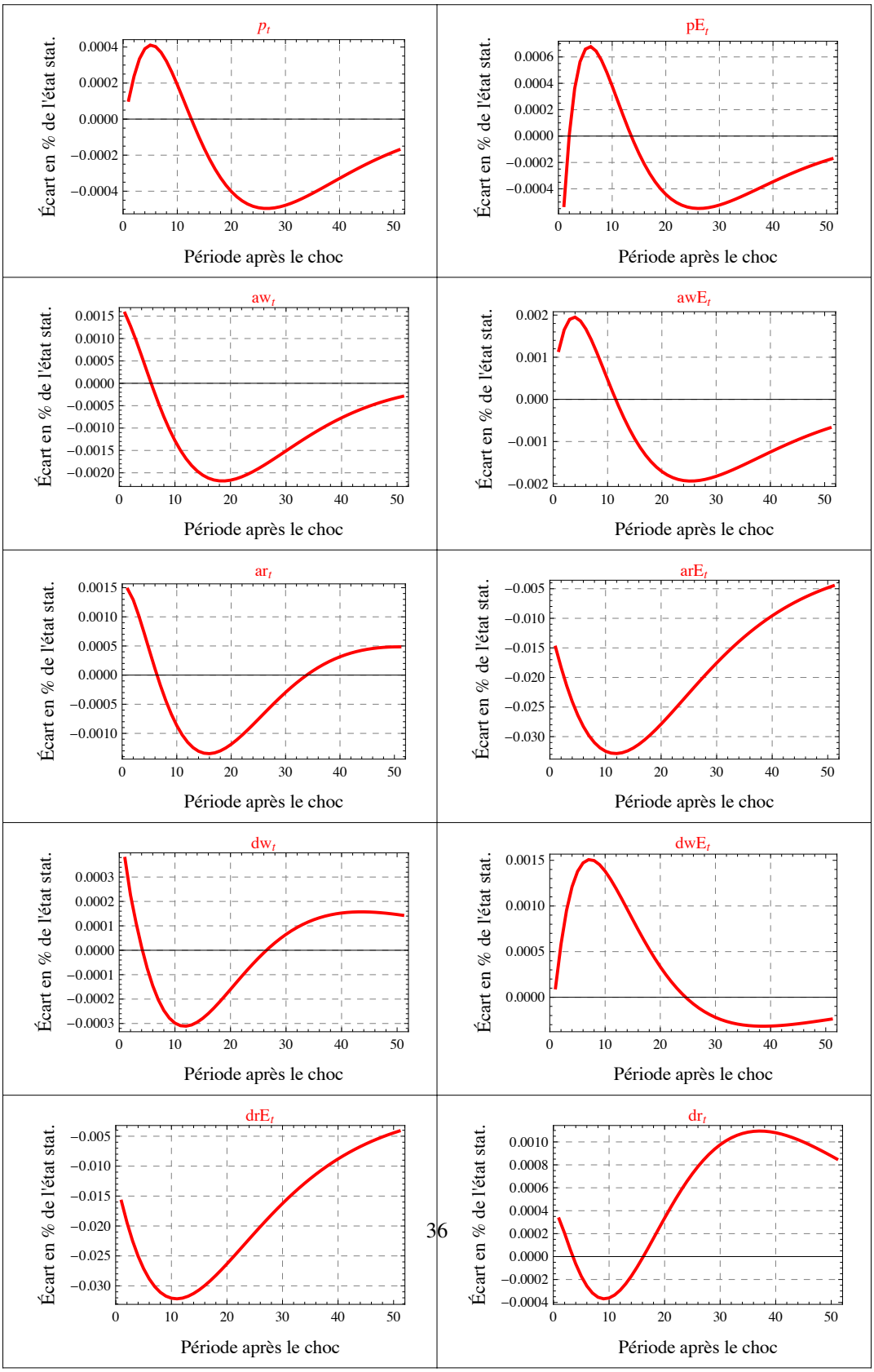


Figure 5.9: Consumer Spending, price and total wealth

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Annex A : Quantitative results of the steady state

A.1: Decreasing the retirement life expectancy in the South

Table 5: Decreasing the retirement life expectancy

Decreasing the retirement life expectancy		
	Scenario	
Variable	symmetrical	asymmetrical
P	1.898	1.899
P^E	1.898	1.899
W	0.948	0.9485
W^E	0.948	0.9485
l^r	0.243	0.245
$(l^r)^E$	0.243	0.157
l^w	0.378	0.378
$(l^w)^E$	378	0.376
kh	2.872	2.879
$(kh)^E$	2.872	2.879
h	0.406	0.406
h^E	0.406	0.386
h^r	0.745	0.747
$(h^r)^E$	0.745	0.237
h^w	10.296	10.325
$(h^w)^E$	10.296	8.189
R	1.065	1.064
R^E	1.065	1.064
k	1.165	1.169
k^E	1.165	1.110
q	1	1
q^E	1	1
af	1.240	1.232
$(af)^E$	1.204	1.194

Decreasing the retirement life expectancy		
	Scenario	
Variable	symmetrical	asymmetrical
λ	0.153	0.152
λ^E	0.153	0.096
ϵ	1.732	1.733
ϵ^E	1.732	2.296
π	0.086	0.087
π^E	0.086	0.106
Ω	1.046	1.046
ΩE	1.046	1.077
y	0.576	0.577
y^E	0.576	0.548
d^r	0.1036	0.104
$(d^r)^E$	0.1036	0.0605
d^w	0.517	0.518
$(d^w)^E$	0.517	0.519
c^r	0.549	0.055
$(c^r)^E$	0.549	0.032
c^w	0.393	0.393
$(c^w)^E$	0.393	0.395
i	0.128	0.129
i^E	0.128	0.122
xn	0	0.000506
xn^E	0	-0.000506
f	0	-0.0114
f^E	0	0.0114

A.2: Decrease the period of activity in the North

Table 6: Decreasing the period of activity in the North

Decreasing the period of activity		
	Scenario	
Variable	symmetrical case	asymmetrical case
P	1.898	1.898
P^E	1.898	1.898
W	0.948	0.948
W^E	0.948	0.984
l^r	0.243	0.253
$(l^r)^E$	0.243	0.243
l^w	0.378	0.378
$(l^w)^E$	378	0.3775
kh	2.872	2.870
$(kh)^E$	2.872	2.870
h	0.406	0.411
h^E	0.406	0.405
h^r	0.745	0.882
$(h^r)^E$	0.745	0.745
h^w	10.296	10.506
$(h^w)^E$	10.296	10.287
R	1.065	1.065
R^E	1.065	1.065
k	1.165	1.178
k^E	1.165	1.164
q	1	1
q^E	1	1
af	1.240	1.250
$(af)^E$	1.204	1.243

Decreasing the period of activity in the North		
	Scenario	
Variable	cas symétrique	asymmetrical case
λ	0.153	0.164
λ^E	0.153	0.153
ϵ	1.732	1.665
ϵ^E	1.732	1.731
π	0.086	0.085
π^E	0.086	0.086
Ω	1.046	1.043
ΩE	1.046	1.045
y	0.576	0.583
y^E	0.576	0.576
d^r	0.1036	0.114
$(d^r)^E$	0.1036	0.104
d^w	0.517	0.517
$(d^w)^E$	0.517	0.518
c^r	0.549	0.0605
$(c^r)^E$	0.549	0.055
c^w	0.393	0.392
$(c^w)^E$	0.393	0.393
i	0.1281	0.129
i^E	0.1281	0.128
xn	0	0.00018
xn^E	0	-0.00018
f	0	-0.00397
f^E	0	0.00397

Productivity shock in the north

Choc de productivité

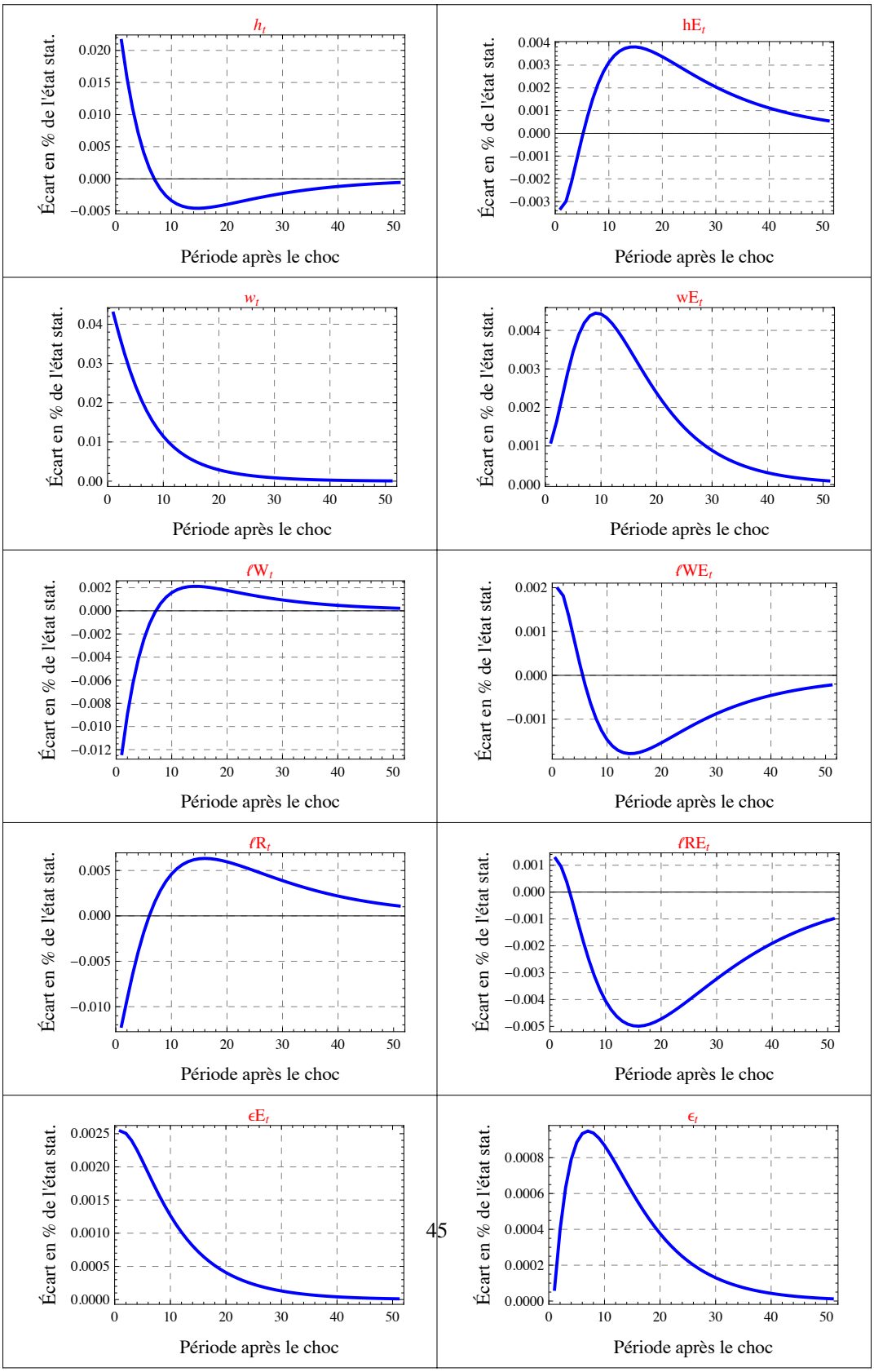

Figure 5.10: Employment and leisure

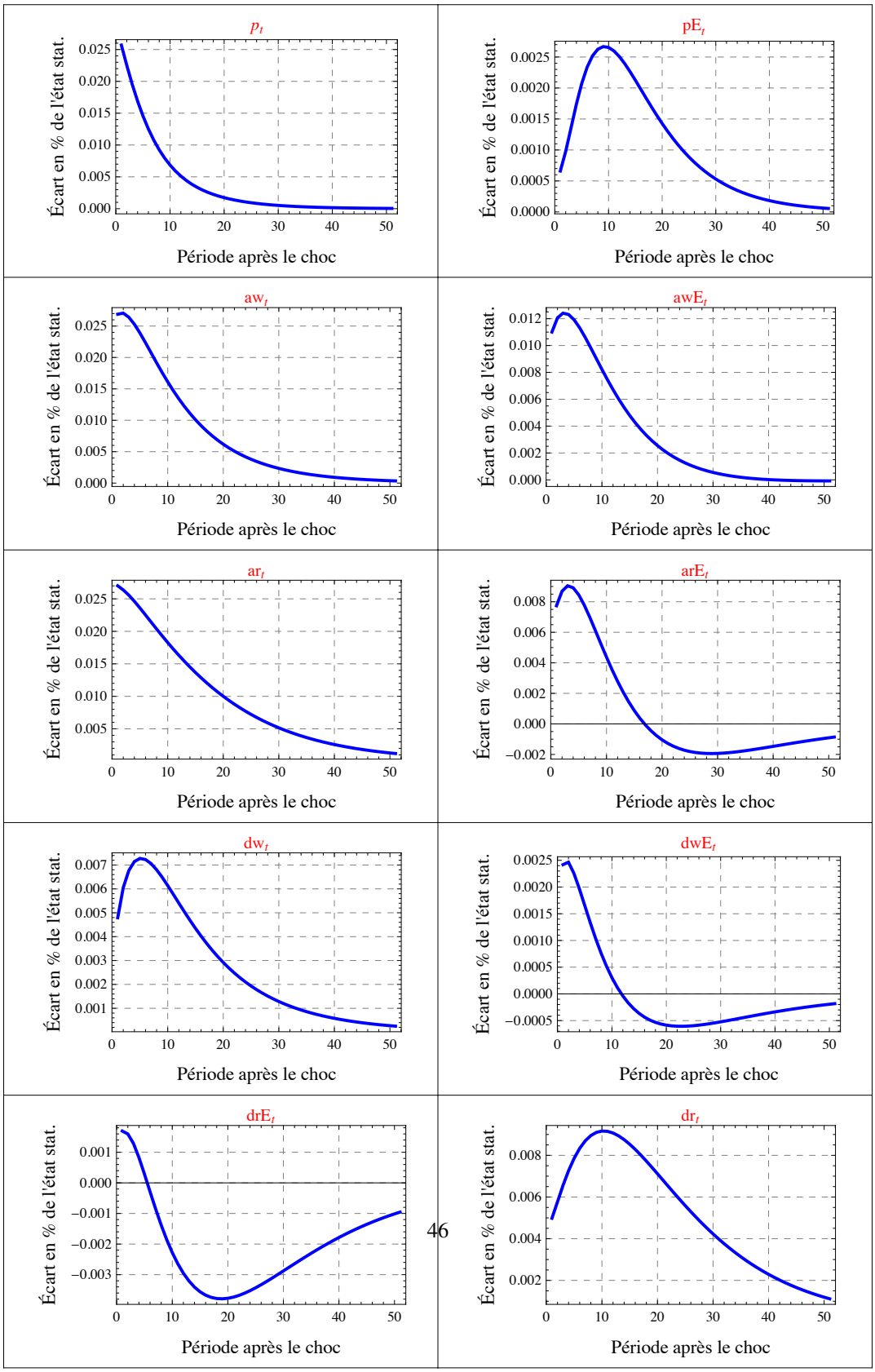

Figure 5.11: Consumer spending, price and total wealth

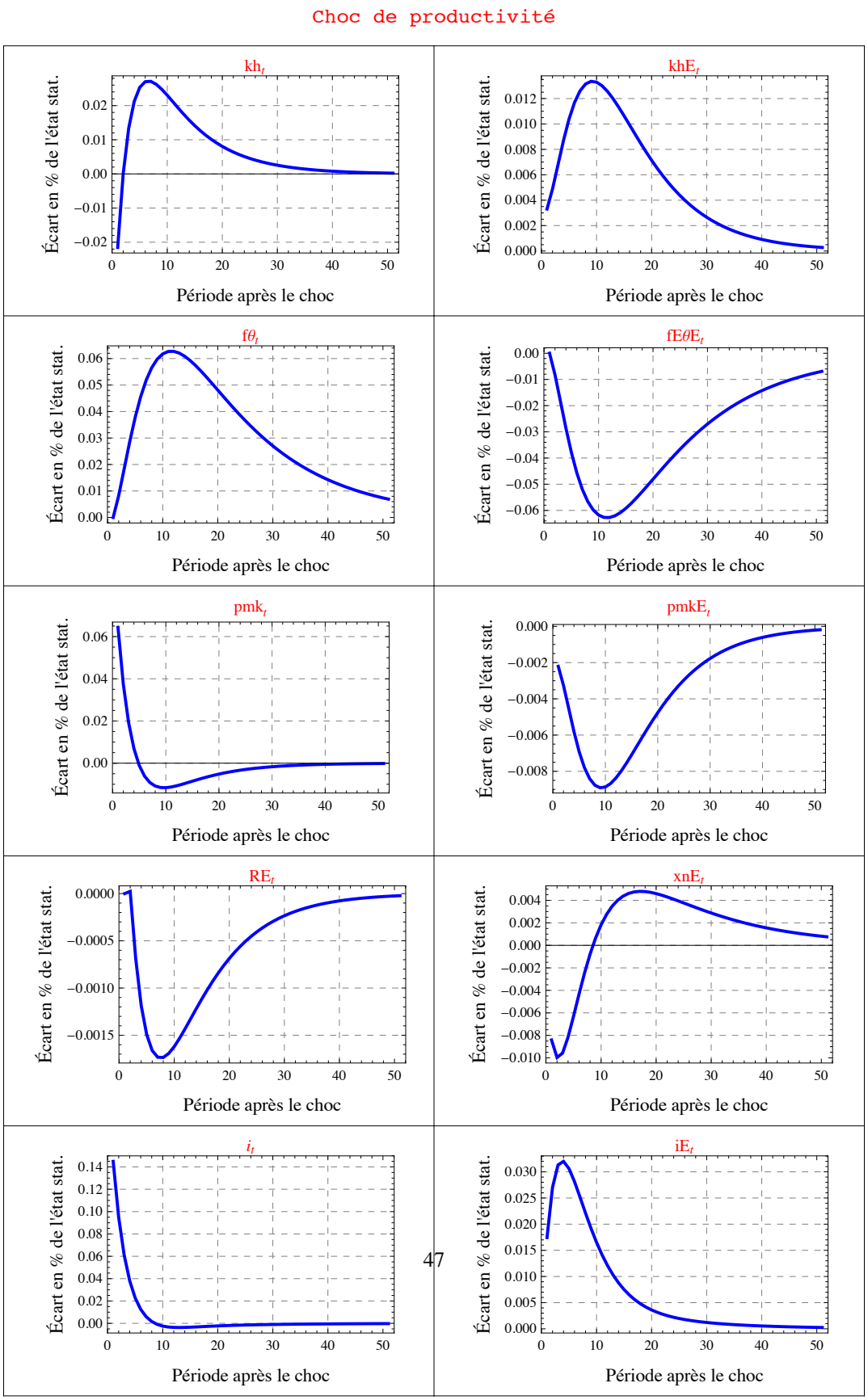

Figure 5.12: External balances, investment and financial wealth

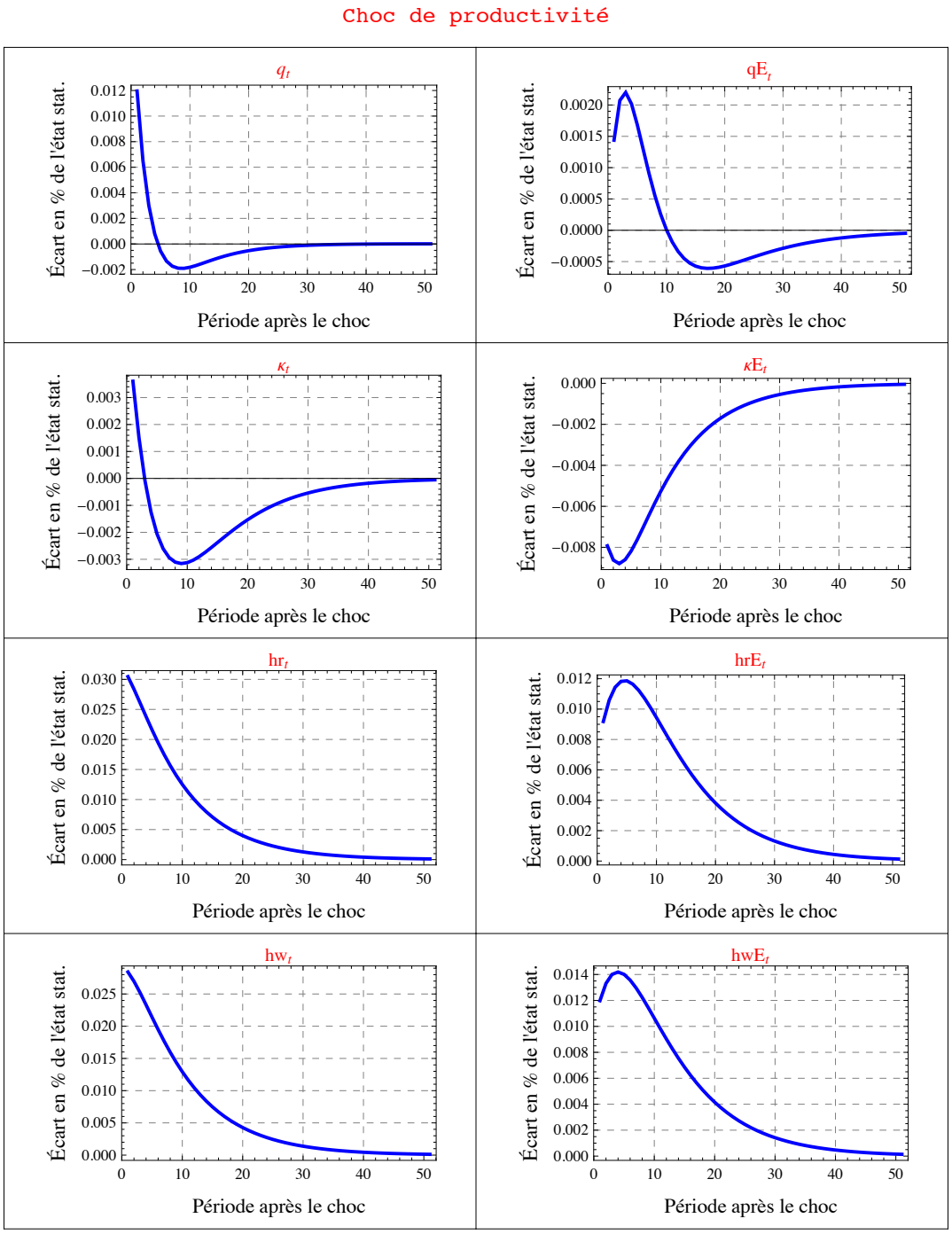

Figure 5.13: Tobin's Q and human resources

Annex B : Demographic shock

B.1 Demographic shock

Choc démographique

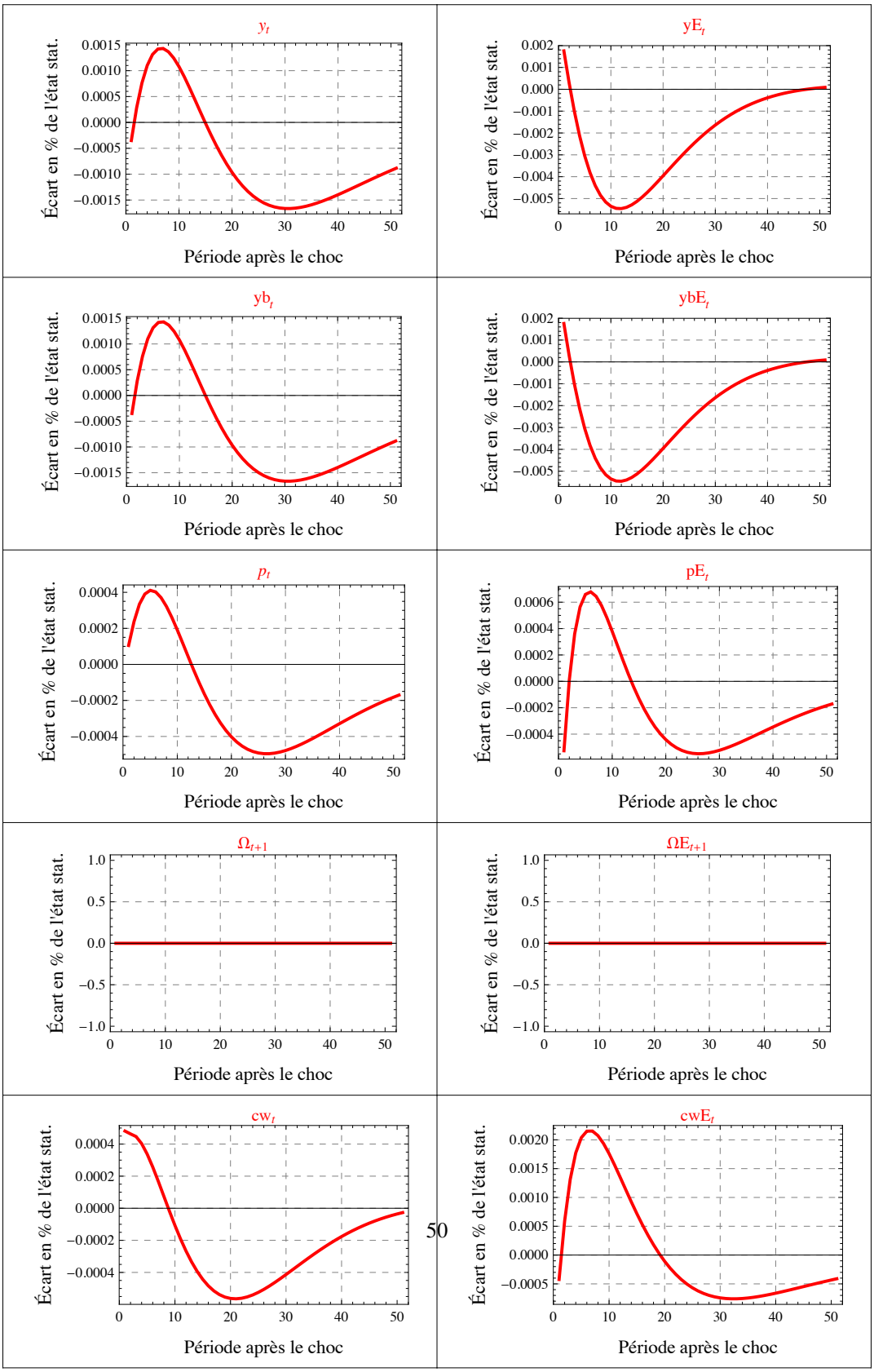

Figure 5.14: Production, consumption and prices

Choc démographique

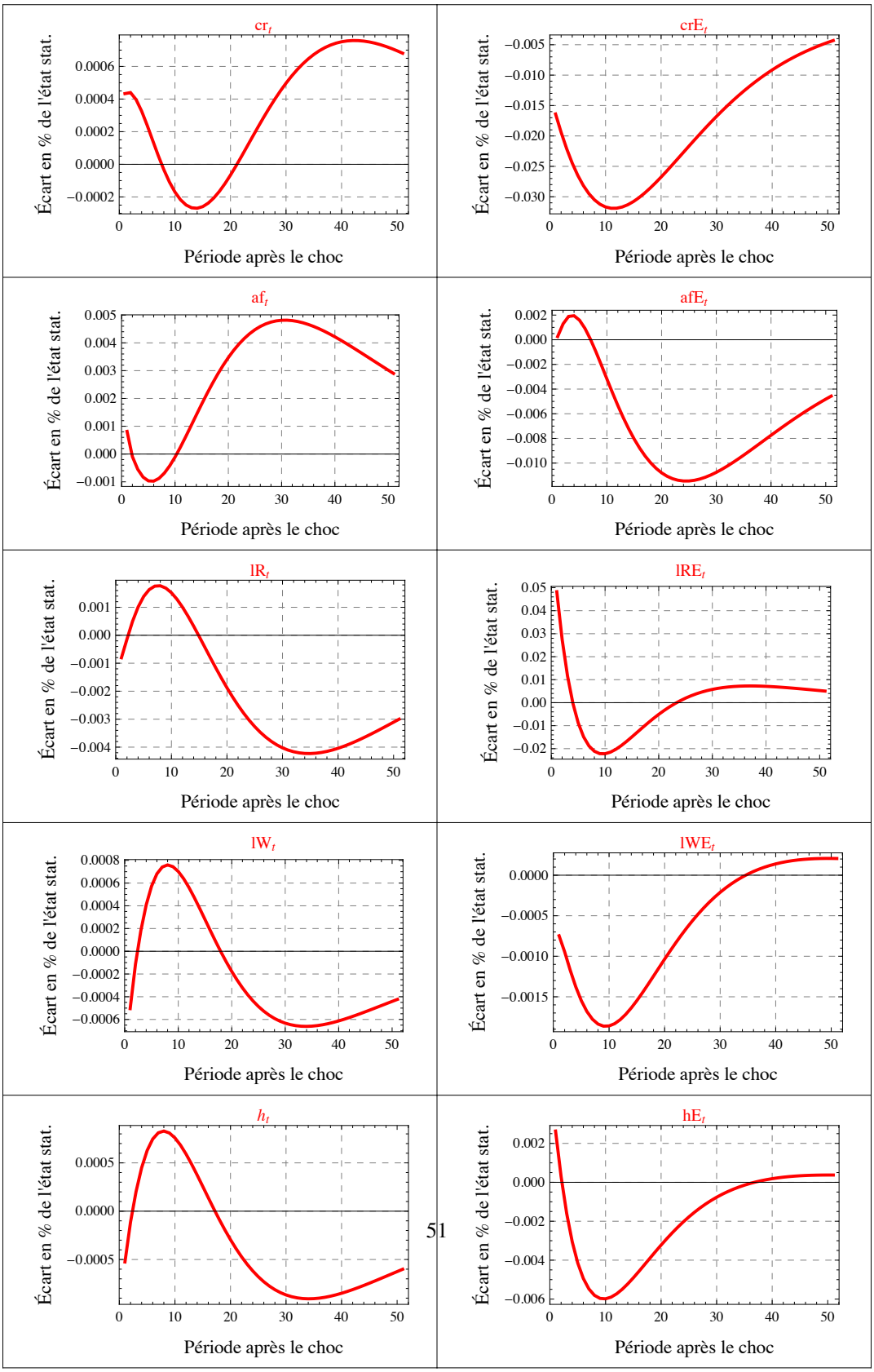

Figure 5.15: Consumption, financial wealth and employment

B.2 Productivity shock

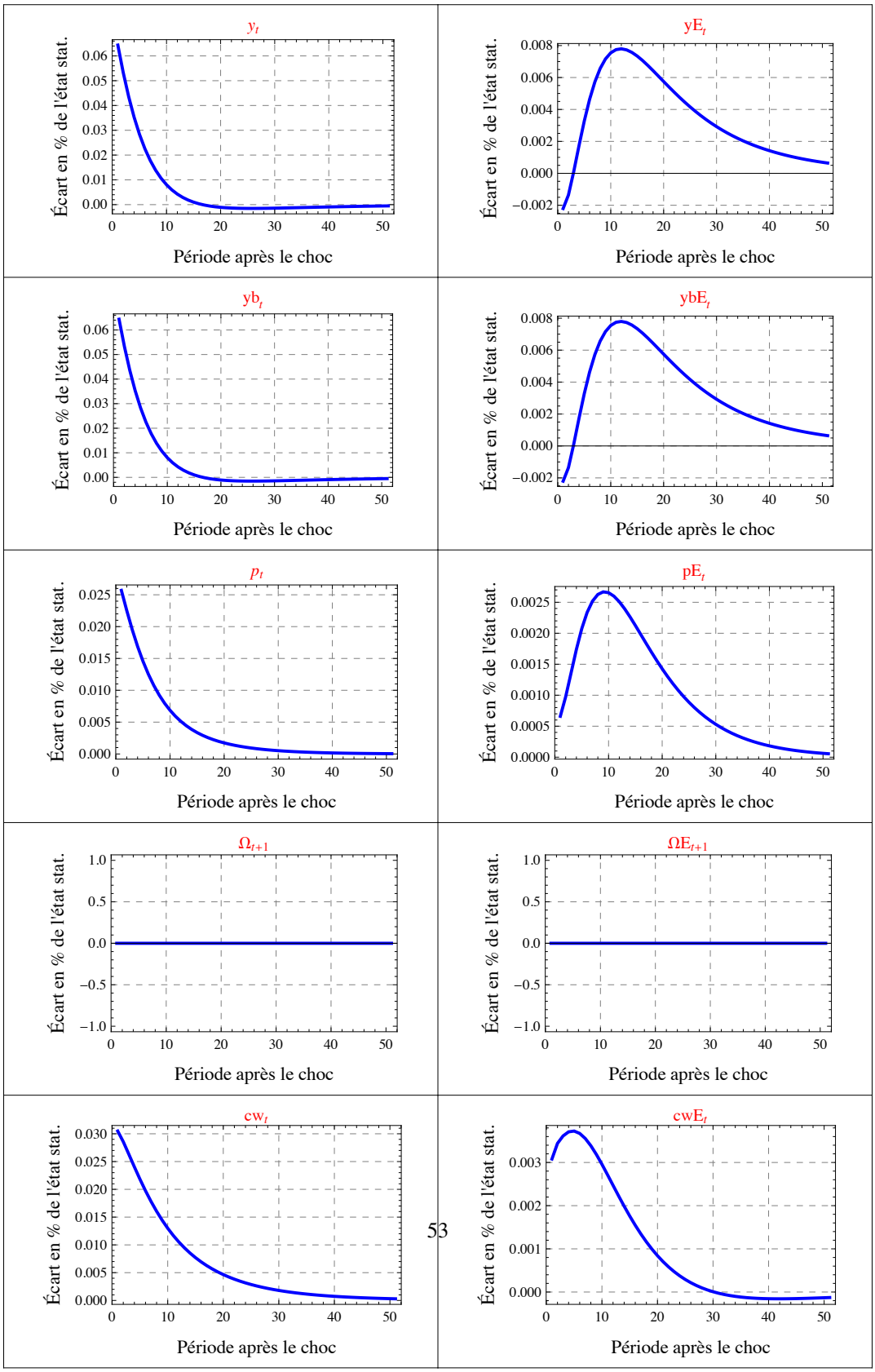

Figure 5.16: Production, consumption and prices

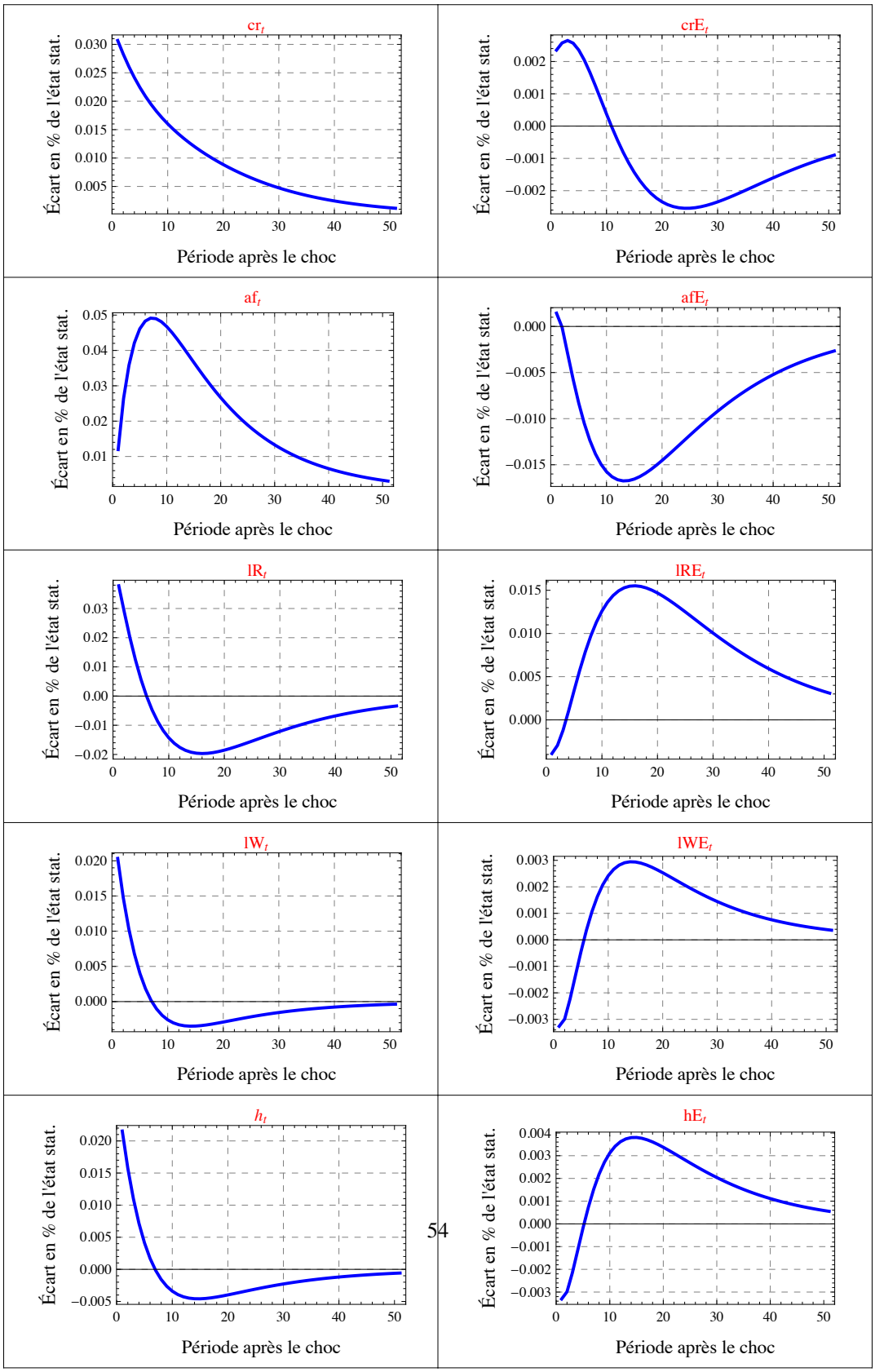

Figure 5.17: Consommation, richesse financière et emploi

Choc de productivité

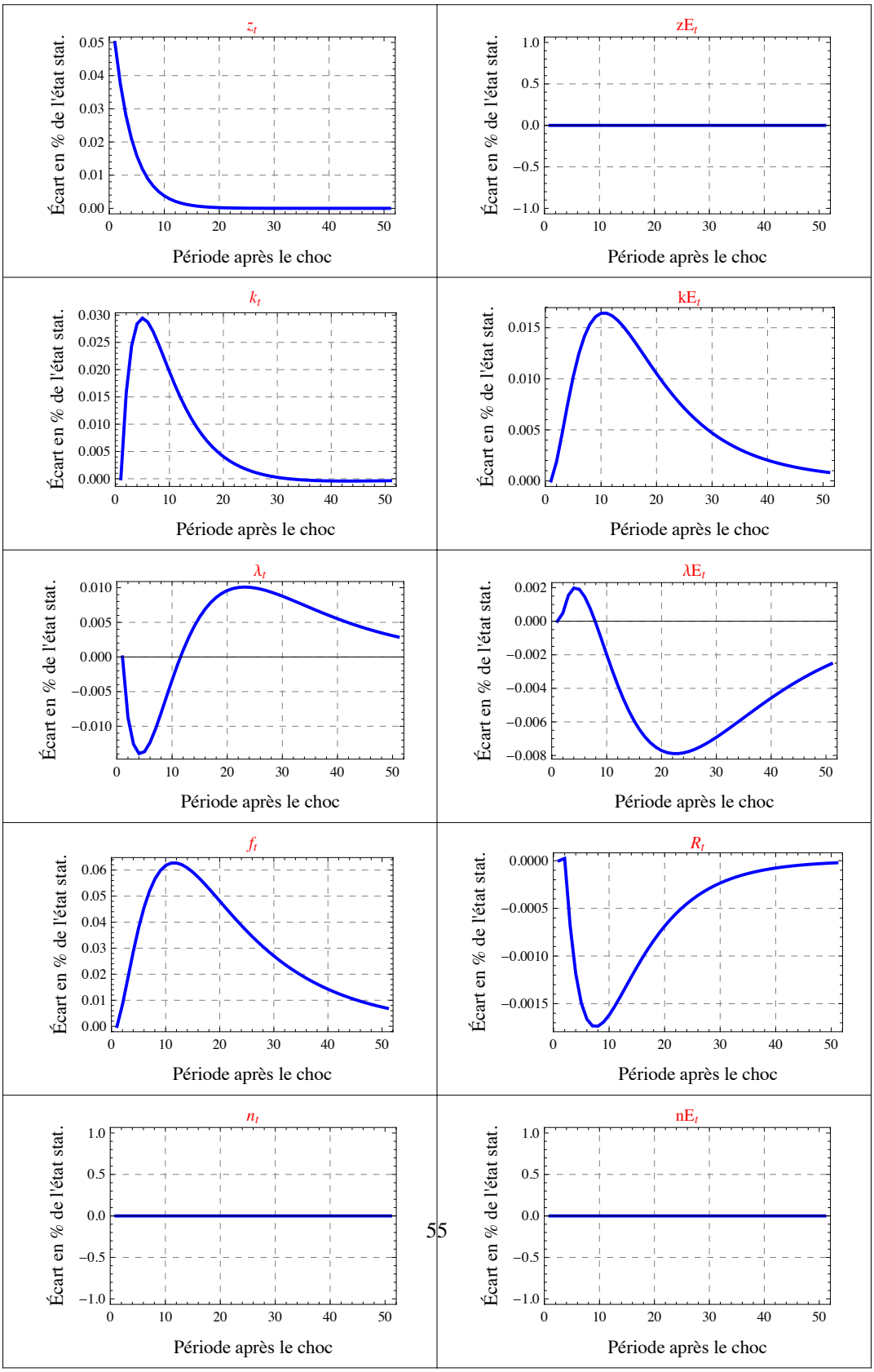

Figure 5.18: Productivity, wealth and real interest rate