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The "capabilities approach" due to Amartya Sen has become influential in the field of economic development, and it is up to a point the theoretical background of the Human Development Index. While the approach provides a rich conceptual framework to define the goals of development, its analysis of the means to achieve them seems lacking. Building on Kuklys and Robeyns interpretation of Sen's theory, I show how take a first step to link goals and means in the capabilities approach using a simple modified model grounded on standard growth theory.

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1. Introduction

Amartya Sen's "capabilities approach" is one of the most sophisticated elaborations of what the goals of development should be about (Sen, 1989). For Sen, what is most relevant is not the goods and services that a person has or may have, but what a person does or can do, is or may be. In Sen's terminology, what matters is the conversion of goods and services in personal achievements or "functionings". In addition, functionings are conditioned by the "capabilities" of a person, the freedom she has to choose between functionings.

The way to make this approach operational has been to consider three basic dimensions of human development: to enjoy a long and healthy life, to acquire knowledge and be creative, and to have a decent standard of living thanks to access to material resources (Anand and Sen, 2000). The first two dimensions refer directly to people's capabilities, while the third refers to their command over resources.

To measure those dimensions, empirical proxies are built by means of indices of health(I_H), education (I_E) and income(I_I). Then, the Human Development Index (HDI) is computed as the geometric mean of those indices (UNDP, 2010).

1.1 $$HDI = {I_H}^{1/3} \,.\; {I_E}^{1/3} \,.\; {I_I}^{1/3}$$

2. Extending Modern Welfare Analysis to Account for the Capabilities Approach

Following Sen (1985), a person's functionings $\boldsymbol{b}_i$ (that is, a person's activities or states of being) can be formalized as:

2.1 $$\boldsymbol{b}_i = f_i[c(\boldsymbol{q}_i), \boldsymbol{z}_i]$$

where:

$\boldsymbol{q}_i$ is a vector of commodities chosen by person i ;

$c(\boldsymbol{q}_i)$ is a function that transforms commodities into characteristics;

$\boldsymbol{z}_i$ are personal characteristics and societal and environmental circumstances;

$f_i[c(\boldsymbol{q}_i), \boldsymbol{z}_i]$ is a personal utilization function that converts commodities characteristics, given personal characteristics and circumstances, into a vector of functionings.

A number of refinements and extensions of Sen's formalization has been developed along the years (Basu and Lopez Calva, 2011). Particularly interesting is the formalization by Kuklys and Robeyns (2005). Building on the work of consumer theory pioneered by Becker (1965) and Atkinson and Stern (1981), Kuklys and Robeyns extend the standard welfare economics framework to account for Sen's theory. They define a utility function u over outcomes $\boldsymbol{o}$, which are a function of commodities $\boldsymbol{q}$ and of conversion factors $\boldsymbol{z}$ (defined as personal, societal and environmental factors that affect the conversion of available resources into outcomes).[1]

2.2 $$u[\boldsymbol{o}(\boldsymbol{q}, \boldsymbol{z}_i)]$$

The individual's problem is:

2.2 $$max\ u_i = u[\boldsymbol{o}(\boldsymbol{q}, \boldsymbol{z}_i)] \quad s.t. \quad \boldsymbol{p}.\boldsymbol{q} = m_i$$

and the resulting indirect utility function:

2.3 $$max\ v_i = v(\boldsymbol{p}, m_i, \boldsymbol{z}_i).$$

Thus, we can build the social welfare function:

2.4 $$W = W[v_1(\boldsymbol{p}, m_1, \boldsymbol{z}_i), \dots , v_n(\boldsymbol{p}, m_n, \boldsymbol{z}_i)].$$

Aggregating over incomes and conversion factors, and given prices, we obtain an aggregate social welfare function:

[1] Kuklys and Robeyns also include the following inputs to the utility function: public goods, rationed goods and non-market goods, as well as a "choice" variable to account for the "intrinsic value of choice" emphasized by Sen (1997). In order to keep things simple, and since the HDI does not include these inputs, I will not include them. As we will see later, I will assume that the conversion factors are variables related to health and education, to follow closely the specification of the HDI.

2.5 $$W = W(m, \boldsymbol{z})$$

where m is aggregate income and $\boldsymbol{z}$ are aggregate conversion factors.

To see how close these extensions of the standard welfare approach are to the Human Development approach, notice that the outcome function $\boldsymbol{o}$ used in 2.2 and the personal utilization functions f_i defined in 2.1 are analogous. Notice also that we can see aggregate conversion factors $\boldsymbol{z}$ in 2.5 as aggregate levels of health and education. Therefore, we can see the HDI in 1.1 as a particular functional form of the welfare function in 2.5.[2]

From a policy point of view, a welfare function is a set of policy targets. To perform policy analysis, we need a model of the dynamic interactions generating the time evolution of the target variables included in the welfare function. In Human Development terminology, we need a model of the way in which resources and functionings are transformed into resources and functionings. In other words, we need an explicit representation of the intertemporal interactions between command over resources (income) and functionings (health and education), something that so far is not provided by the Human Development paradigm, since it is mostly a theory of the goals of development.

3. A Simple Expanded Standard Intertemporal Model

Today's canonical form of modeling intertemporal dynamic interactions of growth processes is by means of Ramsey-Cass-Koopmans type models. An example of this kind of models, useful for our discussion since it includes not only physical capital but also two specific forms of human capital (health and educational capital), is the following one (Barro, 1996):[3]

3.1 $$\text{Max } W = \int_0^\infty u(c)\, \mathrm{e}^{\rho t} \; dt \qquad s.t.$$

3.2 $$y = a\, f(k, h, e, l)$$

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[2] For a discussion of other issues implicit in the interpretation of the HDI as a welfare function, see Mercado (2013).

[3] Time sub-indices are not shown to save notation.

3.3 $$y = c + i_k + i_h + i_e$$

3.4 $$\dot{k} = \delta k + i_k$$

3.5 $$\dot{h} = d h + i_h$$

3.6 $$\dot{e} = d e + i_e$$

where W is welfare; u is instantaneous utility; c is consumption; ρ is the time preference; y is income; a is technological progress; k is physical capital; h is health capital (measured, for instance, by life expectancy); e is educational capital (measured, for instance, by school enrolment); i is investment; l is labor; δ is the depreciation parameter for physical capital; d is the depreciation parameter for health and educational capital; and where the model is subject to suitable initial and transversality conditions.

The depreciation parameter of physical capital is standard. However, the depreciation parameter for education and health depends on the mortality rate and the burden of disease, since high mortality and disease burden rates will deteriorate the stock of health and educational capital more quickly.

For the Human Development approach, health and education, as proxies for sets of capabilities, are not only means to achieve human development, but also goals with intrinsic value. This is not captured by a standard model such as the one presented earlier. In that model, health and education are only means to make possible higher levels of income and consumption. However, we can deal with this limitation by adding health (h) and education (e) as arguments into the intertemporal welfare function:

3.7 $$W = \int_0^{\infty} u(c, h, e) \ \mathrm{e}^{\rho t} \ dt$$

In what follows, I will show how these changes affect some of the analytical results usually obtained for standard models. I will also provide a simulation example.[4]

To solve 3.7 subject to 3.1-3.6, given initial and transversality conditions, we form de Lagrangian:

3.8 $L = u(c) + \mu_k\,(i_k - \delta_k) + \mu_h\,(i_h - d_h) + \mu_e\,(i_e - d_e) + w_1(y - c - i_k + i_h + i_e)$

From the first order conditions, we obtain the following Euler equations:

3.9 $$\frac{\dot{u}_c}{u_c} = \delta_k + \rho + y_k'$$

3.10 $$\frac{\dot{u}_c}{u_c} + \frac{u_h'}{u_c} = d + \rho + y_h'$$

3.11 $$\frac{\dot{u}_c}{u_c} + \frac{u_e'}{u_c} = d + \rho + y_e'$$

Notice that the equation for k (3.9) is the same as in standard models without h and e in the welfare function. However, the equations for h and e (3.10 and 3.11) are different: each contains an extra term: $\frac{u_h'}{u_c}$ in the h equation and $\frac{u_e'}{u_c}$ in the e equation.

Now assume that the utility function takes the form of the HDI:[5]

3.11 $$W = \int_0^{\infty} (c^{1/3}\, h^{1/3}\, e^{1/3})\ \mathrm{e}^{\rho t}\ dt$$

Assume also that de production function is a Cobb-Douglass function with constant returns to scale:

[4] The model uses a time discount factor in the intertemporal welfare function, something that may be a matter of controversy, as in fact it is since the early classic discussion by Ramsey.

[5] In what follows, all variables are in per capita terms.

3.12 $$y = k^{\alpha} h^{\beta} e^{\gamma}$$

Solving for the steady state of the model, we obtain the following system of equations:

3.13 $$\alpha\, k^{\alpha-1}\, h^{\beta}\, e^{\gamma} = \rho + \delta$$

3.14 $$k^{\alpha}\beta\, h^{\beta-1} e^{\gamma} + \frac{c}{h} = \rho + d$$

3.15 $$k^{\alpha}\beta\, h^{\beta} e^{\gamma-1} + \frac{c}{e} = \rho + d$$

3.15 $$k^{\alpha} h^{\beta} e^{\gamma} = c + \delta k + dh + de$$

I now parameterize the model with the following values:

$\rho = 0.05 \quad \alpha = 0.3 \quad \beta = 0.25 \quad \gamma = 0.1 \quad \delta = 0.1 \quad d = 0.05$

Solving for the steady state of the models with and without h and e in the welfare function, we obtain:

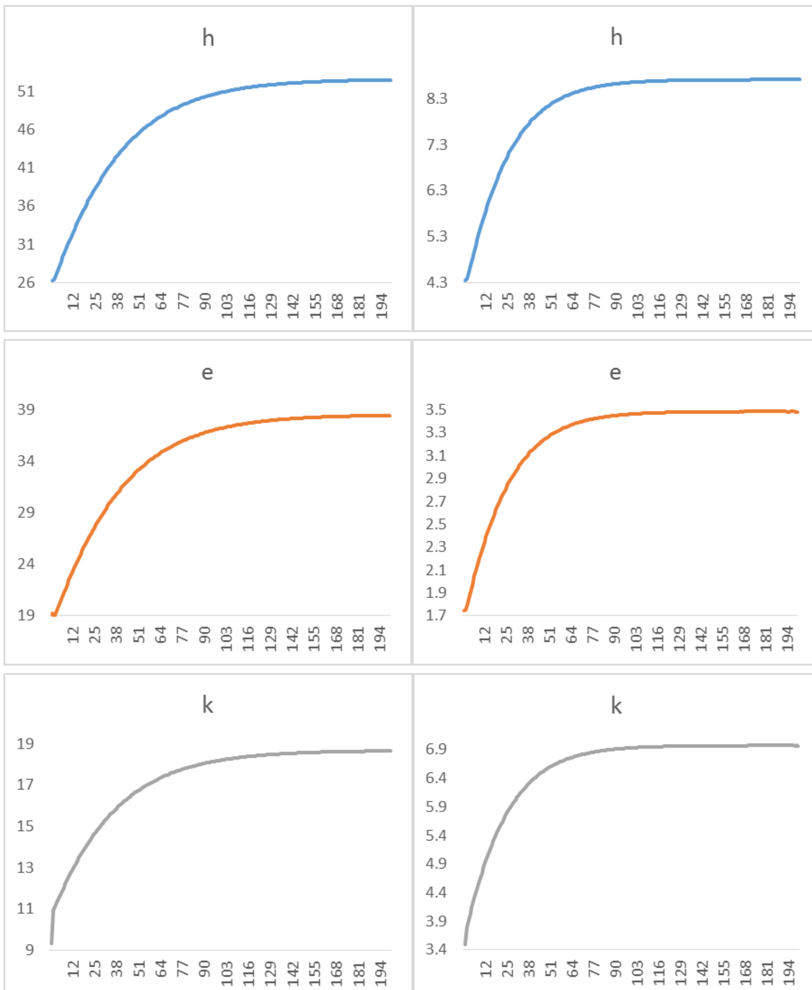
	With **h and e**	**Without** **h and e**
h	52.5	8.7
e	38.5	3.5
k	18.7	7.0

We see that the inclusion of h and e in the welfare function implies higher steady state values for these variables, and even for k.[6] Finally, I simulate the models for 200 periods,

[6] If we include population growth and labor augmenting technical progress, higher steady-state values will not affect the model's growth rates. In the steady state, all variables in both models (with and without h and e in

starting from initial values 50% lower than the steady state values. In the graphs below, we can see that the speed of convergence to the steady state is slower in the model with h and e in the welfare function, than in the model without them.[7]

With h and e **Without h and e**



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the welfare function) will grow at the same rate, equal to the rate of growth of population plus the rate of growth of technical progress.

[7] For a model with only c and k in the welfare function, it can be shown analytically (Duczynski, unpublished) that the steady state is higher and the speed of convergence slower than for a model with only c in the welfare function. For a model like the one in this paper, with c, h and e in the welfare function, an analytical demonstration would be much more difficult, if it is indeed possible.

4. Conclusions

The Human Development paradigm is a sophisticated view of the goals of development. However, it is silent with respect to the means to achieve them in a systematic way, something problematic from a policymaker's point of view. Standard growth models can be extended, going beyond consumption, to account for Human Development goals such as health and education. In addition, they can link those goals to a set of means in a systematic and measureable way. I showed in this paper how to take a first step in that direction.

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