Power Style Contracts Under Asymmetric Lévy Processes

Fajardo, José

FGV/EBAPE

31 May 2016

Power Style Contracts under Asymmetric Lévy Processes

José Fajardo*

May 31, 2016

Abstract

In this paper we present new pricing formulas for some Power style contracts of European type when the underlying process is driven by an important class of Lévy processes, which includes CGMY model, generalized hyperbolic Model and Meixner Model, when no symmetry properties are assumed, extending and complementing in this way previous findings in the literature. Also, we show how to implement our new formulas.

Keywords: Skewness; Lévy processes; Absence of symmetry; Power contracts

JEL Classification: C52; G12

1 Introduction

It is well known that implied volatility symmetry and put-call symmetry are equivalent, Fajardo and Mordecki (2006) and Carr and Lee (2009) prove this equivalence for Lévy process and local and stochastic volatility models, respectively. Also, Fajardo and Mordecki (2014) have shown the relationship among the skewness premium and symmetry properties. More recently, Fajardo (2015a) has shown how to price barrier style contracts when symmetry properties holds and when it is possible to transform the process into a symmetric one.

*Brazilian School of Public and Business Administration. Getulio Vargas Foundation. Praia de Botafogo 190. 22253 900 - Rio de Janeiro. RJ. Brazil. E-mail address: jose.fajardo@fgv.br

Also, we know that symmetry properties are hard to verify in the market, as Bates (1997) and Carr and Wu (2007) have reported, among others. In such context some contracts can be priced, as Carr and Lee (2009) have shown by constructing semi-static hedging for a class of barrier options, but also when it is possible to transform the asymmetric process into symmetric ones, which of course is not always the case.

In the context of Lévy processes, the pricing of exotic contracts is a delicate issue. Fourier transform methods have been used to price them under Lévy processes; see Eberlein, Glau, and Papapantoleon (2010), Eberlein, Glau, and Papapantoleon (2011) and Carr and Crosby (2010). There are many contributions in the literature, we refer the reader to the excellent textbook by Schoutens and Cariboni (2009).

In this paper focusing on pure jump Lévy process with exponential dampening controlling the skewness and using the implied volatility specification proposed by Fajardo (2015a), we show how to price some Power contracts. This allow us to consider any asymmetric dynamic, in the set of Lévy process described above, extending in this way findings presented in the literature, in particular in Fajardo (2015a). It is worth noting that Power style contracts can be used to improve executive compensation efficiency. More precisely, Bernard, Boyle, and Chen (2016) have shown that using Power contracts in stead of Average type contracts (commonly used in executive compensation) we can attain the same utility level for the executive with a lower cost.

The paper is organized as follows. in Section 2 we introduce our model. In Section 3 we show how to price some Power contracts. In Section 4 present some empirical results. Last section concludes.

2 Market Model

Consider a real valued stochastic process $X = \{X_t\}_{t\geq 0}$, defined on a stochastic basis $\mathcal{B} = (\Omega, \mathcal{F}, \mathbf{F} = (\mathcal{F}_t)_{t\geq 0}, \mathbb{Q})$, being càdlàg, adapted, satisfying $X_0 = 0$, and such that for $0 \leq s < t$ the random variable $X_t - X_s$ is independent of the σ-field $\mathcal{F}_s$, with a distribution that only depends on the difference $t - s$. Assume also that the stochastic basis $\mathcal{B}$ satisfies the usual conditions (see Jacod and Shiryaev (1987)). The process X is a Lévy process, and is also called a process with stationary independent increments. For Lévy processes in finance see Schoutens (2003) and Cont and Tankov (2004). Also, let $\mathbf{E}$ denote the expectations taken with respect to $\mathbb{Q}$.

In order to characterize the law of X under $\mathbb{Q}$, consider, for $q \in \mathbb{R}$ the Lévy-Khinchine formula, which states that

$$\mathbf{E}\, e^{iqX_t} = \exp\Big\{t\Big[iaq - \frac{1}{2}\sigma^2 q^2 + \int_{\mathbb{R}} \big(e^{iqy} - 1 - iqh(y)\big)\Pi(dy)\Big]\Big\}, \tag{1}$$

with

$$h(y) = y\mathbf{1}_{\{|y|<1\}}$$

a fixed truncation function, a and $\sigma \geq 0$ real constants, and Π a positive measure on $\mathbb{R}\setminus\{0\}$[1] such that $\int(1 \wedge y^2)\Pi(dy) < +\infty$, called the *Lévy measure*. The triplet (a, σ^2, Π) is the *characteristic triplet* of the process, and completely determines its law.

Now we use the extension to the complex plane used by Lewis (2001), that is, we define the Lévy-Khinchine formula in the strip $\{z := c_1 < \mathrm{Im}(z) < c_2\}$, so that c_1 and c_2 are defined in a way such that $z \in \mathbb{C}_*$, defined by:

$$\mathbb{C}_* = \Big\{z = p + iq \in \mathbb{C}\colon \int_{\{|y|>1\}} e^{-qy}\Pi(dy) < \infty\Big\}. \tag{2}$$

[1] $\Pi(\{0\})$ could be defined as 0. Here I followCont and Tankov (2004).

The exact expressions of c_1 and c_2 as functions of parameter distributions for some important cases of Lévy processes can be found in Table 2.1 in Lewis (2001). Then we can define the *characteristic exponent* of the process X, in $z \in \mathbb{C}_*$, by:

$$\psi(z) = azi - \frac{1}{2}\sigma^2 z^2 + \int_{\mathbb{R}} \big(e^{izy} - 1 - izh(y)\big)\Pi(dy), \tag{3}$$

having $\mathbf{E}\,|e^{izX_t}| < \infty$ for all $t \geq 0$, and $\mathbf{E}\,e^{izX_t} = e^{t\psi(z)}$. The finiteness of this expectation follows from Theorem 21.3 in Sato (1999). Formula (3) reduces to formula (1) when $\mathrm{Im}(z) = 0$.

Remark 2.1. *The set $\mathbb{C}_*$ is a vertical strip in the complex plane and consists of all complex numbers $z = p + iq$ such that $\mathbf{E}\,e^{-qX_t} < \infty$ for some $t > 0$[2]. Moreover, $\mathbf{E}\,|e^{izX_t}| = \mathbf{E}\,e^{-qX_t} < \infty$, $\forall z \in \mathbb{C}_*$.*

2.1 Lévy Market Model

By a *Lévy market* we mean a model of a financial market with two assets: a deterministic savings account $B = \{B_t\}_{t\geq 0}$, with

$$B_t = e^{rt}, \qquad r \geq 0,$$

where $B_0 = 1$ for simplicity and a stock $S = \{S_t\}_{t\geq 0}$, modelled by

$$S_t = S_0 e^{X_t}, \qquad S_0 = e^x > 0, \tag{4}$$

where $X = \{X_t\}_{t\geq 0}$ is a Lévy process.

In this model we assume that the stock pays dividends, with constant rate $\delta \geq 0$, and that the given probability measure $\mathbb{Q}$ is the chosen equivalent martingale measure. In other words, prices are computed as expectations with respect to $\mathbb{Q}$, and the discounted and reinvested

[2] Equivalently for all t, see Th. 25.17 in Sato (1999).

process $\{e^{-(r-\delta)t}S_t\}$ is a $\mathbb{Q}$-martingale.

In terms of the characteristic exponent of the process this means that

$$\psi(-i) = r - \delta, \tag{5}$$

based on the fact that $\mathbf{E}\, e^{-(r-\delta)t+X_t} = e^{-t(r-\delta-\psi(-i))} = 1$, and condition (5) can also be formulated in terms of the characteristic triplet of the process X as

$$a = r - \delta - \sigma^2/2 - \int_{\mathbb{R}} \big(e^y - 1 - \mathbf{1}_{\{|y|<1\}}\big)\Pi(dy). \tag{6}$$

Then,

$$\psi(z) = z(r - \delta - \frac{\sigma^2}{2}) + z^2\frac{\sigma^2}{2} + \int_{-\infty}^{+\infty} [z(1-e^y) + (e^{zy}-1)]\Pi(dy) \tag{7}$$

2.2 Market Symmetry

Here we investigate whether the risk neutral distribution of the discounted asset, under the new risk-neutral measure, denoted by $\tilde{\mathbb{Q}}$[3], remains the same when we change the numéraire of our market. We define a Lévy market to be *symmetric* when the following relation holds:

$$\mathcal{L}\big(e^{X_t-(r-\delta)t} \mid \mathbb{Q}\big) = \mathcal{L}\big(e^{-X_t-(\delta-r)t} \mid \tilde{\mathbb{Q}}\big), \tag{8}$$

meaning equality in law. As Fajardo and Mordecki (2006) pointed out, a necessary and sufficient condition for (8) to hold is

$$\Pi(dy) = e^{-y}\Pi(-dy), \tag{9}$$

where by $-[a,b]$ we mean $[-b,-a]$ and for all interval $A \in \mathbb{R}$ we have $\Pi(-A) = \int_A e^y d\Pi(y)$. This ensures $\tilde{\Pi} = \Pi$. Moreover, in Lévy markets with jump measure of the form

[3] More precisely, $\frac{d\tilde{\mathbb{Q}}_t}{d\mathbb{Q}_t} = e^{X_t-(r-\delta)t}, \;\; t \geq 0$.

$$\Pi(dy) = e^{\beta y}\Pi_0(dy), \tag{10}$$

where $\Pi_0(dy)$ is a symmetric measure, i.e., $\Pi_0(dy) = \Pi_0(-dy)$ and β is a parameter that describes the asymmetry of the jumps.

As a consequence of (9), Fajardo and Mordecki (2006) found that the market is symmetric if and only if $\beta = -1/2$. Thus follows from the fact that

$$e^{\beta y}\Pi_0(dy) = \Pi(dy) = e^{-y}\Pi(-dy) = e^{-y}e^{-\beta y}\Pi_0(-dy)$$

Remark 2.2. *It is worth mentioning that the market symmetry property is equivalent to the put-call symmetry relationship to hold and implied volatility to be symmetric with respect to log-moneyness, as was proved by Fajardo and Mordecki (2006) and Carr and Lee (2009) for Lévy process and positive martingales, respectively.*

Now substituting (10) in eq. (7), one can observe the dependence of the characteristic exponent on the parameter β:

$$\psi(z) = -\frac{\sigma^2}{2}iz - z^2\frac{\sigma^2}{2} + \int_{-\infty}^{+\infty}[iz(1-e^y) + (e^{izy}-1)]e^{\beta y}\Pi_0(dy), \tag{11}$$

Remark 2.3. *If the Lévy process has Lévy measure given by (10), then $z_2 = 2\beta i \in \mathbb{C}_*$ and $z_1 = \beta i \in \mathbb{C}_*$. It follows from:*

$$\int_{\{|y|>1\}} e^{-2\beta y}\Pi(dy) = \int_{\{|y|>1\}} \Pi(dy) < \int (1 \wedge y^2)\Pi(dy) < \infty.$$

From here $\psi(2\beta i) < \infty$, analogously for $\psi(\beta i) < \infty$. Of course, this is for all $\beta \in \mathbb{R}$ satisfying the parameter distribution restrictions given in the definition of each particular Lévy process.

Now we present our main results

3 Results

3.1 Power style contracts

Before state our main results lets recall an useful theorem

Theorem 3.1 (Fajardo (2015a)). *Let $X_t = \log(S_t/S_0)$ be a Lévy process with characteristic triplet (a, σ^2, Π) and characteristic exponent denoted by ψ, with Lévy measure given by $\Pi(dy) = e^{\beta y}\Pi_0(dy)$, where $\beta \neq -0.5$ (absence of symmetry) and define*

$$\gamma := \begin{cases} -\frac{\psi(2\beta i)}{2\beta}, & \beta \neq 0; \\ a, & \beta = 0. \end{cases}$$

Then for any payoff function[4] *f, we have*

$$Ef(S_T) = E\left[\left(\frac{S_T}{S_0 e^{\gamma T}}\right)^{-2\beta} f\left(\frac{S_0^2 e^{2\gamma T}}{S_T}\right)\right] \tag{12}$$

Now we price some down-and-in power options[5]. In what follows we assume that we are in the context of the above Th. 3.1. and also that the distribution of S_T has no atoms.

Corollary 3.1 (down-and-in power option). *The price of the contract $f_1(S_T) = S_T^{-\beta} 1_{\{S_T \geq S_0 e^{\gamma T}\}}$, is given by*

$$f_1 = e^{-rT} E[S_T^{-\beta} 1_{\{S_T \geq S_0 e^{\gamma T}\}}] = \frac{e^{-rT} E(S_T^{-\beta})}{2} = \frac{S_0^{-\beta} e^{(\psi(\beta i) - r)T}}{2}.$$

Proof. Applying Th. 3.1 to the function $f_1(S_T) = S_T^{-\beta} 1_{\{S_T \geq S_0 e^{\gamma T}\}}$, we obtain the result. □

At the same time for the same fixed β the next pricing holds

Corollary 3.2 (down-and-in power option-2). *The price of the contract $f_2(S_T) = S_T^{-2\beta} 1_{\{S_T \geq K_x\}}$, is given by*

[4] A payoff function is a nonnegative Borel function on $\mathbb{R}$.

[5] Here we use the same denomination used in example 5.15 by Carr and Lee (2009).

$$f_2 = S_0^{-2\beta} e^{-(r-\psi(2\beta i))T}[1 - \mathbb{Q}(S_T \geq K_x^*)],$$

where $K_x^* = \frac{S_0^2 e^{2\gamma T}}{K_x}$ *and* $\mathbb{Q}(S_T \geq K_x^*)$ *can be computed using implied volatility approximations, as we show in the next section.*

Proof. Applying Th. 3.1 to the function $f_2(S_T) = S_T^{-2\beta} 1_{\{S_T \geq K_x\}}$, we obtain the result. □

Remark 3.1. *The last Corollary 3.2 extends Corollary 3.3 in Fajardo (2015a) for any* β *and to a larger set of moneyness.*

In general we can have the following

Corollary 3.3 (down-and-in power option-n)**.** *The price of the contract* $f_n(S_T) = S_T^{-n\beta} 1_{\{S_T \geq K_x\}}$, $\forall n$ *such that* $(2-n)\beta i \in \mathbb{C}_*$, *is given by:*

$$f_n = (S_0 e^{\gamma T})^{(-2n+2)\beta} \left[S_0^{(-2+n)\beta} e^{(\psi((2-n)\beta i)-r)T} - f_{2-n}^* \right],$$

where f_n^* *is the price of a contract with barrier* $K_x^* = \frac{S_0^2 e^{2\gamma T}}{K_x}$. *When* $K_x = S_0 e^{\gamma T}$ *we have* $f_n^* = f_n$.

Proof. Applying Th. 3.1 to the function $f_n(S_T) = S_T^{-n\beta} 1_{\{S_T \geq K_x\}}$, we obtain

$$E(S_T^{-n\beta} 1_{\{S_T \geq K_x\}}) = (S_0 e^{\gamma T})^{(-2n+2)\beta} E(S_T^{(-2+n)\beta} 1_{\{S_T \leq K_x^*\}}),$$

from here the result follows. □

Now we show how to implement our formulas.

4 Numerical Examples

The Market Symmetry parameter (β) is estimated for the normal inverse Gaussian (NIG) process, we made this choice since these models have shown a very good fit with financial returns, see Eberlein and Prause (2002) and Fajardo and Farias (2004).

We consider daily returns from Bloomberg, we consider the randomly picked sample period 12/01/2006 to 12/01/2011. As, we need the risk-neutral parameters, we use the density given by the Esscher Transform. To compute this density we need the interest rate, so we use the interest rate given by the U.S. Treasury on that date 12/01/2011, $r = 0.0012$.

For the NIG model we have $\beta = -1.998$. The other risk-neutral parameters are given by

$$(\mu, \alpha, \delta, \beta) = (0.0016, 31.66, 0.0089, -1.998). \tag{13}$$

Now to compute the probability $\mathbb{Q}(S_T \geq K_x^*)$, we proceed in two steps: first, from the Black and Scholes model we know that

$$e^{-rT}\mathbb{Q}(S_T \geq K_x) = e^{-rT}N(d_2(x)) - e^{-(r-\delta)T-x}\sqrt{T}N'(d_1(x))\frac{\partial \sigma_{imp}(x,\beta)}{\partial x},$$

with $d_1(x) = d_2(x) + \sigma_{imp}\sqrt{T}$ and $d_2(x) = -\frac{x+\frac{\sigma_{imp}^2 T}{2}}{\sigma\sqrt{T}}$. Second, using the following implied volatility approximation suggested by Fajardo (2015b)

$$\sigma_{imp}(x_i) \approx \gamma_0 + \gamma_1 d_i + \gamma_2 d_i^2 + \gamma_3(d_i+1)^{\beta+0.5}, \tag{14}$$

where $d_i = \frac{x_i}{\bar{\sigma}\sqrt{T}}$, is the standarized moneyness, $\bar{\sigma}$ is an average volatility. Then we obtain,

$$e^{-rT}\mathbb{Q}(S_T \geq K_x) \approx e^{-rT}N(d_2(x)) - e^{-(r-\delta)T-x-\frac{d_1(x)^2}{2}}\left[\bar{\gamma_1} + 2\bar{\gamma_2}(\frac{x}{\bar{\sigma}\sqrt{T}}) + (\beta+0.5)\bar{\gamma_3}(\frac{x}{\bar{\sigma}\sqrt{T}}+1)^{\beta-0.5}\right], \tag{15}$$

with $\bar{\gamma_i} = \frac{\gamma_i}{\bar{\sigma}\sqrt{2\pi}}, i = 1.2.$

To compute the probabilities we use options on S&P500 from Bloomberg, quoted on the date 12/01/2011, the sample is described in Table (1). Also, the resulting implied volatility term structure is presented in figure (1).

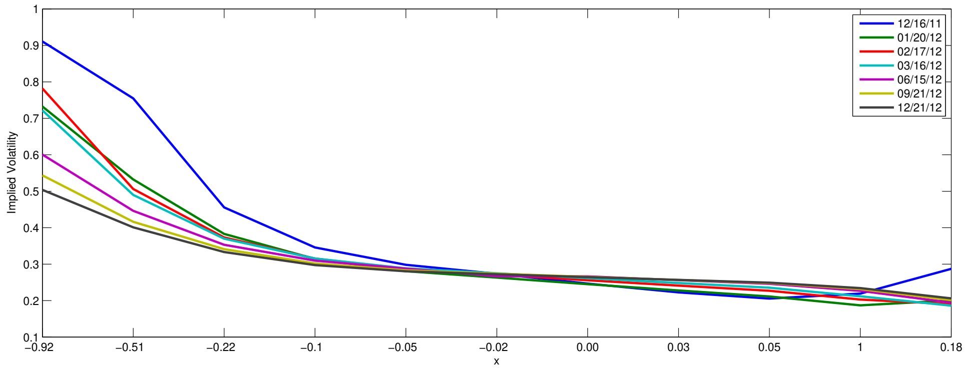

Figure 1: Implied Volatility Term Structure

For each maturity, the implied forward price F is determined based on at-the-money option price using the following formula

$$F = Strike\ price + exp(rT)(Call\ price - Put\ price)$$

where r is the interest risk-free rate determined by the U.S. treasury bill yield curve rates on December 1, 2011. A linear extrapolation technique is used to calculate the relevant rate for the different maturities. The resulting implied volatility approximations are given in Table (2).

[Table (2) about here]

Finally, we compute the probabilities for the first fourth maturities, the results are presented in Table (3).

[Table (3) about here]

Now we price contracts on S&P500 in our randomly picked date 12/01/2011, on that date $S_0 = 1244.59$.

4.1 Down-in-Power Options

The normal inverse Gaussian(NIG) distribution has the following characteristic function:

$$\psi(z) = iz\mu + \delta\left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + iz)^2}\right), \; \beta - \alpha < \mathrm{Im}(z) < \beta + \alpha. \tag{16}$$

Recall that the definition of NIG distribution requires $\delta > 0$ $\alpha > 0$ and $|\beta| < \alpha$. We can verify that $2\beta i$ and βi belongs to the strip $\{z \in \mathbb{C} : \beta - \alpha < \mathrm{Im}(z) < \beta + \alpha\}$ and, as pointed out in Remark 2.3, $\psi(2\beta i)$ and $\psi(\beta i)$ are well defined and finite: $\psi(2\beta i) = -2\beta\mu$ and $\psi(\beta i) = -2\beta\mu + \delta\left(\sqrt{\alpha^2 - \beta^2} - \alpha\right)$. Remembering also that from the martingale condition (5) we have

$$\psi(z) = \delta\left(iz\sqrt{\alpha^2 - (\beta + 1)^2} + (1 - iz)\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + iz)^2}\right).$$

Now using the risk-neutral parameters estimated in (13)

$$(\mu, \alpha, \delta, \beta) = (0.0016, 31.66, 0.0089, -1.998),$$

we obtain $\psi(-3.996i) = 0.0064$ and $\psi(-1.998i) = 0.0058$. Then using our data sample we compute the prices for barrier contracts with $K_x^* = F$ and we present the results in Tables (4) and (5).

[Table (4) and (5) about here]

5 Conclusions

We show how to price some barrier contracts when no symmetry properties are observed or no symmetry transformation is used, extending and complementing in this way previous results in the literature. Then, we show how to implement our new formulas.

Some extensions are of interest, as for example the pricing of more exotic derivatives. We left it for future research.

References

Bates, D. (1997): "The skewness premium: Option Pricing under Asymmetric Processes," *Advances in Futures and Options Research*, 9, 51–82.

Bernard, C., P. Boyle, and J. Chen (2016): "Power options in executive compensation," *Journal of Derivatives*, 23(3), 9–20.

Carr, P., and J. Crosby (2010): "A Class of Levy Process Models with almost exact calibration of both barrier and vanilla FX options," *Quantitative Finance*, 10(10), 1115–1136.

Carr, P., and R. Lee (2009): "Put Call Symmetry: Extensions and Applications," *Math. Finance.*, 19(4), 523–560.

Carr, P., and L. Wu (2007): "Stochastic Skew in Currency Options," *Journal of Financial Economics*, 86(1), 213–247.

Cont, R., and P. Tankov (2004): *Financial Modelling with Jump Processes*. Chapman & Hall /CRC Financial Mathematics Series.

Eberlein, E., K. Glau, and A. Papapantoleon (2010): "Analysis of Fourier transform valuation formulas and applications," *Applied Mathematical Finance*, 17(3), 211–240.

——— (2011): "Analyticity of the Wiener–Hopf factors and valuation of exotic options in Lévy models.," in *Advanced Mathematical Methods for Finance*, ed. by G. D. Nunno, and B. Øksendal. Springer Verlag.

Eberlein, E., and K. Prause (2002): "The Generalized Hyperbolic Model: Financial

Derivatives and Risk Measures," in *Mathematical Finance-Bachelier Congress 2000*, ed. by S. P. T. V. H. Geman, D. Madan. Springer Verlag.

FAJARDO, J. (2015a): "Barrier Style Contracts under Lévy Processes: An Alternative Approach," *Journal of Banking and Finance*, 53, 179–187.

——— (2015b): "A New Factor to Explain Implied Volatility Smirk," Available at SSRN: http://ssrn.com/abstract=2557624.

FAJARDO, J., AND A. FARIAS (2004): "Generalized Hyperbolic Distributions and Brazilian Data," *Brazilian Review of Econometrics*, 24(2), 249–271.

FAJARDO, J., AND E. MORDECKI (2006): "Symmetry and Duality in Lévy Markets," *Quantitative Finance*, 6(3), 219–227.

——— (2014): "Skewness Premium with Lévy Processes," *Quantitative Finance*, 14(9), 1619–1626.

JACOD, J., AND A. SHIRYAEV (1987): *Limit Theorems for Stochastic Processes*. Springer, Berlin, Heidelberg.

LEWIS, A. L. (2001): "A simple option formula for general jump-diffusion and other exponential Lévy processes," Working paper. Envision Financial Systems and OptionCity.net Newport Beach, California, USA. Available at http://www.optioncity.net.

SATO, K.-I. (1999): *Lévy Processes and Infinitely Divisible Distributions*. Cambridge University Press, Cambridge, UK.

SCHOUTENS, W. (2003): *Lévy Processes in Finance: Pricing Financial Derivatives*. Wiley, New York.

SCHOUTENS, W., AND J. CARIBONI (2009): *Lévy Processes in Credit Risk*. Wiley, New York.

Table 1: Option Data

Maturity	T	F	σ_{imp}^{ATM}	r	δ
12/16/2011	0.0411	1242.87	24.67%	0.208%	1.619%
01/20/2012	0.1370	1242.06	24.52%	0.344%	1.826%
02/17/2012	0.2137	1239.90	25.58%	0.454%	2.218%
03/16/2012	0.2904	1237.97	26.16%	0.498%	2.328%
06/15/2012	0.5397	1232.28	26.64%	0.540%	2.367%
09/21/2012	0.8082	1226.62	26.52%	0.580%	2.358%
12/21/2012	1.0575	1221.18	26.45%	0.604%	2.370%

Source: Bloomberg

Table 2: Implied Volatility Approximations for each Maturity and Log-Moneyness

T	1182.4	1213.5	1244.6	1275.7	1306.8	1369	1493.5
12/16/2011	0.2968	0.2580	0.2298	0.2114	0.2021	0.2087	0.3071
01/20/2012	0.2533	0.2351	0.2198	0.2072	0.1972	0.1842	0.1823
02/17/2012	0.2800	0.2619	0.2459	0.2317	0.2192	0.1990	0.1748
03/16/2012	0.2714	0.2566	0.2431	0.2309	0.2199	0.2011	0.1747
06/15/2012	0.2552	0.2474	0.2398	0.2322	0.2247	0.2100	0.1815
09/21/2012	0.1901	0.1930	0.1949	0.1957	0.1957	0.1931	0.1794
12/21/2012	0.2487	0.2418	0.2351	0.2288	0.2228	0.2116	0.1921

Table 3: $I(x, -1.998) = e^{-rT}\mathbb{Q}(S_T \geq K_x^*)$ for Different Strikes

	K						
T	1182.4	1213.5	**1244.6**	1275.7	1306.8	1369	1493.5
12/16/2011	0.9945	0.5916	0.4119	0.2392	0.1066	0.0113	0.0015
01/20/2012	0.6003	0.4989	0.3923	0.2901	0.1993	0.0685	0.0028
02/17/2012	0.5555	0.4578	0.3720	0.2940	0.2244	0.1116	0.0094
03/16/2012	0.4886	0.4193	0.3520	0.2883	0.2299	0.1315	0.0198

Table 4: Down-in-Power Prices 1 for NIG model

Maturity	T	r	f_1
12/16/2011	0.0411	0.0021	$S_0^{1.998}$*0.50008
01/20/2012	0.1370	0.0034	$S_0^{1.998}$*0.50016
02/17/2012	0.2137	0.0045	$S_0^{1.998}$*0.50014
03/16/2012	0.2904	0.0050	$S_0^{1.998}$*0.50012
06/15/2012	0.5397	0.0054	$S_0^{1.998}$*0.50012
09/21/2012	0.8082	0.0058	$S_0^{1.998}$*0.50001
12/21/2012	1.0575	0.0060	$S_0^{1.998}$*0.49989

Table 5: Down-in-Power Prices 2 for NIG model

Maturity	T	r	x	$I(x, -1.998)$	f_2
12/16/2011	0.0411	0.0021	0.00059	0.4120	$S_0^{3.996}$*0.5881
01/20/2012	0.1370	0.0034	0.00204	0.3923	$S_0^{3.996}$*0.6077
02/17/2012	0.2137	0.0045	0.00378	0.3720	$S_0^{3.996}$*0.6279
03/16/2012	0.2904	0.0050	0.00534	0.3520	$S_0^{3.996}$*0.6478
06/15/2012	0.5397	0.0054	0.00995	0.3450	$S_0^{3.996}$*0.6543
09/21/2012	0.8082	0.0058	0.01455	0.3331	$S_0^{3.996}$*0.6657
12/21/2012	1.0575	0.0060	0.01900	0.3143	$S_0^{3.996}$*0.6840