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Abstract

In this paper we find empirical evidence of a new smirk factor, obtained from the jump structure of the risk neutral distribution of the underlying Lévy process. As an application we show how to price a barrier style contract.

Keywords: Skewness; Lévy processes; Implied volatility smirk.

JEL Classification: C52; G10

1 Introduction

Since Black and Scholes (1973), many attempts to capture the real behavior of the implied volatility have been realized. The most well known facts are the volatility smile and smirk, it shows that depending on moneyness and maturity we can observe a determined behavior. As for example the fact that out-of-the money put options in equity markets are more expensive than the corresponding out-of-the money call options, this fact has been extensively address by many authors, among them we have the work of Foresi and Wu (2005) whom establish the

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above fact for a large data set of option prices for equity indexes in twelve countries. Also, Carr and Wu (2003) analyze the pattern of implied volatility smirk across maturities using S&P500 index options. Their findings implies an asymmetric risk neutral distribution for the index.

On the other hand, the relationship between the implied volatility symmetry and the market symmetry, also known as put-call symmetry has been recently established by Fajardo and Mordecki (2006) and Carr and Lee (2009) for Lévy process and local and stochastic volatility models, respectively. Also, Fajardo and Mordecki (2014) have shown the relationship among the skewness premium and the market symmetry parameter. More recently, Fajardo (2015) has shown how to price some barrier contracts using symmetry properties.

In this paper focusing on pure jump Lévy process with exponential dampening controlling the skewness we propose a new smirk factor to explain the implied volatility smirk. We test our specification using S&P500 options data, obtaining a very good fit. Although, there is in the literature more general data-generating process. including stochastic volatility models, by focusing on a particular class we can learn a bit more insights about how this particular process generates the skew. More exactly, the market symmetry parameter is deeply connected with the risk neutral excess of kurtosis, which allow us to relate the risk neutral skewness and kurtosis with the implied volatility skew. In that sense, Tédongap, Feunou, and Fontaine (2009) also tries to relate skewness and excess kurtosis of the risk neutral distribution with the skewness of the implied volatility, but instead of suggesting another factor they use a quadratic model, similar to the one used by Foresi and Wu (2005) but with non constant parameters.

Also, we show how to price digital call options using our specification. This allow us to consider any asymmetric dynamic, in the set of Lévy process described above, and any moneyness, extending in this ways findings of Fajardo (2015). Although it is well known that given plain vanilla call or put prices at sufficiently many strikes, the prices of this kind of barrier contract can be obtained as limits of combinations of such call/put prices, without needing any model at all. We think that our application can be useful from a practical point of view since it can help to understand better the relationship between digital call option prices and implied volatility slopes, and from a regulatory point of view it can be used to compute probabilities of trigger events, such as the probability of a stock price cross a determined barrier at the end of some given period of time. The calculation of such probabilities are needed for example in the pricing of some kinds of CoCo bonds, see Schoutens and De Spiegeleer (2011) for a deep discussion.

The paper is organized as follows. in Section 2 we introduce our model. In Section 3 we present our specification. In Section 4 we describe our sample. In Section 5 we present the main results. In Section 6 we present an application and last section concludes.

2 Lévy Market Model

Consider a real valued stochastic process $X = \{X_t\}_{t\geq 0}$, defined on a stochastic basis $\mathcal{B} = (\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$, being càdlàg, adapted, satisfying $X_0 = 0$ and such that for $0 \leq s < t$ the random variable $X_t - X_s$ is independent of the σ-field $\mathcal{F}_s$, with a distribution that only depends on the difference $t - s$. Assume also that the stochastic basis $\mathcal{B}$ satisfies the usual conditions (see Jacod and Shiryaev (1987)). The process X is a Lévy process, and is also called a process with stationary independent increments. For general reference on Lévy processes see Jacod and Shiryaev (1987). Skorokhod (1991). Bertoin (1996). Sato (1999). For Lévy process in Finance see Boyarchenko and Levendorskiĭ (2002), Schoutens (2003) and Cont and Tankov (2004).

In order to characterize the law of X under $\mathbb{Q}$ consider for $q \in \mathbb{R}$ the Lévy-Khinchine formula, that states

$$\mathbf{E}\, e^{iqX_t} = \exp\Big\{t\Big[iaq - \frac{1}{2}\sigma^2 q^2 + \int_{\mathbb{R}} \big(e^{iqy} - 1 - iqh(y)\big)\Pi(dy)\Big]\Big\}, \tag{1}$$

with

$$h(y) = y\mathbf{1}_{\{|y|<1\}},$$

a fixed truncation function, a and $\sigma \geq 0$ real constants. and Π a positive measure on $\mathbb{R}\setminus\{0\}$[1] such that $\int(1 \wedge y^2)\Pi(dy) < +\infty$, called the *Lévy measure*. The triplet (a, σ^2, Π) is the *characteristic triplet* of the process and completely determines its law.

Consider the set

$$\mathbb{C}_0 = \Big\{z = p + iq \in \mathbb{C}\colon \int_{\{|y|>1\}} e^{py}\Pi(dy) < \infty\Big\}. \tag{2}$$

The set $\mathbb{C}_0$ is a vertical strip in the complex plane, contains the line $z = iq$ ($q \in \mathbb{R}$), and consists of all complex numbers $z = p+iq$ such that $\mathbf{E}\, e^{pX_t} < \infty$ for some $t > 0$. Furthermore, if $z \in \mathbb{C}_0$, we can define the *characteristic exponent* of the process X. by

$$\psi(z) = az + \frac{1}{2}\sigma^2 z^2 + \int_{\mathbb{R}} \big(e^{zy} - 1 - zh(y)\big)\Pi(dy) \tag{3}$$

this function ψ is also called the *cumulant* of X. having $\mathbf{E}\,|e^{zX_t}| < \infty$ for all $t \geq 0$, and $\mathbf{E}\, e^{zX_t} = e^{t\psi(z)}$. The finiteness of this expectations follows from Theorem 25.3 in Sato (1999). For $t = 1$, formula (3) reduces to exponent of eq. (1) with $\mathrm{Re}(z) = 0$.

By a *Lévy market* we mean a model of a financial market with two assets: a deterministic

[1] $\Pi(\{0\})$ could be defined as 0. Here we follows Cont and Tankov (2004).

savings account $B = \{B_t\}_{t\geq 0}$, with

$$B_t = e^{rt}, \qquad r \geq 0,$$

where $B_0 = 1$ for simplicity and a stock $S = \{S_t\}_{t\geq 0}$, modelled by

$$S_t = S_0 e^{X_t}, \qquad S_0 = e^x > 0, \tag{4}$$

where $X = \{X_t\}_{t\geq 0}$ is a Lévy process.

In this model we assume that the stock pays dividends, with constant rate $\delta \geq 0$, and that the given probability measure $\mathbb{Q}$ is the chosen equivalent martingale measure. In other words, prices are computed as expectations with respect to $\mathbb{Q}$, and the discounted and reinvested process $\{e^{-(r-\delta)t} S_t\}$ is a $\mathbb{Q}$-martingale.

In terms of the characteristic exponent of the process this means that

$$\psi(1) = r - \delta, \tag{5}$$

based on the fact that $\mathbf{E}\, e^{-(r-\delta)t + X_t} = e^{-t(r-\delta-\psi(1))} = 1$, and condition (5) can also be formulated in terms of the characteristic triplet of the process X as

$$a = r - \delta - \sigma^2/2 - \int_{\mathbb{R}} \left(e^y - 1 - \mathbf{1}_{\{|y|<1\}}\right)\Pi(dy). \tag{6}$$

Then,

$$\psi(z) = z(r - \delta - \frac{\sigma^2}{2}) + z^2\frac{\sigma^2}{2} + \int_{-\infty}^{+\infty} [z(1-e^y) + (e^{zy} - 1)]\Pi(dy) \tag{7}$$

Henceforth, we denote this exponent by ψ_β due to its future dependence on parameter β of our jump structure.

2.1 Market Symmetry

Here we use the symmetry concept introduced in Fajardo and Mordecki (2006). We define a Lévy market to be *symmetric* when the following relation holds

$$\mathcal{L}\big(e^{-(r-\delta)t+X_t} \mid \mathbb{Q}\big) = \mathcal{L}\big(e^{-(\delta-r)t-X_t} \mid \tilde{\mathbb{Q}}\big), \tag{8}$$

meaning equality in law. Here $\tilde{\mathbb{Q}}$ is defined by $d\tilde{\mathbb{Q}}_t = e^x d\mathbb{Q}_t$. where $\tilde{\mathbb{Q}}_t$ and $\mathbb{Q}_t$ denotes the restrictions of $\tilde{\mathbb{Q}}$ and $\mathbb{Q}$ to $\mathcal{F}_t$, respectively. As Fajardo and Mordecki (2006) pointed out, a necessary and sufficient condition for (8) to hold is

$$\Pi(dy) = e^{-y}\Pi(-dy). \tag{9}$$

In Lévy markets with jump measure of the form

$$\Pi(dy) = e^{\beta y}\Pi_0(dy). \tag{10}$$

where $\Pi_0(dy)$ is a symmetric measure, i.e. $\Pi_0(dy) = \Pi_0(-dy)$ and β is a parameter that describe the asymmetry of the jumps, everything with respect to the risk neutral measure $\mathbb{Q}$.

As a consequence of (9). Fajardo and Mordecki (2006) found that the market is symmetric if and only if $\beta = -1/2$.

Recently, De Oliveira, Fajardo, and Mordecki (2015) and Gerhold and Gülüm (2014) proved that $\beta \gtrless -1/2 \Leftrightarrow \frac{\partial \sigma_{imp}(0,\beta)}{\partial x} \gtrless 0$, locally (any maturity) and globally (small maturity), respectively. With this evidence in mind we propose our new smirk factor in the next section.

3 New Smirk Factor

The quadratic implied volatility approximation presented by Foresi and Wu (2005) and Zhang and Xiang (2008) test the below quadratic approximation[2].

$$\sigma_{imp}(x_i) = \gamma_0 + \gamma_1 d_i + \gamma_2 d_i^2 + e_i, \tag{11}$$

where $d_i = \frac{x_i}{\bar{\sigma}\sqrt{T}}$, is the standarized moneyness, $\bar{\sigma}$ is an average volatility and e_i is a normal distributed error. They called γ_0, γ_1 and γ_2, level, slope and curvature, respectively. We introduce a new factor that we call *torsion*[3] and propose to test the following specification:

$$\sigma_{imp}(x_i) = \gamma_0 + \gamma_1 d_i + \gamma_2 d_i^2 + \gamma_3 (d_i + 1)^{\beta+0.5} + e_i, \tag{12}$$

With this specification we capture previous findings that relate the at-the-money volatility slope with the $\beta - 0.5$ sign, when $\gamma_1 = 0$ and $\gamma_3 > 0$.

Also, to avoid negative values with squares exponents we consider only options with $d > -1$.

4 Sample

To test our specification we use options on SP500 from Bloomberg quoted on a randomly picked date 12/01/2011. To estimate the quadratic curve proposed by Zhang and Xiang (2008), the lowest strike is selected from the first out-of-the-money put with non-zero bid price. The highest price is selected from the first out-of-the-money call with non-zero bid price, see Table (1). Also, the call and put prices are the mid-value of closing bid and ask

[2] Henceforth FW.

[3] The intuition for this name cames from the fact that when we plot implied volatility vs. moneyness and β we can obtain a figure with a torsion, as in Fig. 1, build using Brazilian asset Petróbras (PETR4).

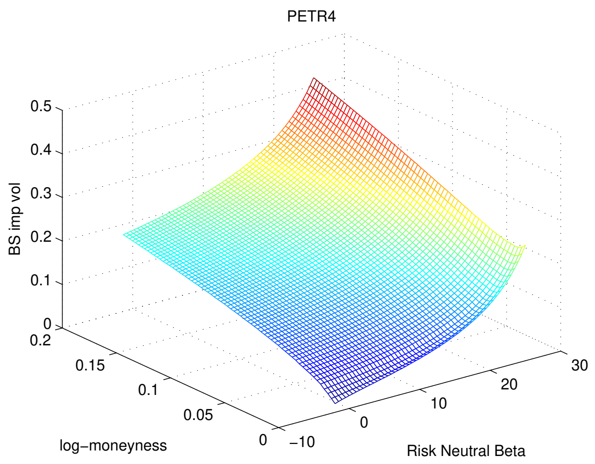

Figure 1: BS implied vol vs. Log-moneyness vs. Risk Neutral Beta

prices. The closing index on 12/01/2011 was $S_0 = 1.444.23$.

[Table (1) about here]

We use the closing VIX of 12/01/2011[4], as a proxy for the average volatility that is $\bar{\sigma} = 27.41\%$. The resulting implied volatility term structure is presented in figure (2).

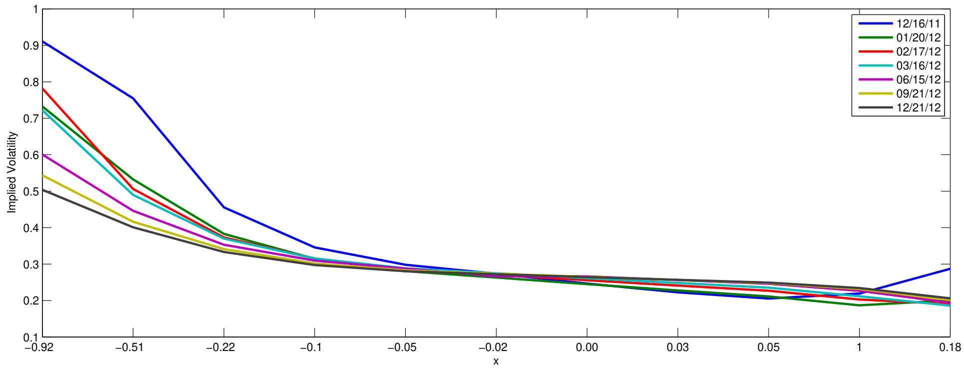

Figure 2: Implied Volatility Term Structure

For each maturity, the implied forward price F_0 is determined based on at-the-money option price using the following formula

$$F_0 = Strike\ price + exp(rT)(Call\ price - Put\ price)$$

where r is the interest risk-free rate determined by the U.S. treasury bill yield curve rates on December 1, 2011. A linear extrapolation technique is used to calculate the relevant rate for the different maturities. The resulting sample is presented in Table (2) below.

[Table (2) about here]

Remember that in order to include the torsion factor we will need to restrict d to be higher than -1, resulting in less options as presented in Table (3). In FW case it is not necessary.

[4]Taken from http://www.cboe.com/data/mktstat.aspx

[Table (3) about here]

5 Results

The Market Symmetry parameter (β) was estimated for two particular processes: the normal inverse Gaussian (NIG) and the generalized hyperbolic (GH) process, we made this choice since these models have shown a very good fit with financial returns, see Eberlein and Prause (2002) and Fajardo and Farias (2004). Also, two spans for the daily return (2 and 5 years) were considered.

We consider daily returns and implied volatilities of S&P500 extracted from Bloomberg. Then, we consider the sample periods: 12/01/2009 to 12/01/2011 and 12/01/2006 to 12/01/2011. As, we need the risk-neutral parameters, we use the density given by the Esscher Transform. To compute this density we need the interest rate so we use the interest rate given by the U.S. Treasury on that date 12/01/2011. $r = 0.0012$. Under this transformation we obtain four possible β, presented in Table (4) below.

[Table (4) about here]

For each maturity we calculate the FW specification and the proposed specification (with each of the four β). For almost all estimated β's the Torsion factor is statistically significant, mainly for the most liquid maturities, see Tables (5) to (9). Also, the FW model is dominated in the adjusted R^2 criteria by the specifications which include a torsion factor.

[Tables (5) to (9) about here]

5.1 Implied volatility shapes

Now we propose a criteria to choose a β in terms of more significant values, excellent R^2 and parameter interpretation, for the most liquid options. Then we have $\beta = -1.998$.[5] The other risk-neutral parameters are given by

$$(\mu, \alpha, \delta, \beta) = (0.0016, 31.66, 0.0089, -1.998). \tag{13}$$

The resulting implied volatility approximations are given in Table (10).

[Table (10) about here]

Another good choice in terms of R^2 can be $\beta = -0.0397$ estimated for the GH model, but we loose the interpretation of the factors, for example the first factor can not be understood as a realistic initial level of implied volatility. It is important to mention that we are not claiming to have the specification with the best fit among all the possible specifications.

6 Application: Digital Call Option

In our Lévy market, introduced in Section 2, consider a European style digital call option with maturity T and barrier K_x, i.e. at maturity derivative pays off $f(y) = 1_{\{e^y \geq K_x\}}$. We can take $y = X_T + \log(S_0) + (r - \delta)T$. then $f(X_T) = 1_{\{e^{X_T} \geq e^x\}}$.

Now denote by $I(\beta, x)$ the integral defined by:

$$I(x, \beta) = -\frac{e^{-rT}}{2\pi} \int_{iv+\mathbb{R}} e^{izx} \frac{1}{iz} e^{T\psi(-z)} dz. \forall x. \;\; v > 0. \tag{14}$$

which is the price of the introduced digital call option at log-moneyness x and symmetry

[5] It is also important to mention that this choice of β allow us to guarantee $\sigma_{imp} > 0$ for the sample set of moneyness.

parameter β. This integral can be computed using Fast Fourier transformation techniques. Instead we will use the implied volatility approximation presented in this paper, to this end observe that $I(x,\beta) = e^{-rT}E^{\mathbb{Q}}(1_{\{X_T \geq x\}}) = e^{-rT}\mathbb{Q}(X_T \geq x)$. Henceforth, we will denote this price by f_0.

In Fajardo (2015) the above integral is computed for the case $\beta = -0.5$ and then it is used as a short-cut to price some barrier style contracts under a particular set of moneyness and asymmetric dynamics that are transformed into symmetric ones. Here we can consider any dynamic (any β) of our set of Lévy processes without the need of such transformation.

Now let BS denote the price of a call option under Black and Scholes model and V the respective price under our Lévy market model, then

$$\frac{\partial V(x,\sigma_{imp}(x,\beta))}{\partial x} = \frac{\partial BS(K_x,\sigma_{imp}(x,\beta))}{\partial x} = -N(d_2(x))K_x e^{-rT} + \frac{\partial BS(K_x,\sigma_{imp}(x,\beta))}{\partial \sigma}\frac{\partial \sigma_{imp}(x,\beta)}{\partial x},$$

using the fact that[6]

$$\frac{\partial V(x,\sigma_{imp}(x,\beta))}{\partial x} = -K_x e^{-rT}\mathbb{Q}(S_T \geq K_x),$$

and with the BS model vega, we obtain:

$$e^{-rT}\mathbb{Q}(S_T \geq K_x) = e^{-rT}N(d_2(x)) - e^{-(r-\delta)T-x}\sqrt{T}N'(d_1(x))\frac{\partial \sigma_{imp}(x,\beta)}{\partial x},$$

with $d_1(x) = d_2(x) + \sigma_{imp}\sqrt{T}$ and $d_2(x) = -\frac{x+\frac{\sigma_{imp}^2 T}{2}}{\sigma_{imp}\sqrt{T}}$, for shorten notation we use σ_{imp} instead of $\sigma_{imp}(x,\beta)$. The left hand side is equal to the integral value in (14).

Then using specification (12) we obtain the following approximation for the digital call option price

[6] Here we need the law of S_T be absolutely continuous. The absolute continuity holds in all Lévy models of interest, see Theorem 27.4 in Sato (1999).

$$I(\beta, x) \approx e^{-rT} N(d_2(x)) - e^{-(r-\delta)T - x - \frac{d_1(x)^2}{2}} \left[\bar{\gamma_1} + 2\bar{\gamma_2}(\frac{x}{\bar{\sigma}\sqrt{T}}) + (\beta + 0.5)\bar{\gamma_3}(\frac{x}{\bar{\sigma}\sqrt{T}} + 1)^{\beta - 0.5} \right], \tag{15}$$

with $\bar{\gamma_i} = \frac{\gamma_i}{\bar{\sigma}\sqrt{2\pi}}, i = 1.2.$

Now we can price our call digital options, as jumps are more important for short-maturity options we obtain the price for the first fourth maturities, the results are presented in Table (11).

[Table (11) about here]

If we compare the prices obtained using Monte Carlo simulation[7] we can see that our prices produce prices for the first fourth maturities, where the number of options is near to 30, near to the ones obtained using Monte Carlo simulation. As it is known in the literature the skew due to jumps is stronger in short maturities[8]. The comparison of prices for near ATM contracts is presented in Table (12).

[Table (12) about here]

If we estimate the generalized hyperbolic model (GH) we would have $\beta = -0.0397$ and we have obtained similar results, see Table (13).

[Table (13) about here]

7 Conclusions

We find empirical evidence of a new factor to explain the implied volatility smirk. This new factor that we called *t*orsion factor, considers the skewness observed on the jump risk neutral distribution. As expected this factor has most of the time a positive impact on the implied

[7]We simulate realizations of a NIG distribution with the estimated parameters.
[8]see Table 15.2 in Cont and Tankov (2004).

volatility skew.

As an application we show how to use this implied volatility specification to price a call digital option, from this price it is easy to obtain the probability that the stock price cross a fixed barrier at the end of a period. It would be interesting to extend this result to other kind of barriers such as the ones considered by Corcuera, Fajardo, Schoutens, Jonsson, Spiegeleer, and Valdivia (2014) and Corcuera, Fajardo, Schoutens, and Valdivia (2016) to model contingent convertibles.

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Table 1: Log-Moneyness x for each Maturity and Strike

T	F	497.836	746.754	995.762	1120.1	1182.4	1213.5	1244.6	1275.7	1306.8	1369	1493.5
01/20/2012	1242.06	-0.9143	-0.5088	-0.2210	-0.1034	-0.0492	-0.0233	0.0020	0.0267	0.0508	0.0973	0.1844
02/17/2012	1239.90	-0.9125	-0.5071	-0.2193	-0.1016	-0.0475	-0.0215	0.0038	0.0285	0.0526	0.0990	0.1861
03/16/2012	1237.97	-0.9110	-0.5055	-0.2177	-0.1001	-0.0459	-0.0200	0.0053	0.0300	0.0541	0.1006	0.1876
06/15/2012	1232.28	-0.9064	-0.5009	-0.2131	-0.0954	-0.0413	-0.0154	0.0099	0.0346	0.0587	0.1052	0.1923
09/21/2012	1226.62	-0.9017	-0.4963	-0.2085	-0.0908	-0.0367	-0.0108	0.0146	0.0392	0.0633	0.1098	0.1969
12/21/2012	1221.18	-0.8973	-0.4918	-0.2041	-0.0864	-0.0323	-0.0063	0.0190	0.0437	0.0678	0.1143	0.2013

Table 2: Option Data

Maturity	T	F	σ_{imp}^{ATM}	r	δ
12/16/2011	0.0411	1242.87	24.67%	0.208%	1.619%
01/20/2012	0.1370	1242.06	24.52%	0.344%	1.826%
02/17/2012	0.2137	1239.90	25.58%	0.454%	2.218%
03/16/2012	0.2904	1237.97	26.16%	0.498%	2.328%
06/15/2012	0.5397	1232.28	26.64%	0.540%	2.367%
09/21/2012	0.8082	1226.62	26.52%	0.580%	2.358%
12/21/2012	1.0575	1221.18	26.45%	0.604%	2.370%

Source: Bloomberg

Table 3: Standarized log-Moneyness d for each maturity and Strike

T	1182.4	1213.5	1244.6	1275.7	1306.8	1369	1493.5
01/20/2012	-0.4852	-0.2293	0.0201	0.2634	0.5008	0.9592	1.8172
02/17/2012	-0.3747	-0.1699	0.0299	0.2246	0.4147	0.7817	1.4686
03/16/2012	-0.3109	-0.1352	0.0362	0.2032	0.3663	0.6811	1.2704
06/15/2012	-0.2052	-0.0763	0.0494	0.1720	0.2916	0.5225	0.9547
09/21/2012	-0.1490	-0.0436	0.0591	0.1592	0.2570	0.4457	0.7989
12/21/2012	-0.11449	-0.02238	0.067394	0.154953	0.240404	0.405368	0.714164

Table 4: Estimated Market Symmetry parameters (β)

	5 years	2 years
GH	-0.0397	-0.2210
NIG	-1.998	-4.1792

Table 5: Maturity in x days -Weighted volume. $\beta = -0.5$ (FW)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	mat15	mat50	mat78	mat106	mat197	mat295	mat386
Level	0.2291***	0.2195***	0.2388***	0.2433***	0.2459***	0.2445***	0.2392***
	(0.0009)	(0.0007)	(0.0003)	(0.0003)	(0.0001)	(0.0005)	(0.0005)
Slope	-0.0400***	-0.0494***	-0.0609***	-0.0713***	-0.0670***	-0.0803***	-0.0661***
	(0.0016)	(0.0016)	(0.0014)	(0.0014)	(0.0009)	(0.0018)	(0.0019)
Curvature	0.0135***	0.0142***	0.0095***	0.0119***	0.0003	0.0207*	-0.0012
	(0.0012)	(0.0012)	(0.0015)	(0.0019)	(0.0011)	(0.0067)	(0.0029)
N	40	46	36	35	20	12	12
r2_a	0.9396	0.9631	0.9950	0.9948	0.9992	0.9950	0.9930
F	304.3285	587.7539	3450.6704	3248.3396	11233.2848	1094.0280	782.7124
ll	157.3740	193.4916	181.9909	189.6965	127.4655	63.8486	62.7464

Standard errors in parentheses

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$

Table 6: Maturity in x days -Weighted volume, $\beta = -0.0397$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	mat15	mat50	mat78	mat106	mat197	mat295	mat386
Level	0.0928***	0.0669	-0.1476*	0.1289	0.3389***	1.7178***	0.1309*
	(0.0183)	(0.0338)	(0.0585)	(0.0747)	(0.0531)	(0.1908)	(0.0493)
Slope	-0.1162***	-0.1303***	-0.2409***	-0.1256**	-0.0245	0.6063***	-0.1227**
	(0.0092)	(0.0169)	(0.0272)	(0.0347)	(0.0255)	(0.0889)	(0.0258)
Curvature	0.0336***	0.0326***	0.0432***	0.0241**	-0.0078	-0.1318***	0.0187
	(0.0020)	(0.0035)	(0.0052)	(0.0071)	(0.0062)	(0.0199)	(0.0094)
Torsion	0.1376***	0.1546***	0.3873***	0.1148	-0.0930	-1.4749***	0.1086
	(0.0184)	(0.0341)	(0.0586)	(0.0749)	(0.0532)	(0.1910)	(0.0494)
N	24	33	34	34	19	12	12
r2_a	0.9910	0.9796	0.9979	0.9960	0.9996	0.9993	0.9951
F	843.0112	512.0774	5218.5720	2736.0443	15958.7152	5498.9105	745.7024
ll	123.2410	152.9147	187.3914	189.4751	129.6487	76.6545	65.5822

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 7: Maturity in x days -Weighted volume. $\beta = -1.998$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	mat15	mat50	mat78	mat106	mat197	mat295	mat386
Level	0.2303***	0.2209***	0.2482***	0.2459***	0.2428***	0.1939***	0.2401***
	(0.0006)	(0.0008)	(0.0024)	(0.0020)	(0.0013)	(0.0061)	(0.0005)
Slope	-0.0520***	-0.0572***	-0.0764***	-0.0766***	-0.0618***	0.0021	-0.0740***
	(0.0019)	(0.0027)	(0.0040)	(0.0036)	(0.0029)	(0.0099)	(0.0034)
Curvature	0.0229***	0.0198***	0.0189***	0.0165***	-0.0034	-0.0464***	0.0099
	(0.0015)	(0.0019)	(0.0027)	(0.0030)	(0.0027)	(0.0084)	(0.0048)
Torsion	-0.0006**	-0.0004	-0.0088***	-0.0023	0.0031*	0.0490***	-0.0007*
	(0.0001)	(0.0003)	(0.0022)	(0.0019)	(0.0012)	(0.0059)	(0.0003)
N	24	33	34	34	19	12	12
r2_a	0.9800	0.9685	0.9966	0.9959	0.9997	0.9994	0.9957
F	376.2852	328.8694	3256.0732	2663.4758	18906.0418	6257.3355	859.9674
ll	113.6776	145.7776	179.3921	189.0198	131.2582	77.4294	66.4348

Standard errors in parentheses

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$

Table 8: Maturity in x days -Weighted volume. $\beta = -0.221$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	mat16	mat51	mat79	mat107	mat198	mat296	mat387
Level	0.0930***	0.0701	-0.1883**	0.1195	0.3517***	1.8972***	0.1291*
	(0.0198)	(0.0362)	(0.0674)	(0.0832)	(0.0582)	(0.2126)	(0.0489)
Slope	-0.0911***	-0.1012***	-0.1821***	-0.1075***	-0.0377*	0.3891***	-0.1036***
	(0.0064)	(0.0115)	(0.0192)	(0.0235)	(0.0173)	(0.0604)	(0.0167)
Curvature	0.0312***	0.0297***	0.0387***	0.0226**	-0.0070	-0.1155***	0.0168
	(0.0019)	(0.0031)	(0.0047)	(0.0063)	(0.0055)	(0.0177)	(0.0084)
Torsion	0.1373***	0.1513***	0.4280***	0.1242	-0.1058	-1.6544***	0.1104
	(0.0200)	(0.0365)	(0.0676)	(0.0833)	(0.0583)	(0.2128)	(0.0490)
N	24	33	34	34	19	12	12
r2_a	0.9898	0.9781	0.9978	0.9960	0.9996	0.9993	0.9952
F	745.6018	476.9556	4964.7221	2725.0741	16169.0434	5567.6743	759.6918
ll	121.7798	151.7648	186.5453	189.4070	129.7730	76.7290	65.6934

Standard errors in parentheses

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$

Table 9: Maturity in x days -Weighted volume. $\beta = -4.1792$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	mat16	mat51	mat79	mat107	mat198	mat296	mat387
Level	0.2296***	0.2203***	0.2394***	0.2437***	0.2455***	0.2361***	0.2394***
	(0.0006)	(0.0007)	(0.0005)	(0.0004)	(0.0002)	(0.0010)	(0.0004)
Slope	-0.0504***	-0.0556***	-0.0639***	-0.0737***	-0.0647***	-0.0445***	-0.0729***
	(0.0019)	(0.0025)	(0.0020)	(0.0017)	(0.0014)	(0.0041)	(0.0029)
Curvature	0.0220***	0.0188***	0.0120***	0.0149***	-0.0021	-0.0224**	0.0091
	(0.0015)	(0.0018)	(0.0019)	(0.0021)	(0.0017)	(0.0053)	(0.0044)
Torsion	-0.0000**	-0.0000	-0.0004	-0.0001	0.0003**	0.0069***	-0.0000*
	(0.0000)	(0.0000)	(0.0002)	(0.0001)	(0.0001)	(0.0008)	(0.0000)
N	24	33	34	34	19	12	12
r2_a	0.9771	0.9668	0.9955	0.9959	0.9997	0.9995	0.9959
F	328.7946	311.1946	2414.5261	2654.9499	23361.7705	6992.6160	886.3738
ll	112.0887	144.8929	174.3269	188.9655	133.2681	78.0957	66.6157

Standard errors in parentheses

* $p < 0.05$. ** $p < 0.01$. *** $p < 0.001$

Table 10: Implied Volatility Approximations for each Maturity and Log-Moneyness

T	1182.4	1213.5	1244.6	1275.7	1306.8	1369	1493.5
12/16/2011	0.2968	0.2580	0.2298	0.2114	0.2021	0.2087	0.3071
01/20/2012	0.2533	0.2351	0.2198	0.2072	0.1972	0.1842	0.1823
02/17/2012	0.2800	0.2619	0.2459	0.2317	0.2192	0.1990	0.1748
03/16/2012	0.2714	0.2566	0.2431	0.2309	0.2199	0.2011	0.1747
06/15/2012	0.2552	0.2474	0.2398	0.2322	0.2247	0.2100	0.1815
09/21/2012	0.1901	0.1930	0.1949	0.1957	0.1957	0.1931	0.1794
12/21/2012	0.2487	0.2418	0.2351	0.2288	0.2228	0.2116	0.1921

Table 11: Call Digital Prices for Different Strikes

	K						
T	1182.4	1213.5	**1244.6**	1275.7	1306.8	1369	1493.5
12/16/2011	1.0094	0.5916	0.4119	0.2392	0.1066	0.0113	0.0015
01/20/2012	0.6003	0.4989	0.3923	0.2901	0.1993	0.0685	0.0028
02/17/2012	0.5555	0.4578	0.3720	0.2940	0.2244	0.1116	0.0094
03/16/2012	0.4886	0.4193	0.3520	0.2883	0.2299	0.1315	0.0198

Table 12: Comparison of Digital Call Prices using MC Simulation and Imp. Vol. approximation for near-at-the-money Options

T	x	I^{MC}	I^{impvol}
0.0411	0.000587	0.52307	0.4119
0.1370	0.002043	0.45168	0.3923
0.2137	0.003783	0.37546	0.3720
0.2904	0.005341	0.31903	0.3520

Table 13: Digital Call Prices for NIG and GH models

T	x	$I^{impvol}(x, -1.998)$	$I^{impvol}(x, -0.0397)$
0.0411	0.000587	0.4119	0.4073
0.1370	0.002043	0.3923	0.3763
0.2137	0.003783	0.3720	0.4435
0.2904	0.005341	0.3520	1.4108