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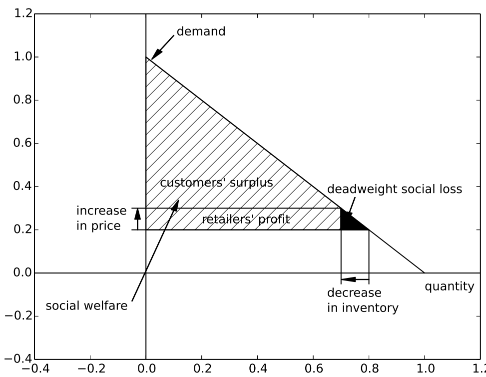
**Abstract:** Legal studies usually treat a policy of a manufacturer or retailer as socially harmful if it reduces product output and increases the price. We consider a two-period model where the first-period price is fixed by resale price maintenance (RPM) and resellers endogenously decide to use another "collusion suspect," meet-the-competition clause with a most-favored-customer clause (MFC), to counteract strategic customer behavior. As a result of MFC, second-period (reduced) price increases, and resellers' inventories decrease. However, customer surplus may increase and aggregate welfare increases in the majority of market situations. MFC can not only decrease the losses in welfare and resellers' profits due to strategic customers but, under reseller competition, may even lead to higher levels of these values than with myopic customers, i.e., to gains from increased strategic behavior. MFC may create "MFC-traps" for resellers, where one of possible market outcomes yields a gain from increased strategic behavior while another results in a reseller profit less than the worst profit in any stable outcome without MFC. With growing competition, benefits or losses from MFC can be higher than losses from strategic customer behavior.

**Keywords:** most favored customer, strategic customer behavior, quantity competition, limited-lifetime product

# 1 Introduction

Legal studies usually treat a policy of a manufacturer or reseller as socially harmful if it reduces product output and increases the price; see, e.g., Edlin (1997), MacKay and Smith (2014), and Christensen and Løvbjerg (2014). Figure 1, which is similar to Figure 1 in Edlin (1997), provides a simple illustration of such an argument. Part of the customer surplus transfers to retailer profit and another part, which corresponds to the shaded triangle area, is the "deadweight loss" for the aggregate welfare. This area benefits neither customers nor retailers.

Figure 1: Welfare loss in a one-period model




In particular, legal studies are increasingly concerned about low-price-guarantees (LPG) as potential anticompetitive practices. Hay (1982) was the first to argue that a price protection or *most-favored-customer clause* (MFC) can facilitate noncompetitive pricing by reducing the incentives of competing retailers to cut prices. As Christensen and Løvbjerg (2014) claim, "We expect increased focus on [MFC] clauses in the future." MFC guarantees that a customer will effectively pay the lowest price suggested to any customer. Quoting Hay (1982), the clause implies also that "a customer who pays list price today may receive a rebate if another customer is offered a lower price within a specified future period."

A *meet-the-competition* clause (MC) is another type of LPG, which guarantees to match a competitor price at the time of purchase. Edlin (1997) concludes that MC can be "much more socially costly than an ordinary monopoly or cartel." This conclusion can be extended to MFC using the same argument because "high prices can persist even when new firms enter the industry." MC and MFC are also called, respectively, concurrent and posterior price matching in the operations management literature; see e.g., Lai, Debo, and Sycara (2010).

An empirical study by Arbatskaya, Hviid, and Shaffer (2004) shows that the use of various forms of LPG, including MC and MFC, "is widespread, with no obvious missing retail sectors." In practice, customers can also exploit retailer return policy to achieve the effects of MFC. Quoting a customer forum RedFlagDeals (2008), this behavior can be "referred to as 'rebuy and return' – you buy the new item at the lower price first, then go return that same physical item with your OLD receipt." Customers can exhibit rebuy and return behavior for any product such that individual items cannot be distinguished by the retailer even if this retailer is out of stock. According to `www.pricematching.us`, all the retailers offering MC have either directly stated "price protection period," i.e., MFC, or return policy, which covers a period up to one year, or both. Thus, meeting a competitor's price during the full-price season is often combined with matching the firm's own price in the future. Our study considers this combination of MC with MFC.

MFC attracts a particular attention of legal bodies when resellers' prices are almost the same, which may result from *resale price maintenance* (RPM). European Commission (2010) defines RPM as an agreement between manufacturer and reseller or concerted practice intended for the "establishment of a fixed or minimum resale price or a fixed or minimum price level to be observed by the buyer." European Commission (2010) stresses that price fixing is more effective when combined with policies, such as MFC, which may reduce reseller's incentives to lower the price.

As Butz (1996) pointed out, MC may also serve as an RPM-facilitating tool: "a manufacturer coordinates retailers' efforts by financing some or all of the meet-the-competition-related rebates they make, so in equilibrium all retailers adopt the 'suggested' price [MSRP]." An individual retailer has no incentive to markdown during the full-price season because the competitors' MC policy does not allow to increase market share by unilateral price cut. Such a deviation from MSRP can lead only to a lower profit margin while the margins of the competitors are supported by the manufacturer. Therefore, the resellers in our model do not deviate from MSRP explicitly in the full-price season. A long history and empirical evidence of RPM is provided in Appendix.

Potentially high social cost of mistakes in determining a legal status of LPG and RPM requires a comprehensive study of these phenomena. At the same time, a fixed price (MSRP) for the full-price season, which is the same for all resellers, allows for a more detailed analysis of other effects, not connected with the choice of this price. Our study focuses on the intertemporal effects of forward-looking or *strategic customer behavior* and responses of competing resellers to this behavior on aggregate welfare and resellers' profits. These effects are an important and underestimated source of misidentifying a legal status of resellers' and manufacturers' policies because legal studies usually do not take them into account.

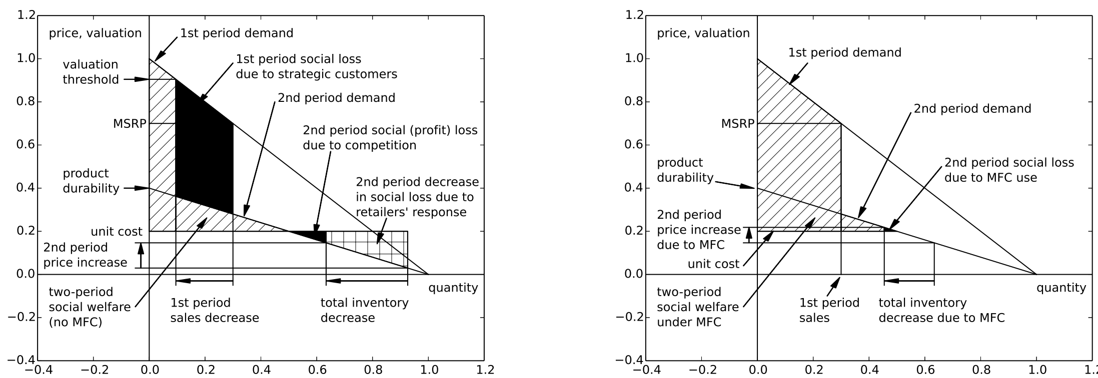
Extensive research, e.g., Shen and Su (2007), Aviv and Vulcano (2010), Aviv, Levin, and Nediak (2009), and Lai, Debo, and Sycara (2010), confirms that strategic behavior can essentially affect resellers' profits and total customer surplus when customers are timing their purchases in anticipation of price markdowns over the course of a sales season. The level of strategic behavior can be different, e.g. within a business cycle, leading to various market outcomes. When economy is expanding, more customers prefer to buy now than wait, and vice versa – an average customer is more inclined to delay the purchase when economy shrinks. For example, a study of a Fortune 500 retailer sales by Allenby, Jen, and Leone (1996) shows that even "fashion-forward consumers who purchase apparel early in the season are more sensitive to economic conditions and expectations than previously believed."

On the reseller side of the market, firms compete for the market shares and, eventually, maximize profits. Therefore, resellers can compete under RPM not only in total-demand-enhancing services but in other tools that attract customers. For example, "the adoption of point cards or loyalty programs has been widely adopted by Japanese retailers, who use them to effectively circumvent the [RPM] system by indirect discounting of product"; see Nippop (2005). In response to strategic customer behavior, "companies are choosing to shorten the length of time before some new releases or re-releases can be discounted, thus reducing prices for consumers who don't mind waiting a while before they buy"; see Nippop (2005).

In this context, MFC can be primarily viewed as a tool to increase market share. However, it also serves as a response of resellers to strategic customer behavior. A theoretical idea of such response was suggested by Coase (1972), who was the first to explain the negative impact of strategic customer behavior for a monopolistic supplier of a durable good. Coase's reseller offered customers a buy-back agreement. Specifically, if the product is offered at any time in the future at a lower price, the monopolist agrees to accept product returns and to issue the customers full refund. In fact,

Figure 2: Welfare change in a two-period model

(a) Due to strategic customers (b) Due to MFC




in Coase's model, this is equivalent to paying the customers the difference between their purchase prices and the offered discounted price at any time. The rationale behind this strategy is that it ties the hands of the monopolist's future "replicas." Since the customers know that future price discounts require the reseller to pay back early purchasers, they do not anticipate the monopolist to offer significant discounts. Consequently, such rational perception drives strategic customers to purchase at premium prices. At the same time, when the first-period price is fixed by RPM, MFC clause allows resellers to circumvent RPM system when the suggested price and the level of strategic behavior are high. Thus, MFC can alleviate a decrease in customer surplus caused by RPM using only market levers, without socially costly interventions of legal bodies.

Our study partly confirms that MFC can soften competition, i.e., increase prices and shrink resellers' inventories. However, when a product loses its value in time and customers are strategic, changes in welfare include intertemporal effects. The two parts of Figure 2 illustrate these effects for the cases with and without MFC. When MFC is not used (Figure 2(a)), there exists a valuation threshold, which is higher than the first-period price (MSRP), such that strategic customers with valuations below this threshold wait for the second-period price drop. This strategic behavior leads to a first-period social loss depicted by the shaded area. Profit-maximizing retailers respond to the strategic behavior by contracting inventory and increasing the second-period price. In this example, second period price is below the unit cost, which results from retailer competition (10 retailers) and a drop in customer valuations to 0.4 of the initial levels.

Keeping all market parameters fixed, the use of MFC (Figure 2(b)) eliminates customer incentive for waiting. The first-period social loss caused by strategic customers vanishes, and the second-period loss shrinks compared to the one in Figure 2(a). In this example, based on the results provided in the paper, the use of MFC doubles the aggregate welfare despite the increase in the second-period price and decrease in the total inventory. Moreover, for high levels of competition, MFC leads to a *welfare gain from increased strategic behavior*, i.e., welfare under MFC with strategic customers is greater than the one with no MFC and myopic customers. This effect can be seen by comparing the areas of welfare loss in Figures 2(b) and (a), where the latter includes the area in crosses for myopic customers. Thus, a higher price and lower output may be welfare-improving if they result from a policy that leads to an intertemporal redistribution of demand.

We find that *MFC is aggregate welfare improving in the majority of market situations* in our setting. A comparative welfare gain from MFC increases in the level of competition keeping other market parameters fixed. This result can shift a balance in the legal treatment of a specific MFC case in favor of MFC under the total welfare standard. More generally, our study implies that a conventional legal argument illustrated in Figure 1 may lead to socially costly mistakes when product value depreciates and customers are strategic.

Lai, Debo, and Sycara (2010), using a two-period model with one retailer and strategic customers, argue that MFC may increase both customer surplus and seller's profit only when the uncertainty in the number of customers with high valuations is high. We exclude uncertainty, which allows for closed-form analysis, and show in a slightly different setup (see §2.6) that MFC indeed may increase the surplus, and the welfare-improving effect of MFC is quite robust under reseller competition. However, for the majority of market parameters, MFC is not surplus-increasing.

Intuitively, the efficacy of MFC as a tool to mitigate strategic customer behavior depends on the degree to which the customers are strategic. Such behavior exerts a downward pressure on the equilibrium product quantity in the market effectively becoming a force opposing competition, which tends to increase the supply. We show that whenever MFC changes the equilibrium structure, the aggregate quantity (total inventory) decreases, leading to a higher second-period price. It is known in the literature that a monopolist using MFC can reduce, i.e. *mitigate*, the loss from strategic customer behavior. Our study identifies when MFC leads to a higher reseller profit than that with myopic customers and without MFC, i.e., to a *gain from increased strategic behavior*. To the best of our knowledge, this result is new. The gain is possible only under reseller competition, which was not considered in previous studies in settings with MFC and strategic customers. The gain happens because MFC-equilibria, for majority of market situations, can be realized only when customers are strategic. This endogenous behavior of resellers is consistent with the empirical results of Arbatskaya, Hviid, and Shaffer (2004) who conclude that "a firm might be more likely to combine its low-price guarantee with a most-favored-customer clause ... if it believes that consumers are concerned about the product going on sale in the future," i.e., if customers are strategic.

We show that beneficial effects of MFC for resellers increase in the level of competition. Additionally, we provide the conditions when MFC results in sales in both periods, i.e., MSRP is effectively void; thus, signaling to the manufacturer that the first-period price is too high. Finally, in §5, we show that the cases of reseller monopoly and oligopoly have essential qualitative differences for market participants. We enhance qualitative insights by showing that, depending on the levels of competition and strategic behavior, different types of MFC equilibria can lead to reseller profit gains or losses exceeding the direct losses from strategic customer behavior. In particular, there are sets of market parameters, which we call *"MFC-traps" for resellers*, that, due to multiple equilibria, may result either in a gain from increased strategic behavior or in a profit less than the worst equilibrium profit without MFC.

## 2 Model description

We consider a two-period model of a competitive market with $n$ resellers. We call them "retailers" when we assume that they sell a product to the end customers or consumers. The product has limited lifetime, and, for clarity, we assume that the resellers are identical and demand is deterministic. Under the first-period price, $p_1$, determined by RPM, the resellers select their profit-maximizing inventory levels in anticipation of the market outcome. The unit cost of inventory is $c$, which obviously includes the cost charged by the manufacturer, but also embeds the reseller's effort to increase its *"market attraction."* We assume that the unit cost is equal for all resellers, driven in part by our market structure in which the manufacturer is common, the product is undifferentiated, and the resellers operate under similar conditions; see, e.g., §4.4 in Liu and Ryzin (2008).

Since the resellers know the market and consider MFC as a strategic-behavior mitigating tool, we assume that they set their MFC policies at the same time they select their inventory levels. Let $y^i$ denote the inventory (capacity) of reseller $i$ at the beginning of the season, and $m^i \in \{0, 1\}$ be reseller $i$'s decision on utilizing an MFC policy (where 0 and 1 mean "no" and "yes," respectively); define the vectors $y = \left(y^1, \dots, y^n\right)$ and $m = \left(m^1, \dots, m^n\right)$ accordingly. In the second period, the resellers are "free" to select their own prices, but we assume that under the competitive market structure, they converge to a price that clears the market; see, e.g., Dixon (2001). To this end, we utilize a Cournot model to predict the second (clearance) period price as a function of the remaining inventory at the end of the first period.

In the first period, the market consists of *regular* customers with a mass normalized to 1 without loss of generality (change of scale). The first-period valuations of these regular customers are uniformly distributed on the interval $[0, 1]$ with 1 being a normalized highest valuation. Two essential parameters affect customers' behavior. First, we use parameter $\beta \in [0, 1]$ to capture a typical decrease in valuations for seasonal and limited lifetime products. For example, a product may lose 25% of its value (i.e., $\beta = 0.75$) – from a customer's standpoint – if that customer purchases the product at the end (second period) rather than at the beginning (first period) of the season. We refer to this parameter as *product durability* (we interpret durability as a measure of *useful* product lifetime). We confine our analysis to the interesting case of $\beta > c$, which means that, in the second period, a customer with the highest valuation $\beta$ may be willing to purchase at a price that is above cost. If that is not the case, it is easy to show that any MFC-equilibrium would have to result in the first-period sales only.

Second, recall that a strategic customer is one that considers the possibility of postponing the time of purchase to the second period, by taking into account the possible price reduction and product availability in that period, as well as the MFC payback (that would become irrelevant if the customer postpones the purchase). We use the parameter $\rho \in [0, 1)$, to which we refer as the *level of strategic behavior*, as a discount factor that the customers apply to the expected second-period surplus and to any refunds from MFC. In particular, a value of $\rho = 1$, which can be considered as a limiting case, means that the market consists of customers that are "fully" strategic, whereas $\rho = 0$ means that the customers are *myopic*, i.e., they always purchase in the first period unless their valuations are less than $p_1$. The customers are homogeneous in their level of strategic behavior.

Similarly to Lai, Debo, and Sycara (2010) and Cachon and Swinney (2009), we assume that in addition to the regular customers, there is an infinite number of bargain-hunting customers who can buy any number of units in the second period, at salvage value $s < c$. Alternatively, one can think of this situation as a market setting in which remaining inventory can be returned to the supplier for a reimbursement of $s$ per unit, e.g., through buyback agreements, or the ability of the supplier to divert the product to a secondary market channel. Thus, we analyze the market using a game theoretical framework that follows the sequence of events listed below.

First, customers form rational expectations about the second period market for all possible combinations of resellers' MFC offerings. Second, given the customers' expectations, the resellers determine their MFC policies and inventories. Then, the first-period sales are realized (see §2.2 for details). Finally, in the second period, if the product remains on shelf, the resellers engage in clearance sales and reimburse the difference in prices between two periods if they use MFC.

## 2.1 Customer behavior

Contingent on the MFC offering information $m$, the customers form rational expectations about the product availability and the price in the second period, and make a decision about purchasing at $p_1$ or waiting for the second period. We model their behavior according to the following lines. Since customers do not observe inventories, they form expectations via two key parameters: first, similar to Su and Zhang (2008), is the *expected* availability, $\bar{\alpha}(m) \in \{0,1\}$, which indicates whether inventory will be left at the end of the first period and hence will be cleared. Second, if inventory is left ($\bar{\alpha}(m) = 1$), is the expected clearance price $\bar{p}_2(m)$. For brevity of exposition, when there is no risk of confusion, we may avoid the explicit functional notation, using $\bar{\alpha}$ and $\bar{p}_2$ in short.

Confined to the pair $(\bar{\alpha}, \bar{p}_2)$ for given $m$, the customers make their buy-or-wait decisions using a hierarchical procedure, as follows. At first, customers would compare their valuations $v$ with the price $p_1$. If $v < p_1$, the customer will *wait* for the second period. Otherwise, the customer who can gain an immediate surplus of $(v - p_1)$ will bring into consideration the second period – the essence of strategic behavior. Specifically, a customer who considers buying from an MFC reseller will calculate the *net gain* that can be achieved in the *second period* by postponing the purchase; i.e., in addition to the loss of the immediate surplus $(v - p_1)$. That net gain consists of two values: (i) the expected MFC payback that will be *forgone* due to the wait; and (ii) the expected surplus that would be *gained* in the second period. Altogether, we have

$$\Delta(v) \triangleq -\bar{\alpha}(p_1 - \bar{p}_2)^+ + \bar{\alpha}(\beta v - \bar{p}_2)^+ .$$

Recall that we use the parameter $\rho$ to express the degree of strategic behavior in the market. Following this approach, a customer with valuation $v \geq p_1$ will attempt to purchase a unit from an MFC reseller in the first period if $\rho\Delta(v) - (v - p_1) \leq 0$. The left-hand side is always decreasing in $v$, and hence, because $\rho\Delta(p_1) - (p_1 - p_1) \leq 0$, we conclude that any customer with valuation not less than the threshold

$$v_1^{\min}(\bar{\alpha}, \bar{p}_2) = \text{constant} = p_1 \tag{1}$$

will attempt to buy a unit from an MFC reseller. Since $v_1^{\min}$ does not depend on expectations, we drop its functional dependence on $(\bar{\alpha}, \bar{p}_2)$ in the rest of the paper.

In reality, some customers with $v \in (\bar{p}_2, p_1)$ might take a risk to buy in the first period in hope to obtain the second-period refund, which, as they expect, may be greater than the first-period negative surplus. However, real customers know that they may not receive the refund due to reasons that may not depend on the reseller, e.g., canceled credit card. Therefore, we assume that since customers strongly dislike negative surplus, they buy in the first period only if $v \geq p_1$. Relaxation of this assumption would only reinforce the conclusion about welfare-improving property of MFC.

The first-period customers always prefer to buy at an MFC reseller rather than at a no-MFC one because, in addition to the surplus $v - p_1$ of the first-period purchase, they anticipate to gain the expected value of refund $\bar{\alpha}\rho(p_1 - \bar{p}_2)^+$. The customers are indifferent only when they are myopic ($\rho = 0$) or expect no sales in the second period ($\bar{\alpha} = 0$). Therefore, only when the unit is not available at an MFC reseller, a customer considers purchasing from a non-MFC one. In such case, the decision would be based on whether $(v - p_1) \geq \bar{\alpha}\rho(\beta v - \bar{p}_2)^+$. Here, it is easy to verify that there are three cases of interest: (i) $p_1 \leq \bar{p}_2/\beta$, for which the valuation threshold $p_1$ would be adopted; (ii) $\bar{p}_2/\beta \leq p_1 \leq 1 - \bar{\alpha}\rho(\beta - \bar{p}_2)$, for which the threshold $(p_1 - \bar{\alpha}\rho\bar{p}_2)/(1 - \bar{\alpha}\rho\beta)$ would be adopted; and (iii) $p_1 \geq 1 - \bar{\alpha}\rho(\beta - \bar{p}_2)$, for which no customer would buy in the first period, effectively using the threshold 1. In summary, any customer with valuation larger than the threshold

$$v_0^{\min}(\bar{\alpha}, \bar{p}_2) = \max\left\{p_1, \min\left\{\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1 - \bar{\alpha}\rho\beta}, 1\right\}\right\} \tag{2}$$

will attempt to buy a unit from a non-MFC reseller in the first period.

The above analysis demonstrates that an opportunity to buy from an MFC reseller completely eliminates strategic customer behavior ($v_1^{\min} = p_1$), regardless of its level $\rho$, the durability of the product $\beta$, or the expectations $(\bar{\alpha}, \bar{p}_2)$. However, it is not clear at this point when such elimination of strategic behavior is beneficial for the participants in the market.

### 2.2 First-period sales distribution among resellers

This section presents a sales allocation mechanism, which allows for the calculation of the first-period sales, denoted by the vector $q = \left(q^1, \ldots, q^n\right)$. Since the first-period customers prefer MFC over no-MFC resellers, the first-period demand is allocated among the resellers with MFC and then, the unsatisfied demand (due to stockouts) is split among no-MFC resellers. Inside each group of resellers, customers buy in the order of their valuations. We assume also, similar to §6.5 in Cachon (2003), that the ability of a reseller to attract sales is proportional to its level of inventory.

Let $n_1, Y_1$, and $Q_1$ be, respectively, the number of MFC resellers, aggregate inventory, and first-period sales for those resellers; similarly, define $n_0, Y_0$, and $Q_0$ for the non-MFC resellers. Moreover, consider specific expectations $(\bar{\alpha}, \bar{p}_2)$ for the corresponding MFC policies $m$. The total demand that the MFC resellers experience in the first period is $(1 - p_1)$ as driven by the threshold value $v_1^{\min} = p_1$. Therefore, there are three cases of interest that depend on the aggregate inventory $Y_1$. (i) If $Y_1 \geq 1 - p_1$, the MFC resellers satisfy all of the demand, each selling a quantity $q^i = (1 - p_1) \cdot y^i / Y_1$, whereas the non-MFC resellers do not make any sales. Additionally, the regular customers remaining for the second period would have valuations uniformly distributed in the range $[0, \beta p_1]$ at that time. (ii) If $1 - v_0^{\min}(\bar{\alpha}, \bar{p}_2) \leq Y_1 < 1 - p_1$, the MFC resellers cannot satisfy all of the demand. Thus, the sales for the MFC resellers are given by $q^i = y^i$, serving the valuation segment $[1 - Y_1, 1]$. Next, since the latter segment turns its demand to the MFC resellers, and since $v_0^{\min}(\bar{\alpha}, \bar{p}_2) \geq 1 - Y_1$, it is easy to see that the non-MFC resellers will experience no demand. Consequently, the regular customers remaining for the second period would have valuations uniformly distributed in the range $[0, \beta(1 - Y_1)]$ at that time. (iii) If $Y_1 \leq 1 - v_0^{\min}(\bar{\alpha}, \bar{p}_2)$, the situation with the MFC resellers remains the same as in case (ii). However, it is easy to verify that the non-MFC resellers would sell the quantities $q^i = \min\left(\left(1 - Y_1 - v_0^{\min}(\bar{\alpha}, \bar{p}_2)\right) \frac{y^i}{Y_0},\ y^i\right)$. The regular customers remaining for the second period would have valuations uniformly distributed in the range $\left[0, \beta \cdot \max\left\{1 - Y_1 - Y_0, v_0^{\min}(\bar{\alpha}, \bar{p}_2)\right\}\right]$ at that time.

### 2.3 Second-period clearance sales

Since the product offerings are undifferentiated, the resellers lower their prices until all remaining inventory is cleared; i.e., the second period price $p_2$ (identical for all resellers) would be set to a sufficiently low level that would make demand equal to the total remaining inventory. Since MFC and inventory decisions are made at the same time and the demand is deterministic, a reseller would never have to withhold previously acquired inventory from clearance because of the MFC. Instead, a rational reseller simply avoids stocking any inventory that is not eventually sold.

Let $Y \triangleq Y_1 + Y_0$ and $Q \triangleq Q_1 + Q_0$. Following the previous section, we anticipate that inventory will be left only if $v_0^{\min}(\bar{\alpha}, \bar{p}_2) > 1 - Y$. In such case, clearance of the inventory (i.e., *completing* the sales of all of the original inventory $Y$) can be made either by targeting the customer with the original valuation of $(1 - Y)$, by setting $p_2 = \beta(1 - Y)$, or, turning to the stream of bargain-hunters, by setting $p_2 = s$. Obviously, the second-period price that would maximize revenue is

$$p_2 = \max\{s, \beta(1 - Y)\}, \tag{3}$$

which is independent of the MFC offers present in the market.

In the rest of the paper, we focus on situations in which the second period valuations are sufficiently high so that $\beta > s/p_1$ (a condition similar to the logical restriction $\beta > c$). If this condition does not hold, it is possible to show that, in a two-period equilibrium, the second period price cannot exceed $s$, $v_0^{\min} = p_1$ under rational expectations, and strategic customer behavior has no effect on any of the possible equilibria.

### 2.4 The first period inventory and MFC decisions

We continue our analysis by looking at the resellers' profit optimization problems in the first period. Recall that since the second-period market is cleared, each reseller's second period inventory (equals to its sales) is $y^i - q^i$. Obviously, because of the interactions among the resellers, we must describe any given reseller's profit as a function of the other resellers' decisions as well as the customers' expectations $(\bar{\alpha}(m), \bar{p}_2(m))$. To this end, define $y^{-i}$, $m^{-i}$ as the vectors of inventories and MFC decisions of all resellers except $i$. We can now present the objective functions for the resellers:

$$\begin{aligned} &r^i(y^i, m^i, y^{-i}, m^{-i}, \bar{\alpha}(m^i, m^{-i}), \bar{p}_2(m^i, m^{-i})) \\ &\qquad = -cy^i + p_1 q^i + p_2(y^i - q^i) - q^i(p_1 - p_2)^+ \cdot \mathbb{1}\left\{\{Y > Q\} \cap \{m^i = 1\}\right\} \end{aligned} \tag{4}$$

where $p_2$ depends on $y$, and the $q^i$-values depend on the values $(y^i, m^i, y^{-i}, m^{-i}, \bar{\alpha}(m^i, m^{-i}), \bar{p}_2(m^i, m^{-i}))$, as explicitly described in §2.2. We conclude that the best response of reseller $i$ belongs to a set of $(y^i, m^i)$ pairs:

$$BR^i\left(y^{-i}, m^{-i}, \bar{\alpha}(\cdot, m^{-i}), \bar{p}_2(\cdot, m^{-i})\right) \triangleq \operatorname*{Argmax}_{y^i, m^i}\left\{r^i(y^i, m^i, y^{-i}, m^{-i}, \bar{\alpha}(m^i, m^{-i}), \bar{p}_2(m^i, m^{-i}))\right\},$$

where notation $\bar{\alpha}(\cdot, m^{-i}), \bar{p}_2(\cdot, m^{-i})$ emphasizes the dependence of the best response set on the expectations corresponding to either value of $m^i \in \{0, 1\}$ but only the given value of $m^{-i}$.

Using the set of best responses, one can proceed to characterize general Nash equilibria in the reseller game. However, our primary focus is on two levels: the level of competition and the level of strategic behavior. The resellers are identical and it is natural to consider cases when they behave in the same way. As we show in §3 below, the resulting symmetric equilibria cover almost 100% of all inputs. One cannot rule out the existence of asymmetric equilibria and they may provide some additional insights, but, given a rich collection of results obtained for the symmetric case, the asymmetry would merely distract from the main effects considered in this paper.

In a *symmetric* pure-strategy Nash equilibrium each reseller makes the same MFC decision $\hat{m}$ and procures the same fraction $\frac{1}{n}\hat{Y}$ of the total inventory $\hat{Y}$. Additionally, we consider *symmetric* expectations characterized by only two pairs of values $(\bar{\alpha}(\hat{m}, \hat{m}, \ldots, \hat{m}), \bar{p}_2(\hat{m}, \hat{m}, \ldots, \hat{m}))$ and $(\bar{\alpha}(1-\hat{m}, \hat{m}, \ldots, \hat{m}), \bar{p}_2(1-\hat{m}, \hat{m}, \ldots, \hat{m}))$ corresponding to, respectively, the equilibrium MFC profile $(\hat{m}, \hat{m}, \ldots, \hat{m})$ and any possible one-reseller deviation. Formally, since expectations depend only on the first argument, we drop the remaining arguments in the rest of the paper. For $n > 2$, customers directly observe the firm that deviates from a symmetric MFC profile. For duopoly, symmetry of expectations requires an assumption that customers can identify the MFC decision that constitutes a deviation. For given symmetric expectations $\bar{\alpha}(\cdot), \bar{p}_2(\cdot)$, a symmetric equilibrium is a pair $(\hat{m}, \hat{Y})[\bar{\alpha}(\cdot), \bar{p}_2(\cdot)]$ (a pair $(\hat{m}, \hat{Y})$ as a function of $\bar{\alpha}(\cdot), \bar{p}_2(\cdot)$) such that $(\hat{m}, \frac{1}{n}\hat{Y})$ provides a best response to a symmetric strategy profile of other resellers, i.e., $(\hat{m}, \frac{1}{n}\hat{Y}) \in BR^i\left((\frac{1}{n}\hat{Y}, \ldots, \frac{1}{n}\hat{Y}), (\hat{m}, \ldots, \hat{m}), \bar{\alpha}(\cdot), \bar{p}_2(\cdot)\right)$, where $(\frac{1}{n}\hat{Y}, \ldots, \frac{1}{n}\hat{Y})$ and $(\hat{m}, \ldots, \hat{m})$ are $n-1$ dimensional vectors, which stand for $y^{-i}$ and $m^{-i}$ respectively.

### 2.5 Stable market outcomes

In order to gauge the effects of strategic customer behavior and resellers' responses, a participant of the market (a reseller, a manufacturer, or a local regulator) must first understand which equilibria are possible for that particular market scenario. Cachon and Swinney (2009), finding the outcomes of the interaction between a reseller and strategic customers, assume that all players in the game can form beliefs about the actions of the other players including customers' beliefs about resellers' inventories. For some products, however, customers may not form beliefs about resellers' inventories even when new versions of the product repeatedly emerge in the market. For example, a buyer of a music or video record usually does not know the number of particular records in the market and the number of customers interested in buying this record. This buyer, however, may form beliefs about the availability of the product on sale and the clearance price depending on resellers' MFC policy because this information is observable ex post over multiple realizations of the market.

In our setting, each customer knows only his/her own valuation, the durability level $\beta$, and the level of strategic behavior $\rho$, and observes the MSRP $p_1$ prior to forming expectations and making wait or buy decisions. *Ex post* customers observe only the second-period availability $\alpha$ and price $p_2$, *not* the inventory levels or market size. Given all available information, customers *cannot even infer* the inventory levels. In such an environment, customer expectations in terms of directly observable quantities such as the second-period availability and price are a natural model.

While there may be equilibria in which customer expectations are not rational, such equilibria would not result in stable market outcomes. Therefore, we focus our attention on equilibria with rational expectations. Specifically, we identify the set of decisions, made by the resellers, such that they are optimal in the sense described earlier, but are also consistent with the customers' expectations $(\bar{\alpha}(\cdot), \bar{p}_2(\cdot))$. That is, the equilibrium inventory levels and MFC decisions of the resellers must lead to precisely the same observed product availability and clearance prices as expected by the customers. Recall that, according to (3), the observed second period price corresponding to the total inventory $\hat{Y}$ is equal to $\max\{s, \beta(1-\hat{Y})\}$. Moreover, if the total first-period sales corresponding to $(\hat{m}, \hat{Y})$ are $\hat{Q}$, then the observed second-period availability is $\mathbb{1}\{\hat{Y} > \hat{Q}\}$. Thus, we define *rational expectations symmetric equilibrium* (RESE) in pure strategies as follows:

**Definition 1.** *The tuple $(m^*, Y^*, \alpha^*(\cdot), p_2^*(\cdot))$ is a RESE if*

- *$m^*$ and $Y^*$ are a symmetric equilibrium MFC decision and a total inventory level corresponding to symmetric expectations $\alpha^*(\cdot)$ and $p_2^*(\cdot)$, i.e., $(m^*, Y^*) = (\hat{m}, \hat{Y})[\alpha^*(\cdot), p_2^*(\cdot)]$;*
- *the expected and the observed equilibrium second-period availabilities and prices coincide, i.e., for the corresponding first-period sales $Q^*$, $\alpha^*(m^*) = \mathbb{1}\{Y^* > Q^*\}$ and $p_2^*(m^*) = \max\{s, \beta(1-Y^*)\}$;*
- *and, for a single reseller deviating from $m^*$ into a different MFC strategy $1-m^*$ and this reseller's optimal inventory decision $y'$, we have, for the corresponding first-period sales $Q'$ under the deviation, $\alpha^*(1-m^*) = \mathbb{1}\left\{\frac{n-1}{n}Y^* + y' > Q'\right\}$ and $p_2^*(1-m^*) = \max\left\{s, \beta(1-\frac{n-1}{n}Y^* - y')\right\}$.*

The last requirement clarifies why expectations have to depend on the MFC profile. In the absence of such dependence, expectations may not match the availability and clearance price observed under the deviations. Thus, a deviating reseller may be able to take advantage of these irrational expectations breaking the equilibrium as the result. On the other hand, if customers adjust expectations when they see an MFC deviation, the deviator no longer has this unfair advantage.

For resellers, it is important to know which outcomes can emerge depending on the market situation. From the model perspective, the market situation is described by particular model inputs and potential outcomes correspond to the equilibria that exist in the reseller game. In the next section, we characterize all possible equilibria in closed form starting with those using MFC. This characterization facilitates analysis of the impact of MFC on resellers, customers, and the local economy. Moreover, switches between equilibrium types due to changes in the inputs (such as the levels of strategic behavior or competition) inform market participants about potential jumps in profits, customer surplus, and welfare.

## 2.6 Discussion of model assumptions

A key challenge in the theoretical study of markets with strategic customers is the identification and characterization of stable market outcomes (or equilibria in game-theoretic terms). To this end, one must pay careful attention to the assumptions regarding the information available to the decision makers: the customers and the resellers. For instance, in mature markets, where manufacturers regularly launch new versions of similar products, the resellers are typically able to conduct comprehensive customer behavior studies. An empirical analysis in Fisher and Raman (1996) shows that demand uncertainty in the fashion apparel industry can be significantly reduced by analyzing preliminary sales of the product. Our research questions are not connected directly to demand uncertainty. In this vein, for clarity of exposition and to avoid excessive generality, we assume that the resellers know demand with certainty and can determine how MFC decisions affect the first-period demand. Uncertain demand is an important element, e.g., in Cachon and Swinney (2009), who study the value of quick inventory response after demand realization. Another example is Lai, Debo, and Sycara (2010), who managed to shows that in their model posterior price-matching (MFC) may benefit consumers only when the market uncertainty is high. The deterministic approach allows us to confirm the robustness of this finding in a different setting as well as provide a detailed closed-form analysis, which otherwise is not possible.

Additionally, we assume that customers can form stable expectations regarding the information that is repeatedly observable in mature markets: price changes and product availability in the second period. Yet, similar to Lai, Debo, and Sycara (2010), the customers observe only resellers' MFC-policies but not the inventory levels. Other similarities with Lai, Debo, and Sycara (2010) include "no hassle cost to process the refund claim," declining valuations in the second period, and, unlike Cachon and Swinney (2009), no chances to replenish inventory during the selling season. Our setup, besides retailers' competition, differs from Lai, Debo, and Sycara (2010) in that we study the effects of changes in the levels of competition and strategic behavior rather than changes in the shares of fully strategic and myopic high-end customers with homogeneous valuations. The difference in research questions motivate different modelling choice. In particular, valuations of our regular customers are continuously distributed, but discount factor is the same.

The customer discount factor, similarly to Cachon and Swinney (2011), is a constant that belongs to the range from zero to one, which is supported empirically by a review in Frederick, Loewenstein, and O'Donoghue (2002). Typically, customers are myopic for inexpensive products. Some studies, e.g., Hausman (1979), claim that the discount rate depends on income, i.e., customers are heterogeneous. Other studies, however, argue that this dependence is not significant, see, e.g., Houston (1983).

For our study of MFC, we use the benchmark game where MFC is not available. Obviously, the benchmark setup is identical to the one presented above except the MFC option. Some of the results that refer to this benchmark are obtained in Bazhanov, Levin, and Nediak (2015) and briefly cited in §3.2 and some proofs.

The assumption about sales proportional to the level of inventory is in congruence with our uniform unit cost assumption, discussed in the beginning of the section: resellers that bring larger capacity could possibly enjoy economies of scale in procurement costs, but on the other hand may want to spend more on sales efforts in order to attract reasonable demand. This assumption stems also from the literature on endogenous demand. Wolfe (1968) was the first who studied an empirical evidence of sales proportional to inventory levels in apparel industry. The proportional allocation mechanism is not always the standard assumption. Admittedly, we utilize this assumption in order to enable us to obtain clear and relatively-elegant theoretical results. For similar reasons of gaining analytical tractability, papers such as Zhao and Atkins (2008), Liu and Ryzin (2008) and Bazhanov, Levin, and Nediak (2015) have considered alternative allocation schemes.

In the presence of RPM, the main reseller decision is the quantity of the product. The literature underlines that quantity decisions are of particular importance in the presence of strategic customers. Coase (1972) suggests that the seller can make a contractual arrangement with the customers in which he agrees not to sell more than a given quantity of the product. This *capacity rationing* proposition has been studied in papers such as Liu and Ryzin (2008). The authors find that when the market consists of a large number of high-valuation risk-averse customers, capacity rationing is useful; otherwise, the firm should serve the entire market at a low price. Under competition, the effectiveness of capacity rationing is reduced, and there exists a critical number of firms beyond which rationing never occurs in a stable market outcome (equilibrium). Levin, McGill, and Nediak (2010) and Cachon and Swinney (2009) demonstrate the effectiveness of capacity decisions, and both provide a sharper understanding of the intricate relationship between the pricing and quantity decisions; see also Su (2007) and Su and Zhang (2008). Since quantity competition of resellers is another important characteristic of our setup, we model clearance sales as Cournot competition. The Cournot model, which we use for the second period, is one of the approximations for real markets; e.g., Flath (2012) shows that products such as music records, bicycles, and thermos bottles are appropriately described by this model. In our study, this model helps to concentrate on the intertemporal effects of strategic customer behavior and resellers' responses (quantity and MFC) without distracting effects of the second-period resellers' price competition.

There are different opinions in the literature regarding the criterion of legality of a reseller policy. Currently, a policy is treated as legal if it improves consumer welfare, which is often estimated as total customer surplus. Many authors, mostly economists, argue that "the consumer welfare standard has a number of shortcomings vis-à-vis the total welfare standard"; see Cseres (2007). In particular, this measure counts only short-term customer benefits and discriminates among different groups in society. In our paper, we characterize the "consumer welfare" in terms of the total customer surplus and the "total welfare" (or "social welfare") for the local economy in terms of the aggregate welfare defined as the sum of the customer surplus and the reseller profits while excluding a transnational manufacturer. Our results, which show that MFC is mostly socially beneficial under the total welfare standard and harmful under the consumer surplus standard, indicate that evaluation criteria used by lawyers, economists, and policymakers must be carefully selected to match strategic policy objectives of the respective countries.

# 3 Characterization of stable market outcomes

There are two fundamental types of market outcomes that can potentially arise in the proposed model: with MFC and without MFC. We will refer to them, respectively, as M and either N if no-MFC is the reseller's decision or NA if MFC is not available for other reasons. Each of these principal types is further classified into subtypes based on the structure of the market outcome. In particular, whether sales occur in both or only in one of the periods, and in which period they occur. We discuss MFC first and then contrast it with no-MFC equilibria.

## 3.1 Stable market outcomes with MFC

When MFC is used by all resellers, there are two types of equilibria which differ in how customers interpret the MFC offers: whether or not the clearance sales should be expected. As we show, the equilibrium with (without) the second-period sales is characterized by relatively high (low) MSRP. The reader will also see that, in competitive markets ($n \geq 2$), for sufficiently high level of strategic behavior and cost-to-durability ratio there is even an interval of MSRP where the equilibria of both types exist. This indicates that consumer expectations is the only determinant of MFC equilibrium structure in such markets. The persistence of equilibria is ensured, per standard game theory reasoning, because it is not rational for a profit-maximizing reseller to deviate unilaterally. Overall, the characterization drives the point that strategic customer behavior critically affects the equilibrium type and the resulting profit.

Following the general logic of Nash equilibrium in the reseller game, we consider two types of one-reseller deviations: into a no-MFC strategy with its corresponding best-possible inventory decision and an MFC and inventory strategy that also changes the availability of the product. The second type of deviation is possible because the profit function is discontinuous at the point where $Y = Q$. For example, in the first part of the theorem below, customers rationally expect that the product is available in the second-period under the equilibrium MFC and inventory strategies, i.e., $\alpha^*(1) = 1$ and $Y^* > Q^* = 1 - p_1$. The MFC-deviation by reseller $i$ in that case would result in a smaller total inventory level $Y' = y^i + \frac{n-1}{n}Y^* = 1 - p_1 = Q'$ and no availability in the second period: $\alpha = \mathbb{1}\{Y' > Q'\} = 0$. The comparison of the associated profits leads to a quadratic inequality in $p_1$ (keeping all other inputs fixed) resulting in case (M1.2). Similarly, the comparison with a no-MFC deviation leads to (M1.1) under the additional condition of rationality of clearance price expectations $p^*(0)$ in a no-MFC deviation. We provide a point-by-point discussion of the conditions immediately following the theorem. In the rest of the section, $v^*$ is the equilibrium value of $v_1^{\min}$, which, along with other equilibrium values, may be explicitly identified with the type of equilibrium, e.g., $v^{*,M1}$ or $Y^{*,M2}$ if necessary.

**Theorem 1.** *If MFC is possible, the MFC-equilibria with the following structure exist if and only if (iff) the respective conditions hold:*

**M1 (Clearance)** $\alpha^* = 1, v^* = p_1, p_2^* = c + \frac{\beta - c}{n+1}, Y^* = \frac{n}{n+1}(1 - c/\beta), r^* = \frac{(\beta - c)^2}{(n+1)^2\beta}$ *iff* $p_1 \geq P_1^M$, *where*

$$P_1^M = \begin{cases} P_{11} \triangleq 1 - \frac{n-1+\rho\beta}{n+1}(1 - c/\beta) \text{ if } c/\beta < CB_1(\rho, \beta, n) \triangleq \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2+(1-\beta)\rho[n-(1-\rho\beta)]}, & \text{(M1.1)} \\ P_{12}(c, \beta, n) \text{ otherwise,} & \text{(M1.2)} \end{cases}$$

*where $P_{12}$ is the larger root of a quadratic equation (formula (10) in Appendix);*

**M2 (No clearance)** $\alpha^* = 0, v^* = p_1, Y^* = 1 - p_1$, *and* $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$ *iff* $p_1 \leq P_2^M$, *where*

$$P_2^M = \begin{cases} P_{21} \triangleq \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} \text{ if } c/\beta < CB_2(\rho, \beta, n) \triangleq \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2+(1-\beta)n\rho^2\beta}, & \text{(M2.1)} \\ P_{22}(c, \beta, n) \text{ otherwise,} & \text{(M2.2)} \end{cases}$$

*where $P_{22}$ is the larger root of a quadratic equation (formula (19) in Appendix).*

*All bounds $P_{11}, P_{12}, P_{21}$, and $P_{22}$ are greater than $c/\beta$ if $n < \infty, \rho > 0$, and $\beta < 1$; $P_{11}, P_{12}, P_{22} \to c/\beta$ as $n \to \infty$, and $P_{21} = c/\beta$ if either $\rho = 0$ or $\beta = 1$; $P_{11}, P_{12}, P_{22} \to 1$ as $c/\beta \to 1$.*

Condition $p_1 \geq P_{11}$ in case (M1.1) characterizes a scenario that a possible deviator into no-MFC has sales only in the second period *under rational expectations in a deviation*, i.e., $\alpha^*(0) = 1$ and $v_0^{\min}(\alpha^*(0), p_2^*(0)) \geq 1 - \frac{n-1}{n}Y^*$. As a result, the effective price in this case is the same for both MFC and no-MFC resellers implying that the best deviator profit and inventory level remain the same as before the deviation. By (2), valuation threshold associated with the demand of a deviating no-MFC reseller $v_0^{\min}(\alpha^*(0), p_2^*(0)) = \frac{p_1 - \rho p_2^*(0)}{1-\rho\beta}$ depends on $\rho$. The resulting rational expectation of clearance price $p_2^*(0) = \beta(1 - Y^*)$ (by (3)) changes with the level of strategic behavior leading to the dependence of the bound $P_{11}$ on $\rho$. As shown in the proof, no-MFC deviations under other scenarios would dominate.

Condition $p_1 \geq P_{12}$ in case (M1.2) results from a quadratic inequality stating that MFC deviator profit with sales only in the first period does not exceed the equilibrium one: $(p_1 - c)y^i = (p_1 - c)(1 - p_1 - \frac{n-1}{n}Y^*) \leq r^*$. The threshold value $P_{12}$ is the larger root of the corresponding quadratic equation. Intuitively, an increase in competition reduces the ability of a single reseller to control the availability of the stock in the second period. Therefore, $p_1 \leq P_{12}$ becomes less restrictive with an increase in $n$ as shown in Corollary 1 below and illustrated in Figure 4 (the area of inputs where M1 exists increases in $n$).

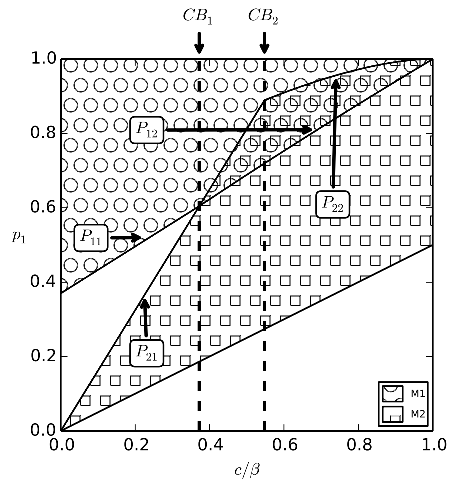
In case (M2), customers rationally expect no sales in the second period when all resellers use MFC. Condition $p_1 \leq P_{21}$ in (M2.1) guarantees that the reseller's equilibrium profit is not dominated by the profit of a deviator into no-MFC with sales in both periods. Similarly to (M1.1), the level of strategic behavior enters this condition through the dependence of $v_0^{\min}$ on $\rho$, which affects the rational expectation of clearance price $p_2^*(0)$ under deviations into no-MFC. Condition $p_1 \leq P_{22}$ in (M2.2) guarantees that the profit of a deviator into MFC with sales in both periods does not exceed the equilibrium one (similarly to (M1.2), this profit comparison leads to a quadratic inequality). Any other forms of deviations do not dominate equilibrium profits.

The conditions of Theorem 1 obviously do not depend on salvage value $s$ because either there are no second-period sales (M2) or the second-period price is above the unit cost to avoid negative profits due to refunds (M1). Theorem 1 also points to a special role played by the cost-to-durability ratio $c/\beta$. Indeed, low values of this ratio in combination with a relatively high first-period price lead to high profitability of the second-period sales, which is one of the key determinants of the equilibrium structure. Cost-to-durability thresholds $CB_1$ and $CB_2$ provided in the statement are the intersection points of the pairs of $p_1$-boundaries $P_{11}$, $P_{12}$ and $P_{21}$, $P_{22}$, respectively. In particular, when $c/\beta < CB_1$, the condition $p_1 \geq P_{11}$ (comparison with a no-MFC deviation) is more restrictive than $p_1 \geq P_{12}$ (comparison with MFC deviation). We illustrate the areas of existence of M1 and M2 equilibria in the $(p_1, c/\beta)$ cross-section of the parameter space for $\rho = 0.3$, $n = 4$, and $\beta = 0.5$ in Figure 3. Both $CB_1$ and $CB_2$ are simultaneously positive, zero, or negative depending on the level of strategic behavior (since both denominators are positive, and the numerator is positive if and only if $\rho < \left(1 - \sqrt{1-\beta}\right)/\beta$). When $CB_1$ and $CB_2$ are positive, $CB_1 \leq CB_2 \leq 1$ where the first inequality is strict unless $\beta = 1$, $\rho = 0$, or $n = 1$. Moreover, when $\beta = 1$ or $\rho = 0$, both $CB_1$ and $CB_2$ equal one. As a result, positive $CB_1$ and $CB_2$ split cost-to-durability ratio values into relatively low, intermediate, and high ranges $(0, CB_1), [CB_1, CB_2)$, and $[CB_2, 1)$ that determine the functional forms of the equilibrium boundaries. For a specific cost-to-durability ratio, the classification depends on other inputs because, as follows from above, a given $c/\beta$ can be less than $CB_1$ only for small levels of competition (if $\beta < 1$ and $\rho > 0$) and strategic customer behavior.

Equilibrium M1 includes the cases with relatively low cost-to-durability ratio and relatively high first-period price leading to attractive sales in the second period. All customers with $v \geq p_1$ buy in the first period and obtain reimbursement $p_1 - p_2^*$ in the second one. The customers with $v \in [p_2^*, p_1)$ wait for clearance sales. Since the effective price for all customers is $p_2^*$, we call this a "clearance" MFC equilibrium.

In the case of M2, the relatively high cost-to-durability ratio, as well as relatively low $p_1$, make two-period sales with reimbursement less attractive than first-period sales only. All customers with valuations $p_1$ or higher buy in the first period. Resellers divide the profit associated with

Figure 3: M1 and M2 regions for fixed $\rho = 0.3$, $n = 4$, and $\beta = 0.5$




the total inventory that is just enough to cover the first-period market. Since the inventory is determined by externally set MSRP, reseller competition is effectively eliminated and this case can be interpreted as an MSRP-facilitated collusion. Since there are no second-period sales we refer to M2 as a "no-clearance" equilibrium. M2 cannot exist if customer valuations remain constant ($\beta = 1$ implies $P_{21} = c$ and $CB_2 = 1$). This outcome is intuitive because the less the decrease in customer valuations between two periods, the more profitable the second-period sales. In both MFC-equilibria, customers behave as if they are myopic ($v^* = p_1$) and, consequently, inventory level $Y^*$ and profit $r^*$ do not depend on the level of strategic behavior.

The fraction of model inputs where MFC-equilibria exist is illustrated in Figure 4 as a function of $1 \leq n \leq 1,000$. The fraction is computed by volume in the region of all inputs $(\rho, \beta, c, s, p_1)$ satisfying the feasibility constraints $0 \leq \rho < 1$, $0 \leq s < c < \beta \leq 1$, and $\max\{s/\beta, c\} < p_1 \leq 1$. The figure is an area plot that shows the fractions of inputs resulting in a particular equilibrium (M1 only, both M1 and M2, M2 only, and neither M1 nor M2) as the heights of the respective shaded areas for each $n$. As $n$ increases, the fraction of inputs where M1 exists increases whereas the fractions of inputs with M2 only, both M1 and M2, as well as neither equilibrium decrease.

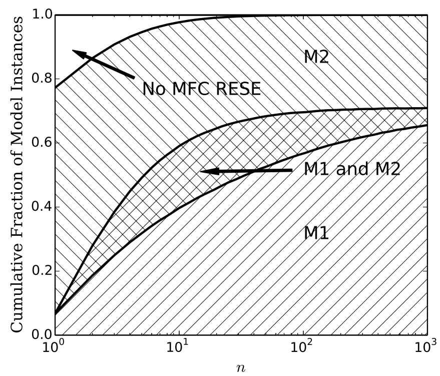
The corollary below augments Figure 4 by establishing a full set of monotonic properties of the M1 and M2 regions. In particular, it characterizes the overlap of M1 and M2 as well as the area where neither MFC equilibrium exists.

**Corollary 1.** *1. If $CB_1 > 0$, for low cost-to-durability ratio $\frac{c}{\beta} < CB_1$, we have $P_{21} < P_{11}$ and there are no MFC-equilibria for $P_{21} < p_1 < P_{11}$. Moreover, $P_{11} = P_{12} = P_{21}$ if $\frac{c}{\beta} = CB_1$.*

*2. For any $\frac{c}{\beta}$, we have $P_{12} < P_{22}$ if $n > 1$ and $P_{12} = P_{22} = P_2 \triangleq \frac{1}{2}\left[1 + c + \sqrt{(1-\beta)(1-c^2/\beta)}\right]$ if $n = 1$. Therefore, for high cost-to-durability ratio $\frac{c}{\beta} \geq CB_2$ (possible only if $\beta < 1, \rho > 0$) there is an overlap $P_{12} \leq p_1 \leq P_{22}$ in the MSRP-range of M1 and M2 existence. In this overlap, $r^{*,M1} < r^{*,M2}$ if $n > 1$ and $r^{*,M1} = r^{*,M2}$ if $n = 1$.*

*3. If $\frac{c}{\beta} > CB_1 > 0$, then $P_{12} < P_{21}$. Thus, for $n > 1$ and intermediate cost-to-durability ratio $CB_1 \leq \frac{c}{\beta} < CB_2$ (possible only if $\beta < 1, 0 < \rho < (1 - \sqrt{1-\beta})/\beta$), there is an overlap $P_{12} \leq p_1 \leq P_{21}$ in the MSRP-range of M1 and M2 existence. In this overlap, $r^{*,M1} < r^{*,M2}$.*

Figure 4: Fractions of model inputs where a particular MFC equilibria structure exists for given $n$




4. *Inequality* $\frac{c}{\beta} \geq CB_1$ *is equivalent to a lower bound on* $\rho$.

5. *The lower* $p_1$*-bounds* $P_{11}, P_{12}$ *and upper* $p_1$*-bounds* $P_{21}, P_{22}$ *depend on inputs as follows:*

| | $c$ | $\beta$ | $\rho$ | $n$ |
|---|---|---|---|---|
| $P_{11}$ | $\nearrow$ | $\searrow$ | $\searrow$ | $\searrow$ |
| $P_{12}$ | $\nearrow$ | $\searrow$ | $\equiv$ | $\searrow$ |
| $P_{21}$ | $\nearrow$ | $\searrow$ | $\nearrow$ | $\equiv$ |
| $P_{22}$ | $\nearrow$ | $\searrow$ | $\equiv$ | $\searrow$ |

The overlap in MSRP ranges of M1 and M2 equilibria established in Corollary 1 (Parts 2 and 3) is exclusive for oligopolistic MFC-resellers and intermediate values of MSRP. That is natural because the monopolist optimally chooses whether to supply the product in one or both periods. For competitive settings, very low MSRP means that the second-period regular-customer market is small and has an extremely low margin. In contrast, a very high MSRP means that the first-period market is very small. Thus, the possibility of either type of equilibrium arises only for intermediate MSRP. Moreover, by Part 4, since $c/\beta \geq CB_1$ in the overlap, it can take place only if customers are sufficiently strategic. Thus, it is natural that customer expectations start to affect the equilibrium outcome. While M2 is always better for competing resellers in the overlap, the magnitude of its difference with M1 deserves a further study and we return to it in subsequent sections. The overlap does not exist for a monopoly, the highest-possible level of durability, or myopic customers.

While Theorem 1 provides a complete characterization of MFC equilibria for a given market situation, market participants may want to forecast adjustments to equilibrium structure when the market situation changes. By Part 5 of Corollary 1, the areas where M1 or M2 exist *expand* when customers become *more* strategic. The changes in other parameters affect the areas of M1 and M2 existence in the opposite way. For example, when the level of competition *increases*, the area of M1 *expands* while the area of M2 *shrinks*.

The knowledge of possible shifts in the equilibrium structure is of particular importance when a market situation is close to the boundary between equilibria with notably different profits. In such situations, equilibrium can be unstable with respect to parameter changes or misestimations. Part 1 of Corollary 1 implies that for small levels of strategic behavior, i.e., $\rho < \left(1 - \sqrt{1-\beta}\right)/\beta$, which yields $CB_1 > 0$, MFC equilibria may not exist. This observation stimulates an interest in the properties of equilibria when the MFC-option is not available or when MFC is available but remains unused. These equilibrium structures are considered below.

### 3.2 Stable market outcomes when MFC is not allowed (NA)

There are four types of symmetric rational expectations NA equilibria identified in Theorems 2 and 3 cited from Bazhanov, Levin, and Nediak (2015) below. Comparative statics, summarized in Table 1, show that resellers' profits, customer surplus, and aggregate welfare can be non-monotonic in $\rho$.

| | Monotonicity in $n$ | | | | Monotonicity in $\rho$ | | | |
|---|---|---|---|---|---|---|---|---|
| NA | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $Y^*$ | $\nearrow$ | $\equiv$ | $\nearrow$ | $\nearrow$ | $\equiv$ | $\equiv$ | $\searrow$ | $\searrow$ |
| $v^*$ | $\equiv$ | $\equiv$ | $\nearrow$ | $\equiv$ | $\equiv$ | $\equiv$ | $\nearrow$ | $\nearrow$ |
| $nr^*$ | $\searrow$ | $\equiv$ | $\searrow$ | $\searrow$ | $\equiv$ | $\equiv$ | $\searrow$, min | $\searrow$ |
| $\Sigma^*$ | $\nearrow$ | $\equiv$ | $\nearrow$ | $\equiv$ | $\equiv$ | $\equiv$ | $\nearrow$, $\searrow$, max | $\nearrow$ |
| $W^*$ | $\nearrow$ | $\equiv$ | $\nearrow$, $\searrow$, max | $\searrow$ | $\equiv$ | $\equiv$ | $\nearrow$, $\searrow$, max | $\nearrow$, $\searrow$, max |

Table 1: Summary of monotonic properties in $n$ and $\rho$ by equilibrium form

Here and in other no-MFC equilibria, we use $v^*$ to denote the equilibrium value of $v_0^{\min}$. Similarly, the theorems below provide the equilibrium expectations $\alpha^*(0)$ and $p_2^*(0)$ in the absence of MFC. When MFC is not available, the expectations $\alpha^*(1)$ and $p_2^*(1)$ corresponding to a one-reseller deviation into MFC are undefined.

**Theorem 2.** *A unique NA with the stated structure exists iff the respective conditions hold:*

**NA1 (No sales in the first period)** $v^* = 1, \alpha^*(0) = 1, p_2^*(0) = c + \frac{\beta - c}{n+1}, Y^* = \frac{n}{n+1}(1 - c/\beta)$, *and* $r^* = \frac{(\beta-c)^2}{(n+1)^2\beta}$ *under condition* $p_1 \geq 1 - \frac{n}{n+1}\rho(\beta - c) \triangleq P_1^N$.

**NA2 (No sales in the second period)** $v^* = p_1, \alpha^*(0) = 0, Y^* = 1 - p_1$, *and* $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$ *under condition* $p_1 \leq \frac{nc}{n-1+\beta} \triangleq P_2^N$.

**NA3 (Sales in both periods, $p_2^* > s$)** $v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta}, \alpha^*(0) = 1, p_2^*(0) = \beta(1 - Y^*)$, *and* $r^* = \frac{1}{n}\left[(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)\right]$, *where* $Y^*$ *is the larger root of a quadratic equation (Eq. (22) in Appendix), under condition* $P_2^N < p_1 < P_1^N$ *and one of the following:* *(a)* $\frac{n-1}{n}(p_1 - s)(1 - v^*)Y^* \leq (c - s)(1 - s/\beta)^2$, *or (b) condition (a) does not hold,* $Y^* < 1 - s/\beta$, *and* $r^* \geq \tilde{r}^i$, *where* $\tilde{r}^i$ *is the maximum profit of a firm deviating from this equilibrium in such a way that* $p_2 = s$ *(the total inventory is greater than* $1 - s/\beta$*).*

*The equilibrium characteristics* $Y^*$, $v^*$, *and* $r^*$ *are continuous on the boundaries between these forms of NA. Moreover, under NA3,* $Y^* > \max\left\{\frac{n}{n+1}(1 - c/\beta), (1 - p_1)\right\}$.

The following proposition shows the relationships between $p_1$-bounds in NA and MFC-equilibria.

**Proposition 1.** *(1) The area of NA2 existence is always inside the area of M2 existence, i.e.,* $P_2^N \leq \min\{P_{21}, P_{22}\}$.

*(2) For* $n > 1$, *the area of NA1 existence is always inside the area of M1 existence, i.e.,* $P_1^N > P_{11}$ *and* $P_1^N \geq P_{12}$ *(strict for* $\beta < 1$*). For* $n = 1$, *the area of M1 existence is always inside the area of NA1 existence, i.e.,* $P_1^N = P_{11}$ *and, for* $c/\beta > CB_1, P_1^N < P_{12}$.

Notably, there are inputs for which either MFC equilibrium can be realized if resellers use MFC, or a price-discriminating equilibrium NA3 is realized if MFC is not available. Due to differences in equilibrium structures, the change in profit can be discontinuous when MFC becomes available.

For a monopoly ($n = 1$), Theorem 2 exhaustively covers all feasible parameter values. Starting from a duopoly, there is an area of inputs where none of the equilibria described in Theorem 2 may exist. At the same time, for oligopoly resellers with strategic customers, by the theorem below, there exists one more form of NA with sales in both periods and $p_2^* = s$ (NA4). This form exists only inside the $p_1$-range of NA3, i.e., there exists a non-empty set of input parameters where both NA3 and NA4 may exist and, by Proposition 1, either M1 or M2 may exist if MFC is available.

**Theorem 3** ("Salvaging" NA4)**.** *NA with* $v^* = \frac{p_1 - \rho s}{1-\rho\beta}, \alpha^*(0) = 1, p_2^*(0) = s, Y^* = \frac{n-1}{n}\frac{p_1-s}{c-s}(1-v^*)$, *and* $r^* = \frac{p_1-s}{n^2}(1-v^*)$ *exists iff one of the following mutually exclusive conditions hold:*

*(a) salvaging is forced on resellers, i.e.,* $\frac{n-1}{n}Y^* \geq 1 - \frac{s}{\beta}$;

*(b) condition (a) does not hold and the deviator profit is strictly increasing in the interval corresponding to* $p_2 > s$, *which is equivalent to* $1 - \frac{s}{\beta} > \frac{n-1}{n}Y^* \geq \left(1 - \frac{s}{\beta}\right)^2 \frac{c+\beta v^* - 2s}{\beta(1-s/\beta)^2 + (p_1-\beta)(1-v^*)}$;

*(c) conditions (a) and (b) do not hold,* $Y^* > 1 - \frac{s}{\beta}$, *and either the deviator profit is strictly decreasing in the interval corresponding to* $p_2 > s$ *(in this case the deviator profit never exceeds* $r^*$*), or* $r^* \geq \tilde{r}^i$, *where* $\tilde{r}^i$ *is the maximum deviator profit in this interval.*

Conditions of NA3 and NA4 existence indicate proximity of the market situation to a boundary of the area of existence. Namely, if the equilibrium exists only because the equilibrium profit $r^*$ exceeds the profit of a potential deviator $\tilde{r}^i$ (condition (b) for NA3 and (c) for NA4), the equilibrium can be very sensitive to parameter changes.

**Proposition 2.** *(1) NA4 exists only if* $c - s < \frac{n-1}{n}\frac{\beta(1-s)^2}{4(\beta-s)}$ *(otherwise, there are no* $p_1$ *and* $\rho$ *leading to* $p_2^* = s$*) and* $p_1 < P_4^N \triangleq 1 - \rho(\beta - s)$ *(otherwise, NA4 form of* $v^*$ *does not permit sales in the first period). Moreover,* $P_4^N < P_1^N$ *and* $p_1$*-bounds are equivalent to upper bounds on* $\rho$, *with* $\rho_4^N \triangleq \frac{1-p_1}{\beta-s}$ *and* $\rho_1^N \triangleq \frac{n+1}{n}\frac{1-p_1}{\beta-c} > \rho_4^N$. *(2) For any inputs where both NA3 and NA4 exist,* $r^{*,NA4} < r^{*,NA3}$. *Moreover,* $r^{*,NA4} < \frac{1}{n}(p_1 - c)(1 - p_1)$.

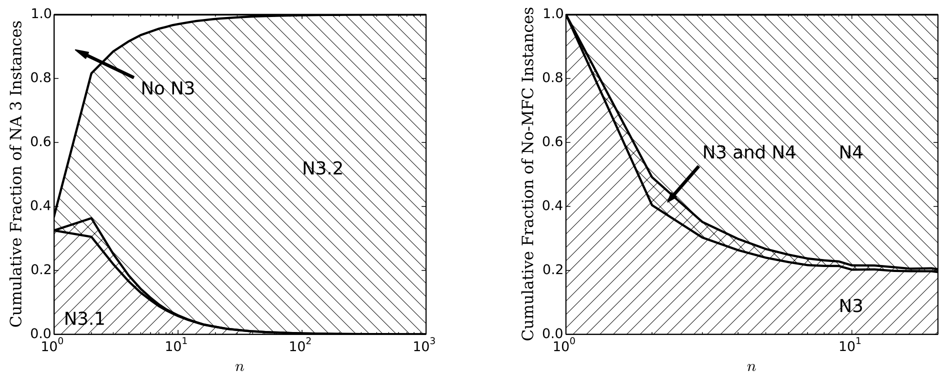
Proposition 2 implies, first, that NA4 exists only when the unit salvage value is relatively close to the cost and when the first-period price is relatively low, resulting in first-period sales that are enough to compensate for the second-period loss. Since the $p_1$-upper bound in NA4 is strictly below $P_1^N$, the $p_1$-upper bound in NA3, NA4 never coexists with NA3 if $p_1 \in [P_4^N, P_1^N)$. If NA4 exists for $\rho = 0$, keeping other inputs fixed, it may also exist for $\rho < \rho_4^N$. Thus, part (2) of Proposition 2 in conjunction with a nonempty range $[\rho_4^N, \rho_1^N)$ may lead to a substantial "discontinuous" gain from increasing strategic behavior. Indeed, for $\rho \in [\rho_4^N, \rho_1^N)$, NA4 does not exist and no other NA-equilibria may exist except NA3, whose profit is higher than that of NA4.

### 3.3 Stable market outcomes without MFC when MFC is allowed

The introduction of MFC decision into resellers' game increases the set of possible strategies. Thus, in the MFC-game, no-MFC equilibria may still exist but under more restrictive conditions than in the no-MFC game since a reseller has an additional dimension to deviate. We denote by N an equilibrium where MFC option is available but not used. A formal statement (Proposition 9, Appendix), illustrated in Figure 5, shows that the additional flexibility for resellers in the form of

Figure 5: For given $n$, fractions of inputs resulting in different types of

(a) N3 within NA3 (b) N within no-MFC




MFC-option indeed restricts the areas of existence of N-equilibria (except for N2 and for N1 with $n > 1$) in comparison with the corresponding areas of NA-equilibria. These additional restrictions can be interpreted as conditions of "stability" of NA-equilibria with respect to MFC option.

The information about MFC-policy gives an additional signal for customer expectations. For example, if $p_1$ is relatively high, implying sales in the second period, the declaration of MFC by a profit-maximizing reseller may lead to a higher $p_2$ than without MFC since, under MFC, $p_2$ cannot be below unit cost. On the other hand, if $p_1$ and $\beta$ are relatively low, any second-period sales may result in $p_2 < c$. In this case, the declaration of MFC implies the absence of second-period sales.

According to the definition of RESE, customer expectations need to be specified both for a symmetric MFC decision profile and for all one-reseller deviations into no-MFC. In this section, $\alpha^*(0)$ and $p_2^*(0)$ specify equilibrium expectations for a symmetric no-MFC strategy profile, while $\alpha^*(1)$ and $p_2^*(1)$ – for a one-reseller deviation into MFC. Rational customer expectations associated with a deviation determine two different subtypes of N3, which we call N3.1 (for $\alpha^*(1) = 0$) and N3.2 (for $\alpha^*(1) = 1$). Both these subtypes correspond to an otherwise identical NA3 structure. A summary of these outcomes is presented visually in Figure 5(a) as fractions of NA3 instances. There is a very small area of inputs where both N3.1 and N3.2 can exist. The incidence of N3.1 and no-N3 quickly diminishes and tends to zero as the market approaches perfect competition ($n \to \infty$). On the other hand, N3.2 type becomes dominant and absorbs the entire NA3 area as $n \to \infty$.

The behavior of fractions of NA4 instances is similar but the decreases in N4.1, an overlap of N4.1 and 4.2, and no-N4 are more rapid. The highest values of these fractions occur in a duopoly and are, respectively, 7.0%, 0.17%, and 0.26%.

By Propositions 1 and 9, the areas of N1 and N2 existence are inside the areas of M1 and M2 existence respectively, i.e., N1 and N2 cannot exist in an area where MFC-equilibria do not exist. On the other hand, by Theorems 2, 3, and Proposition 9, N3 and N4 can exist in this area. We examine the fractions of no-MFC model instances, where N3 and/or N4 exist, visually in the area plot of Figure 5(b). The overwhelming majority of no-MFC instances corresponds to N-equilibria. The remaining fraction of no-MFC instances where neither N3 nor N4 exist is too small to be seen on the figure (its maximum over $n$ is just 0.044%).

For brevity, we use N(A) to refer to either N-equilibrium if MFC is available but resellers do not use it or NA if MFC is not available due to other reasons. Visualizations of possible equilibrium types across model inputs are provided in Figures 7 and 8 (illustrating Examples 1 and 3 in §5).

## 4 When is MFC beneficial for participants in the market?

By affecting the equilibrium, MFC impacts all market participants. Thus, in this section, we consider MFC effects on resellers in terms of their profit, on the manufacturer in terms of the total inventory, and on the customers in terms of the surplus, as well as on the local economy in terms of the aggregate welfare, which is the sum of the customer surplus and the reseller profits.

### 4.1 MFC effect on reseller profits

As shown above, the availability of the MFC option does not always lead to the existence of MFC equilibria. But even if M1 or M2 exists, there are areas of inputs where MFC-equilibria coexist with various forms of N(A), and it is not obvious that MFC-profits are always greater in these areas. Indeed, it turns out that MFC leads sometimes to a lower total profit than N(A)3 and/or N(A)4.

Assuming that, for given inputs, equilibria $X$ and $Y$ exist (possibly in different games), we say that $X$ is *beneficial (equivalent, detrimental) for resellers* compared to $Y$ if benefit $B^{X,Y} \triangleq r^{*,X} - r^{*,Y} > 0$ ($B^{X,Y} = 0, < 0$). Equilibrium $X$ is beneficial (equivalent, detrimental) in an area of inputs if it is beneficial (equivalent, detrimental) for any inputs in this area.

Figure 6 (a) displays the area plot of fractions of M1 inputs where N3 and/or N4 may also exist (implying the existence of NA3 and/or NA4 for these inputs). The overlap is quite large, and Figure 6 (b) shows that M1 is detrimental compared to N3 and/or N4 in approximately 30% of the model inputs where M1 coexists with either M2 or N3 and/or N4. Recall that, depending on $n$, as illustrated in Figure 4, M1 exists in approximately 10% to 70% of the volume of model inputs. Hence, up to 20% of possible model inputs may lead to an MFC equilibrium that is detrimental compared to a no-MFC equilibrium.

The plot for the overlap of M2 with N3 and/or N4 is similar to Figure 6 (a) with the only difference that the cumulative fraction of the overlap is around 80% for $n = 2$ and approaches 100% for $n$ closer to 100. However, unlike M1, M2 is either beneficial or equivalent to *all* other equilibria in the areas of coexistence. For a monopolist, M1 coexists only with N1, therefore the $n$-axes in the plots of Figure 6 start from $n = 2$.

The proposition below provides conditions for the dominance of an equilibrium profit either under MFC, or N(A)3 and N(A)4. For the convenience of exposition, we use $r^{*,N3}, r^{*,N4}$ instead of $r^{*,N(A)3}, r^{*,N(A)4}$, and we let $w^2$ denote the ratio of profits $r^{*,M1}$ over $r^{*,N4}$, normalized by $n^2/(n+1)^2$, i.e., $w^2 \triangleq (\beta - c)^2(1-\rho\beta)/\{\beta(p_1 - s)[1 - p_1 - \rho(\beta - s)]\}$.

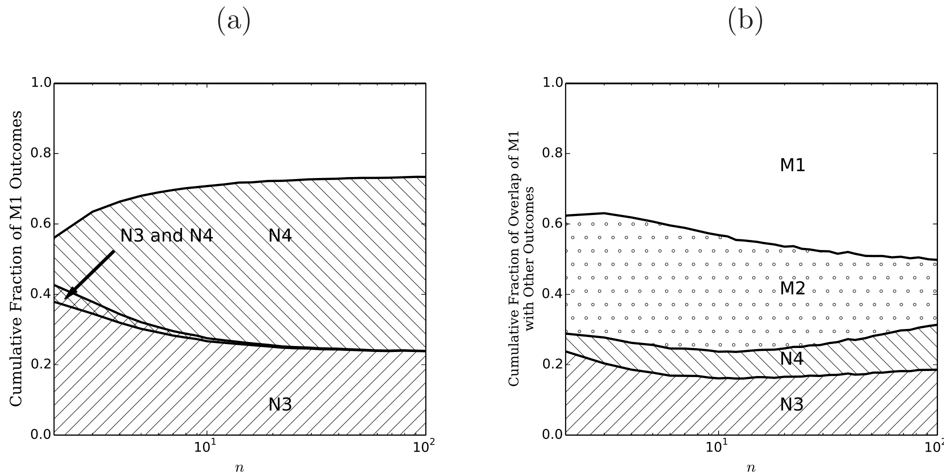
**Proposition 3.** *(1) For any inputs in the overlap of M1 and the corresponding N(A),*

> *(1.1)* $r^{*,M1} < r^{*,N3}$ *if* $p_1 > 1 - \frac{n}{n+1}(\beta - c)$ *and* $c \geq 3\beta - 2\left(1 + \frac{1-\beta}{n}\right)$;
>
> *(1.2)* $r^{*,M1} > r^{*,N4}$ *iff either* $w > \frac{3}{2}$, *or* $1 < w \leq \frac{3}{2}$, *and* $n > \frac{1}{w-1}$ *(w increases in* $\rho$*).*

*(2) For any inputs in the overlap of M2 and the corresponding N(A),*

> *(2.1)* $r^{*,M2} \geq r^{*,N3}$ *with strict inequality if* $n > 1$ *or* $n = 1$ *and* $p_1 < P_{21}$;
>
> *(2.2)* $r^{*,M2} > r^{*,N4}$.

Figure 6: For given $n$, (a) fractions of M1 outcomes where N3 or N4 are possible; (b) fractions of M1 outcomes where M2, N3, or N4 are possible and the respective profit is the greatest




Part (1.1) implies that M1 can be less profitable than N(A)3 when the product is not durable because, as we mentioned above, low durability decreases the second-period profits and, consequently, the attractiveness of M1. Indeed, the lower bound on $c$ in part (1.1) holds for any $c$ and $n$ if $\beta \leq \frac{2}{3}$ and never holds for $\beta > \frac{4+c}{5}$.

By part (1.2), since $w$ increases in $\rho$, the more strategic customers are, the lower the minimum level of competition $n$ when M1 is beneficial compared to N(A)4. The necessary condition $w > 1$ for M1 to be beneficial is equivalent to a lower bound on M1 profit, i.e. $\frac{(\beta-c)^2}{\beta} > \frac{(p_1-s)(1-p_1-\rho(\beta-s))}{1-\rho\beta}$.

## 4.2 MFC effect on the total inventory

An important part of this investigation is the MFC effects on reseller inventory policies with the associated impact on all participants in the market. The total inventory, in turn, affects the existence of the second-period sales and, when these sales exist, the second period price. The results are summarized in the following proposition.

**Proposition 4.** *For the same inputs except MFC-policy, in the areas where an MFC-equilibrium and N(A) coexist, the total inventory under MFC is not greater than under N(A), namely,*

*(1) MFC total inventory and prices are the same as under N(A) if M1 coexists with N(A)1 or M2 coexists with N(A)2;*

*(2) MFC total inventory is less than under N(A) if M1 or M2 coexists with N(A)3 or N(A)4.*

Hence, when the introduction of MFC changes the realized equilibrium structure, the total inventory decreases. This result is consistent with the literature that shows that MFC, by encouraging early purchases, allows resellers to increase prices (Png (1991), Lai, Debo, and Sycara (2010)).

Given that the wholesale price is fixed, the smaller inventory reduces the manufacturer's profit. Therefore, a current-profit-maximizing manufacturer, that is able to set the first-period price, may want to prevent the use of MFC by resellers. On the other hand, a branded product manufacturer may prefer resellers to sell only at MSRP to maintain product reputation (e.g., Orbach (2008)), supporting M2 as a result. The manufacturer's benefits from this support depend on the particular conditions of M2 because the first-period price of branded products is usually high whereas M2 exists for relatively low $p_1$.

Compared to N(A)3 or N(A)4, M1 does not bring any benefits even for a branded product manufacturer. If M1 is realized in the areas of equilibria coexistence, it means that resellers, using MFC, avoid too high an MSRP. This situation is a signal for the manufacturer to target a lower first-period price. Alternatively, the manufacturer may negotiate a restriction against using MFC. This no-MFC restriction may benefit resellers, because, as shown in the previous subsection, reseller profits under M1 may be even lower than under "salvaging" N(A)4, which is the worst equilibrium for resellers in a no-MFC game (Figure 6 (b) and part (1.2) of Proposition 3).

### 4.3 MFC effects on customers and the local economy

The above results partially support the findings in the literature that MFC may be used as an anti-competitive practice. Indeed, recall that the price in M2 is regulated by MSRP and, when another equilibrium with a lower second-period price is also possible, M2-profit is always higher. If MFC is not available and NA2 (which is equivalent to M2 in profit) exists, then, as $n$ increases, the outcome changes to NA3 or NA4 with $p_2 < p_1$. On the other hand, M2 is guaranteed to exist for such inputs. Thus, a declaration of MFC under M2 merely serves as a tool to avoid competitive pricing. Customers do not receive any reimbursements. Nevertheless, as we show below, M2 can improve the aggregate welfare.

The consequences of MFC for customers and resellers are not that obvious for another MFC-equilibrium M1. In this equilibrium, customers *do* obtain reimbursements in the second period. This RESE is the most beneficial for resellers in approximately 40% to 50% of inputs for which the other RESE may also exist. On the other hand, there is a significant share of inputs (Figure 6 (b)) where M1 is detrimental compared to no-MFC equilibria N(A)3 and even "salvaging" N(A)4. When M1 is indeed detrimental for resellers, it is not clear whether it is beneficial for customers compared to N(A)3 or N(A)4. M1 is indeed better than N(A)3 or N(A)4 for high-valuation customers who buy in the first period because their surplus is larger under M1 due to reimbursements. In contrast, the low-valuation customers, who would buy in the second period under N(A)3 or N(A)4, are worse off under M1 because, by Proposition 4, the MFC-price is always higher than the second-period price under N(A)3 or N(A)4. Such mixed effects of MFC raise a non-trivial question: is it possible that an MFC-equilibrium is beneficial for the total customer surplus and/or aggregate welfare?

The total equilibrium customer surplus is the sum of the actual first-period and second-period surpluses. We consider the actual or realized surplus, which is greater than the expected surplus. In the latter one, the second-period surplus would be discounted by $\rho$ similarly to the individual second-period surplus used to determine the customer choice of buying or waiting. In contrast, the actual surplus measures the realized customer benefits depending on a type of equilibrium. The present value of the surplus is not an adequate measure for this purpose because it ignores or underestimates the realized second-period surplus of customers if they are myopic or, respectively, have low $\rho$. The result below shows that M1 is better for customers than N(A)1 and M2.

**Proposition 5.** *For the same inputs, the change of equilibrium structure from N(A)1 to M1 increases the total customer surplus except for a durable product ($\beta = 1$) and $p_1 = 1$ when the surplus remains the same; the change from any structure to M2 decreases the surplus.*

Moreover, MFC is beneficial for the local economy in terms of aggregate welfare for the inputs where N(A)1 and M1 coexist. Indeed, the total profit $nr$ is the same in both equilibria, whereas, under MFC, part of sales occurs in the first period where the consumers enjoy fresh product effectively paying the reduced second-period price due to the reimbursements. For $n \geq 2$, M1 can exist for the same inputs as N(A)3 or N(A)4. For $n = 2$, the fraction of inputs with a welfare-increasing switch from N(A)3 to M1 is 81.5% (of the inputs where both NA3 and M1 exist), and this fraction increases in $n$. Figure 2 in Introduction illustrates this welfare-increasing switch for $n = 10, \rho = 0.95, p_1 = 0.7, \beta = 0.4, c = 0.2$, and $s = 0$. For N(A)4, the fraction of inputs where M1 improves welfare is even higher.

This result complements the finding of Lai, Debo, and Sycara (2010) for a single retailer, who show, contrary to the literature, that posterior price matching (i.e., MFC) can increase customer surplus when the uncertainty in the number of customers with high valuations is high. In our setting, MFC can be surplus- and welfare-improving even without uncertainty, e.g., when $p_1$ and $\rho$ are sufficiently high leading to N(A)1 and M1 existence.

Unlike M1, M2 is always disadvantageous for customers. However, by Proposition 3, M2 is profitable for resellers, which raises a non-trivial question about the welfare-improving ability of M2. M2 improves welfare for $n = 1$ in 78% of inputs where M2 and NA3 exist: intermediate $p_1$, relatively high difference $c - s$, *high* $\rho$ and small $\beta$. Similarly to M1, this share increases in $n$. In the area where M2 and NA4 exist: $s$ close to $c$, *high* $\rho, n$, and low $\beta, p_1$. This share starts from 99.9987% for $n = 2$ and increases in $n$. Thus, the local policymakers may help the retailers to escape from the "salvaging" N(A)4 by encouraging the use of MFC.

Hence, when retailers operate under allegedly anti-competitive RPM, another "collusive" suspect, MFC, improves social welfare for the local economy in most of the cases when customers are strategic. This effect results, first, from the fixed first-period price and, second, from an increase in the first-period sales. The latter effect increases the first-period surplus and the total profit — always under M2 and, in some cases, under M1 despite reimbursements since the second-period price is higher under M1 than under no-MFC equilibria. When MFC is welfare improving, these increases exceed the loss of the second-period surplus under M2 or its decrease under M1.

# 5 Effectiveness of MFC in counteracting strategic behavior

While previous sections provided qualitative description of MFC-effects, the results below show that possible benefits or losses from MFC can be essentially higher than losses from strategic customers. We contrast the cases of monopoly and oligopoly because the effects of MFC are more pronounced under competition and can be qualitatively different in these two cases.

## 5.1 MFC performance

This subsection introduces a suitable measure of MFC performance as a profit-increasing tool relative to the effect on profit from an increase in the level of strategic behavior. Assume that all inputs except $\rho$ are fixed and customers are more strategic for $\rho^H$ than for $\rho^L < \rho^H$. Moreover, in the no-MFC game, one of NA equilibria (denoted as NAL) is realized for $\rho^L$ and, possibly, another NA equilibrium (denoted as NAH) is realized for $\rho^H$. Finally, an MFC equilibrium is realized when customers are more strategic (Figure 7(b)), while the theoretical existence of a no-MFC equilibrium with the same structure as NAH in the MFC-game is not excluded. The corresponding no-MFC profits are $r^{*,NAH}$ at $\rho^H$ and $r^{*,NAL}$ at $\rho^L$. The MFC-profit at $\rho^H$ is $r^{*,MFC}$.

Suppose the increase in $\rho$ leads to a loss in the no-MFC game, namely, $r^{*,NAH} - r^{*,NAL} < 0$. The *performance of MFC as a tool for mitigating the loss from customer strategic behavior* is the ratio of the benefit from MFC at $\rho^H$ to the absolute value of the loss, i.e., $\eta(\text{NAL,NAH,MFC}) \triangleq \frac{r^{*,MFC} - r^{*,NAH}}{r^{*,NAL} - r^{*,NAH}}$. For brevity, we omit the arguments of the measure when it does not lead to confusion. This measure is negative when MFC is detrimental, $\eta \in (0, 1]$ when MFC leads to a *mitigation*, and $\eta > 1$ when MFC results in a *gain*. For example, $\eta = 1$ means that MFC mitigates 100% of the loss from increase in $\rho$. Theoretically, $\eta$ can go to infinity when the change in $\rho$ is close to zero, the profit in the no-MFC game is continuous in $\rho$, and $r^{*,MFC} - r^{*,NAH}$ is separated from zero due to discontinuous changes in the equilibrium structure resulting from the introduction of MFC.

Similarly, suppose the increase in $\rho$ leads to a gain in the no-MFC game, e.g., as a result of the switch from NA4 to NA3 or under NA3 for large $\rho$ and $\beta$. The *performance of MFC as a tool for enhancing the gain from increasing strategic behavior* is the ratio of the benefit from MFC at $\rho^H$ to the absolute value of the gain, i.e., $\eta(\text{NAL,NAH,MFC}) \triangleq \frac{r^{*,MFC} - r^{*,NAH}}{r^{*,NAH} - r^{*,NAL}}$.

We keep to the following refinement of the notion "the gain from increasing strategic behavior." If equilibria $A$ and $B$ exist for both $\rho^H$ and $\rho^L < \rho^H$ with $r^{*,A}|_{\rho=\rho^H} < r^{*,A}|_{\rho=\rho^L} < r^{*,B}|_{\rho=\rho^H}$, $A$ is realized only for $\rho^L$, and $B$ is realized only for $\rho^H$, the difference $r^{*,B}|_{\rho=\rho^H} - r^{*,A}|_{\rho=\rho^L} > 0$ *cannot be conclusively considered a gain* from increased $\rho$ because the reason for the switch to $B$ is not necessarily related to the increase in $\rho$. This difference may be a gain from another undetermined factor causing the switch.

Since equilibria can be multiple for both MFC and no-MFC, and for both $\rho^H$ and $\rho^L$, the analysis below is concentrated on the cases when the benefits from MFC are maximal as well as on the cases when MFC is detrimental with the description of the corresponding areas of inputs.

## 5.2 Monopoly

We first consider the case of monopoly because of its analytical simplicity and qualitative differences from oligopoly. In particular, MFC benefits neither the reseller nor customers if customers are myopic. Indeed, by Theorems 1 and 2 with myopic customers, a two-period MFC-equilibrium (M1) and a no-MFC equilibrium without first-period sales (N(A)1) exist only in a degenerate case with $p_1 = 1$. An MFC-equilibrium without second-period sales (M2) exists only for a non-durable product ($\beta < 1$) and overlaps only with a no-MFC equilibrium that has the same structure (N(A)2). Thus, when customers are myopic and the drop in valuations is relatively low, the major area of inputs belongs to a no-MFC price-discriminating equilibrium N(A)3 ("salvaging" equilibrium N(A)4 does not exist for a monopolist).

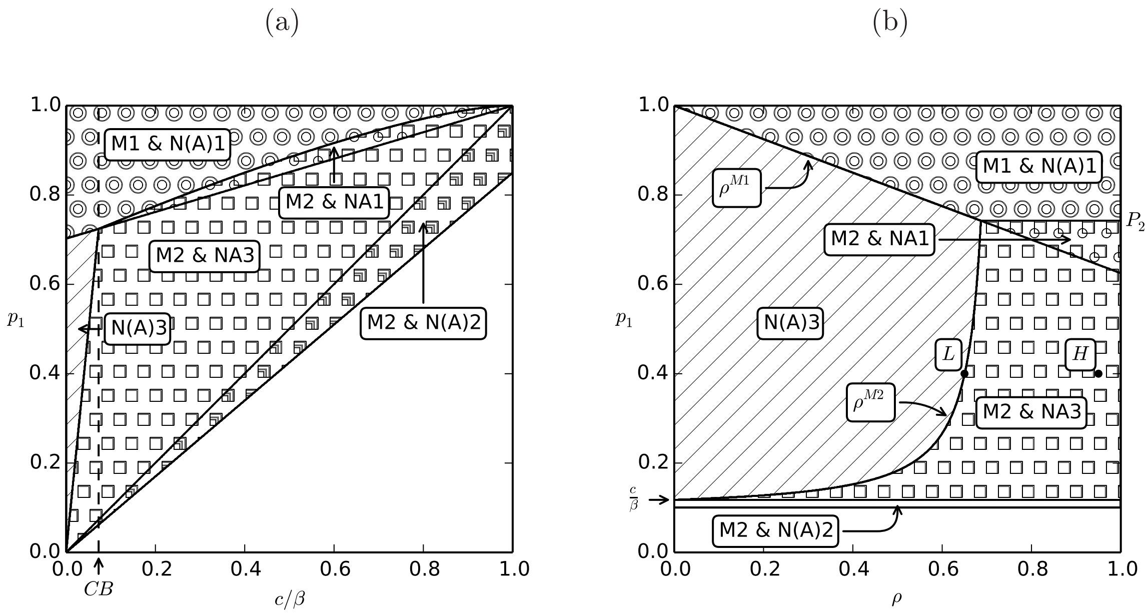
The situation changes when customers are strategic and the product is not durable. Findings below, illustrated in Figure 7, specify the dependence of MFC-benefits on the market parameters. In particular, there is an area leading to an MFC-benefit only for the monopolist. This area, the overlap of M2 and NA1, is not covered by Proposition 3 and exists only for high levels of strategic behavior. Indeed, the following lemma (illustrated in Figure 7(b)) shows that MFC-equilibria exist only for sufficiently high $\rho$.

**Lemma 1.** *For $n = 1$, the conditions of MFC-equilibria existence $p_1 \geq P_{11}$ and $p_1 \leq P_{21}$ are equivalent to lower bounds on $\rho$ : $\rho \geq \frac{2(1-p_1)}{\beta - c} \triangleq \rho^{M1}$, and $\rho \geq \frac{1}{\beta}\left[1 - \sqrt{p_1(1-\beta)/(p_1 - c)}\right] \triangleq \rho^{M2}$ respectively, where $\rho^{M2} \in (0, 1]$, $\rho^{M2} \to 0$ if $p_1\beta \to c + 0$, and $\rho^{M2} = 1$ if $\beta = 1$.*

Recall that the boundaries of MFC-equilibria (Theorems 1 and Corollary 1) and their intersection points are such that $P_{12}|_{n=1} = P_{22}|_{n=1} = P_2$ and $CB_1|_{n=1} = CB_2|_{n=1} = CB$.

**Proposition 6.** *(1) An MFC-equilibrium is beneficial for a monopolist compared to a no-MFC equilibrium in and only in the union of (1.1) the overlap of M2 and NA1 leading to benefit $B^{M2,NA1} = (p_1 - c)(1 - p_1) - \frac{(\beta - c)^2}{4\beta} > 0$, which is constant in $\rho$, decreasing in $\beta$*

Figure 7: Monopoly ($n = 1$) market outcomes with $\beta = 0.85$ in (a) $(c/\beta, p_1)$ cross-section of inputs for fixed $\rho = 0.7$; and (b) $(\rho, p_1)$ cross-section for fixed $c = 0.1$




*and increasing in c; and (1.2) the overlap of M2 and NA3 leading to benefit* $B^{M2,NA3} = \frac{\beta p_1 - c}{2-\rho\beta}\left[p_1(1-\beta) - (1-\rho\beta)^2(p_1 - c)\right] > 0$, *which is increasing in* $\rho$.

(2) *Reseller is indifferent between MFC and no-MFC equilibria in and only in the union of the overlaps of M1 and N(A)1, M2 and N(A)2, and the boundary between N3 and M2.*

(3) *MFC is less profitable than price discrimination (and, consequently, MFC is not used) iff* $\frac{c}{\beta} < CB, p_1 > \frac{c}{\beta}$ *and* $\rho < \min\{\rho^{M1}, \rho^{M2}\}$.

*Moreover, MFC-equilibria do not lead to a gain from an increase in* $\rho$.

The proposition illustrates the nature of the relations between $p_1$-bounds in MFC and N(A)-equilibria. When the second-period sales are relatively attractive, i.e., cost-to-durability ratio is low ($\frac{c}{\beta} < CB$), MFC can be preferred to price discrimination (N3) only if the level of strategic behavior is high. When the second-period sales are less attractive, i.e., $\frac{c}{\beta} \geq CB$, MFC is always no worse than price-discrimination. In this case, bound $P_2$, which does not depend on $\rho$, separates two forms of MFC-equilibria — with sales in both periods (M1) and only in the first one (M2).

For a monopolist, MFC-equilibria, when they exist, are never detrimental. However, MFC never leads to a gain from increased strategic behavior. The following example illustrates Proposition 6.

**Example 1.** $p_1 = 0.4, \beta = 0.85, c = 0.1, \rho^L = 0.65, \rho^H = 0.95$.

Price-discriminating no-MFC equilibrium NA3 exists for both $\rho$ (Figure 7(b)) and, if MFC is not available, the loss from increased $\rho$ is $r^{*,NA3}|_{\rho=0.95} - r^{*,NA3}|_{\rho=0.65} = 0.170294 - 0.180010 = -0.009716$. If MFC is available at $\rho^H$, M2 is realized ($\rho^H > \rho^{M2} = \frac{1-\sqrt{0.2}}{0.85} = 0.6503$) with $r^{*,M2} = 0.18$, which mitigates almost all the loss. The performance of MFC is $\eta$(NA3,NA3,M2) $= 0.9989$. The performance decreases with the difference $\rho^{M2} - \rho^L$ ($\rho^L$ moves to the left from $\rho^{M2}$).

## 5.3 Oligopoly

This subsection shows that in the oligopoly, unlike monopoly, MFC can lead to substantial gains from strategic behavior as well as amplify losses depending on the market situation.

The competitive case has the following major differences from monopoly: "salvaging" equilibrium N(A)4, which may coexist with N(A)3, and the area of coexistence of M1 and M2. These differences lead to a much richer pattern of overlaps of MFC and no-MFC equilibria than under monopoly. The analysis of MFC in the overlaps is simplified by the following results obtained above: (i) M2-profit always exceeds the profit of M1 (Corollary 1); (ii) N(A)3-profit always exceeds the profit of N(A)4 (Proposition 2); (iii) M2 is always beneficial compared to N(A)3 and 4 (Proposition 3). By (ii) and (iii), the maximum benefit from M2 belongs to the area where M2 overlaps with N(A)4, which is specified in the following proposition.

**Proposition 7.** *For given inputs with $\rho^H > 0$, let M2 and N(A)4 exist and, additionally, N(A)4 exists for the same inputs except $\rho^L < \rho^H$. Then M2 at $\rho^H$ leads to a gain from increased strategic behavior, bounded from below by $\eta(NA4{,}NA4{,}M2)|_{\rho^L=0}$ as follows: $\eta(NA4{,}NA4{,}M2) \geq \eta(NA4{,}NA4{,}M2)|_{\rho^L=0} = 1 + \frac{(1-\rho^H\beta)(1-p_1)[n(p_1-c)-(p_1-s)]}{(p_1-s)\rho^H(p_1\beta-s)} > 1$.*

The following example illustrates the minimum value of $\eta(\text{NA4,NA4,M2})|_{\rho^L=0}$ in $n$ (attained at $n = 2$) for moderate values of other parameters (the existence of the equilibria in the examples below is shown in the appendix).

**Example 2.** $n = 2, \rho^L = 0, p_1 = \beta = \rho^H = 0.5, c = 0.1, s = 0.05$.

The performance of MFC is $\eta(\text{NA4,NA4,M2})|_{\rho^L=0} = 1 + \frac{3}{4} \cdot \frac{0.5\cdot(0.8-0.45)}{0.45\cdot 0.5\cdot 0.2} = 1 + \frac{3\cdot 0.35}{0.36}$, i.e., the increase in profit due to the introduction of M2 at $\rho = 0.5$ is almost four times greater than the loss of profit under NA4 due to increased strategic behavior from $\rho = 0$ to $\rho = 0.5$. This gain is impossible without strategic customers because, for these data and small $\rho$, M2 does not exist.

The case $p_1 = \beta$ used in Example 2 also provides a simple characterization of inputs where M1 is beneficial compared to NA3:

**Proposition 8.** *Under the conditions of M1 (Theorem 1) and NA3 (Theorem 2) with $p_1 = \beta$, $r^{*,M1} > r^{*,NA3}$ iff $\beta > \frac{1+c}{2}$ and either $\rho > 2\frac{1-\beta}{\beta-c}$, or $\rho \in \left(\frac{1-\beta}{\beta-c}, 2\frac{1-\beta}{\beta-c}\right]$ and $n > \frac{1-\beta}{(\beta-c)\rho-1+\beta}$.*

This proposition shows that M1 is never beneficial compared to NA3 for $\beta$ close to $c$, namely, for $\beta \leq \frac{1+c}{2}$. If the necessary conditions $\beta > \frac{1+c}{2}$ and $\rho > \frac{1-\beta}{\beta-c}$ hold, M1 *may be* better for resellers than NA3. M1 *is* better for any level of competition $n$ if the level of strategic behavior is quite high, i.e., $\rho > 2\frac{1-\beta}{\beta-c}$, and for sufficiently high $n > \frac{1-\beta}{(\beta-c)\rho-1+\beta}$, if the level of strategic behavior is moderate i.e., $\rho \in \left(\frac{1-\beta}{\beta-c}, 2\frac{1-\beta}{\beta-c}\right]$.

M1 is beneficial compared to NA4 for $p_1 = \beta$ only if $(\beta - c)^2 > \beta(\beta - s)(1 - \frac{\beta-\rho s}{1-\rho\beta})$, which, by part (1.2) of Proposition 3, is equivalent to $w > 1$. This condition never holds for $\beta$ close to $c$ and, when it holds (for large $\rho$), M1 is beneficial for large $n$.

There is an important qualitative difference between M1 and M2 that should not be neglected by resellers. The difference is that, under competition, M1 may be detrimental compared to no-MFC equilibria N(A)3 or 4. This property leads to the situation that can be called an *MFC-trap for resellers*. Assume, for the following data, that resellers are not using MFC and N(A)4 is realized.

**Example 3.** $n = 3, \rho = 0.5, c = 0.1, s = 0.05, p_1 = 0.7, \beta = 0.25$.

Figure 8: Market outcomes near inputs of (a) Example 3: $(\rho, p_1)$ cross-section for fixed $n = 3$, $c = 0.1$, $s = 0.05$ and $\beta = 0.25$; and (b) Example 4: Equilibrium profit as a function of $\rho$ for fixed $p_1 = \beta = 0.5$, $n = 4$, $c = 0.1$, $s = 0$.

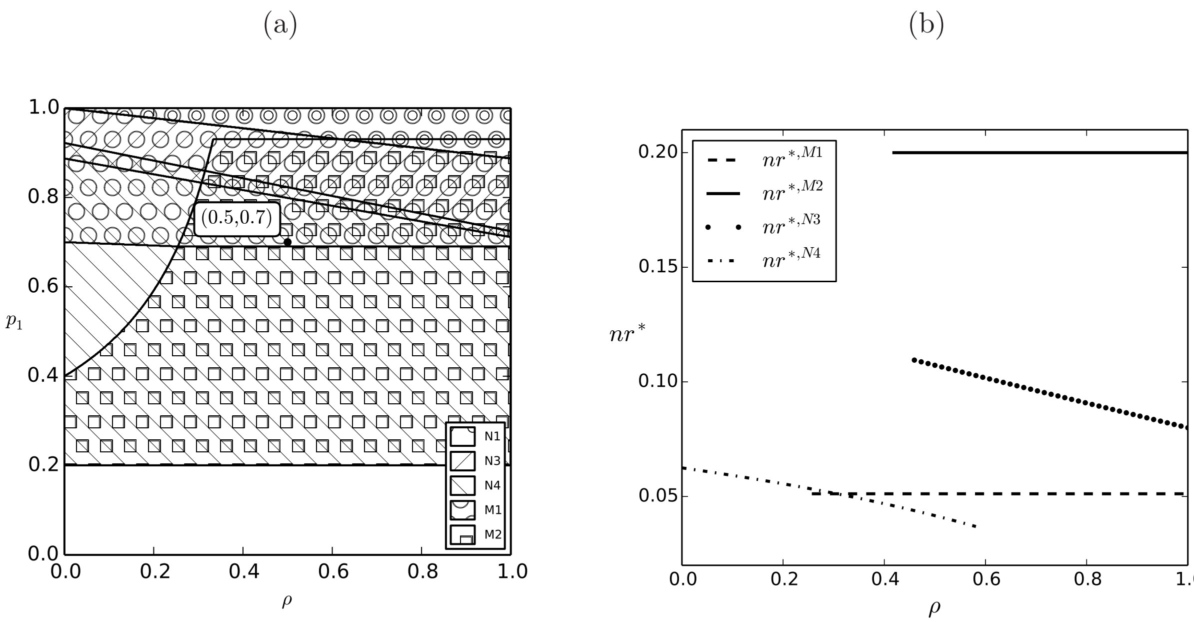



Figure 8(a) shows RESE types in the neighborhood of these inputs in a $(\rho, p_1)$ cross-section of the feasible inputs for fixed $n = 3$, $c = 0.1$, $s = 0.05$ and $\beta = 0.25$. For these inputs, $r^{*,N(A)4} = 0.0165$, and MFC can be beneficial since M2 exists and $r^{*,M2} = 0.06$, which is 3.6 times higher than $r^{*,N(A)4}$. However, the attractive comparison of $r^{*,M2}$ with $r^{*,N(A)4}$ may work as a bait in a trap. For these inputs, there also exists M1 with the profit $r^{*,M1} = 0.0056$, which is approximately 1/3 of $r^{*,N(A)4}$. The example illustrates Theorems 1, 3 and Corollaries 1, 3 showing that, depending on customer expectations, MFC can lead to losses when theoretically, for the same inputs, a beneficial equilibrium exists.

The differences in MFC-trap profits may be even higher, e.g., for the same data except $p_1 = 0.85$ and $\beta = 0.15$ (the equilibria exist by the same conditions), namely, $r^{*,N(A)4} = 0.0096, r^{*,M2} = 0.0375$ (3.9 times higher than $r^{*,N(A)4}$), and possible outcome $r^{*,M1} = 0.00104$ is 9.2 times less than the initial profit $r^{*,N(A)4}$. The area of the MFC-trap shrinks with $n$ since $CB_1$ and $CB_2$ decrease to zero in $n$ and both $P_{12}$ and $P_{22}$ go to $\frac{c}{\beta}$, reducing to zero the area of coexistence of M1 and M2.

If resellers are trapped in the detrimental M1, it is profitable for the manufacturer to help them out by adding to the contract a "no-MFC" condition since, by Proposition 4, $Y^{*,N4}$ always exceeds both $Y^{*,M1}$ and $Y^{*,M2}$. However, local policymakers may counteract manufacturer activity because the aggregate welfare is greater for M1 in both examples. Besides the MFC-trap, the overlap of N(A)4 with M1 and M2 contains the inputs where $r^{*,M1} > r^{*,N4}$. By part (1.2) of Proposition 3, these inputs correspond to larger $\rho, n$, and differences $\beta - c$. In this area, the welfare is still greater for M1, and the only part of the market suffering from MFC is the manufacturer.

Example 3 quantifies the result of Proposition 3 showing the amount by which M1-profit may be less than the least profit without MFC. Compared to no-MFC equilibria, M1 can become detrimental when the level of strategic customer behavior is increasing even if it was beneficial for lower values of $\rho$. The following example shows the extent of the negative effect in this case.

**Example 4.** $n = 4, p_1 = \beta = 0.5, \rho^H = 0.65, \rho^L = 0.2, c = 0.1, s = 0.$

This example illustrates another MFC-trap for resellers, which can be called a "regulator-facilitated MFC-trap." The regulators may encourage resellers to switch to another equilibrium (with MFC) at $\rho = 0.32$ (Figure 8(b)) since, under M1, both resellers' profit and welfare are higher than under N4 (the M1-welfare is 0.34 and N4-welfare is 0.26). For larger $\rho$, there also exist equilibria M2 and N3 with higher profits than under M1, but the regulators may discourage reseller switching away from M1 since the welfare attains maximum under M1 in this example. The manufacturer, who is also worse off under M1 than under N3, could initiate the switch to no-MFC for large $\rho$. However, the regulators may restrict manufacturer's interventions since, from their point of view, a socially-optimal outcome is realized. The negative performance of MFC as a tool for enhancing the gain from increasing $\rho$, when $\rho$ increases from 0.2 to 0.65, is $\eta(\text{NA4,NA3,M1}) = -1.1$. That is, the loss from MFC is greater than the gain from increased strategic behavior without MFC. On the other hand, by Proposition 3, the performance of M1 as a mitigating tool can be positive and, as the following example shows, M1 may even lead to a notable gain.

**Example 5.** $n = 4, p_1 = 0.4, \beta = 0.65, \rho^H = 0.4, \rho^L = 0.3, c = 0.05, s = 0.$

NA4-profit decreases in $\rho$ by 8.7% from $r^{*,NA4}|_{\rho=0.3} = 0.01258$ to $r^{*,NA4}|_{\rho=0.4} = 0.01149$. If MFC becomes available and M1 is realized at $\rho^H = 0.4$ (M1 does not exist at $\rho^L = 0.3$), profit at $\rho^H$ almost doubles to $r^{*,M1}|_{\rho=0.4} = 0.02215$, which in terms of MFC performance is $\eta(\text{NA4,NA4,M1}) = 9.776$ — the increase in profit due to use of MFC is almost ten times greater than the loss from increased strategic behavior under NA4. Moreover, the aggregate welfare of 0.346 in M1 is greater than in NA4, where it is 0.277.

# 6 Conclusions

The fundamental effects of the most-favored-customer clause (MFC) under resale price maintenance (RPM) in the presence of strategic customers include a reduction in the total inventory and the corresponding increase in the clearance prices or even elimination of the clearance sales. Legal studies treat such cases as socially harmful. However, a higher price and lower output of a limited lifetime product can be welfare-improving if they result from a policy that leads to an intertemporal redistribution of demand. We show that MFC with clearance sales (M1) can improve the total customer surplus since a high MSRP is effectively void in this case. Thus, M1 can alleviate a decrease in customer surplus caused by RPM using only market levers, without socially costly interventions of legal bodies. In addition, MFC is welfare-improving for the majority of model inputs. This welfare gain increases in the level of competition.

Since resellers also tend to decrease inventories in response to strategic customer behavior, MFC can amplify this reduction in the presence of strategic customers. As a result, by using MFC, resellers can mitigate losses from strategic customer behavior and even gain from an increase in its level, i.e., obtain a profit level higher than with myopic customers. This gain, however, is impossible for a monopolistic reseller. When MSRP is relatively high, the resellers cannot take advantage of it in a stable market outcome with MFC and clearance (M1) because of reimbursements. As a result, reseller profit in M1 can be less than without MFC for low levels of strategic behavior, competition, and product durability as well as for a high unit cost. In particular, there may exist an "MFC-trap" for resellers — an input area leading to the worst "salvaging" stable outcome (N4) without MFC.

Depending on expectations, an MFC outcome in this area can be either no-clearance M2 (with a greater profit than in the "salvaging" N4) or the clearance M1 (with the profit less than in N4 as in Example 3).

These effects of MFC and strategic behavior lead to the following implications for *resellers*. (i) Stable market outcome with MFC and no clearance is always efficient as a strategic-customer mitigating tool and always better than clearance stable outcomes with and without MFC for any given inputs. (ii) MFC can hurt resellers when multiple stable outcomes are possible and the clearance outcome with MFC is realized. (iii) A profit increase due to MFC can be smaller than a profit gain without MFC due to a switch to a more profitable equilibrium (Example 4). This drawback of MFC may arise when "salvaging" no-MFC outcome is possible and the level of strategic behavior is increasing.

There are several implications for a *manufacturer* that is able to influence reseller MFC policy. (i) Since MFC is never beneficial for a current-profit-maximizing manufacturer, it may use contract terms to discourage resellers from this policy. (ii) A branded product manufacturer may support the no-clearance MFC outcome. This support is possible only for a sufficiently low first-period price when this outcome is possible.

The closed-form analysis provided in the paper can be used for further research, e.g., for studying a game between manufacturer and resellers, which may endogenize MSRP, wholesale price, and buy-back value. Alternatively, the results may serve as a benchmark in a game where resellers compete in prices and MFC-policies. Both extensions require complicated analysis and deserve consideration in separate papers.

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Table 2: Main abbreviations and notation

| Symbol | Definition |
|---|---|
| $p_1, p_2$ | first- and second-period price |
| $c, s$ | unit cost and salvage value |
| $I = \{1, \dots, n\}$ | set of all resellers |
| $y^i, q^i$ | reseller $i$ inventory and sales in the first period |
| $m^i$ | most-favored-customer (MFC) decision: $m^i = 1$ — MFC, $m^i = 0$ — no-MFC |
| $Y_k, Q_k$ | combined first-period inventories and sales of resellers with $m^i = k$ |
| $Y, Q$ | total first-period inventories and sales |
| $v$ | first-period customer valuation of the product |
| $\beta$ | factor of decrease of customer valuation by the second period |
| $\bar{\alpha}, \bar{p}_2$ | customer expectations about product availability and price in the second period |
| $\rho$ | the level of customer strategic behavior |
| $v_k^{\min}$ | minimum valuation level of customers who, given a choice, would purchase from resellers with decision $m^i = k$ in the first period |
| $y^{-i}, m^{-i}$ | vectors of inventories and MFC decisions of all resellers except $i$ |
| $r^i$ | profit of reseller $i$ |
| $BR^i$ | best response of reseller $i$ |
| $\hat{m}, \hat{Y}$ | symmetric equilibrium MFC and inventory decision |
| $(m^*, Y^*, \alpha^*, p_2^*)$ | rational expectations symmetric equilibrium (RESE) |
| $r^*, \Sigma$ | RESE-profit and total customer surplus |
| $CB_1, CB_2$ | $c/\beta$-bounds for RESE M1 and M2 |
| $P_{11}, P_{12}$ | $p_1$-low bounds for M1 |
| $P_{21}, P_{22}$ | $p_1$-upper bounds for M2 |
| $P_1^N$ | $p_1$-bound between RESE NA1 and NA3 |
| $P_2^N$ | $p_1$-bound between RESE NA2 and NA3 |
| $P_4^N$ | $p_1$-upper bound (necessary) for RESE NA4 |
| $\eta$ | MFC performance as a tool for mitigating the loss from customer strategic behavior |
| $a \vee b, a \wedge b$ | $\max\{a, b\}$ and $\min\{a, b\}$ respectively |

# 7 Supplemental material

## 7.1 History and empirical evidence of RPM

The problem of a proper legal treatment of minimum and fixed resale price maintenance (RPM) has an old and controversial history across countries. The legal status of this practice fluctuated in the USA from per se legal to per se illegal since the first half of the nineteenth century; see Orbach (2008). In 2007, the Supreme Court, by a narrow 5–4 vote, decreed to consider RPM under the rule of reason, which recognizes that the social effects of RPM can be either beneficial or adverse. As of December 2015, RPM was still treated as per se illegal in Australia, New Zealand, UK, Japan, EU, and some states in the USA. For some products, exceptions are made but, in general, not consistently across countries. For example, the use of RPM is legal in Japan for copyrighted materials such as music records and books. Despite several attempts to prohibit RPM completely, the Japan Fair Trade Commission announced in 2001 “that for the time being it would retain the saihan [RPM] system, thus ending the discussion about the controversial system” explaining the decision by “widespread public support for resale price maintenance”; see Nippop (2005).

A review in Overstreet (1983) refers to empirical evidence of RPM prevalence when it was considered as legal. For example, RPM affected up to 10 percent of retail sales in the USA, 44 percent of consumer expenditures in the UK, 20 percent of goods sold through grocery stores and 60 percent of goods sold through drug stores in Canada. When RPM was per se illegal, Butz (1996) quoted antitrust authorities arguing that RPM is "ubiquitous" and "endemic", "but based upon 'winks and nods' rather than written agreements that could be used in court." An empirical support of this viability of RPM can be found in Overstreet (1983), e.g., for Sweden and Denmark, where RPM developed during 1930's, was abolished in 1954 and 1955 respectively, "but suggested retail prices continued to be followed on a large number of articles." There are a number of explanations for this phenomenon. For example, Buehler and Gärtner (2013) claim that retailers may follow the *manufacturer suggested retail price* (MSRP) under repeated interactions with the manufacturer, even if this price is not binding since the manufacturer may use it to communicate private information on marginal cost and consumer demand to the retailers. Using the data before and after the Supreme Court decision, MacKay and Smith (2014) show that, after 2007, RPM became more common in the USA.

## 7.2 Proof of Theorem 1 (MFC-equilibria)

The proof uses the following technical lemma.

**Lemma 2.** *The roots $(p_1)_{1,2}(x)$ of the following equation exist for $x \geq c/\beta$ :*

$$p_1^2 - (x+c)p_1 + \frac{\beta}{4}(x+c/\beta)^2 = 0. \tag{5}$$

*Moreover, $(p_1)_2(x)$ is increasing in $x$ if $x > c/\beta$, and $(p_1)_1(x) \leq \frac{1}{2}(x+c/\beta) \leq (p_1)_2(x) \leq x$ with strict inequalities if $x > c/\beta$ and, for the first two inequalities, if $\beta < 1$.*

Assume that expectations are defined when all resellers use MFC as well as when any reseller $i$ deviates into no-MFC, which, recall, we denote as $\bar{\alpha}(0), \bar{p}_2(0)$.

The proof of Theorem 1 is based on the lemma below that provides the necessary and sufficient conditions for MFC response with positive inventory to consistent MFC strategies of others. This consideration excludes the following trivial case where the MFC best response can only have zero inventory: $Y^{-i} \geq 1 - c/\beta$ and $Y^{-i} \geq 1 - p_1$. Indeed, under these conditions the second-period sales are always below cost and it is impossible for reseller $i$ to have positive sales in the first period only. The conditions in the form of $p_1$-bounds with $(p_1)_2$ in parts (a.2) and (b) of Lemma 3 guarantee that the maximum profit of an MFC-reseller with the same product availability as other resellers (no second period sales or there are second-period sales) is not dominated by the profit of a deviator who also uses MFC but has a different inventory policy: with second period sales in part (a.2) and without second-period sales in part (b). Condition (6) guarantees that the maximum profit of an MFC-reseller without second-period sales is not dominated by the profit of a deviator into no-MFC with sales in both periods.

**Lemma 3.** *There exists BR with MFC and positive inventory to consistent MFC strategies of others iff one of the following hold*

*(a) (no second-period sales) $1 - Y_1^{-i} > p_1$ and either of*

*(a.1) $\frac{c}{\beta} \geq 1 - Y_1^{-i}$, or*

*(a.2) $\frac{c}{\beta} < 1 - Y_1^{-i}$, $v_0^{\min} \notin \left(\frac{c}{\beta}, 1 - Y_1^{-i}\right)$ and $p_1 \leq (p_1)_2$ (where $(p_1)_2 = (p_1)_2(x)\big|_{x=1-Y_1^{-i}} > \frac{c}{\beta}$), or*

*(a.3)* $\frac{c}{\beta} < 1 - Y_1^{-i}$, $v_0^{\min} \in \left(\frac{c}{\beta}, 1 - Y_1^{-i}\right)$ *and*

$$p_1^2 - (v_0^{\min} + c)p_1 + \frac{\beta}{4}\Big(v_0^{\min} + c/\beta\Big)^2 \le 0. \tag{6}$$

*(b) (there are second-period sales)* $\frac{c}{\beta} < 1 - Y_1^{-i}, v_0^{\min} \notin \left(\frac{c}{\beta}, 1 - Y_1^{-i}\right)$, *and* $p_1 \ge (p_1)_2$ *(where* $(p_1)_2 > \frac{c}{\beta}$*).*

*The BR level of inventory* $y_1^i$ *is: in case (a)* $y_1^i = 1 - p_1 - Y_1^{-i}$, *and in case (b)* $y_1^i = \frac{1}{2}(1 - Y_1^{-i} - c/\beta)$*; the optimal inventory of a deviator into no-MFC in case (a.3) is*

$$\tilde{y}_0^i = 1 - Y_1 - \frac{1}{2}\Big(v_0^{\min} + c/\beta\Big). \tag{7}$$

**M1.** By part (b) of Lemma 3, the symmetric inventory of a reseller is $\frac{1}{n}Y_1^* = \frac{1}{2}(1 - \frac{n-1}{n}Y_1^* - c/\beta)$, resulting in $Y_1^* = \frac{n}{n+1}(1 - c/\beta)$ and $Y_1^{-i} = \frac{n-1}{n+1}(1 - c/\beta)$. Then condition $c/\beta < 1 - Y_1^{-i}$ is $\frac{n-1}{n+1}(1 - c/\beta) < 1 - c/\beta$, which always holds. Inequality $v_0^{\min} \le c/\beta$, as a part of $v_0^{\min} \notin (c/\beta, 1 - Y_1^{-i})$, is not relevant because $v_0^{\min} = v_0^{\min}(\bar{\alpha}(0), \bar{p}_2(0)) \le c/\beta$ implies (see the proof of Lemma 3) that a possible deviator into no-MFC selects $y_0^i = \breve{y}_0^i$ (no second-period sales). Then, by rationality, $\bar{\alpha} = 0$ and $v_0^{\min} = p_1 \le c/\beta$, which cannot hold together with $p_1 \ge (p_1)_2 > c/\beta$.

Hence, the existence of M1 is determined only by $v_0^{\min} \ge 1 - Y_1^{-i}$ and $p_1 \ge (p_1)_2$. Inequality $v_0^{\min} \ge 1 - Y_1^{-i}$ means that a possible deviator into no-MFC has sales only in the second period with $r_0^i = [\beta(1 - Y_1 - y_0^i) - c]y_0^i$, the first-order condition $\frac{\partial r_0^i}{\partial y_0^i} = -2\beta y_0^i + \beta(1 - Y_1) - c = 0$, and the resulting profit-maximizing inventory

$$\tilde{y}_0^i = \frac{1}{2}(1 - Y_1 - c/\beta) = \tilde{y}_1^i, \tag{8}$$

giving $\bar{\alpha} = 1, \bar{p}_2 = p_2^* = \beta(1 - Y_1^*)$, and $v_0^{\min} = \frac{p_1 - \rho\beta(1 - Y_1^*)}{1 - \rho\beta}$. Then inequality $v_0^{\min} \ge 1 - Y_1^{-i}$ is

$$\begin{aligned}
\frac{p_1 - \rho\beta[1 - \frac{n}{n+1}(1 - c/\beta)]}{1 - \rho\beta} &\ge 1 - \frac{n-1}{n+1}(1 - c/\beta) \Leftrightarrow \\
p_1 - \rho\beta + \frac{n\rho\beta}{n+1}(1 - c/\beta) &\ge 1 - \rho\beta - \frac{n-1}{n+1}(1 - c/\beta) + \rho\beta\frac{n-1}{n+1}(1 - c/\beta) \Leftrightarrow \\
p_1 &\ge 1 - \frac{n - 1 + \rho\beta}{n+1}(1 - c/\beta) = P_{11}.
\end{aligned} \tag{9}$$

$P_{11}$ is less than $(p_1)_2(x)|_{x = 1 - \frac{n-1}{n+1}(1 - c/\beta)} = P_{12}$ if after substitution of $P_{11}$ for $p_1$ and $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$ into (5) the LHS of (5) becomes negative. This condition takes the form of a quadratic inequality in $c$ with the coefficient in front of $c^2$ equal to $\frac{(1-\rho\beta)^2 + (1-\beta)\rho[n - (1-\rho\beta)]}{\beta(n+1)^2} > 0$ and the roots $\{\beta \cdot CB_1, \beta\}$, The first root is not greater than $\beta$ (strictly less if $\beta < 1$). Therefore, $P_{11} \le P_{12}$ if $c/\beta \ge CB_1$. Then M1 exists under condition

$$p_1 \ge P_{12} = \frac{1}{2}\left[x + c + \sqrt{(1 - \beta)(x^2 - c^2/\beta)}\right] \text{ (part (1.2) of the Theorem).} \tag{10}$$

Now we show that the substitution of $P_{11}$ and $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$ into (5) results in positive LHS of (5) *only if* $P_{11} > P_{12}$. This conclusion results from the following chain of inequalities that is proved below:

$$P_{11} = 1 - \frac{n - 1 + \rho\beta}{n+1}(1 - c/\beta) > \frac{1}{2}(x + c/\beta) \ge (p_1)_1(x),$$

where $(p_1)_1(x)$ is the smaller root of (5) and $x = 1 - \frac{n-1}{n+1}(1 - c/\beta)$. Indeed, the first inequality is

$$\frac{2-\rho\beta}{n+1} + \frac{n-1+\rho\beta}{n+1}\frac{c}{\beta} > \frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta} \Leftrightarrow \frac{1-\rho\beta}{n+1} > \frac{1-\rho\beta}{n+1}\frac{c}{\beta},$$

which always holds, and the second inequality $\frac{1}{2}(x+c/\beta) \geq (p_1)_1(x)$ holds by Lemma 2. Therefore, whenever $c/\beta < CB_1$, we have $P_{11} > P_{12}$ yielding case (M1.1) of the Theorem. Moreover, $P_{11} = P_{12}$ when $c/\beta = CB_1$.

**M2.** By part (a) of Lemma 3, the symmetric inventory in this case is $Y_1^* = 1-p_1$ and condition $1 - Y_1^{-i} > p_1$ holds for $y^i > 0$. The condition of part (a.1), resulting in the existence of M2, takes the form $c/\beta \geq 1 - \frac{n-1}{n}(1-p_1) \Leftrightarrow p_1 \leq 1 - \frac{n}{n-1}(1-c/\beta) = c/\beta - \frac{1}{n-1}(1-c/\beta) < c/\beta$. The complementary case to this inequality is covered by parts (a.2) and (a.3) of Lemma 3.

*When* $v_0^{\min} \leq c/\beta$, a possible deviator into no-MFC has no sales in the second period, and $v_0^{\min} = p_1 \leq c/\beta$ implying $p_1 \leq (p_1)_2$ by part (a.2) of Lemma 3. Therefore, M2 exists for any feasible $p_1 \leq c/\beta$.

*Consider* $v_0^{\min} \in (c/\beta, 1-Y_1^{-i})$ (part (a.3) of Lemma 3). This condition in combination with (6) excludes $v_0^{\min} = p_1$ because (6) becomes $p_1^2-(p_1+c)p_1+\frac{\beta}{4}(p_1+c/\beta)^2 \leq 0 \Leftrightarrow \frac{\beta}{4}(p_1-c/\beta)^2 \leq 0$, which is impossible for $v_0^{\min} = p_1 > c/\beta$. The case $v_0^{\min} = 1$ is also irrelevant here because it contradicts $v_0^{\min} < 1-Y_1^{-i}$. Since the range $v_0^{\min} \in (c/\beta, 1-Y_1^{-i})$ means that a possible deviator into no-MFC has sales in both periods (sales only in the first period yield profit that is not greater than under M2), i.e., $\bar{\alpha} = 1$, the only relevant case for $v_0^{\min}$ is $v_0^{\min} = \frac{p_1-\rho p_2}{1-\rho\beta}$, where $p_2 = \beta(1-Y_1-y_0^i)$, and, by (7), $y_0^i = \tilde{y}_0^i = 1-Y_1-\frac{1}{2}(v_0^{\min}+c/\beta)$, yielding

$$\begin{aligned} v_0^{\min} &= \frac{p_1 - \rho\beta\frac{1}{2}(v_0^{\min}+c/\beta)}{1-\rho\beta} \Leftrightarrow (1-\rho\beta)v_0^{\min} = p_1 - \rho\beta v_0^{\min}/2 - \rho\beta c/2 \Leftrightarrow \\ (1-\rho\beta/2)v_0^{\min} &= p_1 - \rho c/2 \Leftrightarrow v_0^{\min} = \frac{2p_1-\rho c}{2-\rho\beta}. \end{aligned}$$

Then condition $v_0^{\min} < 1 - Y_1^{-i}$ becomes

$$\begin{aligned} \frac{2p_1-\rho c}{2-\rho\beta} &< 1 - \frac{n-1}{n}(1-p_1) \Leftrightarrow 2p_1 - \rho c < (2-\rho\beta)\Big(\frac{1}{n} + \frac{n-1}{n}p_1\Big) \Leftrightarrow \\ \Big(2 - \frac{n-1}{n}(2-\rho\beta)p_1\Big)p_1 &< \frac{2-\rho\beta}{n} + \rho c \Leftrightarrow p_1 < \frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}, \end{aligned} \tag{11}$$

and condition $v_0^{\min} > c/\beta$ is $\frac{2p_1-\rho c}{2-\rho\beta} > c/\beta \Leftrightarrow 2p_1 - \rho c > 2c/\beta - \rho c \Leftrightarrow p_1 > c/\beta$. Under the combination of this inequality with (11), by part (a.3) of Lemma 3, an equilibrium with $Y_1^* = 1-p_1$ exists iff inequality (6) holds. With the rational "symmetric" $v_0^{\min}$ this inequality becomes

$$p_1^2 - \Big(\frac{2p_1-\rho c}{2-\rho\beta} + c\Big)p_1 + \frac{\beta}{4}\Big(\frac{2p_1-\rho c}{2-\rho\beta} + \frac{c}{\beta}\Big)^2 \leq 0. \tag{12}$$

The coefficient in front of $p_1^2$ is $a_2 = 1 - \frac{2}{2-\rho\beta} + \frac{\beta}{(2-\rho\beta)^2} = \frac{\beta(1-2\rho+\rho^2\beta)}{(2-\rho\beta)^2}$, which is positive iff $\rho < (1-\sqrt{1-\beta}/\beta$ (the larger root of $1-2\rho+\rho^2\beta = 0$ is $(1+\sqrt{1-\beta})/\beta > 1$ – irrelevant here). If $a_2 = 0$, (which means that $1-2\rho+\rho^2\beta = 0$) the inequality above becomes $p_1 \geq -a_0/a_1$, where $a_0 = \frac{\beta}{4}\Big(c/\beta - \frac{\rho c}{2-\rho\beta}\Big)^2$ and $a_1 = -(2-2\rho\beta-2\rho+\rho^2\beta+\rho^2\beta^2)/(2-\rho\beta)^2 = -[(1-\rho\beta)^2+1-2\rho+\rho^2\beta]/(2-\rho\beta)^2$ $= -(1-\rho\beta)^2/(2-\rho\beta)^2$, yielding $p_1 \geq c/\beta$, which always holds in this case.

If $a_2 > 0$, the reduced form of (12), after collecting the terms with $p_1$ and dividing by $a_2$, is

$$p_1^2 - \frac{c}{\beta}\Big(1 + \frac{(1-\rho\beta)^2}{1-2\rho+\rho^2\beta}\Big)p_1 + \Big(\frac{c}{\beta}\Big)^2\Big(\frac{(1-\rho\beta)^2}{1-2\rho+\rho^2\beta}\Big) \leq 0 \tag{13}$$

with the roots of the corresponding equation $(p_1)_{1,2} = \{\frac{c}{\beta}, \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\rho^2\beta}\}$, which can be seen by observing that $-[(p_1)_1+(p_1)_2]$ equals the coefficient in front of $p_1$ and $(p_1)_1(p_1)_2$ – the free coefficient of (13). The roots are distinct iff $\beta < 1$. Both (12) and (13) hold if $p_1$ is between the roots:

$$\frac{c}{\beta} \leq p_1 \leq \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} = P_{21}. \tag{14}$$

In the case $a_2 < 0$ (implying $1-2\rho+\rho^2\beta < 0$), inequality (13) is inverted and holds if $p_1$ does not exceed the smaller root, which is irrelevant since $P_{21} < 0$ in this case, or if $p_1$ is not less than the larger root: $p_1 \geq c/\beta$, which always holds in this case.

Hence, the case $v_0^{\min} \in (c/\beta, 1-Y_1^{-i})$ yields two upper bounds on $p_1$ that guarantee M2 existence, namely, conditions (11) and (14). The bound on $c/\beta$ below shows when $P_{21}$ is less than the bound from (11).

$$P_{21} = \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} < \frac{2-\rho\beta+n\rho\beta\frac{c}{\beta}}{2-\rho\beta+n\rho\beta} \Leftrightarrow \frac{c}{\beta}\left[\frac{(1-\rho\beta)^2(2-\rho\beta+n\rho\beta)}{1-2\rho+\rho^2\beta} - n\rho\beta\right] < 2-\rho\beta \Leftrightarrow$$

$$\frac{c}{\beta}\frac{(1-\rho\beta)^2(2-\rho\beta)+n\rho\beta[(1-\rho\beta)^2-1+2\rho-\rho^2\beta]}{(2-\rho\beta)(1-2\rho+\rho^2\beta)} < 1 \Leftrightarrow \frac{c}{\beta}\frac{(1-\rho\beta)^2+n\rho^2\beta(1-\beta)}{1-2\rho+\rho^2\beta} < 1 \Leftrightarrow \frac{c}{\beta} < CB_2. \tag{15}$$

*Consider* $v_0^{\min} \geq 1 - Y_1^{-i}$ (part (a.2) of Lemma 3). Since a possible deviator to no-MFC has no first period sales, the optimal inventory, by (8), is $\tilde{y}_0^i = \frac{1}{2}(1-Y_1^{-i}-c/\beta)$, and inequality $v_0^{\min} \geq 1-Y_1^{-i}$ is

$$\begin{aligned}\frac{p_1 - \rho\beta[1-Y_1^{-i}-\frac{1}{2}(1-Y_1^{-i}-c/\beta)]}{1-\rho\beta} &\geq 1-Y_1^{-i} \Leftrightarrow\\ p_1 - \rho\beta(1-Y_1^{-i}) + \rho\beta(1-Y_1^{-i}-c/\beta)/2 &\geq (1-Y_1^{-i})(1-\rho\beta) \Leftrightarrow p_1 + \rho\beta(1-Y_1^{-i})/2 - \rho c/2 \geq 1-Y_1^{-i},\end{aligned}$$

which with $1-Y_1^{-i} = \frac{1}{n}+\frac{n-1}{n}p_1$ is

$$p_1 + \frac{\rho\beta}{2}\Big(\frac{1}{n}+\frac{n-1}{n}p_1\Big) - \frac{\rho c}{2} \geq \frac{1}{n}+\frac{n-1}{n}p_1 \Leftrightarrow \Big(\frac{1}{n}+\frac{\rho\beta(n-1)}{2n}\Big)p_1 \geq \frac{\rho c}{2}+\frac{1}{n}\Big(1-\frac{\rho\beta}{2}\Big)$$

yielding $[2+\rho\beta(n-1)]p_1 \geq \rho cn + 2 - \rho\beta$, which gives inequality complementary to (11). $v_0^{\min} = 1$ is included in this condition since $v_0^{\min} = 1 \geq 1-Y_1^{-i}$ always holds; $v_0^{\min} = p_1$ is irrelevant here because $v_0^{\min} = p_1 \geq 1-Y_1^{-i}$ contradicts the necessary condition of part (a) Lemma 3.

Hence, when $v_0^{\min} \geq 1-Y_1^{-i} = x > c/\beta$, the existence of M2 is guaranteed by condition

$$\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} \leq p_1 \leq (p_1)_2(x)|_{x=\frac{1}{n}+\frac{n-1}{n}p_1}. \tag{16}$$

Similar to above, the resulting condition for $c/\beta$ below is equivalent to the non-emptiness of this range. Consider inequality

$$p_1 \leq (p_1)_2(x)|_{x=\frac{1}{n}+\frac{n-1}{n}p_1}, \tag{17}$$

which, by (5), is equivalent to $2p_1 - (x+c) \le \sqrt{(x+c)^2 - \beta(x+c/\beta)^2}$. The LHS is $2p_1 - \Big(\frac{1}{n} + \frac{n-1}{n}p_1 + c\Big) = \frac{n+1}{n}p_1 - \frac{1+nc}{n}$, which is non-negative for $p_1 \ge (1+nc)/(n+1)$, implying that (17) is equivalent to

$$\left[p_1^2 - (x+c)p_1 + \beta(x+c/\beta)^2/4\right]\Big|_{x=\frac{1}{n}+\frac{n-1}{n}p_1} \le 0 \tag{18}$$

with the coefficient in front of $p_1^2$ equal to $1 - \frac{n-1}{n} + \beta\left(\frac{n-1}{n}\right)^2/4 > 0$. Moreover, for $p_1 = (1+nc)/(n+1)$, conditions (17) and (18) hold trivially. Therefore, $p_1 = (1+nc)/(n+1)$ is between the roots of (5) with $x = \frac{1}{n} + \frac{n-1}{n}p_1$.

Observe also that the LHS of range (16) is

$$\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} = \frac{2-\rho\beta+n\rho\beta-n\rho\beta+n\rho\beta c/\beta}{2-\rho\beta+n\rho\beta} = 1 - \frac{n\rho\beta(1-c/\beta)}{2-\rho\beta+n\rho\beta} = 1 - \frac{n(1-c/\beta)}{2/\rho\beta-1+n},$$

which is decreasing in both $\rho$ and $\beta$ implying that $\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} > \frac{1+nc}{n+1}$ for any feasible $\rho$ and $\beta$. Therefore, $\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}$ is greater than the smaller root of (5) with $x = \frac{1}{n} + \frac{n-1}{n}p_1$. Hence, the condition of non-emptiness of range (16) follows from inequality (18) with $p_1 = \frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}$. The resulting condition takes the form of a quadratic inequality in $c$

$$\Big(\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}\Big)^2 - \Big(\frac{1}{n}+\frac{n-1}{n}\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}+c\Big)\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta} + \frac{\beta}{4}\Big(\frac{1}{n}+\frac{n-1}{n}\frac{2-\rho\beta+\rho cn}{2+(n-1)\rho\beta}+\frac{c}{\beta}\Big)^2 \le 0$$

with the coefficient in front of $c^2$ equal to $\frac{\rho^2\beta n(1-\beta)+(1-\rho\beta)^2}{\beta[2+(n-1)\rho\beta]^2} > 0$ and the roots $\{\beta \cdot CB_2, \beta\}$, where $CB_2 = \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2+n\rho^2\beta(1-\beta)} \le 1$ implying that range (16) is not empty iff $c$ is between these roots. The denominator of $CB_2$ is always positive. Therefore, if $1-2\rho+\rho^2\beta \le 0$, we have $CB_2 \le 0$ and range (16) is not empty for any feasible $c : 0 < c < \beta$.

If $1-2\rho+\rho^2\beta > 0$, range (16) is not empty for any $\frac{c}{\beta}$ such that $CB_2 \le \frac{c}{\beta} \le 1$. The LHS of this condition is complementary to (15), which means that when it does not hold, range (16) is empty and M2 exists if $p_1 \le P_{21}$ (part (2.1) of the Theorem); and when it holds, M2 exists if (17) is satisfied.

It remains to specify condition (17) by expressing the larger root of (5) with $x = \frac{1}{n} + \frac{n-1}{n}p_1$. After the substitution and collection of terms, (5) becomes

$$\left[\frac{1}{n} + \frac{\beta}{4}\left(\frac{n-1}{n}\right)^2\right]p_1^2 + \left[\frac{1}{2n}\left(\beta\frac{n-1}{n} + c(n-1)\right) - \frac{1}{n} - c\right]p_1 + \frac{\beta}{4}\left(\frac{1}{n}+\frac{c}{\beta}\right)^2 = 0,$$

which, multiplied by $4n^2$, is $\left[4n+\beta(n-1)^2\right]p_1^2 + 2\left[\beta(n-1) - cn(n+1) - 2n\right]p_1 + \beta\left(1+nc/\beta\right)^2 = 0$. The larger root is

$$\frac{2[cn(n+1)+2n-\beta(n-1)]+\sqrt{D}}{2[4n+\beta(n-1)^2]},$$

where $D = 4\left\{[\beta(n-1)-cn(n+1)-2n]^2 - \left[4n+\beta(n-1)^2\right]\beta\left(1+nc/\beta\right)^2\right\}$, where the squared bracket $[\cdot]^2$ is $\beta^2(n-1)^2 + c^2n^2(n+1)^2 + 4n^2 - 2\beta(n-1)cn(n+1) - 4\beta(n-1)n + 4cn^2(n+1)$, and the second term in the bracket $\{\cdot\}$ is $-\left[4n+\beta(n-1)^2\right]\beta\left(1+2nc/\beta+n^2c^2/\beta^2\right)$, which can be written as $-\left[4n\beta + 8n^2c + 4n^3c^2/\beta + \beta^2(n-1)^2 + 2\beta nc(n-1)^2 + (n-1)^2n^2c^2\right]$. After simplifications,

$$\begin{aligned} D &= 4\left\{4n^3c^2 + 4n^2 - 2\beta cn(n^2-1+n^2-2n+1) - 4\beta n^2 + 4cn^2(n+1) - 8n^2c - 4n^3c^2/\beta\right\} \\ &= (4n)^2\left\{nc^2 + 1 - \beta c(n-1) - \beta + c(n+1) - 2c - nc^2/\beta\right\} \\ &= (4n)^2\left\{nc[c-\beta+1-c/\beta] + 1 - \beta + \beta c - c\right\} = (4n)^2(1-\beta)\left\{nc\left(1-c/\beta\right)+1-c\right\} \end{aligned}$$

and the expression for the larger root is

$$P_{22} = \frac{n(n+1)c + 2n - \beta(n-1) + 2n\sqrt{(1-\beta)\left[nc(1-c/\beta) + 1 - c\right]}}{4n + \beta(n-1)^2}. \tag{19}$$

The fact $P_{11}, P_{12}, P_{22} \to 1$ as $\frac{c}{\beta} \to 1$ can be shown by direct substitution of $\frac{c}{\beta} = 1$ into the formulas for $P_{11}, P_{12}$, and $P_{22}$.

### 7.3 Proof of Corollary 1 (profit M2 exceeds M1)

*Part (1).* When $\frac{c}{\beta} < CB_1$, equilibria M1 or M2 exist (Theorem 1) if, respectively, $p_1 \geq P_{11}$ or $p_1 \leq P_{21}$. Therefore, MFC-equilibria do not exist for $P_{21} < p_1 < P_{11}$ if $P_{11} > P_{21}$. By the definition of $P_{11}$ and $P_{21}$, inequality $P_{11} \geq P_{21}$, multiplied by $n+1$ is

$$\begin{aligned} n + 1 - n + 1 - \rho\beta + (n - 1 + \rho\beta)\frac{c}{\beta} &\geq \frac{c}{\beta}\frac{(1-\rho\beta)^2(n+1)}{1 - 2\rho + \beta\rho^2} \Leftrightarrow \\ 2 - \rho\beta &\geq \frac{c}{\beta}\frac{(1-\rho\beta)^2(n+1) - (n-1+\rho\beta)(1-2\rho+\beta\rho^2)}{1-2\rho+\beta\rho^2}. \end{aligned}$$

Since $1 - 2\rho + \beta\rho^2 = (1-\rho\beta)^2 - \rho(1-\beta)(2-\rho\beta)$, the numerator of the second fraction can be written as $(1-\rho\beta)^2(2-\rho\beta) + (n-1+\rho\beta)\rho(1-\beta)(2-\rho\beta)$. Then, after dividing both sides by $2-\rho\beta$ and expressing $\frac{c}{\beta}$, the inequality becomes $\frac{c}{\beta} \leq \frac{1-2\rho+\rho^2\beta}{(1-\rho\beta)^2 + (1-\beta)\rho[n-(1-\rho\beta)]} = CB_1$. Hence, $P_{11} \geq P_{21}$ is equivalent to $\frac{c}{\beta} \leq CB_1$. By the proof of part M1 of Theorem 1, we also know that $P_{11} = P_{12}$ if $\frac{c}{\beta} = CB_1$.

*Part (2).* By Theorem 1, both $P_{12}$ and $P_{22}$ are greater than $\frac{c}{\beta}$ if $n < \infty$, and, by Lemma 3, both are the larger roots of (5) at different (for $n > 1$) $x$, namely $x_{12} = 1 - \frac{n-1}{n+1}(1 - c/\beta)$ and $x_{22} = \frac{1}{n} + \frac{n-1}{n}p_1$. For $n = 1$, $x_{12} = x_{22} = 1$, and the expression for $P_{12} = P_{22} = P_2$ results from direct substitution. For $n > 1$, by Lemma 2, $x_{12} > \frac{c}{\beta}$ and $x_{22} > \frac{c}{\beta}$, and inequality $P_{12} < P_{22}$ follows from $x_{12} < x_{22}$ since the larger root of (5) increases in $x$. Inequality $x_{12} < x_{22}$ is

$$\frac{2}{n+1} + \frac{n-1}{n+1}\frac{c}{\beta} < \frac{1}{n} + \frac{n-1}{n}p_1 \Leftrightarrow p_1 > \frac{n}{n-1}\left[\frac{2}{n+1} - \frac{1}{n} + \frac{n-1}{n+1}\frac{c}{\beta}\right] = \frac{\beta + nc}{(n+1)\beta}.$$

This inequality holds for any $p_1$, corresponding to M1 (including the overlap with M2) if $\frac{\beta+nc}{(n+1)\beta} < P_{11}$, i.e., $\frac{\beta+nc}{(n+1)\beta} < 1 - \frac{n-1+\rho\beta}{n+1}\left(1 - \frac{c}{\beta}\right) \Leftrightarrow 1 + n\frac{c}{\beta} < n + 1 - n + 1 - \rho\beta + (n - 1 + \rho\beta)\frac{c}{\beta}$, which is equivalent to $(1-\rho\beta)c/\beta < 1 - \rho\beta$ and always true.

Assume, for $n \geq 1$, that $P_{12} \leq p_1 \leq P_{22}$, which determines the overlap of M1 and M2 only if $\frac{c}{\beta} \geq CB_2$, i.e., $CB_2 < 1$ must hold ($\beta < 1, \rho > 0$). Inequality $r^{*,M1} \leq r^{*,M2}$ is equivalent to

$$p_1^2 - (1+c)p_1 + c + \frac{n(\beta - c)^2}{(n+1)^2\beta} \leq 0. \tag{20}$$

It can be shown that the LHS of (20) equals the LHS of (5) at $x = 1$ for $n = 1$ and strictly less for $n > 1$. This property implies, first, by Lemma 2, that for any $\beta < 1$ there are two distinct roots of the equation corresponding to (20), and, second, that, for $n > 1$, the larger root is greater, and the smaller root is less than the corresponding roots of (5) at $x = 1$ (for $n = 1$ the equations coincide). Indeed, the free coefficient in the LHS of (20) can be written as

$$c + \frac{n(\beta-c)^2}{(n+1)^2\beta} - \frac{\beta}{4}\left(1 + \frac{c}{\beta}\right)^2 + \frac{\beta}{4}\left(1 + \frac{c}{\beta}\right)^2 = \frac{\beta}{4}\left(1 + \frac{c}{\beta}\right)^2 + \frac{1}{4\beta}\left[\frac{4n(\beta-c)^2}{(n+1)^2} - (\beta+c)^2 + 4\beta c\right],$$

where the bracket $[\cdot] = \frac{4n(\beta-c)^2}{(n+1)^2} - (\beta-c)^2$ equals zero for $n=1$ and decreases in $n$.

It can be shown also that range $[P_{12}, P_{22}]$ is strictly (for $n>1$) between the roots of the equation, corresponding to (20). First, by Lemma 2, $P_{22}$ is not greater than the greater root of (5) at $x=1$ since $\frac{c}{\beta} < x_{22} \leq 1$. For $n=1$, $P_{22}$ equals this root and equals $P_{12}$, implying that (20) holds as equality yielding $r^{*,M1} = r^{*,M2}$ if $p_1 = P_{12} = P_{22} = P_2$. Second, for $n>1$, $P_{12}$ is always greater than the smaller root of the equation, corresponding to (20), which follows from the chain of inequalities: first, by Lemma 2, $P_{12} = (p_1)_2(x)|_{x=x_{12}} \geq \frac{1}{2}(x+c/\beta)\big|_{x=x_{12}}$ and, second, $\frac{1}{2}(x+c/\beta)\big|_{x=x_{12}}$ is greater than the smaller root of the equation, corresponding to (20). The last inequality holds if the LHS of (20) becomes negative after the substitution of $\frac{1}{2}(x+c/\beta)\big|_{x=x_{12}} = \frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta}$ for $p_1$. Indeed, this substitution yields

$$\left(\frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta}\right)^2 - (1+c)\left(\frac{1}{n+1} + \frac{n}{n+1}\frac{c}{\beta}\right) + c + \frac{n(\beta-c)^2}{(n+1)^2\beta},$$

which, multiplied by $(n+1)^2\beta^2$ becomes $(\beta+nc)^2 - (1+c)(n+1)\left(\beta^2+nc\beta\right) + c(n+1)^2\beta^2 + n\beta(\beta-c)^2$ $= n\{n[c^2-(1+c)c\beta+c\beta^2] + c\beta - \beta^2 + \beta^3 - c\beta^2\}$, where $\{\cdot\}$ is $n[c^2(1-\beta) - c\beta(1-\beta)] + \beta(c-\beta) - \beta^2(c-\beta) = nc(1-\beta)(c-\beta) + \beta(c-\beta)(1-\beta) < 0$ for any $n>1$. Hence, $p_1 \in [P_{12}, P_{22}]$ results in satisfaction of (20) as a strict inequality and $r^{*,M1} < r^{*,M2}$.

*Part (3).* This part is relevant only for $n>1, \beta<1$, and $0 < \rho < (1-\sqrt{1-\beta})/\beta$ leading to $0 < CB_1 < CB_2$. By part (1), $P_{12} = P_{21}$ if $\frac{c}{\beta} = CB_1 > 0$. Then $P_{12} < P_{21}$ for $\frac{c}{\beta} > CB_1$ if $\frac{\partial P_{12}}{\partial(c/\beta)} < \frac{\partial P_{21}}{\partial(c/\beta)}$ for all $\frac{c}{\beta} > CB_1$. Denoting $x = 1 - \frac{n-1}{n+1}(1-c/\beta)$, the derivatives $\frac{\partial P_{12}}{\partial(c/\beta)}$ and $\frac{\partial^2 P_{12}}{[\partial(c/\beta)]^2}$ are

$$\begin{aligned}
\frac{\partial P_{12}}{\partial(c/\beta)} &= \frac{1}{2}\left\{\frac{n-1}{n+1} + \beta + (1-\beta)\left(x\frac{n-1}{n+1} - \beta\frac{c}{\beta}\right)/\sqrt{(1-\beta)\left[x^2 - c^2/\beta\right]}\right\},\\
\frac{\partial^2 P_{12}}{[\partial(c/\beta)]^2} &= \frac{1}{2}\left\{\left(\frac{n-1}{n+1}\right)^2 - \beta - \left(x\frac{n-1}{n+1} - c\right)^2/\left[x^2 - c^2/\beta\right]\right\}\sqrt{(1-\beta)/\left[x^2-c^2/\beta\right]}.
\end{aligned}$$

Since $P_{12}$ is a branch of a second-order curve, and such a branch is either convex or concave in its entire domain, the concavity of $P_{12}(c/\beta)$ can be shown at $c/\beta = 0$, where $x|_{c/\beta=0} = \frac{2}{n+1}$. Namely, $\frac{\partial^2 P_{12}}{[\partial(c/\beta)]^2}\Big|_{c/\beta=0} = -\frac{\beta}{4}(n+1)\sqrt{1-\beta} \leq 0$. Since $P_{12}(c/\beta)$ is concave, $P_{12}|_{c/\beta=0} = \frac{1+\sqrt{1-\beta}}{n+1} > P_{21}|_{c/\beta=0} = 0$, and $P_{12} = P_{21}$ at $c/\beta = CB_1$, we have $\frac{\partial P_{12}}{\partial(c/\beta)} < \frac{\partial P_{21}}{\partial(c/\beta)}$ at $c/\beta = CB_1$. The last inequality combined with the concavity of $P_{12}(c/\beta)$ implies that $\frac{\partial P_{12}}{\partial(c/\beta)} < \frac{\partial P_{21}}{\partial(c/\beta)}$ for all $c/\beta > CB_1$.

Inequality $r^{*,M1} < r^{*,M2}$ for $P_{12} \leq p_1 \leq P_{21}$ follows from part (2) since $P_{21} < P_{22}$ for $c/\beta < CB_2$.

*Part (4).* $c/\beta \geq CB_1$ is $1 - 2\rho + \rho^2\beta \leq [1 - 2\rho\beta + \rho(1-\beta)(n-1) + \rho^2\beta]c/\beta$ or $\rho^2(\beta-c) - \rho b + 1 - c/\beta \leq 0$, where $b \triangleq 2 + [n(1-\beta)-1]c/\beta - c$. The discriminant of the corresponding equation is $D = b^2 - 4(\beta-c)^2/\beta$, which is non-negative since $\frac{\partial D}{\partial\beta} = 2b(-nc+c)/\beta^2 - 4[2(\beta-c)-(\beta-c)^2]/\beta^2 < 0$ and $D|_{\beta=1} = 0$. The larger root is irrelevant since $\frac{b}{2(\beta-c)}\Big|_{n=1} \geq 1$ and $b$ increases in $n$. Therefore, $c/\beta \geq CB_1 \Leftrightarrow \rho \geq \bar{\rho}$, where $\bar{\rho} \triangleq \frac{1}{2(\beta-c)}\left[b - \sqrt{b^2 - 4(\beta-c)^2/\beta}\right]$ is the smaller root, which goes to zero with $n \to \infty$.

*Part (5).* The monotonicity of $P_{11}$ and $P_{21}$ in $c, \rho, \beta$, and $n$ follows directly from the definitions of these bounds given in Theorem 1. In particular,

$$\frac{\partial P_{21}}{\partial\rho} = \frac{c}{\beta(1-2\rho+\beta\rho^2)^2}\left[-2\beta(1-\rho\beta)(1-2\rho+\beta\rho^2) - (1-\rho\beta)^2(-2+2\rho\beta)\right],$$

where $[\cdot] = 2(1-\rho\beta)\{(1-\rho\beta)^2 - (\beta - 2\rho\beta + \rho^2\beta^2)\}$, where $\{\cdot\} = 1-\beta$, leading to $\frac{\partial P_{21}}{\partial \rho} = \frac{c}{\beta}\frac{2(1-\rho\beta)(1-\beta)}{(1-2\rho+\beta\rho^2)^2} \geq 0$.

Both $P_{12}$ and $P_{22}$ are decreasing in $n$ since they are the larger roots of (5), which, by Lemma 2, are increasing in $x$ for any $x > \frac{c}{\beta}$, and both $x_{12} = 1 - \frac{n-1}{n+1}(1-c/\beta)$ and $x_{22} = \frac{1}{n} + \frac{n-1}{n}p_1$ are greater than $c/\beta$ and decreasing in $n$.

$$\frac{\partial P_{12}}{\partial c} = \frac{1}{2}\left[\frac{\partial x}{\partial c} + 1 + \frac{1}{2}\sqrt{1-\beta}\cdot 2\left(x\frac{\partial x}{\partial c} - \frac{c}{\beta}\right)/\sqrt{x^2 - c^2/\beta}\right],$$

where $\frac{\partial x}{\partial c} = \frac{n-1}{\beta(n+1)}$ is increasing in $n$, and the last fraction in $[\cdot]$ is also increasing in $n$ either for any $c$ and $n \leq 3$ or for $n > 3$ and $c \geq c^0 = \frac{1}{2} - \frac{1}{n-1}$. This monotonicity follows from the expression for $x\frac{\partial x}{\partial c} = \frac{(n-1)[2+c(n-1)]}{\beta(n+1)^2}$ and the derivative

$$\begin{aligned}\frac{\partial}{\partial n}\left(x\frac{\partial x}{\partial c}\right) &= \frac{2(n+1)}{\beta(n+1)^4}\left([1+c(n-1)](n+1) - (n-1)[2+c(n-1)]\right)\\ &= \frac{2}{\beta(n+1)^3}(2-(n-1)+2c(n-1)) = \frac{2[2+(n-1)(2c-1)]}{\beta(n+1)^3},\end{aligned}$$

where the last bracket $[\cdot] \geq 0$ under the above conditions on $c$ and $n$. Then, denoting $d(x) \triangleq x^2 - c^2/\beta$,

$$\frac{\partial}{\partial n}\left(x\frac{\partial x}{\partial c}/\sqrt{d(x)}\right) = \frac{1}{d(x)}\left\{\frac{2[2+(n-1)(2c-1)]}{\beta(n+1)^3}\sqrt{d(x)} - x\frac{\partial x}{\partial n}\frac{(n-1)[2+c(n-1)]}{\beta(n+1)^2}/\sqrt{d(x)}\right\},$$

where $\frac{\partial x}{\partial n} = -\frac{2(1-c/\beta)}{(n+1)^2} < 0$. Consider $n > 3$ and $c < c^0$. Then $\{\cdot\}$, multiplied by $\frac{\beta(n+1)^3}{2}\sqrt{d(x)} > 0$, becomes $[2+(n-1)(2c-1)]d(x) + [2+c(n-1)]\frac{n-1}{n+1}(1-c/\beta)x$, where $\frac{n-1}{n+1}(1-c/\beta) = 1-x$. Collecting the terms with $x$, we have $x\{x[(n-1)(2c-1) - c(n-1)] + 2 + c(n-1)\} - c^2[2+(n-1)(2c-1)]/\beta$, where the last term is positive for $n > 3$ and $c < c^0$, and the bracket $\{\cdot\}$ in the first term is $\{\cdot\} = (n-1)[x(c-1)+c] + 2$ is minimized at $c = 0$. Namely, $\{\cdot\}|_{c=0} = (n-1)\left[-1 + \frac{n-1}{n+1}\right] + 2 = 2\left[1 - \frac{n-1}{n+1}\right] > 0$. Hence, since $\frac{\partial P_{12}}{\partial c}$ is increasing in $n$, it remains to show that it is positive at $n = 1$. Indeed, $\frac{\partial P_{12}}{\partial c}|_{n=1} = \frac{1}{2}\left(1 - \frac{c}{\beta}\sqrt{\frac{1-\beta}{1-c^2/\beta}}\right)$, and since $\frac{c^2}{\beta} < \beta$, leading to $\sqrt{\frac{1-\beta}{1-c^2/\beta}} < 1$, we obtain $\frac{\partial P_{12}}{\partial c}|_{n=1} > \frac{1}{2}(1-c/\beta) > 0$.

Using (19) for $P_{22}$, we have

$$\frac{\partial P_{22}}{\partial c} = \frac{1}{4n+\beta(n-1)^2}\left[n(n+1) + \frac{n(1-\beta)(n(1-2c/\beta)-1)}{\sqrt{(1-\beta)(nc(1-c/\beta)+1-c)}}\right],$$

where $[\cdot] = n(n+1) + n(n(1-2c/\beta)-1)\sqrt{\frac{1-\beta}{nc(1-c/\beta)+1-c}}$ and $\sqrt{\frac{1-\beta}{nc(1-c/\beta)+1-c}} < 1$ since the LHS is decreasing in $n$ and less than one for $n = 1$. Then $\frac{\partial P_{22}}{\partial c} > 0$ if $n(n+1) + n(n(1-2c/\beta)-1) > 0$. The last inequality is equivalent to $n^2(1+1-2c/\beta) > 0$, which always holds.

Consider $\beta < 1$ since $CB_1|_{\beta=1} = CB_2|_{\beta=1} = 1$ and both $P_{12}$ and $P_{22}$ are irrelevant — MFC-equilibria are determined either by condition (1.1) or (2.1) of Theorem 1. Since $P_{12}$ and $P_{22}$ are the larger roots of (5), $\frac{\partial P_{12}}{\partial \beta}$ and $\frac{\partial P_{22}}{\partial \beta}$ can be found from the differentiation of (5):

$$2p_1\frac{\partial p_1}{\partial \beta} - (x+c)\frac{\partial p_1}{\partial \beta} - \frac{\partial x}{\partial \beta}p_1 + \frac{1}{4}\left[(x+c/\beta)^2 + 2\beta(x+c/\beta)(-c/\beta^2)\right] = 0,$$

which can be written as $\frac{\partial p_1}{\partial \beta}[2p_1-(x+c)]=\frac{\partial x}{\partial \beta}p_1-\frac{1}{4}\left(x^2-c^2/\beta^2\right)$. The RHS is negative since both $x_{12}$ and $x_{22}$ are greater than $\frac{c}{\beta}$, $\frac{\partial x_{22}}{\partial \beta}=0$, and $\frac{\partial x_{12}}{\partial \beta}=-\frac{n-1}{n+1}c/\beta^2\leq 0$. The bracket $[\cdot]$ in the LHS is positive since, by Lemma 2, $2p_1>x+\frac{c}{\beta}>x+c$ for $\beta<1$. Therefore, $\frac{\partial P_{22}}{\partial \beta}=-\frac{1}{4}\frac{\left(x_{22}^2-c^2/\beta^2\right)}{2P_{22}-(x_{22}+c)}<0$ and $\frac{\partial P_{12}}{\partial \beta}=-\left[\frac{n-1}{n+1}cP_{12}/\beta^2+\frac{1}{4}\left(x_{12}^2-c^2/\beta^2\right)\right]/\ [2P_{12}-(x_{12}+c)]<0$.

## 7.4 Proof of Proposition 1 ($p_1$-bounds of MFC and NA)

*Part (1)* follows from $\max_n P_2^N=\frac{c}{\beta}\leq\min\{P_{21},P_{22}\}$. The inequality is shown in Theorem 1.

*Part (2).* $P_1^N\geq P_{11}$ is $1-\frac{n}{n+1}\rho(\beta-c)\geq 1-\frac{n-1+\rho\beta}{n+1}(1-c/\beta)\Leftrightarrow n\rho\beta\leq n-1+\rho\beta\Leftrightarrow 1-\rho\beta\leq n(1-\rho\beta)$, which is strict for $n>1$. When $n=1$, we have $P_1^N\equiv P_{11}$, and, for $\frac{c}{\beta}>CB_1$, the $p_1$-boundary of M1 dominates $P_1^N$ since $P_{12}>P_{11}$. For $n>1$, $P_1^N\geq P_{12}$ for any $\rho\in[0,1)$ if $\inf_\rho P_1^N\geq P_{12}$, which is $1-\frac{n}{n+1}(\beta-c)\geq\frac{1}{2}\left[x+c+\sqrt{(1-\beta)(x^2-c^2/\beta)}\right]$. Substitution for $x=1-\frac{n-1}{n+1}(1-c/\beta)$ leads to $\sqrt{\cdot}-1+c-\frac{(\beta-c)[n(1-2\beta)-1]}{\beta(n+1)}\leq 0$, where $\sqrt{\cdot}$ and the last term are decreasing in $n$. Indeed, considering $n$ as a continuous variable, the derivative of the last term w.r.t. $n$ is $\frac{-1}{\beta(n+1)^2}\{(\beta-c)(1-2\beta)\beta(n+1)-\beta(\beta-c)[n(1-2\beta)-1]\}=\frac{-(\beta-c)}{(n+1)^2}\{(1-2\beta)(n+1)-n(1-2\beta)+1\}=\frac{-2(\beta-c)}{(n+1)^2}(1-\beta)\leq 0$. Therefore, inequality $\inf_\rho P_1^N\geq P_{12}$ holds if it holds for $n=2$, which is

$$\sqrt{(1-\beta)(x^2-c^2/\beta)}\leq 1-c+(\beta-c)(1-4\beta)/(3\beta), \tag{21}$$

where $x|_{n=2}=(2\beta+c)/(3\beta)$ and $\left(x^2-c^2/\beta\right)|_{n=2}=[(2\beta+c)^2-9\beta c^2]/(3\beta)^2$. Then (21), multiplied by $3\beta$, can be written as $\sqrt{(1-\beta)[\cdot]}\leq 4\beta+\beta c-4\beta^2-c$ or $\sqrt{(1-\beta)[\cdot]}\leq(1-\beta)(4\beta-c)$, which holds as equality for $\beta=1$. Consider $\beta<1$. Then (21), squared, is $(2\beta+c)^2-9\beta c^2\leq(1-\beta)(4\beta-c)^2$, which, divided by $\beta$, can be written as $(4\beta-c)^2-12\beta+12c-9c^2\leq 0$ or $12\beta^2-12c^2+4(\beta-c)^2-12(\beta-c)\leq 0\Leftrightarrow(\beta-c)(\beta+c-3)+(\beta-c)^2\Leftrightarrow 2\beta-3\leq 0$, which holds strictly.

## 7.5 Proof of Proposition 2 (NA4-profit less than NA3)

*Part (1).* NA4 exists only if $p_2^*=s$ or $Y^*>1-\frac{s}{\beta}$, which can be written as $c-s<\frac{n-1}{n}\frac{\beta(p_1-s)}{\beta-s}(1-v^*)$. The RHS is maximal at $\rho=0$ ($v^*=p_1$ is minimal) and $p_1=\frac{1+s}{2}$ yielding $c-s<\frac{n-1}{n}\frac{\beta(1-s)^2}{4(\beta-s)}$. NA4 can also exist (profit is positive) only if there are first-period sales, i.e., $v^{*,NA4}<1\Leftrightarrow p_1-\rho s<1-\rho\beta\Leftrightarrow p_1<1-\rho(\beta-s)=P_4^N$, which is less than $P_1^N=1-\frac{n}{n+1}\rho(\beta-c)$. These bounds can be written as $\rho<\frac{1-p_1}{\beta-s}=\rho_4^N$ and $\rho<\frac{n+1}{n}\frac{1-p_1}{\beta-c}=\rho_1^{N\cdot}$ respectively.

*Part (2)* follows from the observations: (a) the second-period sales under NA4 are always at loss with $p_2^{*,NA4}=s<p_2^{*,NA3}$ while $Y^{*,NA4}>Y^{*,NA3}$; and (b) since, by (2) and rationality of expectations in equilibrium, $v^*$ is nonincreasing in $p_2^*$, we have $v^{*,NA4}\geq v^{*,NA3}$ implying that the first-period profit is less under NA4. A similar argument leads to $r^{*,NA4}<\frac{1}{n}(p_1-c)(1-p_1)$ for any inputs where NA4 exists. The RHS of this inequality is the first-period profit in NA4 for $\rho=0$, coinciding with the expression for profit under NA2 or M2. Since $v^{*,NA4}$ is increasing in $\rho$, the first-period profit in NA4 is decreasing $\rho$. Consequently, because the second-period sales are at loss, the equilibrium profit is strictly less than the RHS.

## 7.6 Proposition 9 (RESE N)

The conditions of N-equilibria existence result from comparison of profit in various options, which leads to quadratic inequalities in $z \triangleq 1 - \frac{n-1}{n} Y_0^* - c/\beta$. When $z$ is non-negative, one can interpret it as inventory of a reseller deviating from a symmetric equilibrium that makes a clearance price equal to the unit cost. Thus, if $z \leq 0$, the second-period price, by (3), cannot be above cost regardless of a one-reseller deviation from a symmetric strategy profile. Several thresholds on $z$ result from quadratic inequalities and, below, we denote $z_1 \triangleq \frac{2}{\beta}\left[p_1 - c - \sqrt{(p_1-c)p_1(1-\beta)}\right]$ and

$$\tilde{z}_1 \triangleq \begin{cases} z_1, & \text{if } n=1, \text{ or } \rho = 0, \text{ or } \beta = 1, \text{or } p_1 = c/\beta, \\ \frac{(p_1-c)(2-\rho\beta)}{\beta(1-\rho\beta)}\left[1 - \sqrt{1 - \frac{4(\beta p_1 - c)(1-\rho\beta)}{(p_1-c)(2-\rho\beta)^2}}\right] & \text{otherwise,} \end{cases}$$

as the smaller roots of the corresponding equations, and $\hat{z}_2 \triangleq \hat{z}_2(\beta, c, n, s)$ – as the larger one.

**Proposition 9.** *If MFC is available, N-equilibria with the following structure exist iff the respective conditions apply. The set of necessary and sufficient conditions is given in each case by the combination of the conditions in the corresponding NA-equilibrium and the additional conditions listed below.*

**N1** *requires no additional conditions for* $n > 1$ *and, for* $n = 1$, *the condition* $p_1 \geq P_2$; $\alpha^*(1) = 1$.

**N2** *requires no additional conditions and* $\alpha^*(1) = 0$.

**N3** *under the following additional conditions, where* $Y^*$ *is the larger root of the quadratic equation*

$$Y^2 - \frac{(\beta-c)n(1-\rho\beta) + \beta(1-p_1)n - (p_1-\beta)\rho\beta(n-1)}{\beta(n+1-\rho\beta)} Y - \frac{(p_1-\beta)(1-p_1)(n-1)}{\beta(n+1-\rho\beta)} = 0. \tag{22}$$

*and* $r^{*,N3}$ *is defined by part NA3 of Theorem 2:*

*(3.1) inequality* $r^{*,N3} \geq \breve{r}_1^i = (p_1 - c)\left(1 - p_1 - \frac{n-1}{n}Y^*\right)$ *holds and either* $p_1 \leq \frac{c}{\beta}$ *and* $\frac{n-1}{n}Y^* \leq 1 - p_1$, *or* $p_1 > \frac{c}{\beta}$ *and* $\frac{n-1}{n}Y^* \leq 1 - \frac{c}{\beta} - z_1$ *with the corresponding rational expectations* $\alpha^*(1) = 0$; *or*

*(3.2) either* $p_1 \leq \frac{c}{\beta}$ *and* $\frac{n-1}{n}Y^* > 1 - p_1$, *or* $p_1 > \frac{c}{\beta}$ *and either* $\frac{n-1}{n}Y^* \geq 1 - \frac{c}{\beta}$, *or* $1 - \frac{c}{\beta} - \min\left\{\tilde{z}_1, 2\sqrt{r^{*,N3}/\beta}\right\} \leq \frac{n-1}{n}Y^* < 1 - \frac{c}{\beta}$ *with the corresponding rational expectations* $\alpha^*(1) = 1$.

**N4** *under the following additional conditions, where* $Y^* = \frac{n-1}{n}\frac{p_1-s}{c-s}\left(1 - \frac{p_1-\rho s}{1-\rho\beta}\right)$:

*(4.1) inequality* $(1-p_1)\left[1 + \frac{c-s}{(n-1)^2(p_1-c)}\right]^{-1} \leq \frac{n-1}{n}Y^*$ $(r^{*,N4} \geq \breve{r}_1^i)$ *holds and either* $p_1 \leq \frac{c}{\beta}$ *and* $\frac{n-1}{n}Y^* \leq 1 - p_1$, *or* $p_1 > \frac{c}{\beta}$ *and* $\frac{n-1}{n}Y^* \leq 1 - \frac{c}{\beta} - z_1$ *with the corresponding rational expectations* $\alpha^*(1) = 0$; *or*

*(4.2) either* $p_1 \leq \frac{c}{\beta}$ *and* $\frac{n-1}{n}Y^* > 1 - p_1$, *or* $p_1 > \frac{c}{\beta}$ *and either* $\frac{n-1}{n}Y^* \geq 1 - \frac{c}{\beta}$, *or* $1 - \frac{c}{\beta} - \min\{\hat{z}_2, \tilde{z}_1\} \leq \frac{n-1}{n}Y^* < 1 - \frac{c}{\beta}$ *with the corresponding rational expectations* $\alpha^*(1) = 1$.

Equation (22) is derived in the proof of Theorem 2. This proof can be found in Bazhanov, Levin, and Nediak (2015).

*Proof* of Proposition 9 is based on the following lemma, which uses the notations $z_0 \triangleq 2\,(p_1 - c/\beta)$.

**Lemma 4.** *Consider reseller $i$ using MFC and a profile of competitor strategies not using MFC with combined inventory $Y_0 \geq 0$. There exist optimal inventory of reseller $i$ and the corresponding rational expectations of customers in one of the forms given below. No other forms can exist.*

*(a)* $\breve{y}_1^i = 1 - p_1 - Y_0 = z - \frac{z_0}{2}$ *(no second-period sales) with positive profit* $\breve{r}_1^i = \breve{y}_1^i(p_1 - c)$ *and rational expectations* $\bar{\alpha} = 0$ *iff either of the two conditions holds: (a.1)* $p_1 \leq \frac{c}{\beta}$ *and* $z > \frac{z_0}{2}$ *(*$Y_0 < 1 - p_1 \Rightarrow p_1 < 1$*), or (a.2)* $p_1 > \frac{c}{\beta}$ *and* $z \geq z_1$ *(*$\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$*);*

*(b)* $\tilde{y}_1^i = \frac{z}{2}$ *(with second-period sales) with positive profit* $\tilde{r}_1^i = \beta(\tilde{y}_1^i)^2$ *and rational expectations* $\bar{\alpha} = 1, \bar{p}_2 = p_2 = \beta(1 - Y_0 - \tilde{y}_1^i)$ *iff* $p_1 > \frac{c}{\beta}, z > 0$ *and either of the two conditions holds: (b.1)* $p_1 = 1$*, or (b.2)* $p_1 < 1$*, and* $z \leq \tilde{z}_1$ *(*$\breve{r}_1^i|_{\bar{\alpha}=1} \leq \tilde{r}_1^i$*) for* $Y_0 > 0$ *or* $z \leq z_1$ *for* $Y_0 = 0$*;*

*(c) the optimal inventory and profit are zero iff* $z \leq 0 \wedge \frac{z_0}{2}$*. Rational expectations are* $\bar{\alpha} = 0$ *if* $z = \frac{z_0}{2}$ *and* $\bar{\alpha} = 1, \bar{p}_2 = \beta(1 - Y_0)$ *if* $z < \frac{z_0}{2}$*.*

*Moreover, under the conditions of part (a.2),* $\frac{z_0}{2} < z_1 \leq z_0$*; part (b),* $c < p_2 < \beta p_1$*; part (b.2),* $\tilde{z}_1 \leq z_0$*,* $p_1 > \frac{c}{\beta} + \frac{z}{2}$*, and* $v_0^{\min}$ *is not decreasing in* $\rho$*; under the conditions of both (a.2) and (b.2),* $z_1 \leq \tilde{z}_1$ *with strict inequality if* $\bar{\alpha} = 1, \beta < 1, p_1 > \frac{c}{\beta}$*, and* $\rho > 0$*; the condition of part (c) never holds for* $Y_0 = 0$ *(a monopoly). The profit value* $\tilde{r}_1^i$ *and the corresponding* $\tilde{y}_1^i$ *do not depend on the specific values of rational expectations and are identical in parts (a) and (b). The general expression for the optimal inventory of a reseller who limits the sales to the first period is* $\breve{y}_1^i = 1 - v_0^{\min} - Y_0$*.*

The three parts of lemma correspond to the following mutually exclusive cases. Parts (a) and (b) describe positive optimal inventory decisions corresponding to rational customer expectations of sales, respectively, only in the first period and in both periods. Part (c) describes a trivial (zero) optimal inventory decision and corresponding rational expectations. Necessary and sufficient conditions of parts (a) and (b) allow an overlap of input parameter regions when $p_1 > c/\beta$ and $z_1 \leq z \leq \tilde{z}_1$. In this case, a potential form of reseller $i$ decision, (a) or (b), depends on customer expectations, i.e., both $\bar{\alpha} = 0$ and $\bar{\alpha} = 1$ can be rational.

When MFC is possible, a RESE N with the corresponding expectations for a one-reseller deviations into MFC, $\alpha^*(1)$, exists if and only if either of the two conditions hold: (i) both possible deviations of a reseller $i$ into MFC are trivial (part (c) of the lemma) or (ii) at least one of the deviations is not trivial (parts (a) or (b) of the lemma) and optimal deviator's profit does not exceed the equilibrium profit under the corresponding RESE NA. Since $Y_0 = \frac{n-1}{n}Y^*$ and $z = 1 - \frac{n-1}{n}Y^* - c/\beta$, case (i) is characterized by $\frac{n-1}{n}Y^* \geq (1 - p_1) \vee (1 - c/\beta)$. Rational expectations under deviation are $\alpha^*(1) = 0$, when $\frac{n-1}{n}Y^* \leq 1 - p_1$, and $\alpha^*(1) = 1$, otherwise.

*N1.* By part NA1 of Theorem 2, $r^{*,N1} = \frac{(\beta-c)^2}{(n+1)^2\beta}$ and $Y^* = \frac{n}{n+1}(1 - c/\beta)$, yielding $Y_0 = \frac{n-1}{n+1}(1 - c/\beta)$. Since $p_1 > c/\beta$, this situation is covered by case (ii). By Lemma 4, $\tilde{y}_1^i = \frac{1}{2}(1 - Y_0 - c/\beta) = \frac{z}{2} = \frac{1}{n+1}(1 - c/\beta)$, which is strictly positive (i.e., $z > 0$). The resulting total inventory would be the same as in NA1, therefore, the rational expectations under deviation into $\tilde{y}_1^i$ would lead to $v_0^{\min} = 1$ for $n > 1$. Under this scenario $\breve{y}_1^i = 1 - v_0^{\min} - Y_0 \leq 0$ is infeasible. Since the optimal deviator's profit $\tilde{r}_1^i = \beta(\tilde{y}_1^i)^2 = \frac{(\beta-c)^2}{(n+1)^2\beta}$ coincides with $r^{*,N1}$, N1 with $\alpha^*(1) = 1$ exists without any additional conditions. For $n = 1$, $\breve{y}_1^i = 1 - p_1 \geq 0$ is feasible, and N1, by part (b) of Lemma 4 exists if and only if $z \leq z_1$ ($Y_0 = 0$). This inequality is $1 - c/\beta \leq \frac{2}{\beta}\left[p_1 - c - \sqrt{(p_1 - c)p_1(1 - \beta)}\right] \Leftrightarrow \sqrt{(p_1 - c)p_1(1 - \beta)} \leq p_1 - \frac{\beta+c}{2} \Leftrightarrow p_1^2 - p_1(1+c) + \frac{1}{4\beta}(\beta+c)^2 \geq 0$. The smaller root of the corresponding equation is irrelevant since $\frac{1+c}{2}$ is less than the $p_1-$lower bound in NA1: $\frac{1+c}{2} < 1 - \frac{\rho}{2}(\beta - c)$. This inequality holds for any $\rho < 1$ since the RHS is minimized at $\rho \to 1$ and $1+c \le 2-\beta+c$. Therefore, the additional condition of existence of N1 for $n=1$ is $p_1 \ge (p_1)_2$, where $(p_1)_2$ is the larger root of the equation, corresponding to the quadratic inequality above, and $(p_1)_2 = \frac{1}{2}\left[1+c+\sqrt{(1+c)^2-\frac{1}{\beta}(\beta+c)^2}\right]$. The expression under the square root is $1+2c+c^2-\beta-2c-c^2/\beta = (1-\beta)\left(1-c^2/\beta\right)$.

N1 cannot exist with $\alpha^*(1)=0$ because the rationality of expectations would imply optimality of $\breve{y}_1^i$ (part (a) of Lemma 4). Under the conditions of the lemma, this means that $r^{*,N1} = \tilde{r}_1^i$ is dominated by $\breve{r}_1^i$.

*N2.* The equilibrium profit in NA2 is not dominated by a possible deviation into MFC with $y_1^i = \breve{y}_1^i$ because the prospective deviator's profit coincides with the equilibrium one: $\breve{r}_1^i\big|_{v_0^{\min}=p_1} = r^{*,N2} = \frac{1}{n}(p_1-c)(1-p_1)$. The equilibrium profit is also not dominated by a possible deviation into MFC with $y_1^i = \tilde{y}_1^i$ because, by Lemma 4, a non-trivial $\tilde{y}_1^i$ is optimal only if $z \le \tilde{z}_1 \le 2\left(p_1 - c/\beta\right)$, which is incompatible with the condition $p_1 \le \frac{nc}{n-1+\beta}$ of the existence of NA2. Indeed, in this case, $Y_0 = \frac{n-1}{n}(1-p_1)$ implying $z = 1-Y_0-c/\beta = \frac{1}{n}+\frac{n-1}{n}p_1 - c/\beta$. Inequality $z \le 2\left(p_1-c/\beta\right)$ yields the following lower bound on $p_1$ :

$$\frac{1}{n}+\frac{n-1}{n}p_1 - c/\beta \le 2\left(p_1-c/\beta\right) \Leftrightarrow \frac{n+1}{n}p_1 \ge c/\beta + \frac{1}{n} \Leftrightarrow p_1 \ge \frac{nc+\beta}{(n+1)\beta},$$

which exceeds the upper bound given in NA2:

$$\frac{nc+\beta}{(n+1)\beta} > \frac{nc}{n-1+\beta} \Leftrightarrow (nc+\beta)(n-1+\beta) > nc(n+1)\beta \Leftrightarrow n^2c(1-\beta)+n(\beta-c)-\beta(1-\beta) > 0.$$

The last inequality always holds because the LHS is increasing in $n$ and positive for $n=1$ : $c(1-\beta)+\beta-c-\beta(1-\beta) = (\beta-c)-(\beta-c)(1-\beta) = \beta(\beta-c) > 0$.

*N3 and N4.* By part (c) of Lemma 4, both possible deviations into MFC are trivial if and only if $z \le 0 \wedge \frac{z_0}{2}$. Therefore, N3 and N4 exist under this condition, part of which, $z = \frac{z_0}{2} \le 0$, corresponding to $\alpha^*(1)=0$, is included into the conditions of parts (3.1) and (4.1), and the remaining part, corresponding to $\alpha^*(1)=1$, — into the conditions of parts (3.2) and (4.2).

By part (a) of Lemma 4, a pair of optimal deviator's inventory $\breve{y}_1^i$ with the corresponding $\alpha^*(1)=0$ exists if and only if either $p_1 \le c/\beta$ and $\frac{n-1}{n}Y^* < 1-p_1$ ($z > \frac{z_0}{2}$), or $p_1 > c/\beta$ and $z \ge z_1 \Leftrightarrow \frac{n-1}{n}Y^* \le 1-c/\beta-z_1$. Inequality $z \ge z_1$ implies, by Lemma 4, $z > \frac{z_0}{2}$. In this case, N3 and 4 exist if the corresponding equilibrium profit is not dominated by the profit of potential deviator: $r^{*,N3} \ge \breve{r}_1^i = \left(1-p_1-\frac{n-1}{n}Y^*\right)(p_1-c)$ for N3 and $r^{*,N4} \ge \breve{r}_1^i$ for N4. By Theorem 3, $Y^* = \frac{n-1}{n}\frac{p_1-s}{c-s}(1-v^*)$ and $r^{*,N4} = \frac{c-s}{n(n-1)}Y^*$; therefore inequality $\breve{r}_1^i \le r^{*,N4}$ becomes

$$1-p_1-\frac{n-1}{n}Y^* \le \frac{c-s}{n(n-1)(p_1-c)}Y^* \Leftrightarrow (1-p_1)\left[1+\frac{c-s}{(n-1)^2(p_1-c)}\right]^{-1} \le \frac{n-1}{n}Y^*. \quad (23)$$

The combination of the conditions $p_1 \le \frac{c}{\beta}$ ($z_0 \le 0$) and $\frac{n-1}{n}Y^* = 1-p_1$ ($z = \frac{z_0}{2}$), which is the part of the condition $z \le 0 \wedge \frac{z_0}{2}$ that corresponds to $\alpha^*(1)=0$, can be trivially included into the conditions of parts (3.1) and (4.1) because $\breve{r}_1^i = 0$ in this case while $r^{*,N3}$ and $r^{*,N4}$ are strictly positive.

When $\bar{\alpha}(1)=1$, the conditions of the existence of N3 and N4 do not include the comparison of the equilibrium profit with the profit of a deviator if both possible deviations into MFC are trivial, which is covered by the remaining conditions of part (c), Lemma 4. Namely, if $z \le 0$ ($\frac{n-1}{n}Y^* \ge 1-\frac{c}{\beta}$) and $z < \frac{z_0}{2}$ ($\frac{n-1}{n}Y^* > 1-p_1$). The case $p_1 = \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* = 1-\frac{c}{\beta}$ is included into parts (3.1) and (4.1). Therefore, the remaining combination of the conditions, yielding the existence of N3 and 4 with trivial deviations into MFC and $\bar{\alpha}(1) = 1$, is $p_1 \le c/\beta$ and $\frac{n-1}{n}Y^* > 1-p_1$ or $p_1 > c/\beta$ and $\frac{n-1}{n}Y^* \ge 1 - c/\beta$.

Recall that $p_1 < 1$ under the conditions of both NA3 and NA4. Therefore, by part (b.2) of Lemma 4, the pair of positive optimal deviator's inventory $\tilde{y}_1^i = \frac{z}{2}$ with profit $\tilde{r}_1^i = \frac{\beta}{4}\left(1 - c/\beta - \frac{n-1}{n}Y^*\right)^2$ and $\bar{\alpha}(1) = 1$ exists if and only if $p_1 > c/\beta$ and $0 < z \le \tilde{z}_1$, where the last condition is equivalent to $1 - c/\beta - \tilde{z}_1 \le \frac{n-1}{n}Y^* < 1 - c/\beta$. Then N3 and 4 exist if the corresponding equilibrium profit is not dominated by the profit of the deviator: $r^{*,N3} \ge \tilde{r}_1^i = \frac{\beta}{4}z^2$, which can be written as $z \le 2\sqrt{r^{*,N3}/\beta}$, and $r^{*,N4} \ge \tilde{r}_1^i$. The last inequality is $\frac{c-s}{n(n-1)}Y^* - \frac{\beta}{4}\left(1 - c/\beta - \frac{n-1}{n}Y^*\right)^2 \ge 0$ or, in terms of $z$, $\frac{\beta}{4}z^2 - \frac{c-s}{(n-1)^2}\left(1 - z - c/\beta\right) \le 0 \Leftrightarrow \frac{\beta}{4}z^2 + \frac{c-s}{(n-1)^2}z - \frac{c-s}{(n-1)^2}\left(1 - c/\beta\right) \le 0$, which is strict at $z = 0$. Therefore, the condition of N4 existence in this case is $z \le \hat{z}_2$, where $\hat{z}_2$ is the larger root of the corresponding equation, namely,

$$\hat{z}_2 = \frac{2}{\beta}\left[-\frac{c-s}{(n-1)^2} + \sqrt{\frac{(c-s)^2}{(n-1)^4} + \frac{(c-s)(\beta-c)}{(n-1)^2}}\right] = \frac{2(c-s)}{\beta(n-1)^2}\left[\sqrt{1 + \frac{(n-1)^2(\beta-c)}{c-s}} - 1\right].$$

The combination of the inequalities $z \le \hat{z}_2$ and $z \le \tilde{z}_1$ yields $1 - c/\beta - \tilde{z}_1 \wedge 2\sqrt{r^{*,N3}/\beta} \le \frac{n-1}{n}Y^*$.

### 7.6.1 Analysis of additional conditions

The additional conditions of N-equilibria existence nontrivially restrict the parameter regions corresponding to NA3 and NA4, i.e., the conditions may hold or not hold depending on the market situation. For example, conditions of parts (3.1) and (4.1) hold for $\rho = 0$. Indeed, the profit of a deviator from NA3 (Theorem 2) or NA4 (Theorem 3) to no-MFC with sales only in the first period is $(p_1 - c)\left(1 - v^*(\emptyset, I) - \frac{n-1}{n}Y^*\right)$, which coincides with $\breve{r}_1^i$ in both parts if $\rho = 0$. Since equilibrium profits $r^{*,N3}$ and $r^{*,N4}$ are not dominated by deviator profit, inequalities $r^{*,N3} \ge \breve{r}_1^i$ and $r^{*,N4} \ge \breve{r}_1^i$ hold under the corresponding NA if $\rho = 0$.

Combination of $p_1 \le \frac{c}{\beta}$ with $\frac{n-1}{n}Y^* \le 1 - p_1$ in parts (3.1) and (4.1) hold for $p_1$ near the lower bound for both NA3 and NA4 (which is below $\frac{c}{\beta}$ for $n > 1$) since $Y^*$, by continuity, approaches $1 - p_1$. On the other hand, the following lemma illustrates that for large $\rho$ and small $n$, inequality $r^{*,N3} \ge \breve{r}_1^i$ may not hold.

**Lemma 5.** *For $n = 1, c \to 0, \beta \to 1$, and $\rho = \left(1 - \sqrt{1-\beta}\right)/\beta \to 1$, conditions $p_1 > \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* \le 1 - \frac{c}{\beta} - z_1$ of part (3.1) of Proposition 9 hold, while $r^{*,N3} \ge \breve{r}_1^i$ does not hold.*

By parts (3.2) and (4.2), equilibria N3 and N4 exist under the combination of conditions $p_1 \le \frac{c}{\beta}$ and $\frac{n-1}{n}Y^* > 1 - p_1$. The first condition implies $p_2 \le c$, preventing deviation into MFC with sales in both periods, and the second one prevents any deviations with sales only in the first period. The following lemma shows that this combination has a non-empty intersection with the input areas of NA3 and NA4.

**Lemma 6.** *Under NA3 or 4, there exist $\beta < 1$ and $N > 1$ such that the condition $\frac{nc}{\beta+n-1} \vee 1 - \frac{n-1}{n}Y^* < p_1 \le \frac{c}{\beta}$ may hold for any $n \ge N$.*

When condition $1 - \frac{n-1}{n}Y^* < p_1 \le \frac{c}{\beta}$ does not hold, the following lemma provides an example, where inequality $r^{*,N3} \ge \tilde{r}_1^i$ of part (3.2) is satisfied.

**Lemma 7.** *If $p_1 > \frac{1+c}{2}, n = 1, \rho = 0$, and $\beta = 1$, inequalities $p_1 > \frac{c}{\beta}, 1 - \frac{c}{\beta} - \tilde{z}_1 \le \frac{n-1}{n}Y^* < 1 - \frac{c}{\beta}$, and $r^{*,N3} \ge \tilde{r}_1^i$ of part (3.2) hold.*

The opportunity to deviate into MFC is stipulated by the combined inventory of other resellers, i.e., $Y^{-i} = \frac{n-1}{n} Y^*$. Namely, if $Y^{-i}$ is such that, regardless of the inventory of reseller $i$, there are sales in the second period, or these sales are such that $p_2 \leq c$, the corresponding non-trivial forms of deviation into MFC cannot exist. Combination of these conditions for $Y^{-i}$ with the necessary conditions of existence of some NA-equilibria yield simple sufficient conditions of existence of N-equilibria under the conditions of NA.

**Lemma 8.** *If MFC is available, N4 exists under the conditions of NA4 if* $n \geq \frac{\beta-s}{p_1\beta-s} \vee \frac{\beta-s}{c-s}$.

## 7.7 Proof of Proposition 4 (MFC inventory is not greater than no-MFC)

*Part (1)* follows directly from Theorem 1, and parts NA1 and NA2 of Theorem 2.

*Part (2)* follows from the facts that (a) under NA3, $Y^* > \frac{n}{n+1}(1 - c/\beta) \vee (1 - p_1)$ (Theorem 2), where, by Theorem 1, $\frac{n}{n+1}(1 - c/\beta)$ is the total equilibrium inventory under M1 and $1 - p_1$ — under M2; and (b) under N4, $p_2 = s$, which is always less than $p_2$ under N3. Therefore, by formula (3) for $p_2$, the total inventory under N4 is always greater than under N3, which, by the argument above, is always greater than under M1 or M2.

## 7.8 Proof of Proposition 3 (profits of M1, M2 and N(A)3, N(A)4)

*Part (1.1).* The RHS of inequality $p_1 > 1 - \frac{n}{n+1}(\beta - c)$ is the $p_1$-boundary between N(A)3 and N(A)1 for $\rho \to 1$. Therefore, if this inequality holds under N(A)3 ($p_1 < 1$), there exists $\rho_1^N = \frac{n+1}{n}\frac{1-p_1}{\beta-c} \in (0, 1)$ such that $p_1 = 1 - \frac{n}{n+1}\rho_1^N(\beta - c)$ and, for the same given inputs except $\rho$, N(A) can be realized as N(A)3 for $\rho < \rho_1^N$ and as N(A)1 for $\rho \geq \rho_1^N$. By Proposition 4 in Bazhanov, Levin, and Nediak (2015), profit $r^{*,N3}$ is decreasing in $\rho$ for all $p_1 \geq \beta - \frac{n}{2(n+1)}(\beta - c)$, and, by continuity of the profit without MFC, $r^{*,N3} = r^{*,N1}$ at $\rho = \rho_1^N$. The last inequality is implied by $p_1 > 1 - \frac{n}{n+1}(\beta - c)$ if $1 - \frac{n}{n+1}(\beta - c) \geq \beta - \frac{n}{2(n+1)}(\beta - c)$, which is equivalent to $\frac{2(n+1)}{n}(1-\beta) \geq \beta - c$ or $c \geq 3\beta - 2\left(1 + \frac{1-\beta}{n}\right)$. Hence, since $r^{*,N1} \equiv r^{*,M1}$, and $r^{*,M1}$ is constant in $\rho$, we have, under the conditions of part (1.1), that $r^{*,M1} < r^{*,N3}$ for any $\rho < \rho_1^N$. The lower bound on $c$ above holds for any $c$ and $n$ if $\beta \leq \frac{2}{3}$ and never holds for $\beta > \frac{4+c}{5}$. Indeed, the lower bound on $c$ yields an upper bound on $n$, which is less that one if $\beta > \frac{4+c}{5}$. The lower bound on $p_1$ can be written as a lower bound on $n$: $n > \frac{1-p_1}{\beta-c-(1-p_1)}$.

*Part (1.2).* By Theorems 1 and 3, inequality $r^{*,M1} > r^{*,N4}$ is equivalent to

$$\frac{n^2(\beta - c)^2}{(n+1)^2\beta} > (p_1 - s)\Big(1 - \frac{p_1 - \rho s}{1 - \rho\beta}\Big) \Leftrightarrow \frac{n}{n+1} > \sqrt{\frac{\beta(p_1 - s)(1 - p_1 - \rho(\beta - s))}{(\beta - c)^2(1 - \rho\beta)}} = \frac{1}{w},$$

where the expression under the root is always positive. The last inequality implies that $r^{*,M1} > r^{*,N4}$ can hold only if $w > 1$ and holds for any $n \geq 2$ if $w > \frac{3}{2}$. If $w \in (1, \frac{3}{2}]$, inequality $\frac{n}{n+1} > \frac{1}{w}$, which is equivalent to $r^{*,M1} > r^{*,N4}$, can be written as $w > 1 + \frac{1}{n}$ or $n > \frac{1}{w-1}$. By Theorem 3, fraction $\frac{1-p_1-\rho(\beta-s)}{1-\rho\beta}$ is $1 - v^* > 0$, where $v^*$ increases in $\rho$ implying that $w$ increases in $\rho$.

*Part (2.1).* By Proposition 10 in Bazhanov, Levin, and Nediak (2015), $nr^{*,N3}$ decreases in $n$ while $nr^{*,M2}$ is constant. Therefore, $r^{*,M2} \geq r^{*,N3}$ for any $n \geq 1$ and any other inputs that are in the area where M2 overlaps with N(A)3 for $n = 1$ if, in this area, $r^{*,M2} \geq r^{*,N3}$ for $n = 1$. Indeed, for $n = 1$, $r^{*,N3}$ coincides with the profit of the deviator to no-MFC with sales in both periods, and, in the area of M2 existence, this profit does not exceed the equilibrium profit $r^{*,M2}$. By the proof of part M2 of Theorem 1 for $n = 1$, inequality $r^{*,M2} \geq r^{*,N3}$ is equivalent to $p_1 \leq \frac{c}{\beta}\frac{(1-\rho\beta)^2}{1-2\rho+\beta\rho^2} = P_{21}$, where $P_{21}$ does not depend on $n$. Therefore, first, inequality $r^{*,M2} \geq r^{*,N3}$ is strict for $n = 1$ and $p_1 < P_{21}$; second, since $nr^{*,N3}$ is decreasing in $n$ while $nr^{*,M2}$ is constant, $r^{*,M2} > r^{*,N3}$ for $p_1 \leq P_{21}$ if $n > 1$; and third, the $p_1$-bound of the overlap $P_{21}$, which is relevant for $\frac{c}{\beta} < CB_2$, does not change with $n$. For $\frac{c}{\beta} \geq CB_2$, the $p_1$-upper bound for N(A)3 is $P_1^N$ and, for M2, — $P_{22}$. By Proposition 1, $P_1^N$ is the $p_1$-bound of the overlap for $n = 1$ and $\frac{c}{\beta} \geq CB_2$ since $P_1^N < P_{22}$ for $n = 1$ and $\frac{c}{\beta} > CB_1 = CB_2$ ($P_1^N = P_{22}$ at $\frac{c}{\beta} = CB_2$). $P_1^N$ is decreasing in $n$, resulting in shrinking of the overlap. For $n > 1$, the overlap area shrinks also due to conditions (a), (b) of part NA3 of Theorem 2 and additional conditions (3.1) and (3.2) for N3 existence (Proposition 9). For $n = 1$, all these conditions hold trivially.

The only bound that leads to the expansion of the overlap with $n$ is $p_1$-lower bound for N(A)3 $P_2^N = \frac{nc}{\beta+n-1}$ that separates N(A)3 from N(A)2. This bound decreases from $\frac{c}{\beta}$ for $n = 1$ to $c$ for $n \to \infty$. Recall that M2 and N(A)2 exist only if $\beta < 1$. $P_2^N$ is strictly less than $\frac{c}{\beta}$ for any $n > 1$ while the upper $p_1-$bounds for M2, $P_{21}$ and $P_{22}$ are not less than $\frac{c}{\beta}$. Therefore, if $p_1 \leq \frac{c}{\beta}$, the equality $p_1 = \frac{nc}{\beta+n-1}$ yields $n_2 = \frac{p_1(1-\beta)}{p_1-c}$ such that, for the same given inputs except $n$, N(A) can be realized as N(A)2 for all $n \leq n_2$ and as N(A)3 for all $n > n_2$. Since profits are continuous under N(A), i.e., $r^{*,N3} = r^{*,N2}$ at $n = n_2$, and, by Proposition 10 in Bazhanov, Levin, and Nediak (2015), $nr^{*,N3}$ is decreasing in $n$, while $nr^{*,N2} \equiv nr^{*,M2}$ is constant, we have $r^{*,M2} > r^{*,N3}$ for any $n > n_2$, i.e., for $p_1 \in \left(P_2^N, c/\beta\right]$ and any other inputs in the overlap of N(A)3 and M2, inequality $r^{*,M2} > r^{*,N3}$ also holds.

*Part (2.2)* follows from the facts: (a) for N(A)4, $v^*$ is increasing in $\rho$, i.e., the total first-period profit $nr^{*,N4}$ does not exceed $nr^{*,N4}|_{\rho=0} = (p_1 - c)(1 - p_1) = nr^{*,M2}$, and (b) the second period is always at loss under N(A)4 since $s < c$.

## 7.9 Proof of Proposition 5 (customer surplus with MFC vs. no-MFC)

The total equilibrium customer surplus is $\Sigma = \Sigma_1 + \Sigma_2$, where $\Sigma_1$ and $\Sigma_2$ are the first-period and second-period surpluses respectively.

**Lemma 9.** *Under the conditions of the corresponding RESE, the total customer surplus is*

*(1) under M1:* $\Sigma^{M1} = \frac{(1-p_1)^2}{2} + (1 - p_1)(p_1 - p_2^*) + \frac{(\beta p_1 - p_2^*)^2}{2\beta}$;

*(2) under M2 and N(A)2:* $\Sigma^{M2} = \Sigma^{N2} = \frac{(1-p_1)^2}{2}$;

*(3) under N(A)1:* $\Sigma^{N1} = \frac{(\beta-p_2^*)^2}{2\beta}$;

*(4) under N(A)3 and 4, $\Sigma$ has the same form:* $\Sigma = \frac{(1-p_1)^2}{2} - \frac{(v^*-p_1)^2}{2} + \frac{\left(\beta v^*-p_2^*\right)^2}{2\beta}$, *where* $p_2^{*,N4} = s < p_2^{*,N3}$ *and* $v^{*,N4} \geq v^{*,N3}$, *which is strict for any* $\rho \in (0, 1)$.

The value $\Delta\Sigma^{A,B} \triangleq \Sigma^A - \Sigma^B$ below denotes the change in the total surplus that results from the switch from equilibrium $B$ to equilibrium $A$ given the same inputs when both RESE are possible. Consider $\Sigma^{N1}$ as $\Sigma^{N1} = \int_{p_2^*}^{\beta p_1}(\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} + \int_{\beta p_1}^{\beta}(\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta}$ where the first integral is $\Sigma_2^{M1}$. Then

$$\Delta\Sigma^{M1,N1} = \Sigma_1^{M1} - \int_{\beta p_1}^{\beta}(\tilde{v}-p_2^*)\frac{d\tilde{v}}{\beta} = \int_{p_1}^{1}(v-p_2^*)dv - \int_{p_1}^{1}(\beta v-p_2^*)dv = \int_{p_1}^{1} v(1-\beta)dv = \frac{(1-\beta)}{2}(1-p_1^2),$$

which is positive for any $p_1 < 1$ and $\beta < 1$.

The result for $\Delta\Sigma^{M1,M2}$ follows directly from parts (1) and (2) of Lemma 9 after substitution for $p_2^* = p_2^{*,M1} = c + \frac{\beta-c}{n+1}$ leading to $\Delta\Sigma^{M1,M2} = (1-p_1)(p_1 - c - \frac{\beta-c}{n+1}) + \frac{1}{2\beta}(\beta p_1 - c - \frac{\beta-c}{n+1})^2 > 0$.

By Lemma 9, $\Delta\Sigma^{M2,N1} = \frac{1}{2}\left[(1-p_1)^2 - \frac{1}{\beta}(\beta - p_2^*)^2\right] = \frac{1}{2}\left\{(1-p_1)^2 - \frac{1}{\beta}\left[\frac{n}{n+1}(\beta - c)\right]^2\right\}$, because, by part NA1 of Theorem 2, $\beta - p_2^* = (\beta - c)\frac{n}{n+1}$. Since $\Delta\Sigma^{M2,N1}$ decreases in $p_1$, it is always negative if $\Delta\Sigma^{M2,N1} < 0$ at $p_1-$lower bound, which, minimized at $\rho \to 1$, by Theorem 2, is $p_1^{LB} = 1 - \frac{n}{n+1}(\beta - c)$. Indeed, $\Delta\Sigma^{M2,N1}|_{p_1 = p_1^{LB}} = \frac{1}{2}\left[\frac{n}{n+1}(\beta - c)\right]^2(1 - \frac{1}{\beta}) < 0$ for any $\beta < 1$.

By Lemma 9, $\Delta\Sigma^{M2,N3} = \frac{1}{2}\left[(v^* - p_1)^2 - \frac{1}{\beta}(\beta v^* - p_2^*)^2\right]$, where, by part NA3 of Theorem 2,

$$v^* - p_1 = \frac{p_1 - \rho p_2^* - p_1 + p_1\rho\beta}{1-\rho\beta} = \frac{\rho\beta(p_1 - 1 + Y^*)}{1-\rho\beta} \text{ and } \beta v^* - p_2^* = \frac{\beta p_1 - \beta\rho p_2^* - p_2^* + p_2^*\rho\beta}{1-\rho\beta} = \frac{\beta(p_1 - 1 + Y^*)}{1-\rho\beta},$$

yielding $\Delta\Sigma^{M2,N3} = \frac{\beta}{2}\left[\frac{Y^{*,N3} - (1-p_1)}{1-\rho\beta}\right]^2(\rho^2\beta - 1) < 0$.

The sign of $\Delta\Sigma^{M2,N4}$ can be shown in the same way using $p_2^* = s$ and $v^* = v^{*,N4} = \frac{p_1 - \rho s}{1-\rho\beta}$. Then $v^* - p_1 = \frac{\rho(p_1\beta - s)}{1-\rho\beta}$ and $\beta v^* - s = \frac{p_1\beta - s}{1-\rho\beta}$, yielding $\Delta\Sigma^{M2,N4} = \frac{1}{2\beta}\left(\frac{p_1\beta - s}{1-\rho\beta}\right)^2(\rho^2\beta - 1) < 0$.

## 7.10 Proof of Lemma 1 ( $p_1$-bounds are equivalent to $\rho$-bounds)

By the proof of Theorem 1 for $n = 1$, the $p_1$-bounds $P_{11}$ and $P_{21}$ separate M1 and M2 respectively from NA3. These bounds can be written as bounds on $\rho$. Indeed, $p_1 \geq P_{11} \Leftrightarrow 1 - p_1 \leq \frac{\rho}{2}(\beta - c) \Leftrightarrow \rho \geq \rho^{M1} = \frac{2(1-p_1)}{\beta - c}$, and $p_1 \leq P_{21} \Leftrightarrow p_1\left[(1-\rho\beta)^2 - (1-\beta)\right] \leq c(1-\rho\beta)^2 \Leftrightarrow (1-\rho\beta)^2 \leq \frac{p_1(1-\beta)}{p_1 - c} \Leftrightarrow \rho \geq \rho^{M2} = \frac{1}{\beta}\left[1 - \sqrt{\frac{p_1(1-\beta)}{p_1-c}}\right]$, where $\sqrt{\frac{p_1(1-\beta)}{p_1-c}} < 1$ under NA3 since $p_1\beta > c$. Inequality $\rho^{M2} \leq 1$ is equivalent to $1 - \sqrt{\frac{p_1(1-\beta)}{p_1-c}} \leq \beta$, which holds as equality if $\beta = 1$. Consider $\beta < 1$. Then inequality $\rho^{M2} \leq 1$ can be written as $1 - \beta \leq \frac{p_1}{p_1 - c}$ or $-c - \beta(p_1 - c) \leq 0$, which is strict for any feasible $c, p_1$, and $\beta$. In the same way, $\rho^{M2} > 0$ is equivalent to $p_1(1-\beta) < p_1 - c$, which always holds.

## 7.11 Proof of Proposition 6 (benefit from MFC, $n = 1$)

**Lemma 10.** *For $n = 1$ and $\frac{c}{\beta} < p_1 < 1 - \frac{\rho}{2}(\beta - c)$, NA3 exists and unique with* $v^* = \frac{2p_1 - \rho c}{2-\rho\beta}, Y^* = 1 - \frac{\beta p_1 + c(1-\rho\beta)}{\beta(2-\rho\beta)}, p_2^* = c + \frac{\beta p_1 - c}{2-\rho\beta}, r^{*,N3} = \frac{(p_1-c)[2(1-p_1) - \rho(\beta-c)]}{2-\rho\beta} + \frac{(\beta p_1 - c)^2}{\beta(2-\rho\beta)^2}, \Sigma^* = \frac{(1-p_1)^2}{2} + \frac{1}{2}\left(\frac{\beta p_1 - c}{2-\rho\beta}\right)^2\left(\frac{1}{\beta} - \rho^2\right)$.

The proof of the Proposition follows from the properties of the boundaries between RESE, established in Theorems 1, 2, Corollary 1, Proposition 9, and the fact that, for $n = 1$, a monopolist is indifferent between two RESE at the boundary. For $n = 1$, the area where M1 exists is inside the area where NA1 exists because, by Proposition 1, $P_1^N = P_{11}$ and $P_1^N < P_{12}$.

*Part (1.1).* By part M2 of Theorem 1, M2 exists if $\frac{c}{\beta} \geq CB$ and $p_1 \leq P_2$ (if $p_1 = P_2$, the form of a *realized* RESE depends on the expectations: for $\alpha^* = 1$ it is M1, for $\alpha^* = 0$ — M2). By part NA1 of Theorem 2, NA1 exists if $p_1 \geq P_1^N$. The benefit from MFC is $B^{M2,NA1} = r^{*,M2} - r^{*,NA1} = (p_1 - c)(1 - p_1) - \frac{(\beta-c)^2}{4\beta}$, which is increasing in $c$ because $\frac{\partial B^{M2,NA1}}{\partial c} = -(1-p_1) + \frac{1}{2\beta}(\beta - c) = p_1 - \frac{1}{2\beta}(\beta + c) > 0$. The last inequality holds since $p_1 \geq P_1^N = 1 - \frac{\rho}{2}(\beta - c)$ under NA1, and $\frac{1}{2\beta}(\beta + c) < 1 + \frac{\rho}{2}(\beta + c) - \rho\beta \Leftrightarrow \frac{1}{2}\left(1 + \frac{c}{\beta}\right)(1 - \rho\beta) < 1 - \rho\beta \Leftrightarrow \frac{\beta + c}{2} < \beta$ holds for any $c < \beta$. Then $B^{M2,NA1} \geq 0$ for any $c$ if $B^{M2,NA1}|_{c=0} \geq 0$, which is $-p_1^2 + p_1 - \frac{\beta}{4} \geq 0$. This inequality holds between the roots $(p_1)_{1,2} = \frac{1}{2}\left[1 \mp \sqrt{1-\beta}\right]$. Inequality $p_1 \geq (p_1)_1$ always holds if $(p_1)_1 \leq P_1^N|_{c=0} = 1 - \frac{\rho\beta}{2}$, which is satisfied for any $\rho < 1$ since $P_1^N|_{c=0}$ is decreasing in $\rho$ and $(p_1)_1 \leq P_1^N|_{c=0}$ holds for $\rho = 1 : \frac{1}{2}\left[1 - \sqrt{1-\beta}\right] \leq 1 - \frac{\beta}{2} \Leftrightarrow \beta - \sqrt{1-\beta} \leq 1$ (always holds). Inequality $p_1 \leq (p_1)_2$ always holds if $(p_1)_2$ is not less than $p_1$-upper bound for M2, which, for $c = 0$, is $P_2$ because if $0 = \frac{c}{\beta} < CB$ (part 2.1 of Theorem 1), inequality $p_1 \leq P_{21} = 0$ never holds. Equality $P_2|_{c=0} = \frac{1}{2}\left[1 + \sqrt{1-\beta}\right] \equiv (p_1)_2$ implies $B^{M2,NA1}|_{c=0} \geq 0$. Benefit $B^{M2,NA1}$ decreases in $\beta$ since $\frac{\partial B^{M2,NA1}}{\partial \beta} = -\frac{1}{4\beta^2}\left[2(\beta-c)\beta - (\beta-c)^2\right] = -\frac{\beta^2-c^2}{4\beta^2} < 0$.

*Part (1.2).* By Lemma 10, NA3 exists. By part M2 of Theorem 1, M2 exists either if $\frac{c}{\beta} \geq CB$ (since $p_1 < P_1^N < P_2$) or $\frac{c}{\beta} < CB$ and $p_1 \leq P_{21}$, which, by Lemma 1, is equivalent to $\rho \geq \rho^{M2}$. The benefit from MFC is $B^{M2,NA3} = r^{*,M2} - r^{*,NA3} = (p_1-c)(1-p_1) - (p_1-c)\frac{2(1-p_1)-\rho(\beta-c)}{2-\rho\beta} - \frac{(\beta p_1 - c)^2}{\beta(2-\rho\beta)^2}$

$$
\begin{aligned}
&= \frac{1}{2-\rho\beta}\left[(p_1-c)\rho(\beta p_1 - c) - \frac{(\beta p_1-c)^2}{\beta(2-\rho\beta)}\right] = \frac{\beta p_1 - c}{2-\rho\beta}\left[(p_1-c)\rho - \frac{\beta p_1 - c}{\beta(2-\rho\beta)}\right] \\
&= \frac{\beta p_1 - c}{2-\rho\beta}\left[c(1-2\rho\beta+\rho^2\beta^2) - p_1(\beta - 2\rho\beta + \rho^2\beta^2)\right] = \frac{\beta p_1 - c}{2-\rho\beta}\left[p_1(1-\beta) - (1-\rho\beta)^2(p_1-c)\right],
\end{aligned}
$$

which is increasing in $\rho$. Since $\beta p_1 - c > 0$ under NA3 for $n = 1$, $B^{M2,NA3} > 0$ if and only if $1 - \rho\beta < \sqrt{\frac{p_1(1-\beta)}{p_1-c}}$, which is, indeed, equivalent to $\rho > \frac{1}{\beta}\left[1 - \sqrt{\frac{p_1(1-\beta)}{p_1-c}}\right] = \rho^{M2}$. By Lemmas 9, and 10, $\Delta\Sigma^{M2,NA3} = \frac{1}{2}\left(\frac{\beta p_1 - c}{2-\rho\beta}\right)^2\left(\rho^2 - \frac{1}{\beta}\right) < 0$.

*Part (2).* Since the profits in the pairs M1 – N(A)1 and M2 – N(A)2 are identical, reseller is indifferent between these equilibria in the correspondent areas where (2.1) M1 exists: $\frac{c}{\beta} < CB$ and $p_1 \geq P_{11} \Leftrightarrow \rho \geq \rho^{M1}$ or $\frac{c}{\beta} \geq CB$ and $p_1 \geq P_2$; (2.2) N(A)2 exists: $\beta < 1$, any $\rho$ and $\frac{c}{\beta}$, and $p_1 \leq \frac{c}{\beta}$. Part (2.3) follows from the proof of part (1.2) since $B^{M2,NA3} = 0$ at the boundary between N3 and M2 where $\rho = \rho^{M2}$.

*Part (3).* The remaining area with $\frac{c}{\beta} < CB, p_1 > \frac{c}{\beta}, p_1 < P_{11} = P_1^N$ ($\rho < \rho^{M1}$), and $p_1 > P_{21}$ ($\rho < \rho^{M2}$) corresponds to inputs where only price-discriminating N(A)3 exists and MFC-equilibria do not exist because of a lower profit.

MFC never leads to a gain from increased strategic behavior because (i) profit is constant in $\rho$ for both M1 and M2; (ii) profit is continuous at the boundaries between equilibria. Moreover, N(A)3 is realized for any $\rho^L < \rho^{M1} \wedge \rho^{M2}$ and one of MFC-equilibria (denote it as $MFC$) is realized for any $\rho^H \geq \rho^{M1} \wedge \rho^{M2}$. Therefore, since $r^{*,NA3}$ is decreasing in $\rho$ for $n = 1$ (Bazhanov, Levin, and Nediak (2015)), inequality $r^{*,NA3}|_{\rho=\rho^L} > r^{*,MFC}|_{\rho=\rho^H}$ always holds yielding $\eta(\text{NA3,NA3,MFC}) = \frac{(r^{*,MFC} - r^{*,NA3})|_{\rho=\rho^H}}{r^{*,NA3}|_{\rho=\rho^L} - r^{*,NA3}|_{\rho=\rho^H}} < 1$.

## 7.12 Proof of Proposition 7 (gain from M2)

Assume that N(A)4 and M2 exist for the same inputs including $\rho^H > 0$, and N(A)4 exists for these inputs except $\rho^L < \rho^H$. The loss from increased strategic behavior without MFC is $r^{*,NA4}|_{\rho=\rho^H} - r^{*,NA4}|_{\rho=\rho^L} < 0$ and the performance of M2 is $\eta(\text{NA4,NA4,M2}) = \frac{r^{*,M2} - r^{*,NA4}|_{\rho=\rho^H}}{r^{*,NA4}|_{\rho=\rho^L} - r^{*,NA4}|_{\rho=\rho^H}} = 1 + \frac{r^{*,M2} - r^{*,NA4}|_{\rho=\rho^L}}{r^{*,NA4}|_{\rho=\rho^L} - r^{*,NA4}|_{\rho=\rho^H}}$. Since $r^{*,NA4}$ is decreasing in $\rho$, profit $r^{*,M2} = \frac{1}{n}(p_1-c)(1-p_1)$ does not depend on $\rho$, and, by Proposition 2, $\frac{1}{n}(p_1-c)(1-p_1) > r^{*,NA4}$ for any inputs in the area of NA4 existence (implying that $r^{*,M2} > r^{*,NA4}|_{\rho=\rho^L}$), M2 leads to a gain ($\eta > 1$). A lower bound of $\eta$ in $\rho^L$ is at $\rho^L = 0$ where the denominator in the expression for $\eta$ attains maximum. Then, the substitution of the expressions for profits yields

$$\eta(\text{NA4,NA4,M2}) \geq 1+\frac{(1-p_1)\left[n(p_1-c)-(p_1-s)\right]}{(p_1-s)(v^{*,NA4}-p_1)} = 1+\frac{(1-\rho^H\beta)(1-p_1)\left[n(p_1-c)-(p_1-s)\right]}{(p_1-s)\rho^H(p_1\beta-s)}.$$

This measure is unbounded in $n$ since $nr^{*,NA4}$ decreases in $n$ to zero while $nr^{*,M2}$ is constant.

### 7.13 Equilibria existence in Example 2 (gain from M2)

NA4 exists for both $\rho = 0.5$ and $\rho = 0$, and, when MFC is available and used by resellers at $\rho = 0.5$, N4 exists for $\rho = 0$ and M2 — for $\rho = 0.5$. Indeed, by Theorem 3 for $\rho = 0$, $v^{*,N4} = p_1 = \frac{1}{2}, Y^{*,N4} = \frac{1}{4}\frac{0.45}{0.05} = \frac{9}{4}$, and condition (a) of Theorem 3 holds: $\frac{n-1}{n}Y^{*,N4} = \frac{9}{8} > 1 \geq 1 - \frac{s}{\beta} > 1 - \frac{c}{\beta}$. The last inequality means that the additional condition (4.2) of Proposition 9 for the existence of N4 also holds since $p_1 = 0.5 > \frac{c}{\beta} = 0.2$. M2, by Theorem 1, does not exist because, for $\rho = 0, CB_2 = 1 > \frac{c}{\beta}$ and $P_{21} = \frac{c}{\beta} = 0.2 < 0.5 = p_1$. For $\rho = 0.5, v^{*,N4} = \frac{1-0.05}{2(3/4)} = \frac{1.9}{3}$, and $Y^{*,N4} = \frac{1}{2}\frac{0.45}{0.05}\frac{1.1}{3} = \frac{3.3}{2}$. Condition (a) of Theorem 3 does not hold: $\frac{n-1}{n}Y^{*,N4} = \frac{3.3}{4} = 0.825 < 1 - \frac{s}{\beta} = 0.9$, but condition (b) holds: $\frac{n-1}{n}Y^{*,N4}\frac{\beta}{c+\beta v^{*}-2s} = \frac{3.3}{4\cdot 2\cdot(0.1+1.9/6-0.1)} = \frac{9.9}{7.6} > 1$. M2 exists since $CB_2 = \frac{1-1+1/8}{(1-1/4)^2+\frac{1}{2}\cdot 2\cdot\frac{1}{8}} = \frac{2}{11} < \frac{c}{\beta} = \frac{2}{10}$ and $P_{22} = \frac{2\left(4.6-0.5+4\sqrt{53}/10\right)}{17} > \frac{8.2+8\cdot 0.7}{17} > p_1 = 0.5$.

### 7.14 Proof of Proposition 8 (MFC-profit exceeds NA3, $p_1 = \beta$)

The proof uses the following lemma where $p_1$-bounds between NA3, 2, and 1 are written as the bounds on $\frac{c}{\beta}$ with $CB_{N1} \triangleq 1 - \frac{n+1}{n\rho\beta}(1-\beta)$ and $CB_{N2} \triangleq 1 - \frac{1-\beta}{n}$.

**Lemma 11.** *If $p_1 = \beta$, the forms of NA3 and NA4 simplify as follows:*

**NA3** ($p_2^* > s$) $Y^* = \frac{[(1-c/\beta)(1-\rho\beta)+1-\beta]n}{n+1-\rho\beta}$ *and* $r^{*,NA3} = \beta(Y^*/n)^2$; *condition* $P_2^N < p_1 < P_1^N$ *is equivalent to* $\frac{c}{\beta} < CB_{N2}$ *and either* $\beta < 1$ *for* $\rho = 0$ *or, for* $\rho > 0$, $\frac{c}{\beta} > CB_{N1}$; *condition (a) becomes* $\frac{n-1}{n}\frac{\beta^2(1-v^*)Y^*}{(c-s)(\beta-s)} \leq 1$; *and condition* $Y^* < 1 - \frac{s}{\beta}$ *becomes either* $c - s \geq \beta(1-\beta)$ *or* $c - s < \beta(1-\beta)$ *and* $n < \frac{(1-\rho\beta)(1-s/\beta)}{(1-\rho\beta)(1-c/\beta)-\beta+s/\beta}$.

**NA4** ($p_2^* = s$) $Y^* = \frac{n-1}{n}\frac{\beta-s}{c-s}\left(1-\frac{\beta-\rho s}{1-\rho\beta}\right)$ *and* $r^{*,NA4} = \frac{\beta-s}{n^2}\left(1-\frac{\beta-\rho s}{1-\rho\beta}\right)$; *condition (b) becomes* $\beta(n-1)Y^* \geq n(c+\beta v^* - 2s)$.

As to equilibria NA1 and NA2, both of them may exist and have overlaps with M1 and M2 for some feasible inputs when $p_1 = \beta$. NA1 exists if and only if $\frac{c}{\beta} \leq CB_{N1}$ for $\rho > 0$ or $\beta = 1$ for $\rho = 0$; and NA2 — if and only if $\frac{c}{\beta} \geq CB_{N2}$.

By Theorem 1 and Lemma 11, inequality $r^{*,M1} > r^{*,N3}$ is

$$\frac{(\beta-c)^2}{(n+1)^2\beta} > \frac{[\beta(1-\beta)+(\beta-c)(1-\rho\beta)]^2}{\beta(n+1-\rho\beta)^2} \Leftrightarrow n\left[(\beta-c)\rho-1+\beta\right] > 1-\beta, \tag{24}$$

which holds for any $n \geq 1$ if $(\beta-c)\rho > 2(1-\beta)$ since, under this condition, it holds for $n = 1$ and the LHS is increasing in $n$. On the other hand, (24) may hold only if $[\cdot] > 0$, which is equivalent to $\rho > \frac{1-\beta}{\beta-c}$. Then, if $\rho \in \left(\frac{1-\beta}{\beta-c}, 2\frac{1-\beta}{\beta-c}\right]$, inequality $r^{*,M1} > r^{*,N3}$ is equivalent to $n > \frac{1-\beta}{(\beta-c)\rho-1+\beta}$. Condition $\beta > \frac{1+c}{2}$ follows from inequality $\frac{1-\beta}{\beta-c} \geq 1$.

### 7.15 Equilibria existence in Examples 3-5

*Example 3.* Condition (a) of Theorem 3 holds: $Y^{*,NA4} = \frac{208}{105}$ and $\frac{n-1}{n}Y^{*,NA4} = \frac{416}{315} > 1 > 1 - \frac{s}{\beta}$, i.e., "salvaging" is forced on resellers and N(A)4, indeed, exists with $v^* = \frac{27}{35}$ and the profit $r^{*,NA4} = \frac{0.65}{9}(1 - v^*) = \frac{26}{1575} = 0.0165$. M2 exists since $\frac{c}{\beta} = \frac{4}{10} > \frac{4}{58} = CB_2$ and $p_1 < P_{22} = 0.93$ with $r^{*,M2} = 0.06$. M1 exists since $\frac{c}{\beta} = \frac{4}{10} > \frac{4}{100} = CB_1$ and $p_1 > P_{12} = 0.69$, with the profit $r^{*,M1} = \frac{0.15^2}{4^2 \cdot 0.25} = 0.0056$.

*Example 4.* Equilibrium profits in Figure 8 (b) are computed under the existence conditions of the corresponding RESE types. In particular, for $\rho = 0.2$ and $\rho = 0.65$ the existence can be demonstrated as follows. For $\rho = 0.65$, NA3 is realized in no MFC game since, by Lemma 11, $Y^{*,NA3} = 0.89 < 1 = 1 - \frac{s}{\beta}$, condition (b) of Theorem 2 holds: $r^{*,NA3} = 0.0247 > \tilde{r}^i = 0.0188$ (which can be shown using the expression for $\tilde{r}^i = \left\{\sqrt{(p_1 - s)(1 - v^*)} - \sqrt{\frac{n-1}{n}Y^*(c - s)}\right\}^2$ given in Bazhanov, Levin, and Nediak (2015)), and NA4 does not exist because the necessary condition $Y^{*,NA4} > 1 - \frac{s}{\beta}$ does not hold: $Y^{*,NA4} = \frac{105}{108} < 1$. For $\rho = 0.2$, the only existing equilibrium is NA4 without MFC or N4 with MFC. Indeed, $1 - v^{*,NA4} = \frac{4}{9}, Y^{*,NA4} = \frac{3 \cdot 5}{4}\frac{4}{9} = \frac{5}{3}$ and $\frac{n-1}{n}Y^{*,NA4} = \frac{5}{4} > 1 = 1 - \frac{s}{\beta} > 1 - \frac{c}{\beta}$, which means that condition (a) of Theorem 3 holds and additional condition (4.2) of Proposition 9 for existence of N4 holds ($p_1 > \frac{c}{\beta} = 0.2$). At the same time, NA3 does not exist since $Y^{*,NA3}|_{\rho=0.2} = \frac{488}{490}$, condition (a) of part NA3 of Theorem 2 does not hold: $\frac{n-1}{n}\frac{\beta^2(1-v^*)Y^*}{(c-s)(\beta-s)} = \frac{3 \cdot 4.5}{4}\frac{488}{490} > 1$, and condition (b) does not hold: $r^{*,NA3} = 0.152 < \tilde{r}^i = 0.158$. MFC-equilibria also do not exist for $\rho = 0.2$. M1: $CB_1 = \frac{31}{56} > \frac{20}{100} = \frac{c}{\beta}$ and $P_{11} = \frac{2.52}{5} > p_1$; M2: $CB_2 = \frac{62}{85} > \frac{20}{100} = \frac{c}{\beta}$ and $P_{21} = \frac{8.1}{31} < \frac{15.5}{31} = p_1$. When $\rho = 0.65$ and MFC is available, M1 exists since $CB_1 < 0$ and $P_{12} = 0.487 < p_1$. The MFC performance is $\eta(\text{NA4,NA3,M1}) = \frac{\left(r^{*,M1} - r^{*,NA3}\right)|_{\rho=0.65}}{r^{*,NA3}|_{\rho=0.65} - r^{*,NA4}|_{\rho=0.2}} = \frac{0.0128 - 0.0247}{0.0247 - 0.0139} = \frac{0.0119}{0.0108} = -1.102$.

*Example 5.* Condition (a) of Theorem 3 holds for both $\rho^H = 0.4$ and $\rho^L = 0.3$. M1 exists at $\rho^H$ since $CB_1 = \frac{304}{1004} > \frac{100}{1300} = \frac{c}{\beta}$ and $P_{11} = 0.398 < p_1$, and M1 does not exist at $\rho^L$ since $CB_1 = 0.466 > \frac{c}{\beta}$ and $P_{11} = 0.410 > p_1$.

## 8 Proofs of auxiliary statements

### 8.1 Proof of Lemma 2 (roots of equation $\breve{r}^i = \tilde{r}^i$)

Equation (5), i.e. $p_1^2 - (x + c)p_1 + \frac{\beta}{4}\left(x + \frac{c}{\beta}\right)^2 = 0$, originates from comparing the expressions $\breve{r}^i = (p_1 - c)(x - p_1)$ and $\tilde{r}^i = \frac{\beta}{4}\left(x - \frac{c}{\beta}\right)^2$, and collecting the terms in the equation $\tilde{r}^i - \breve{r}^i = 0$. The roots of (5) exist since the discriminant $D = (x + c)^2 - \beta\left(x + \frac{c}{\beta}\right)^2 \geq 0$. Indeed,

$$D = x^2(1 - \beta) + c^2\left(1 - \frac{1}{\beta}\right) = (1 - \beta)\left[x^2 - \frac{c^2}{\beta}\right] \geq (1 - \beta)\left[\frac{c^2}{\beta^2} - \frac{c^2}{\beta}\right] \geq 0.$$

$\frac{1}{2}\left(x + \frac{c}{\beta}\right)$ is between the roots with strict inequalities when $x > \frac{c}{\beta}$ and $\beta < 1$ because substituting $p_1 = \frac{1}{2}\left(x + \frac{c}{\beta}\right)$ into the LHS of (5) we obtain:

$$\begin{aligned} & \frac{1}{4}\left(x + \frac{c}{\beta}\right)^2 - \left[\left(x + \frac{c}{\beta}\right) + c - \frac{c}{\beta}\right]\frac{1}{2}\left(x + \frac{c}{\beta}\right) + \frac{\beta}{4}\left(x + \frac{c}{\beta}\right)^2 \\ = \;& \frac{1}{4}\left\{-\left(x + \frac{c}{\beta}\right)^2 - \frac{2c}{\beta}(\beta - 1)\left(x + \frac{c}{\beta}\right) + \beta\left(x + \frac{c}{\beta}\right)^2\right\} = \frac{1}{4}\left(x + \frac{c}{\beta}\right)(\beta - 1)\left[x - \frac{c}{\beta}\right] \leq 0. \end{aligned}$$

Inequality $(p_1)_2(x) \le x$ follows from $\tilde{r}^i - \breve{r}^i\big|_{p_1=x} \ge 0$, which is strict unless $x = \frac{c}{\beta}$.

The larger root is increasing in $x$ if $x > \frac{c}{\beta}$, which is evident from the implicit differentiation of the equation with respect to $x$

$$[2p_1 - (x+c)]\frac{\partial p_1}{\partial x} = p_1 - \frac{\beta}{2}\Big(x + \frac{c}{\beta}\Big),$$

since, for the larger root $2p_1 > x + c$ and $p_1 - \frac{\beta}{2}\Big(x + \frac{c}{\beta}\Big) = p_1 - \frac{1}{2}(\beta x + c) \ge p_1 - \frac{1}{2}(x+c) > 0$ implying that $\frac{\partial p_1}{\partial x} > 0$.

## 8.2 Proof of Lemma 3 (MFC BR)

When all resellers use MFC, the general expression for reseller $i$ profit, by (4), is

$$r_1^i = \begin{cases} (p_1 - c)y_1^i, & \text{if } Y = Q, \\ (p_2 - c)y_1^i, & \text{if } Y > Q. \end{cases}$$

In this case, $Q = Y \wedge (1 - v_1^{\min})$, where, by (1), $v_1^{\min} = p_1$. Therefore, $Y = Q$ if $Y \le 1 - p_1$ (sales only in the first period) and $p_2$ is not defined. Otherwise ($Y > 1 - p_1$), there are second-period sales and, by (3), $p_2 < p_1$. Thus, $r_1^i$ has a discontinuity at $Y = Q$. Moreover, the profit at $Y = Q + 0$ is strictly less than at $Y = Q$.

Consider two principal cases: (a') the maximum-profit MFC response *without* the second-period sales (i.e., $Y_1 \le 1 - p_1$) is not dominated by any MFC response with the second period sales (i.e., $Y_1 > 1 - p_1$), and (b') the maximum-profit MFC response *with* the second period sales is not dominated by any MFC response without the second period sales.

We start by describing the nontrivial BR candidates for the cases (a') and (b'). The profit $r_1^i = (p_1 - c)y_1^i$ in case (a') is strictly increasing in $y_1^i$, implying that, if BR exists in this region, reseller $i$ sets $y_1^i$ to $\breve{y}_1^i = 1 - p_1 - Y_1^{-i}$ resulting in $Y_1 = 1 - p_1$. The nontrivial BR of this form exists if and only if $\breve{y}_1^i > 0$, i.e., $1 - Y_1^{-i} > p_1$, and the corresponding profit $\breve{r}_1^i = (p_1 - c)(1 - p_1 - Y_1^{-i})$ is not dominated by that of case (b') or by the profit corresponding to the no-MFC response.

The nontrivial responses for case (b') are constrained by $Y_1 > 1 - p_1$ or, equivalently, $y_1^i > 1 - p_1 - Y_1^{-i}$, and $y_1^i > 0$. The profit function in this case, $r_1^i = (p_2 - c)y_1^i = [\beta(1 - Y_1^{-i} - y^i) - c]y^i$, is strictly concave.

The profit-maximizing $y_1^i$ must satisfy the first-order condition $\frac{\partial r_1^i}{\partial y_1^i} = \beta(1 - Y_1^{-i}) - c - 2\beta y_1^i = 0$, yielding $\tilde{y}_1^i \triangleq \frac{1}{2}(1 - \frac{c}{\beta} - Y_1^{-i})$. The profit corresponding to $\tilde{y}_1^i$ is

$$\tilde{r}_1^i = \left\{\beta\left[1 - Y_1^{-i} - \frac{1}{2}\left(1 - \frac{c}{\beta} - Y_1^{-i}\right)\right] - c\right\} \times \frac{1}{2}\left(1 - \frac{c}{\beta} - Y_1^{-i}\right) = \frac{\beta}{4}\left(1 - \frac{c}{\beta} - Y_1^{-i}\right)^2.$$

By feasibility constraints, $\tilde{y}_1^i$ is a nontrivial BR candidate only if $\tilde{y}_1^i > 0$ and $\tilde{y}_1^i > 1 - p_1 - Y_1^{-i}$. The first inequality is equivalent to $1 - Y_1^{-i} > \frac{c}{\beta}$. The second one is equivalent to

$$\frac{1}{2}\Big(1 - \frac{c}{\beta} - Y_1^{-i}\Big) > 1 - p_1 - Y_1^{-i} \Leftrightarrow p_1 > \frac{1}{2}\Big(1 - Y_1^{-i} + \frac{c}{\beta}\Big) \triangleq (p_1)_0.$$

If either of these conditions is violated, the profit function is strictly decreasing in the entire region of case (b'). Thus, a nontrivial BR of this form exists if and only if $1 - Y_1^{-i} > \frac{c}{\beta}$, $p_1 > (p_1)_0$ and $\tilde{r}_1^i$ is not dominated by the profit $\breve{r}_1^i$ of case (a') and the one corresponding to a no-MFC response.

We now establish conditions when the maximum profit within case (a') is not dominated by the maximum profit within case (b') and vice versa. In particular, when either $\frac{c}{\beta} \geq 1 - Y_1^{-i}$ or $p_1 \leq (p_1)_0$, $\tilde{y}_1^i$ is not within the feasible region of case (b') and $\breve{r}_1^i$ dominates the profit corresponding to any response within case (b') as long as $\breve{y}_1^i$ is feasible and nontrivial, i.e., $1 - Y_1^{-i} > p_1$.

If $\tilde{y}_1^i$ is feasible and nontrivial, i.e., $1 - Y_1^{-i} > \frac{c}{\beta}$ and $p_1 > (p_1)_0$, the response of case (a') is not dominated by that of case (b') if and only if $\breve{y}_1^i$ is feasible, nontrivial, and $\breve{r}_1^i \geq \tilde{r}_1^i$. This inequality is $(p_1 - c)(1 - p_1 - Y_1^{-i}) \geq \frac{\beta}{4}\left(1 - c/\beta - Y_1^{-i}\right)^2$, which is equivalent to

$$p_1^2 - (1 + c - Y_1^{-i})p_1 + \frac{\beta}{4}\Big(1 - Y_1^{-i} + c/\beta\Big)^2 \leq 0. \tag{25}$$

By Lemma 2, the roots $(p_1)_{1,2}$ of (5) exist with $x = 1 - Y_1^{-i} \geq \frac{c}{\beta}$. Then (25) holds if and only if $(p_1)_1 \leq p_1 \leq (p_1)_2$. By Lemma 2, $(p_1)_1 \leq (p_1)_0 \leq (p_1)_2 \leq 1 - Y_1^{-i}$ where the last inequality is strict unless $1 - Y_1^{-i} = \frac{c}{\beta}$.

Combining all situations where the maximum profit in case (a') is strictly positive and not dominated by responses in case (b'), we obtain the following conditions: $1 - Y_1^{-i} > p_1$ (i.e., $\breve{y}_1^i$ is feasible and nontrivial) and either (a'.1) $\frac{c}{\beta} \geq 1 - Y_1^{-i}$ (i.e., $\tilde{y}_1^i \leq 0$ because the second-period sales are always at $p_2 \leq c$) or (a'.2) $\frac{c}{\beta} < 1 - Y_1^{-i}$ (i.e., $\tilde{y}_1^i > 0$) and $p_1 \leq (p_1)_0$ (i.e., $\tilde{y}_1^i \leq \breve{y}_1^i$ because the profit function decreases for all $y_1^i > \breve{y}_1^i$) or $(p_1)_0 < p_1 \leq (p_1)_2$ (i.e., even though $\tilde{y}_1^i$ is feasible, $\breve{r}_1^i \geq \tilde{r}_1^i$). Since $\frac{c}{\beta} < 1 - Y_1^{-i}$ implies $(p_1)_0 \leq (p_1)_2$, the subcase (a'.2) can be compactly described by the pair of conditions $\frac{c}{\beta} < 1 - Y_1^{-i}$ and $p_1 \leq (p_1)_2$.

Symmetrically, combining all situations where the maximum profit in case (b') is strictly positive and not dominated by responses in case (a'), we obtain the following conditions: $1 - Y_1^{-i} > \frac{c}{\beta}$ (i.e., $\tilde{y}_1^i$ is nontrivial) and either (b'.1) $1 - Y_1^{-i} \leq p_1$ (i.e., $\breve{y}_1^i$ is infeasible or trivial) or (b'.2) $1 - Y_1^{-i} > p_1 \geq (p_1)_2$. Indeed, for case (b'.1), there is no need to compare $\tilde{r}_1^i$ with $\breve{r}_1^i$ and the feasibility condition $p_1 > (p_1)_0$ is implied by $p_1 \geq 1 - Y_1^{-i} > \frac{c}{\beta}$ since, then, $(p_1)_0 < 1 - Y_1^{-i}$. For case (b'.2), $\tilde{r}_1^i \geq \breve{r}_1^i$ if and only if $p_1 \geq (p_1)_2$ or $p_1 \leq (p_1)_1$, but the feasibility condition $p_1 > (p_1)_0$ cannot be satisfied together with $p_1 \leq (p_1)_1$ because $(p_1)_1 \leq (p_1)_0$. On the other hand, $p_1 > (p_1)_0$ is implied by $p_1 \geq (p_1)_2$ and $1 - Y_1^{-i} > c/\beta$ (recall that the latter implies $(p_1)_2 > (p_1)_0$).

We now determine when the maximum profit of case (a') is not dominated by responses without MFC, implying that $Y_1$ is equivalent to $Y_1^{-i}$ above. Recall that these responses correspond to expectations $\bar{\alpha}(0), \bar{p}_2(0)$. First, response that results in first-period sales only cannot lead to profits higher than $\breve{r}_1^i$ because the potential first-period demand under such response does not exceed the first-period demand under MFC. Second, a response with sales only in the second period results in all stock sold at $p_2$ and profit no higher than $\tilde{r}_1^i \leq \breve{r}_1^i$. The third remaining case is a response with sales in both periods characterized by $v_0^{\min} = v_0^{\min}(\bar{\alpha}(0), \bar{p}_2(0)) < 1 - Y_1$ and $y_0^i > 1 - Y_1 - v_0^{\min}$. The profit in this case is concave quadratic of the form

$$r_0^i = (p_1 - c)(1 - Y_1 - v_0^{\min}) + (\beta(1 - Y_1 - y_0^i) - c)(y_0^i - [1 - Y_1 - v_0^{\min}])$$

with $\frac{\partial r_0^i}{\partial y_0^i} = -2\beta y_0^i + \beta[1 - Y_1 - v_0^{\min}] + \beta(1 - Y_1) - c$, the unique solution to the first-order condition $\tilde{y}_0^i = 1 - Y_1 - \frac{1}{2}(v_0^{\min} + c/\beta)$, yielding total inventory $\tilde{y}_0^i + Y_1 = 1 - \frac{1}{2}(v_0^{\min} + c/\beta) < 1 - s/\beta$ (since $c > s$ and $v_0^{\min} \geq p_1 > \frac{s}{\beta}$) and profit

$$\tilde{r}_0^i = (p_1 - c)(1 - Y_1 - v_0^{\min}) + \frac{\beta}{4}(v_0^{\min} - c/\beta)^2. \tag{26}$$

If $v_0^{\min} \leq c/\beta$, then $\tilde{y}_0^i \leq 1 - Y_1 - v_0^{\min}$ and $r_0^i$ is decreasing for all $y_0^i \geq 1 - Y_1 - v_0^{\min}$. Thus, the profit-maximizing level of inventory without MFC is $\breve{y}_0^i = 1 - Y_1 - v_0^{\min}$ that results only in the first-period sales. In this case, we have already established that no-MFC response does not dominate $\breve{r}_1^i$.

If $v_0^{\min} > c/\beta$, we need to check when $\breve{r}_1^i = (p_1 - c)(1 - p_1 - Y_1) \geq \tilde{r}_0^i$ which is equivalent to inequality $(p_1 - c)(v_0^{\min} - p_1) \geq \frac{\beta}{4}(v_0^{\min} - c/\beta)^2$ and, in turn, (6). The corresponding equation is (5) with $x = v_0^{\min}$, and, by Lemma 2, its roots exist for $v_0^{\min} \geq c/\beta$. Moreover, relation $v_0^{\min} < 1 - Y_1$ implies that the larger root is less than $(p_1)_2|_{x=1-Y_1^{-i}}$. Thus, when $c/\beta < v_0^{\min} < 1 - Y_1$, there is a non-empty interval of $p_1$ in which (6) holds and, for any $p_1$ in this interval, $p_1 \leq (p_1)_2$ holds.

Summarizing all conditions where no-MFC responses cannot dominate $\breve{r}_1^i$ we obtain: either (i) $v_0^{\min} \geq 1 - Y_1$, or (ii) $v_0^{\min} < 1 - Y_1$ and $v_0^{\min} \leq c/\beta$, or (iii) $c/\beta < v_0^{\min} < 1 - Y_1$ and (6). Combining these conditions with those of case (a'), we obtain the conditions of case (a) in the lemma. Indeed, in case (a'.1) $c/\beta \geq 1 - Y_1$ implies that either (i) or (ii) holds. In the complementary case (a'.2), subcases $v_0^{\min} \geq 1 - Y_1$ or $v_0^{\min} \leq c/\beta$ require only additional condition $p_1 \leq (p_1)_2$ ($v_0^{\min} < 1 - Y_1$ is implied by $v_0^{\min} \leq c/\beta$ and $c/\beta < 1 - Y_1$. An additional useful observation is that since $(p_1)_0 > c/\beta$ in this case, we have $(p_1)_2 > c/\beta$. On the other hand, if $c/\beta < v_0^{\min} < 1 - Y_1$, condition $p_1 \leq (p_1)_2$ is superseded by a stronger condition (6).

We complete the proof by describing when the maximum profit of case (b') is not dominated by responses without MFC. Two out of three possibilities are ruled out in a way almost identical to the reasoning for the case (a'). First, response with the first-period sales only cannot lead to profits higher than $\tilde{r}_1^i$ because the potential first-period demand under such response does not exceed the first-period demand under MFC while the latter would result in $\breve{r}_1^i \leq \tilde{r}_1^i$. Second, a response with sales only in the second period results in all stock sold at $p_2$ and profit no higher than $\tilde{r}_1^i$. The remaining case is a response with sales in both periods characterized by $v_0^{\min} = v_0^{\min}(\bar{\alpha}(0), \bar{p}_2(0)) < 1 - Y_1$ and $y_0^i > 1 - Y_1 - v_0^{\min}$.

Similarly to a comparison with $\breve{r}_1^i$, no-MFC response cannot dominate $\tilde{r}_1^i$ if either (i) $v_0^{\min} \geq 1 - Y_1$ or (ii) $v_0^{\min} < 1 - Y_1$ and $v_0^{\min} \leq c/\beta$. The condition $v_0^{\min} < 1 - Y_1$ in (ii) is always satisfied for (b') because $1 - Y_1 > c/\beta$. Examine $c/\beta < v_0^{\min} < 1 - Y_1$. The MFC BR with inventory level $\tilde{y}_1^i$ exists in this case if and only if $\tilde{r}_1^i \geq \tilde{r}_0^i$:

$$\begin{aligned}
\frac{\beta}{4}(1 - c/\beta - Y_1)^2 &\geq (p_1 - c)(1 - Y_1 - v_0^{\min}) + \frac{\beta}{4}(v_0^{\min} - c/\beta)^2 \quad \Leftrightarrow \\
\frac{\beta}{4}(1 - Y_1 - v_0^{\min})(1 - 2c/\beta - Y_1 + v_0^{\min}) &\geq (p_1 - c)(1 - Y_1 - v_0^{\min}) \quad \Leftrightarrow \\
\frac{\beta}{4}(1 - 2c/\beta - Y_1 + v_0^{\min}) &\geq p_1 - c \quad \Leftrightarrow \quad \frac{\beta}{4}(1 + 2c/\beta - Y_1 + v_0^{\min}) \geq p_1
\end{aligned}$$

(recall that $Y_1$ in $\tilde{r}_0^i$ and $Y_1^{-i}$ in $\tilde{r}_1^i$ are equivalent here). The left-hand-side of the last inequality does not exceed $(p_1)_2$ because $\frac{\beta}{4}(1 + 2c/\beta - Y_1 + v_0^{\min}) < \frac{\beta}{4}[2(1 - Y_1) + 2c/\beta] = \beta(p_1)_0 < (p_1)_2$. However, this implies that $p_1 < (p_1)_2$ which is incompatible with case (b') because it requires $p_1 \geq (p_1)_2$. Thus, there is a no-MFC BR that dominates $\tilde{r}_1^i$ when $\frac{c}{\beta} < v_0^{\min} < 1 - Y_1$. Combining (i) and (ii) with conditions of case (b'), we get the statement of the lemma.

## 8.3 Proof of Lemma 4 (deviation from no-MFC RESE into MFC)

The form of $r_1^i$ follows from general formula (4) and the expressions for the first-period sales given in §2.2 with $Y_1 = y_1^i$. There are two cases: (1) $1 - y_1^i \geq v_0^{\min}$ with $Q_1 = y_1^i$ (MFC-reseller $i$ has no sales in the second period) and $Q_0 = (1 - y_1^i - v_0^{\min}) \wedge Y_0$, and (2) $1 - y_1^i < v_0^{\min}$ with $Q_1 = (1 - p_1) \wedge y_1^i$ and $Q_0 = 0$.

(1.1) If $1 - y_1^i - v_0^{\min} \geq Y_0$, then $Q_0 = Y_0$ implying $Q = Y$ (sales in the first period only) with $r_1^i = (p_1 - c)y_1^i$.

(1.2) If $1 - y_1^i - v_0^{\min} < Y_0$, which is possible only if $Y_0 > 0$, we have $Q_0 = 1 - y_1^i - v_0^{\min}$ and $Y > 1 - v_0^{\min} = Q$. This subcase implies sales in the second period with $p_2 = s \vee [\beta(1-Y)]$, which exceeds $p_1$ if $\beta(1-Y) \geq p_1 \Leftrightarrow y_1^i \leq 1 - Y_0 - p_1/\beta$. This inequality may hold for a non-trivial $y_1^i$ only if $Y_0 < 1 - p_1/\beta$, which, in turn, is possible in this subcase if $v_0^{\min} > p_1/\beta$. Then $r_1^i$ is

$$r_1^i = \begin{cases} (p_1 - c)y_1^i, & \text{if } y_1^i \leq 1 - Y_0 - p_1/\beta, \\ (p_2 - c)y_1^i, & \text{if } y_1^i > 1 - Y_0 - p_1/\beta, \end{cases} \tag{27}$$

which is continuous in $y_1^i$ (since $p_2 = \beta(1 - Y_0 - y_1^i) = p_1$ at $y_1^i = \breve{y}_1^i \triangleq 1 - Y_0 - p_1/\beta$) and concave. Since $(p_1 - c)y_1^i$ increases in $y_1^i$, a profit maximizing reseller would not consider inventory levels below $\breve{y}_1^i$ implying $p_2 \leq p_1$.

(2.1) If $1 - p_1 \geq y_1^i$, then $Q_1 = y_1^i$. If $Y_0 = 0$, there are no sales in the second period and $r_1^i = (p_1 - c)y_1^i$. If $Y_0 > 0$, profit $r_1^i$ is the same as in (1.2).

(2.2) If $1 - p_1 < y_1^i$, then $Q_1 = 1 - p_1$, $Q_0 = 0$, and there are sales in the second period with $r_1^i = (p_2 - c)y_1^i$ and $p_2 < \beta p_1$.

Summarizing all cases, we conclude that $r_1^i$ is defined by (27) if $v_0^{\min} > \frac{p_1}{\beta}$ and, otherwise, by

$$r_1^i = \begin{cases} (p_1 - c)y_1^i, & \text{if } y_1^i \leq 1 - Y_0 - v_0^{\min}, \\ (p_2 - c)y_1^i, & \text{if } y_1^i > 1 - Y_0 - v_0^{\min}. \end{cases}$$

If $Y_0 = 0$ (reseller $i$ is a monopolist), $v_0^{\min}$ in the formula above for $r_1^i$ is substituted by $p_1$ because sales in the second period occur only when $y_1^i$ exceeds $1 - p_1$ (unlike the case of $Y_0 > 0$ for which the second period sales occur whenever $y_1^i$ is not less than $1 - v_0^{\min}$).

Throughout the proof, we use the following notation:

$$\breve{y}_1^i \triangleq 1 - Y_0 - v_0^{\min}, \qquad \breve{r}_1^i \triangleq (p_1 - c)\breve{y}_1^i,$$
$$\tilde{y}_1^i \triangleq (1 - Y_0 - c/\beta)/2 = z/2, \qquad \tilde{r}_1^i \triangleq \beta(\tilde{y}_1^i)^2.$$

Quantity $\breve{y}_1^i$ is the maximizer of $(p_1 - c)y_1^i$ on the interval $y_1^i \leq 1 - Y_0 - v_0^{\min}$, and $\tilde{y}_1^i$ is an unconstrained maximizer of $(p_2 - c)y_1^i = [\beta(1 - Y_0 - y_1^i) - c]y_1^i$.

We can rule out any $y_1^i \leq 1 - Y_0 - p_1$ as a candidate for the optimal solution under rational expectations leading to $v_0^{\min} > p_1$ (which may take place only if $\rho > 0$ and $Y_0 > 0$). Indeed, for such $y_1^i$, we have $1 - Y \geq p_1$, resulting in $p_2 \geq \beta p_1$ and rational $v_0^{\min} = \left(p_1 \wedge \frac{p_1 - \rho p_2}{1 - \rho\beta}\right) \vee 1 = p_1$, a contradiction. On the other hand, any $y_1^i > 1 - Y_0 - p_1$ would result in $p_2 < \beta p_1$.

Hence, under rational expectations, an optimal inventory level of reseller $i$ that deviates into MFC may lead to the following three principal cases:

(a) Reseller $i$ has positive inventory but sales occur only in the first period and $\bar{\alpha}(1) = 0$, leading to $v_0^{\min}(\bar{\alpha}(1), \bar{p}_2(1)) = p_1$. The inventory and profit are $y_1^i = \breve{y}_1^i|_{\bar{\alpha}=0} = 1 - Y_0 - p_1 = z - \frac{z_0}{2}$ and $r_1^i = \breve{r}_1^i|_{\bar{\alpha}=0} = (p_1 - c)\breve{y}_1^i|_{\bar{\alpha}=0}$. This inventory level can be a candidate for optimum *only if* it is positive, i.e., $Y_0 < 1 - p_1$ or $z > \frac{z_0}{2}$. Since $\breve{y}_1^i|_{\bar{\alpha}=0} \geq \breve{y}_1^i|_{\bar{\alpha}=1} \vee \breve{y}_1^i$, the necessary and sufficient conditions for $\breve{y}_1^i$ to be the maximizer, include $z > \frac{z_0}{2}$ and either $\breve{y}_1^i|_{\bar{\alpha}=0} \geq \tilde{y}_1^i$ (i.e., $z - \frac{z_0}{2} \geq \frac{z}{2} \Leftrightarrow z \geq z_0$) or $\breve{y}_1^i|_{\bar{\alpha}=0} < \tilde{y}_1^i$ (i.e., $z < z_0$) and $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$.

(b) Reseller $i$ has positive inventory while sales occur in both periods, $\bar{\alpha}(1) = 1$, and any $v_0^{\min}(\bar{\alpha}(1), \bar{p}_2(1))$ from the interval $[p_1, 1]$ is plausible *a priori*. Since, under rational expectations, $p_2 < \beta p_1 \leq p_1$, case (b) involves reimbursements and the general expression for the profit of reseller $i$ is $r_1^i = (p_2 - c)y_1^i$ regardless of the specific value of $v_0^{\min}$. The maximum must be internal, can only be at $\tilde{y}_1^i$, and the corresponding profit is $\tilde{r}_1^i$. Rationality of expectations requires that $\tilde{y}_1^i > 1 - Y_0 - p_1$, which implies $p_2 < \beta p_1$, and can be written either as $p_1 > \frac{c}{\beta} + \frac{z}{2}$, or $z >$

$2(1-Y_0-p_1) = 2z - 2(p_1 - \frac{c}{\beta})$, or $z < z_0$. Since $v_0^{\min} \geq p_1$ and $p_1 \leq p_1/\beta$, inequality $z < z_0$ implies that $\tilde{y}_1^i$ does belong to the range of inventory levels where the profit function has the form $r_1^i = (p_2 - c)y_1^i$, i.e., $\tilde{y}_1^i > \breve{y}_1^i|_{\bar{\alpha}=1} = 1 - v_0^{\min} - Y_0$ and $\tilde{y}_1^i > \check{y}_1^i$. Thus, the inventory $\tilde{y}_1^i$ is the maximizer under rational expectations if and only if

- (feasibility) $\tilde{y}_1^i > 0$ or, equivalently, $Y_0 < 1 - c/\beta \Leftrightarrow z > 0$;
- (rationality) $z < z_0$; and
- (optimality) either $\breve{y}_1^i|_{\bar{\alpha}=1} \leq (\check{y}_1^i)^+$ (i.e., profit function is continuous and concave, and there is no need to compare profits), or $\breve{y}_1^i|_{\bar{\alpha}=1} > (\check{y}_1^i)^+$ (profit is discontinuous at $\breve{y}_1^i|_{\bar{\alpha}=1}$) and $\tilde{r}_1^i \geq \breve{r}_1^i|_{\bar{\alpha}=1} = \breve{y}_1^i|_{\bar{\alpha}=1}(p_1 - c)$.

(c) MFC reseller $i$ chooses to exit the market by setting $y_1^i = 0$ if and only if *neither* $\breve{y}_1^i|_{\bar{\alpha}=0} > 0$ nor $\tilde{y}_1^i > 0$ can be a candidate for the optimal solution. This outcome is possible if and only if $z \leq \frac{z_0}{2}$ ($Y_0 \geq 1 - p_1$ — positive $\breve{y}_1^i|_{\bar{\alpha}=0}$ with sales only in the first period is impossible) *and* either $z \leq 0$ ($Y_0 \geq 1 - c/\beta$), or $z \geq z_0$, or both (if $p_1 \leq c/\beta$) hold — positive $\tilde{y}_1^i$ is impossible under rational expectations. For $\frac{z_0}{2} > 0$, $z_0 \leq z \leq \frac{z_0}{2}$ cannot hold, and only $z \leq 0$ is compatible with a weaker condition $z \leq \frac{z_0}{2}$. For $\frac{z_0}{2} \leq 0$, at least one of $z \leq 0$ or $z \geq z_0$ holds for any $z \leq \frac{z_0}{2}$. A combination of these two cases yields the condition of part (c). Any $z < \frac{z_0}{2}$ results in the second period sales and rational expectations $\bar{\alpha} = 1$. If $z = \frac{z_0}{2}$, sales take place in the first period only with $\bar{\alpha} = 0$. For $Y_0 = 0, y_1^i = 0$ is never optimal since $Y_0 \geq 1 - p_1$ may hold only for $p_1 = 1$ and then $\tilde{y}_1^i = \frac{1}{2}(1 - c/\beta) > 0$ satisfies $z < z_0$, which becomes $1 - c/\beta > 0$.

It remains to show the equivalence of the above necessary and sufficient conditions in parts (a) and (b) to the corresponding conditions in the statement of the lemma.

*Part (a.1)* If $z_0 \leq 0$ (i.e., $p_1 \leq c/\beta$), then $z > \frac{z_0}{2}$ implies $z \geq z_0$ and there is no need to compare profits.

*Part (a.2).* If $z_0 > 0$ (i.e., $p_1 > c/\beta$), it is still possible that $z \geq z_0$ and there is no need to compare profits. Consider $\frac{z_0}{2} < z < z_0$, where the profits need to be compared. In this case, $\breve{y}_1^i|_{\bar{\alpha}=0}$ is not less profitable than $\tilde{y}_1^i$ if and only if $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$, which is a quadratic inequality in $z$ : $\frac{\beta}{4}z^2 - z(p_1 - c) + \frac{z_0}{2}(p_1 - c) \leq 0$ with the discriminant $(p_1-c)^2 - \beta\,(p_1 - c/\beta)\,(p_1 - c) = (p_1 - c)p_1(1-\beta) \geq 0$ (strict inequality if $\beta < 1$), and the roots of the corresponding equation $z_{1,2} = \frac{2}{\beta}\left[p_1 - c \mp \sqrt{(p_1-c)p_1(1-\beta)}\right]$, implying that $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$ is equivalent to $z_1 \leq z \leq z_2$. The roots and $z_0$ are such that $\frac{z_0}{2} < z_1 \leq z_0 \leq z_2$. Indeed, the LHS of the quadratic inequality in $z$ is $\frac{\beta}{4}z_0^2 > 0$ at $z = \frac{z_0}{2}$ and non-positive at $z = z_0 : \beta(p_1 - c/\beta)^2 - 2(p_1 - c/\beta)(p_1 - c) + (p_1 - c/\beta)(p_1 - c) \leq 0 \Leftrightarrow p_1\beta - c \leq p_1 - c$, which always holds. Hence, since in case (a) the comparison of $\breve{r}_1^i|_{\bar{\alpha}=0}$ with $\tilde{r}_1^i$ is relevant only in the range $\frac{z_0}{2} < z \leq z_0$, we can conclude that $\breve{r}_1^i|_{\bar{\alpha}=0} \geq \tilde{r}_1^i$ if $z \geq z_1$. This inequality includes as a particular case the condition $z \geq z_0$ for $p_1 > c/\beta$, when $\breve{y}_1^i|_{\bar{\alpha}=0}$ is optimal without comparing the profits.

*Part (b), possible values of* $v_0^{\min}$. As shown above, feasibility of $\tilde{y}_1^i$ and rationality of expectations require $z$ be in the range $0 < z < z_0$. It remains to specify the conditions of optimality of $\tilde{y}_1^i$. These conditions depend on $\breve{y}_1^i|_{\bar{\alpha}=1} = 1 - Y_0 - v_0^{\min}$, which equals $1 - p_1$ if $Y_0 = 0$. In this case, $\breve{y}_1^i|_{\bar{\alpha}=1} = \breve{y}_1^i|_{\bar{\alpha}=0}$ and, by part (a), the condition of optimality is $z \leq z_1$.

*Consider* $Y_0 > 0$. Denote $V(z) \triangleq \frac{p_1 - \rho c - \rho\beta z/2}{1-\rho\beta}$. Then, in part (b),

$$v_0^{\min} = p_1 \vee \left(\frac{p_1 - \rho\beta(1 - Y_0 - z/2)}{1-\rho\beta} \wedge 1\right) = p_1 \vee [V(z) \wedge 1]\,. \tag{28}$$

Given $0 < z < z_0$, the possible values of $v_0^{\min}$ include the following subcases.

$v_0^{\min} = p_1$ if $p_1 - \rho c - \rho\beta z/2 \le p_1 - \rho\beta p_1 \Leftrightarrow \rho\beta p_1 - \rho c \le \rho\beta z/2$, which holds either if $\rho = 0$ or $\rho > 0$ and $\rho\beta\,(p_1 - c/\beta) \le \rho\beta z/2$. The last inequality contradicts $z < z_0$, therefore $v_0^{\min} = p_1$ may hold only if $\rho = 0$. Thus, for $\rho = 0$, $\breve{y}_1^i|_{\bar\alpha=1} = \breve{y}_1^i|_{\bar\alpha=0}$, and, again, by part (a), the condition of optimality is $z \le z_1$.

*Consider* $\rho > 0$. Then $v_0^{\min} > p_1$ if $p_1 < 1$ and $v_0^{\min} = 1$ iff either $p_1 = 1$ or $p_1 < 1$ and

$$p_1 - \rho c - \rho\beta z/2 \ge 1 - \rho\beta \Leftrightarrow z \le 2(p_1 - \rho c + \rho\beta - 1)/(\rho\beta). \tag{29}$$

In this case, $\breve{y}_1^i|_{\bar\alpha=1} = -Y_0 < 0$.

*Part (b), condition $z \le \tilde{z}_1$ ($\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$).* Recall that in the range $0 < z < z_0$, inventory $\tilde{y}_1^i$ is optimal iff (I) there is no need to compare profits ($\breve{y}_1^i|_{\bar\alpha=1} \le (\tilde{y}_1^i)^+$), or (II) $\breve{y}_1^i|_{\bar\alpha=1} > (\tilde{y}_1^i)^+$ and $\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$.

(I). *Consider* $\tilde{y}_1^i \le 0$. Condition $\breve{y}_1^i|_{\bar\alpha=1} \le 0$ trivially holds for $v_0^{\min} = p_1 = 1$.

For $p_1 < 1$, condition $\breve{y}_1^i|_{\bar\alpha=1} \le 0$ is equivalent to $v_0^{\min} \ge 1 - Y_0$, or, in terms of $z$, $v_0^{\min} - c/\beta \ge z$, which, for $V(z) \in (p_1, 1]$, becomes

$$p_1 - \rho c - \rho\beta z/2 \ge (z + c/\beta)(1 - \rho\beta) \Leftrightarrow z(1 - \rho\beta + \rho\beta/2) \le p_1 - c/\beta \Leftrightarrow z \le \frac{z_0}{2 - \rho\beta}.$$

Combining the last inequality with (29), we obtain that inequality $\breve{y}_1^i|_{\bar\alpha=1} \le 0$ is equivalent to $z \le \frac{2}{\rho\beta}(p_1 - \rho c + \rho\beta - 1) \vee \frac{z_0}{2-\rho\beta}$, where the RHS is the maximum from the two bounds because $z \ge 2(p_1 - \rho c + \rho\beta - 1)/(\rho\beta)$ is equivalent to $V(z) \in (p_1, 1]$. Both bounds are always strictly less than $z_0$. Indeed, $2 - \rho\beta > 1$, and

$$2[p_1 - \rho c + \rho\beta - 1]/(\rho\beta) < z_0 \Leftrightarrow p_1 - \rho c + \rho\beta - 1 < \rho\beta p_1 - \rho c \Leftrightarrow 1 - \rho\beta > p_1(1 - \rho\beta),$$

which holds for any $p_1 < 1$.

*Consider* $\tilde{y}_1^i > 0$. Inequality $\breve{y}_1^i|_{\bar\alpha=1} \le \tilde{y}_1^i$ is $1 - Y_0 - v_0^{\min} \le 1 - Y_0 - p_1/\beta \Leftrightarrow p_1/\beta \le v_0^{\min}$, which, for $\rho > 0$, may hold only if $p_1 \le \beta$. Under this condition, $p_1/\beta \le v_0^{\min}$ is equivalent to $p_1(1 - \rho\beta)/\beta \le p_1 - \rho c - \rho\beta z/2$ or $z \le 2\,(p_1 - \rho c + \rho p_1 - p_1/\beta)\,/(\rho\beta)$.

(II). This subcase contains two conditions: $z \in (0, z_0) \cap \left\{z : \breve{y}_1^i|_{\bar\alpha=1} > (\tilde{y}_1^i)^+\right\}$ and $\breve{r}_1^i|_{\bar\alpha=1} \le \tilde{r}_1^i$. The last inequality, after the substitution of $\breve{y}_1^i|_{\bar\alpha=1} = z + c/\beta - v_0^{\min}$

$$= z + \frac{c}{\beta} - \frac{p_1 - \rho c - \rho\beta z/2}{1 - \rho\beta} = \frac{c}{\beta} - \frac{p_1 - \rho c}{1 - \rho\beta} + \frac{z}{2}\frac{2 - \rho\beta}{1 - \rho\beta}$$

into $\breve{r}_1^i|_{\bar\alpha=1} = \breve{y}_1^i|_{\bar\alpha=1}(p_1 - c)$, becomes $\left(\frac{z}{2}\frac{2-\rho\beta}{1-\rho\beta} + \frac{c/\beta - \rho c - p_1 + \rho c}{1-\rho\beta}\right)(p_1 - c) \le \frac{\beta}{4}z^2$ or

$$z^2 - \frac{2z(p_1 - c)(2 - \rho\beta)}{\beta(1 - \rho\beta)} + \frac{2z_0(p_1 - c)}{\beta(1 - \rho\beta)} \ge 0. \tag{30}$$

The LHS of (30) at $z = z_0$ is $z_0\left\{z_0 - 2(p_1 - c)/\beta\right\} = 2z_0\left\{\beta p_1 - c - p_1 + c\right\}/\beta \le 0$. Inequality is strict if $\beta < 1$, i.e., the roots $\tilde{z}_1$ and $\tilde{z}_2$ of the corresponding equation always exist and $\tilde{z}_1 \le z_0 \le \tilde{z}_2$.

It is easy to show that $\frac{z_0}{2-\rho\beta} < \tilde{z}_1$. Indeed, the LHS of (30) with $z = \frac{z_0}{2-\rho\beta}$ becomes $\left(\frac{z_0}{2-\rho\beta}\right)^2 > 0$. The bound $\frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1]$ that corresponds to $v_0^{\min} = 1$ is also strictly below $\tilde{z}_1$ because, by (28) with $z = \tilde{z}_1$, we have $v_0^{\min} < 1$. Otherwise, by (29) with $z = \tilde{z}_1$, $v_0^{\min} = 1$ implying $\breve{r}_1^i|_{v_0^{\min}=1} < 0$, and equality $\breve{r}_1^i|_{v_0^{\min}=1} = \tilde{r}_1^i$ cannot hold.

The bound $\frac{2}{\rho\beta}\left(p_1 - \rho c + \rho p_1 - p_1/\beta\right)$, which results from $p_1/\beta \le v_0^{\min}$, also does not exceed $\tilde{z}_1$ since the LHS of inequality $\breve{r}_1^i|_{\bar{\alpha}=1} \le \tilde{r}_1^i$, i.e., $\left(z + c/\beta - v_0^{\min}\right)(p_1 - c)$, is decreasing in $v_0^{\min}$, and the inequality holds for $v_0^{\min} = p_1/\beta$. Indeed,

$$\left(z + \frac{c}{\beta} - \frac{p_1}{\beta}\right)(p_1 - c) \le \frac{\beta}{4}z^2 \Leftrightarrow \frac{\beta}{4}z^2 - z(p_1 - c) + \frac{(p_1 - c)^2}{\beta} \ge 0 \Leftrightarrow \left(\frac{z\sqrt{\beta}}{2} - \frac{p_1 - c}{\sqrt{\beta}}\right)^2 \ge 0.$$

Hence, we have $0 < \frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1] \vee \frac{z_0}{2-\rho\beta} \vee \frac{2}{\rho\beta}\left(p_1 - \rho c + \rho p_1 - p_1/\beta\right) < \tilde{z}_1 \le z_0 \le \tilde{z}_2$, and, combining all the conditions in case (b) for $\rho > 0$ and $Y_0 > 0$, $\tilde{y}_1^i$ is optimal iff $z > 0$ and either $p_1 = 1$ or $p_1 < 1$ and $z \le \tilde{z}_1$. Namely, for $p_1 < 1$, a positive $\tilde{y}_1^i$ is optimal in the subrange $z \le \bar{z}$, where

$$\bar{z} = \begin{cases} \frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1] \vee \frac{z_0}{2-\rho\beta}, & \text{if } p_1 > \beta, \\ \frac{2}{\rho\beta}[p_1 - \rho c + \rho\beta - 1] \vee \frac{z_0}{2-\rho\beta} \vee \frac{2}{\rho\beta}\left(p_1 - \rho c + \rho p_1 - p_1/\beta\right), & \text{if } p_1 \le \beta \end{cases}$$

because in this subrange a positive $\breve{y}_1^i|_{\bar{\alpha}=1}$ cannot be feasible and rational; and $\tilde{y}_1^i$ is optimal in the subrange $z \in (\bar{z}, \tilde{z}_1]$ because both positive $\tilde{y}_1^i$ and $\breve{y}_1^i|_{\bar{\alpha}=1}$ are feasible and rational, and $\breve{r}_1^i|_{\bar{\alpha}=1} \le \tilde{r}_1^i$.

When positive $\tilde{y}_1^i, \breve{y}_1^i|_{\bar{\alpha}=0}$, and $\breve{y}_1^i|_{\bar{\alpha}=1}$ are feasible and rational, it can be shown that $z_1 \le \tilde{z}_1$ with equality only if $\bar{\alpha} = 0, Y_0 = 0$, or $\rho = 0$. Indeed, if we assume that $z_1 > \tilde{z}_1$ for $\bar{\alpha} = 1, Y_0 > 0$, and $\rho > 0$, then, for any $z \in (\tilde{z}_1, z_1)$, we have $\breve{r}_1^i|_{\bar{\alpha}=1} > \tilde{r}_1^i$ and, at the same time, $\breve{r}_1^i|_{\bar{\alpha}=0} < \tilde{r}_1^i$, contradicting the fact that $\breve{r}_1^i|_{\bar{\alpha}=1}$ is decreasing in $v_0^{\min}$. Therefore, $\tilde{y}_1^i$ is optimal for any $z \in [z_1, \tilde{z}_1]$ when $\bar{\alpha} = 1$, and $\breve{y}_1^i|_{\bar{\alpha}=0}$ is optimal for $z \ge z_1 = \tilde{z}_1$ when $\bar{\alpha} = 0$.

The expression for $\tilde{z}_1$ (for $n > 1$) is $\tilde{z}_1 = \frac{1}{2}\left[\frac{2(p_1-c)(2-\rho\beta)}{\beta(1-\rho\beta)} - \sqrt{\frac{4(p_1-c)^2(2-\rho\beta)^2}{\beta^2(1-\rho\beta)^2} - \frac{8z_0(p_1-c)}{\beta(1-\rho\beta)}}\right]$, which can be written as $\tilde{z}_1 = \frac{(p_1-c)(2-\rho\beta)}{\beta(1-\rho\beta)}\left[1 - \sqrt{1 - \frac{2z_0\beta(1-\rho\beta)}{(p_1-c)(2-\rho\beta)^2}}\right]$. For $\beta = 1$, this formula yields $\tilde{z}_1 = z_1 = z_0 = z_2$. Indeed, $\tilde{z}_1|_{\beta=1} = \frac{(2-\rho)(p_1-c)}{1-\rho}\left[1 - \sqrt{1 - \frac{4(1-\rho)}{(2-\rho)^2}}\right]$, where the expression under the square root is $\rho^2/(2-\rho)^2$ resulting in $\tilde{z}_1|_{\beta=1} = \frac{p_1-c}{1-\rho}[2-\rho-\rho] = 2(p_1-c) = z_1|_{\beta=1} = z_0|_{\beta=1} = z_2|_{\beta=1}$. In the general case, $\tilde{z}_1|_{\beta=1} \ne \tilde{z}_2|_{\beta=1}$.

If $p_1 = c/\beta$, then $\tilde{z}_1 = z_1 = z_0 = 0$, where $\tilde{z}_1 = z_1 = 0$ since the free coefficient in both quadratic equations for $z_{1,2}$ and for $\tilde{z}_{1,2}$ contains $z_0$, which is zero in this case.

$v_0^{\min}$ from part (b) is not decreasing in $\rho$ if $V(z) \in (p_1, 1]$ since $\frac{\partial v_0^{\min}}{\partial\rho} = \frac{A}{(1-\rho\beta)^2}$, where $A = \{(-c - \beta z/2)(1 - \rho\beta) + \beta(p_1 - \rho c - \rho\beta z/2)\} = (c + \beta z/2)(\rho\beta - 1 - \rho\beta) + \beta p_1 = \beta\left(p_1 - z/2 - c/\beta\right)$, which, as shown above, is positive for $v_0^{\min} \ge p_1$. If $V(z) \ge 1$, $v_0^{\min}$ is constant in $\rho$ and equals one.

## 8.4 Proof of Lemma 5 (condition of N3.1 does not hold)

For $n = 1$, $\breve{r}_1^i \equiv r^{*,M2}$ (Theorem 1) and $r^{*,N3}$ is equal to the profit of a deviator from M2 into no-MFC with sales in both periods (see (26) with $Y^{-i} = 0$ in the proof of Lemma 3). By Theorem 1, M2 exists in the area that intersects with the area of NA3 existence, which, for $n = 1$, requires $p_1 > c/\beta$, and, by part (2.2) of Theorem 1, M2 exists for $\beta \to 1 - 0$ and $\rho = (1 - \sqrt{1-\beta})/\beta \to 1$, implying $CB_2 = 0$, and $p_1 \le P_{22}$, where $P_{22} > c/\beta$, yielding a non-empty range $c/\beta < p_1 \le P_{22}$, where $P_{22}|_{n=1} = \frac{1}{2}\left[1 + c + \sqrt{(1-\beta)(1 - c^2/\beta)}\right]$ (Corollary 1), which, for $c \to 0$ and $\beta \to 1$ goes to $\frac{1}{2}$. For these inputs, inequality $\frac{n-1}{n}Y^* \le 1 - c/\beta - z_1$ is $z_1 \le 1$, which is equivalent to $p_1 \le P_{22} = \frac{1}{2}$. Indeed, $z_1 \le 1 \Leftrightarrow p_1 - c - \sqrt{(p_1-c)p_1(1-\beta)} \le \frac{\beta}{2} \Leftrightarrow p_1 \le \frac{1}{2}$.

At the same time, by Lemma 2 with $x = v_0^{\min} > c/\beta$, inequality $r^{*,M2} \ge \tilde{r}_0^i$ may be strict for $\beta < 1$, resulting, by continuity of $r^{*,N3}$ and $\breve{r}_1^i$, in violation of $r^{*,N3} \ge \breve{r}_1^i$ in the vicinity of the area with $\beta = \rho = 1$ and $c = 0$.

## 8.5 Proof of Lemma 6 (condition of N3.2 and N4.2 holds)

Under NA3, by Theorem 2, $Y^* > 1 - p_1$ for any $n \geq 1$, and, under NA4, $Y^* \geq 1 - s/\beta > 1 - p_1$ for any $n > 1$ since $p_2 = s$ and $p_1 > s/\beta$. Therefore, there exists $N > 1$ such that $\frac{n-1}{n} Y^* > 1 - p$ for any $n \geq N$. If $\beta < 1$ and $n > 1$, the lower bound for $p_1$ in both NA3 and NA4, $\frac{nc}{\beta+n-1}$, is strictly less than $c/\beta$ and approaches $c$ with $n \to \infty$. Hence, for $\beta < 1$ and $n \geq N$, there exist a non-empty range for $p_1$ such that $\frac{nc}{\beta+n-1} \vee 1 - \frac{n-1}{n} Y^* < p_1 \leq c/\beta$.

## 8.6 Proof of Lemma 7 (condition for profits of N3.2 holds)

For $n = 1, \rho = 0$, and $\beta = 1$, the NA3 $p_1-$range is $p_1 \in (c, 1)$, hence the condition $p_1 > c/\beta$ holds and inequality $\frac{n-1}{n} Y^* \geq 1 - c/\beta - \tilde{z}_1$ becomes $p_1 - c \geq \frac{1-c}{2}$ or $p_1 \geq \frac{1+c}{2}$. Since, in this case, $v^* = p_1$, the inventory, $p_2^*$, and the profit are $Y^* = \frac{1}{2}(2 - c - p_1)$, $p_2^* = \frac{1}{2}(c + p_1), r^{*,N3} = (p_1 - c)(1 - p_1) + (p_2^* - c)[p_1 - \frac{1}{2}(c + p_1)]$. Then inequality $r^{*,N3} \geq \tilde{r}_1^i$ takes the form

$$(p_1 - c)(1 - p_1) + \frac{1}{4}(p_1 - c)^2 \geq \frac{1}{4}(1 - c)^2 \Leftrightarrow 4(p_1 - c)(1 - p_1) \geq (1 - p_1)(1 + p_1 - 2c)$$

or $p_1 \geq \frac{1+2c}{3}$, which holds for any $p_1 \geq \frac{1+c}{2}$ since $c < 1$.

## 8.7 Proof of Lemma 8 (N4 existence, sufficient conditions)

Lemma 4 yields necessary conditions of existence of positive $\breve{y}_1^i$ and $\tilde{y}_1^i$ as upper bounds on $Y_0$. Namely, $Y_0 < 1 - p_1$ for $\breve{y}_1^i$, and $Y_0 < 1 - c/\beta$ $(z > 0)$ for $\tilde{y}_1^i$. At the same time, by Theorem 3, $Y^* > 1 - s/\beta$, which is a lower bound: $Y_0 > \frac{n-1}{n}(1 - s/\beta)$. Hence, a deviation *from* NA4 *into* one of the forms of MFC is impossible if both $Y_0$-ranges are empty, i.e., $\left(1 - \frac{1}{n}\right)(1 - s/\beta) \geq (1 - p_1) \vee (1 - c/\beta)$, which can be written as $\frac{1}{n}(1 - s/\beta) \leq (p_1 - s/\beta) \wedge \frac{c-s}{\beta}$ or $n \geq \frac{\beta-s}{p_1\beta-s} \vee \frac{\beta-s}{c-s}$.

## 8.8 Proof of Lemma 9 (Total equilibrium customer surplus)

The total surplus is $\Sigma = \Sigma_1 + \Sigma_2$, where $\Sigma_1 = \int_{v^*}^1 (v - p_1)dv$ without MFC and $\Sigma_1 = \int_{p_1}^1 (v - p_2^*)dv$ with MFC. When there are second-period sales, $\Sigma_2 = \int_{\frac{p_2^*}{\beta}}^{v^*} (\beta v - p_2^*)dv = \int_{p_2}^{\beta v^*} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta}$ without MFC and $\Sigma_2 = \int_{p_2}^{\beta p_1} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta}$ with MFC. Straightforward integration specifies $\Sigma$ as follows.

*Part (1).* $\Sigma_1^{M1} = \int_{p_1}^1 (v - p_2^*)dv = \left(v^2/2 - p_2^* v\right)\big|_{p_1}^1 = \frac{1}{2} - p_2^* - p_1^2/2 + p_2^* p_1 + p_1 - p_1 + p_1^2/2 - p_1^2/2$ $= (1-2p_1+p_1^2)/2 + p_1(1-p_1) - p_2^*(1-p_1) = (1-p_1)^2/2 + (1-p_1)(p_1 - p_2^*)$, and $\Sigma_2^{M1} = \int_{p_2^*}^{\beta p_1} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} =$ $\left(\tilde{v}^2/2 - p_2^*\tilde{v}\right)/\beta\big|_{p_2^*}^{\beta p_1} = \left[\beta^2 p_1^2/2 - \beta p_2^* p_1 - (p_2^*)^2/2 + (p_2^*)^2\right]/\beta = (\beta p_1 - p_2^*)^2/(2\beta)$.

*Part (2).* Since $v^* = p_1$ under N2, $\Sigma^{M2} = \Sigma^{N2} = \Sigma_1^{M2} = \int_{p_1}^1 (v - p_1)dv = 1/2 - p_1 - p_1^2/2 + p_1^2 = (1 - p_1)^2/2$.

*Part (3).* Since $v^* = 1$ under N1, $\Sigma^{N1} = \Sigma_2^{N1} = \int_{p_2^*}^{\beta} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} = (\beta - p_2^*)^2/(2\beta)$.

*Part (4).* Under N3 and 4, $\Sigma_1 = \int_{v^*}^1 (v - p_1)dv = \frac{1}{2} - p_1 - (v^*)^2/2 + p_1 v^* + p_1^2/2 - p_1^2/2 =$ $(1-p_1)^2/2 - (v^* - p_1)^2/2$, and $\Sigma_2 = \int_{p_2^*}^{\beta v^*} (\tilde{v} - p_2^*)\frac{d\tilde{v}}{\beta} = (\beta v^* - p_2^*)^2/(2\beta)$. Under N3 and N4, inequality $p_2^{*,N4} = s < p_2^{*,N3}$ always hold. In both N3 and N4, $v^* = \frac{p_1 - \rho p_2^*}{1 - \rho\beta}$, which is decreasing in $p_2^*$ except the case $\rho = 0$ when $v^{*,N3} = v^{*,N4} = p_1$. Therefore, $v^{*,N4} > v^{*,N3}$ for any $\rho \in (0, 1)$.

## 8.9 Proof of Lemma 10 (NA3, $n = 1$)

By part NA3 of Theorem 2 with $n = 1$, condition (a) always holds, and the equation (22) in $Y$ reduces to $Y\left[Y - \frac{(\beta-c)(1-\rho\beta)+\beta(1-p_1)}{\beta(2-\rho\beta)}\right] = 0$ yielding $Y^* = \frac{(\beta-c)(1-\rho\beta)+\beta(1-p_1)}{\beta(2-\rho\beta)} = 1 - \frac{\beta p_1+c(1-\rho\beta)}{\beta(2-\rho\beta)}$, $v^* = \frac{1}{1-\rho\beta}\left[p_1 - \rho\frac{\beta p_1+c(1-\rho\beta)}{2-\rho\beta}\right] = \frac{1}{1-\rho\beta}\frac{2p_1-p_1\rho\beta-\rho\beta p_1-c\rho(1-\rho\beta)}{2-\rho\beta} = \frac{2p_1-\rho c}{2-\rho\beta}$, and $p_2^* = \beta(1-Y^*) = \frac{\beta p_1+c(1-\rho\beta)}{2-\rho\beta} = c+\frac{\beta p_1-c}{2-\rho\beta}$. Substitution into the formula for $r^{*,N3}$ results in $r^{*,N3} = (p_1-c)\frac{2(1-p_1)-\rho(\beta-c)}{2-\rho\beta}+\frac{\beta p_1-c}{2-\rho\beta}\left(\frac{2p_1-\rho c}{2-\rho\beta} - \frac{\beta p_1+c(1-\rho\beta)}{\beta(2-\rho\beta)}\right)$, where the bracket in the last term is $\frac{2\beta p_1-\rho\beta c-\beta p_1-c(1-\rho\beta)}{\beta(2-\rho\beta)} = \frac{\beta p_1-c}{\beta(2-\rho\beta)}$, leading to the expression in the lemma. The expression for $\Sigma^*$ results from direct substitution of $v^*$ and $p_2^*$ into the general formula (Lemma 9).

## 8.10 Proof of Lemma 11 (NA3, NA4, $p_1 = \beta$)

*NA3.* The equation in $Y$ with $p_1 = \beta$ yields the expression for $Y^*$. With this $Y^*$ and $p_1 = \beta$ we have $v^* = \beta\frac{n+1-\rho\beta-\rho[1-\rho\beta-n(1-c/\beta)(1-\rho\beta)+n\beta]}{(1-\rho\beta)(n+1-\rho\beta)} = \beta\frac{1-\rho+n[1+\rho(1-c/\beta)]}{n+1-\rho\beta}$. Then $r^{*,N3}$ is

$$r^{*,N3}|_{p_1=\beta} = \frac{1}{n}\left[(\beta-c)(1-v^*) + (\beta(1-Y^*)-c)(Y^*-1+v^*)\right] = \frac{Y^*}{n}\left[\beta - c - \beta(Y^*-1+v^*)\right],$$

which after substitutions for $Y^*$ and $Y^*-1+v^* = \frac{1}{n+1-\rho\beta}\{n(1-c/\beta)-(1-\beta)\}$ becomes

$$r^{*,N3}|_{p_1=\beta} = \frac{(1-c/\beta)(1-\rho\beta)+1-\beta}{(n+1-\rho\beta)^2}\{(\beta-c)(n+1-\rho\beta) - \beta[n(1-c/\beta)-(1-\beta)]\},$$

where $\{\cdot\} = (1-\rho\beta)(\beta-c)+\beta(1-\beta)$, yielding the expression for $r^{*,N3}|_{p_1=\beta}$. Condition $c/\beta < CB_{N2}$ results from the $p_1$-lower bound $p_1 > \frac{nc}{\beta+n-1}$; the upper bound is $\beta < 1$ for $\rho = 0$ and, for $\rho > 0$, it can be written as $c/\beta > 1 - \frac{n+1}{n\rho\beta}(1-\beta) = CB_{N1}$. Condition (a) is specified for $p_1 = \beta$. Using the expression for $Y^*$, inequality $Y^* < 1 - s/\beta$ is equivalent to $n[(1-c/\beta)(1-\rho\beta) - \beta + s/\beta] < (1-\rho\beta)(1-s/\beta)$, which always holds if $[\cdot] \le 0$ or $\beta^2 - s - (\beta-c)(1-\rho\beta) \ge 0$. Since the LHS is increasing in $\rho$, this inequality holds for any $\rho \ge 0$ if $c - s \ge \beta(1-\beta)$. Otherwise, $[\cdot] > 0$ and $Y^* < 1 - s/\beta$ for any $n < (1-\rho\beta)(1-s/\beta)/[\cdot]$.

*NA4.* The expressions for $Y^*, v^*, r^{*,N4}$ and condition (a) follow directly from Theorem 3 with $p_1 = \beta$. Condition (b) is $\frac{n-1}{n}\frac{\beta Y^*}{c+\beta v^*-2s} \ge 1$, which, after substitution for $Y^*$ and $v^*$, becomes $(c-s)\left(c-s+\frac{\beta^2-s}{1-\rho\beta}\right) \le \left(\frac{n-1}{n}\right)^2\beta(\beta-s)\left(1-\frac{\beta-\rho s}{1-\rho\beta}\right)$.

The requirement in condition (c) that conditions (a) and (b) do not hold and the deviator profit is strictly decreasing in the interval corresponding to $p_2 > s$ is equivalent, as shown in Bazhanov, Levin, and Nediak (2015), to the following: "there are no real roots of equation

$$2Y^3 - \left(2 - v^* - c/\beta + \frac{n-1}{n}Y^*\right)Y^2 + (1-p_1/\beta)(1-v^*)\frac{n-1}{n}Y^* = 0 \tag{31}$$

in the interval $(1-v^*, 1-s/\beta)$." If $p_1 = \beta$, the single root of (31) is $\tilde{Y} = \frac{1}{2}\left(2 - v^* - c/\beta + \frac{n-1}{n}Y^*\right)$. This root is not in the interval $(1-v^*, 1-s/\beta)$ if and only if either $\tilde{Y} \le 1 - v^*$, which, using $\frac{n-1}{n}Y^* = \left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}(1-v^*)$, becomes $1 - c/\beta \le (1-v^*)\left[1 - \left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}\right]$, or $\tilde{Y} \ge 1 - s/\beta$, which, in the same way, becomes $1 + \frac{c-2s}{\beta} \le (1-v^*)\left[1 + \left(\frac{n-1}{n}\right)^2\frac{\beta-s}{c-s}\right]$. When $\tilde{Y} \in (1 - v^*, 1-s/\beta)$, NA4 exists if $r^{*,N4} \ge \tilde{r}^i = \left(\tilde{Y} - \frac{n-1}{n}Y^*\right)\left[\beta\left(2 - v^* - \tilde{Y}\right) - c + \frac{(p_1-\beta)(1-v^*)}{\tilde{Y}}\right]\Big|_{p_1=\beta} = \left(\tilde{Y} - \frac{n-1}{n}Y^*\right)\left[\beta(2-v^*-\tilde{Y}) - c\right]$, where $\beta(2-v^*-\tilde{Y}) - c = \frac{\beta}{2}\left(2 - v^* - c/\beta - \frac{n-1}{n}Y^*\right)$ and

$\tilde{Y} - \frac{n-1}{n}Y^* = \frac{1}{2}\left(2 - v^* - c/\beta - \frac{n-1}{n}Y^*\right) = \frac{1}{2}\left[(1-v^*)\left(1 - \left(\frac{n-1}{n}\right)^2 \frac{\beta - s}{c-s}\right) + 1 - c/\beta\right]$. These expressions yield
$\tilde{r}^i = \frac{\beta}{4}\left[(1-v^*)\left(1 - \left(\frac{n-1}{n}\right)^2 \frac{\beta-s}{c-s}\right) + 1 - c/\beta\right]^2$.

## Supplementary References


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