Testing for Non-Fundamentalness

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1 June 2016

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Abstract

Non-fundamentalness arises when observed variables do not contain enough information to recover structural shocks. This paper propose a new test to empirically detect non-fundamentalness, which is robust to the conditional heteroskedasticity of unknown form, does not need information outside of the specified model and could be accomplished with a standard F-test. A Monte Carlo study based on a DSGE model is conducted to examine the finite sample performance of the test. I apply the proposed test to the U.S. quarterly data to identify the dynamic effects of supply and demand disturbances on real GNP and unemployment.

Keywords: Non-Fundamentalness; Invertibility; Vector Autoregressive.

JEL classification: C5, C32, E3.

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1 Introduction

Structural Vector Autoregressive (SVAR) models have been used extensively for economic analysis. The underlying assumption of SVAR, known as fundamentalness, is that we are able to recover the structural shocks driving the process from linear combinations of observed present and past values of the process, by imposing proper identification restrictions. Once the representation is non-fundamental, all identification schemes fail to recover the true structural shocks.

In this paper, I propose a test to empirically detect whether the shocks recovered from the estimation of a VAR are truly fundamental. I prove that the reduced form residuals are predictable if and only if the model is non-fundamental. The test is simple to implement with common econometrics packages, since the test statistic is F-distributed under the null of fundamentalness. Finally, I apply the proposed test to the U.S. quarterly data to identify the dynamic effects of supply and demand disturbances on real GNP and unemployment. I test whether the small scale SVAR model considered by Blanchard and Quah (1989) (hereafter BQ) is fundamental. I find that the baseline VAR model of BQ is non-fundamental, and therefore, the impulse responses and variance decompositions obtained from this model is not reliable.

2 Characterization of non-fundamental VARMA representations

Consider the following d-variate zero mean VARMA(p,q) model in standard representation:

$$x_t = \sum_{i=1}^{p} \phi_i x_{t-i} + \xi_t + \sum_{j=1}^{q} \theta_j \xi_{t-j}.$$

The vectors x_t and ξ_t contain the d univariate time series: $x_t = [x_{1t}, x_{2t}, \cdots, x_{dt}]$ and $\xi_t = [\xi_{1t}, \xi_{2t}, \cdots, \xi_{dt}]$. We can also write the previous equation in lag operators:

$$\Phi(L)x_t = \Theta(L)\xi_t \ , \qquad t = 0, \pm1, \pm2, \cdots \tag{2.1}$$

where $\Phi_p \neq 0$ and $\Theta_q \neq 0$ and L is the lag operator, i.e., $Lx_t = x_{t-1}$. The polynomials $\Phi(\cdot)$ and $\Theta(\cdot)$ have no common roots and neither of the roots is on the unit circle. Moreover, $\{\xi_t\}$ is an unpredictable process, also known as martingale difference. A real-valued stationary time series $\{\xi_t\}_{t=-\infty}^{\infty}$ is a martingale difference (MD) process if $E[\xi_t|\xi_{t-1}, \xi_{t-2}, \cdots] = 0$.

A VARMA process defined by (2.1) is said to be fundamental, also known as invertible, if and only if all the roots of $\det[\Theta(z)]$ lie outside the unit circle in the complex plane.[1] One can show that if non-fundamental representation is excluded by mistake, the resulting process has a representation given by

$$\Phi(L)x_t = \tilde{\Theta}(L)\tilde{\xi}_t \tag{2.2}$$

where $\tilde{\Theta}(L)$ has the same order as $\Theta(L)$ but all its roots are outside the unit circle and $\{\tilde{\xi}_t\}$ are the Wold innovations related to the original innovations, $\{\xi_t\}$, through

$$\tilde{\xi}_t = \tilde{\Theta}^{-1}(L)\Theta(L)\xi_t \tag{2.3}$$

where $\tilde{\Theta}^{-1}(L)\Theta(L)$ is the *Blaschke* factor (Lippi and Reichlin, 1994). Therefore, (2.2) can be written as a VAR(∞) form:

$$\tilde{\Theta}(L)^{-1}\Phi(L)x_t = \sum_{j=0}^{\infty} \gamma_j x_{t-j} = \tilde{\xi}_t \tag{2.4}$$

In this paper, I use the information available in the higher order moments to propose a new test which is robust to the conditional heteroskedasticity of unknown form.

Assumption 1. Let $\{\xi_t\}$ be a vector of shocks and $\{\xi_{jt}\}$ denote the jth element of this vector. Then, (a) for all j, $\{\xi_{jt}\}$ is a m.d.s., continuously distributed with a non-Gaussian distribution such that $(a+1)$st moment finite with $(a+1)$st cumulant nonzero for some $a \geq 2$, and (b) there exists a $j \in 1, \cdots, d$ such that $\phi_{\xi_{jt}+\xi_{jt'}}(\tau) = \phi_{\xi_{jt}}(\tau)\phi_{\xi_{jt'}}(\tau)$ for any

[1] Fundamentalness is slightly different from invertibility, since invertibility requires that no roots of the MA component be on or inside the unit circle. In this framework, they are equivalent since unit root in the MA polynomial is ruled out.

$t \neq t'$, where $\phi_{\xi_t}(\tau)$ denote the characteristic function of $\{\xi_t\}$.

Proposition.1: Let Assumption 1 hold. The VARMA model (2.1) is non-fundamental if and only if the Wold innovations are predictable.

For the proof see Hamidi Sahneh (2014). Proposition 1 implies that one can detect non-fundamentalness by testing if the residuals of the reduced form VAR are unpredictable, i.e.,

$$\mathrm{E}[\hat{\xi}_t|\hat{\xi}_{t-j}] \neq 0, \text{ for some} j \geq 1. \tag{2.5}$$

In this paper, I take advantage of the powerful result of Bierens (1990) to propose a simple test for (2.5). This result essentially states that

$$\mathrm{E}[\hat{\xi}_t|\hat{\xi}_{t-j}] \neq 0, \text{ if and only if } \mathrm{E}[\hat{\xi}_t\Psi(\hat{\xi}_{t-j})] \neq 0 \text{ for some} j \geq 1 \tag{2.6}$$

where Ψ belongs to the class of *generically comprehensively revealing* (GCR) functions (Stinchcombe and White, 1998). An important class of functions of the GCR class includes second and higher order moments. Proposition 1 and equation (2.6) together imply that one can detect non-fundamentalness by testing for the joint significance of squares and cubes of the past residuals.

3 Monte Carlo evidence and empirical application

3.1 Simulation study

In this section I examine the finite sample performance of the proposed test based on artificial data generated from the DSGE model with fiscal foresight of Leeper et al. (2013). Assuming that agents have one period fiscal foresight, i.e.,

$$\hat{\tau} = \xi_{t,\tau} + b\xi_{t-1,\tau}$$

the equilibrium solution for capital and tax rate is:

$$\begin{bmatrix} (1-\alpha L)k_t \\ \hat{\tau} \end{bmatrix} = \begin{bmatrix} 1 & -\kappa_y b \\ 0 & 1+bL \end{bmatrix} \begin{bmatrix} \xi_{t,a} \\ \xi_{t,\tau} \end{bmatrix} \tag{3.1}$$

The determinant of (3.1) vanishes for $|b| < 1$. For the simulation exercise, I set parameter $b = (0.1, \cdots, 0.9)$ for the fundamental representation and $b = (2, \cdots, 10)$ to generate data from a non-fundamental representation. Following Leeper et al. (2013), I set $\alpha = 0.36$, $\beta = 0.95$, and $\tau = 0.25$. The structural shocks $\xi_{a,t}$ and $\xi_{\tau,t}$ are generated as *iid* $lognormal(0,1)$, mutually independent at all leads and lags. To examine the impact of the conditional heteroskedasticity of unknown form on the performance of the test, I also consider the following GARCH process: $\xi_{a,t} = \sigma_t^{\frac{1}{2}} z_t$ where z_t is *iid* $N(0,1)$ and $\sigma_t = 0.01 + 0.05\xi_{t-1}^2 + 0.95\sigma_{t-1}$. I estimate a VAR with four lags included based on a sample size of 200 which is about the size of most postwar data sets. The number of Monte Carlo replication is 1000.

The auxiliary regression that we consider is as follows:

$$\hat{\xi}_t = c + \beta_1 \hat{\xi}_{t-1}^2 + \beta_2 \hat{\xi}_{t-1}^3 + e_t, \tag{3.2}$$

and null of fundamentalness can be stated as

$$\mathbb{H}_0 : \beta_1 = \beta_2 = 0 \tag{3.3}$$

which can be tested using an standard F-test. Although we are using the estimated residuals on the right hand side of the regression, generated regressors is not an issue. This is because under the null hypothesis each fitted value has zero population coefficient, and therefore usual limit theory applies.

INSERT TABLE 1 HERE

INSERT TABLE 2 HERE

Tables 1 and 2 report the rejection rates of the tests at the 10%, 5% and 1% levels. Overall, our proposed test has good power against the alternative hypothesis while controlling the size. The performance is slightly worst when one of the roots is close to the unit circle and when we consider the GARCH process.

4 Empirical Application

In order to investigate the performance of the test, I consider the model of Blanchard and Quah (1989) which represents the origin of the debate on non-fundamentalness. The equilibrium solution for unemployment rate U_t and output growth ΔY_t has the structural form:

$$\begin{bmatrix} \Delta Y_t \\ U_t \end{bmatrix} = \begin{bmatrix} 1-L & d(L)+(1-L)a \\ -1 & -a \end{bmatrix} \begin{bmatrix} \xi_{t,d} \\ \xi_{t,s} \end{bmatrix} \tag{4.1}$$

BQ assume no dynamics in productivity except for the instantaneous response to the supply shock, i.e., $d(L) = 1$. This assumption implies that the determinant of the MA polynomial equals to unity and the model is invertible. Lippi and Reichlin (1993), however, argue that economic theory does not provide sufficient conditions on the roots of $d(L)$ and the invertibility of (4.1) is not automatically guaranteed.

I now apply the test to empirically evaluate the validity of the invertibility assumption of the model (4.1). The quarterly time series data is obtained from the St. Louis Fed website, using the seasonally adjusted series GNPC96 and UNRATE. Here, I extend the sample to cover the period 1948:1 to 2010:4 as opposed to the period 1948:1-1989:4 in the original paper.

Following BQ, I estimate a bivariate VAR system in real GNP growth and unemployment rate, allowing for eight lags and obtain the residuals, $\hat{\xi}_t$, which is the inputs of our test. Since the test relies on a non-Gausianity assumption of the estimated residuals, I first provide some empirical evidence of it. We reject the null of normality in the residuals at 1% level, since the p-values for the multivariate Jarque-Bera test with the null of Gausianity is 0.0001. Applying our proposed test to this data set, the p-value is 0.0053, which implies that the model is non-invertible and therefore the impulse response functions and variance decompositions obtained from a VAR is not reliable.

5 Conclusions

This paper provides a new testing procedure to empirically detect fundamentalness, convert the fundamentalness testing problem into one of predictability of the Wold innovations. The proposed test is simple to apply since it only needs model residuals and fitted values as input and can be implemented using a simple F-test. The test is robust to the conditional heteroskedasticity of unknown form and does not need information outside of the specified model. The Monte Carlo study based on a DSGE model with fiscal foresight exhibits a satisfactory finite-sample performance of the proposed test.

Table 1: Size Performance

b	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
					Panel A: *IID*				
10%	1.50	1.50	1.30	1.50	1.40	1.60	1.20	1.80	5.30
5%	0.90	1.20	1.00	1.10	1.20	1.20	0.30	1.30	3.60
1%	0.50	0.40	0.40	0.40	0.60	0.50	0.20	0.30	1.30
					Panel B: *GARCH*				
10%	2.50	1.80	1.40	1.30	2.90	2.10	2.70	2.90	9.60
5%	1.20	0.90	1.00	1.00	1.50	1.00	1.40	1.40	5.20
1%	0.40	0.30	0.30	0.50	0.60	0.10	0.20	0.30	1.80

Notes: Percentage of rejections across 1000 experiments for the null of fundamentalness of the VAR with 4. Sample size is 200.

Table 2: Power Performance

b	2	3	4	5	6	7	8	9	10
					Panel A: *IID*				
10%	92.7	99.8	99.9	100	99.7	98.8	97.9	95.9	91.4
5%	89.9	99.6	99.9	100	99.5	98.2	97.0	94.3	88.4
1%	80.5	98.5	99.9	100	98.6	96.2	93.4	88.6	79.3
					Panel B: *GARCH*				
10%	63.3	85.4	90.7	95.0	95.1	93.3	89.5	86.9	83.7
5%	57.6	81.9	88.8	92.3	93.1	90.2	84.8	80.7	77.4
1%	47.9	76.2	84.1	87.0	87.0	81.7	74.0	67.5	62.5

Notes: Percentage of rejections across 1000 experiments for the null of fundamentalness of the VAR with 4 lags. Sample size is 200.

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