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Abstract

This study provides a new approach for implied volatility indices forecasting. We assess whether non-parametric techniques provide better predictions of implied volatility compared to standard forecasting models, such as AFRIMA and HAR. A combination of Singular Spectrum Analysis (SSA) and Holt-Winters (HW) model is applied on eight implied volatility indices for the period from February, 2001 to July, 2013. The findings confirm that the SSA-HW provides statistically superior one trading day and ten trading days ahead implied volatility forecasts world widely. Model-averaged forecasts suggest that the forecasting accuracy is further enhanced, for the ten-days ahead, when the SSA-HW is combined with an ARI(1,1) model. Additionally, the trading game reveals that the SSA-HW and the ARI-SSA-HW are able to generate significant average positive net daily returns in the out-of-sample period. The results are important for option pricing, portfolio management, value-at-risk and economic policy.

Keywords: Implied Volatility, Volatility Forecasting, Singular Spectrum Analysis, ARFIMA, HAR, Holt-Winters, Model Confidence Set, Combined Forecasts.

JEL codes: C14; C22; C52; C53; G15.

1. Introduction and review of the literature

Volatility refers to the dispersion of the returns around their average value over time. Thus, the notion of volatility refers to the amount of risk about the size of changes in a stock's value. The extant literature has long established the importance of studying and forecasting volatility of financial markets (see, *inter alia*, Andersen et al., 2003,2005; Christodoulakis, 2007; Fuertes et al., 2009; Charles, 2010; Barunik et al., 2016). Its importance lies on the fact that volatility forecasting is important for investors, portfolio managers, asset valuation, hedging strategies, risk management purposes, as well as, policy makers. Investors and portfolio managers seek a prediction of their future uncertainty in order to estimate a specific upper limit of risk that are willing to accept, to reach optimal portfolio decisions and to form appropriate hedging strategies.

Forecasting volatility is the single most important component for pricing derivative products, such as option contracts. Unless derivatives contracts are priced correctly, hedging strategies can be expensive and not yield the desired outcome. Nowadays, volatility can be the underlying asset of derivatives products, such as in the VIX futures contracts. Thus, forecasting the expected volatility of the underlying asset helps for the correct valuation of these contracts.

The Basel accords have made volatility forecasting a key component for risk management purposes. According to Basel II, financial institutions are required to estimate their capital requirements and for such estimates the calculation of the Value-at-Risk (VaR) is necessary. One of the most important inputs in the VaR estimations is the volatility forecast.

Forecasting volatility is also important for policy makers. Stock market volatility informs monetary policy decisions of central banks, such as the Federal Reserve Bank and the Bank of England. Similarly, volatility forecast is able to measure the expectations of the financial markets regarding the (un)successful outcome of fiscal and/or monetary policy decisions. The aforementioned arguments deem the importance of forecasting volatility accurately.

The finance literature has extensively examined the concept of stock market volatility forecasting. The vast majority of the volatility forecasting studies have concentrated their attention on the use of models which are variants of GARCH models (see, *inter alia*, Bollerslev et al., 1994; Degiannakis, 2004; Hansen and Lunde, 2005), stochastic volatility models (see, among others, Deo, 2006; Yu, 2012) or realized volatility models (Andersen et al., 2003, Andersen et al., 2005). These models forecast current looking volatility and they demand the use of past stock prices.

Nevertheless, a strand in the literature maintains that implied volatility indices are better predictors of the future volatility and thus, forecasting implied volatility rather than conditional or realized volatility is more important. This superior predictive ability of implied volatility has been pointed out since the late 70s and early 80s by the studies of Chiras and Manaster (1978) and Beckers (1981). In addition, studies by Fleming et al. (1995), Christensen and Prabhala (1998), Fleming (1998), Blair et al. (2001), Simon (2003) and Giot (2003) have provided evidence that implied volatility is more informative when we forecast stock market volatility. More recently, findings by Degiannakis (2008a) and Frijns et al. (2010) second the aforementioned claims.

Methodologically, on one hand, the literature provides evidence that the fractionally integrated autoregressive moving average models outperform the volatility forecasts that are produced by the GARCH and stochastic volatility models (Koopman et al., 2005). Degiannakis (2008b) also maintains that due to the long memory property of volatility, the ARFIMA framework is suitable for estimating and forecasting the logarithmic transformation of volatility. On the other hand, some argue that heterogeneous autoregressive models (HAR) are more successful in forecasting volatility due to the fact that they are parsimonious and they are able to capture the long-memory that is observed in volatility (see, *inter alia*, Andersen et al., 2007; Corsi, 2009; Busch et al., 2011; Fernandes et al., 2014, Sevi, 2014). Nevertheless, Angelidis and Degiannakis (2008) provide evidence that there is not a unique model that is offering better predictive ability than others in all instances.

The aim of this study is to assess whether a new approach, namely a non-parametric framework such as the Singular Spectrum Analysis (SSA) type model, can provide better forecast of the implied volatility. More specifically, we use an SSA-type model to forecast several implied volatility indices and we compare these forecasts against those made by HAR and ARFIMA models, as well as by four naïve models; i.e. I(1), ARI(1,1), FI(1) and ARFI(1,1) and model-averaging.

SSA is regarded as a powerful non-parametric technique for time series analysis and forecasting. In short, SSA decomposes a time series into the sum of a small number of independent and interpretable components such as a slowly varying trend, oscillatory components and noise (Hassani et al., 2009). The main advantage of SSA-type models is that they do not require any statistical assumptions in terms of the stationarity of the series or the distribution of the residuals. In fact, SSA uses bootstrapping to generate the confidence intervals that are required for the evaluation of the forecasts (Hassani and Zhigljavsky, 2009; Vautard et al., 1992).

Overall, SSA has been applied widely is various disciplines, such as biology, medical studies, physics (see, for example, Sanei et al., 2011; Ghosi et al., 2009). Recently, this method has attracted a considerable attention in the economic literature, (see for example, Hassani et al., 2009; Beneki et al., 2012). The limited empirical applications of SSA on economic and financial series provide significant evidence of its superior predictive ability against the standard forecasting models, such as the ARIMA-type and GARCH-type models.

Interestingly enough, no studies have utilised this method to forecast stock market volatility, despite the fact that since the early 2000 Thomakos et al. (2002) maintain that SSA is able to decompose volatility series more effectively, capturing both the market trend and a number of market periodicities, and thus an important extension to the existing literature would be to assess the forecasting ability of SSA in the context of volatility modeling.

Therefore, the aim of this study is to assess the 1-day and 10-days ahead forecasting ability of an SSA-type model (SSA-HW) on a series of implied volatility indices, competing against two conventional model frameworks, namely, an ARFIMA-type and a HAR-type model and four naïve models. The 1-day and 10-days ahead predictions are chosen, given that these time horizons apply to certain investors and portfolio managers, as well as, the Basel II requirements for VaR forecasting.

The contribution of the paper is described succinctly. First, we provide an alternative model to forecast implied volatility; second, we open new avenues for the use of SSA-type in finance and third, we contribute to the non-parametric literature of financial markets.

The study provides empirically significant evidence that the SSA-HW model achieves more accurate forecasts for the 1-day and 10-days ahead, compared to the ARFIMA, HAR, SSA and HW models, as well as, four naïve models. Model-averaged forecasts reveal that the forecasting accuracy of the SSA-HW is enhanced for the 10-days ahead if it is combined with the ARI(1,1) model. The predictive accuracy is assessed by the Mean Squared Error (MSE) and the Mean Absolute Error

(MAE) loss functions, the Model Confidence Set forecasting evaluation procedure, as well as, the Direction-of-Change criterion. Finally, we assess the forecasting ability of the models by means of a trading game. The results reveal that investors are able to generate significant positive average net profits using the SSA-HW and the ARI-SSA-HW models.

The rest of the paper is structured as follows. In section 2, we describe the data of the study. Section 3 illustrates the forecasting models. Section 4 provides a detailed explanation of the implied volatility forecasts estimation procedure and section 5 describes the adopted forecasting evaluation method. Section 6 analyses the empirical findings, whereas Section 7 concludes the study.

2. Data description

We use daily data from 1st of February, 2001 up to 9th of July, 2013 (i.e. 3132 trading days) from eight implied volatility indices. The implied volatilities are the following: VIX (S&P500 Volatility Index – US), VXN (Nasdaq-100 Volatility Index – US), VXD (Dow Jones Volatility Index – US), VSTOXX (Euro Stoxx 50 Volatility Index – Europe), VFTSE (FTSE 100 Volatility Index – UK), VDAX (DAX 30 Volatility Index – Germany), VCAC (CAC 40 Volatility Index – France) and VXJ (Japanese Volatility Index - Japan). The stock markets under consideration represent six out of the ten most important stock markets of the world, in terms of capitalisation. In addition, these markets are among the most liquid markets of the world. Thus, we maintain that their implied volatility indices are representative of the world's stock market uncertainty. The data have been extracted from *Datastream*®. As we aim for a common sample of the aforementioned implied volatility indices, the starting data of the sample period was dictated by the availability of the data of the VXN index.

Figure 1 and Table 1 exhibit the series under consideration and list their descriptive statistics, respectively.

[FIGURE 1 HERE]
[TABLE 1 HERE]

From Figure 1 we observe that all implied volatility indices display very similar patterns. For example, it is evident that during the Great Recession of 2007-2009 all indices reached their highest level over the sample period. In addition, the magnitude of these peaks is comparable across indices. Furthermore, we observe two more peaks in 2003 and 2011. The volatility spikes in 2003 can be attributed to the second war in Iraq, whereas a plausible explanation of the 2011 peak in stock market volatilities can be found in the European debt crisis which initiated in Greece but spread to other countries such as Ireland, Spain and Portugal, as well. The US debt-ceiling crisis of the same year could have aggravated higher uncertainty in world stock markets.

From Table 1 we notice that average volatility is of similar size across indices, with the exception being the VXN and VXD indices, which exhibit the highest and lowest average volatility, respectively. Furthermore, the VXN index also exhibits the highest level of standard deviation, suggesting that it is the most volatile index. All series under examination are stationary and heteroscedastic, as suggested by the ADF and ARCH LM tests, respectively.

3. Methodology and Models

3.1. IV-ARFIMA Model

The long memory property of implied volatility indices makes the Autoregressive Fractionally Integrated Moving Average, or ARFIMA, model an appropriate framework for multiple-step-ahead implied volatility index, IV_t, predictions. The IV-ARFIMA(*k,d,l*) model for the discrete time t real-valued process $\log\left(IV_t\right)$ is utilized in the form[1]:

$$\begin{gathered}\left(1-C(L)\right)\left(1-L\right)^d\left(\log\left(IV_t\right)-\boldsymbol{\beta}'\mathbf{x}_{t-1}\right)=\left(1+D(L)\right)\varepsilon_t,\\ \varepsilon_t\sim N\left(0,\sigma_\varepsilon^2\right),\end{gathered}\tag{1}$$

where $\mathbf{x}_{t-1}=\left[1\quad y_{t-1}\quad d_{t-1}y_{t-1}\right]'$ is the vector of explanatory variables, $\boldsymbol{\beta}$ is a vector of unknown parameters, and $C(L)=\sum_{i=1}^{k}c_iL^i$, $D(L)=\sum_{i=1}^{l}d_iL^i$ are polynomials with the parameters $c_1,...,c_k,d_1,...,d_l$ for estimation. The y_t denotes the log-returns of the underlying stock index and the d_t is a binary dummy variable, i.e. $d_t=1$, if $y_t>0$ and zero otherwise[2].

[1] The ARFIMA model was initially developed by Granger and Joyeux (1980).

[2] The dummy variable models the asymmetric relationship between volatility and lagged log-return; i.e. Degiannakis (2008b).

3.2. IV-HAR Model

The Heterogeneous Autoregressive, or HAR, model relates the current trading day's implied volatility with the daily, weekly and monthly implied volatilities. The autoregressive structure of the volatility over different interval sizes attempts to replicate the different perspectives that market participants may have on their investment horizon, which is the basic idea of the heterogenous market hypothesis in economic theory; see Müller et al. (1997).

The IV-HAR, model for the discrete time real-valued process $\log(IV_t)$ is defined as[3]:

$$\log(IV_t) =$$

$$w_0 + w_1 \log(IV_{t-1}) + w_2\left(5^{-1}\sum_{j=1}^{5}\log(IV_{t-j})\right) + w_3\left(22^{-1}\sum_{j=1}^{22}\log(IV_{t-j})\right) + \varepsilon_t, \quad (2)$$

$$\varepsilon_t \sim N(0, \sigma_\varepsilon^2),$$

where the w_0, w_1, w_2, w_3 are the unknown parameters to be estimated[4].

3.3. IV-SSA-HW Model

The idea underlying the combination forecast of SSA-HW is the exploitation of SSA's sound decomposition capabilities which can then be combined with HWs' non-parametric forecasting capability. Whilst it is possible to build a combination forecast using any other time series analysis and forecasting technique, here we opted for SSA in combination with HW for three main reasons. Firstly, based on past experience in forecasting time series with increased volatility, HW has always been a close contender to SSA as it is able to model the fluctuations in past data and then provide sound predictions (see i.e. Hassani et al., 2013). The second reason is the fact that HW, like SSA, is a non-parametric technique. Accordingly, by combining together two non-parametric techniques we are able to clear out the need for assumptions that must be considered when adopting parametric techniques. Thirdly, the analysis of implied volatility time series shows its nonlinear in nature and as such, we had no reservations in selecting HW.

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3 The HAR model initially developed by Corsi (2009).

4 The HAR model could be extended to accommodate heteroscedasticity in the error term, as in Corsi et al. (2005). However, the modeling of volatility of realized volatility is out of the scope of the paper.

In this paper we decompose the implied volatility series using SSA and then we forecast each of the decomposed series using the HW model[5]. A description of the decomposition stage is presented below and Section 4.3 presents the HW forecasting algorithm. In the decomposition stage, the first step is referred to the embedding process and the construction of the trajectory matrix. Consider the implied volatility index time series IV_t of length $\breve{T}$. Embedding process maps the one dimensional time series IV_t into a multidimensional time series $X_1,..., X_K$ with vectors $X_i = [IV_i, IV_{i+1}, IV_{i+2},..., IV_{i+\mathbf{L}-1}]'$, where $\mathbf{L}$ is an integer such that $2 \le \mathbf{L} \le \breve{T} - 1$. The selection of the optimal window length $\mathbf{L}$ for decomposing the time series is based on the RMSE criterion[6]. The trajectory matrix, $\mathbf{X}$, is constructed such that $K = \breve{T} - \mathbf{L} + 1$; $\mathbf{X}$ is a Hankel matrix, i.e. elements along the diagonal *i+j* are equal:

$$\mathbf{X} = [X_1,..., X_r,..., X_K] = (x_{i,j})_{i,j=1}^{\mathbf{L},K} = \begin{pmatrix} IV_1 & IV_2 & IV_3 & \cdots & IV_K \\ IV_2 & IV_3 & IV_4 & \cdots & IV_{K+1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ IV_{\mathbf{L}} & IV_{\mathbf{L}+1} & IV_{\mathbf{L}+2} & \cdots & IV_{\breve{T}} \end{pmatrix}. \quad (3)$$

The second step of the decomposition stage is known as singular value decomposition (SVD). In order to obtain the SVD of the trajectory matrix $\mathbf{X}$, we calculate $\mathbf{XX}'$ for which $\lambda_1,..., \lambda_{\mathbf{L}}$ denote the eigenvalues in decreasing order, and $U_1,..., U_{\mathbf{L}}$ represent the corresponding eigenvectors. The SVD step then provides the singular values r (the second parameter of SSA), such that $\mathbf{X} = X_1 + ... + X_r$. Thereafter, we use diagonal averaging to transform the components of the matrix $\mathbf{X}$ into a Hankel matrix which can then be converted into time series $IV_{t,1} \ldots . IV_{t,r}$, where $IV_{t,r}$ refers to the decomposed time series from the original implied volatility index. Having decomposed the implied volatility series, we apply the HW algorithm (Hyndman et al., 2013) to forecast the decomposed series from $IV_{t,1} \ldots . IV_{t,r}$.

[5] The SSA-HW model is estimated in R software.

[6] The implied volatility series is divided into training and test sets. Decomposition of the training set is evaluated for different window lengths and eigenvalues. The results from the best decomposition as determined via the training approach is then used to decompose the test set of each index and then forecasted individually with HW prior to combining these decomposed forecasts for which the out-of-sample forecasting errors are reported.

4. Forecasting IV indices

4.1. IV-ARFIMA model

We define the orders of *k* and *l* of the IV-ARFIMA(*k,d,l*) model based on the Schwarz (1978) information criterion (for the total sample)[7], which is reported in Table 2.

[TABLE 2 HERE]

The IV-ARFIMA(2*,d,1*) model is estimated for all the IV indices, except for the VCAC, VXN and VXJ, for which the IV-ARFIMA(*2,d,2*) has been selected.

For the ARFIMA(2*,d,1*) model the one-step-ahead logarithmic implied volatility, $\log\left(IV_{t+1|t}\right)$, is estimated as:

$$\log\left(IV_{t+1|t}\right)=\left(\hat{c}_1+\hat{c}_2L\right)\log\left(IV_t\right)+\left(1-\hat{c}_1L-\hat{c}_2L^2\right)\hat{\boldsymbol{\beta}}'\mathbf{x}_t+\sum_{j=1}^{\infty}A_jL^{j-1}\varepsilon_{t|t}+\sum_{j=0}^{\infty}A_jL^j\hat{d}_1\varepsilon_{t|t} \quad (4)$$

where $A_j\equiv\dfrac{\Gamma\left(j+\hat{d}\right)}{\Gamma\left(\hat{d}\right)\Gamma\left(j+1\right)}$, and $\varepsilon_{t|t}$ denotes the residual term at time *t* estimated based on the information set at time *t*, or $\varepsilon_{t|t}=\log\left(IV_t\right)-\left(\hat{c}_1+\hat{c}_2L\right)\log\left(IV_{t-1}\right)$ $-\left(1-\hat{c}_1L-\hat{c}_2L^2\right)\hat{\boldsymbol{\beta}}'\mathbf{x}_{t-1}-\sum_{j=0}^{\infty}A_jL^j\varepsilon_{t|t}-\sum_{j=0}^{\infty}A_jL^j\hat{d}_1\varepsilon_{t-1|t}$.[8] The infinite expansion of the fractional differencing operator is approximated as (see Xekalaki and Degiannakis, 2010, Baillie, 1996): $\sum_{j=0}^{\infty}\left(\dfrac{\Gamma\left(j+d\right)}{\Gamma\left(d\right)\Gamma\left(j+1\right)}L^j\right)=1+\dfrac{1}{1!}dL+\dfrac{1}{2!}d\left(1+d\right)L^2-\dots$. The parameters of the models $[\boldsymbol{\beta},d,c_1,\dots,c_k,d_1,\dots,d_l]$ are re-estimated at each trading day.

For $\varepsilon_t\sim N\left(0,\sigma_\varepsilon^2\right)$, the $\exp\left(\varepsilon_t\right)$ is log-normally distributed. Thus, the (unbiased) estimator of $IV_{t|t}$ equals to $\exp\left(\log\left(IV_{t|t}\right)+\tfrac{1}{2}\hat{\sigma}_\varepsilon^2\right)$. Consequently, the one-trading-day-ahead implied volatility is predicted as:

$$IV_{t+1|t}=\exp\left(\log\left(IV_{t+1|t}\right)+\tfrac{1}{2}\hat{\sigma}_\varepsilon^2\right). \quad (5)$$

[7] The models were estimated in the ARFIMA package of Ox; see Doornik and Ooms (2006). The Schwarz information criterion (SBC) is computed from the Akaike information criterion (AIC) provided by ARFIMA package: $SBC=AIC+\breve{T}^{-1}q\left(\log\left(\breve{T}\right)-2\right)$, for $\breve{T}$ and q denoting the number of observations and parameters of the models (including the residuals' variance), respectively.

[8] Accordingly, the $\varepsilon_{t-1|t}$ denotes the residual term at time *t-1* estimated based on the information set at time *t*.

The 10-step-ahead logarithmic implied volatility is estimated as[9]:

$$\log\left(IV_{t+10|t}\right) = \left(\hat{c}_1 + \hat{c}_2 L\right)\log\left(IV_{t+9|t}\right) + \sum_{j=10}^{\infty} A_j L^{j-10}\varepsilon_{t|t} + \sum_{j=9}^{\infty} A_j L^{j-9}\hat{d}_1\varepsilon_{t|t}\,. \tag{6}$$

4.2. IV-HAR model

Correspondingly, the IV-HAR model forecast is computed as:

$$IV_{t+1|t} = \exp\left(\hat{w}_0 + \hat{w}_1\log\left(IV_t\right) + \hat{w}_2\left(5^{-1}\sum_{j=1}^{5}\log\left(IV_{t-j+1}\right)\right) + \hat{w}_3\left(22^{-1}\sum_{j=1}^{22}\log\left(IV_{t-j+1}\right)\right) + \tfrac{1}{2}\hat{\sigma}_\varepsilon^2\right) \tag{7}$$

The 10-days-ahead logarithmic implied volatility, based on IV-HAR model, is computed as:

$$\log\left(IV_{t+10|t}\right) = \hat{w}_0 + \hat{w}_1\log\left(IV_{t+9|t}\right) + \hat{w}_2\left(5^{-1}\sum_{j=1}^{5}\log\left(IV_{t-j+10|t}\right)\right) + \hat{w}_3\left(22^{-1}\left(\sum_{j=1}^{9}\log\left(IV_{t-j+10|t}\right) + \sum_{j=10}^{22}\log\left(IV_{t-j+10}\right)\right)\right). \tag{8}$$

4.3. IV-SSA-HW model

We aggregate the Holt-Winters forecasts obtained for time series $IV_{t,1} \ldots . IV_{t,r}$ to arrive at the SSA-HW forecasts. We propose the combination of the forecasts attained via HW for each decomposed component via aggregation. The underlying idea behind this approach is to decompose first a given series, so to enables us to identify the various fluctuations, which were previously hidden under the overall series. Secondly, the approach is concerned with forecasting each of these decompositions with HW so that the model is able to capture all fluctuations, which were hidden previously, and then combine all these forecasts via aggregation to come up with the SSA-HW forecast. Depending on the characteristics of the time series, the Hyndman et al. (2013) algorithm automatically selects either the multiplicative or the additive HW method. The additive HW framework for forecasting implied volatility is presented as:

$$\hat{l}_t = \hat{\alpha}\left(IV_t - \hat{s}_{t-m}\right) + \left(1 - \hat{\alpha}\right)\left(\hat{l}_{t-1} + \hat{b}_{t-1}\right) \tag{9}$$

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[9] The s-step-ahead forecast, for s>2, is $\log\left(IV_{t+s|t}\right) = \left(\hat{c}_1 + \hat{c}_2 L\right)\log\left(IV_{t+s-1|t}\right) + \sum_{j=s}^{\infty} A_j L^{j-s}\varepsilon_{t|t} + \sum_{j=s-1}^{\infty} A_j L^{j-s+1}\hat{d}_1\varepsilon_{t|t}$.

$$\hat{b}_t = \hat{\beta}\left(\hat{l}_t - \hat{l}_{t-1}\right) + \left(1 - \hat{\beta}\right)\hat{b}_{t-1}$$

$$\hat{s}_t = \hat{\gamma}\left(IV_t - \hat{l}_t - \hat{b}_{t-1}\right) + \left(1 - \hat{\gamma}\right)\hat{s}_{t-m} ,$$

where $\hat{l}_t$ is the smoothing equation for the level, b_t is for the trend, s_t is the seasonal equation and *m* is used to denote the period of seasonality. The alternative, which is the multiplicative HW method has the form:

$$\hat{l}_t = \hat{\alpha}\left(IV_t / \hat{s}_{t-m}\right) + \left(1 - \hat{\alpha}\right)\left(\hat{l}_{t-1} + \hat{b}_{t-1}\right)$$

$$\hat{b}_t = \hat{\beta}\left(\hat{l}_t - \hat{l}_{t-1}\right) + \left(1 - \hat{\beta}\right)\hat{b}_{t-1} \tag{10}$$

$$\hat{s}_t = \hat{\gamma}\left(IV_t / \left(\hat{l}_t - \hat{b}_{t-1}\right)\right) + \left(1 - \hat{\gamma}\right)\hat{s}_{t-m} .$$

The additive HW one-step-ahead, $IV_{t+1|t}$, and 10-days-ahead, $IV_{t+10|t}$, implied volatility forecasts are computed as:

$$IV_{t+1|t} = \hat{l}_t - \hat{b}_t + \hat{s}_{t+1-m} \tag{11}$$

$$IV_{t+10|t} = \hat{l}_t - 10\hat{b}_t + \hat{s}_{t+10-m} , \tag{12}$$

respectively. By contrast, the multiplicative HW one-step-ahead, $IV_{t+1|t}$, and 10-days-ahead, $IV_{t+10|t}$, implied volatility forecasts are computed as:

$$IV_{t+1|t} = (\hat{l}_t - \hat{b}_t) * \hat{s}_{t+1-m} \tag{13}$$

$$IV_{t+10|t} = (\hat{l}_t - 10\hat{b}_t) * \hat{s}_{t+10-m} , \tag{14}$$

respectively.

4.4. Naïve models & Model-averaged Forecasts

As mentioned in section 1, apart from the three models presented in this section we further employ four naïve models, namely, the I(1), ARI(1,1), FI(1) and ARFI(1,1), which serve as benchmarks, as well as, the HW and SSA models, separately. For brevity, we do not develop these models here.

Furthermore, the intention of this study is not to develop a *horse-race* forecasting exercise, thus we employ model-averaged forecasts combining only the best naïve model with the HAR, ARFIMA and SSA-HW. In addition, since the aim of the study is to assess whether the non-parametric models of SSA and HW, as well as their combination, can outperform the parametric models we also proceed with the model-averaged forecast of the HAR-ARFIMA model. Forecasting literature states (i.e. Favero and Aiolfi, 2005, Samuels and Sekkel, 2013, Timmermann, 2006) that model-averaged forecasts improve upon forecasts based on a single model i) with equal weight averaging working particularly well and ii) fewer models included in the combination provides more accurate forecasts.

5. Forecasting Evaluation

5.1. Model Confidence Set

The training period of the models is $\tilde{T}$ =1000 days, i.e. from 02/02/2001 until 28/01/2005. The remaining T =2132 days are used for the evaluation period of the out-of-sample forecasts. In order to proceed to the first out-of-sample forecast (i.e. $t+1$ forecast or day 1001) we train the models using the initial 1000 days. The use of a restricted sample size of 1000 trading days incorporates changes in trading behaviour more efficiently. For example Angelidis et al. (2004), Degiannakis et al. (2008) and Engle et al. (1993) provide empirical evidence that the use of restricted samples captures better the changes in market activity[10,11]. The total number of observations is $\breve{T} = \tilde{T} + T$. The forecasting accuracy of the models is gauged using two established loss functions, the MSE and the MAE, as presented in Table 3.[12]

[TABLE 3 HERE]

In addition, we employ the Model Confidence Set (MCS) procedure of Hansen et al. (2011). The MCS test determines the set of models that consists of the best models where best is defined in terms of a predefined loss function. In our case two loss functions are employed, namely the MSE and the MAE. The MCS compares the predictive accuracy of an initial set of M^0 models and investigates, at a predefined level of significance, which models survive the elimination algorithm. For $L_{i,t}$ denoting the loss function of model i at day t, and $d_{i,j,t} \equiv L_{i,t} - L_{j,t}$ is the evaluation differential for $i, j \in M^0$ the hypotheses that are being tested are:

$$H_{0,M} : E\left(d_{i,j,t}\right) = 0 \tag{15}$$

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[10] We have used various window lengths for the rolling window approach and the results remain qualitatively unchanged.

[11] We have also used a recursive approach, where for each subsequent forecast after the $t+1$ forecast we added to the training period an additional day. For example for the $t+2$ forecast we used $\tilde{T}+1$ daily observations. The results are qualitatively similar and they are available upon request.

[12] An alternative forecasting evaluation method is the Mincer and Zarnowitz (1969) regression, where the future VIX is regressed against the three different forecasts. The coefficients of the regressions are interpreted as the amount of information embedded in the different forecasts. The results are qualitatively similar.

for $\vee i, j \in M$, $M \subset M^{0}$ against the alternative $H_{1,M} : E(d_{i,j,t}) \neq 0$ for some $i, j \in M$. The elimination algorithm based on an equivalence test and an elimination rule, employs the equivalence test for investigating the $H_{0,M}$ for $\forall M \subset M^{0}$ and the elimination rule to identify the model i to be removed from M in the case that $H_{0,M}$ is rejected.[13]

5.2. Direction-of-Change

Furthermore, we consider the Direction-of-Change (DoC) forecasting evaluation technique. The DoC is particularly important for trading strategies as it provides an evaluation of the market timing ability of the forecasting models. The DoC criterion reports the proportion of trading days that a model correctly predicts the direction (up or down) of the volatility movement for the 1-day and 10-days ahead.

5.3. Portfolio performance

Finally, we compare the performance of each forecasting method based on a simple day-trading game. For the 1-day ahead forecasts, the trader takes a long position when the $t+1$ forecasted volatility of model i is higher compared to the actual volatility at time t. By contrast, if the $t+1$ forecasted volatility of model i is lower compared to the actual volatility at time t, then the trader takes a short position. Similarly, we construct the trading game for the 10-days ahead forecasts. Portfolio returns are computed as the average net daily returns over the investment horizon, which coincides with our out-of-sample forecasting period of T =2132 days. The transaction costs per unit for each trade are estimated to be between 0.6%-1.2% (see Jung, 2015).

6. Empirical findings

We consider the models' forecasting performance at two different horizons, namely 1-day and 10-days ahead. The MSE and MAE loss functions are presented in Tables 4 and 5, whereas Tables 6 and 7 display the MCS p-values.

[13] The Superior Predictive Ability (SPA) test of Hansen (2005) was also used to evaluate the forecasting accuracy of the competing models. Initially, the benchmark model for the SPA test was the ARI(1,1), which is the best naïve model. Subsequently, we used the IV-HAR and the IV-ARFIMA as benchmark models against the SSA-HW. The results confirm the MCS findings and they are available upon request.

[TABLE 4 HERE]
[TABLE 5 HERE]
[TABLE 6 HERE]
[TABLE 7 HERE]

Tables 4 and 5 provide evidence that the forecasts of the SSA-HW model outperform these produced by all naïve, SSA, HW, ARFIMA and HAR models. We observe that this holds true for both time horizons, i.e. 1-day and 10-days ahead, and all indices. The only exception for the 1-day ahead forecasts is the VFTSE, which according to the MAE the best forecast is achieved by the SSA. In addition, for the 10-days ahead forecast, the MAE (MSE) suggests that for the VCAC index the best forecast is obtained by the IV-ARFIMA (HW), whereas according to the MSE the best forecasts for the VTFSE and VXD are generated by the HW.

Despite the exceptions, it is clear that the use of the SSA-HW model, as opposed to the naïve, SSA, HW, ARFIMA or HAR models, provides a considerable improvement in the forecasting accuracy for all indices.

Next we compare the forecasting accuracy of the models using the MCS procedure. The results for the 1-day ahead forecasts (Table 6) suggest that in both the cases of the MAE and the MSE loss functions, the model that belongs to the confident set of the best performing models is only the SSA-HW. The only exception is the forecasts for VFTSE, where in the case of the MAE the best performing model is only the SSA, whereas in the case of MSE it is also the SSA that belongs to the set of the best performing models. For the 10-days ahead forecasts (Table 7), the SSA-HW is the only best one for VXJ and VXN, according to the MSE, whereas for all the other cases, SSA-HW belongs to the set of best models. Based on the MAE, the SSA-HW is the only best model for all the cases except the VCAC. For VCAC, the SSA-HW is among the ones that belong to the set of the best models.

Overall, evidence suggests that the use of the SSA-HW model gains a substantial improvement in forecasting accuracy, compared to the naïve, SSA, HW, ARFIMA and HAR models.

6.1. Model-averaged Forecasts

Next, we proceed with model-averaged forecasts in order to assess whether the inclusion of a naïve model could improve the performance of the competing models. According to Tables 4 and 5 the best naïve model is the ARI(1,1) model. Thus, we consider the following model-averaged forecasts, ARI-IV-ARFIMA, ARI-IV-HAR and ARI-SSA-HW. In addition, we also use the combined forecast of the ARFIMA-HAR models. Table 8 summarizes the results for the 1-day and 10-days ahead forecasts for both the MSE and the MAE.

[TABLE 8 HERE]

For the 1-day ahead forecasts, we observe that apart from the VFTSE forecast based on the MAE criterion, in all other cases none of the model-averaged forecasts is able to outperform the best performing single model, which is the SSA-HW. However, for the 10-days ahead forecasts, we notice that the inclusion of the ARI(1,1) model in the SSA-HW is able to produce superior predictions.

The MCS test including the model-averaged forecasts also verifies the findings of Table 8. More specifically, Table 9 suggests that for the 1-day ahead forecasts it is only the SSA-HW model that belongs to the set of the best performing models. Thus, none of the model-averaged forecasts improves the forecasting accuracy of the SSA-HW model. The only exception is the case of VFTSE where according to the MSE the ARI-SSA-HW also belongs among the best performing models and based on the MAE the ARI-SSA-HW is the only model that belongs to the best performing models.

Table 10, which reports the MCS results for the 10-days ahead forecasts, reveals that it is the ARI-SSA-HW model that is always among the best performing models, yet the SSA-HW also belongs to the set of the best models in four cases (VDAX, VFTSE, VIX and VSTOXX), whereas HW is also among the best models for the case of VFTSE. Our study presents empirical evidence that in the case of multi-days-ahead volatility forecasts the predictive accuracy of the model-averaged method is statistically significant improving.

[TABLE 9 HERE]

[TABLE 10 HERE]

Scatter plots in Figure 2 provide a visual representation of the relationship between actual and predicted implied volatility indices, indicatively, for the VIX index only. Panel A corresponds to the 1-day ahead forecasts, whereas Panel B exhibits the 10-days ahead forecasts. It is clear from these figures that for the 1-day ahead forecast it is the SSA-HW that produces rather slimmer plots (middle column), whereas for the 10-days ahead forecast it is the ARI(1,1)-SSA-HW (right column). The worse forecasts are produced by the FI(1,1) for both forecasting horizons. In addition, the SSA-HW for the 1-day ahead and the ARI(1,1)-SSA-HW model for the

10-days ahead forecasts are observed to have fewer outliers. In addition, it is worth noting that at the higher levels of volatility the SSA-HW (for the 1-day ahead) and the ARI(1,1)-SSA-HW (for the 10-days ahead) models are showing to produce less scattered points.

[FIGURE 2 HERE]

Overall, the SSA-HW model is superior to its competitors, especially for the 1-day ahead forecast, whereas the combination of SSA-HW with the ARI(1,1) is the best model for the 10-days ahead. We also assess the forecasting performance of our models in three sub-periods (pre-crisis period: January 2005 – November 2007, crisis period: December 2007 – June 2009, post-crisis period: July 2009 – July 2013) and the results are qualitatively similar. Due to brevity, these results are available upon request.

The ability of the SSA-HW to generate superior forecasts stems from the fact that it is able to utilise the advantages of each of the model's components. The SSA has the ability to decompose volatility indices into interpretable components. By decomposing the series using SSA, the interpretable components capture the dynamics of volatility indices, which can then be forecasted individually using HW. In turn, HW has the ability to provide accurate forecasts of trend and signal via exponentially weighted moving averages (Holt, 2004). Thus, HW's modelling capability is enhanced by the SSA filtering, which reduces the noise of the series. Therefore, instead of forecasting the index itself, we forecast each decomposed series prior to combining these forecasts.

In more simple terms, the superior performance reported by SSA-HW can be attributed to the fact that in the absence of filtering with SSA the trend and other signals within the index would be distorted owing to the noise. When one decomposes the series we are able to separate all such components into individual time series which will have its own and varying structure which was earlier hidden underneath the overall series. Thereby forecasting these individual series which have its own and varying individual structure with HW enables the model to capture the underlying fluctuations which would have been more difficult to capture in the absence filtering via SSA. This is further evident in the fact that neither SSA nor HW by itself is able to outperform the forecasts from SSA-HW at both horizons with the exception of once in each horizon.

Furthermore, SSA is more popular as a filtering technique as opposed to a forecasting technique. This can be one reason underlying its poor performance by itself as the SSA forecasting algorithm appears to encounter problems with modelling implied volatility even after filtering for noise. Note that when SSA filters for noise it forecasts the signal alone and this signal is not decomposed further like we do in the SSA-HW approach. At the same time, HW's poor performance is attributable to the fact that there is no filtering involved and as a result it encounters problems in picking up the true underlying signal which is distorted by the noisy implied volatility indices.

6.2. Direction of change

The DoC results are shown in Tables 11 and 12 for the 1-day and 10-days ahead, respectively. Table 11 shows that all forecasting models exhibit a good prediction of the DoC, since all scores are above the 50% level (with the only exception being the I(1) model), nevertheless the forecasting model with the highest prediction ability is the SSA-HW, followed by the ARI-SSA-HW and the SSA. More specifically, the SSA-HW and ARI-SSA-HW are capable of predicting accurately the DoC in 65-80% of the cases, depending on the volatility index. Similar findings are reported for the 10-days ahead forecasts (as shown in Table 12), where the SSA-HW and ARI-SSA-HW exhibit a very high predictive ability of the DoC, although the highest precision is attributed to the SSA-HW. In particular, the models are able to predict 65-88% of the directional changes on the implied volatilities. These results confirm the findings of the MCS, which provided evidence that the best model is the SSA-HW, followed by the ARI-SSA-HW.

[TABLES 11 and 12 HERE]

6.3. Portfolio performance

The results of the trading game are reported in Tables 13 and 14 for the 1-day and 10-days ahead, respectively.

[TABLES 13 and 14 HERE]

For the 1-day ahead (see Table 13), it is evident that the SSA, SSA-HW and the ARI-SSA-HW provide positive net returns, which are significantly higher than zero. The largest figures are observed for the SSA-HW, followed by the ARI-SSA-HW and then the SSA. Turning our attention to the 10-days ahead (see Table 14), we can make similar inference, as the only forecasting models that yield positive net returns are those of the HW, SSA-HW and ARI-SSA-HW. Nevertheless, we observe that statistically significant net returns are only feasible for the VIX and VSTOXX indices. Hence, these findings confirm the superior predictive ability of the SSA-HW.

7. Conclusions

The aim of this paper is to assess whether better forecasts for implied volatility indices can be obtained using an SSA-type model. More specifically, we generate 1-day and 10-days ahead forecasts based on the SSA-HW, ARFIMA and HAR models, as well as, four naïve models and compare their forecasting accuracy using the MSE and MAE evaluation criteria, the MCS procedure and the Direction-of-Change. In addition, we assess the forecasting ability of the models using a trading game. The data consisted of eight implied volatility indices for the period February, 2001 until July, 2013.

The results show that SSA-HW is a powerful tool for predicting implied volatility indices as it is able to exploit the advantages of two non-parametric methods. The forecasting accuracy tests reveal that the forecasts generated by the SSA-HW model outperform these by naïve, ARFIMA and HAR models. These findings hold for both the 1-day and 10-days ahead forecasts and for all implied volatility indices. When we proceed to model-averaged forecasts we reveal that the SSA-HW is still the best performing model for the 1-day ahead forecasts, whereas the inclusion of an ARI(1,1) model to the SSA-HW improves further its forecasting accuracy. The results of the trading game reveals that the SSA-HW and the ARI-SSA-HW could provide significant positive net returns over the out-of-sample period, although this primarily holds for the 1-day ahead and for the VIX and VSTOXX for the 10-days ahead. Overall, we maintain that this superior forecasting ability of the SSA-HW model is important to investors (e.g. for portfolio allocation decisions), portfolio managers (e.g. for Global Tactical Asset Allocation strategies), derivatives pricing, risk management purposes (e.g. for VaR calculations), as well as, policy makers (e.g. monetary policy decisions).

The use of SSA-HW enables users to overcome the parametric assumptions which restrict the applicability of many parametric models when applied to real world scenarios. As such we believe this proposed combination forecast which combines a renowned forecasting technique with an equally renowned filtering technique will enable users to achieve better outcomes in general when considered as a solution for other real world forecasting problems which go beyond implied volatility forecasts. In a world where the emergence of Big Data and the related noise continues to distort the signal in time series, the SSA-HW approach proposed and proven through this paper can be a useful tool in attaining reliable and accurate forecasts in the future. An interesting avenue for further study is to assess SSA forecasting ability using intra-day data.

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FIGURES

Figure 1: Implied Volatility Indices. The sample period runs from January, 2001 to July, 2013.

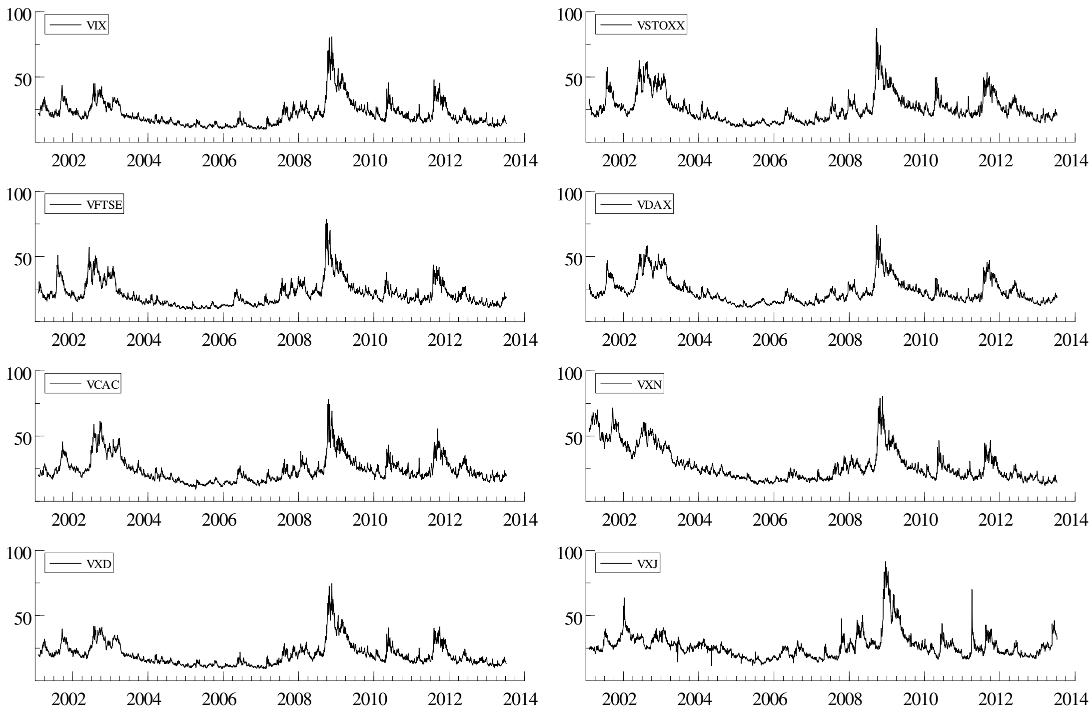
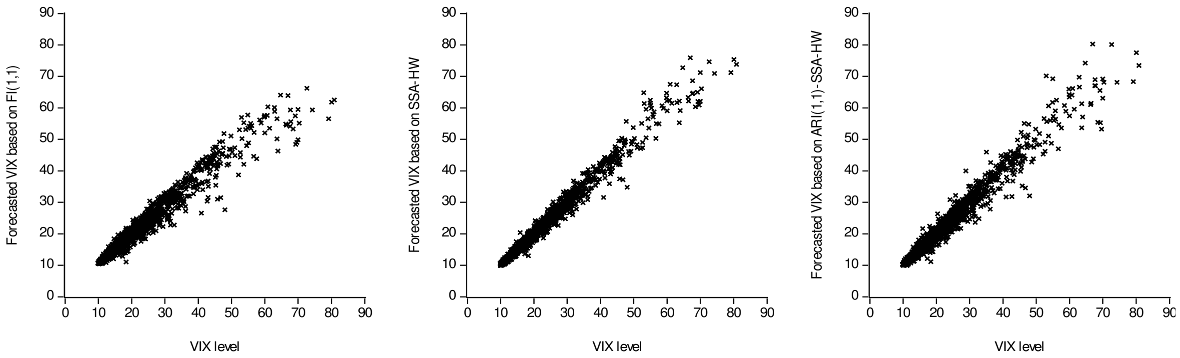
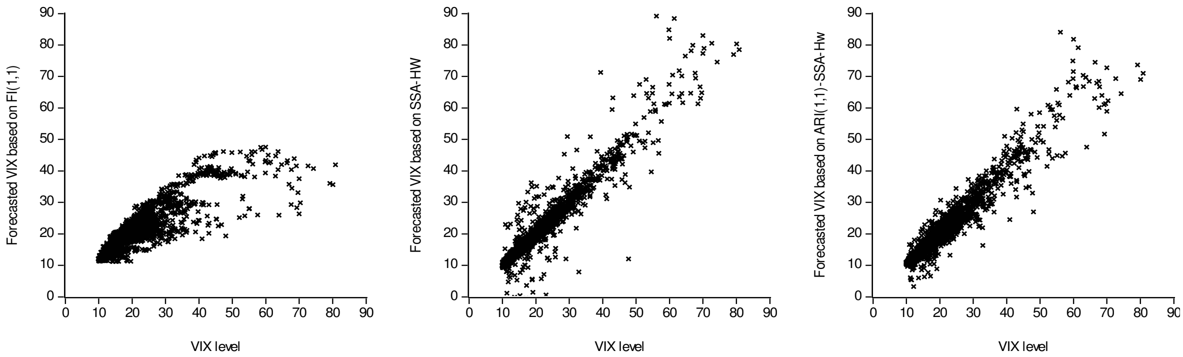


Figure 2: One-day and 10-days ahead forecasts scatter plots of the models for the VIX index. The sample period runs from January, 2005 to July, 2013.

1-day ahead forecasts



10-days ahead forecasts



Note: Columns from left to right present the scatter plots for FI(1,1), SSA-HW and ARI(1,1)-SSA-HW, respectively. The y-axes (x-axes) show the actual (predicted) values.

TABLES

Table 1: Descriptive Statistics of Implied Volatility Indices (January, 2001 to July, 2013).

	Mean	Min	Max	Std.Dev	Jarque-Bera	ADF-statistic	ARCH LM Test
VIX	21.52	9.89	80.86	9.48	6174.43 ***	-3.23 **	5288.04 ***
VSTOXX	25.99	11.60	87.51	10.78	1655.11 ***	-3.63 ***	5759.33 ***
VFTSE	21.19	9.10	78.69	9.45	3829.52 ***	-3.89 ***	5535.42 ***
VDAX	23.32	10.98	74.00	9.54	1578.59 ***	-3.16 **	8317.23 ***
VCAC	24.31	9.24	78.05	9.76	2250.23 ***	-3.69 ***	4588.81 ***
VXN	27.92	12.03	80.64	13.01	929.13 ***	-2.98 **	12370.04 ***
VXD	19.98	9.28	74.60	8.80	5205.14 ***	-3.17 **	6263.71 ***
VXJ	26.66	11.53	91.45	9.70	12706.03 ***	-4.10 ***	5620.22 ***

***,**,* indicate significance at 1%, 5% and 10% level, respectively.

Table 2: The SBC criterion for various orders of the IV-ARFIMA(*k,d,l*) model.

	k=0 *l*=0	*k*=0 *l*=1	*k*=1 *l*=0	*k*=1 *l*=1	*k*=2 *l*=1	*k*=1 *l*=2	*k*=2 *l*=2	*k*=3 *l*=2	*k*=2 *l*=3
VIX	-2.338	-2.528	-2.607	-2.650	**-2.664**	-2.656	-2.661	-2.659	-2.659
VSTOXX	-2.415	-2.683	-2.817	-2.844	**-2.863**	-2.853	-2.861	-2.858	-2.858
VFTSE	-2.292	-2.549	-2.690	-2.724	**-2.739**	-2.730	-2.735	-2.733	-2.735
VDAX	-2.609	-2.906	-3.077	-3.108	**-3.130**	-3.114	-3.127	-3.125	-3.125
VCAC	-2.400	-2.609	-2.714	-2.759	-2.763	-2.760	**-2.766**	-2.758	-2.764
VXN	-2.606	-2.848	-2.966	-3.006	-3.018	-3.011	**-3.019**	-3.013	-3.016
VXD	-2.372	-2.564	-2.650	-2.698	**-2.712**	-2.702	-2.709	-2.706	-2.706
VXJ	-2.353	-2.483	-2.547	-2.618	-2.622	-2.620	**-2.622**	-2.619	-2.620

Bold face fonts present the best order of the IV-ARFIMA(*k,d,l*) model.

Table 3: Loss functions for the evaluation of forecasting accuracy.

Loss functions	Formula
Mean squared error	$MSE = T^{-1}\sum_{t=1}^{T}\left(IV_{t+n\mid t} - IV_{t+n}\right)^2$
Mean absolute error	$MAE = T^{-1}\sum_{t=1}^{T}\left|IV_{t+n\mid t} - IV_{t+n}\right|$

Note: $IV_{t+n\mid t}$ is the implied volatility forecast, whereas IV_{t+n} is the actual implied volatility

Table 4: Forecast accuracy tests: One-day ahead forecasts (January, 2005 to July, 2013).

Model	Loss Function	Implied Volatility Indices							
		VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	MSE	4.18	2.21	2.92	3.81	3.76	2.91	4.67	3.12
	MAE	1.21	0.90	1.06	1.15	1.17	1.03	1.24	1.10
IV-ARFIMA	MSE	4.20	2.19	2.90	3.84	3.77	2.96	4.67	3.18
	MAE	1.22	0.90	1.06	1.16	1.17	1.04	1.25	1.10
HW	MSE	4.65	2.76	3.54	4.42	4.90	3.36	5.46	4.18
	MAE	1.37	1.11	1.28	1.34	1.49	1.19	1.45	1.44
SSA	MSE	2.55	1.67	2.39	2.92	2.87	2.09	2.71	2.41
	MAE	0.99	0.81	**0.98**	1.04	1.05	0.91	0.97	0.99
SSA-HW	MSE	**1.46**	**1.29**	**2.28**	**2.18**	**2.20**	**1.49**	**1.46**	**1.86**
	MAE	**0.79**	**0.73**	1.02	**0.91**	**0.94**	**0.79**	**0.75**	**0.89**
I(1)	MSE	4.28	2.21	2.94	3.96	3.81	3.00	4.64	3.16
	MAE	1.22	0.90	1.06	1.16	1.18	1.04	1.24	1.10
ARI(1,1)	MSE	4.26	2.22	2.93	3.86	3.81	2.94	4.70	3.15
	MAE	1.22	0.90	1.06	1.16	1.18	1.03	1.25	1.10
FI(1)	MSE	6.11	3.98	5.23	6.07	6.29	4.75	8.22	5.20
	MAE	1.45	1.17	1.32	1.39	1.45	1.26	1.54	1.35
ARFI(1,1)	MSE	4.37	2.33	3.10	4.28	3.96	3.27	5.14	3.42
	MAE	1.24	0.92	1.07	1.19	1.18	1.06	1.30	1.13

Bold face fonts present the best performing model.

Table 5: Forecast accuracy tests: Ten-days ahead forecasts (January, 2005 to July, 2013).									
		Implied Volatility Indices							
Model	Loss Function	VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	MSE	21.22	13.86	19.85	18.94	22.17	15.60	29.57	18.88
	MAE	2.92	2.39	2.77	2.72	2.96	2.50	3.20	2.74
IV-ARFIMA	MSE	21.27	13.47	19.41	19.32	21.89	15.56	29.18	19.61
	MAE	**2.90**	2.34	2.73	2.69	2.93	2.44	3.19	2.76
HW	MSE	**17.77**	13.36	**14.04**	13.98	19.03	**13.51**	21.90	18.66
	MAE	2.91	2.27	2.38	2.38	2.73	2.49	2.74	3.04
SSA	MSE	45.80	19.78	33.12	26.24	36.10	24.52	54.05	34.66
	MAE	4.26	2.72	3.46	3.20	3.58	3.22	4.32	3.69
SSA-HW	MSE	20.41	**12.12**	14.99	**13.13**	**15.49**	14.40	**19.00**	**12.70**
	MAE	3.10	**1.89**	**2.29**	**1.79**	**1.66**	**2.21**	**2.39**	**2.22**
I(1)	MSE	22.22	13.77	20.15	18.56	22.56	14.93	30.19	18.37
	MAE	3.05	2.42	2.83	2.74	3.08	2.50	3.26	2.77
ARI(1,1)	MSE	21.98	13.75	20.11	18.35	22.49	14.81	30.11	18.29
	MAE	3.03	2.42	2.83	2.74	3.08	2.50	3.25	2.77
FI(1)	MSE	28.12	21.69	27.82	31.20	32.24	25.22	42.89	27.89
	MAE	3.21	2.82	3.10	3.23	3.38	2.93	3.78	3.22
ARFI(1,1)	MSE	26.55	19.69	25.65	29.43	29.84	23.72	41.37	26.03
	MAE	3.11	2.67	2.97	3.13	3.25	2.84	3.69	3.09

Bold face fonts present the best performing model.

Table 6: MCS *p*-values: One-day ahead forecasts (January, 2005 to July, 2013).

Model	Loss Function	Implied Volatility Indices							
		VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	MSE	0.0000	0.0000	0.0002	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
IV-ARFIMA	MSE	0.0001	0.0000	0.0001	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HW	MSE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SSA	MSE	0.0001	0.0000	0.1245*	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	1.0000*	0.0000	0.0000	0.0000	0.0000	0.0000
SSA-HW	MSE	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
	MAE	1.0000*	1.0000*	0.0005	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
I(1)	MSE	0.0000	0.0000	0.0002	0.0000	0.0002	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI(1,1)	MSE	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FI(1)	MSE	0.0001	0.0000	0.0004	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARFI(1,1)	MSE	0.0001	0.0005	0.0030	0.0000	0.0005	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

* denotes that the model belongs to the confidence set of the best performing models. The interpretation of the MCS *p*-value is analogous to that of a classical *p*-value; a $(1-a)$ confidence interval that contains the 'true' parameter with a probability no less than $(1-a)$.

Table 7: MCS *p*-values: Ten-days ahead forecasts (January, 2005 to July, 2013).

		Implied Volatility Indices							
Model	Loss Function	VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	MSE	0.1796*	0.2192*	0.0052	0.0161	0.0810	0.3407*	0.0013	0.0199
	MAE	0.7881*	0.0000	0.0000	0.0000	0.0000	0.0038	0.0000	0.0000
IV-ARFIMA	MSE	0.1796*	0.5671*	0.0105	0.0162	0.0810	0.4632*	0.0013	0.0199
	MAE	1.0000*	0.0000	0.0000	0.0000	0.0000	0.0104	0.0000	0.0000
HW	MSE	1.0000*	0.1245*	1.0000*	0.6193*	0.2634*	1.0000*	0.0528	0.0001
	MAE	0.9280*	0.0000	0.0855	0.0000	0.0000	0.0007	0.0000	0.0000
SSA	MSE	0.0001	0.0206	0.0001	0.0000	0.0076	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SSA-HW	MSE	0.1597*	1.0000*	0.2748*	1.0000*	1.0000*	0.5806*	1.0000*	1.0000*
	MAE	0.1324*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
I(1)	MSE	0.0274	0.5671*	0.0017	0.0128	0.0810	0.3237*	0.0002	0.0199
	MAE	0.0028	0.0000	0.0000	0.0000	0.0000	0.0037	0.0000	0.0000
ARI(1,1)	MSE	0.0917	0.5671*	0.0017	0.0157	0.0810	0.4632*	0.0002	0.0199
	MAE	0.0061	0.0000	0.0000	0.0000	0.0000	0.0037	0.0000	0.0000
FI(1)	MSE	0.0001	0.0000	0.0002	0.0000	0.0000	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARFI(1,1)	MSE	0.0068	0.0001	0.0017	0.0006	0.0014	0.0007	0.0001	0.0015
	MAE	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

* denotes that the model belongs to the confidence set of the best performing models. The interpretation of the MCS *p*-value is analogous to that of a classical *p*-value; a $(1-a)$ confidence interval that contains the 'true' parameter with a probability no less than $(1-a)$.

Table 8: Forecast accuracy tests: Model-averaged forecasts (January, 2005 to July, 2013).									
		Implied Volatility Indices							
Model	Loss Function	VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
		One-day ahead							
ARI-IV-HAR	MSE	4.21	2.21	2.92	3.82	3.78	2.91	4.64	3.12
	MAE	1.21	0.90	1.06	1.15	1.18	1.03	1.24	1.10
ARI-IV-ARFIMA	MSE	4.19	2.19	2.89	3.82	3.77	2.92	4.64	3.14
	MAE	1.21	0.90	1.06	1.15	1.17	1.03	1.24	1.10
HAR-ARFIMA	MSE	4.17	2.19	2.89	3.82	3.76	2.92	4.66	3.14
	MAE	1.21	0.90	1.06	1.15	1.17	1.03	1.24	1.10
ARI-SSA-HW	MSE	2.43	1.60	2.32	2.82	2.76	2.02	2.66	2.31
	MAE	0.94	0.78	**0.97**	1.00	1.02	0.88	0.96	0.95
		Ten-days ahead							
ARI-IV-HAR	MSE	21.15	13.56	19.67	18.34	21.90	14.92	29.23	18.30
	MAE	2.94	2.38	2.76	2.70	2.99	2.47	3.18	2.72
ARI-IV-ARFIMA	MSE	20.64	13.26	19.26	18.32	21.55	14.56	28.85	18.19
	MAE	2.91	2.35	2.73	2.67	2.96	2.42	3.17	2.70
HAR-ARFIMA	MSE	20.94	13.59	19.54	18.99	21.93	15.35	29.29	18.96
	MAE	2.90	2.36	2.74	2.70	2.94	2.46	3.19	2.73
ARI-SSA-HW	MSE	**13.48**	**9.83**	14.45	**8.39**	**8.14**	**10.69**	**16.86**	**10.41**
	MAE	**2.48**	1.95	2.32	1.80	1.83	**2.09**	**2.24**	**2.10**

Bold face fonts present the model that outperforms the best performing models of Table 4 and 5 for the 1-day and 10-days ahead, respectively.

Table 9: MCS *p*-values: Model-averaged forecasts, one-day ahead (January, 2005 to July, 2013).

Model	Loss Function	Implied Volatility Indices							
		VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	MSE	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
IV-ARFIMA	MSE	0.0001	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HW	MSE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SSA	MSE	0.0001	0.0000	0.0008	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0018	0.0000	0.0000	0.0000	0.0000	0.0000
SSA-HW	MSE	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
	MAE	1.0000*	1.0000*	0.0000	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
I(1)	MSE	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI(1,1)	MSE	0.0000	0.0000	0.0000	0.0000	0.0001	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FI(1)	MSE	0.0001	0.0000	0.0004	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARFI(1,1)	MSE	0.0001	0.0005	0.0010	0.0000	0.0005	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI-IV-HAR	MSE	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI-IV-ARFIMA	MSE	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-ARFIMA	MSE	0.0000	0.0000	0.0000	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI-SSA-HW	MSE	0.0001	0.0003	0.6171*	0.0000	0.0002	0.0000	0.0001	0.0000
	MAE	0.0000	0.0000	1.0000*	0.0000	0.0000	0.0000	0.0000	0.0000

* denotes that the model belongs to the confidence set of the best performing models. The interpretation of the MCS *p*-value is analogous to that of a classical *p*-value; a $(1-a)$ confidence interval that contains the 'true' parameter with a probability no less than $(1-a)$.

Table 10: MCS *p*-values: Model-averaged forecasts, Ten-days ahead (January, 2005 to July, 2013).

		Implied Volatility Indices							
Model	Loss Function	VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	MSE	0.0004	0.0009	0.0014	0.0000	0.0002	0.0013	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
IV-ARFIMA	MSE	0.0006	0.0024	0.0063	0.0000	0.0002	0.0032	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HW	MSE	0.0002	0.0033	1.0000*	0.0000	0.0002	0.0032	0.0000	0.0000
	MAE	0.0000	0.0000	0.1955*	0.0000	0.0000	0.0000	0.0000	0.0000
SSA	MSE	0.0000	0.0010	0.0000	0.0000	0.0003	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
SSA-HW	MSE	0.0000	0.0247	0.5147*	0.0003	0.0190	0.0052	0.0631	0.0168
	MAE	0.0000	1.0000*	1.0000*	1.0000*	1.0000*	0.0284	0.0199	0.0161
I(1)	MSE	0.0001	0.0000	0.0002	0.0000	0.0000	0.0009	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI(1,1)	MSE	0.0001	0.0000	0.0002	0.0000	0.0000	0.0010	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
FI(1)	MSE	0.0001	0.0000	0.0002	0.0000	0.0002	0.0000	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARFI(1,1)	MSE	0.0004	0.0001	0.0012	0.0000	0.0002	0.0005	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI-IV-HAR	MSE	0.0003	0.0001	0.0008	0.0000	0.0001	0.0017	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI-IV-ARFIMA	MSE	0.0004	0.0010	0.0019	0.0000	0.0001	0.0032	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
HAR-ARFIMA	MSE	0.0006	0.0013	0.0033	0.0000	0.0002	0.0032	0.0000	0.0000
	MAE	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
ARI-SSA-HW	MSE	1.0000*	1.0000*	0.5296*	1.0000*	1.0000*	1.0000*	1.0000*	1.0000*
	MAE	1.0000*	0.2480*	0.5695*	0.9885*	0.0171	1.0000*	1.0000*	1.0000*

* denotes that the model belongs to the confidence set of the best performing models. The interpretation of the MCS *p*-value is analogous to that of a classical *p*-value; a $(1-a)$ confidence interval that contains the 'true' parameter with a probability no less than $(1-a)$.

Table 11: Direction-of-Change - One-day ahead (January, 2005 to July, 2013).

	Implied Volatility Indices							
Model	VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	0.5397	0.5270	0.5211	0.5336	0.5276	0.5315	0.5220	0.5202
IV-ARFIMA	0.5244	0.5347	0.5216	0.5364	0.5318	0.5315	0.5258	0.5164
HW	0.5077	0.4868	0.5053	0.5002	0.4995	0.5081	0.5158	0.5088
SSA	0.6789	0.6437	0.6207	0.6397	0.6336	0.6492	0.7204	0.6456
SSA-HW	0.7373	0.6992	0.6547	0.7044	0.6887	0.7169	0.7973	0.6922
I(1)	0.5785	0.4840	0.4646	0.4584	0.4577	0.4628	0.4618	0.4637
ARI(1,1)	0.5780	0.4926	0.4799	0.5296	0.4748	0.5243	0.4914	0.4907
FI(1)	0.5900	0.5318	0.5259	0.5450	0.5347	0.5372	0.5325	0.5287
ARFI(1,1)	0.5780	0.5122	0.5292	0.5093	0.5247	0.5148	0.5191	0.5059
ARI-IV-HAR	0.5431	0.5088	0.5005	0.5250	0.5157	0.5291	0.5105	0.5221
ARI-IV-ARFIMA	0.5258	0.5265	0.5115	0.5250	0.5166	0.5338	0.5096	0.5164
HAR-ARFIMA	0.5411	0.5328	0.5220	0.5393	0.5276	0.5372	0.5249	0.5164
ARI-SSA-HW	0.7340	0.6872	0.6379	0.6844	0.6811	0.6930	0.7677	0.6770

Table 12: Direction-of-Change - Ten-days ahead (January, 2005 to July, 2013).

	Implied Volatility Indices							
Model	VCAC	VDAX	VFTSE	VIX	VSTOXX	VXD	VXJ	VXN
IV-HAR	0.5630	0.5441	0.5598	0.5564	0.5779	0.5764	0.5488	0.5638
IV-ARFIMA	0.5749	0.5488	0.5655	0.5645	0.5703	0.5517	0.5360	0.5642
HW	0.6559	0.6498	0.6902	0.6545	0.6678	0.6300	0.6635	0.6406
SSA	0.4829	0.5005	0.5161	0.5308	0.5418	0.4720	0.4967	0.4917
SSA-HW	0.7180	0.7223	0.6917	0.8308	0.8783	0.6689	0.7739	0.7411
I(1)	0.4867	0.4512	0.4715	0.4564	0.4743	0.4568	0.4408	0.4661
ARI(1,1)	0.4905	0.4521	0.4682	0.4739	0.4796	0.4782	0.4673	0.4827
FI(1)	0.5820	0.5602	0.5740	0.5654	0.5827	0.5583	0.5445	0.5533
ARFI(1,1)	0.5815	0.5531	0.5802	0.5635	0.5822	0.5574	0.5427	0.5505
ARI-IV-HAR	0.5687	0.5275	0.5460	0.5488	0.5703	0.5697	0.5365	0.5614
ARI-IV-ARFIMA	0.5754	0.5531	0.5645	0.5602	0.5775	0.5398	0.5299	0.5505
HAR-ARFIMA	0.5763	0.5531	0.5669	0.5592	0.5798	0.5659	0.5398	0.5657
ARI-SSA-HW	0.7166	0.7133	0.6874	0.8265	0.8788	0.6618	0.7716	0.7378

Table 13: Trading game results - One-day ahead (January, 2005 to July, 2013).

	Implied Volatility Indices															
Model	VCAC		VDAX		VFTSE		VIX		VSTOXX		VXD		VXJ		VXN	
IV-HAR	-0.0021		-0.0045		-0.0035		0.0000		-0.0039		-0.0012		-0.0033		-0.0030	
IV-ARFIMA	-0.0026		-0.0041		-0.0034		0.0001		-0.0029		-0.0009		-0.0037		-0.0032	
HW	-0.0065		-0.0079		-0.0068		-0.0059		-0.0076		-0.0059		-0.0060		-0.0053	
SSA	0.0213	***	0.0112	***	0.0113	***	0.0179	***	0.0128	***	0.0183	***	0.0225	***	0.0139	***
SSA-HW	0.0273	***	0.0167	***	0.0148	***	0.0249	***	0.0190	***	0.0255	***	0.0280	***	0.0193	***
I(1)	0.0016		-0.0067		-0.0056		-0.0053		-0.0066		-0.0054		-0.0055		-0.0076	
ARI(1,1)	0.0014		-0.0065		-0.0068		-0.0003		-0.0074		-0.0021		-0.0063		-0.0067	
FI(1)	0.0023		-0.0037		-0.0030		-0.0005		-0.0024		-0.0006		-0.0032		-0.0019	
ARFI(1,1)	0.0013		-0.0060		-0.0032		-0.0047		-0.0040		-0.0042		-0.0033		-0.0040	
ARI-IV-HAR	-0.0018		-0.0062		-0.0049		-0.0010		-0.0034		-0.0007		-0.0050		-0.0031	
ARI-IV-ARFIMA	-0.0027		-0.0046		-0.0029		-0.0009		-0.0037		-0.0006		-0.0053		-0.0033	
HAR-ARFIMA	-0.0013		-0.0045		-0.0032		0.0007		-0.0039		0.0000		-0.0029		-0.0038	
ARI-SSA-HW	0.0271	***	0.0155	***	0.0132	***	0.0225	***	0.0178	***	0.0227	***	0.0256	***	0.0179	***

Note: The numbers denote net average daily profits having deducted the transaction costs. *** denotes significance at 1% level.

Table 14: Trading game results - Ten-days ahead (January, 2005 to July, 2013).

	Implied Volatility Indices									
Model	VCAC	VDAX	VFTSE	VIX		VSTOXX		VXD	VXJ	VXN
IV-HAR	-0.0005	-0.0014	-0.0009	-0.0012		-0.0011		-0.0007	-0.0012	-0.0014
IV-ARFIMA	-0.0005	-0.0012	-0.0008	-0.0009		-0.0014		-0.0012	-0.0016	-0.0013
HW	0.0019	0.0016	0.0028	0.0023		0.0024		0.0010	0.0021	0.0008
SSA	-0.0050	-0.0041	-0.0042	-0.0020		-0.0018		-0.0046	-0.0056	-0.0038
SSA-HW	0.0041	0.0027	0.0034	0.0070	***	0.0074	***	0.0024	0.0046	0.0039
I(1)	-0.0035	-0.0044	-0.0044	-0.0035		-0.0039		-0.0037	-0.0043	-0.0039
ARI(1,1)	-0.0035	-0.0043	-0.0047	-0.0033		-0.0037		-0.0036	-0.0037	-0.0037
FI(1)	-0.0002	-0.0011	-0.0004	-0.0009		-0.0006		-0.0009	-0.0015	-0.0012
ARFI(1,1)	-0.0003	-0.0011	-0.0001	-0.0010		-0.0006		-0.0010	-0.0013	-0.0012
ARI-IV-HAR	-0.0005	-0.0016	-0.0014	-0.0012		-0.0012		-0.0008	-0.0015	-0.0015
ARI-IV-ARFIMA	-0.0005	-0.0010	-0.0007	-0.0011		-0.0010		-0.0014	-0.0016	-0.0016
HAR-ARFIMA	-0.0004	-0.0012	-0.0007	-0.0011		-0.0010		-0.0009	-0.0014	-0.0013
ARI-SSA-HW	0.0041	0.0026	0.0033	0.0069	***	0.0074	***	0.0024	0.0046	0.0039

Note: The numbers denote net average daily profits having deducted the transaction costs. *** denotes significance at 1% level.