Long Run Current Account through theoretical Intertemporal Model

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Abstract

This paper analyses the current account in the present value model (PVMCA) framework. Based on Obstfeld and Rogoff's book (1996), we aim to model the current account (CA) to GDP ratio in the long run. Since there is scarce theory-based empirical modeling, this paper provides evidence for main determinants of the current account. Firstly, we criticize the tautological approach in the paper of Cerrato et al. (2014) when using a simple relation that the output growth is the sum of the population growth and the per capita GDP growth. This relation leads to identical equations of aggregate and per capita CA-to-GDP ratio. Secondly, we consider the overlapping generations to determine the equation of per-capita CA using relevant variables. This model appears more interesting and testable. It allows to verify empirically the validity of the PVMCA through the quasi-elasticity of CA-to-GDP with respect to the per capita growth rate of output and consumption.

Keywords: Current account, Consumption, Intertemporal Model, Long-run, Per-capita GDP, Quasi-elasticity.

JEL codes: E21, F32

1 Introduction

The theoretical intertemporal model considers the current account as tool to smoothing in the short-run or "tilting" in the long-run the consumption when economy faces shocks on the output, private investment and government expenditures through lending or borrowing processes from the international financial markets. This model is relying on the society behavior as a consumer and a producer in achieving the required adjustments leading to a long-run equilibrium of the economy. Also, it depends on the relevant intervention of the government authorities to control the repercussions of any external or domestic shock especially permanent ones. We focus on how to reveal a testable model through the quasi-elasticity of the ratio of the current account to the output growth and consumption growth.

There are number of empirical papers testing the validity of the present value model of the current account (PVMCA) for many countries and regions. Many of such papers adopted a simple version of this model by assuming that the change in the net output is the only determinant on the current account, this leads to finding rejecting the intertemporal model (Otto 1992). The PVMCA is improved by analyzing the global transitory and permanent shocks on current account in the short and long-run through the expected and unexpected fluctuations as in the global interest rate and the return rate of the global equity markets. Among the recent research on the theoretical and empirical analysis of the current account via the net output, real exchange rate, global interest rate and the ratio of the current account to the net output, we have Hoffmann (2013), Souki and Enders (2008) and Kano (2008). But, these papers did not consider the per capita macroeconomic level for relevant economic and social variables.

The paper of Cerrato, Kalyoncu, Naqvi and Tsoukis (hereafter CKNT 2014) deliberately included the population growth rate without significant justification. They consider that per capita GDP (consumption) growth $g_{y(c)}$ plus the population growth rate n corresponds to aggregate output (consumption) growth $g_{Y(C)} \coloneqq g_{y(c)} + n$. Due to the result that the CA-to-GDP ratio can only be negative and the positive case appears to be unstable (Obstfeld and Rogoff 1966), they introduce the overlapping generations to overcome this limitation, but we will show that CKNT (2014) reach in fine the same previous result. In this paper, we determine the model of per capita current account to per capita output ratio more evidently than the attempt of CKNT (2014). We start to explain that the limitation of the PVMCA at the macro level could be escaped partially by considering the relevant per capita variables; this approach leads to a more generalized current account model. On this basis, we make some hypotheses about the modeling of PVMCA to determine the extent of the interaction between per capita current account to the output ratio relatively to per capita growth rates of GDP and consumption. Such modeling provides to understand if the per capita consumption growth rate will be with high level tomorrow, it will lead to more saving now, which generates more surpluses in the current account process. While if the per capita output growth rate will have a high level tomorrow, this will lead to less resources today, which may cause deficits in the current account process.

But, according to Hoffmann (2013) the PVMCA explains most of the variability in the current account of China's economy and shows that the permanent global shocks influence significantly the Chinese economy. This outcome fits with the expectation that there are some factors related to Chinese financial development which lead to current account surplus in China. Except the findings of Hoffmann (2013) about the Chinese economy, we find that most of papers support that the local impact on the current account dominates. So we suggest modeling of the per capita PVMCA to highlight the importance of the individual economic and financial behavior. The most basic information about such behavior is inherent in the per capita real GDP, real consumption, population growth rate and the return rate on foreign assets.

Section 2 addresses some basics of the PVMCA briefly to model the long-run equation. Section 3 deals in detail with the importance of overlapping generations and the per capita dimension of the relevant variables. Section 4 determines the quasi-elasticities of the per capita current account ratio with respect to the per capita growth rates of output and consumption. We conclude by Section 5.

2 A Model of long-run current account

The most used utility function in the PVMCA framework depends on the infinity time horizon, generalizing the utility function for lifetime as $s = [t, T]$ (Obstfeld and Rogoff 1996) as follows:

$$U_t = \lim_{\mathrm{T}\to\infty}(u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \cdots) = \sum_{s=t}^{\infty}(1+\delta)^{t-s}u(C_s) \quad (1)$$

where β is a positive subjective discount rate; because it is related to the consumer state of mind indicating his/her future credence compared to the current values. It can be measured by $\beta = 1/(1+\delta)$ where δ represents a discount rate $(0 < \delta < 1)$, also called subjective time preference rate. From the identity of current account CA_t at real values:

$$CA_t \coloneqq B_{t+1} - B_t = Y_t + \tau B_t - C_t - G_t - I_t \quad (2a)$$

The sequential constraint serving to maximize the utility, through the investment and consumption processes by supposing the return rate on foreign assets τ with $0 < \tau < 1$, will be as follows:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}(C_s + I_s + G_s) + \lim_{\mathrm{T}\to\infty}(1+\tau)^{-T}B_{t+T+1} = (1+\tau)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}Y_s \quad (2b)$$

We rewrite the constraint (2b) to regulate in the case of surplus the hypotheses of the relationship between the return rate on foreign asset holdings, output growth rate and the consumption growth rate. By assuming the constancy of growth rates in the steady state, all output and its components except consumption grow at the same rate g_Y, we get the following:

$$\frac{1+\tau}{\tau - g_C}C_t + \lim_{\mathrm{T}\to\infty}(1+\tau)^{-T}B_{t+T+1} = (1+\tau)B_t + \frac{1+\tau}{\tau - g_Y}Y_t - \frac{1+\tau}{\tau - g_I}I_t - \frac{1+\tau}{\tau - g_G}G_t \quad (2c)$$

Under the assumption that $\tau > g_C$, the consumption function can be modeled as:

$$C_t = \begin{cases} (\tau - g_C)\left[B_t + \frac{1}{\tau - g_Y}Y_t - \frac{1}{\tau - g_I}I_t - \frac{1}{\tau - g_G}G_t - \lim_{\mathrm{T}\to\infty}(1+\tau)^{-T-1}B_{t+T+1}\right] \\ (\tau - g_C)\left[B_t + \frac{1}{\tau - g_Y}Y_t\big(1 - \gamma - g_K(K_t/Y_t)\big) - \lim_{\mathrm{T}\to\infty}(1+\tau)^{-T-1}B_{t+T+1}\right] \end{cases} \quad (2d)$$

The intertemporal approach usually based on the permanent income model affirms that the function of the current account consists on smoothing only transitory volatility of an economy's net resources through intertemporal trade. In this paper, we focus on the permanent component of the current account modeling allowing for consumption "tilting". It is usual to consider the domestic GDP net of investment and government expenditures corresponding to resources available for current and future consumption. Then, we assume that $I_t \coloneqq g_K\left(\frac{K_t}{Y_t}\right)Y_t$ i.e. capital growth rate and capital coefficient $k_Y \coloneqq \frac{K}{Y}$ are supposed constant. Also, we assume that government spending is a fraction of the GDP, $G_t \coloneqq \gamma Y_t$. To guarantee non-negative sign of the consumption, we suppose that the coefficient of the net output is positive $\frac{\tau - g_C}{\tau - g_Y} > 0$. Besides, the constraint (2b) requires the well-known condition of the no-cheater-Ponzi-game, meaning that there is no exhaustion of all resources during all periods of life, but there are savings for future generations (For more details see Obstfeld and Rogoff 1996, pages 63-66):[1]

$$\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1}\geq 0 \tag{3}$$

Maximizing the utility $function$ (1) under the resources conditions (2b) in addition to the condition (3) leads to the same Euler consumption equation (Gourinchas and Parker 2002) for each period $s\geq t$ after differentiating U_t on C_t and C_{t+1}. The utility maximization consists on

$$\max\sum_{s=t}^{\infty}(1+\delta)^{t-s}u(C_s)$$

under the sequential constraints $B_{s+1}=(1+\tau)B_s+Y_s-C_s-G_s-I_s$ with $s\geq t$, and a constraint ruling out Ponzi games. This constraint (3) makes right to assume that there is a function, called the value function, which leads to the maximal constrained value of U_t as a function of overall initial resources $W_t\equiv(1+r)B_t+\sum_{s=t}^{\infty}(1+r)^{-s+t}Y_s$ by supposing from (2b) that $I=G=0$. Writing the value function like $J(W_t)$ which is differentiable (Stocky and Lucas 1989). According to a simple dynamic equation of the initial wealth, we have:

$$W_{t+1}=(1+\tau)W_{t+1}+\sum_{s=t+1}^{\infty}(1+\tau)^{-s+t+1}Y_s=(1+\tau)(W_t-C_t)$$

The dynamic programming is based on recursive equation involving the value function named Bellman equation (1957) which describes inter-temporally the maximizing path of the utility from consumption. The optimal consumption path from the standpoint of time t should maximize U_{t+1} under the constraint of future wealth W_{t+1} which is generated from present consumption decision C_t. Bellman equation can be as

$$J(W_t)=\max_{C_t}\{u(C_t)+\beta J[(1+\tau)(W_t-C_t)]\}$$

Then, the necessary first order condition (FOC) is: $u'(C_t)-(1+\tau)\beta J'(W_{t+1})=0$.

To transform this condition into a familiar expression, we apply the envelope theorem, by considering that the change in the wealth corresponds to the change in the optimal utility,. We assume that an increase to wealth on any time has the same effect on the lifetime utility regardless that the wealth is allocated for consumption or saving. By using that $C=C(W)$, we can easily show that $J'(W)=u'(C)$ at each time during the maximizing consumption path. This leads to the same consumption Euler equation: $u'(C_t)=\beta(1+\tau)u'(C_{t+1})$. With isoelastic utility function, we have to find the best guesswork of the value function using Bellman's equation, and we reach the optimal consumption function (Obstfeld and Rogoff 1996). By using the dynamic programming, we obtain that

$$u'(C_s)=\beta(1+\tau)u'(C_{s+1})=\frac{1+\tau}{1+\delta}u'(C_{s+1}) \tag{4a}$$

[1] If the present value of what the economy consumes and invests exceeds the present value of its output i.e. $\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1}<0$, then the economy continues to borrow and pays the increased interests on the external debt instead of converting their real resources to foreign borrowers. This process can be done by reducing $(C+I)$ to less than $(Y-G)$. While if the present value of the output exceeds the present value of what the economy consumes and invests i.e. $\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1}>0$, then the economy does not utilize their resources completely. This implies that the economy will be in excess resources state, which could be invested in foreign financial markets. Besides, from the available resources the economy can increase its utilities by improving slightly the consumption level. When the economy is close to $\lim_{T\to\infty}(1+\tau)^{-T}B_{t+T+1}=0$, the present value of the output will be equal to the present value of what the economy consumes and invests.

with

$$u(C) \coloneqq \begin{cases} C^{1-1/\sigma}/(1-1/\sigma) & if\ \sigma \neq 1, \sigma > 0 \\ \ln(C_t) & if\ \sigma = 1 \end{cases}$$

where the positive parameter σ stands for elasticity of intertemporal substitution.[2] The utility function is as follows:

$$U_t = \sum_{s=t}^{\infty} (1+\delta)^{t-s} \frac{C_s^{1-1/\sigma}}{1-1/\sigma} = \sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-1/\sigma}}{1-1/\sigma} \qquad (4b)$$

We obtain the Euler equation for consumption:

$$C_{s+1} = \beta^{\sigma}(1+\tau)^{\sigma} C_s \Leftrightarrow 1 + g_C = (1+\delta)^{-\sigma}(1+\tau)^{\sigma} \qquad (5)$$

In the equation (5) and at steady state, the consumption growth rate g_C is assumed constant. At the stable growth process, due to Bellman equation, the optimal consumption function $(2d)$ can be modeled as follows:

$$C_t = \frac{\tau - g_C}{1+\tau}\left[(1+\tau)B_t + \frac{1+\tau}{\tau - g_Y} Y_t\big(1 - \gamma - g_K(K_t/Y_t)\big)\right] \qquad (6a)$$

The second term inside the square brackets corresponds to the present value, discounted by $(1+\tau)$, of the net resource which grows at rate g_Y. In terms of ratio to GDP, we obtain[3]:

$$\frac{C_t}{Y_t} = (\tau - g_C)\frac{B_t}{Y_t} + \frac{\tau - g_C}{\tau - g_Y}\left(1 - \gamma - g_K k_{Y_t}\right) \qquad (6b)$$

From the result $(6b)$, it appears that the average propensity to consume (APC) is supported by financial returns payments from net foreign assets and a fraction of net domestic resources. In the case of a closed economy, we have $B = 0$ and $CA = 0$, then the APC is supported only by the net domestic resources according to the coefficient $\frac{g_C}{g_Y}$.

By relating this analysis to the current account (CA), defined as the net accumulation of foreign assets, we use the identity $(2a)$ and the optimal equation $(6b)$ to obtain the steady-state current account to GDP ratio:

$$\frac{CA}{Y} = g_C \frac{B}{Y} + \frac{g_C - g_Y}{\tau - g_Y}(1 - \gamma - g_K k_Y) = \sigma(\tau - \delta)\frac{B}{Y} + \frac{g_C - g_Y}{\tau - g_Y}(1 - \gamma - g_K k_Y) \qquad (7a), (7b)$$

This macro equation shows that there is net saving or dissaving depending on the sign of the RHS of $(7a)$. Its first term[4] indicates a fraction of the financial returns payments on its net foreign asset holdings; it will be positive if the net assets are positive and the return rate on the foreign assets is greater than the discount rate. The second term represents a fraction of current resources and its sign depends on the difference between consumption and GDP growth.

Considering a "patient" economy where $\beta(1+\tau) > 1$, such economy saves more than "impatient" economy and could tend to realize CA surpluses. It would start from a low level of consumption and save early on; after that in tendency it is possible that consumption growth will

[2] It corresponds to the degree of response of consumption growth to changes in return rate τ on saving. It is defined by $\sigma = -\frac{u'(C)}{C\,u''(C)}$ where $u'(C)$ is determined from the well-known Euler equation for consumption. Knowing that the utility function u_t has a constant relative risk aversion (CRRA), as measured by Arrow-Pratt (1965, 1964), and we have $u'''(C) > 0$. This result indicates that there is a positive motivation for precautionary saving, as measured by Kimball (1990) by the relative prudence $p(C) = -\frac{C\,u'''(C)}{u''(C)} = 1 + \sigma^{-1}$. If $\sigma = 1$, the utility function is logarithmic, a relative risk aversion $a(C) = 1$ and a relative prudence $p(C) = 2$.

[3] This result appears in Obstfeld and Rogoff (1996) at page 118. Also, it corresponds to the equation $(6')$ in CKNT's paper (2014) at page 7.

[4] Knowing that each of g_C, δ and τ are between 0 and 1, by using the approximate value around zero of the elements of equation (5), we find that $g_C \approx \sigma(\tau - \delta)$.

be higher than GDP growth ($g_C > g_Y$), this allows using up all intertemporal resources. In fact, the economy could save a fraction of its current resources, and then the second term of the RHS of (7) will be positive if the return rate on foreign assets is greater than the output growth. By supposing that g_C is positive i.e. $\tau > \delta$, the current account to GDP ratio should indicate current surplus. Besides, through the identity (2a) the foreign assets to GDP ratio will be as follows:

$$\frac{B_{s+1}}{Y_{s+1}} = \frac{1+g_C}{1+g_Y}\frac{B_s}{Y_s} + \frac{g_C - g_Y}{(1+g_Y)(\tau - g_Y)}\left(1-\gamma-g_K k_{Y_s}\right) \tag{8a}$$

The sequential equation (8a) shows that the foreign-assets-to-GDP ratio path will be unstable if its slope is greater than one i.e. $1+g_C > 1+g_Y$. However, the equation will be stable when $1+g_C < 1+g_Y$.[5] But this last condition makes the coefficient of net output negative in the current account equation (7) if the return rate on foreign assets exceeds the GDP growth. This result exhibits the importance to link the current account path to that of borrowing growth especially in the globalization agenda, which liberalizes more the capital movement as increases quickly the loans cycle. According to Jordà, Schularick and Taylor (2011), such financial and economic dynamics amplify the risks of global instability. In the steady-state and by treating the foreign-assets-to-output ratio as exogenous, the equation (8a) will be equal to (Obstfeld and Rogoff 1996):

$$\frac{B}{Y} = \frac{-1}{\tau - g_Y}\left(1-\gamma-g_K k_Y\right) \tag{8b}$$

If the return rate on foreign assets is lesser than the GDP growth i.e. $\tau < g_Y$, the foreign-assets-to-output ratio becomes positive. By inserting (8b) into (7a), we obtain:

$$\frac{CA}{Y} = \frac{-g_Y}{\tau - g_Y}\left(1-\gamma-g_K k_Y\right) \tag{7c}$$

There is rigorous limitation and contradiction of the PVMCA, because in the steady state a small open economy can only support debt. Also, there is no motivation pushing to invest the current account surplus through foreign assets, because the home resources allocation will be more fruitful domestically. This limitation induces to improve the intertemporal model through the overlapping generations (as in Blanchard 1985, Weil 1989, Obstfeld and Rogoff 1996). Such overlapping influences the consumption efforts and then the current account. Cerrato and *al*. (2014) introduced the relation $g_Y \equiv g_y + n$, where n is population growth rate and g_y is per capita output growth rate, and g_Y is aggregate GDP growth rate.[6] But, this relation makes its equation ($8'$) in page 8 exactly equivalent to its equation ($13'$) in page 9. There is confusion, because even if they consider per capita level, the outcome does not differ from the equation of aggregate level. In the steady-state, by assuming $\tau - \beta = g_C$ and $g_c + n \equiv g_C$, CKNT's paper (2014) reach the erroneous equation ($13'$); the correct equation is:

$$\frac{CA}{Y} = \left(\frac{g_Y}{g_Y - g_c - n}\right)\left(\frac{g_C - g_y - n}{g_C + \beta - g_y - n}\right)\left(1-\gamma-g_K k_Y\right) = \frac{-g_Y}{g_C + \beta - g_Y}\left(1-\gamma-g_K k_Y\right)$$

This outcome indicates that the population growth does not change the CA-to-GDP ratio.

3 Overlapping generations and the long-run PVMCA

By integrating the overlapping generations in the PVMCA (Weil 1989, Obstfeld and Rogoff 1996), we overcome the limitation of equation (7c), and we can reach more generalized

[5] Such case is close to the rational consumer behavior, which -even if the banking system encourages loans- does not push his/her family to a high increase in consumption exceeding his/her income growth.

[6] They consider similar relation for the consumption process.

outcomes. We suppose that an individual born on date v, living eternally and on any time t he maximizes U_t^v defined as follows:

$$U_t^v = \sum_{s=t}^{\infty} (1+\delta)^{t-s} \ln(c_s^v) = \sum_{s=t}^{\infty} \beta^{s-t} \ln(c_s^v)$$

where c_s^v represents the individual consumption in time s. Assuming that the number of individuals in the economy is N_t and growing with positive growth rate n:

$$N_t = (1+n)N_{t-1} = (1+n)^t \qquad t \geq 0 \qquad (t=0, N_0 = 1)$$

We also suppose that the successive generations would transmit dynamic wealth, through inheritance or bequest, for instance, to face the economic life. The main assumption is that there is no financial wealth or assets holding at birth i.e. $b_v^{P,v} = 0$, where P stands for the parent. The budget constraint for the individual v at time $t \geq v$ is defined by (Obstfeld and Rogoff 1996, page 182):

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+\tau}\right)^{s-t} c_s^v = (1+\tau) b_t^{P,v} + \sum_{s=t}^{\infty} \left(\frac{1}{1+\tau}\right)^{s-t} (y_s - z_s) \tag{9}$$

where z is the government economic activity. The dynamic equation that governs individual asset accumulation is

$$b_{t+1}^{P,v} = (1+\tau) b_t^{P,v} + y_t - z_t - c_t^v \tag{10a}$$

When we maximize the individual utility subject to the budget constraint, according to the equation of the initial dynamic wealth and supposing $z = 0$, we obtain the wealth functions w_t^v and w_{t+1}^v as follows:

$$w_t^v = (1+\tau) b_t^{P,v} + \sum_{s=t}^{\infty} \left(\frac{1}{1+\tau}\right)^{s-t} y_s \tag{11a}$$

$$w_{t+1}^v = (1+\tau) b_{t+1}^{P,v} + \sum_{s=t+1}^{\infty} \left(\frac{1}{1+\tau}\right)^{s-t-1} y_s \tag{11b}$$

$$= (1+\tau) b_{t+1}^{P,v} - (1+\tau) y_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+\tau}\right)^{s-t-1} y_s = (1+\tau)(w_t^v - c_t)$$

Using the consumption Bellman equation $J(w_t^v)$ and with the FOC, we obtain

$$u'(c_s^v) = \beta(1+\tau) u'(c_{s+1}^v) = \frac{1+\tau}{1+\delta} u'(c_{s+1}^v) \tag{12a}$$

This equation is similar to the Euler equation. By the logarithmic utility function, we have

$$c_{s+1}^v = \beta(1+\tau) c_s^v \Leftrightarrow 1 + g_c = \beta(1+\tau) \tag{12b}$$

Inserting this result in the individual budget constraint, we get

$$c_t^v = (1-\beta) \left[(1+\tau) b_t^{P,v} + \sum_{s=t}^{\infty} \left(\frac{1}{1+\tau}\right)^{s-t} (y_s - z_s) \right] \tag{13a}$$

Focusing on the aggregate consumption behavior, we have to sum the consumptions of all age-groups (vintages) born since $t = 0$; for age-group $v = 0$ born at $t = 0$, the number of population members is $N_0 = 1$. At $t = 1$, the number is N_1; with a constant population growth rate, we have $N_1 - N_0 = (1+n) - 1 = n$ as members of the age-group $v = 1$. Similarly, we determine the members' number of the second, third cohort, and so on. For any age-group $v > 0$, the population number is $n(1+n)^{v-1}$. Hence, the aggregated consumption per capita on date t, as macro weighted average consumption, is

$$c_t = \frac{1c_{t,0} + nc_{t,1} + n(1+n)c_{t,2} + \cdots + n(1+n)^{t-1}c_{t,t}}{(1+n)^t} = \frac{c_{t,0} + n\sum_{s=1}^{t}(1+n)^{s-1}c_{t,s}}{N_t} \quad (14)$$

We can apply such aggregation to any other individual variable to obtain an aggregate per capita variable, which is just the macro variable divided by total population. We deduce, from the RHS of the previous equation, an expression for b_{t+1}^P and knowing that $b_{t+1}^{P,t+1} = 0$, we get

$$b_{t+1}^P := \frac{b_{t+1}^{P,0} + n\sum_{s=1}^{t+1}(1+n)^{s-1}b_{t+1}^{P,s}}{(1+n)^{t+1}} \Rightarrow (1+n)b_{t+1}^P = \frac{b_{t+1}^{P,0} + n\sum_{s=1}^{t}(1+n)^{s-1}b_{t+1}^{P,s}}{N_t} \quad (10b)$$

where $b_t^P := \frac{B_t^P}{N_t}$ represents the average per capita value at time t of the net financial assets that the individuals own from time $t-1$. From the equation (10a) and the last expression of (10a), we can write that

$$(1+n)b_{t+1}^P = (1+\tau)b_t^P + y_t - z_t - c_t \Rightarrow b_{t+1}^P = \frac{(1+\tau)b_t^P + y_t - z_t - c_t}{1+n} \quad (10c)$$

Also, the equation of the aggregate per capita consumption is simply related to b_t^P; we get

$$c_t = (1-\beta)\left[(1+\tau)b_t^P + \sum_{s=t}^{\infty}\left(\frac{1}{1+\tau}\right)^{s-t}(y_s - z_s)\right] \quad (13b)$$

Now, from the equations (10c) and (13b), we determine the dynamic equation that governs aggregated private assets accumulation:

$$b_{t+1}^P = \frac{\beta(1+\tau)}{1+n}b_t^P + \left[\frac{(y_t - z_t) - (1-\beta)\sum_{s=t}^{\infty}(1+\tau)^{-s+t}(y_s - z_s)}{1+n}\right] \quad (10d)$$

By assuming that $z_t := \zeta y_t$ and $b_t^P := b_t$, and concentrating on the steady-state balanced growth path, we can rewrite the aggregate per capita consumption (13b) under the hypothesis $\frac{1+g_y}{1+\tau} < 1$:

$$c_t = (1-\beta)\left[(1+\tau)b_t + \frac{1+\tau}{\tau - g_y}y_t(1-\zeta)\right] \quad (13d)$$

Therefore, the relationship that governs the private dynamic assets accumulation would be as follows

$$b_{t+1} = \frac{\beta(1+\tau)}{1+n}b_t + \left[\frac{\beta(1+\tau) - (1+g_y)}{(1+n)(\tau - g_y)}\right]y_t(1-\zeta) \quad (10e)$$

The coefficient $\beta(1+\tau)$ can be interpreted as inclination or tilt of an individual's consumption path. In the framework of small-open-economy hypothesis and according to the outcome (12b), if $\beta(1+\tau) > 1+n$, then the individual can during his age-period accumulate financial assets over time. The per capita aggregate assets would continue to increase in tandem with the positive world real economic growth, and even though the consumption growth rate is greater than the population growth rate i.e. despite the instability of the dynamic equation of per capita foreign financial assets. While if $\beta(1+\tau) < 1+n$, the new age-group members, even though with no inheritance or bequest, they come in the economic activities suitably more rapidly that per capita macro foreign assets reach a stable steady-state. Besides, whenever the consumption growth rate is positive, then the population growth rate should be positive, and per capita aggregate foreign assets path converges and will be stable if $g_c < n$. Since there is positive real economic growth, we can convert the equation (10e) to stationary form by dividing both sides by y_{t+1}, we find

$$\frac{b_{t+1}}{y_{t+1}} = \left[\frac{\beta(1+\tau)}{(1+n)(1+g_y)}\right]\frac{b_t}{y_t} + \left[\frac{\beta(1+\tau) - (1+g_y)}{(1+n)(1+g_y)(\tau - g_y)}\right](1-\zeta) \qquad (10f)$$

where $\frac{b_t}{y_t}$ represents net foreign assets to GDP ratio. When $\beta(1+\tau) > (1+g_y)$, the economy will have positive net foreign assets. The slope of the equation $(10f)$ shows that a rise in the per capita real output growth rate g_y lowers the aggregate long-run net-foreign-asset-to-GDP ratio. It seems that per capita income increases along his/her life horizon or that earnings are expected to happen later in life, this belief makes the individual more inclined to reduce his/her saving efforts during both the first and last period of his/her economic life. We can intuitively understand this result by the fact that faster GDP growth incites all age-groups to save less.

Also, the equation $(10f)$ shows that the path of net-foreign-asset-to-GDP ratio becomes unstable becomes unstable if $\beta(1+\tau) > (1+n)(1+g_y)$. While if $\frac{1+g_c}{1+n} < 1+g_y$ i.e. the growth of the average propensity to consume for each generation is less than $(1+n)$, the previous dynamic path will be stable.[7] But, if the slope of the equation $(10f)$ is less than one, and considering that the process $\frac{b_t}{y_t}$ is stationary, we obtain its long-run equation

$$\frac{b}{y} = \frac{(1+g_c) - (1+g_y)}{[(1+n)(1+g_y) - (1+g_c)](\tau - g_y)}(1-\zeta) \qquad (10g)$$

This exhibits that the coefficient of the net output depends on the sign of the difference between per capita growth rates of consumption and GDP. When such difference is positive, it corresponds to the consumption "tilt" factor, which could be in fact amplified through the borrowing from banks. The equation $(10g)$ indicates that in the economy the members of each age-group could have loans when $\frac{1+\tau}{1+\delta} > 1+g_y > 1$, because they hold foreign assets and take advantage of profitability in international financial markets in particular when the return rate is greater than the expected discount rate.

In the steady state, we can now write the equation of per capita current account ca_t to per capita output y_t by determining the equation of the APC, from equation $(13d)$, as follows

$$\frac{c_t}{y_t} = (1-\beta)\left[(1+\tau)\frac{b_t}{y_t} + \frac{1+\tau}{\tau - g_y}(1-\zeta)\right] \qquad (13e)$$

Using the current account identity, we obtain

$$\frac{ca_t}{y_t} := (1+g_y)\frac{b_{t+1}}{y_{t+1}} - \frac{b_t}{y_t} \Rightarrow \frac{ca}{y} = g_y\frac{b}{y}$$

where the last RHS represents the long-run current account to GDP ratio. By using $(10g)$, we obtain a long-run equation

$$\frac{ca}{y} = \frac{g_y[(1+g_c) - (1+g_y)]}{[(1+n)(1+g_y) - (1+g_c)](\tau - g_y)}(1-\zeta) \qquad (15)$$

In the steady state, requiring the stability condition i.e. $\tau > g_y$ and the veracity of double inequality $\beta(1+\tau) > 1+g_y > 1$, from the version (15) of the PVMCA, the long-run factor of the current-account-output ratio could have any sign, and there is no sign presumption as in the equation $(7c)$. We can now derive the effects of the per capita (or aggregate) consumption and

[7] This case is close to the rational behavior, which does not push the individual to replicate the pattern consumption of his/her generation even if the banking system incites the families to borrow more. Normally, the "tilt" factor should be reduced when the individual expects that his/her consumption growth exceeds his/her income.

output growth rates on the CA-to-GDP ratio. By supposing that the population growth is zero i.e. per capita and aggregate growth rates will be equal, then the current-account-GDP ratios modeled in equations (7c) and (15) are equivalent.[8]

When the population growth rate is increasing, the first factor of the denominator in RHS will be positive. Accordingly, there are many economic motives pushing to invest the current account surplus through foreign assets. But, if $\beta(1+\tau) < 1 + g_y$, there will be current account deficit; and the economy borrows from abroad or increase its domestic loans to continue the productive activities and finance new economic projects. But, such borrowing process will support a higher consumption than current resources tolerate.

4 Quasi-elasticities of the long-run current account

Assuming that the first factor of the denominator in RHS of (15) is positive and knowing that the second factor is positive. We can derive the effects of per capita GDP and consumption growth rates, and population growth on the current account to output ratio. Firstly, we derive the per capita output multiplier:

$$\frac{\partial\left(\frac{ca}{y}\right)}{\partial g_y} = \left[\frac{V_1 - (1+n)U_1}{V_1^2}\right]\left(\frac{U_2}{V_2}\right) + \left[\frac{U_2 - V_2}{V_2^2}\right]\left(\frac{U_1}{V_1}\right) = \frac{(n-g_c)U_2}{V_1^2 V_2} + \frac{(g_c - \tau)U_1}{V_2^2 V_1} \tag{16a}$$

where $U_2 \coloneqq (1+g_c) - (1+g_y)$, $U_1 \coloneqq g_y$, $V_1 \coloneqq (1+n)(1+g_y) - (1+g_c)$; and with the stability condition of the output path, we have $V_2 \coloneqq \tau - g_y > 0$. Assuming current account surplus, we get $V_1 > 0$ and $U_2 > 0$. Also, we suppose $U_1 > 0$. Since $n < g_c$, the first term of the last RHS of the equation (16a) has a negative sign. In addition, if $g_c < \tau$, the sign of the partial derivative is negative meaning that the increase in per capita GDP growth leads to a decline in per capita current account to per capita output i.e. in CA-to-GDP ratio. An early economic growth, allowing that the financial resources would be more available increasingly through time, may drive to deficits in current account particularly if the return rate on foreign assets exceeds the per capita consumption growth. Similarly, as indicated by Cerrato et al. (2014), a smaller economic growth could mean that there are more resources available early on, thus the tendency for a CA deficit early on shrinks. Equivalently, an economic growth, leading to a saving growth and generating lately less available resources, could drive to negative effects. Considering that the fluctuations in savings, and congruously in investment, reflect the GDP fluctuations, these latter affect the current account (Blanchard and Giavazzi 2002). We could not state that such effects are minor or not, the empirical exploration could help to identify some direct and reversal implications of consumption and saving behaviors on current account dynamics.

While, if $g_c > \tau$, then the multiplier sign will depend on the interaction between the population growth and per capita growth rates of consumption and output with U_2V_2 and U_1V_1. We find three negative and five positive terms;[9] by assuming that return rate on foreign assets is closer to per capita GDP growth rate compared to per capita consumption growth rate, then by adding first negative to third positive terms, and second negative to second positive terms we obtain negative result. While adding third negative to fourth positive terms leads to smaller positive result compared to negative one. The final outcome depends on the effects of the

[8] By using equations (12b) and (15), we can determine a more compatible form with data through the aggregate variables instead of per capita ones.

[9] The negative terms are $-g_c[g_c(\tau - g_y)]$, $g[g_c(\tau - g_c)]$ and $n[g_y(g_y - \tau)]$, respectively. The positive terms are $g_y[g_y(g_c - \tau)]$, $g_c[g_y(\tau - g_y)]$, $n[g_c(\tau - g_y)]$, $n[g_y(g_c - \tau)]$ and $n[g_y^2(g_c - \tau)]$, respectively.

remaining first and fifth positive terms. Due to that these latter values are the smallest ones, the negative multiplier hypothesis dominates. To corroborate this outcome from the literature, Aizenman and Sun (2010) confirms that, despite the speed or slower growth in Chinese economy, its surplus current account remains constrained by the limited growth of the partner economies supporting deficits current account, which reversely could slowdown their economic growth.

Secondly, we determine the multiplier of per capita consumption growth on $\frac{ca}{y}$:

$$\frac{\partial\left(\frac{ca}{y}\right)}{\partial g_c}=\left(\frac{U_1}{V_1^2}\right)\left(\frac{U_2}{V_2}\right)+\left(\frac{V_2}{V_2^2}\right)\left(\frac{U_1}{V_1}\right)=\frac{g_y\left(g_c-g_y\right)}{V_1^2V_2}+\frac{g_y\left(\tau-g_y\right)}{V_2^2V_1}=\frac{g_yU_2}{V_1^2V_2}+\frac{\left(\tau-g_y\right)U_1}{V_2^2V_1} \quad (16b)$$

The sign of this multiplier is positive meaning positive effect of the consumption growth rate on the CA-to-GDP ratio. As g_c increases, the economy becomes more "patient" with a smaller early consumption and higher later economic growth. Equivalently, this economy saves more initially and then holds dynamically foreign asset due to its positive current account.

The previous findings, that $\partial(ca/y)/\partial g_y<0$ and $\partial(ca/y)/\partial g_c>0$, indicate that there is no parallel fluctuations between per capita consumption and GDP growth rates. Irrespective to their signs, the two multipliers would have different coefficients, and consequently the dynamic paths of per capita real GDP and per capita real consumption are not homogeneous.

Lastly, we have to determine the effect sign of the population growth rate on per capita current account:

$$\frac{\partial\left(\frac{ca}{y}\right)}{\partial n}=\left[\frac{-\left(1+g_y\right)U_1}{V_1^2}\right]\left(\frac{U_2}{V_2}\right)=\frac{-g_y\left(1+g_y\right)U_2}{V_1^2V_2} \quad (16c)$$

The population growth multiplier has a negative sign; this result is expected because a rise in population growth rate expands the proportion of dependent children, dependent overage parents and young savers. This outcome is exhibited in many empirical works as Karras (2009). The new young population takes advantage from the economic efforts of the previous generations, and would lately boost the output growth. In such case, we reach the outcome as discussed about the GDP growth multiplier. This means that the dynamic interaction between new population through the overlapping generation, consumption and saving could generate lately less available resources, and drive to negative effects on CA-to-GDP ratio growth.

In light of the above outcomes, we can build theoretical models by focusing on a limited number of random variables leading to find an optimal level of foreign assets Sachs (1982). We can derive an estimable model by linearizing the equation (15) as follows:

$$\frac{ca_t}{y_t}=\beta_0+\beta_1g_{y_t}+\beta_2g_{c_t}+\beta_3g_{n_t}+u_t,\quad \beta_1<0,\ \beta_2>0,\ \beta_3<0 \quad (17a)$$

where the parameters β_i with $i=1,2,3$ are initially the partial derivatives of equations (16). Also, the intercept β_0 will be estimated using the multipliers of the partial derivatives and the sample means of the related variables. Other regressors are highlighted in some previous literature but without offering theoretical consensus as openness index, budget-balance-to-GDP ratio, M2-to-GDP ratio.[10] The GDP and consumption multipliers of equations (16a) and (16b) provide a testable restriction between the two partial derivatives named β_1 and β_2, respectively; we can write that:

[10] As Sheffrin and Woo (2000), Lee and Chinn (2002), Blanchard and Giavazzi (2002), Chinn and Prasad (2003), Corcetti and Müller (2006) and Cerrato et al. (2014).

$$\frac{\frac{ca}{y}}{g_y} = \beta_1 + \beta_2 \qquad \text{or} \qquad \frac{\frac{ca}{y}}{g_c} = \beta_1 + \beta_2 \tag{17b}$$

This restriction allows testing empirically the validity of the long-run PVMCA. We shows by using appropriate elasticities that such restriction could be expressed as

$$\frac{g_y}{g_c} E\left(\frac{ca}{y}, g_c\right) + E\left(\frac{ca}{y}, g_y\right) = 1 \qquad \text{or} \qquad \frac{g_c}{g_y} E\left(\frac{ca}{y}, g_y\right) + E\left(\frac{ca}{y}, g_c\right) = 1 \tag{17c}$$

then, by using the restriction (17b) the long-run quasi-elasticities of the current account to GDP respecting to per capita output and consumption growth rates should add up to one. According to the opposite signs of each multiplier and then the related elasticities interact in opposite paths. This interaction means that a higher growth rate of consumption tomorrow i.e. later on involves more saving yesterday i.e. earlier and bring-up a positive current account balance. According to Yang, Zhang and Shaojie (2010), such interaction happens in Chinese economy and leading to surplus current account path. Whereas, a higher output growth tomorrow i.e. later on implies less resources yesterday i.e. earlier and bringing-up a negative current account balance. In such case, the economy should build precautionary saving to face any negative fluctuation mostly in economic growth rate (Sandri 2011). The issue lies in which among the two dynamic multipliers and their corresponding paths overcomes the other.

5 Conclusion

In the steady state, the long-run per capita CA-to-GDP ratio is related to a positive difference between return rate on foreign assets and output growth rate. Consequently, the sign of the CA-to-output ratio depends on the dynamic interactions between population, consumption and output growth rates. By considering the overlapping generations, there is no sign presumption of CA-to-GDP as with aggregate level variable in PVMCA. We criticize the theoretical findings of Cerrato et al. (2014) because they use a tautological approach leading in fine to equivalents CA-to-GDP ratio at per capita and aggregate levels. As the consumption real interest rate is greater than the real output growth rate, the per capita current account to output ratio will be stable. By considering a "patient" economy, which saves more than "impatient" economy, it could tend to realize surpluses in current account; such economy would start from a low level of consumption and save early on, after that in tendency the consumption growth would be higher that GDP growth allowing for its members to use up all intertemporal resources. In the steady state, the per capita income increases along his/her life horizon or that earnings are expected to happen later in life. Such belief makes the individual more inclined to reduce his/her saving efforts during both the first and last period of his/her economic life. We can intuitively understand this result by the fact that faster GDP growth incites all age-groups to save less. Such incitation will work in particular when an increased population growth drives to reverse the sign difference between per capita consumption and output growth rates and generates dynamically surplus in current account through foreign assets.

From the CA-to-GDP ratio using per capita growth rates of GDP and consumption in addition to population growth rate, we determine the related multipliers. In the framework of the PVMCA, the economic growth rate multiplier is dominated by a negative effect. So, an economic growth, leading to a saving growth and generating lately less available resources, could drive to negative effects on the current account. Blanchard and Giavazzi (2002) shows that the fluctuations in saving, and congruously in investment, reflect the GDP fluctuations, which affect the current account. While the consumption growth rate multiplier affects negatively the long-run current account to output ratio. As there is a consumption "small tilt" factor, the economy becomes more "patient" with a smaller early consumption and higher later economic growth; this economy saves more initially and then holds dynamically foreign asset due to its positive current account. Consequently, the dynamic paths of per capita real GDP and real consumption are not homogeneous. The output and consumption multipliers provide a testable restriction stating that long-run quasi-elasticities of the current account to GDP with respect to per capita output and consumption growth rates should add up to one. According to the opposite signs of each multiplier, the related elasticities interact in opposite paths, meaning that a higher growth rate of consumption tomorrow i.e. later on involves more saving yesterday i.e. earlier and bring-up a positive current account balance.

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