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## **Forecasting Inflation using Functional Time Series Analysis**

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# 1 Introduction

In every real life phenomenon there is an uncertainty involved. We want to model this uncertainty for some reasons. One of the reason is forecasting for the purpose of decision making. If we have some knowledge of the future then we can make decisions to meet our objectives. Econometrics is used for modeling this uncertainty, specially in Economics. Making a right decision on right time is very important to tackle the complex system of Economy. There are mainly four objectives of the Econometric modeling. First is Describing a real life phenomenon using some model, secondly Forecasting, thirdly Casual Analysis and last but not least Policy Analysis, as suggested by Hendry (1995).

Consumer Price Index (*CPI*) is a key variable which indicate the overall price level of basket of goods and services in a country. It has many useful motives, some are suggested by Khalid and Asghar (2010), as measuring cost of living, indexation of monetary flows, indexation of wages, indexation of rents, indexation of social security benefit and inflation. Inflation is one of the important variable that can influence the whole economy and government policy defined as rate of change in *CPI*. Inflation not only affects the

individual as well as economy. To maintain a rapid growth in the economy it is very important to maintain inflation at threshold level. High inflation with a momentum can be an indication of diluting economy. On the other hand low inflation may also have negative impact on growth.

Inflation forecasting is equally important for monetary policy as well as fiscal policy. Private agents, labor market and financial market also react to the change in inflation. Irrespective of having expectation about inflation, so it is of grave importance to have accurate forecast to some horizons. Inflation is very important indicator of a country's economy. It provides understanding of the state of economy. Generally economist alleged that high inflation is caused by the extensive supply of money. It is a main objective of the central bank to maintain a moderate level of inflation for steady growth of economy. In Pakistan various factors effect inflation including money supply and fuel prices. Qayyum (2006) studied some determinants of inflation and found that in Pakistan it is highly effected by the monetary policy. Recent rise in inflation has many causes but excessive printing of money by democratic governments, rise in prices of fuel in international market, rise in price of electricity, shortage of electricity and rise in production cost are the important factors influencing the inflation rate. Although there are different studies to define a threshold level of inflation for which there is a positive impact on growth and beyond which has a negative impact on growth. Generally in

case of developing countries 7-11 percent of inflation is claim to be a level for which there is a positive impact as noted by Khan and Senhadji (2001). In case of Pakistan it is found that inflation around 8 percent is best .

Forecasting is one of the main objectives of Econometric modeling. There are mainly two types of forecasting. Statistical forecasting *i.e.* based on historical data (Univariate models also called extrapolation ) and Economic forecasting *i.e.* based on Multivariate models (VAR models and cointegration). Univariate models are used for short term forecasting where as multivariate models are renowned for long term forecasting. Both type of models have their pros and cons. One may argue that univariate models are same like a guess which may or may not be true (rather its a scientific guess with some confidence interval). On the other hand multivariate models are more complex and errors like model misspecification can arise. Another factor which may add accuracy to forecasting is frequency of the data. High frequency data can be used to have more accurate forecast. High frequency data has an advantage of accurate forecast which may induce complexity specially to multivariate models.

In the present study our main objective is to forecast inflation in case of Pakistan as accurate as possible. Secondary one is comparing forecasting performance. For this purpose we are using highest frequency data of national price level available in case of Pakistan monthly data. We will forecast

the inflation using classic Box-Jenkins methodology and Functional Time Series Analysis using data of CPI general from 2002-2011 of Pakistan.

## 2 Literature Review

In forecasting inflation many efforts have been made using different variables and methodology. Many authors have used univariate models (ARIMA and ARCH models), famous Philips curve (VAR models) and cointegration. Many determinants like unemployment, Economic growth and Monetary variables like money supply have been used for this purpose. If we study the evaluation of time series methodologies we found that Wiener (1964) had pioneer work ,which laid the foundation of modern time series analysis and forecasting. This work was actually done to resolve some problems in engineering related to frequency domain. He was trying to estimate underlying signal for set of time series observations corrupted by noise. These signal extraction models work almost similar to complex econometric models in forecasting, developed at different time.

In forecasting time series based on purely statistical or models based on historical data, many types of modification like , linear, non linear, smoothing parametric and non-parametric models were used in this regard. X-12

ARIMA models developed by US Bureau forecast after extracting the cyclical and seasonal component. Exponential smoothing is another method used to forecast variable's future values. Originally it was developed by Holt (1957) and then Winters (1960) applied this technique to forecast sales. He also introduced a slope term in the model. The main idea behind exponential smoothing was to give exponential weights to the observations based their time of occurrence. The most recent observation has the maximum weight and rest have exponentially decaying weights with the minimum weight to the first observation. The advantage of this technique is it minimizes the inherited noise in the observation which leads to a smooth curve. This curve made by exponential smoothing is then used to forecast future values. The main disadvantage of this method is that it lacks the theoretical consideration due to which it doesn't allow for prediction interval. Brown and Meyer (1961) studied the least square estimated of coefficients of exponential smoothing model. His contribution added the flavor of econometrics in the model. Many other contributions were made till 1970 in existing as well as new techniques. In the same year Box and Jenkins (1970) came up with a methodology that address many practical issues for modeling and forecasting time series. In case of multivariate forecasting techniques first one is a multivariate extension of ARIMA model i.e. Vector ARIMA models. Vector Autoregressive (VAR) is special case of VARIMA. VAR was first derived by the Quenouille

(1968). In general The main disadvantage of the VAR model is, that it has too many extra estimated parameters which are usually insignificant , as a result out of sample forecasting is poor as noted by Simkins (1995) although in sample forecasting is good. Performance of VAR models were even worst at level when variables involved are non stationary. Engle and Granger (1987) comes up with a solution called cointegration and Error Correction models. These models have good out sample forecasting on the long horizon. Some modified and improved versions of the method are also available. Besides these some non linear models like regime switching models, Artificial Neural Network models, GARCH models and ARFIMA models are also used for forecasting purpose.

Comparison of forecast efficiency of different type of models have been made in many studies. Cooper (1972) and Naylor et al. (1972) concluded that ARIMA models performance is superior then Wharton Economic model. Armstrong (1985) in his famous book compared the performance of both type of models and concluded that time series models are superior then Economic models. Specifically in univariate vs economic forecasting. Meyler et al. (1998)found that ARIMA models outperform all other models in forecasting Irish inflation. They also investigated the performance of VAR models in Bayesian paradigm. Ang et al. (2007)showed that univarite models out-

perform models like Philip's curve. Stockton and Glassman (1987) showed that time series ARIMA models outperform macro models. In a paper by Lee (2012), predictive performance of the univariate time series and some economic forecast models were compared. He found that univariate models are the optimal ones for forecasting inflation.

In the early history of Pakistan till 1990 inflation was not a big issue. After the adoption of Structural Adjust Program in 90's the focus shifted on this side. The main debate started after 2000 about inflation, monetary variables and policy. Before 2000 we have few studies in this context. After it many studies have been made specially focused on inflation and monetary. The main focus of the studies was to determining the possible variable responsible for change in inflation. These multivariate studies was of two type. Theoretical (Economic Forecasting) and models just for forecasting inflation with out any theory based analysis (Statistical Forecasting). Most of the work done in theory based model was to test monetary theory or checking possible determinants of inflation.

For example Ahmed [1991] found Gross National product (GNP), growth rate of imports, growth in ratio of M1 and M2 as important variables in determining inflation. Dhakala[1993] found M1 , interest rate import prices as important variables. Hasen [1995] put forwards the view of using fiscal



variables. Ahmed and Ali [1999] finds interest rate as important one. Khan [2002] finds Dollar exchange rate, Khalid 2005 suggest imported inflation and openness important. Malik [2006] founds money supply as more influential. Aleem [2007] conclude adaptive expectation private sector credit and imported prices as one of possible causes of inflation. Omer and Farooq [2008] and Khan et. al [2008] tested and concluded political instability as most influential on unstable price level. Some of the researchers tested some theories. Akhter [1990] supported monetarist point of view in determining inflation. Bayer[1993], Qayyum [2006] and Kemal[2006] found evidence in favor of Quantity Theory of Money. On the other hand some studies are also their for multivariate case in forecasting inflation of Pakistan. Different models have been used for example Ahmed [1999] used 2sls method. Hyder and Shah used Recursive VAR model. Qayyum [2005] built a p-star model. Khan [2006] and Kemal [2006] used VECM for the purpose of forecasting. Gul [2012] using data 1992-2010 finds Philips Curve still good to forecast inflation in Pakistan. Tahir et. al [2014] used data from 1991-1 to 2013-11 and Bayesian VAR modle to forecast inflation of Pakistan. He used three priors for this purpose. Minnesota, Independent Normal-Wishart and Independent Minnesota Wishart priors.

In the past fifteen years there are numerous fields in which Functional Data Analysis is applied. Bio medicine , signal processing, Economic, Finance,

Demographic forecasting, Human growth, multi-stream web data, Ecology , Environment studies and many more see (ullah [2013]). It is mainly consist of smoothing techniques, data reduction techniques, functional linear models and forecasting methods. In demography age specific mortality rates, Hyndman and Shang (2010), fertility, mortality and Hyndman and Booth (2008), monthly non-durable goods index Ramsay and Ramsey (2002) and income distribution.

Particularly in foretasting inflation Chaudhuri et al. applied the functional data method on forecasting inflation. He used Sector wise disaggregated data of CPI in two and more levels. He used data of CPI disaggregated at 2nd and 3rd level from January 1997 to February 2008. Authors modeled Each series of disaggregated data as a functional observation. After smoothing the data using kernel smoothing methods, Functional Autoregressive model(FAR) was fitted. Recomposing each level of forecasted data series to get next level series using some weights and finally getting CPI. Authors fitted many model to forecast the series after smoothing. FAR(1), fitting FAR after differencing (DFAR), FAR on 3 month moving average (FAR-3m). Authors found FAR-3m model to be optimal in forecasting inflation.

In case of Pakistan many attempts have been made for forecasting inflation. Forecasting inflation using time series models were attempted by many authors. Salam et al. (2007) foretasted inflation used data od CPI from 1993-7

to 2004-6. He fitted many ARIMA models using Box-Jenkins methodology. Author compared performance of all these univariate models using performance evaluation criteria. Haider and Hanif (2009) foretasted inflation using Artificial neural networks. Iftakhar and Amin (2013) modeled the data 1961-2013 of CPI using ARIMA models foretasted the inflation of Pakistan to be 8.83 in year 2013. They found ARIMA(1,1,1) model the best for forecasting inflation. Sultana et al. (2013) used monthly data of CPI from 2008-7 to 2013-6. Authors modeled the data using two methods . ARIMA models and decomposing CPI to its time series components and then recomposing after projection to forecast inflation. They concluded that the ARIMA models gives better results.

### **3 Methodology and Data**

In this section we will briefly explain the functional data analysis and its application to forecasting Functional Time Series Analysis (FTSA). In forecasting inflation we have used CPI of Pakistan from JAN-2002 to DEC-2011.

### 3.1 ARMA Models

The autoregressive model is a kind of linear regression in which independent variables is actually the response variable from the previous period (i.e. lag of the series). For example for a series  $y_t$ , the autoregressive of order  $p$  (i.e. AR( $p$ )) is of the form

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2}, \dots, \phi_p y_{t-p} + \epsilon_t \quad (1)$$

where  $\epsilon_t$  is noise in the data series assumed to be identically independently distributed with 0 mean. The reason to denote AR( $p$ ) is that  $y_t$  is regressed on its own subsequent previous years values. The moving average is a linear model of a time series dependent on the past errors. So the MA model of order  $q$  can be expressed as

$$y_t = \theta_1 \epsilon_{t-1} + \theta_2 \epsilon_{t-2} + \dots + \theta_q \epsilon_{t-q} \quad (2)$$

#### 3.1.1 Test for Structural Break

We have used in this thesis Bai and Perron (1998) test for structural breaks. The standard model considered by authors is a multiple linear regression model with  $T$  time periods and  $m$  potential break points, which results in

$m + 1$  regimes. The model is given by

$$Y = X\beta + Z\delta + \epsilon \quad (3)$$

where  $Z$  is a set of variables whose coefficient vary across regimes and  $X$  is set of variable whose coefficients remain same across the regimes. We have used here sequential procedure so we will only define it. The steps involved in the testing are following. First full sample is considered and consistency of parameters of variables is tested. If the hypothesis is rejected then the data is divided into two subsets and possible presence of structural breaks is tested. The data is dived and the test for more and more regimes are performed until the hypothesis accept. Following F test is used for testing

$$F = \frac{1}{T} \left( \frac{T - (l + 1)q}{lq} \right) \hat{\delta} R (RV(\hat{\delta})R)^{-1} R \hat{\delta} \quad (4)$$

where  $q$  is number of restrictions,  $l$  is number of potential break points  $R$  is restriction matrix and  $V$  is variance-covariance matrix.

### 3.1.2 Non-Stationarity, its Detection and Remedy

If distribution of a series (say)  $y_t$  dose not changes over time the series is called Stationary. If we able to test such kind of stationarity, then this kind

of stationarity is called strong stationarity. The actual problem in testing such kind of stationarity is that we are unable to find the distribution of a time dependent variable. Due to this difficulty we move toward weak stationarity. A series is called weakly stationary if its mean variance and covariance are constant over time. In time series analysis, the stationarity of a series is tested assuming different time series data generating process (DGP) of series. For example the pioneer work of Dickey and Fuller (1981) assumed the DGP AR(1) model with drift and trend. The test was developed with a null hypothesis of unit root in the series. There are many modification of unit root test is available including non-parametric and Bayesian techniques. In testing seasonal unit root the first and simplest approach motivates by his own non-seasonal test was proposed by Dickey et al. (1984). An extension and general form of the test was proposed by Hylleberg et al. (1990). Our focus is mainly on seasonal monthly unit root so we move towards the objective instead of discussing nonseasonal unit root and seasonal unit root of quarterly data. First test for seasonal unit root for monthly data was proposed by Franses (testing for seasonal unit root in monthly data rotterdam). Beaulieu and Miron (1993) proposed an alternative test which is an extension of HEGY test for quarterly data. Smith and Taylor (1998) and Taylor (1998) have generalized the regression-based approach of HEGY and BM to allow for differential seasonal drift in quarterly and monthly scenarios respectively.

### 3.1.3 Box-Jenkins Methodology

In order to select a good model to forecast univariate time series Box and Jenkins (1970) proposed a three step methodology. The steps are (a) Identification (b) Estimation and (c) Diagnostic .

#### 1. Identification

In selecting a appropriate model we have to identify the parameters  $(p,d,q)$  as more the one model can exist for a particular series. Due to a property called invertibility i.e. an infinite AR model can be represented by an MA model, we may have more then one model. After testing the stationarity of the series, we examine the Auto-Correlation (ACF) and Partial Auto-Correlation (PACF) functions plot to decide the model.

#### 2. Estimation

The observed ACF and PACF are used to identify the model, then we estimate the model for significant spikes observed in ACF and PACF. For different models we use Akaike information criteria and Bayesian information criteria to choose a model. A parsimonious models is always desired in the class of models.

#### 3. Diagnostics

After estimation we have to check the model adequacy. In this stage we apply some test on the residuals and estimated coefficients to check the goodness of fit of the model. We plot the residual to check the outliers, plot ACF to check autocorrelation in the residuals, apply test to check normality of the residuals and significance of the coefficients. These tests are applied to select an appropriate model.

### **3.2 Functional Data Analysis and its Application**

In statistics we deal with sequence of observation of vectors, but in FDA we deal with functions. Each function is a set of observations which is called a functional observation. So in this analysis we need more than one (functional) observation. For example growth curve of children in terms of height measured at different ages, each child growth curve can be treated as a functional observation. Another example is temperature of different cities. Now in this case there may be two types of functional observation, we can deal with the temperature of whole year of a station as functional observation if we have data of many years or for year we can model the data of many stations in functional data. Also as in these curves the first and second derivative (difference) can reveal interesting features of growth.

The functional observation can be a single long record which could be broken



at finer scale for analysis. e.g. the temperature of a station could be broken at each year to treat as a functional observation. The functional data can also be an input output system, it can also be used in linear models. we can also fit distribution on a data using functional data analysis. In statistics fitting a pdf to data means estimating its parameters , which determines its shape. The purpose of fitting a probability distribution is actually some kind of forecasting,e.g. calculating probability of a certain value of a variable. Now in real world normality is myth. So instead proving data to be normal and fitting a distribution, we can go for curve fitting which has only assumption of smoothness. Almost all of statistical analysis can be done in FDA without any prior assumption and with greater accuracy. The main features of the FDA are

1. Model the data.
2. Highlight Characteristic of the data.
3. Study the patterns in the data
4. Prediction

### 3.3 Defining FDA

Let  $y_{ij}$  be the observation and  $t_{ij}$  be the continuum for each record  $i$ , where  $j = 1, 2, 3, \dots, n$  and  $i = 1, 2, 3, \dots, N$  then the pair  $(y_{ij}, t_{ij})$  can be represented as

$$y_{ij} = x_i(t_{ij}) + \epsilon_{ij} \quad (5)$$

where  $x_i$  is a function and  $\epsilon_{ij} \sim N(0, \sigma^2)$  is noise. We are assuming here that  $t_{ij}$  is same for all records but it can vary for different records. The construction of functional model is estimated separately for each record but when signal to noise ratio is low we can use the same information for each record. We are treating  $t_{ij}$  as time but it can be any continuum e.g. time, space, age frequency wave etc. In functional data form model can be written as

$$y_j = x(t_j) + \epsilon_j \quad (6)$$

where  $y_j$  represents one functional observation. The variance of the error term is

$$\text{Var}(y_{ij}) = \sigma^2 I \quad (7)$$

but it allows to vary the variance for each record. If the observations are without noise, then the model is simple interpolation of observations. Thus  $\epsilon_i$  adds roughness to the data. Our goal in the modeling is to minimize this noise to get efficient fit as much as possible.

### **3.4 Basis System to fit a Functional Model**

The function  $x_i$  is modeled using some basis function. Basis are the set of independent vectors whose linear combination is used to model hundreds and thousands of data points simultaneously. More it is also based on some interpolating techniques and matrix algebra which we enjoy in Econometrics. Due to linearity inference is also convenient for testing parameters. There many types of basis function available such as constant basis, polynomial basis, power basis, exponential basis, Fourier basis and B-spline basis. we will use B-spline basis in our study. First of all we will introduce the splines, which is basic building block of the smoothing spline regression model. The splines method is basically the numerical approximation of a curve. If we want approximate a curve by a polynomial function we can approximate through numerical methods. One of the possible option is splines.

### 3.4.1 B-Splines Basis

Let us have a sequence of non-decreasing real numbers as  $t_0, t_1, t_2, \dots, t_{n-1}, t_n, t_{n+1}$  such that  $t_1 < t_2 < t_3 < \dots < t_{n-1} < t_n < t_{n+1}$ . We define the knots as  $t_{-m-1} = \dots = t_0, t_1, t_2, \dots, t_{n-1}, t_n, t_{n+1} = \dots = t_{n+m}$ . We appended the sequence backward as well as forward due to recursive nature of B-Splines. If we need we can use the index due to recursiveness in the first element as  $t_{-m-1}$  and a total of  $n + 2m$  recursive knots. Now the index of the knots  $t_i = 0, 1, 2, \dots, n + 2m - 1$ . For each of the knots  $t_i$ , where  $i = 0, 1, 2, \dots, n + 2m - 1$  we can define a set of real values function recursively  $B_{ik}$  where  $k = 0, 1, 2, \dots, K$  is the degree of B-Splines basis as follows

$$B_{i0} = \begin{cases} 1, & \text{if } t_i < t < t_{i+1} \\ 0, & \text{otherwise} \end{cases}$$

Also the general form of B-Spline is  $B_{ik+1}(x) = \alpha_{i,k+1}(x)B_{ik}(x) + [1 - \alpha_{i+1,k+1}(x)B_{i+1,k}(x)]$

Where,

$$\alpha_{i,k} = \begin{cases} \frac{x-t_i}{t_{i+k}-t_i}, & \text{if } t_{i+k} \neq t_i \\ 0, & \text{otherwise} \end{cases}$$

Using above definition we have to define some terminologies.

1. The sequence  $a$  is known as knot sequence, and each individual term represents knot.
2.  $B_{ik}(x)$  is called  $i$ th B-Spline basis function of order  $k$ , and the relation is called d'Boor recursive relation named after Carl de Boor (2001).
3. For any non negative integer  $k$ , the vector space  $V_i(t)$  over  $\mathbb{R}$  by set of all B-Spline basis functions of order  $k$  is called B-Spline of order  $k$ .
4. An element  $V_i(t)$  is B-Spline function of order  $k$ .
5. The first term  $B_{n+1,0}$  is called the intercept

for  $k > 1$  and  $i = 1, 2, 3, \dots, n$  B-Splines have following properties

1. Positive  $\beta_{i,k} \geq 0$
2. Local Support  $\beta_{i,k} = 0$  for  $t_0 < t < t_i$  and  $t_{i+k} < t < t_{n+k}$
3. Unity  $\sum \beta_{i,k} = 1$  for  $t \in [t_0, t_n]$
4. Recursive (as defined above)
5. Continuity i.e.  $\beta_{i,k}$  has  $n - 2$  continuous derivatives.

### 3.5 Functional Time Series Analysis

In fact FTSA is an application of FDA in forecasting a high dimensional time series data. It uses all the tools describe by FDA and some conventional time

series models to forecast. As mentioned earlier many authors applied FTSA methodology for forecasting purpose using different methodologies. We have modified algorithm defined by Hyndman et al. (2007) . The modified algorithm will be is as follows.

1. detect and remedy outliers.
2. Transform the observed data  $y_j(t)$  using some appropriate transformation, which control heterogeneity in the data and also out of sample variance.
3. Estimate the smooth function  $x_i(t_j)$  using some smoothing technique on transformed data for whole continuum.

$$y_j = x_i(t_j) + \epsilon_j$$

4. Estimate the mean  $\mu(x)$  across years of the functional observation, and estimate FPCA decomposition to estimate

$$x_i(t_j) = \hat{\mu}(x) + \sum_{k=1}^K \beta_{j,k} \phi_k(x) + e_j(x)$$

5. fit a time series model to these coefficients  $\beta_{j,k}$ .
6. forecast  $\beta_{j,k}$  and recombine to forecast functional values  $x_i(t_j)$ .

### 3.6 Evaluation of Forecast and Forecast Accuracy

After fitting an appropriate model to the data , the ultimate objective of the model fitting is to forecast. So after checking all the diagnostics, if forecasting is not good then the model is useless. To evaluate the forecasting of the models we have many measures. Let we have a data set  $y_1, y_2, y_3, \dots, y_t$  and we partitioned the data such that some of the data is used for modeling (usually initial 80 percent data, this percentage may vary depending upon the size of data) and some of the data is hold for checking the accuracy of the forecast (usually 20 percent of the data at the end). First data can be said as training data and second can be said as test data Hyndman and Koehler (2006). The difference between training data forecast and actual test data values is called forecast errors.

$$e_t = y_t - \hat{y}_t$$

. Most of the measures of forecast accuracy are based on this error. There are mainly scale-dependent errors, percentage error, symmetric measures, relative errors, relative measures and scaled measures. The pioneer and core

measures are scale dependent measure, consisting of

$$\text{MeanSquareError}(MSE) = \text{mean}(e_t^2)$$

$$\text{RootMeanSquareError}(RMSE) = \sqrt{MSE} = \sqrt{\text{mean}(e_t^2)}$$

$$\text{MeanAbsoluteError}(MAE) = \text{mean}(|e_t|)$$

$$\text{MedianAbsoluteError}(MdAE) = \text{median}(|e_t|)$$

All other type of measures have same idea , with the tackling scale and unit of data. In our case as we have scale free and unit free data, more over data of one variable so we shall use these basic measures.

## 4 Result and Discussion

Numerous studies have been made to forecast inflation using different models. There are mainly two types of models used, theoretical and statistical. Present study belongs to the types of model which are statistical. These models are purely based on data. Among these models *SARIMA* models are famous. We also used another method to model the data in our study. Functional time series analysis is new and emerging field of statistics, which is used to model high frequency data. In present article we will model the



monthly data of both types of series using SARIMA models and Functional time series analysis. Then comparison would be made to asses the superiority of the models.

#### 4.1 Modeling General CPI

We tried to model general CPI by using SARIMA and FTSA. We spliced the data in homogeneous parts. Each functional observation consist of 1 year data. So we have 9 functional observations. We used 9 functional observations in FTSA to Model general CPI. The possible structural break points dates are

Table 1: Structural breaks in General CPI

Series	Structural Break Points Dates				
CPI General	2004(4)	2005(8)	2006(12)	2008(4)	2009(8)

We will use the dummies of these break points in the model to get a stable estimates of parameters. The order of integration of the series was tested using seasonal unit root test. The result shows the annual unit root of order 2 i.e. is integrated of order 2. The estimated models using SARIMA and FTSA are

Gen.jpeg

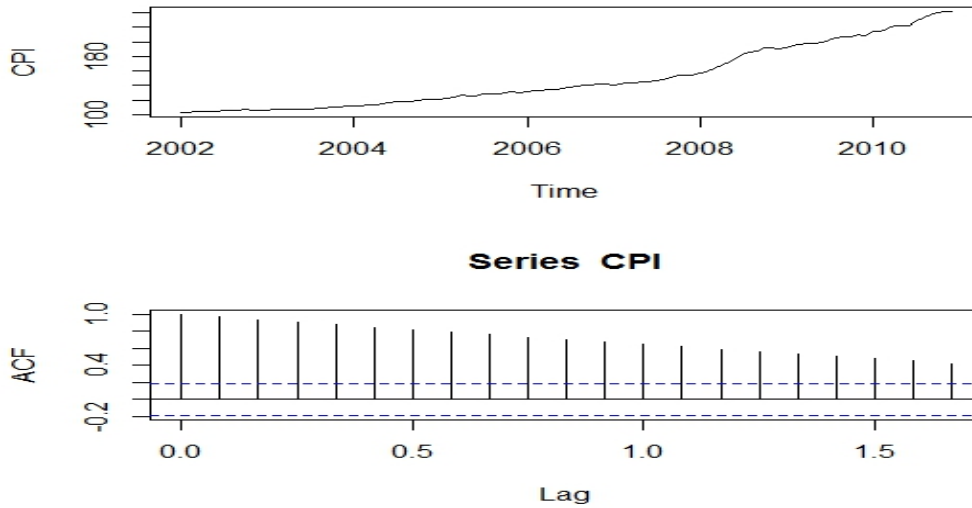


Figure 1: Time series and ACF plot of General CPI

## 4.2 Results and Discussion

The comparison using forecast accuracy measure is given in the following table.

In above table we can see that modeling CPI general using FTSA is far far better than modeling using SARIMA. MSE shows FTSA is almost 7 times better than SARIMA. RMSE shows almost double efficiency over SARIMA. MAE almost 3 times and MEDAE shows that forecast accuracy of FTSA is 5 times better than SARIMA in modeling general CPI.

We can see that forecasted value using FTSA has less difference than forecasted value using SARIMA model. Which also shows the superiority of

Table 2: SARIMA model of General CPI

Coefficients	Estimates	Standard Error	t-Statistics	P-Value	
ma1	-0.8015	0.0826	-9.70339	0	
sar1	0.9486	0.0728	13.03022	0	
sma1	-1.0407	0.1463	-7.11347	0	
sma2	0.2471	0.1488	1.660618	0.099587	
AIC	-724.74	AICc	-724.14	BIC	-711.42
$\sigma^2$	5.217e-05:	Log Likelihood	367.37		
Ljung-Box Q Statistics			Q(4)	4.3717,	0.358
			Q(8)	6.9608,	0.5409
			Q(12)	11.4945,	0.4871

FTSA over SARIMA models using data of general CPI. So modelling using

FTSA is recommended for forecasting inflation.

Table 3: FTSA estimated parameters  $\mu$ ,  $\phi$  and  $\beta$  of CPI general

$\mu$	$\phi$	$\beta$
0.007447	0.317661	-0.01372
0.005575	-0.02043	-0.01737
0.009984	0.400729	-0.00225
0.011648	0.407582	-0.00429
0.00376	0.451007	-0.00447
0.006184	0.29089	-0.00443
0.015616	0.379315	0.040919
0.011357	0.258099	-0.00305
0.008024	0.107805	0.008672
0.009976	0.196118	
0.006021	-0.07827	
-0.00105	-0.13129	

Table 4: Forecast accuracy comparison of General CPI

Method	MSE	RMSE	MAE	MEDAE
SARIMA-Gen	28.39617	5.328806	4.813637	5.168776
FTSA-GEN	4.125	2.031009	1.492074	1.004106

Table 5: Comparison based on forecast of 2011

Actual	SARIMA	FTSA
254.844	259.48	257.94