Endogenous Leadership in a Federal Transfer Game

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Abstract

The paper explores the issue of leadership in central transfer within a federation. In a federal country, provinces, in anticipation of the ultimate federal bailout, may spend more than what is optimal. Such behaviour creates negative fiscal externalities and harms the central government. To counter such tendencies, it is suggested by policymakers that central authority should always be a first mover in the transfer game: once the grant (presumably formulaic) is dispensed, it should refrain from any ex post transfer. In spite of such recommendations, central governments, in almost all countries, chooses to be the second mover from time to time. We explore the conditions, other than the familiar political economy arguments, under which the central government optimally chooses to be the second mover. The key determinants of the first or second mover advantages in such situations is the nature of spillover effects of public goods between the two tiers of government.

Key words: Federalism, Transfer Game, First and Second Mover Advantages

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1 Introduction

Due to inherent vertical and horizontal imbalances, central (or federal) authority need to transfer funds to provinces in a federal economy. A point of concern is the degree of control wielded by the central government over the subnational units regarding disbursement and utilization of such funds. The debate goes back to the foundation of USA, the first federal (as pr the current usage) country in modern world. Madison (1887) argued that, in a heterogeneous country, some freedom for the local level of government is necessary so that they can choose their level and composition of public good in an effective manner. Others (Hamilton, *ibid.*1887) have raised the fear that reckless spending by provinces and subsequent bailouts by centre can prove disastrous for the federation as a whole. Translated in modern terms, the debate centred around the following issue: in order to maximize its beenefit, should a central government in a federal economy choose to make *ex-post* (disbursed *after* the provinces make their tax-expenditure decision) grants or take recourse to ex-ante (disbursed, once-and-for-all, *before* the provinces make their tax-expenditure decision) transfers? In this paper, we try to answer this question.

Hamilton's fear later realized in Brazil and Argentina during the 1990's. In Europe, Bailouts occurred in post war Germany in the provinces of Bremmen, Saarland and Berlin (Rodden, 2006). The issue has assumed importance in the light of recent events in Eurozone (a quasi federal setup). For theoretical analyses of such "common pool" problem, see Wildasin (1997), Velasco (2000) and Goodspeed (2002).

Recent policy prescriptions are heavily tilted towards the Hamiltonian paradigm. Anwer Shah (2006), for example, prescribes that "Grants to finance subnational deficits, which create incentives for running higher deficit in future" (page 47) are to be avoided. In India, the Twelfth Finance Commission, a constitutional body which regulates centre state financial relation to a large extent, had recommended (2004) the termination of central governments' role in assuming the states' debt. In other words, centre should dispense with the *ex post* grants.

In spite of this intellectual onslaught, bailout by centre (which is, almost and always, *ex post* in nature) still persists, ostensibly, in name of equity. One can argue that such transfers are inherently politically motivated. Evidence of political motivation behind ex post grants, either partisan or strategic, is well documented in, say, Solé-Ollé and Sorribas-Navarro (2008) or Arulampalam *et. al.*(2008). What we attempt in this paper is to explain the persistence of *allegedly* inefficient behavior from efficiency calculations. In other words, due to sequential nature of grant dispensation in a federation, the cost-benefit calculus (of ex post vis-à-vis ex ante) should be done in a sequential game framework, in which the provinces and the centre are the two sets of players. Depending on the nature of the game and transfer, it is possible that there exists a *second movers advantage* that the centre wishes to exploit. The basic assumption is centre acting as a dictator, writes the fiscal constitution in order to maximize federal welfare. If it happens that the centre maximizes its welfare by being a second mover in the grant dispensation game, constitution may have to have a provision for soft budgets and bailouts.

The reason is not hard to explain. The Hamiltonian paradigm places emphasis of fiscal solvency at various levels of Government, including centre. Thus, the only benefit that accrues to centre is the savings (to be spent in various central projects), net of transfer to the provinces (which is used to finance provincial budget) out of a fixed revenue resource. Thus fiscal stress in provinces (and, left unchecked, this is what the provinces do), will ultimately transferred to the centre and the latter (or the nation as a whole) will suffer. However, the above view of centre in a federal country misses one important aspect of federalism: that the central authority derives benefit not only from its own projects, but provincial projects as well. To the extent that provincial projects are important to centre, it may benefit from higher output from provincial project.

We assume that centre has a fixed fund which finances a central project as well as transfer to provinces. Transfers are earmarked for provincial projects. Thus, provincial public good is financed partly by provincial taxation. There also exist two way "spillover" between provincial and central public good, that is, each level of government value outcome of the project at other sphere. Under this set up, we show that, for both modes of transfer, there is a possibility of second movers' advantage for the central government. The result depends degree to which central project benefits provinces.

It is to be noted that other authors (e.g. Besfamille and Lockwood, 2008) have already explored the relative inefficiency of hard budgets (centre as first mover) vis-a-vis soft budget (mostly ex post grants) constraints in a federal set up. A key tool for their analysis is imperfect information and the associated moral hazard problem. Provincial revenue raising activities are not observable by the central authority. In real life, however, due to availability of budgetary documents in public domain and continuous scrutiny by media, such an assumption is hard to maintain.[1] Koethenbuerger (2008) also demonstrates possibility of welfare improvement when either centre or provinces can precommit: this is achieved by putting a brake to the 'race to the bottom'. Silva(2015) considers the regime of earmarked grants (central transfers which are tied to a specific public project) show that such grants may improve overall efficiency if provinces have the ability to commit (as a first mover in a sequential game). In this strand of literature,

Given that the present analysis centres around the first mover and second mover advantages, the research contribution also spills over to the timing game paradigm in a federal set up. Timing games have been well researched in Industrial Organization literature (e.g. Gale-Or 1985; Dowrick 1986; Hamilton and Slustky 1990; Aamir and Stepanova 2006). The chief research problem of timing games is to figure out conditions under which a

[1] That is, provinces are too big to escape notice of media or the central government etc.

leader or follower in a von-Stackelberg game (usually, a duopoly) is identified endogenously. The methodology was first used in context of a federation by Kemph and Graziosi (2010). In this paper, the authors address the issue of leadership between countries with transboundary externalities (e.g. environmental externalities) in a perfect information set up. However, the emphasis is on the interaction between countries, not between hierarchical governments, which is a feature of federal economies.

In sum, there are two different strands of literature within fiscal federalism regarding commitment. One preoccupies itself with the consequences of different commitment protocols within a federation with hierarchical governments, but does not explain how the protocols arise. The second strand (Kemph and Graziosi, *ibid*) discusses the origin of such protocols under different contexts, but does not include the hierarchical framework typical in a federation. The present study attempts to build a bridge between these two strands: by providing an explanation of first and second mover advantages in a federation with central and provincial governments.

The current paper (like Kemp and Graziosi,*ibid*) uses the taxonomy of strategic variables provided by Eaton (2004).[2] Eaton shows that second mover advantage in a general duopoly game is present only when both players have upward sloping reaction function. However, one important difference is the following. In his treatment, Eaton has assumed players with symmetric payoffs. Since the players in the current analysis are province and centre, the payoff of each agent is asymmetric in nature. In the present paper, a second movers' advantage can be detected even if the players have downward sloping reaction function.

Our paper is divided into the following parts. Section 2 describes the basic model. Section 3 characterizes different equilibria that may arise. Section 4 provides a discussion on endogeneity of different constitutional protocols in the context. Section five compares the protocols in terms of

[2] See appendix A1.

central transfer, provincial taxation and project outcomes at two levels of government. Section 5 concludes.

2 Model

We assume a simple federal set up consisting of two provinces ($i = 1, 2$) and a centre. Provinces derive utility from a local project (outcome p_i) and local consumption. Central authority gets utility from a central project (outcome P). In addition, both central and provincial projects *may be* valued by each other. Centre faces a budget constraint: $P + T_1 + T_2 = M$, where M , the total amount of central fund, is exogenous.

Provincial welfare is

$$w^i\left(p_i, c_i, P\right) = u\left(p_i\right) + v\left(c_i\right) + \beta f(P) \tag{1}$$

Where c_i is provincial consumption (or net income after taxes). Provincial income, y_i, is given. Here, $f(.)$ represents provincial benefit from central project. The parameter β is the weight (or, equivalently, a parameter that captures marginal benefit of central project) that the province puts on benefit from central project. Different values of β and different forms of f allows us to capture many facets of reality. It also recognizes the fact that it is possible a central project is evaluated subjectively by the province.[3] For example, if $f(.) \doteq P$ and $\beta = 1$, then the centrally produced good assumes the nature of a national public good (e.g. a lighthouse) within the federation.

Similarly, central welfare depends on p_i and P:

$$W\left(p_1, p_2, P\right) = V\left(P\right) + \gamma\left(F(p_1) + F(p_2)\right) \tag{2}$$

Here, V is the benefit that centre receives from central project. $F(.)$ is the benefit that the centre receives from provincial project and γ is the

[3] If P is armament import, provinces may perceive that the benefit is close to 0. If P is federally sponsored road network, β could be quite high. There is no upper limit on β.

weight on associated benefits (equivalently, a parameter characterizing marginal benefit).[4] Again, this formulation allows us to capture many facets of reality. To focus on the hierarchical behavior of centre and provinces, we assume away inter-provincial benefit from p_i. We also make the familiar assumptions: $u', v', V' > 0$ and $u'', v'', V'' < 0$. As mentioned in section 1, a further assumption is the center has an overriding presence, such that its best interest is will be protected in the constitution and institutionalized.

Notice that provinces are identical (save in y_i). We need this assumption in order to induce identical protocols for both provinces, i.e. if centre is leader (follower) with respect to one province, it can not behave differently towards other province.

3 Characterizing Equilibrium Protocols

Now we put more structure to the model by explicitly bringing in the nature of central grants. Grants are conditional in the sense that these are tied to a specific provincial project. Province raises a tax, say θ_i, to finance the public good project. Centre provides the transfer T_i such that $T_1 + T_2 + P = M$. Provincial public good is $p = \theta_i + T_i$. Provincial consumption is $c_i = y_i - \theta_i$.[5] Incorporating these information in the utility functions, we can write (1) and (2) as functions of θ_i and T_i's, where $i = 1, 2$.

[4] Traditionally, in Fiscal Federalism literature, centre is seen as a Benevolent dictator which has sum of provincial utilities as objective function. However, such formulation may not address Hamilton's fear that reckless spending by provinces will squeeze central fund. The extent to which Central government cares for provinces is captured by the $\gamma \sum F_i(.)$ term. It may be noted that such non-Benthamite formulation of Federal welfare is not without precedence, e.g. see Snowdon and Wen (2003). In their formulation, provincial cost reduces centres' welfare. In our formulation, provincial project outcome increases central benefit.

[5] It is possible that central fund depends on federal income tax, i.e. $M = \tau(y_1 + y_2)$, where τ is the rate of tax and $c_i = (1 - \tau) y_i - \theta_i$. This will not change our results, as long as τ is exogenously given. We assume away from the issue.

3.1 Reaction Functions

Province i chooses θ_i to maximize[6]

$$w(\theta_i, T_1, T_2) = u\left(\theta_i + T_i\right) + v\left(y_i - \theta_i\right) + \beta f\left(M - T_1 - T_2\right) \tag{3}$$

From the first order condition,

$$u'\left(\theta_i + T_i\right) = v'(y_i - \theta_i) \tag{4}$$

The slope of provincial reaction function is

$$\theta'\left(T_i\right) = -\frac{u''}{u'' + v''} < 0 \tag{5}$$

Thus T_i is strategic substitute for θ_i. Since raising θ_i is costly for the province, higher T_i will reduce θ_i. Note that $\frac{\partial p_i}{\partial T_i} = \frac{v''}{u''+v''} > 0$ on the reaction function of the province.

Similarly, centre chooses T_1, T_2 to maximize[7]

$$W(T_1, T_2, \theta_1, \theta_2) = V(M - T_1 - T_2) + \gamma\left[F\left(\theta_1 + T_1\right) + F(\theta_2 + T_2)\right] \tag{6}$$

The first order conditions are, for $i = 1, 2$

$$-V'(M - T_1 - T_2) + \gamma F'\left(\theta_i + T_i\right) = 0 \tag{7}$$

From the first order conditions, we can express the reaction function of centre as $T_i(\theta_1, \theta_2)$. It can be shown that

$$\begin{aligned} \frac{\partial T_i}{\partial \theta_i} &= -\frac{\gamma F''\left(V'' + \gamma F''\right)}{\left(V'' + \gamma F''\right)^2 - \left(V''\right)^2} < 0 \\ \frac{\partial T_j}{\partial \theta_i} &= \frac{\gamma F'' V''}{\left(V'' + \gamma F''\right)^2 - \left(V''\right)^2} > 0 \end{aligned} \tag{8}$$

[6] It can be shown that the associated Hessian matrix is negative definite (irrespective of the value of β). Thus the problem of non convexiety does not arise.

[7] We can not assume the benefits from provinces to be $F\left(p_1 + p_2\right) = \sum\left(\theta_i + T_i\right)$ since that would leave transfer to individual provinces indeterminate.

Similarly for θ_j. Thus, an increase in θ_i will reduce T_i but increase T_j. Notice that, following an increase in θ_i, the sum $(T_1 + T_2)$ falls, and hence an increase in provincial taxation reduces *total* central transfer and *quid pro quo*, increases output from central project on the reaction function of centre.

The slope of central reaction function is *negative* on $T_i - \theta_i$ plane.[8] The reason is as follows. Higher T_i is costly for the centre (as its own public good production decreases). At the same time, with higher provincial taxation, provincial welfare from public good will increase. Since centre cares for the provincial public good, $(\gamma \neq 0)$ centre's response is to reduce the transfer as θ_i increases.

Example 3.1 *Let* $u(.) = p_i - \frac{\delta}{2}p_i^2$, $v(.) = c_i - \frac{\rho}{2}c_i^2$, $f(.) = P - \frac{\lambda}{2}P^2$, $V(.) = P - \frac{\eta}{2}P^2$; $F(.) = p_i - \frac{\phi}{2}p_i^2$. *We assume that the parameters* $\delta, \rho, \lambda, \eta, \phi$ *are small enough to always guarantee positive marginal utility.*

For province i*, the FOC is*

$$\begin{aligned} 1 - \delta(\theta_i + T_i) &= 1 - \rho(y_i - \theta_i) \\ &\implies \theta_i(T_i) = \frac{\rho y_i - \delta T_i}{\delta + \rho} \end{aligned}$$

For centre, the FOC's are

$$\begin{aligned} -1 + \eta(M - T_1 - T_2) + \gamma(1 - \phi(\theta_1 + T_1)) &= 0 \\ -1 + \eta(M - T_1 - T_2) + \gamma(1 - \phi(\theta_2 + T_2)) &= 0 \end{aligned}$$

Solving this, we get

$$T_i(\theta_i, \theta_j) = \frac{\gamma + M\eta - 1}{2\eta + \gamma\phi} - \frac{\eta + \gamma\phi}{2\eta + \gamma\phi}\theta_i + \frac{\eta}{2\eta + \gamma\phi}\theta_j$$

On central reaction function, total transfer is $T_1 + T_2 = \frac{2(\gamma + M\eta - 1)}{2\eta + \gamma\phi} - \frac{\gamma\phi}{2\eta + \gamma\phi}(\theta_1 + \theta_2)$

[8] Higher θ_j will 'blow' the reaction function to right.

$$\textit{Outcome of central project } P = \frac{M\gamma\phi + 2(1-\gamma)}{2\eta + \gamma\phi} + \frac{\gamma\phi}{2\eta + \gamma\phi}(\theta_1 + \theta_2)$$

The structure of the problem allows us to treat interaction between different provinces and centre separately. For example, the reaction functions of centre and province i can be plotted in the $\theta_i - T_i$ plane, keeping θ_j as a parameter, which is determined in the $\theta_j - T_j$ plane.

3.2 Nash Protocol

The Nash outcome $(\theta_1^N, \theta_2^N, T_1^N, T_2^N)$ is solution of the following equations:

$$\begin{aligned} \frac{\partial w(\theta_i^N, T_1^N, T_2^N)}{\partial \theta_i} &= 0;\ i = 1, 2 \\ \frac{\partial W(T_1^N, T_2^N, \theta_1^N, \theta_2^N)}{\partial T_i} &= 0;\ i = 1, 2 \end{aligned} \quad (9)$$

As it is clearly demonstrated, we have a Cournot type game with downward sloping reaction functions. We assume existence of a Nash equilibrium.

The relation between θ_i and T_i can be shown in the following graph:

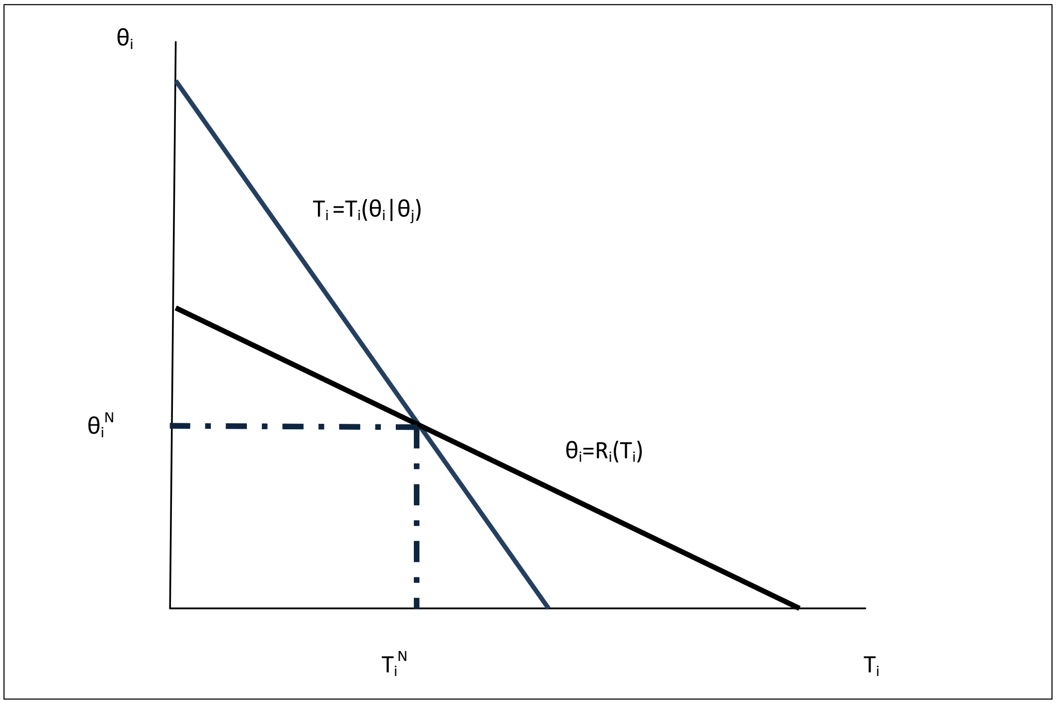

Figure 1: Nash Equilibrium in a Province

It can be shown that the equilibrium is stable. A Nash outcome is more likely if neither provinces, nor centre are unable to commit or reach a binding, enforceable constitution.

3.3 Stackelberg Protocols(s)

Here, we define the problems first.

If centre is the first mover, then it chooses T_1 and T_2 in such a way that

$$W(T_1, T_2) = V(M - T_1 - T_2) + \gamma \left[F\left(\theta_1\left(T_1\right) + T_1\right) + F(\theta_2\left(T_2\right) + T_2)\right]$$

is a maximum. Here, $\theta_i\left(T_i\right)$ is the reaction function of the province (obtained from 4).

In symbols, the first order condition at the optimum can be written as, for $i = 1, 2$

$$\frac{\partial W(\theta_1^F,\theta_2^F,T_1^L,T_2^L)}{\partial T_i}+\frac{\partial W(\theta_1^F,\theta_2^F,T_1^L,T_2^L)}{\partial \theta_i}*\frac{\partial \theta_i}{\partial T_i}=0 \tag{10}$$

Solution to two equations will yield T_i^L. Plugging into provincial reaction functions, we get θ_i^F.

We show how γ alters T_i. In order to demonstrate the result, we continue with the LQ example.[9]

Lemma 3.1 *If γ increases, then T_i^L increases. As a result, P^L falls and p_i^F increases.*

Proof. See appendix A2. ■

If province i (given the symmetry of provinces, similar conditions can be derived for province j) is leader vis-a-vis the centre, then it has to choose θ_i in such a way that

$$w^i=u(\theta_i+T_i(\theta_1,\theta_2))+v(y-\theta_i)+\beta f(M-T_1(\theta_1,\theta_2)-T_2(\theta_1,\theta_2))$$

is a maximum. Here, $T_i(\theta_i,\theta_j)$ is the reaction function of the centre (obtained from .

In symbols, the first order condition can be written as, for $i=1,2$

$$\frac{\partial w^i(\theta_1^L,\theta_2^L,T_1^F,T_2^F)}{\partial \theta_i}+\sum_{j=1,2}\left[\frac{\partial w(\theta_1^L,\theta_2^L,T_1^F,T_2^F)}{\partial T_j}*\frac{\partial T_j}{\partial \theta_i}\right]=0 \tag{11}$$

Solutions to above equations will yield $\theta_i=\Theta^i(\theta_j)$ and $\theta_j=\Theta^j(\theta_i)$. Since our main concern is hierarchical structure *between the centre and provinces*, we refrain from inter provincial commitment issues that may arise when provinces are first mover. That is, $\left(\theta_1^L,\theta_2^L\right)$ is determined by a simultaneous move game. Continuing with the LQ example, we prove a couple of

[9] If we do not assume LQ functional form, then, in the comparative statics analysis, second derivatives of the reaction function such as $\frac{\partial^2\theta_i}{\partial T_i^2}$ etc. involve third derivatives of the utility functions. This becomes difficult to interpret.

lemmas to characterize provincial interaction. We focus on the parameter β because it takes a central stage in our model.

Lemma 3.2 *If β is sufficiently high, then θ_1, θ_2 are strategic substitutes in the simultaneous move game.*

Note that, if $f(.) = M - T_1 - T_2$ so that the second derivative is zero, then θ_1 and θ_2 are always strategic complements.

Lemma 3.3 *If β increases, then equilibrium θ_i^L increases.*

Proof. See appendix A2. ■

It can be shown that the partial effect of β on θ_i is positive. In case of strategic complementarity between θ_1 and θ_2, the total effect of β on θ_i (consisting of the partial effect as well as the indirect effect via θ_j) is unambiguously positive. In case of strategic substitutability between θ_i and θ_j, the total effect is ambiguous. However, as the above lemma proves, the effect is also positive.

4 Determination of Outcomes

To determine the first and/or the second movers' advantage in the Stackelberg game, we need to figure out the shape of iso-welfare curves of the province as well as the centre. That is, we need to determine whether the strategic variables chosen by one tier of government are plain substitutes/complements.

4.1 Provincial and Central Iso-welfare Curves

Let us first look at the provinces. The iso-welfare curve is defined by

$$\bar{w}_i = u(\theta_i + T_i) + v(y_i - \theta_i) + \beta f(M - T_1 - T_2) \tag{12}$$

The slope of the curve in the $\theta_i - T_i$ plane[10] is $\frac{dT}{d\theta}|_{\bar{w}} = -\frac{w_\theta}{w_T}$ and $\frac{d^2T}{d\theta^2}|_{\bar{w}} = -\frac{w_{\theta\theta}}{w_T}$ *on the reaction function of province.* Therefore, $sign\left(\frac{d^2T}{d\theta^2}|_{\bar{w}}\right) = sign\,(w_T)$ at the critical point. Now $w_T = u'(p) - \beta f'(P)$. Theoretically, higher central transfer leads to higher output from provincial project, but the amount spent on central output reduces *quid-pro-quo*, and hence there is loss of provincial utility. For low values of $\beta\,(\approx 0)$, higher T is likely to be associated with higher provincial welfare. Hence T is plain complement (PC)[11] for province and the iso-welfare curves achieve a *minimum* on the reaction function. On the other hand, if β is sufficiently high, then the provincial iso-welfare curves achieve a maximum on the reaction functions, and higher T is associated with lower provincial welfare.

Example 4.1 *Let us illustrate the point using the LQ specification. We have $w_T = 1 - \delta p - \beta(1 - \lambda P)$. Suppose $\lambda = 0$. Then, T_i will always be a plain substitute for province i if β is close to* 1.

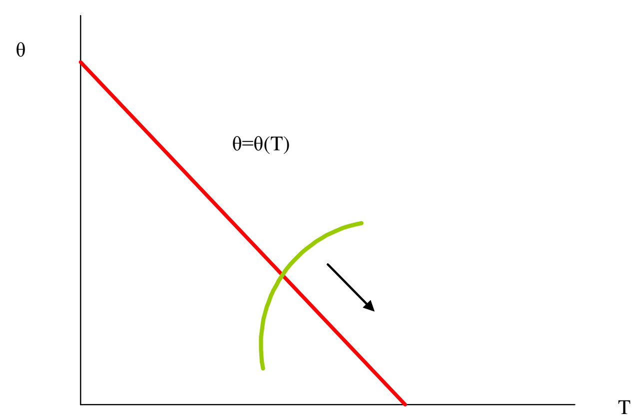

Fig 2: T is Plain Complement for province, $w_T > 0$, low β

[10] So as to avoid division by zero. We have also removed subscripts to retain notational clartity.

[11] See appendix A1.

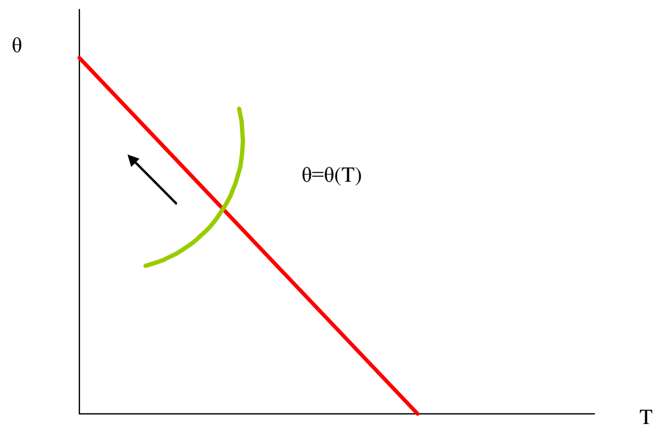

Fig 3: T is Plain Substitute (PS) for province, $w_T < 0$, high β

4.1.1 Central Iso-welfare Curve

The Central Iso Welfare curve is

$$\bar{W} = V(M - T_1 - T_2) + \gamma \left[F(\theta_1 + T_1) + F(\theta_2 + T_2) \right]$$

Following similar logic, we find that, *on* the central reaction function facing province i, $\frac{d^2\theta}{dT^2}|_{\bar{W}} = -\frac{W_{TT}}{W_\theta}$, such that $sign\left(\frac{d^2\theta}{dT^2}|_{\bar{W}}\right) = sign\,(W_\theta)$. Now $W_\theta = \gamma F'() > 0$. hence $W_\theta > 0$. *i.e.* the central iso-welfare curve always has a minimum on the reaction function of the centre and higher θ implies higher welfare for centre (provincial tax is always a plane . These possibility is depicted in the following diagram.

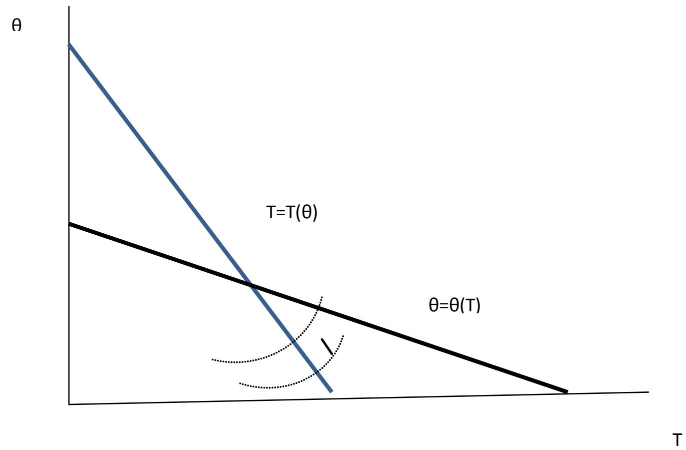

Fig 5: $W_\theta > 0$: Central isowelfare curves are convex to the origin.

4.2 Second Movers' Advantage

Now we are ready to state our main proposition.

Proposition 4.1 *Assume that central grants are targeted to a specific provincial project. If the weight attached to central good by the provinces (β) is high, then central transfer is plain substitute for the province. As a result, centre is better off as a second mover. Otherwise, centre is better off as a first mover.*

Proof. See appendix A2. ■

This discussion can be summarized in the following diagram:

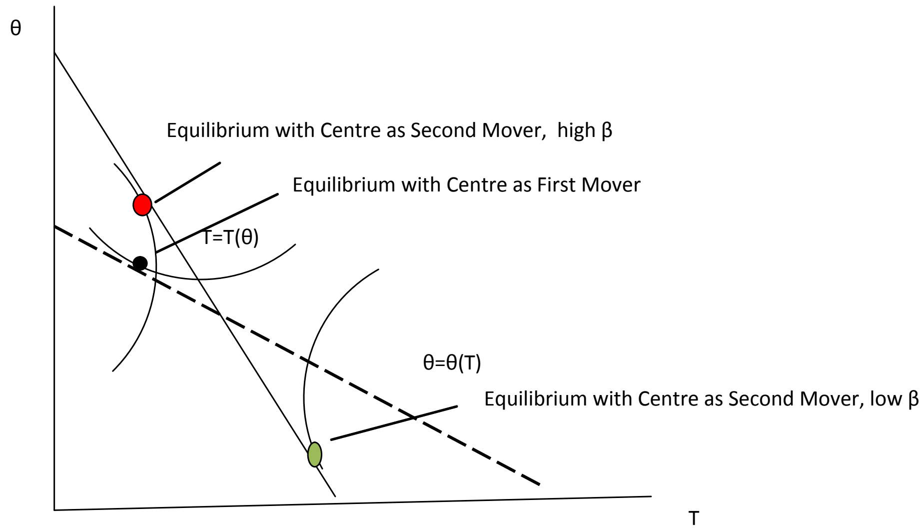

Fig 6: Centre Gains as Second Mover if $w_T < 0$ and $W_\theta > 0$.

Note that, given $W_\theta > 0$, centre wish to settle for θ as high as possible. It is evident that, if β is low, the provincial authority would like to have a high central transfer-low provincial taxes regime (because the cost of public good can be effectively shifted to the centre and it does not depend on central public good so much). In that case, centre gains by being the first mover and restrict the transfer.

In a similar way, if $w_T < 0$, then province also prefers to be the second mover (the proof is similar to that of proposition 3.1). In other words, both players have an incentive to be the second mover. The deadlock is broken by the overriding authority of the centre.

5 Comparison of Two Equilibria

Here. we compare the two equilibria in terms of project outcomes, transfers and federal welfare. The main results are derived through two propositions.

Assuming identical provinces, provincial will be identical and can be represented by a common notation θ. Similarly, central transfer, in equilibrium, will be same for both provinces and can be denoted by a generic T. Let the three equilibria (Nash, and two Stackelberg points) be given by $\langle \theta^N, T^N \rangle$, $\langle \theta^F, T^L \rangle$ and $\langle \theta^L, T^F \rangle$, respectively. We now have the following proposition.

Proposition 5.1 *(i) Assume β to be small such that $w_T > 0$. Then, $\theta^F > \theta^N > \theta^L$ and $T^F > T^N > T^L$.*

(ii) If β is large enough, then $\theta^L > \theta^F > \theta^N$ and $T^F > T^L > T^N$ with the LQ example.

Proof. See appendix A2. ■

As a corollary, we immediately have the following, for small β (when the constitutional outcome is centre as first mover)

Corollary 5.1 *(ai) Central transfers are lowest when centre is a leader. That is, central project has highest output when centre is leader: $P^L > P^F$.*

(aii) Central leadership point is associated with highest provincial taxes, that is $\theta^F > \theta^L$. The effect on provincial public good, however, is ambiguous. As T decreases from T^F to T^N, θ increases along the central reaction function and the marginal response is more than 1,*so that provincial public good increases, $p^N > p^L$. As T further increases from T^N to T^L, θ increases along the provincial reaction function and the marginal response is less than* 1, *which means lower provincial public good. The ultimate effect will depend on (given the slope of reaction functions), the relative distances between T^F, T^N and T^L.*

The next corollary summarizes that with (sufficiently) higher β (when the constitutional outcome is centre as follower)

Corollary 5.2 *(bi) Central transfers are higher with centre as second mover (that is $T^F > T^L$). As a result, central project yields lower output ($P^F > P^L$)*

(bii) Provincial tax efforts are higher when provinces are lead $\theta^F > \theta^L$. Thus $p^L > p^F$.

6 Conclusion

In this paper, we have provided a general scenario under which the central government gains by being a follower in the grant dispensation game.

If central government grant is tied up with the public project of the province, provincial tax and central transfer are strategic substitutes. Higher central transfer lowers the marginal utility of public project to the province and province responds by cutting down taxes. If central welfare is increasing in provincial taxes, then centre should choose a mode of transfer which generates high provincial taxes.

The work can be extended to several dimensions. First, we have demonstrated the result with one type of grant. In a federal economy, the grants may just augment provincial budget instead of being tied to a project. Second, a key assumption of the paper is centre has an over-riding presence in dictating the mode of transfer. In many economies, the balance of power between centre and provinces are determined by bargaining, e.g. as in the nascent years of the USA. This opens up the possibility of a timing game to resolve the tie of leader/follower. Third, we have assumed that for each province, β is same. But suppose it is not: some provinces, due to political alognment with centre, has a high β, other provinces have low perceived β. Then centre may be a first(second) mover with the latter (former) group of provinces: we would expect provincial debt services being assumed by the central government in the politically aligned provinces. This suggests that a familiar effect of partisan behaviour turns out not from the fact that centre putting different weight on provinces (as assumed in traditional political economy literature, e.g. Sengupta 2012), but from provinces placing different weights on central project

Thus there exists future scope of research based on the current work.

Appendix A1
Strategic and Plain Complements

Briefly, suppose $\pi^i(a_i, a_j)$ is benefit function for agent i, while a_i and a_j are own actions and other agents' action, respectively. Then a_j is plain complement (PC) for agent i if $\dfrac{\partial \pi^i}{\partial a_j} > 0$, and plain substitutes (PS) if $\dfrac{\partial \pi^i}{\partial a_j} < 0$. Similarly, a_i and a_j are strategic substitutes ,SS (complements, SC) if $\dfrac{\partial^2 \pi^i}{\partial a_i \partial a_j} < (>) 0$. Similarly for agent j.

The first order condition for agent i is

$$\frac{\partial \pi^i}{\partial a_i} = 0$$

Differentiating with respect to a_j, we have

$$\begin{aligned} \frac{\partial^2 \pi^i}{\partial a_i^2} \frac{da^i}{da_j} + \frac{\partial^2 \pi^i}{\partial a_i \partial a_j} &= 0 \\ \rightarrow \quad \frac{da^i}{da_j} &= -\frac{\frac{\partial^2 \pi^i}{\partial a_i \partial a_j}}{\frac{\partial^2 \pi^i}{\partial a_i^2}} \end{aligned}$$

If SOC holds, the sign of cross (double) derivative determines the slope of reaction functions. In the same vein, sign of cross(single) derivative determine the shape of isoprofit curves near the reaction function.

Let the isoprofit curve for agent i in $a_i - a_j$ plane be $\pi^i(a_i, a_j) = \bar{k}$. The slope is

$$\frac{da_j}{da_i} = -\frac{\pi_i^i}{\pi_j^i}.$$

Here, the lower subscripts denote partial derivatives, i.e. $\frac{\partial \pi^i}{\partial a_i} = \pi_i^i$. On the reaction function, $\pi_i^i = 0$, so that is a critical point of the isoprofit(or iso-welfare) curve. Notice that

$$\begin{aligned}\frac{d^2 a_j}{da_i^2} &= -\frac{\pi_{ii}^i \left(\pi_j^i\right)^2 + \pi_{jj}^i \left(\pi_i^i\right)^2 - 2\pi_{ij}^i \pi_i^i \pi_j^i}{\left(\pi_j^i\right)^3} \\ &= -\frac{\pi_{ii}^i}{\pi_j^i} \text{ at } \pi_i^i = 0\end{aligned}$$

Thus, $sign\left(\frac{d^2 a_j}{da_i^2}\right) = sign\left(\pi_j^i\right)$ at the critical point.

Appendix A2

Proof of Lemma 3.1

From equation 10 , we can write the Hessian matrix

$$
\begin{aligned}
H &= \begin{bmatrix} \frac{\partial^2 W}{\partial T_1^2} & \frac{\partial^2 W}{\partial T_1 \partial T_2} \\ \frac{\partial^2 W}{\partial T_1 \partial T_2} & \frac{\partial^2 W}{\partial T_2^2} \end{bmatrix} \\
&= \begin{bmatrix} V'' + \gamma F'' k^2 & V'' \\ V'' & V'' + \gamma F'' k^2 \end{bmatrix}
\end{aligned}
$$

Here $k = 1 + \dfrac{\partial \theta_i}{\partial T_i} = \dfrac{\rho}{\delta + \rho}$. Of course, $V'' + \gamma F'' k^2 < 0$ and $|H| = \left(V'' + \gamma F'' k^2\right)^2 - \left(V''\right)^2 > 0$. Thus $W(.)$ is concave.

Differentiating the FOC with respect to γ, we get the following matrix equation

$$
\begin{bmatrix} V'' + \gamma F'' k^2 & V'' \\ V'' & V'' + \gamma F'' k^2 \end{bmatrix} \begin{bmatrix} \frac{dT_1}{d\gamma} \\ \frac{dT_2}{d\gamma} \end{bmatrix} = \begin{bmatrix} -kF' \\ -kF' \end{bmatrix}
$$

$$
\Longrightarrow \frac{dT_i^L}{d\gamma} = -\frac{kF'(\gamma k^2 F'')}{|H|} > 0 \text{ for } i = 1, 2
$$

Thus $\frac{dP^L}{d\gamma} = -\frac{d(\sum T_i)}{d\gamma} < 0$, $\frac{d\theta_i^F}{d\gamma} = \theta'\,()\,\frac{dT_i}{d\gamma} < 0$ and $\frac{dp_i^F}{d\gamma} = \left(1 + \theta'(T_i)\right) \frac{dT_i^L}{d\gamma} > 0$.

Proof of Lemma 3.2

Equation (11) can be written as, say for province 1:

$$
\frac{\partial w^1}{\partial \theta_1} = u'(\theta_1 + T_1)\left(1 + \frac{\partial T_1}{\partial \theta_1}\right) - v'\left(y_1 - \theta_1\right) + u'(\theta_1 + T_1)\frac{\partial T_1}{\partial \theta_1} - \beta f'(P)\left(\frac{\partial T_1}{\partial \theta_1} + \frac{\partial T_2}{\partial \theta_1}\right) = 0
$$

Assuming LQ functional forms, $\dfrac{\partial T_1}{\partial \theta_1}$ and $\dfrac{\partial T_1}{\partial \theta_1}$ are constant numbers. Differentiating above expression with respect to θ_2, we get

$$
\frac{\partial^2 w^1}{\partial \theta_1 \partial \theta_2} = u'' \frac{\partial T_1}{\partial \theta_2}\left(1 + 2\frac{\partial T_1}{\partial \theta_1}\right) + \beta f''(P)\left(\frac{\partial T_1}{\partial \theta_1} + \frac{\partial T_2}{\partial \theta_1}\right)\left(\frac{\partial T_1}{\partial \theta_2} + \frac{\partial T_2}{\partial \theta_2}\right)
$$

Notice that the first term is positive, since $u'' = -\delta < 0$, $\frac{\partial T_1}{\partial \theta_2} = \frac{\eta}{2\eta+\gamma\phi} > 0$ and $1 + 2\frac{\partial T_1}{\partial \theta_1} = 1 - 2\frac{\eta+\gamma\phi}{2\eta+\gamma\phi} = -\frac{\gamma\phi}{2\eta+\gamma\phi} < 0$. On the other hand, $f'' = -\lambda < 0$ and sum of $T_1 + T_2$ falls as θ_1 or θ_2 increases. So the second term is definitely negative. If β is large enough, then, even if the first term is positive, the whole expression $\frac{\phi\gamma}{(2\eta+\gamma\phi)^2}(\delta\eta - \beta\lambda\gamma\phi)$ is negative. Hence θ_1,θ_2 are strategic substitute since $\frac{\partial\theta_1}{\partial\theta_2} = -\frac{\left(\frac{\partial^2 w^1}{\partial\theta_1\partial\theta_2}\right)}{SOC} < 0$.[12]

Proof of Lemma 3.3

To show this, we proceed in two steps. First, we show the (partial) effect of β on θ_i^L. Second, we demonstrate the total effect of β on θ_i^L(consisting of the direct effect as well as the indirect effect via θ_j^L.

In order to do so, we write the reaction functions (implicitly defined by 11) as $\theta_i = \Theta^i(\theta_j;\beta)$. Notice that, we have already demonstrated the fact that (lemma 2.1), if β is above a threshold then $-1 < \frac{\partial\Theta^i}{\partial\theta_j} < 0$.

To find the direct effect of β on θ_i, we need to find $\frac{\partial\Theta^i}{\partial\beta} = -\frac{\left(\frac{\partial^2 w^1}{\partial\theta_1\partial\beta}\right)}{SOC}$. Given our assumptions, $\frac{\partial^2 w^1}{\partial\theta_1\partial\beta} = -f'(P)\left(-\frac{\gamma\phi}{2\eta+\gamma\phi}\right) > 0$, so an increase in β increases θ_i, if nothing else changes.

To obtain the total effect (because of the ambiguity that higher β increases both θ_i and θ_j, but higher θ_j reduces θ_i) one has to differentiate the equations $\theta_i = \Theta^i(\theta_j;\beta)$ and $\theta_j = \Theta^j(\theta_i;\beta)$ and obtain $\frac{d\theta_i}{d\beta}$ etc. The resulting matrix equation is

$$\begin{bmatrix} 1 & -\frac{\partial\Theta^i}{\partial\theta_j} \\ -\frac{\partial\Theta^j}{\partial\theta_i} & 1 \end{bmatrix} \begin{bmatrix} \frac{d\theta_i}{d\beta} \\ \frac{d\theta_j}{d\beta} \end{bmatrix} = \begin{bmatrix} \frac{\partial\Theta^i}{\partial\beta} \\ \frac{\partial\Theta^j}{\partial\beta} \end{bmatrix}$$

Here, $\Delta = \begin{vmatrix} 1 & -\frac{\partial\Theta^i}{\partial\theta_j} \\ -\frac{\partial\Theta^j}{\partial\theta_i} & 1 \end{vmatrix} = 1 - \left(\frac{\partial\Theta^i}{\partial\theta_j}\right)\left(\frac{\partial\Theta^j}{\partial\theta_i}\right) > 0$

[12] The SOC is $\frac{\partial^2 w_1}{\partial\theta_1^2} = -\frac{\delta\eta^2}{(2\eta+\gamma\phi)^2} - \rho - \beta\lambda\left(\frac{\gamma\phi}{2\eta+\gamma\phi}\right)^2 < 0$. It can be shown that the magnitude of the slope of the reaction function in $\theta_i - \theta_j$ space is less than one (Similarly for the other province. That is, the Nash equilibrium is stable.

Thus,

$$\frac{d\theta^i}{d\beta} = \frac{\begin{vmatrix} \frac{\partial\Theta^i}{\partial\beta} & -\frac{\partial\Theta^i}{\partial\theta_j} \\ \frac{\partial\Theta^j}{\partial\beta} & 1 \end{vmatrix}}{\Delta} = \frac{\frac{\partial\Theta^i}{\partial\beta} + \frac{d\Theta^j}{d\beta}\frac{\partial\Theta^i}{\partial\theta_j}}{\Delta}$$

If we have LQ example (where provinces only differ by income level), $\frac{\partial\Theta^i}{\partial\beta} = \frac{\partial\Theta^j}{\partial\beta}$. So the numerator is $\frac{\partial\Theta^i}{\partial\beta}\left(1 + \frac{\partial\Theta^i}{\partial\theta_j}\right) > 0 \rightarrow \frac{d\theta_i^L}{d\beta} > 0$. Similarly for $\frac{d\theta_j^L}{d\beta}$

Thus, $\frac{d(\sum T_i^F)}{d\beta} = -\frac{\gamma\phi}{2\eta+\gamma\phi}\frac{d(\sum\theta_i^L)}{d\beta} < 0 \rightarrow \frac{dP^F}{d\beta} = -\frac{d(\sum T_i^F)}{d\beta} > 0$ and $\frac{dp_i^L}{d\beta} = \frac{d\theta_i^L}{d\beta} - \frac{\eta+\gamma\phi}{2\eta+\gamma\phi}\frac{d\theta_i^L}{d\beta} + \frac{\eta}{2\eta+\gamma\phi}\frac{d\theta_j^L}{d\beta}$. Since $\frac{d\theta_j^L}{d\beta} \equiv \frac{d\theta_i^L}{d\beta}$ we have $\frac{dp_i^L}{d\beta} = 2\left(2\eta+\gamma\phi\right)^{-1}\eta\frac{d\theta^L}{d\beta} > 0$. Thus transfers go down, but local taxation rises to compensate such that local project outputs increase.

Proof of Proposition 4.1

In this and subsequent proofs, We are omitting other variables (θ_j, T_j) for notational clarity. Also, the subscripts are dropped: that is $w^i(\theta_i^E, T_i^E | T_j^E, \theta_j^E) \equiv w^i(\theta_i, T_i^E)$[13] $\equiv w(\theta^E, T^E)$ (subscripts are dropped). Here, the superscript E stands for different equilibria, e.g. Nash or Stackelberg. Similarly for the centre.

Second part of the proposition is evident: so only the first part is proved. The proof[14] follows Dowrick (1986). Notice that, we need to show that the Centre is better off as a Stackelberg follower than leader.

Proof. The proof proceeds in two stages. In the following figure, we have drawn reaction function of the centre and a Stackelberg point B (θ_L, T_F) such that province is the leader (the iso-welfare curve is not necessary).

[13] T_j being a parameter in the equilibrium

[14] The proof does not depend on linear quadratic assumption.

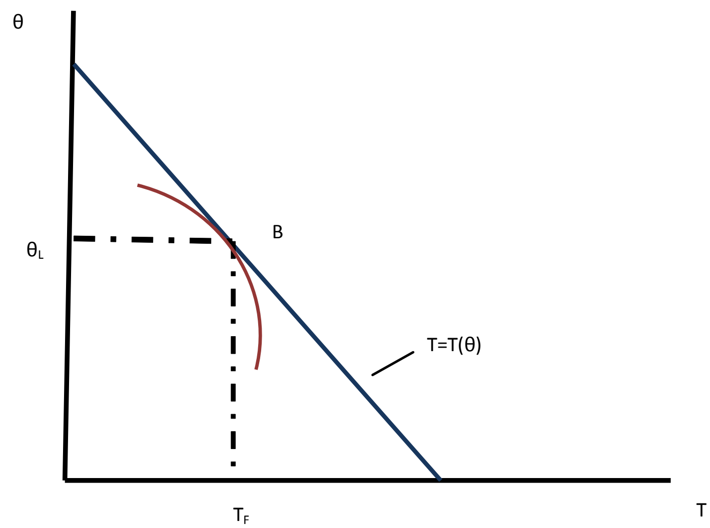

Fig A1: A Stackelberg Point of Province

First, we show that the reaction function of the province , $\theta = \theta(T)$ must be below B, i.e. $\theta(T_F) < \theta_L$.

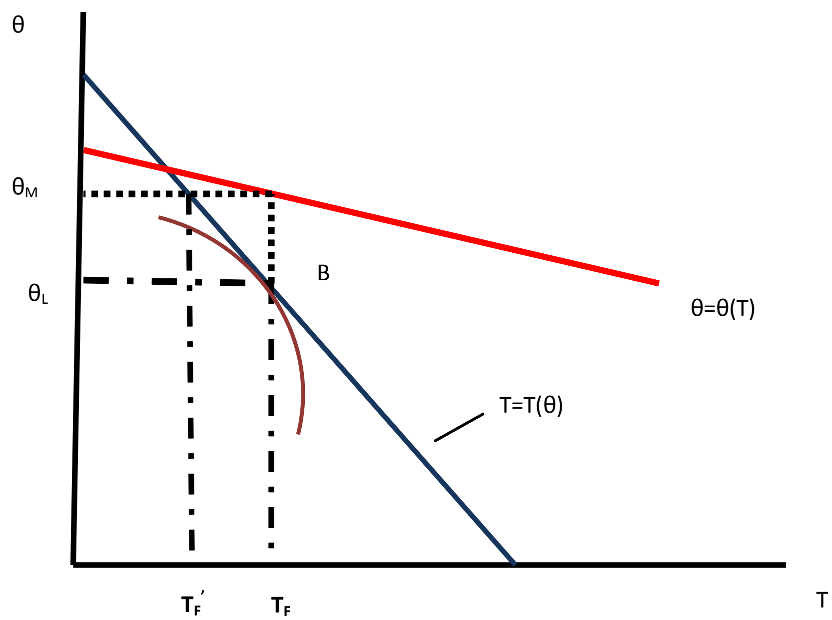

Fig A2: Position of the Provincial Reaction Function

Suppose not. Then $\theta_M = \theta(T_F) > \theta_L$. Thus we have, for the province, $w(\theta_M, T_F) > w(\theta_L, T_F)$ (by the definition of reaction function). Let, T_F' be the best response to θ_M, that is, $T_F' = T(\theta_M)$. But, then since $w_T < 0$, we have $w(\theta_M, T_F) < w(\theta_M, T_F')$. Combining these inequalities, we get $w(\theta_M, T_F') > w(\theta_m, T_F) > w(\theta_L, T_F)$. But then,, (θ_L, T_F) cannot be the Stackelberg leadership point of the province on the reaction function of centre.

Second, $\theta = \theta(T)$ cannot pass through the Stackelberg leadership point B. Notice that the isoprofit curve of the province must have zero slope on $\theta = \theta(T)$. For simultaneous tangency on $T = T(\theta)$ a critical point of $\theta = \theta(T)$, the reaction function $T = T(\theta)$ must be positively sloped. But this case is ruled out either. Thus, $\theta = \theta(T)$ must be below B, the Stackelberg leadership point of the province. So, $\theta_M < \theta_L$.

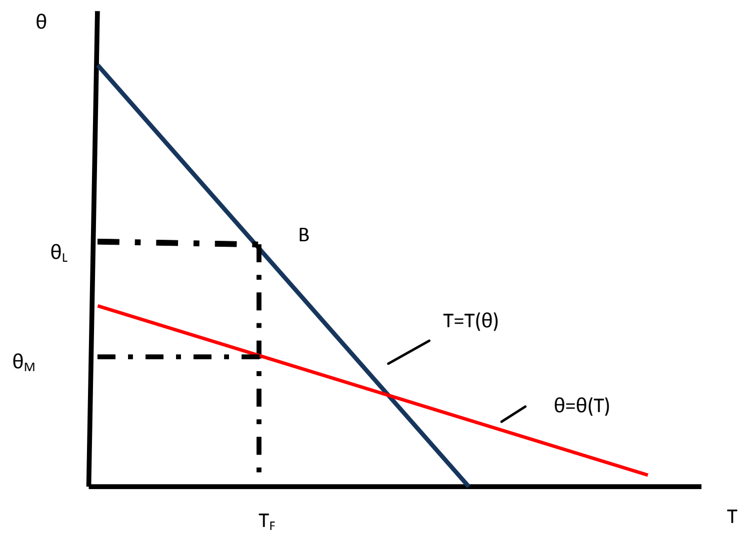

Fig A3: Centre Prefers to be Stackelberg Leader.

Note that, $W(\theta_L, T_F) > W(\theta_M, T_F)$, since $W_\theta > 0$. But, (θ_M, T_F) is one of the set of points which the centre can choose as a Stackelberg leader. Therefore, centre must prefer to be a Stackelberg follower if $W_\theta > 0$ and $w_T < 0$. ■

Proof of Proposition 5.1

As before, we are focussing on the interaction between one province and the corresponding transfer. The lne of reasoning follows Kemp and Graziosi, *ibid.*

Proof. *Part (i)* Comparing provincial Nash and Leadership position, we have

$$w\left(\theta^L, T^F\left(\theta_L\right)\right) \geq w(\theta^N, T^N) \tag{A2.1}$$

by definition of Nash equilibrium and Stackelberg leadership.

When province is follower in a Stackelberg game or under Nash protocol, equilibrium occurs *on* provincial reaction function. Thus

$$\frac{\partial w(\theta^N, T^N)}{\partial \theta} = \frac{\partial w(\theta^F, T^L)}{\partial \theta} = 0 \qquad \text{A2.2}$$

by definition of reaction function. Given $w_{T\theta} < 0$

$$\theta^N > \theta^F \Longleftrightarrow T^N < T^L \tag{A2.3}$$

Again, by definition of Nash equilibrium:

$$w(\theta^N, T^N) \geq w\left(\theta^L, T^N\right)$$

Here, we are considering the subcase where $w_T > 0$. Suppose $T^N > T^F$. Then

$$w(\theta^N, T^N) \geq w\left(\theta^L, T^N\right) > w\left(\theta^L, T^F\right)$$

But this contradicts with the definition of Stackelberg leadership. Therefore, we must have $T^N < T^F$. Notice that T^N and T^F are on the same (downward sloping) central reaction function. The $\theta^N > \theta^L$. Similarly, for the centre ($W_{T\theta} < 0$ and $W_\theta > 0$), we must have $\theta^F > \theta^N$. But θ^N and θ^F are two points on the (downward sloping) provincial reaction function. So we must have $T^L < T^N$. Combining these observations, we have result 5.1 (i).

For part (ii), notice that β is high, so that $w_T < 0$. Applying the same methodology as above, we have $T^N > T^F$, i.e. $\theta^N < \theta^L$. At the same time, $\theta^F > \theta^N$ i.e. $T^L < T^N$. That is, we have $\left(\theta^F, \theta^L\right) > \theta^N$ and $\left(T^F, T^L\right) < T^N$. However, the relative magnitudes of the leadership positions with the follower position is not known.

Let us assume $T^L > T^F$.

We compare the first order conditions of the equilibrium of the centre. We already know that, when centre is the follower, as well as in Nash outcome

$$\frac{\partial W(\theta^L, T^F)}{\partial T} = \frac{\partial W(\theta^N, T^N)}{\partial T} = 0$$

We compare this with the case when centre is the leader. It solves the problem

$$\max_T W(T, \theta(T)) = 0$$

The FOC yields

$$\frac{\partial W(\theta^F, T^L)}{\partial T} + W_\theta * \frac{\partial \theta}{\partial T} = 0$$

Note that $\frac{\partial \theta}{\partial T} = -\frac{w_{\theta T}}{w_{\theta\theta}} < 0$. Given $W_\theta > 0$, we must have $\frac{\partial W(\theta^F, T^L)}{\partial T} > 0$. Thus

$$\frac{\partial W(\theta^F, T^L)}{\partial T} > \frac{\partial W(\theta^L, T^F)}{\partial T} = \frac{\partial W(\theta^N, T^N)}{\partial T}$$

If $T^L > T^F$, then $\theta^F < \theta^L$.

Notice $\frac{\partial W(\theta^L, T^F)}{\partial T} > \frac{\partial W(\theta^L, T^L)}{\partial T}$ since MU falls with T and $T^L > T^F$. Second $\frac{\partial W(\theta^L, T^L)}{\partial T} > \frac{\partial W(\theta^F, T^L)}{\partial T}$ since MU falls with θ ($W_{T\theta} < 0$)and $\theta^L > \theta^F$. Combining these two statements, $\frac{\partial W(\theta^L, T^F)}{\partial T} > \frac{\partial W(\theta^F, T^L)}{\partial T}$ which is a contradiction.

Thus $T^F > T^L > T^N$.

Similarly, when province is the follower, and under Nash

$$\frac{\partial w(\theta^F, T^L)}{\partial \theta} = \frac{\partial w(\theta^N, T^N)}{\partial \theta} = 0$$

when province is leader, it maximises

$$\max_\theta w(\theta, T_i(\theta, .).T_j(., \theta))$$

FOC is

$$w_\theta + \left[w_{T_i} * \frac{\partial T_i}{\partial \theta} + w_{T_j} * \frac{\partial T_j}{\partial \theta} \right] = 0$$

Notice that, $w_{T_i} < 0$ and $\frac{\partial T_i}{\partial \theta_i} < 0$. On the other hand, $w_{T_j} < 0$ and $\frac{\partial T_j}{\partial \theta} > 0$ (Note: $w_{T_i} = u' - \beta f'$ and $w_{T_j} = -\beta f'$). Using the LQ specification, $\frac{\partial T_i}{\partial \theta_i} = -\frac{\eta + \gamma\phi}{2\eta + \gamma\phi}$ and $\frac{\partial T_j}{\partial \theta_i} = \frac{\eta}{2\eta + \gamma\phi}$. Thus

$$\begin{aligned}
& w_{T_1} * \frac{\partial T_1}{\partial \theta_1} + w_{T_2} * \frac{\partial T_2}{\partial \theta_1} \\
= & -\left(u' - \beta f'\right) \frac{\eta + \gamma\phi}{2\eta + \gamma\phi} + \frac{\eta}{2\eta + \gamma\phi} \left(-\beta f'\right) \\
= & \beta f' \left(\frac{\eta}{2\eta + \gamma\phi} \right) - u' \left(\frac{\eta + \gamma\phi}{2\eta + \gamma\phi} \right)
\end{aligned}$$

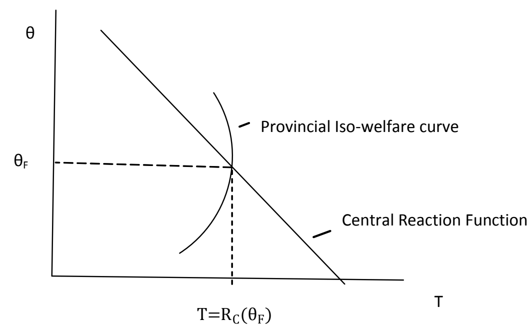

Figure 1: Fig A4: Condition for $\theta^L > \theta^F$.

We already know that β is sufficiently large such that $\beta f' > u'$. We need to impose a mildly stringent restriction on β, i.e. $\beta f' > u'\left(1 + \frac{\gamma\phi}{\eta}\right)$ such that expressions under square brackets is positive and

$$\frac{\partial w(\theta^L, T^F)}{\partial \theta} < \frac{\partial w(\theta^F, T^L)}{\partial \theta} = \frac{\partial w(\theta^N, T^N)}{\partial \theta}$$

A comparison between the first two terms imply that one cannot rank θ^L and θ^F from the condition stated above. However, if the isowelfare curve of the province cuts the reaction function of the centre at the point $\left(\theta^F, \tilde{T} = R_C(\theta^F)\right)$, then the tangency (which defines θ^L) between provincial isoprofit curve and central reaction function occurs at a *higher* point, that is $\theta^L > \theta^F$. The possibility is shown in the following diagram.

Slope of the provincial isoprofit curve $\frac{d\theta}{dT}|_{(\theta^F,\tilde{T})} = -\dfrac{u' - \beta f'}{u' - v'}$, while that of central reaction function in the $\theta_i - T_i$ plane is A_i (e.g. for LQ assumption, it is $\dfrac{2\eta + \gamma\phi}{\eta + \gamma\phi}$). So we need $\dfrac{u' - \beta f'}{u' - v'}|_{(\theta^F,\tilde{T})} > A_i$. Since the point $\left(\theta^F, \tilde{T}\right)$ is above the reaction function of the province, we must have $u' - v' < 0$. The inequality then suggests $\beta > \hat{\beta} = \dfrac{u' - A_i * (u' - v')}{f'}|_{(\theta^F,\tilde{T})}$ is required for $\theta^L > \theta^F > \theta^N$. This is also partially corroborated by lemma 3.3.

It is easy to (but tedious) figure out $\hat{\beta}$ for the LQ example. As we know, $T_i^L = \dfrac{M\eta - 1 + \alpha\gamma - (\eta + \alpha\gamma\phi)\, y_i + \eta y_j}{2\eta + \alpha\gamma\phi} \rightarrow \theta_i^F = \alpha y_i - (1-\alpha)\, T_i^L$, where $\alpha = \frac{\rho}{\rho+\delta}$ This, in turn, implies that

$$\begin{aligned}
\hat{T}_i &= \frac{\gamma + M\eta - 1}{2\eta + \gamma\phi} - \frac{\eta + \gamma\phi}{2\eta + \gamma\phi}\theta_i^F + \frac{\eta}{2\eta + \gamma\phi}\theta_j^F \\
\hat{p}_i &= \theta_i^F + \hat{T}_i \\
\hat{P} &= \frac{M\gamma\phi + 2(1-\gamma)}{2\eta + \gamma\phi} + \frac{\gamma\phi}{2\eta + \gamma\phi}\left(\theta_1^F + \theta_2^F\right)
\end{aligned}$$

From here, we can calculate $u' = 1 - \delta\hat{p}_i$, $v' = 1 - \rho(y_i - \theta_i^F)$ and $f' = -1 + \lambda\left(\hat{P}\right)$. ■

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