Another Solution for Allais Paradox: Preference Imprecision, Dispersion and Pessimism

Bayrak, Oben

Centre for Environmental and Resource Economics

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Oben K. Bayrak

Abstract

Although there are alternative models which can explain the Allais paradox with non-standard preferences, they do not take the emerging evidence on preference imprecision into account. The imprecision is so far incorporated into these models by adding a stochastic specification implying the errors that subjects make. However, there is also the inherent part of the preference imprecision which does not diminish with experience provided in repeated experiments and these stochastic specifications cannot explain a significant portion of the observed behavior in experiments. Moreover, evidence on imprecision suggests that subjects exhibit higher imprecision for a lottery with a higher variance. This paper presents a new model for decision under risk which takes into account the findings of the literature. Looking at the indifference curves predicted by the new model, the new model acts like a mixture of Expected Utility Theory and Rank Dependent Utility Theory depending on which part of the probability triangle the lottery is located.

Keywords: Allais Paradox, Independence Axiom, Preference Imprecision, Anomalies, Decision Theory, Decision under Risk and Uncertainty, Alternative Models

1 Introduction

Allais Paradox was first proposed by Maurice Allais (1953) to challenge the Expected Utility Theory (EUT). The paradox is usually described by two choice problems between pair of lotteries shown in Table 1. The first pair includes choosing one of the two lotteries: S_1 = ($1M, 1) or R_1 = ($5M, 0.1; $1M, 0.89; 0, 0.01). The second pair includes: S_2 = ($1M, 0.11; 0, 0.89) and R_2 = ($5M, 0.1; 0, 0.9). A closer look would reveal that S_1 and R_1 includes a common consequence of $1M with probability of 0.89, and that S_2 and R_2 are derived by subtracting this common consequence from S_1 and R_1, respectively. An individual whose preferences are compatible with EUT would either choose 'S' or 'R' type of lotteries in both choice problems; common consequences added or subtracted to the two prospects should have no effect on the desirability of one prospect over the other; because according to EUT formulation which treats the probabilities linearly, common consequences cancel out. However, Allais argued that most people might opt for S_1 in the first problem lured by the certainty of winning $1M, and R_2 in the second problem since the odds of winning are very similar but the winning prizes are very different; $1M and $5M. This problem is known as 'common consequence effect'[1]. Allais's ideas challenged the independence axiom of EUT-the idea of expected utility being linear in probabilities- and finally contributed to the development of alternative models such as Fanning-out hypothesis, Rank Dependent Utility Theory, Prospect Theory and its variants.

Table 1: Bets in Allais Paradox.

Outcome	S_1	R_1	S_2	R_2
$0	-	0.01	0.89	0.90
$1M	1.00	0.89	0.11	-
$5M	-	0.10	-	0.10

[1] Another related phenomenon is the 'common ratio effect': There are two choice tasks and each task includes a pair of lotteries. The first pair includes: M_1 = ($3000, 1) or N_1 = ($4000, 0.8; $0, 0.2) whereas the second pair includes M_2 = ($3000, 0.25; $0, 0.75) or N_2 = ($4000, 0.2; $0, 0.8). The common choice pattern is choosing M_1 and N_2 is inconsistent with the predictions of EUT because the second pair is formed by multiplying the probabilities of the first pair's winning prizes by a common ratio of 0.25.

However, there is an emerging literature on preference imprecision which challenges the validity of these alternative theories as well. Experimental studies in this new strand of literature suggest that even intelligent and numerate individuals find it hard to know their own preferences precisely and are not able to state their choices and subjective valuations for goods and risky prospects with perfect confidence (See Bayrak & Kriström, 2016; Butler & Loomes, 1988, 2011, 2007; Cohen, Jaffray, & Said, 1987; Cubitt, Navarro-Martinez, & Starmer, 2015; Morrison, 1998). Although alternative theories model individual behavior in a non-standard way to explain these observed anomalies, they share a common implicit assumption with EUT that individuals can articulate their subjective valuations for goods and make choices in a precise manner. Therefore, the issues raised by this recently emerging literature are not covered by the existing models in the literature including both the EUT and its alternatives. These recent findings have critical implications as well: if, for example, consumers' preferences are imprecise and prone to being manipulated, this may be used against consumers' own best interests. Moreover, if the inherent characteristics of economic preferences are imprecise, the validity of the studies that evaluate and analyze the policies and/or market schemes based on the existing models of precise preferences should also be reconsidered. In order to reach solid conclusions about all of these issues, it is vital to have a better understanding and a better model of the imprecise preferences.

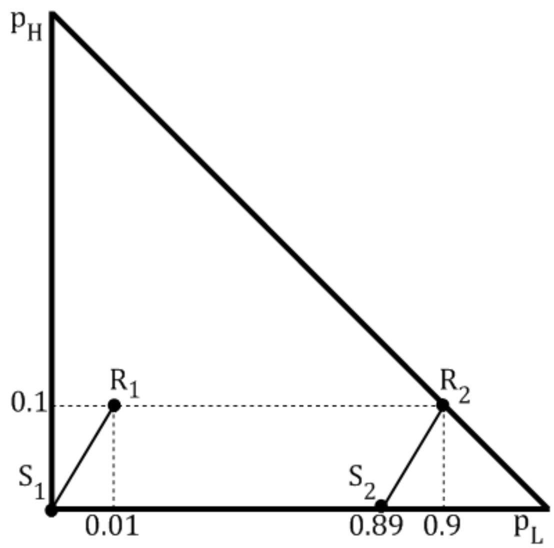
Imprecision, so far is incorporated in economic preferences by modelling it as a stochastic component of a deterministic theory such as EUT, Rank Dependent Utility Theory and Prospect Theory. However, it seems that no single combination of deterministic core and stochastic specification can explain the significant portion of the behavior in the experiments (Loomes, 2005). More specifically, evidence suggests that a theory which predicts parallel and linear indifference curves inside the probability triangle and non-conventional patterns on the boundaries would fit the data better and the alternative models seem to allow patterns that are rarely observed (Harless & Camerer, 1994). Although several different error specifications for existing deterministic models have been developed in the literature, there has not much progress made in modelling the imprecise preferences in a deterministic manner. The existing stochastic models are limited to treat the imprecision as errors that subjects make, however, the evidence show that there is also an inherent and deterministic part which does not decay with experience. Loomes et al. (2002) provides the evidence for why we should make a distinction between errors and imprecision: In a repeated experiment each subject presented with randomly ordered 45 pairwise choice questions, after a short break, the same 45 pairwise choices were presented again, in a different random order. They find that stochastic variation associated with errors decay with experience, but there is also a stable part which can be seen as an inherent part of the preferences. This paper focuses on the stable part and introduces a deterministic core model which explains how imprecision is formed and a particular expected utility is drawn from the imprecision range. To develop such a model, I pay attention to the recent experimental findings and particularly about the determinants of imprecision: The experimental evidence provided by Butler and Loomes (1988) and Cubitt et al. (2015) suggest that the higher the variance of a lottery, the broader the imprecision range for a lottery. Taking these into account I assume that the imprecision range is proportionate to dispersion. The paper is organized as follows: Section 2 provides a literature review by presenting: i. the existing explanations of the Allais paradox using probability triangle, i. experimental studies on Allais paradox, iii. a review of preference imprecision literature. Section 3 presents the new model and its explanation for Allais paradox.

2. Related Literature

2.1. Probability Triangle, Expected Utility Theory and Alternatives

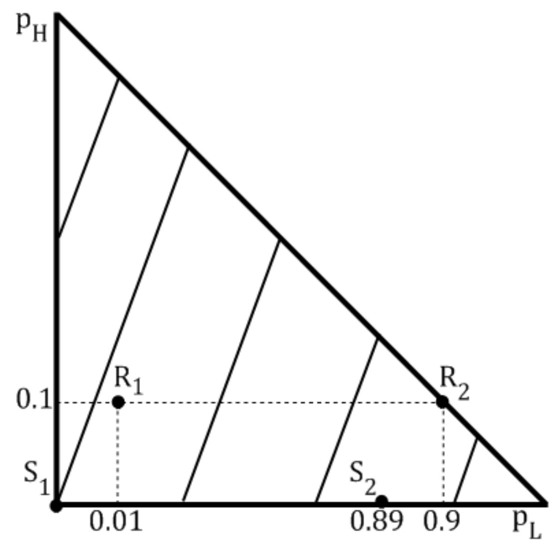
In order to see the differences between EUT and the alternatives, it will be helpful to use the probability triangle and demonstrate the Allais type of bets on the triangle. These bets are characterized as three outcome lotteries where

Figure 1: Probability Triangle



the outcomes are x_1, x_2 and x_3, which have the following order in terms of magnitude: $x_L < x_M < x_H$. The corresponding probabilities of these outcomes are a vector of probabilities: (p_L, p_M, p_H). For the original version of the Allais problem the outcomes are \$0, \$1M, and \$5M. Figure 1 demonstrates the problem in a probability triangle where the vertical edge shows the probability of best consequence, whereas the horizontal edge measures the probability of the worst consequence. The probability of winning \$1M (p_M) is implicit on the graph because $p_M = 1 - p_H - p_L$. The bets that are located on the triangle edges (excluding the corners) assign positive probabilities only for two consequences out of three. S_1 gives \$1M with a probability of 1, it is centered in the corner where the probabilities of other consequences are zero $(p_H = p_L = 0)$. In addition, since S_2 has positive probabilities for the consequences such as 0 and \$1M, it lies on the horizontal axis. Similarly, R_2 does not assign a positive probability for winning \$1M therefore it is on the hypotenuse, which depicts the probability of winning \$1M. The interior of the triangle includes the bets that assign positive probability to all three consequences; such as R_1. Next we derive the slope of the indifference curves of EUT, to do that we use the following expression:

Figure 2: EUT Indifference Curves



$$U^* = p_L \cdot u(x_L) + p_M \cdot u(x_M) + p_H \cdot u(x_H) \quad (1)$$

Equation (1) simply implies the collection of the lotteries which give the same utility level, U^*. Next, we substitute $1-p_H-p_L$ for p_M and take the derivative of the expression with respect to p_L. After rearranging we find that the slope of the indifference curves under EUT is[2]:

$$\frac{\partial p_H}{\partial p_L} = \frac{u(x_M) - u(x_L)}{u(x_H) - u(x_M)} \quad (2)$$

Since the slope only depends on the utilities of the outcomes, indifference curves under EUT are parallel straight lines with a constant slope. They are increasing in terms of desirability towards the northwest of the triangle because the best outcome is located on the vertical axis and the worst outcome is on the horizontal axis. Figure 2 shows an example of indifference curves drawn according to EUT.

The slope of the indifference curves implies the risk attitude of the individuals: the steeper the slope, the more risk averse the individual is, as shown in Figure 3. The solid line in the figure implies relatively more risk aversion compared to the dashed line: x on the figure gives \$1M with

[2] $\frac{\partial U^*}{\partial p_L} = 0$, by definition of indifference curves.

Figure 3: Risk Attitudes

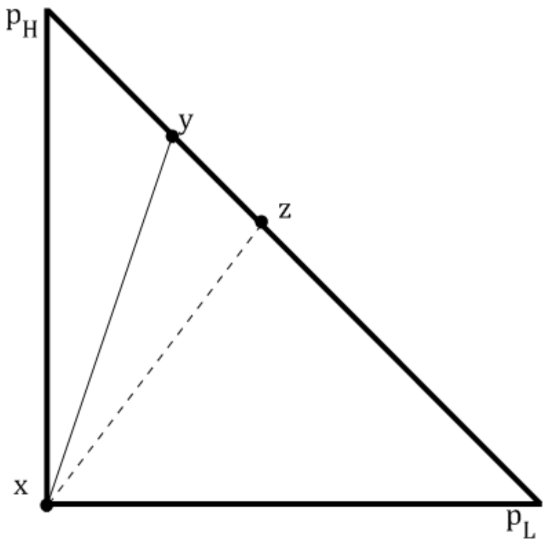
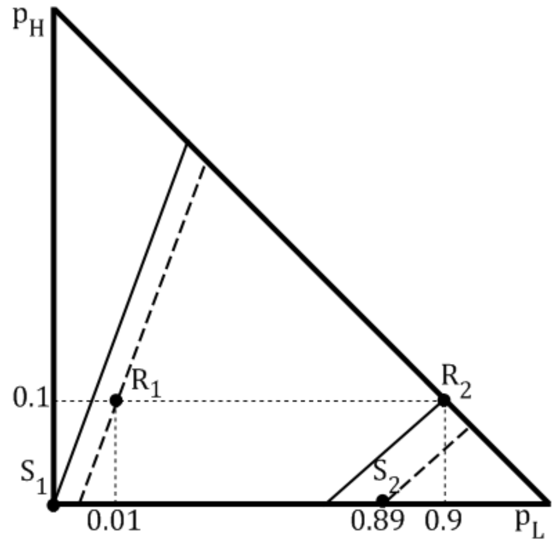


Figure 4: Paradoxical Behavior



certainty, whereas y and z are the risky prospects that assign positive probability to the worst (\$0) and the best consequences (\$5M), but zero for the middle-ranked consequence (\$1M). Furthermore, y assigns a higher probability to \$5M than z. Therefore the solid line belongs to an individual who demands a higher probability of getting \$5M to be indifferent between the risky prospect and \$1M with certainty. Under EUT, throughout the triangle the individual maintains the risk attitude by having the parallel indifference curve covering the triangle. However, the actual behaviour observed in the literature contradicts the prediction of EUT. Figure 4 demonstrates the observed behaviour: the individual choosing S_1 in the first question signals an indifference curve similar to c_1, which means that the indifference curve that passes through R_1 lies somewhere below c_1, which is in the less desirable region. On the other hand, if individual chooses R_2 in the second question it means that the indifference curve passes through S_2 and lies somewhere below c_2. It is easy to see that c_1 and c_2 are not parallel which means that individual acts as though less risk averse while making a choice between S_2 and R_2 as compared to when making the choice between S_1 and R_1. This behavior is inconsistent with EUT, because it implies that the risk attitude of the individual does not remain the same across the choices between two pairs.

This pattern of unstable risk attitudes is hypothetised as indifference curves being fanning out from the bottom-left corner of the triangle. To maintain transitivity it is assumed that the starting point of fanning out is located outside the triangle as shown in Figure 5. The figure shows the typical linear but fanning out indifference curves under the Weighted Utility Theory developed by Chew and MacCrimmon (1979). There are also different patterns produced by alternative theories, which allow for Allais paradox. Figure 6 shows the indifference curves of Rank-Dependent Utility Theory with a concave probability weighting function. The curves are steepest in the bottom-right corner where the probability of the middle-ranked outcome (\$1M) equals one. They get flatter as we move along the horizontal and vertical axes and finally become parallel close to the hypotenuse where the probability of the middle-ranked outcome equals zero.

Overall, alternative theories treat the probabilities in a nonlinear manner, which then relaxes the linearity and/or parellelism of the indifference curves (see Camerer (1989) for a detailed analysis and the derivation of the indifference curves of different preference functionals).

2.2. Experiments on Allais Paradox

Besides the theoretical advances in the literature to explain Allais Paradox, there are also studies that empirically question and test its robustness. One strand of literature can be seen as the

Figure 5: Fanning Out Hypothesis

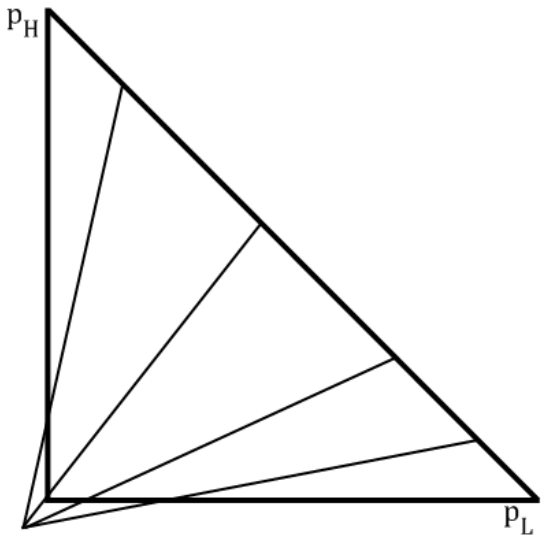
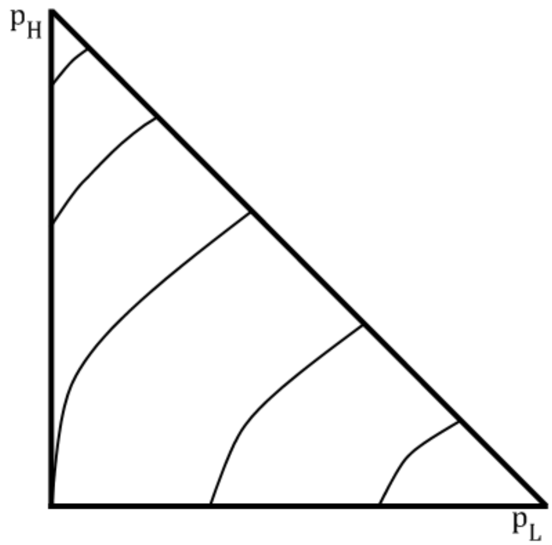


Figure 6: Rank Dependent Utility



defenders of EUT that claim that the violations can be explained by misunderstandings and inattentiveness (M. Allais, 1990; Amihud, 1979a, 1979b; Morgenstern, 1979). In an experimental study, Savage (1954) modifies the representation of the lotteries in order to highlight the similarity of the bets in two questions, as shown in Table 2.

Table 2: Savage's representation of the Allais bets

	1	2-11	12-100
M	1000	1000	1000
N	0	5000	1000
M'	1000	1000	0
N'	0	5000	0

The last three columns include the different way of presenting the probabilities associated with the three outcomes. For example, suppose a subject chooses N in the first question and the random number drawn equals 9, then the subject wins 5000, since it is between 2 and 11. This representation facilitates understanding the similarity between the first and the last two lotteries shown in the table: discarding the common consequence of winning 1000 if the random number is between 12 and 100 from M and N produces M' and N'. Although this modification in the presentation of the lotteries decreases the inconsistencies from 60% to 40%, they do not disappear (Incekara-Hafalir & Stecher, 2012). Conlisk (1989) also focuses on the presentation of the lottery tickets and finds that the inconsistencies decrease from 50% to 28%. In addition to the subject misunderstandings, Harrison (1994) criticises the hypothetical nature of the surveys that document the inconsistencies and suggests that it would be premature to discard EUT based on them. Burke et al. (1996) takes the critics of Harrison into account and use real monetary payoffs in an experimental study which again reduces the inconsistent preference statements but does not eliminate them completely (See Camerer, 1989 for another example with real payoffs).

2.3. Imprecision

EUT and the alternatives assume that every risky prospect has a certainty equivalent; a precise amount of money that is equally desirable and more importantly individuals can state this amount with confidence. This might be true for an individual who has sufficient familiarity and expertise in risky situations, but ordinarily it is more likely that the certainty equivalent would be a range of rounded numbers rather than a precise estimation.

The idea of imprecision dates back to 19th century, investigated in the works of Fechner and Weber who are considered to be the founders of the psychophysics and experimental psychology (Gescheider, 2013). They investigate the relation between stimulus and sensation, particularly focusing on judgments about stimuli such as light, sound, weight, and distance. Those early works suggest that human judgement of stimuli is subject to errors, therefore expecting a perfect evaluation from individuals is not realistic (Fechner, 1966). Moreover, upon comparing, e.g., the weight of two objects, the probability of making a mistake is higher when the weights are very close, such as 1 kg and 1.05 kg. Psychophysics studies focus on the physical stimuli, however in the realm of economics, individuals deal with evaluations of risky prospects, which are the main focus of this study. One can see the risky prospects or lotteries in economics are the counterparts of the physical stimuli concept in psychophysics. Finally, the 'preference imprecision hypothesis' is the idea which claims that as individuals' judgements about objects are subject to mistakes, i.e., the choices among options and valuations of the goods are also liable to imprecision and noise.

Two prominent findings of experimental literature lead economists to focus on imprecise preferences. The first is that when subjects face the same pairwise choice more than once, a considerable portion of the subjects seem to be behaving inconsistently on different occasions in a given experiment (Ballinger & Wilcox, 1997; Camerer, 1989; Hey & Orme, 1994; Starmer & Sugden, 1989). Second, the existing theories of decision under risk seem to be only partially successful in explaining the behaviour observed in experiments (Loomes & Sugden, 1998). Beginning in 1990s, the idea of imprecision began to receive attention by researchers in the form of modelling it as the stochastic component of a deterministic theory such as EUT and/or alternative theories (Harless & Camerer, 1994; Hey & Orme, 1994; Loomes & Sugden, 1995, 1998; Sopher & Gigliotti, 1993). The common approach employed by these studies is to incorporate the imprecision as the stochastic component—the random and/or error part—of a core deterministic theory. The general logic that is employed is to reject a theory if the observed behaviour systematically departs from the core theory, if the anomalies cannot be explained by random errors or deviations from the core theory. However, it seems that no single combination of deterministic core and stochastic specification can explain the significant portion of the behavior in experiments (Loomes, 2005). For example, Harless and Camerer (1994) conducted their analysis on 23 data sets consisting of approximately 8,000 choices that subjects made in Allais type of problems. Overall, they found that none of the existing theories perform significantly better than others: all theories are rejected by a chi-square test. This implies that the variation that is not predicted by the existing core theories can be explained by another theory as yet undeveloped, because the stochastic part is found to be a systematic variation rather than being an error. Most importantly, they find that EUT describes the data better when the lotteries are located inside the triangle and fits poorly when lotteries are on the boundaries of the triangle. This implies a theory which predicts parallel and linear indifference curves inside the triangle and non-conventional patterns on the boundaries would fit the data better. Overall, their results suggest that EUT can be improved by further generalizations to incorporate commonly observed patterns in the literature. Moreover, the alternative models such as Rank-Dependent Utility Theory seem to allow patterns that are rarely observed. Thus, the results suggest not abandoning EUT but extending it.

There is also another strand of literature consisting of experimental studies that use direct elicitation methods of imprecision intervals mainly relying on the subjects' self-reporting. These studies elicit the imprecision range for the subjective valuations by asking subjects to answer a series of binary choice questions between a lottery and a sure amount of money. Additionally, for each question subjects indicate how confident they are about their choice[3]. The common patterns found in this

[3] There is also another method which is known as "iteration procedure". For example Dubourg et al. (1994) used a numbered disk, which has a small window showing only single value at a time. For each value, subjects state their preference by choosing one of the three phrases: definitely willing to pay, definitely not willing to pay, or not sure. If the response was 'willing', the interviewer rotates the disk to reveal a higher value through the window, whereas if the answer is 'not willing', the interviewer reveals a lower amount. The experiment continues until there is a maximum amount that subjects are definitely willing to pay and not willing to pay. If the two amounts are different, then the interviewer asks

strand of literature provide information about the nature of preference imprecision: For example, Butler and Loomes (1988) used four lotteries in their experiment. Their findings support that there is a positive relation between the variance of the lottery and individuals' subjective valuation range. Most recently, Cubitt et al. (2015) provides a more comprehensive treatment of the imprecision issue: their data includes responses for 33 different lotteries which are organized in seven sequences each including 5 lotteries. Lotteries in sequence 1 have equal variances whereas in sequence 2 they do not. They find that the size of the imprecision range is significantly different for the lotteries in sequence 2, increasing with the variance, however in sequence 1 there is not significant change in the size of the imprecision range. This result suggests that subjects exhibit higher imprecision for a lottery with a higher variance. Their design also enables tests of stability, i.e. whether the size of the intervals changes with repetition or not. It is important, because if imprecision is merely a result of errors or unfamiliarity with the experimental mechanisms, it should disappear with repetition and experience. However, they found no evidence for imprecision declining with experience. Their analysis supports that imprecision is stable and not temporary; it seems to be the inherent characteristic of individuals' preferences.

3. Model

In light with the emerging evidence known as preference imprecision and considering the results of Harless and Camerer (1994), I present a new model which assumes that individuals also take into account the dispersion of the lotteries as well while calculating the expected utility. Taking the evidence discussed in Section2.3 into account, I assume that the imprecision range is proportionate to dispersion. The model is formulated as having two stages: in the first stage individual forms the imprecision range and the values of the range is denoted as $EU_k(.)$ which is indexed by the subscript $k \in \{1,...,K\}$ and the values in the range are ranked as follows: $EU_i(.) > EU_j(.)$ if and only if $i > j$. In the second stage model describes how individuals select a single expected utility for decision making. Starting with the first stage, lower $(EU_1(.))$ and the upper bounds $(EU_K(.))$ are calculated in the following way:

$$EU_1(X) = EU(X) - \beta \cdot u(\sigma) \tag{3}$$

$$EU_K(X) = EU(X) + \beta \cdot u(\sigma) \tag{4}$$

where $u(\sigma)$ is the utility of the standard deviation and $\beta \geq 0$, a measure of an individual's ability to be precise about preferences. It describes how individual is familiar with decision under uncertainty. Notice that the individual has precise preferences and behaves in the way that EUT predicts when $\beta = 0$. Therefore the model can be seen as an extension of EUT. As β increases, the imprecision range also increases. The second stage of the model describes how individuals select a single value from the range formed in the first stage: Note that, although individuals are imprecise about their preferences, the situations in real life requires precise judgements; e.g. none of the transactions takes place in terms of interval amounts of money in real markets. Thus, the final decision such as determining a buying or a selling price has a precise unit in real life situations. At the end of the first stage individual forms a range of expected utility values but cannot determine a single value from the range confidently, he or she calculates the expected value of the range, denoted as $\alpha EU(.)$. However, individual does not have a prior knowledge about the distribution of the true expected utility of this range. In other words the decision at the second stage of the model resembles the decision under ambiguity i.e. individual knows the outcomes, $EU_k(.)$, but does not know the probability of those outcomes. Therefore individual forms beliefs about the possible distribution of the true expected utility in this range:

$$\alpha EU(.) = E\left[EU_k(.)\right] = \sum_1^K EU_k(.) \cdot f\left(EU_k(.)\right) \tag{5}$$

for a 'best estimate' of the subject for determining the 'switching point'

Regarding the shape of $f(EU_k(.))$; a pessimist will put more weight to the values towards the lower bound of the range whereas an optimist will put more weight to the values close to the upper bound of the range. Perhaps, an idiom - seeing the glass half-full or half-empty- might be helpful to understand the intuition: It is an expression which indicates that a particular situation can be seen as both potentially good and bad depending on how people perceive it. The optimists will attain extra utility from how much dispersion the prospect has, because they see the dispersion as the opportunity not to be missed: they see the "glass half full". The pessimists want to avoid dispersion, because the dispersion would cause them a disutility; they see the glass half empty.

In order to see how model incorporates Allais Paradox and explore the parameters of the models that allow for Allais type of behaviour, I use CRRA utility function for $u(.)$ as an example and the bets in Table 1. For the purpose of being parsimonious, I specify the simplest specification for the beliefs formed in the second stage, i.e. $f(EU_k(.)$ in equation (3): Using α as the weight attached to the lower bound of the imprecision range and $1-\alpha$ to the upper bound, individual calculates a weighted average of the expected utility range. In this case alpha represents the pessimism level of the individual, e.g. a relatively pessimistic individual have α higher than 0.5 so attaches more weight to the lower bound $(EU_1(.))$. Thus an individual calculates the expected utility of a bet X under the new model as follows:

$$\alpha EU(X)=\alpha\cdot EU_1(X)+(1-\alpha)\cdot EU_K(X) \quad (6)$$

where $EU_1(X)=EU(X)-\beta\cdot u(\sigma)$ and $EU_K(X)=EU(X)+\beta\cdot u(\sigma)$. Using (6) we can now write the expected utilities for the lotteries S_1, R_1, S_2 and R_2[4]:

$$\alpha EU(S_1)=1^a \quad (7)$$

$$\alpha EU(R_1)=\alpha\cdot\left[0.89\cdot 1^a+0.1\cdot 5^a-k\cdot 1.21^a\right] +(1-\alpha)\cdot\left[0.89\cdot 1^a+0.1\cdot 5^a+k\cdot 1.21^a\right] \quad (8)$$

$$\alpha EU(S_2)=\alpha\cdot\left[0.11\cdot 1^a-k\cdot 0.31^a\right] +(1-\alpha)\cdot\left[0.11\cdot 1^a+k\cdot 0.31^a\right] \quad (9)$$

$$\alpha EU(R_2)=\alpha\cdot\left[0.1\cdot 5^a-k\cdot 1.5^a\right] +(1-\alpha)\cdot\left[0.1\cdot 5^a+k\cdot 1.5^a\right] \quad (10)$$

Since S_1 gives \$1M with certainty, the standard deviation is zero, which then reduces to the standard expected utility formulation. For the other three lotteries, the standard deviations are 1.21, 0.31, and 1.5, respectively. Figure 7 shows the combinations of parameters of the model (α,β) which allow for Allais type of behaviour.

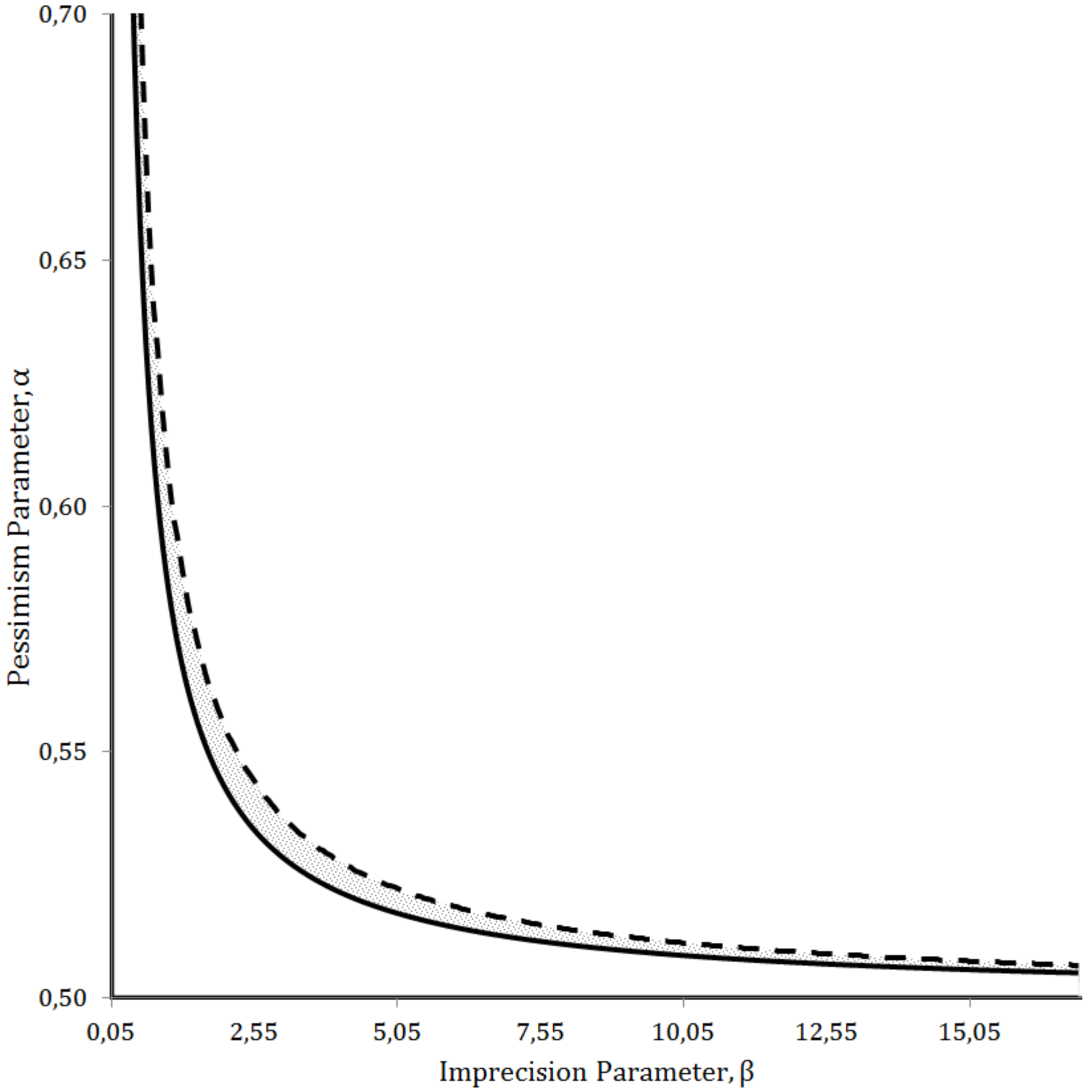
The vertical axis measures α values whereas the horizontal axis lists values for β. The solid curve shows the critical α values for the first task where the individual has to make a choice between S_1 and R_1, and, above this curve, S_1 is chosen over R_1. Second, the dashed curve shows the critical values for α in the second task where the individual has to make a choice between S_2 and R_2, and below this curve, R_2 is chosen over S_2. Thus, below the solid curve the individual prefers R_1 and R_2 in both tasks whereas above the dashed line the individual prefers S_1 and S_2 in both tasks. These regions include the combination of parameters, α and β, which result in consistent behaviour with EUT. On the other hand, the region between these two curves includes the parameter combinations that allow for the paradoxical behaviour: the individual prefers S_1 in the first task and R_2 in the second task. Overall, as the level of imprecision increases, the critical α value that allows for the Allais Paradox converges to 0.5.

Next we look at the indifference curves predicted by the model. When we substitute the expressions for $EU_1(X)$ and $EU_1(X)$ into (6) and simplify, we get:

$$\alpha EU(X)^*=EU(X)+(1-2\cdot\alpha)\cdot\beta\cdot u(\sigma) \quad (11)$$

[4] See the supplementary excel file for the calculations

Figure 7: Model parameters that that allows for Allais Paradox. Solid line shows the critical α values in the first task, whereas the dashed line shows the ones in the second task. Above the solid curve, S1 is chosen over R1, whereas above the dashed curve, S2 is chosen over R2



Similar to what we did while deriving the slope of the EUT indifference curves in Section 2.1, we simply take the derivative of equation (7) with respect to p_L and $\partial p_H / \partial p_L$ equals:

$$\frac{u(x_L)-u(x_M)+P\cdot\left[(x_L-EV)^2-(x_M-EV)^2\right]}{u(x_M)-u(x_H)-P\cdot\left[(x_H-EV)^2-(x_M-EV)^2\right]} \quad (12)$$

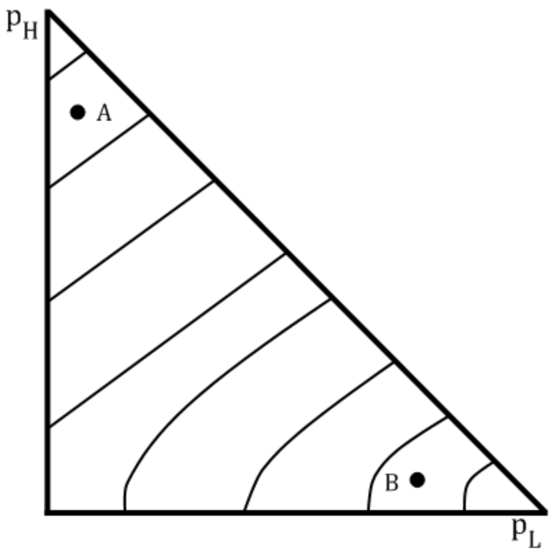
where $P=(0.5\cdot a\cdot\sigma^{a-2})\cdot(1-2\cdot\alpha)\cdot\beta$ and EV is the expected value of the lottery has the following usual formula: $EV=p_L\cdot x_L+p_M\cdot x_M+p_H\cdot x_H$.

Figure 8 shows the pattern of the indifference curves under the new model[5].

Indifference curves of the new model are in line with the findings of Harless and Camerer (1994): Inside the triangle they are parallel as in EUT, and they are nonlinear in the region that is closer to the horizontal edge of the triangle.

[5] An easy way to evaluate such a complicated slope expression is to use vector field method. Mathematica raw output can be found in Appendix.

Figure 8: Indifference curves of the new model



More interestingly, the new model seems to behave as a mix of Rank Dependent Utility Theory and EUT in different regions of the triangle: when we move towards the north-west of the triangle they behave like EUT indifference curves, straight and parallel lines as in Figure 2; whereas when look at the other half of the triangle we see the indifference curves showing a similar pattern to the ones predicted by Rank Dependent Utility Theory (See Figure 6). To understand the intuition behind this pattern, one should look at equation (11) and consider the two lotteries (A and B) in Figure 8 as an example: Both have equal standard deviations, since the model parameters α and β are same for an individual, the following are equal: $(1-2\cdot\alpha)\cdot k\cdot u(\sigma_A)$ and $(1-2\cdot\alpha)\cdot k\cdot u(\sigma_B)$. Since Lottery A assigns a higher probability to the highest consequence x_H than Lottery B, $EU(A)$ is higher than $EU(B)$. As we move towards the north-west of the triangle the standard expected utility of the lotteries $(EU(.))$ increases and outweighs the effect of standard of the standard deviation, $(1-2\cdot\alpha)\cdot\beta\cdot u(\sigma)$. However, when we are close to the horizontal edge the probability assigned for the highest outcome is lower, therefore $EU(.)$ is lower, and this makes the total $(\alpha EU(.))$ to be affected by standard deviation more.

4. Conclusion

This paper presents another explanation for Allais paradox: Previous theories such as Rank Dependent Utility Theory, Prospect Theory and its variants focus on nonlinear probability weighting to explain the paradox. However, emerging literature on preference imprecision challenges these alternative models too: These alternative models share an implicit common assumption which is the precision of preferences i.e., individuals can articulate their preferences confidently in a precise manner. The experimental studies reviewed in Section 2.3 shows that even literate and numerate individuals exhibit imprecision when stating their preferences (valuations and choices) especially for the risky prospects. To incorporate imprecision, researchers modelled the existing deterministic core theories in a stochastic manner by adding an error term to the deterministic part. However, none of these modelling approaches seem to explain a significant portion of the behaviour. Most importantly, literature on stochastic preferences suggests that a model which predicts indifference curves similar to EUT inside the probability triangle, but non-standard patterns for the region close to edges of the triangle would fit better. Secondly, experiments on preference imprecision shows that the imprecision range is proportional to the dispersion of lotteries. The model presented in this paper takes these into account; it predicts indifference curves in line with the evidence: If we draw an imaginary line (from the point where $p_L = p_H = 0$ to the hypotenuse) to divide the triangle into two pieces, in the upper half of the triangle where the effect of standard deviation becomes negligible, indifference curves are similar to the ones that EUT predicts; whereas in the lower part, the curves are similar to the ones that Rank Dependent Utility Theory. Note that the concern of this paper is to present a simple and parsimonious model which incorporates the recent findings of the literature. Central to the model; the satisfaction from the risky prospect depends on both the standard expected utility formulation and the dispersion of the risky prospect. Further extensions are possible, especially to overcome the violation of monotonicity for the lotteries which lie in the region that is close to the right bottom corner, where the disutility from standard deviation of the lottery $(1-2\cdot\alpha)\cdot k\cdot u(\sigma)$ might overweigh the expected utility part $EU(A)$, for pessimistic individuals which implies $\alpha > 0.5$ (See Equation 12). To overcome this problem: (i) one can assume that theory is undefined for this region or (ii) assume that the utility of the lotteries that lie in this region equals to the utility of the lowest outcome (winning nothing). Considering (i), it is not a major disadvantage of the model, since a widely influential model-Prospect Theory-is undefined for a much larger region (See Figure 2A in Appendix). The decision weight function is not defined for the probabilities close to zero and one. The intuition of (ii) is that the lotteries, which give the lowest outcome of zero with a probability close to one, are perceived as a degenerate lottery giving zero with a probability of one. For example, pessimistic individuals will think that if the probability of winning zero is higher than a certain probability such as 0.97, they will definitely win nothing from the lottery.

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Appendix

Below is the Mathematica code which I used to evaluate the slope equation (12) for different combinations of p_L and p_H .

Streamplot command in Mathematica plots streamlines that show the local direction of the vector field at each point. Figure 1A shows the raw output. Figure 8 is drawn according to this output.

Figure A1: Mathematica *Streamplot* Output

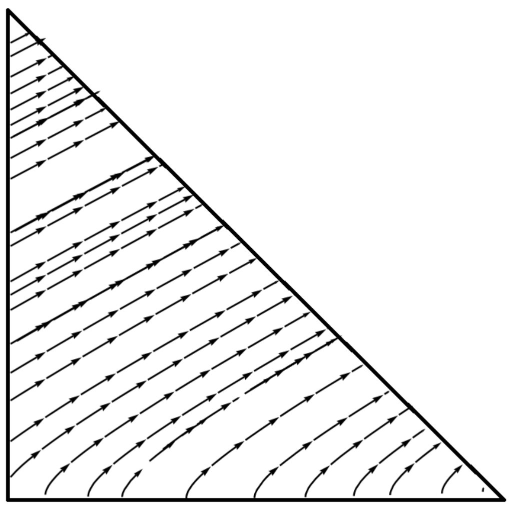

Figure A2: Propect Theory, dashed lines show the undefined region