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# The economics of violence in natural states\*

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## Abstract

Violence is key to understanding human interaction and societal development. The natural state of societal organization is that a subset of the population, capable of mustering organized large-scale violence, forms an elite coalition that restrains both violence and coercive appropriation. We highlight key mechanisms underlying such natural states using insights from the economic literature on conflict and appropriation. Our results show large variations in elite size, appropriation, production levels, and welfare across natural states due to only minor variations in exogenous model parameters. Specifically, unproductive societies tend to have a large elite coalition and a high tax rate. Only when the elite coalition is small (which occurs in societies with high productivity) but still able to control a sizeable share of production, can societies prosper in a natural state.

**JEL Classification:** C72, D02, D60, D72, D74, O17, P48

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# 1 Introduction

Violence is key to understanding human interaction and societal development. A society that is unable to contain violence will be disrupted and cannot be expected to sustain high levels of welfare, as is painfully illustrated by the current situation in Somalia, Afghanistan, or Libya. Violence may deter interaction, exchange, trade, and the benefits of specialization that come with trade, possibly leading to significant welfare losses (Hirshleifer, 1988; Skaperdas and Syropoulos, 2002; Findlay and O'Rourke, 2007). Societies cope with this threat of endemic violence in various ways, intrinsically linked up with, or embedded in, their economic and political systems. These systems structure the distribution of rents in order to contain the destructive effects of unconstrained violence on production, the economy, and society at large. In their seminal study of social and economic development, North et al. (2009) provide a novel framework to interpret this development of economic and political systems in light of the imminent problem of violence in human interaction (see also Seabright, 2010 and Fukuyama, 2011). They advance three social orders that have developed throughout history to successfully restrain such violence. In this paper we provide economic intuition for one of these, the 'limited access order', which is conventionally known as the 'natural state'.

In a natural state there is no government monopoly on the legitimate use of organized large-scale violence. Instead, a subset of the population has the capacity to muster organized large-scale violence or, contrarily, to provide protection from such violence. They have the capacity to do so and they will use this capacity opportunistically (Lane, 1958). We will refer to this subset of the population as a separate class of 'violence specialists'. Importantly, we stress that the nature of violence considered in this paper is of a higher level than the banditry-type of violence commonly considered in the literature on the economics of conflict and appropriation (for an overview of this literature, see Garfinkel and Skaperdas, 2007; Konrad, 2009). Organized and large-scale violence comprises the possibility to assemble small armies of loyal troops or mercenaries. More generally, violence specialists have the skills to generate and maintain a patronage network of followers that can, in turn, be used to exert violence – or coercion – on others. This is why we refer to violence as being both organized and large-scale. A direct implication is that entry and exit of violence specialists is exceptional. Depending on the specific setting, membership of the class of violence specialists is by inheritance or by a combination of wealth, standing, and influence. As such, our model departs from models where agents can switch between production and (offensive or defensive) violence activities (cf. Usher, 1989; Hirshleifer, 1995; Grossman and Kim, 1995; Konrad and Skaperdas, 2012). In our model, producing agents have no

means to engage in organized large-scale violence, and, simultaneously, they have no means to defend against such violence.

This sharp distinction between violence specialists and producing agents allows us to focus on the behaviour of violence specialists (as opposed to e.g. Bates et al., 2002) and its consequences for production and welfare in a natural state. We do so mainly by following North et al. (2009)'s narrative of the limited access order, in which violence can be contained by the formation of a coalition of violence specialists (i.e. the elite). Violence specialists within the elite coalition agree to respect each others' privileges, power base and resource base. They use their joint power to collectively extract rents from the rest of the population, which is what brings the coalition together. Although the coalition utilizes its coercive power against the rest of society, it restricts violence, both by limiting violence between elite-members as well as by limiting their coercive power as compared to warlords, which is further explained below. The result is a social order with an elite that exercises its coercive power to extract rents from the society at large in order to stay in power. Within this elite, violence specialists compete for the distribution of the rents and they also compete, as a group, with violence specialists outside of the elite coalition (who choose not to respect privileges of other violence specialists), termed 'warlords' here, for control of society's rents.<sup>1</sup> Violence specialists choose between joining the elite – sustaining the economic and political system that limits unconstrained violence and coercion – or becoming a warlord<sup>2</sup> and exploiting their violence capacity to extract rents coercively without constraints.

Having introduced both elites and warlords, we can now be more specific on use of the concepts of violence and coercion in this paper. Conflict between violence specialists (i.e. elite vs. warlords and warlords among themselves) is characterized by *unconstrained* violence. In their use of coercion against producers, the two types of violence specialists use violence only as a means for coercion, but the type of coercion differs, reflecting their differing perspectives on appropriation. Coercion by elites is *self-constrained* – and institutionalized to some degree, depending on the maturity of the elite. Elites use coercion collectively and strategically, optimizing the gains from appropriation while respecting each others' privileges and taking into account the effect of appropriation on production decisions. Coercion by warlords, however, is *unconstrained*. Warlords use coercion my-

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<sup>1</sup>Note that, different from the literature that compares conflict and development in anarchy with some form of hierarchy (e.g. Grossman, 2002; Bates et al., 2002), we treat the elite as a composite rather than a monolithic entity.

<sup>2</sup>We use the term warlord rather than the more conventional term 'bandit' to stress the violence capacities that we focus on in our model, i.e. organized and large-scale violence based on a patronage network of followers.

opically, optimizing appropriative income under the uncertainty of any future options for appropriation, and knowing that their individual appropriation has a negligible impact on production decisions.

In light of the above descriptions of natural states, violence, and coercion, we argue that most historical and contemporaneous societies in the world can be characterized as natural states, be it the feudal societies of medieval Europe and Japan, or present-day societies in much of Africa, Latin America, and the Middle East. In these natural states, the violence specialists within the elite coalition each bring into the coalition their political parties, ethnic groups, patronage networks and associated organizations, thus creating a commitment to constrain violence (North et al., 2013). The coalition can create and distribute rents. Rents may be created by way of monopolies, exclusive rights to trade, subsidies, redistribution of taxes, or privileges to exploit natural resources. Especially in resource-rich areas the latter is a crucial source of rents, both in historical and present-day natural states, with the Democratic Republic of Congo forming a clear, recent, example (Kaiser and Wolters, 2013).

Violence specialists outside of the elite coalition, the warlords, may be the leaders of organized criminal or illegal networks and drug cartels (e.g. Mexico), guerilla movements (Colombia), revolting militias (Somalia) or the leaders of independent clans or tribes (Afghanistan). The number of warlords can vary over time. Examples of rapidly growing numbers of warlords are Liberia in the 1980s, as officers with their soldiers left the government army and turned predatory (Bates et al., 2002); the Soviet-Union around 1990, during its disintegration, as violence specialists formerly integrated in the state defence and security system started to loot enterprises, natural resources and public funds; or Congo in the 1990s, as various militias, political movements and ethnic groups militarily competed for power (Bates et al., 2002; Bates, 2008; Kaiser and Wolters, 2013). The growth of the number of warlords in situations like these has a twofold negative effect on the economy. First, the ensuing violence destroys lives and capital goods, and deters trade and investments. Second, warlords do not levy regular taxes but exploit their violence capacity to appropriate as much production as they can – through looting, extortion etc. – realizing that future options for appropriation are highly uncertain or may not even exist. Such appropriation undermines the security of property rights and deters investments, and this to a greater extent than in a stable society with more predictable monopolies and privileges of the elite coalition, a situation that does not fully or necessarily stand in the way of economic growth (North et al., 2013).

Using insights from the economic literature on conflict and appropriation, in which violence is a central element, we propose a model that allows us to better understand the

relation between, on the one hand, opportunities and constraints in the use of organized large-scale violence, and production and distribution of welfare on the other hand. Doing so, we highlight some of the key mechanisms underlying the natural state; mechanisms that determine the extent to which violence is contained and how this affects production and welfare. In a nutshell our model is as follows. Violence specialists either join the elite or become a warlord. Depending on their relative size, the elite and warlords each control a share of society, its population, and production. Both elites and warlords use their violence capacity to appropriate production, but they do so in distinct ways. The elite resembles a ‘stationary bandit’, by enforcing a tax on production to support their coalition, taking into account – in the spirit of McGuire and Olson (1996) – that a high tax rate deters production. This tax rate should be interpreted broadly as encompassing all possible forms of rent extraction, and the optimal level of rent extraction may be very high, for instance when the marginal effect of taxation on production is low. Warlords do not levy taxes but instead they exploit their violence capacity to appropriate as much production as they can. This advantage of warlords over elite-members in terms of rent extraction is partly mitigated by cooperation of the elite in the coalition, based on the mutual agreement to respect each others’ privileges and collectively defend these against warlords. A side-effect, however, is that such cooperation may result in competition over rent distribution within the coalition. In response to the expected level of appropriation, producers decide how much effort to devote to production.<sup>3</sup> Since the aggregate rate of appropriation is largely determined by the number of warlords (in addition to the tax rate), a key outcome of our paper is the ratio of elites to warlords in equilibrium.

Our three main results are the following. First, we identify conditions under which a sizeable elite emerges, capable of limiting the activities of warlords. We interpret such a sizeable coalition as capable of providing order and stability. We find that an elite coalition of substantial size is feasible only when productivity is low<sup>4</sup> or when the elite has strong benefits to cooperation against warlords, offsetting the natural appropriation advantage of warlords. Second, we identify how production affects the behaviour of violence specialists and vice versa. We find that the tax rate levied by the elite coalition decreases with the productivity of the society. Hence, more productive societies face a lower tax rate from the elite coalition, but this implies that more violence specialists will choose to become warlords, increasing the level of appropriation in such productive societies, all of which results in a situation reminiscent of the resource curse (Mehlum et al., 2006). Jointly, our

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<sup>3</sup>‘Effort’ is a stylized term to capture several mechanisms through which appropriation leads to reduced production, see also Section 2.3.

<sup>4</sup>We use the term productivity to refer to the output elasticity of effort, being the only input to the production function introduced in Section 2.3.

first two results imply that societies with low productivity are characterized by large elite coalitions and high tax rates. Such societies gain from order and stability that come with large coalitions but this comes at the price of high tax rates. This brings us to our third result on the interrelations of these outcomes with producer welfare in a natural state. Consistent with our first two results, we find that the benefits of an elite coalition to society are often limited. Only when the elite coalition is small (which occurs in societies with high productivity) but still able to control a sizeable share of production, can societies prosper in a natural state. Summarizing, both productivity and order are necessary conditions for prosperity. This conclusion advances previous work by e.g. Bates et al. (2002) who stress the trade-off between ‘production’ and ‘protection’ in stateless societies. In our analysis we find that this trade-off is mitigated by the incentives of violence specialists and especially those that join forces in the elite.

In general, we offer additional insights to the mechanisms of the natural state, which emerged during the Neolithic Revolution, and is the dominant social order in most of the world still. Adding to North et al. (2009), we find large variations in elite size, appropriation, production levels, and welfare across natural states due to only minor variations in exogenous model parameters, such as productivity and the cost of conflict. This result implies that outcomes may differ substantially across natural states and even across societies with the same level of ‘maturation’. Surprisingly, we obtain this result in a model with an institutional setting that does not foster growth (cf. Acemoglu et al., 2005) and whilst ignoring the impact of human capital (Galor, 2005), the two leading suggested determinants of economic welfare. Our results differ from those by North et al. (2009) mainly because we make the incentives to violence specialists explicit. Whereas North et al. (2009) take it for granted that all violence specialists have an incentive to join the elite coalition, our model shows that this choice depends on a number of factors, including differences in appropriation rates between elites and warlords as well as the behaviour of producers. We emphasize the existence of a substantial number of warlords as a rather permanent situation, whereas North et al. (2009) interpret warlordism only as a transitional situation in times of coalition instability. A consequence of accounting for incentives is that we shift our attention from elite behaviour to violence specialists’ behaviour in general, including implications for elite size, appropriation, production levels, and welfare.

## 2 Model

The salient feature of the natural state as described by North et al. (2009) is the formation of an elite coalition that restrains the imminent problem of unconstrained violence and coercion. This focus on violence is central to our model. The success of the elite in establishing order and stability mainly depends on their capacity to restrain the warlords. The elites have an incentive to fight off the warlords because these also appropriate part of the production of society and do not commit to respecting other specialists' privileges. Therefore, the more the elites restrain the warlords, the more production they can control (and tax) themselves. The question is now whether providing order and stability to extract taxes can be profitable enough to induce violence specialists to refrain from unconstrained coercive appropriation from producers. We approach this question by analysing the payoffs to elites and warlords and their interaction with the producing agents.

Our main simplifying assumption is that we consider violence specialists as individuals, and their capacities as homogeneous. Recall that we consider violence specialists as having the skills to generate and maintain a patronage network of followers. Our assumption of violence specialists as homogeneous individuals has three main implications. First, homogeneity removes any reason to discriminate between violence specialists of the same occupation. As a result, payoffs are equal across elite-members as well as across warlords. Second, we can abstract away from the specificities of the formation and size of patronage networks. Second, we need not explicitly model entry and exit into the elite coalition. In reality, as a result of competition over the tax rents within the coalition, the composition of the coalition may be continuously changing, as power relations between members change, and because members with negligible contributions are weeded out and, potentially, substituted for new members (see Section 4). With homogeneous violence specialists – and given that our model allows violence specialists to freely choose whether or not to join the elite – this process of entry and exit can be ignored and coalitions are characterized by their size only.

Given this simplification, our model allows us to focus on the violence specialists' choice whether or not to join the elite, and its implications for production levels and producer welfare. This choice depends on the relative profitability of each occupation. As a result, each equilibrium features a specific distribution of elites and warlords.

Now, consider a natural state with a population of a fixed size, denoted by the set  $N$ . There are two subsets of individuals in this society: violence specialists, denoted by the set  $V$ , and the remainder of population that we refer to as producers, denoted by its complement  $P = N \setminus V$ . The categories are mutually exclusive and collectively exhaustive



with respect to  $N$ , and in our static model there is no mobility between them. Members of each subset are homogeneous in all relevant aspects.

Violence specialists  $i \in V$  can appropriate production from the producers in two different ways, and this appropriation decision is determined by their choice of occupation. Elite-members cooperate and appropriate by levying a jointly determined tax on their controlled production, while warlords appropriate by stealing all of their controlled production. This model feature is a stylized representation of the warlords' uncertain power base which makes them myopic in their decision on how much to appropriate.<sup>5</sup> The two occupations exert negative externalities, because the amount of production appropriated by warlords decreases the production available for appropriation by elite-members, and vice versa. As a result, elite-members and warlords compete over the share of total production either side controls. From the side of the elite-members, this can be interpreted as either (i) the share of society whereon they effectively impose order, or (ii) the extent to which they succeed in establishing order over the entire population.

The occupation choice by violence specialists is the first stage of our model, which consists of three stages. These are shortly described below and worked out in detail in Section 3. In addition, In Section 4 we will extend the model with an additional stage that features conflict within the coalition, a highly relevant model feature in light of the historical examples cited in the introduction.

## 2.1 Occupation choice (Stage 1)

Each violence specialist  $i \in V$  decides to join the elite coalition or not. We denote this occupation choice by  $\mu_i \in \{1, 0\} \forall i \in V$ . If  $\mu_i = 1$ , the specialist joins the elite coalition. If  $\mu_i = 0$ , the specialist becomes a warlord. The outcome of these decisions is a vector  $\mu = (\mu_i : i \in V)$  that partitions the violence specialists in two subsets: the elite coalition  $E = \{i : \mu_i = 1\}$  consisting of  $e = |E|$  elite-members, and its complement  $W = V \setminus E = \{i : \mu_i = 0\}$  consisting of  $w = |W|$  warlords.

Control over producers by elites and warlords is given deterministically by the ratio  $\frac{e}{w}$  (see (17) below). We use the following function to determine the share of total production that is controlled by elites:

$$\rho(e, w) = \frac{\theta e^m}{\theta e^m + w^m}, \quad (1)$$

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<sup>5</sup> In addition, since warlords operate alone – and given a sufficiently large number of violence specialists – individual appropriation rates have only negligible impact on production so that myopic behaviour is not an assumption but an implication from the model introduced below.

with  $m \in (0, 1)$  and  $\theta \in [1, \infty)$ . It follows that the share of total production that is controlled by warlords equals  $1 - \rho(e, w)$ . A few comments on (1) are appropriate here.

First, its functional form is borrowed from the rent seeking literature (Tullock, 1980). Our specification of  $\rho(e, w)$  is based on a modification of the ratio-form contest success function (CSF), inspired by the axiomatic characterization of group CSFs by Münster (2009). Parameter  $m$  is conventionally interpreted as the effectiveness of conflict, and here we interpret it as the effectiveness of group size, where group refers to either the elite coalition or the aggregate of warlords. Given  $m < 1$ , there are diminishing marginal returns to group formation. Specifically, low  $m$  implies that a small elite coalition is capable of controlling a relatively large share of production. Parameter  $\theta$  represents a fighting asymmetry (cf. Usher, 1989; Clark and Riis, 1998) in favour of the elite coalition that we consider to be better organized than warlords, because of their commitment to respect each others' privileges. In the context of North et al. (2009) one could interpret  $\theta$  as the cooperative quality, or maturity, of the coalition, with more mature coalitions capable of organizing and coordinating power more efficiently.<sup>6</sup> One implication of this functional form is that the elite may control a larger share of production, even if it is smaller in size than the aggregate of warlords.

Second, note that our interpretation of  $\rho(e, w)$  is non-probabilistic in the sense that it represents a share, rather than a winning probability (although the two interpretations are equivalent under the assumption of risk neutrality). For a detailed discussion of CSFs and their interpretations, see Hirshleifer (2000), Garfinkel and Skaperdas (2007), and Konrad (2009).

Third, although we talk freely about the share of production that is controlled by warlords, the degree of cooperation by warlords has not been specified yet. North et al. (2009) are not explicit about cooperative behaviour by warlords, if at all. In the functional form chosen in (1), warlords do work together, but have a fighting disadvantage compared with the elite, through  $\theta$ . The alternative approach is to model warlords as operating alone, using the term  $w \times 1^m$  rather than  $w^m$ , which would imply

$$\rho'(e, w) = \frac{\theta e^m}{\theta e^m + w \times 1^m} = \frac{\theta e^m}{\theta e^m + w}. \quad (2)$$

This alternative specification, however, has two disadvantages: (i) It would give the coalition a fighting disadvantage for any  $m < 1$ , so that the effects of  $m$  and  $\theta$  may cancel each other out. This disadvantage is reversed for  $m > 1$ . In our model set-up, however, if

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<sup>6</sup>See van Bavel et al. (2016) who interpret  $\theta$  as an indicator of a specific type of organizations in the natural state.

$\theta$  is sufficiently large, then  $m > 1$  leads to a corner solution where all specialists end up in the elite coalition.<sup>7</sup> This is a standard feature of the ratio-form CSF, discussed in detail by Hirshleifer (1995) and employed by Skaperdas (1998) to assess coalition formation in a different setting. (ii) Using  $\rho'(e, w)$  to calculate the equilibrium ratio of elites to warlords, as we do for  $\rho(e, w)$  in (14) below, would lead to asymptotic behaviour of this ratio, including discontinuities and negative outcomes. Both features are undesirable and we stick to (1).

Fourth and final, our specification of  $\rho(e, w)$  deviates from the standard approach in the economic literature on conflict and appropriation. Most importantly, this contest is deterministic in the sense that the outcome of the contest depends only on the ratio  $\frac{e}{w}$ . Notably, it does not depend on costly investments in violence capacity. That is, in the Stage 2 contest, elite-members and warlords do not explicitly choose their violence level as is conventional in models that feature a trade-off between own production and appropriation (e.g. Hirshleifer, 1988, 1995; Skaperdas, 1992; Grossman and Kim, 1995) or in rent-seeking models (Nitzan, 1994). We focus, however, on organized large-scale violence and, in our model, the capacity for such violence is restricted to violence specialists, while production is the domain of the separate subset of producers. As a result, violence specialists are not confronted with this trade-off between own production and appropriation. In addition, motivated by the homogeneity of violence specialists, the only effect of *not* ignoring costly investments in violence would be that payoffs of violence specialists would be reduced in the symmetric outcome of such a model, without any qualitative impacts on model results.

## 2.2 Tax (Stage 2)

Given the outcome of Stage 1, the elite controls a share  $\rho(e, w)$  and warlords jointly control a share  $1 - \rho(e, w)$ , which each of them can appropriate as they wish. Following the main features of the limited access order by North et al. (2009), elite-members collectively determine their tax rate  $\tau \in [0, 1]$ , while warlords, by construction, choose to appropriate all production under their control.

## 2.3 Production (Stage 3)

Given the outcome of Stages 1–2, producers decide on their joint production level. We model production  $Y$  by producers with a Von Thünen production function. This function exhibits diminishing marginal returns to costly effort  $\phi$ , the only variable input. In a Von

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<sup>7</sup>Specifically, a corner solution results if  $(1 - \alpha)\theta \geq 1$ , where  $\alpha$  is a productivity parameter that is introduced below in Section 2.3.

Thünen production function, this effort term is conventionally interpreted as a measure of the capital-to-labour ratio. As a result, our production function is similar to a linearly homogeneous Cobb-Douglas production function (Lloyd, 2001), with exponent  $1 - \alpha$  for labour and  $\alpha$  for capital, whilst it allows us to focus on effort only:

$$Y(\phi) = \beta \phi^\alpha. \quad (3)$$

Parameter  $\alpha \in (0, 1)$  denotes the output elasticity of effort, which, in absence of other variable inputs, we will refer to as *productivity*. Parameter  $\beta \in (0, \infty)$  reflects total factor productivity. We will refer to  $\beta$  as a *technology* parameter.

Appropriation by elites and warlords reduces the amount of produce available for consumption. Producers maximize their utility  $U$  which equals aggregate consumption – production net of appropriation – minus the cost of effort.

$$U = (1 - \tau)\rho(e, w)Y(\phi) - \gamma\phi. \quad (4)$$

with cost parameter  $\gamma \in (0, \infty)$ .

## §

Given our main assumption that violence specialists are homogeneous, payoffs  $\pi_i$  are equal across warlords as well as across elite-members. Incorporating all decisions made in Stages 2–4, this implies the following payoff functions:

$$\pi_i = \left(\frac{1}{e}\right) \tau \rho(e, w) Y(\phi) \quad \forall i \in E; \quad (5)$$

$$\pi_j = \left(\frac{1}{w}\right) (1 - \rho(e, w)) Y(\phi) \quad \forall j \in W. \quad (6)$$

The stability concept that we use to evaluate outcomes of the model is a simple equilibrating mechanism that equates payoffs to elites and warlords. That is, in Stage 1 violence specialists choose the most profitable occupation. They make this choice whilst taking into account (i) the optimal tax rate that will be chosen by the elite, and (ii) the optimal response in terms of production by producers. Hence, in equilibrium, payoffs to elite-members and warlords are equal. If not, then a profitable switch of occupation could be made by at least one violence specialist, while taking into account that changing occupations shifts the balance of power between warlords and the elite coalition with subsequent impacts on the outcomes of Stages 2–3. This equilibrating mechanism is reminiscent of more advanced stability concepts applied in alliance models and non-cooperative models of

coalition formation (cf. Skaperdas, 1998; Yi, 2003; Garfinkel, 2004).

### 3 Results

In this section we present the results of our model. Solving the model backwards, we analyse each of the four stages consecutively.

#### 3.1 Production (Stage 3)

Given outcomes of Stages 1–2, producers choose  $\phi$  to maximize their utility (4):

$$\frac{\partial U}{\partial \phi} = (1 - \tau)\rho(e, w)\frac{\partial Y(\phi)}{\partial \phi} - \gamma = 0 \quad (7)$$

By the production function in (3) we have

$$\frac{\partial Y(\phi)}{\partial \phi} = \alpha\beta\phi^{\alpha-1} \quad (8)$$

Substituting this derivative into (7) and solving for  $\phi$ , we obtain:

$$\phi^* = \left( (1 - \tau)\rho(e, w)\frac{\alpha\beta}{\gamma} \right)^{\frac{1}{1-\alpha}}. \quad (9)$$

Substituting this equilibrium level of effort into (3) and solving for  $Y$ , we obtain:

$$Y^* = \beta \left( (1 - \tau)\rho(e, w)\frac{\alpha\beta}{\gamma} \right)^{\frac{\alpha}{1-\alpha}}. \quad (10)$$

We will further assess  $Y^*$  in Section 3.4 below.

#### 3.2 Tax (Stage 2)

Given the outcome of Stage 1, the elite coalition chooses the tax rate  $\tau$ . Since violence specialists are homogeneous, there is no difference between choosing a tax rate that maximizes individual payoffs or one that maximizes the payoff to the coalition as a whole.

Substituting (10) for  $Y$ , the coalition payoff  $\pi_E$  equals  $e$  times individual elite payoff (5):

$$\begin{aligned}\pi_E &= e \left( \frac{1}{e} \right) \tau \rho(e, w) Y(\phi) \\ &= \tau \rho(e, w) \beta \left( (1 - \tau) \rho(e, w) \frac{\alpha \beta}{\gamma} \right)^{\frac{\alpha}{1-\alpha}}\end{aligned}\quad (11)$$

We can now maximize and solve for  $\tau$  to find:

$$\tau^* = 1 - \alpha. \quad (12)$$

Our first result follows directly.

**Proposition 1.** *In equilibrium, the optimal tax rate  $\tau^*$  decreases linearly in productivity  $\alpha$ , and is independent of  $\beta$ ,  $\gamma$ ,  $\theta$ , and  $m$ .*

Given  $\alpha < 1$ , the equilibrium tax rate is strictly smaller than 1. The elite coalition abstains from fully taxing away its controlled production. When  $\alpha$  is low the effect of the tax rate on production is small and hence, it is optimal to set a high tax rate. The opposite holds when  $\alpha$  is high.

### 3.3 Occupation choice (Stage 1)

Each violence specialist chooses his occupation  $\mu \in \{0, 1\}$  to maximize his payoff as given by (5) and (6), taking into account the effects of occupation choice on payoffs via (1) on the Stage 2 tax rate and Stage 3 production. Recall our equilibrating mechanism of equal payoffs to both occupations such that  $\pi_i = \pi$  for all  $i \in V$ . Applying this mechanism, we equate (5) and (6) to find the equilibrium ratio of elites to warlords, which is independent from production:

$$\frac{e}{w} = \frac{\tau \rho(e, w)}{1 - \rho(e, w)}. \quad (13)$$

By the specification of  $\rho(e, w)$  in (1), the size of the elite-controlled production depends positively on the ratio of elite-members to warlords. Since our economy has a population of fixed size and does not allow mobility between violence specialists and producers, the number of violence specialists is also fixed. We have  $|V| = e + w$ : an increase in  $e$  implies a decrease in  $w$  of equal size and vice versa. We use this model feature and also substitute (1) for  $\rho(e, w)$  in the equilibrium ratio (13). After simplification and substitution of (12) for  $\tau$  we obtain the equilibrium ratio of elites to warlords as a function of parameters  $\alpha$ ,  $m$ ,

and  $\theta$ :

$$\frac{e^*}{w^*} = [(1 - \alpha)\theta]^{\frac{1}{1-m}}. \quad (14)$$

By substituting  $w^* = |V| - e^*$ , we also obtain  $e^*$  and  $w^*$  separately:

$$e^* = \left( [(1 - \alpha)\theta]^{\frac{1}{m-1}} + 1 \right)^{-1} |V|, \quad (15)$$

$$w^* = \left( [(1 - \alpha)\theta]^{\frac{1}{1-m}} + 1 \right)^{-1} |V|. \quad (16)$$

The elite-warlord ratio (14) increases with the tax rate. It also increases with  $m$  when  $(1 - \alpha)\theta > 1$  which implies  $\frac{e^*}{w^*} > 1$ . In that case, the elite has an advantage in generating rents as a combined effect of controlling and taxing production. Given  $m < 1$ , profit per occupation, in both occupations, decreases with the size of the group. Therefore, equilibrium group size can be interpreted as the relative profitability of an occupation, with the larger group having an advantage in generating rents.

For the limit case where  $\theta = 1$ , and since  $\alpha \in (0, 1)$ , the elite-warlord ratio is strictly smaller than 1, approaching unity only in the limit where the tax rate (i.e.  $1 - \alpha$ ) approaches 1. Put differently, in absence of an elite fighting advantage, the number of elite-members is never larger than the number of warlords. This cap on elite size follows from our model feature that warlords fully appropriate their controlled production, whereas elite-members do not necessarily. Hence, elite-members, by construction, have a disadvantage in terms of their capacity to generate rents, which can be offset only by their fighting advantage in case  $\theta > 1$ .

In Figure 1 we plot (15), the equilibrium number of elites  $e^*$ , and (14), the equilibrium elite-warlord ratio, as a function of  $(1 - \alpha)\theta$  for different values of parameter  $m$ . This figure illustrates that the presence of a sizeable elite coalition in a natural state is not trivial. An (almost) empty elite coalition is possible for low values of  $(1 - \alpha)\theta$ . In contrast, an elite coalition that contains (nearly) all violence specialists is possible for high values of  $(1 - \alpha)\theta$  and high  $m$ . Note that, by the exponent  $\frac{-1}{1-m}$  in (15) and (16), these are limit results for  $\theta\tau$  going to zero or infinity. Both  $e^*$  and  $w^*$  converge to, but will never reach, 0 or  $|V|$ , ruling out any corner solutions.

From (14) follows our next result.

**Proposition 2.** *In equilibrium, the elite-warlord ratio  $\frac{e^*}{w^*}$  is:*

- (i) *decreasing with productivity  $\alpha$ ;*
- (ii) *independent of the technology-cost ratio  $\frac{\beta}{\gamma}$ ;*
- (iii) *increasing with elite fighting advantage  $\theta$ ;*

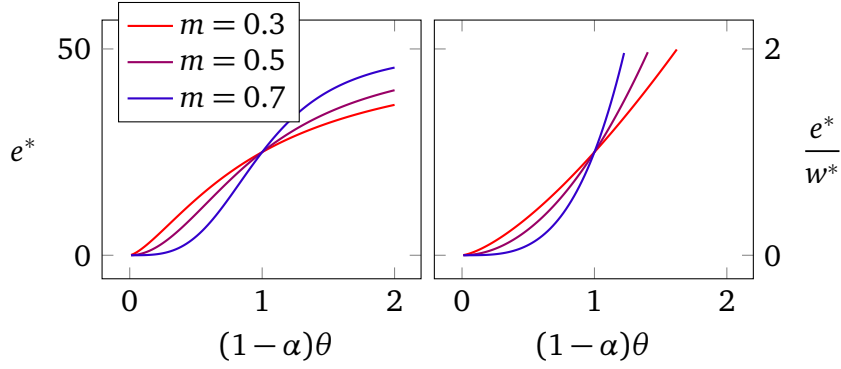


Figure 1: Equilibrium size of the elite coalition  $e^*$  and equilibrium elite-warlord ratio  $\frac{e^*}{w^*}$  as a function of  $(1 - \alpha)\theta$  for  $|V| = 50$ , and different values of parameter  $m$ .

(iv) decreasing with the effectiveness of group size  $m$  if and only if  $(1 - \alpha)\theta < 1$ .

*Proof.* The results follow directly from the relevant first order conditions to (14).  $\square$

### 3.4 Producer welfare (Back to Stage 3)

The negative relation between productivity  $\alpha$  and the equilibrium elite-warlord ratio naturally results in the question whether and, if so, under which conditions producers benefit from the presence of a large elite coalition. Our results show three countervailing effects with respect to  $\alpha$ . First, there is a direct positive effect of  $\alpha$  on production (see (8)). Second, there is an indirect positive effect of  $\alpha$  on production via the tax rate (see Proposition 1). Third, there is an indirect negative effect of  $\alpha$  on production via the elite-warlord ratio (see Proposition 2). We assess this combination of effects on production by evaluating the equilibrium production level through substitution of the equilibrium elite-warlord ratio and the equilibrium tax rate. From this equilibrium production level, we can then proceed to evaluate producer utility (4), our measure of producer welfare.

We first rewrite the CSF in (1) in terms of the elite-warlord ratio by multiplying both RHS fraction terms with  $(\theta e^m)^{-1}$ :

$$\rho(e, w) = \left( 1 + \left( \frac{1}{\theta} \right) \left( \frac{e}{w} \right)^{-m} \right)^{-1}. \quad (17)$$

We then proceed to substitute (14) for  $\frac{e}{w}$  in order to obtain  $\rho(e^*, w^*)$  in equilibrium:

$$\rho(e^*, w^*) = \left( 1 + (1 - \alpha) \left[ (1 - \alpha)\theta \right]^{\frac{1}{m-1}} \right)^{-1}. \quad (18)$$

Consistent with Proposition 2, the comparative statics of  $\rho(e^*, w^*)$  with respect to exogenous



parameters  $\alpha$ ,  $\theta$ , and  $m$  are equivalent to those for the equilibrium ratio of elites to warlords in (14). That is,  $\rho(e^*, w^*)$  is decreasing in  $\alpha$ , increasing in  $\theta$ , and decreasing in  $m$  if and only if  $(1 - \alpha)\theta < 1$ .

Next, we substitute (12) for  $\tau$  and (18) for  $\rho(e, w)$  in the equilibrium production level (10), to obtain equilibrium production as a function of exogenous parameters only:

$$Y^* = \beta \left( \frac{\alpha^2 \beta}{\gamma} \right)^{\frac{\alpha}{1-\alpha}} \left( 1 + (1 - \alpha) [(1 - \alpha)\theta]^{\frac{1}{m-1}} \right)^{\frac{\alpha}{\alpha-1}}. \quad (19)$$

From (19) follows our next result.

**Proposition 3.** *In equilibrium, production  $Y^*$  is:*

- (i) *increasing with the technology-cost ratio  $\frac{\beta}{\gamma}$ ;*
- (ii) *increasing with elite fighting advantage  $\theta$ ;*
- (iii) *decreasing with the effectiveness of group size  $m$  if and only if  $(1 - \alpha)\theta < 1$ .*

*Proof.* The results follow directly from the relevant first order conditions to (19).  $\square$

Note that Proposition 3 does not cover the impact of productivity  $\alpha$  on optimal production. The relevant first order condition to (19) cannot be solved analytically and, instead, we evaluated (19) numerically for a wide range of values for  $\beta$ ,  $\gamma$ ,  $\theta$ , and  $m$ . This evaluation reveals that, for most parameter combinations, production is not very sensitive to  $\alpha$ ; the countervailing effects identified in the beginning of Section 3.4 cancel each other out. There is one exception. When  $\beta$  is sufficiently large and  $m$  is sufficiently small, production peaks for relatively large values of  $\alpha$ . Example plots for different values of  $\beta$  and  $m$  are provided in Figure 2. The increase in production for sufficiently large  $\alpha$  and  $\beta$  is largely driven by a peak in optimal effort (9), which is subsequently offset (for even larger  $\alpha$ ) by the effect of increasing appropriation. Figure 2 also illustrates that for low values of  $\alpha$ , production may be decreasing in productivity  $\alpha$ . This result should be seen in the context of our production function which is qualitatively similar to a two-factor Cobb-Douglas production function with exponent  $1 - \alpha$  for labour and  $\alpha$  for capital (see Section 2.3). Doing so, the negative relation that we find is the conventional negative relation between production and the exponent on capital provided that the capital-to-labour ratio is below unity.

Substituting (12) for  $\tau$ , (18) for  $\rho(e, w)$ , (19) for  $Y(\phi)$  and (9) for  $\phi$  in the producers' utility function (4), we obtain, after substantial simplification:

$$U^* = \left( (1 - \alpha) \frac{\gamma}{\alpha} \right) \left( \frac{\gamma}{\alpha^2 \beta} \right)^{\frac{1}{\alpha-1}} \left( 1 + (1 - \alpha) [(1 - \alpha)\theta]^{\frac{1}{m-1}} \right)^{\frac{1}{\alpha-1}}. \quad (20)$$

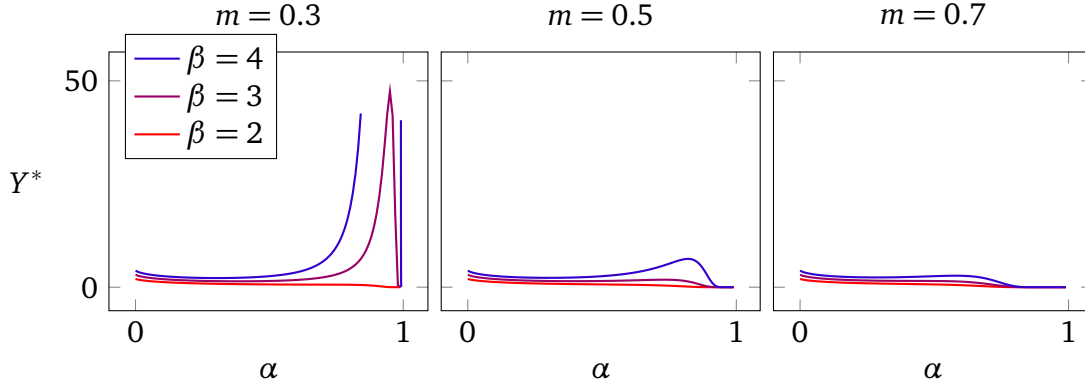


Figure 2: Equilibrium production  $Y^*$  as a function of  $\alpha$ ; example plots for  $\gamma = 1$ ,  $\theta = 2$ , and different values of parameters  $\beta$  and  $m$ .

which is strictly positive under our parameter assumptions.

Based on (20), the effects of model parameters on producer welfare are summarized in our next result.

**Proposition 4.** *In equilibrium, producer utility  $U^*$  is:*

- (i) *increasing with the technology-cost ratio  $\frac{\beta}{\gamma}$ ;*
- (ii) *increasing with elite fighting advantage  $\theta$ ;*
- (iii) *decreasing with the effectiveness of group size  $m$  if and only if  $(1 - \alpha)\theta < 1$ .*

*Proof.* The results follow directly from the relevant first order conditions to (20). □

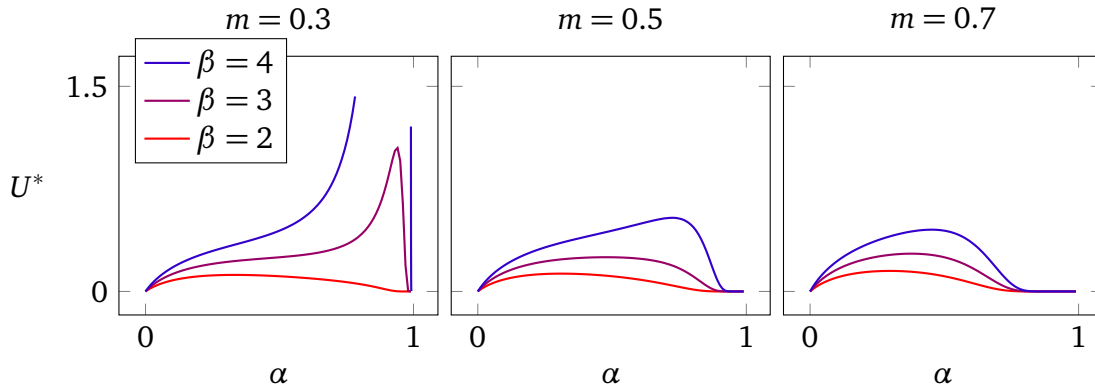


Figure 3: Equilibrium producer utility  $U^*$  as a function of  $\alpha$ ; example plots for  $\gamma = 1$ ,  $\theta = 2$ , and different values of parameters  $\beta$  and  $m$ .

Note that Proposition 4 does not cover the impact of productivity  $\alpha$  on producer utility. The relevant first order condition to (20) cannot be solved analytically and, instead, we evaluated (20) numerically for a wide range of values for  $\beta$ ,  $\gamma$ ,  $\theta$ , and  $m$ . This evaluation

reveals that producer utility is hump-shaped in productivity  $\alpha$ . Example plots for different values of  $\beta$  and  $m$  are provided in Figure 3, which is directly comparable to Figure 2. This figure illustrates the combination of direct and indirect effects of  $\alpha$  on utility – as discussed in the beginning of this section. Utility is low for both low  $\alpha$  and, perhaps surprisingly, high  $\alpha$ . Maximal welfare levels are reached for intermediate values of  $\alpha$ . The explanation for this shape is largely found in the level of the appropriation rate and the return to investments in effort. The appropriation rate equals the sum of taxation by elite and full rent extraction by warlords. It can be expressed as  $\rho(e, w)(1 - \alpha) + (1 - \rho(e, w)) = 1 - \alpha\rho(e, w)$ . From this expression follows immediately that the appropriation rate tends to 1 for low  $\alpha$ . Also, by (18) we know that  $\rho(e^*, w^*)$  is decreasing in  $\alpha$ , which implies increasing rent extraction by warlords. Jointly, these effects make that the appropriation rate follows a *U*-shape with minimal appropriation for intermediate levels of  $\alpha$ .

All in all, producers are best off with intermediate levels of  $\alpha$ , where their production peaks through substantial investments in effort and the level of appropriation is still relatively low. Combined with low  $m$ , the elite coalition is not very large (see Figure 1), but is able to control the bulk of production. Lower levels of  $\alpha$  would decrease the return to investments in effort (higher tax and lower productivity), while higher levels of  $\alpha$  would lead to increased appropriation by warlords. Both effects are detrimental to producer utility.

## 4 Extensions

In this section we consider two extensions to our model, all of which relate to behaviour within the elite coalition. So far, we have assumed a frictionless elite which stands in sharp contrast to some of the historical examples that were referred to in the introduction. Conflict within the elite coalition over the tax rent is likely and barriers to enter for new members are to be expected. These are just two possible examples of frictions within the elite coalition, but they are important ones. In Section 4.1 we assess the impact of conflict over the tax rent by elite coalition members. In Section 4.2 we assess the implications of entry barriers.

### 4.1 Within-coalition conflict

In the natural state, the elite emerges out of the pool of violence specialists, and cooperation of violence specialists in the elite coalition is not self-evident. Given the outcome of Stages 1–3, members of the elite coalition may engage in conflict over the tax rent. There are various

ways to model such conflict and one could even argue that an appropriately designed sharing rule or voting procedure could eliminate the incentive for rent-seeking within the coalition. Yet, given the prominent position of violence in North et al. (2009) who also stress the importance of conflict within the coalition, we follow their perspective here. We proceed with a simple conflict model that we include as stage 4 of our model. This simple set-up is sufficient to demonstrate the impact of the prospect of conflict on elite size.

We do so using a ratio-form CSF, similar to (1), except that it is not deterministic since its outcome depends on deliberate choices by the elites to invest in conflict. Each elite receives a share  $\sigma_i(\mathbf{s})$  of the tax rent, which depends on costly investments in conflict by all elite-members, captured in the vector  $\mathbf{s} = (s_i : i \in E)$ :

$$\sigma_i(\mathbf{s}) = \frac{s_i^n}{\sum_{j \in E} s_j^n}, \quad (21)$$

with  $n \in (0, 1)$  being the effectiveness parameter for within-coalition conflict.

We update the payoff function 5 to elites:

$$\pi_i = \sigma_i(\mathbf{s})\tau\rho(e, w)Y(\phi) - s_i \quad \forall i \in E; \quad (22)$$

Now, in Stage 1 violence specialists choose the most profitable occupation whilst taking into account not only the optimal tax rate and production, but also the severity of conflict within the coalition. Note that the combination of Stage 1 and Stage 4 resembles models of sequential inter- and intra-group resource contest (Wärneryd, 1998; Esteban and Sákovics, 2003; Garfinkel, 2004; Inderst et al., 2007). We add to this the interaction between violence specialists and producers.

Given outcomes of Stages 1–3, each elite-member chooses  $s_i$  to maximize his payoff as given by (22):

$$\frac{\partial \pi_i}{\partial s_i} = \frac{\partial \sigma_i(\mathbf{s})}{\partial s_i} \tau\rho(e, w)Y(\phi) - 1 = 0 \quad \forall i \in E. \quad (23)$$

Note that we exclude the peaceful outcome where  $s_i = 0$  for each agent. Such a peaceful outcome cannot be an equilibrium to the conflict since one elite-member  $j$  could secure the complete resource with a small investment in conflict  $s_j > 0$  (Garfinkel and Skaperdas, 2007). This opportunity would not be left unexploited in equilibrium, which is why we exclude it from our analysis.

By (21) we have:

$$\frac{\partial \sigma_i(\mathbf{s})}{\partial s_i} = \frac{ns_i^{n-1} \sum_{j \in E \setminus \{i\}} s_j^n}{\left( \sum_{j \in E} s_j^n \right)^2}. \quad (24)$$

With homogeneous violence specialists, such that  $s_i = s$  for each  $i \in E$ , we can simplify this derivative to

$$\frac{\partial \sigma_i(\mathbf{s})}{\partial s_i} = \frac{n(e-1)}{se^2}. \quad (25)$$

Substituting this simplified derivative into (23) and solving for  $s$ , we obtain:

$$s = \left( \frac{e-1}{e^2} \right) n \tau \rho(e, w) Y(\phi). \quad (26)$$

Note that we cannot proceed by substituting our previously found equilibrium values, since these may be affected by the prospect of within-coalition conflict on Stage 1–3 decisions. Specifically, this prospect affects the ratio of elites to warlords. It does not affect our result on the optimal tax rate and it also does not affect our results on optimal production or producer welfare, except through this ratio. To show the effect of within-coalition conflict on  $\frac{e}{w}$ , we equate (22) with (6) and we substitute (26) for  $s_i$ , to obtain:

$$\left( \frac{1-n+n/e}{e} \right) \tau \rho(e, w) = \left( \frac{1}{w} \right) (1 - \rho(e, w)). \quad (27)$$

We can now rearrange terms to find the equilibrium ratio of elites to warlords, which is independent from production  $Y$ :

$$\frac{e}{w} = \frac{(1-n+n/e) \tau \rho(e, w)}{1 - \rho(e, w)}. \quad (28)$$

We substitute (1) for  $\rho(e, w)$  and (10) for  $\tau$  in the equilibrium ratio (28). After simplification we obtain the following ratio of elites to warlords as a function of exogenous parameters and  $e$ :

$$\frac{e}{w} = \left( (1-n+n/e)(1-\alpha)\theta \right)^{\frac{1}{1-m}} \quad (29)$$

Comparing (28) with the related ratio in the standard version of our model (14), we see that an additional term  $(1-n+n/e)$  has entered the solution. This term is driven by the anticipation of conflict in the fourth stage of the model (recall  $n$  is the effectiveness

parameter for within-coalition conflict). For  $n \rightarrow 0$ , the effect of Stage 4 within-coalition conflict on Stage 1 occupation choice vanishes; the ratio  $\frac{e^*}{w^*}$  converges to the ratio (14) of the standard version of our model.

We continue to assess the effect of within-coalition conflict on  $\frac{e}{w}$  by substituting  $|V| - e$  for  $w$  in (29) and using implicit differentiation:

$$\frac{d}{dn}e^* = \frac{(e - e^2)(|V| - e)^2}{n(|V| - e)^2 + (1 - m)(|V|e^2)(1 - n + n/e)^{\frac{m}{m-1}}((1 - \alpha)\theta)^{\frac{1}{m-1}}} < 0. \quad (30)$$

All terms of (30) are positive, with the exception of the term  $(e - e^2)$  in the numerator. Hence,  $\frac{d}{dn}e^* < 0$ . As a result, we find that the elite-warlord ratio  $\frac{e^*}{w^*}$  is decreasing with the effectiveness of within-coalition conflict  $n$ . To illustrate this result we reproduce Figure 1 (which features no within-coalition fighting, hence  $n = 0$ ), for the case where  $n = 1$ . Values for  $e^*$  and  $\frac{e^*}{w^*}$  in Figure 4 are computed by solving (29) numerically for  $n = 1$  using the Newton-Raphson method. Comparison of both figures shows that the limit case of the model with  $n = 1$  implies a substantially smaller elite coalition in equilibrium. Choosing the elite occupation has become less attractive compared to the standard version of our model, because of the prospect of within-coalition conflict. Obviously, this difference in elite size is mitigated if we allow for the possibility that warlords also engage in a conflict over their appropriative rents. Such warlord conflict would obviously decrease warlord payoffs. The combined effect of warlord conflict and within-coalition conflict on elite size will depend on the relative effectiveness of both conflicts. Note that we will need much of the above derivations to introduce our results in the remainder of this section, which do not depend on the existence or not of warlord conflict.

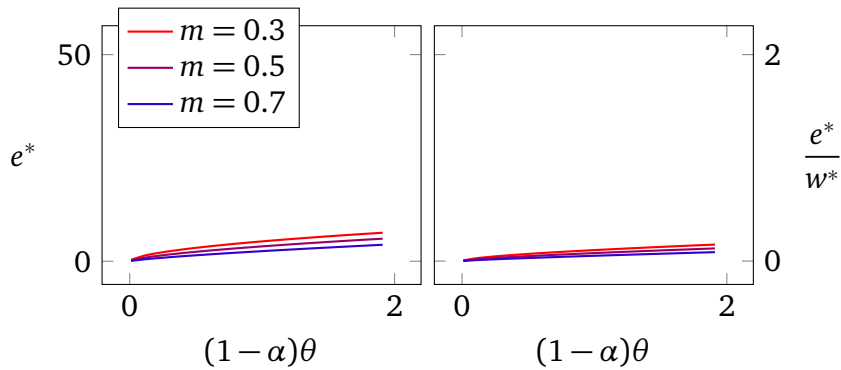


Figure 4: Equilibrium size of the elite coalition  $e^*$  and equilibrium elite-warlord ratio  $\frac{e^*}{w^*}$  as a function of  $(1 - \alpha)\theta$  for  $n = 1$ ,  $|V| = 50$ , and different values of parameter  $m$ .

Our analysis up to here allows us to assess the impact of the number of contestants on investments in conflict. A standard result from rent-seeking models (cf. Garfinkel and

Skaperdas, 2007) is that an increase in the number of contestants *decreases* individual investments in conflict. The intuition for this result is that in the presence of more competitors the expected return to investments in conflict decreases. In the context of our paper, this result would imply that the effect of an increase of  $e$  on  $s^*$  is negative. In conflict models with endogenous production, however, this result is reversed (Hirshleifer, 1995): an increase in the number of contestants *increases* individual investments in conflict. The intuition is that, as the number of contestants increases, a smaller fraction of own production can be retained and hence investments in conflict become more attractive. Our next proposition shows that in the setting of our paper, where production is the domain of a separate subset of producers and subject to interaction with violence specialists, both results are combined. Specifically, an increase in the size of the elite coalition decreases the level of appropriation which provides incentives to increase production. Increased production, in turn, provides incentives to increase investments in conflict. This indirect positive effect may offset the direct negative effect of  $e$  on  $s^*$ .

**Proposition 5.** *In equilibrium, within-coalition investments in costly conflict  $s^*$  are hump-shaped in the size of the elite coalition  $e$  with a global maximum  $\hat{e} \in \mathbb{R} : \hat{e} = \frac{2-Z(\hat{e})}{1-Z(\hat{e})}$ , where  $Z(e) = \frac{m}{1-\alpha}(1 - \rho(e, w))$ .*

*Proof.* Using (26), we first derive the first order condition  $\frac{\partial s^*}{\partial e}$  and solve for  $e$  which yields the implicit function

$$e = \frac{2(1-\alpha)(\theta e^m + w^m) - mw^m}{(1-\alpha)(\theta e^m + w^m) - mw^m} = \frac{2-Z(e)}{1-Z(e)}, \quad (31)$$

where  $Z(e) = \frac{m}{1-\alpha}(1 - \rho(e, w))$ . The second derivative of (26) with respect to  $e$  is rather involved, so we confirmed concavity of  $\frac{\partial s^*}{\partial e}$  numerically. Given the domains of parameters  $\alpha$  and  $m$  and the function  $\rho(e, w)$ , we have that  $Z(e) > 0$ . By (31),  $Z(e) > 0$  implies that  $\hat{e}$  is unbounded (e.g. consider cases with  $\alpha$  and  $m$  such that  $Z(e)$  is close to unity).

To gain further insights into the effect of  $e$  on  $s^*$ , notice that  $e$  not only affects  $s^*$  directly but also through  $\rho(e, w)$  and  $Y(\phi)$ , where  $\phi$  depends on  $e$  through  $\rho(e, w)$ . To evaluate these effects separately we take the total derivative of (26) to  $e$  (to reduce notational clutter we write  $\rho(e, w)$  as  $\rho$  and  $Y(\phi)$  as  $Y$ ):

$$\frac{ds^*}{de} = \frac{\partial s^*}{\partial e} + \frac{\partial s^*}{\partial \rho} \frac{d\rho}{de} + \frac{\partial s^*}{\partial Y} \frac{\partial Y}{\partial \rho} \frac{d\rho}{de}. \quad (32)$$

We find that these three effects are given by:

$$\frac{\partial s^*}{\partial e} = \left[ -\left(\frac{e-2}{e^3}\right) n\tau\rho Y \right]; \quad (33)$$

$$\frac{\partial s^*}{\partial \rho} \frac{d\rho}{de} = \left[ \left(\frac{e-1}{e^2}\right) n\tau Y \right] \times \left[ \rho(1-\rho) \left(\frac{m}{e}\right) \right]; \quad (34)$$

$$\begin{aligned} \frac{\partial s^*}{\partial Y} \frac{\partial Y}{\partial \rho} \frac{d\rho}{de} &= \left[ \left(\frac{e-1}{e^2}\right) n\tau\rho \right] \times \left[ \rho(1-\rho) \left(\frac{m}{e}\right) \right] \\ &\times \left[ \left(\frac{\alpha}{1-\alpha}\right) \frac{\beta}{\rho} \left( (1-\tau)\rho \frac{\alpha\beta}{\gamma} \right)^{\frac{\alpha}{1-\alpha}} \right]. \end{aligned} \quad (35)$$

The direct effect of an increase of  $e$  on  $s^*$  in (33) is negative (as long as  $e > 2$ ), while both indirect effects in (34) and (35) are positive for  $e > 1$ . An increase in  $e$  increases both the share of production controlled by elites as well as (indirectly) the production level. Both of these contribute to a higher tax rent, which makes fighting more attractive. Depending on parameter values the positive or negative effect dominates as stated in the proposition.  $\square$

## 4.2 Limiting access to the coalition

As a final extension, we assess whether the elite has an incentive to limit entry into the coalition in order to avoid dilution of the tax rent. It seems reasonable to assume that access into the coalition should require consent of the coalition members. In the literature on coalition formation in games with externalities it is shown that the rules of coalition formation may impact the coalition size. Specifically, requiring consent on membership – called ‘exclusive membership’ in this literature – is one such rule. In a negative externality game (as we have here), it normally implies smaller coalitions with higher payoffs per member compared to ‘open membership’ (Yi, 1997).

Below we will illustrate that this result does not necessarily hold in our setting. We do not impose a specific membership rule but rather compare the impact of (exogenous) coalition size on elite payoffs, and subsequently compare these payoffs to those under the (endogenous) equilibrium coalition size. Doing so, we find that elite payoffs are not necessarily maximized at the endogenously determined coalition size (15). Instead, members of the elite coalition may prefer an alternative elite size, which may be smaller or larger, depending on parameter combinations in the model. This result is illustrated in Figure 5 for different parameter combinations of  $\alpha$  and  $m$ .

Figure 5 is based on the elite payoff function (15). Substitute (12) for  $\tau$  and (10)



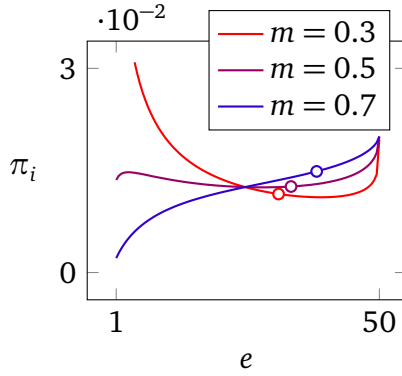


Figure 5: Scaled individual payoffs (as given by the first RHS term of (36)) to members of the elite coalition as a function of coalition size  $e$  for  $|V| = 50$ ,  $\alpha = 0.65$ ,  $\theta = 2$ , and different values of parameter  $m$ . Circles indicate the (endogenous) equilibrium coalition size for each  $m$ .

for  $Y(\phi)$  and rearrange to obtain elite payoffs as a function of exogenous parameters and  $e$ :

$$\pi_i = \left( \frac{(\rho(e, w))^{\frac{1}{1-\alpha}}}{e} \right) (1-\alpha)\beta \left( \frac{\alpha^2 \beta}{\gamma} \right)^{\frac{\alpha}{1-\alpha}}, \quad (36)$$

Only the first RHS term of (36) depends on  $e$  and only this term was used to construct Figure 5, the other terms being constant for given parameter values. The figure shows individual elite payoffs as a function of elite coalition size  $e$  for three values of  $m$ . It also displays, for each  $m$ , the equilibrium coalition size as the outcome of our model. Clearly, elite-members could benefit from a change in the size of the elite coalition. For  $m = 0.3$ , elite-members would benefit from a decrease in elite size. Conversely, for  $m = 0.7$ , elite-members would benefit from an increase in elite size. For  $m = 0.5$ , they are largely indifferent. Only when elites prefer a smaller coalition they would benefit from implementing exclusive membership. When they prefer a larger coalition, then apparently the equilibrium payoffs to warlords are sufficiently large to keep them out of the coalition; no warlord has an incentive to switch and become a member. In such a setting, exclusive membership would not affect coalition size.

The dependence of preferred elite size on  $m$  is due to the extent of diminishing marginal returns to group formation for any  $m < 1$ . If  $m$  is low, group size becomes less relevant for the share of production controlled by the elite than if  $m$  is large. Hence, for low  $m$ , a smaller elite would increase the tax rent per elite-member. The opposite effect holds for high  $m$ .

One question that may arise is what happens to the violence specialists that are rejected

entrance in case of exclusive membership? Our model seems to allow for only one option, they become warlords. In reality, however, such violence specialists may prefer to switch and become a producer instead.<sup>8</sup> In Section 1 we have argued that it is not possible for producers to become violence specialists. Up to here, however, we have not discussed the possibility of a switch in the opposite direction. A proper assessment of the incentives for such a switch requires a comparison of payoffs to warlords and producers. Since, in our model, production is represented only in aggregate terms without any reference to the cardinality of the set of producers  $P$ , any such comparison would be ad-hoc. If the switch from warlord to producer would be allowed, however, we can expect production to go up (perhaps only marginally), as well as the share of production controlled by the elite coalition. Both effects would increase payoffs to elite-members, amplifying the benefits of exclusive membership.

## 5 Discussion and conclusion

In this paper we provide economic intuition for the natural state by analysing the role of organized large-scale violence and how it is contained. We highlight key mechanisms underlying the natural state using insights from the economic literature on conflict and appropriation. In general, we find large variations in the size of elite coalitions, appropriation, production levels, and welfare across natural states, due to only minor variations in exogenous model parameters. The characteristics of a given society at a given moment in time will depend on the power balance between elite and warlords, all other factors equal. Extending and formalizing the analysis by North et al. (2009), by explicitly taking into account the incentive to violence specialists, this result implies that we can expect to find a wide variety of outcomes within the class of limited access orders. Specifically, our results show that unproductive societies tend to have large elite coalitions and high tax rates. Only when the elite coalition is small (which occurs in societies with high productivity) but still able to control a sizeable share of production, can societies prosper in a natural state. Summarizing, both productivity and order are necessary conditions for prosperity. This conclusion puts previous work by e.g. Bates et al. (2002) in perspective. Whereas they stress the trade-off between ‘production’ and ‘protection’ in stateless societies, we find that this trade-off is mitigated by the incentives of violence specialists to join forces in the elite and jointly restrict violence.

This result illustrates the delicate balance between productivity and order (cf. Bates

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<sup>8</sup>Alternatively they may start a second, competing, coalition. Co-existence of multiple coalitions was analysed by a.o. Garfinkel (2004).

et al., 2002). Dal Bó et al. (2015) analyse this balance as a pre-institutional process, arguing that institutions play no role in explaining different outcomes across states. Their setting is different (i.e. a monolithic incumbent, with certain defence and growth capabilities, owns a productive asset and is challenged by a predatory competitor), but their conclusion is similar to ours: both productivity and order are necessary conditions for prosperity. Like Dal Bó et al. (2015), our results challenge the standard explanations for prosperity that focus on the role of institutions (cf. Acemoglu et al., 2005) and human capital (Galor, 2005).

Our model is capable to answer, at least partially, why societies with extractive elites emerge and persist. The traditional answer to this question is that the elite is better off in an extractive and exclusive regime and powerful enough to maintain it (Sokoloff and Engerman, 2000; Acemoglu and Robinson, 2008). In contrast, North et al. (2009) start from the assertion that violence is an endemic threat to the stability of societies, arguing that a society with a small and extractive elite is a natural social order since it guarantees a certain degree of order and stability, through a system of coercive rent extraction rather than by destructive (and chaotic) appropriation. In contrast to most of the literature, the elite coalition considered by North et al. (2009) and modelled in this paper, is not a monolithic entity with absolute power. Instead, it consists of individuals who compete with each other, each having specific but limited power. As a result, the elite coalition is fragile, and the behaviour of individual elite-members is constrained by the threat of other elite-members as well as warlords.

This perspective undermines theories that treat the elite as a powerful monolithic entity, capable of reorganizing society. For instance, Grossman (2002) states that the ruler of a society can induce the population to provide such a level of defence that all violence and appropriation is deterred. In the natural state, however, no elite-member has such power, and the decision-making of the coalition as a whole is much more complex. Also, Acemoglu and Robinson (2006, 2008, 2012) stress that extractive hierarchies are persistent because the monolithic elite will obstruct any change that might undermine its power. In the natural state, however, the elite is not monolithic but elite-members and the rest of society are constrained in their behaviour by the rigidity of the social order.

Several caveats are in place. First, we have modelled one-dimensional relations between the tax rate, production, and within-coalition conflict. These relations are likely to be more involved in practice. For instance, taxation and within-coalition conflict both require effort from the patronage networks of the elite coalition. One consequence is that within-coalition conflict could have detrimental effects on the tax rate that the coalition, as a group, can enforce onto producers. Another consequence is that the decision on a tax rate could be the

source of substantial conflict within the coalition. These, and other, complicating factors are not considered here, since this would require a more explicit consideration of coalition decision-making, where the size and composition of the coalition are endogenous to the decision-making itself, which is beyond the scope of this paper.

Second, the model developed in this paper is a stylized representation of a natural state and, as a result, does not elaborate on how both elites and warlords exercise control over producers and production. Our interpretation is that the size of the elite-controlled production is simply a measure of the elite's success in imposing order and stability. A more realistic interpretation would be that elites and warlords each have a share of territory under control, possibly tied to their patronage network. An implication of this alternative interpretation is that one should take into account migration of producers and distinct production decisions on territory controlled by elites and warlords. Taking the interpretation one step further, one even has to consider competing coalitions in bordering regions.<sup>9</sup> For simplicity, we abstract from such considerations.

Third, we have ignored the changing nature of societies by treating society, its size, and the distribution of capacities as constants. In future work we plan to address such dynamic aspects.

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<sup>9</sup>We argue that, under this alternative interpretation, a setting with two coalitions can be modelled as two separate natural states.

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