Holding an Auction for the Wrong Project

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Abstract

How does the probability of being involved in a renegotiation during the execution of a procurement contract affect the behavior of the interested contractors? What are its implications for the optimal contractual choice made by the buyer? We investigate these issues in a context characterized by uncertainty about the adequateness of the project initially specified by the buyer. We determine under which circumstances the buyer may find it profitable to hold an auction for a project design which ex-ante does not have the highest probability of being adequate.

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JEL classifications: D44, D86, H57.

1 Introduction

Contracts concerning the provision of customized goods or services are often granted through auctions. Auctions are widely advocated since they guarantee transparency and foster competition, allowing the buyer to obtain the desired good at the most favorable economic conditions. However, once awarded, a number of procurement contracts require modifications which may significantly change the design of the good. In a lot of cases the events which trigger the revision of the original agreement could have been anticipated at the time at which the initial contract was drawn up.[1]

If a renegotiation significantly alters the scale and the scope of the original contract, one may question the optimality of the auction outcome. A firm suitable to provide the original service may no longer be the most appropriate operator when the contract is altered. Nevertheless,

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[1] In this regard, Guasch (2004) provides extensive empirical evidence of strategic renegotiation of concession contracts granted in Latin America and the Caribbean in the period 1985-2000.

contract clauses may prevent the buyer from turning to another firm for the provision of the service or may just make it unprofitable.

A compelling example involves the construction of the new railway station in Mons (Belgium). At the time of the call for tenders in 2006, the stated objective of the local authorities was to preserve the original station which dated back to the 1950s. The renowned Spanish architect Santiago Calatrava was granted the contract since his design was the only one which met that requirement. Because of technical problems, the initial winning design could not be undertaken and Calatrava' s architectural firm worked out a new project which would lead to the replacement of the old station. Presumed similarities existing with other project designs rejected at the bidding stage have sparked a lot of criticism: indeed, rival and lesser-known architectural firms have claimed that they could deliver the same project at a lower price.[2]

This example raises the question of whether a buyer would find it advantageous to choose a design which has a high ex-ante probability of being infeasible. Similarly, we may wonder whether the buyer would select a contractor who is not the most appropriate for the project design ultimately implemented. In this paper, we provide an answer to these questions showing the existence of a non-trivial trade-off between the cost of an ill-specification of the project design and the benefits of intensifying competition ex-ante.

We show that a buyer may decide to hold an auction for a project specification which is highly likely to be inappropriate (that is, a *wrong project*) to stiffen competition at the bidding stage when the potential contractors have different design specializations.[3]

Bidders know that with some positive probability the initial design will be flawed and rationally incorporate the rents they expect to earn at the renegotiation stage when they submit their bids. If renegotiation is always successful and the bidders have the same bargaining power, the expected rents enter the firms' bidding functions in the same way and, as a result, the initial choice of the design is neutral as it affects neither the efficiency of the contract allocation nor the expected buyer's payoff. However, the renegotiation rents may not be high enough to compensate some contractors for the higher cost of production that an alternative project design entails. If so, renegotiation fails with some positive probability and, then, there is scope for a strategic choice of the initial design. In this case, the expectation that the starting design of the project may fail causes an asymmetric shift in the bidding functions of the firms. The buyer can thus take advantage of the firms' heterogeneous reactions to countervail the existing cost asymmetry among bidders and receive lower and more aggressive bids. The buyer may optimally choose the wrong specification to intensify competition at the bidding stage even at the expense of going through a costly renegotiation with a higher probability.[4]

In the model we assume that a buyer wishes to procure a good and holds an auction to select the contractor. There are two alternative specifications of the good, A and B, and the prior probability that either design turns out to be flawed, if implemented, is common knowledge to

[2]LeVif.be: *"La gare Calatrava à Mons: un projet contesté et suspect"* - April 20, 2010.

[3]There are many reasons why firms may incur different costs to deliver alternative designs. For instance, they may be heterogeneous for reasons related to their size or to the skills of their human resources.

[4]The assumption of incomplete information plays a critical role. If the buyer knew the firms' cost of production there would be no need to stiffen competition in the auction.

all the players of the game. We assume that there are two bidders, a and b, who are specialized in delivering one specification of the project each. The alternative designs of the project require different capabilities from the engaged contractor and therefore entail different production costs. In particular, we assume that the relative cost advantage enjoyed by one bidder in undertaking project A is reversed when it is the alternative project specification B to be carried out.

Throughout the model we rely extensively on the results of the literature on asymmetric auctions. In the independent private value model, Maskin and Riley (2000b) and Lebrun (1996) prove the existence of an equilibrium, while Maskin and Riley (2003) and Lebrun (1999) show under what conditions the equilibrium is unique. In particular, we draw heavily on the insights provided by Maskin and Riley (2000a) on the bidding behavior of asymmetric bidders in a first-price auction. They show that a weak bidder, namely a bidder who is more likely to have a low valuation for the good, submits more aggressive bids than a strong bidder. Stated differently, this means that the degree of bid-shading of the weak bidder is lower than than of the strong bidder. In a similar vein, Lebrun (1998) shows that if the valuation distribution of one bidder stochastically increases, so does his own bid distribution while his rivals will submit higher bids in equilibrium.

To our knowledge, our paper is the first to highlight how the buyer can strategically use the design of the good she needs to procure to toughen price competition at the tender stage. In a related vein, Esö and White (2004) consider a setting in which there may be pure risk in the bidders' valuations, arising from information that none of the bidders can obtain before the tender takes place. They find that bidders who have decreasing absolute risk aversion are better off when the object sold by the auctioneer is riskier as they will submit less aggressive bids. In contrast, we show that the auctioneer may prefer to auction off a design whose probability of being inadequate is higher so as to stiffen competition at the bidding stage between risk-neutral bidders.

Our paper is also close to Waehrer (1999) who considers a buyer who uses an auction for the purchase of a primary good and may subsequently decide to purchase from the same winning bidder a supplementary good whose price is determined through sequential bargaining. The author shows that the bidders will not follow a separating bidding strategy, namely one which is strictly monotone in their cost of production for the primary good, if this would allow the buyer to learn their cost of production for the supplementary good.[5] We deal with a related problem since we consider a buyer who may want to change the design of the good after awarding the contract to a supplier through an auction. The buyer may use the winning bid to update her belief about the contractor's costs of production and this impacts on the bidders' bidding strategies. To make the model tractable and ensure that the bidders follow a separating bidding strategy in equilibrium, we assume that the winning bidder holds all the bargaining power at the ex-post bargaining stage. Different modeling approaches are pursued by Elyakime et al. (1997) and Estache and Quesada (2002) to guarantee tractability and ensure bidding strategies which are monotonic in the bidders' private information. Elyakime et al. (1997) study a first-price auction in which the auctioneer's reservation price is not announced in advance and bargaining

[5] This is an illustration of the ratchet effect first formalized by Laffont and Tirole (1988).

occurs between the auctioneer and the bidder with the highest bid if the auction fails to produce a transaction. They assume that bargaining occurs under complete information and the parties split evenly the surplus when there are gains from trade. The features of the bargaining stage are anticipated by the bidders at the tender stage. In contrast, Estache and Quesada (2002) assume that the auctioneer and the bidders overlook the possibility of renegotiation when designing the auction and deciding the bids, respectively.

The remainder of the paper is organized as follows. Section 2 presents the model and discusses the features of the renegotiation game. Section 3 analyzes the case in which the buyer holds a first-price auction to award the contract and the contractor is entitled to make a take-it-or-leave-it (TIOLI) offer at the renegotiation stage. Section 4 shows that the main conclusions regarding the strategic choice of the initial design of the project in a first-price auction environment carry over to an English auction format. Section 5 provides a brief discussion of some of the most relevant assumptions of the model. Section 6 concludes.

2 The Model

Consider a risk-neutral female buyer such as a public agency who wishes to procure a good from the outside. The buyer must choose between two alternative designs of the good that we call A and B. The buyer attaches a positive value $v > 0$ to the good. If the initial design turns out to be flawed, then the project yields the buyer utility $v - h \in [0, v]$ if it is not modified. Whereas, if a design change occurs and the alternative specification is adopted, the buyer again attains utility v. Every player knows that the ex-ante probability that A (respectively, B) is the correct design is $1 - \beta$ (β).

There are two risk-neutral male bidders, a and b, who have different project design specializations. In particular, we assume that bidder a bears a low cost of production to deliver design A, c_{aA}, and bears a high cost to deliver design B, c_{aB}. By contrast, bidder b incurs a high cost to produce A, c_{bA} and a low cost to produce B, c_{bB}.

Bidders' low production costs, c_{aA} and c_{bB}, are realizations of a random variable $\tilde{c}_l$, which is distributed according to distribution F_l over the interval $[\underline{c}_l, \overline{c}_l]$. Bidders' high production costs, c_{aB} and c_{bA}, are realizations of a random variable $\tilde{c}_h$ distributed according to distribution F_h over the interval $[\underline{c}_h, \overline{c}_h]$. All realizations are independent and it holds that $\underline{c}_l < \overline{c}_l \leq \underline{c}_h < \overline{c}_h < v$. Each bidder is privately informed about his own costs of production for the two alternative designs. In contrast, the costs' distributions and the buyer's utility function are publicly known. We further denote by $\widetilde{c}$ the random variable $\widetilde{c_h} - \widetilde{c_l}$, whose distribution and density are $F(\widetilde{c})$ and $f(\widetilde{c})$, respectively.[6]

In practice, one may think of a large and a small firm. The former has higher fixed costs but can take advantage of economies of scale, whereas the latter has higher variable costs, but negligible fixed costs. If A and B differ with respect to the size of the project, it is reasonable to suppose that the large firm will bear a lower total cost of production than the small firm when the larger design is undertaken and vice versa.

[6] $f(\widetilde{c})$ is the convolution of the density functions of $\widetilde{c_h}$ and of $\widetilde{c_l}$.

Because of the design specialization, if the buyer holds an auction for project A and this subsequently turns out to be flawed, then bidder a will be reluctant to shift to project B as it involves a higher cost of production while bidder b will eagerly accept the design change as it will allow him to save on the production cost.

Henceforth, we assume that once the tender process has taken place, the buyer is stuck to the selected contractor and cannot hire the other firm.[7] Moreover, we assume that it is not possible to write a contract contingent on the adoption of a different design ex-post, namely, the procurement contract must specify the delivery of either A or B. In practice, ex-ante it may be prohibitively costly to describe both designs in the contract and the buyer must content herself with initially specifying only A or B.

The contractor is selected through a low-bid auctions whose rules also determine the transfer paid by the buyer for the delivery of the good. We start by considering a *first-price reverse auction* in which the lowest bidder is awarded the contract and receives a transfer t from the buyer which is equal to his winning bid. If there is more than one lowest bidder, the tie is broken randomly.

We focus on *fixed-price contract*s where the awardee does not receive any reimbursement for the costs he incurs. We make this choice for several reasons. First, the buyer might be unable to verify the contractor's realized cost. Second, fixed price contracts are most often awarded through auctions. In the United States, the Federal Acquisition Regulations (FARs) recommend the use of auctions of fixed price-contracts for public sector purchases. Finally, Bajari and Tadelis (2001) have emphasized the merits of a cost-plus contract when it comes to renegotiating an agreement, and their argument is mainly based on their greater flexibility to adapt to ex-post adjustments than fixed-price contracts. However, if cost-plus contracts were auctioned off, the bids would not convey any information about the bidders' costs of production. [8] In an auction environment, we aim to investigate how the award of a fixed-price contract affects the buyer's utility and behavior in the presence of a known positive probability that the initial agreement will warrant some changes.

The timing of the game is as follows:

- At time 0, β is observed by all the players of the game. The buyer decides whether to auction off project A or B.
- At time 1, the low-bid auction takes place and the buyer selects the contractor.
- At time 2, uncertainty is resolved. If the project design chosen at time 0 exhibits imperfections, a renegotiation between the buyer and the selected contractor occurs and, if successful, the design changes.
- At time 3, the project is delivered and payoffs are realized.

[7] For concreteness, one may think of a large cost of breaching the initial contract which makes it unprofitable for the buyer to back out. Furthermore, we implicitly assume away the possibility of subcontracting the work to another firm (e.g., the rival firm may no longer be available).

[8] For further details, see McAfee and McMillan (1986). Note also that the standard cost-plus contract provided by The American Institute of Architects, A-102-2007, is not intended for use in auctions.

Note that we analyze what project design would be selected when there is imperfect information about the production costs incurred by the firms. In the absence of information asymmetries, a competition effect could not arise and, as a result, there would not be scope for a strategic choice of the initial design.

2.1 Renegotiation Under Asymmetric Information

The different design specializations of the two bidders clearly impact on their renegotiation claims, and under some circumstances can even prevent a design change from occurring. In turn, these considerations undoubtedly affect the bidding strategies pursued by the two firms. To determine the buyer's utility function and the bidders' profit functions, we need to make an assumption on the way renegotiation takes place.

When the initial specification happens to be flawed, the buyer wishes to change the design not to incur the net loss of welfare h. The cost parameters of the contractors continue to remain their private information. This assumption is in line with Bajari and Tadelis (2001), where modification costs cannot be accurately measured. Since the extent to which the design change has hurt the cost efficiency of the contractor cannot be verified, any agent's reimbursement claim for the increased cost of delivery the good cannot be trusted. However, the contractor is entitled to reject any revision of the original agreement.

In developing a bargaining model in presence of asymmetric information, one comfortable option is to focus on the polar case in which the informed party (i.e., the contractor) can make a take-it-or-leave-it offer to which he can fully commit. This is consistent with the results of the literature on bargaining under asymmetric information (see Samuelson, 1984) which predicts that the parties may sometimes fail to reach an agreement and that the first best is attainable provided that the informed party may commit to the first-and-final offer he makes.

While stark, this assumption on how the bargaining power is allocated ex-post has the benefit of ensuring tractability. The buyer might be able to update her belief about the cost structure of the contractor she is facing at the renegotiation stage. If she holds some bargaining power at stage 2, then the bidders' equilibrium bidding strategies will not be monotonic in their private information, as shown by Waehrer (1999). In our setting, the assumption that ex-post the supplier can make a take-it-or-leave-it offer may be reasonable since we have assumed that, once the contract is awarded, the buyer is stuck with the contractor and the other supplier is no longer available..

In what follows, we study how the potential renegotiation game occurring at time 2 impacts on the firms' bidding behavior at time 1 and, in turn, on the buyer's initial choice of the design at time 0.

3 Renegotiation and Strategic Choice of the Design

A contractor who happens to be involved in a renegotiation with the buyer will make the most of his perfect information about the buyer's utility, asking for h, which is publicly observable. However, in some occurrences the contractor may be still unwilling to accept the design change.

This event occurs when the contractor has to incur a higher cost of production to deliver the new project design and the renegotiation rent, h, is not large enough to reimburse him for the increased cost. Therefore, we must take into account that the renegotiation may break down when an auction for A (B) has been held, a (b) has been awarded the contract, and the buyer requests a change in the design. When the buyer auctions off project design A and, once the uncertainty is resolved, she is willing to adopt the alternative design B, the higher cost of production firm a has to incur may prevent the renegotiation process. This occurs whenever h is greater than the difference in a's cost of production for the two alternative designs.

In what follows, we distinguish between two cases: when the value of the renegotiation rent is so high that a requested design change always succeeds (i.e., $h \geq \bar{c}_h - \underline{c}_l$) and when h is not so high and, as a result, a requested renegotiation may not lead to a design change (i.e., $h < \bar{c}_h - \underline{c}_l$). Such distinction will be crucial to determine whether or not the buyer may decide to strategically choose the design to auction off.

First note that when the buyer initially avails herself of project design A, her expected utility is given by:

$$EU(A) = v - t_w - \beta h. \tag{1}$$

This is so irrespective of whether or not there exists some positive probability that renegotiation breaks down and is due to the buyer's lack of bargaining power at the renegotiation stage. In the expression above, t is the transfer from the buyer to the contractor, and the subscript w in the above equation can be equal to either a or b and denotes the winning bidder.

Whereas, when B is the starting project design, her expected utility is:

$$EU(B) = v - t_w - (1 - \beta)h.$$

Note that the buyer sets a ceiling to the bids she receives. Specifically, when the design she auctions off is A, the buyer turns down all the bids above $v - \beta h$ as they would yield her a negative expected utility. Likewise, when the initial design is B the maximum acceptable bid is $v - (1 - \beta)h$.

3.1 Neutrality of the Initial Design

We first consider the case in which the parties always succeed in renegotiating the contract, regardless of the identity of the winning bidder, i.e. $h \geq \bar{c}_h - \underline{c}_l$. This implies that the maximum cost differential is very small or that the renegotiation loss would be hefty.

We illustrate the profit functions and the bidders' types when the buyer auctions off project design A. It is straightforward to restate the expressions when B is the initial project specification. Let c_{iJ} be i's cost of production for design J, with $i = a, b$ and $J = A, B$. Then, a's expected profit function takes the following form:

$$E\pi_a(A) = \begin{cases} t_a - (1 - \beta)c_{aA} - \beta(c_{aB} - h) & \text{if } a \text{ wins} \\ 0 & \text{otherwise.} \end{cases}$$

In the above expression, t_a denotes a's bid. Analogously, b's expected profit function is:

$$E\pi_b(A) = \begin{cases} t_b - (1-\beta)c_{bA} - \beta(c_{bB} - h) & \text{if } b \text{ wins} \\ 0 & \text{otherwise.} \end{cases}$$

In the above expression, t_b denotes b's bid.

A crucial role in the analysis is played by the bidders' *pseudo-types* θ_{iJ}, for $i = a, b$ and $J = A, B$. These consist of two components. The first is the bidder's expected cost of production which may depend on the design which is initially auctioned off. We denote this expected cost by $E[c_i(J)]$, with $i = a, b$ and $J = A, B$. The second is the expected rent which merely shifts to the left the expected total cost of delivering the good. Note that also the second component of a bidder's pseudo-type may be affected by the initial design of the good.

$$\begin{cases} \theta_{aA} = \overbrace{(1-\beta)c_{aA} + \beta c_{aB}}^{E[c_a(A)]} - \beta h \\ \theta_{bA} = \underbrace{(1-\beta)c_{bA} + \beta c_{bB}}_{E[c_b(A)]} - \beta h \end{cases}$$

The generic bidder's pseudo-type, θ_{iJ}, is then drawn from a distribution Φ_{iJ}, which we assume to be twice continuously differentiable with strictly positive density ϕ_{iJ}, on the interval $[\underline{\theta}_{iJ}, \overline{\theta}_{iJ}]$ for $i = a, b$ and $J = A, B$.[9] Specifically, when the initial design is A, the supports are

$$\begin{cases} \operatorname{Supp} \Phi_{aA} = [(1-\beta)\underline{c}_l + \beta \underline{c}_h - \beta h; (1-\beta)\overline{c}_l + \beta \overline{c}_h - \beta h] = [\underline{\theta}_{aA}; \overline{\theta}_{aA}] \\ \operatorname{Supp} \Phi_{bA} = [(1-\beta)\underline{c}_h + \beta \underline{c}_l - \beta h; (1-\beta)\overline{c}_h + \beta \overline{c}_l - \beta h] = [\underline{\theta}_{bA}; \overline{\theta}_{bA}] \end{cases}$$

As a matter of fact, what distinguishes the bidders is only the cost function while the renegotiation rent they expect to earn is the same. The profit functions we have set out above are correct as long as h is greater than the actual cost of modifying the design of the project c, so that the renegotiation between the buyer and the contractor always proves successful.[10]

Throughout we maintain the following standard assumption:

Assumption 1. *(a) Bidders' cost parameters are drawn independently;*

(b) Bidders' expected profit functions are monotonically decreasing in the firms' own pseudo-types;

(c) Bidders' expected profit functions are weakly supermodular.

Note that the profit functions written above are both monotonically decreasing and weakly supermodular (the latter condition is always satisfied when bidders are risk neutral). The following lemma concerns the firms' bidding behavior:

Lemma 1. *If Assumption 1 holds, bidders bid according to a weakly monotonic bidding function. That is, if $t_i = \gamma_i(\theta_{iJ})$, then $\gamma_i(\theta'_{iJ}) \geq \gamma_i(\hat{\theta}_{iJ})$ if $\theta'_{iJ} > \hat{\theta}_{iJ}$, for $i = a, b$ and $J = A, B$.*

[9] Note that the distributions of $\tilde{c}_l$ and $\tilde{c}_h$ must be such that the above assumptions on the distributions of the pseudo-types are satisfied.

[10] In particular, renegotiation succeeds even if the awardee for the initial project designs A and B are firms a and b, respectively.

Proof. In the Appendix □

We can now proceed with our first result:

Proposition 1. *If*

(i) the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage;

(ii) Assumption 1 holds;

(iii) $h \geq \bar{c}_h - \underline{c}_l$,

then:

1. *Bidders fully compete away h at the bidding stage.*

2. *The buyer is indifferent between auctioning off design A or B.*

Proof. In the Appendix □

The first part of the proposition says that the renegotiation rent does not represent a concern for the buyer. This is because the renegotiation rent is entirely discounted at the bidding stage. The reason is the following. Both bidders know the probability with which they will earn a rent if they are granted the project, and they know the magnitude of the rent itself. The expected value of renegotiation is the same for the two bidders when $h \geq \bar{c}_h - \underline{c}_l$. Therefore, a competitive bidding process will work in a Bertrand fashion, allowing the buyer to extract all the winning bidder's willingness to pay to be granted the right to potentially earn the renegotiation rent, that is, βh for design A and $(1-\beta)h$ for design B.

The second part of the proposition says that when renegotiation is always successful, it does not matter to the buyer which project design is auctioned off. Namely, **the initial choice of the design is neutral** as it does not affect the players' payoffs. To see this, consider that when $h \geq \bar{c}_h - \underline{c}_l$ the expected cost of production of the two bidders is independent of the design initially auctioned off, that is $E[c_i(A)] = E[c_i(B)]$ for $i = a, b$. Therefore the initial design only impacts on the expected renegotiation rent which is entirely competed away at the bidding stage.

3.2 Auction of the Wrong Project

Now we turn to the case in which the renegotiation rent may be lower then the increased cost of production that a design change entails.

Suppose that design A is auctioned off. Relative to the previous section, only bidder a's expected profit function may change as she may refuse to change the design if defective. This happens when $h < c_{aB} - c_{aA}$, in which occurrence a's expected profit function conditional on winning becomes:

$$E\pi_a(A) = t_a - c_{aA}$$

Whereas, if $h \geq c_{aB} - c_{aA}$, the expected profit function of firm a conditional on winning is the same as the one shown in the previous subsection:

$$E\pi_a(A) = t_a - (1-\beta)c_{aA} - \beta(c_{aB} - h)$$

On the contrary, bidder b is always willing to shift to design B as this would entail a reduction in the cost of production.

The buyer anticipates that if a wins the contract to deliver design A and this specification subsequently fails, event which occurs with probability β, there is some positive probability that the design change will not be successful. While the difference in the cost of production $c_{aB} - c_{aA}$ is a's private information, the buyer knows that this is distributed according to the distribution $F(\cdot)$. Thus, from the buyer's perspective, the ex-ante probability that a will be willing to shift from A to B is $F(h)$.

Let us now consider how the bidders' pseudo-types change if the buyer decides to contract out project specification A or B. In the former case, a's pseudo-type is:

$$\theta_{aA} = \begin{cases} (1-\beta)c_{aA} + \beta(c_{aB} - h) & \text{if } h \geq c_{aB} - c_{aA} \\ c_{aA} & \text{if } h < c_{aB} - c_{aA} \end{cases}$$

while b's pseudo-type does not change: $\theta_{bA} = (1-\beta)c_{bA} + \beta(c_{bB} - h)$.

When B is auctioned off, it is bidder b who may not want to shift to project design A, should B turn out to be defective. b's pseudo-type is:

$$\theta_{bB} = \begin{cases} \beta c_{bB} + (1-\beta)(c_{bA} - h) & \text{if } h \geq c_{bA} - c_{bB} \\ c_{bB} & \text{otherwise.} \end{cases}$$

whereas a's pseudo-type is: $\theta_{aB} = \beta c_{aB} + (1-\beta)(c_{aA} - h)$.

Adapting the Maskin and Riley's framework to a procurement auction, we can define the weak bidder (w) as the one whose pseudo-type's distribution first order stochastically dominates that of the strong bidder (s):[11]

$$\Phi_w(\theta) < \Phi_s(\theta), \forall\theta \in (\underline{\theta}_s, \overline{\theta}_w) \tag{2}$$

Note that (2) implies that $\underline{\theta}_s \leq \underline{\theta}_w$ and $\overline{\theta}_s \leq \overline{\theta}_w$.

Since the distributions of bidders' pseudo-types vary with the values of β and h, the identities of the strong and weak bidders are endogenously determined by these parameters. Even though the buyer cannot set either of them, she can decide which project design to auction off at the beginning of the game (whether A or B). This affects the probability of a renegotiation and, in turn, the distributions of the bidders' pseudo-types the buyer faces. In other words, the buyer can influence the competitiveness of the bidding process with her initial decision of the project design. Before drawing some conclusions on the buyer's optimal strategy, we impose a property which is stronger than first order stochastic dominance, hazard rate dominance. We make the following assumption:

[11]Note that we remove the subscript referring to the design auctioned off to save on notation.

Assumption 2. $\Phi_w(\theta)$ *stochastically dominates* $\Phi_s(\theta)$ *according to the Hazard Rate, that is:*

$$\frac{\phi_w(\theta)}{1-\Phi_w(\theta)} < \frac{\phi_s(\theta)}{1-\Phi_s(\theta)},\ \forall\theta \in [\underline{\theta}_w, \overline{\theta}_s]$$

Prior to discuss the implications of hazard rate dominance, we need to introduce some more concepts and notation. The following lemma, established by Maskin and Riley (2000a,b, 2003), shows that, under our assumptions on the bidders' preferences, the distribution of winning bids in equilibrium has a support consisting of a closed interval $[\underline{t}, \bar{t}]$ and is continuous on that support. When these conditions are satisfied, the same authors show that a bidder's equilibrium bid function is strictly monotonic in his own type. This allows us to work with equilibrium inverse bid functions that we call $g_i(t)$ which are evaluated at $t \in [\underline{t}, \bar{t}]$.

Lemma 2. *If Assumption 1 holds, the distribution of winning bids in equilibrium*

(a) is an interval $[\underline{t}, \bar{t}]$*;*

(b) has a continuous c.d.f. on $[\underline{t}, \bar{t}]$.

Proof. See Maskin and Riley (2000b), Proposition 3. □

Let $P_i(t)$ be the ex-ante equilibrium bid distribution function of bidder i. This is the ex-ante probability that bidder i submits a bid lower than t. We say that a bidder i bids *consistently more aggressively* than bidder j if i shades less his bid above his pseudotype than j for any θ in the interior of their common support. In terms of equilibrium inverse bid functions, this implies that $g_i(t) > g_j(t)$ for $t \in (\underline{t}, \bar{t})$ (see Maskin and Riley, 2000a and Kirkegaard, 2009). The following lemma shows that when Assumptions 1 and 2 are fulfilled, the distribution of the equilibrium bids of the weak firm will first order stochastically dominate that of the strong firm. Moreover, the weak bidder will submit consistently more aggressive bids than the strong bidder.[12]

Lemma 3. *If Assumptions 1 and 2 hold, then*

1. $P_w(t) < P_s(t)$ *for all* $t \in (\underline{t}, \bar{t})$*;*

2. *the weak bidder will bid more aggressively than the strong bidder for any bid on the interior of their common support, that is,* $\forall t \in (\underline{t}, \bar{t})$*, it holds that* $g_w(t) > g_s(t)$.

Proof. In the Appendix. □

We now introduce the following definition of wrong project.

Definition 1 (Wrong project). *A wrong project is a project design whose prior probability of being flawed exceeds that of another design specification available to the buyer.*

[12]Two remarks. First, in fact only first-order stochastic dominance of the pseudo-types' distribution is required to induce first-order stochastic dominance of the equilibrium bid distributions. Second, to obtain the analogous result on the bidders' bidding behavior in the more standard high-bid auction, the condition required is that of reverse hazard rate dominance. The relationship between standard and procurement auctions is analyzed by Pesendorfer (2000) and de Castro and de Frutos (2010).

This definition will be helpful in analyzing the buyer's problem of choosing which project design to auction off after observing the value of β. Note that in this model where only two alternative designs are available, the wrong project is the one which has the higher probability of exhibiting imperfections. If $\beta > 1/2$, A is the wrong project. Conversely, if $\beta < 1/2$, B is the wrong project. We can now present our second result:

Proposition 2. *If:*

(i) the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage;

(ii) Assumptions 1 and 2 hold;

(iii) the ratio between the hazard rate of the strong and the weak bidder is lower when the wrong project is auctioned off;

Then, the buyer finds it profitable to auction off the wrong project.

Proof. In the Appendix. □

Proposition 2 determines the conditions under which the buyer prefers to auction off the project specification more likely to be flawed ex-post, that is, the wrong project.

To provide an intuition for the results of this proposition, suppose that β is very high so that A is the wrong project design. If the buyer decides to auction off project specification B, there exists a strong asymmetry between the distributions of the two bidders' pseudo-types. Bidder b, who is the strong bidder, can win the auction by submitting a very high bid which is detrimental to the buyer. If the buyer auctions off design A, both the distributions of the weak and the strong bidders' pseudo-type are shifted to the left. This is because there is a higher probability of renegotiating the design, which is beneficial to the bidders since they hold all the bargaining power at the renegotiation stage. The shift is asymmetric, though, since renegotiation may fail with a positive probability and the bidders' willingness to accept a change in the design of the project depends on which design is initially auctioned off. In particular, when the buyer auctions off A rather than B, the distribution of a's pseudo-type becomes closer to that of b and the identities of the strong and the weak bidders may even change.

From the buyer's standpoint what matters is that, when the wrong project design is auctioned off, she receives lower bids and such reduction in the expected payment more than compensates for the higher expected renegotiation cost. This occurs when the wrong design is initially chosen because even though the strong bidder is made stronger (his pseudo-type's distribution is left-shifted), he faces a rival who is relatively less weak. That is, the weak bidder is strengthened relatively more than the strong bidder when A rather than B is auctioned off. Therefore, the strong bidder will submit lower and more aggressive bids.[13]

The implication is that, if the assumptions of Proposition 2 are satisfied, the buyer is better off when she auctions off project design A when $\beta > \frac{1}{2}$ and design B when $\beta < \frac{1}{2}$. By adopting

[13] This intuition is triggered by Maskin and Riley (2000a): if a strong bidder faces a bidder who is relatively less weak than another one, he will react by submitting more aggressive bids.

the wrong design at the beginning of the game, the buyer is able to stiffen competition at the bidding stage and receive more aggressive bids from the bidder who is more likely to win the auction, namely the strong bidder.

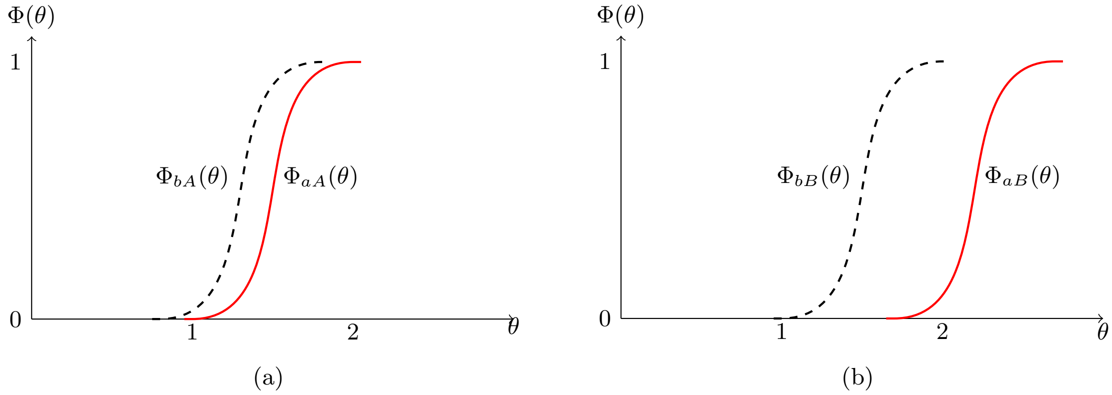

Figure 1: Distribution of bidders' pseudo-types for $\beta = 0.8$ and $h = 0.5$.

A graphical example may help grasp the intuition: in Figure 1 we compare the distributions of bidders' pseudo-types when an auction for A (part (a) of the figure) and B (part (b)) take place under the following assumptions for the distributions and the parameters: $\widetilde{c}_l \sim U[1, 2]$, $\widetilde{c}_h \sim U[2, 3]$, $h = 0.5$, and $\beta = 0.8$. The buyer clearly gains from auctioning off the wrong specification, A, as the firms are made far less asymmetric in so strengthening the competition at the bidding stage. Such benefit is not outdone by the higher rent he expects to pay.

4 Second-price Auction and Strategic Choice of the Design

In the first part of the paper we have shown that the buyer may be willing to auction off the wrong project design to induce more aggressive bidding behavior in a first-price procurement auction. Another auction format widely popular is that of the English auction and in this section we investigate whether also in this different context the buyer may have similar incentives to strategically choose the initial design of the project. The English auction is strategically equivalent to a second-price auction where the bidder who submits the lowest bid wins the auction and receives from the buyer an amount equivalent to the second-lowest bid.

In this section we build on Cantillon (2008) who studies how asymmetries between bidders impact on the expected revenues. Cantillon shows that an auction environment can be fully characterized by a *configuration*, namely the cumulative distributions of bidders' valuations (the bidders' pseudo-types in our model). Recall that in a second-price auction the distribution of the opponents' pseudo-types does not affect the equilibrium bidding strategy of a player: bidding one's own expected cost net of the expected renegotiation gain is a weakly dominant strategy. Thus, the expected winning bid for the auction of design J equals the expected value of the second-order statistics of the configuration (Φ_{aJ}, Φ_{bJ}), for $J = A, B$. Following Cantillon (2008), we can denote by S_J the cumulative distribution function of the second-order statistics of configuration (Φ_{aJ}, Φ_{bJ}) as follows (arguments are omitted):[14]

$$S_J = \Phi_{aJ}\Phi_{bJ} + (1-\Phi_{aJ})\Phi_{bJ} + (1-\Phi_{bJ})\Phi_{aJ} = \Phi_{aJ} + \Phi_{bJ} - \Phi_{aJ}\Phi_{bJ} \quad (3)$$

The choice of the initial design to auction off affects the distributions of the pseudo-types the buyer faces and therefore is analogous to selecting one out of two alternative configurations. For any value of β the buyer will select the configuration which minimizes the expected procurement cost, knowing h and the distributions of $\widetilde{c}_h$ and $\widetilde{c}_l$.

Note that if for all θ it holds that $S_A \geq S_B$ when $\beta > 1/2$ and strictly so for some values of θ, then the expected payment to the winning bidder is lower when the wrong design is put out for tender.[15] Since the wrong auction entails a higher expected loss for the buyer associated with the potential inadequacy of the design, this condition is only necessary for the buyer to be willing to auction off the wrong design under the English auction format.

To determine whether or not the buyer will auction off the wrong project design, it is useful to deal with two cases separately as we did in the previous section. When renegotiation is always successful, the following proposition shows that the neutrality of the project design found for a first-price auction is preserved in a second-price auction. Even though the auction for the design more likely to be flawed leads to a lower expected payment to the winning bidder, this benefit is exactly offset by the rise in the expected renegotiation cost borne by the buyer. Moreover, as in the first-price auction, the renegotiation rent per se is not an issue for the buyer as its value is entirely discounted at the bidding stage.

Proposition 3. *If*

(i) A second-price auction is held to award the contract;

(ii) the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage;

(iii) $h \geq \overline{c}_h - \underline{c}_l$;

then:

1. *Bidders fully compete away h at the bidding stage.*
2. *The buyer is indifferent between auctioning off design A or B.*

Proof. In the Appendix □

Suppose now that the renegotiation may fail as $h < \overline{c}_h - \underline{c}_l$. The following lemma shows that when the buyer holds an auction for the wrong project, the expected payment to the winning bidder decreases.

[14] In a general setting with N bidders whose types are independently distributed with distribution Φ_i, the configuration is $(\Phi_1, ..., \Phi_N)$. Letting S be the cumulative distribution function of the second-order statistics of $(\Phi_1, ..., \Phi_N)$, this amounts to:

$$S = \sum_{i=1}^{N} \left[(1-\Phi_i) \prod_{j \neq i} \Phi_j \right] + \prod_{i=1}^{N} \Phi_i$$

[15] An analogous condition can be derived when $\beta < 1/2$.

Lemma 4. *If*

(i) A second-price auction is held to award the contract;

(ii) the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage;

(iii) $h < \overline{c}_h - \underline{c}_l$*;*

Then, the expected payment to the winning bidder is lower for the wrong project design.

Proof. In the Appendix □

We now provide some intuitions as to when it is more likely that auctioning off the wrong project is profitable for the buyer. This occurs when the auction of the wrong design involves a decrease in the expected payment to the winning bidder that suffices to compensate the buyer for the higher expected loss due to the inadequacy of the design. To this end, we consider the distribution of the weak bidder's pseudo-type. Consider that in a second-price reverse auction with two bidders the expected payment to the winning bidder is weakly higher than the expected value (pseudo-type) of the weak bidder.

Suppose that the identities of the weak and strong bidders are affected by the project design auctioned off. This happens when the probability of renegotiating the initial design is not too high. When this is the case, the following lemma shows that auctioning off the wrong rather than the right design shifts to the left the distribution of the weak bidder's pseudo-type by more than the increase in the expected renegotiation rent.

Lemma 5. *If*

(i) A second-price auction is held to award the contract;

(ii) the contractor is entitled to make, and can commit to, a take-it-or-leave it offer to the buyer at the renegotiation stage;

(iii) $h < \overline{c}_h - \underline{c}_l$*;*

(iv) the identities of the weak and strong bidders are determined by the design auctioned off.

The distribution of the weak bidder's pseudo-type shifts to the left by more than the increase in the expected renegotiation rent when the wrong rather than the right design is auctioned off.

Proof. In the Appendix □

5 Discussion

Discrimination. In the model we have abstracted away from other possible tools the buyer might use to strengthen competition between bidders. Building on Myerson's work on optimal auction design (Myerson, 1981), McAfee and McMillan (1989) show that it is optimal to discriminate against those firms which have on average a lower cost of production. This is desirable as it induces a more balanced competition between bidders. Also in our setting, through an appropriate use of handicaps the buyer could toughen competition at the bidding stage by reducing the ex-ante asymmetry existing between bidders. Therefore, instead of using the project design more likely to be inadequate ex-post, the buyer could induce the bidders to bid more aggressively with handicaps.[16] Very often the rules which govern how public purchases should be tendered tend to favor some subset of bidders, e.g. domestic against foreign bidders. However, these procurement policies are likely due to the pressure of certain interest groups and do not normally have any solid economic justification. Relative to handicaps, a public agency may have more freedom in the initial selection of a project design. This may then be used as a tool to stiffen competition without drawing much criticism and controversy.

Complete contract. One critical assumption that is maintained throughout the paper is that the buyer is unable to hold a multiple-design auction, that is an auction in which each bidder specifies a vector of prices, one for any possible design that could be implemented. To justify our assumption, we have invoked the presence of costs that the buyer must incur to contractually describe the designs ex-ante which make it impossible to specify both A and B at time 0. When this is the case, the buyer must choose which project design to auction off. In this environment we have highlighted how the buyer could select a design more unlikely to be adequate to stiffen competition at the bidding stage.

In a complete contracting setting there is no room for such a strategic choice. It is possible to show that the buyer can devise a multiple-design auction which (i) induces the suppliers to reveal their true costs of production for the alternative designs and (ii) preempts ex-post renegotiation by automatically implementing a design change whenever it is efficient.[17] It is worth noticing that the buyer does not benefit from being unable to write a complete contract.

6 Conclusion

Renegotiation of procurement contracts is a widespread practice. In a number of cases, renegotiation is apparently unrelated to any contract incompleteness explanation, namely, it is not the emergence of ex-ante unforeseeable contingencies to trigger substantial contract modifications.

In this paper, we have shown that if the prior probability of a failure of a project specification is known to all the parties to a contract, the buyer may act strategically when choosing the design of the project to auction off. In particular, the buyer may decide to hold an auction for the project design which has a lower probability of being appropriate, in an effort to toughen competition at the bidding stage.

Our model can be best applied to the *Design-Bid-Build (DBB)* project delivery system which is the traditional and, to date, the most widely used method to organize and design

[16] To provide an example of such a discriminatory auction, consider a First Price Handicap Auction with Handicap Δ and two bidders. A supplier i who is discriminated against a supplier j wins only if $t_i \leq t_j + \Delta$. There are other forms of discrimination which are closely related, such as bonuses and cost shifts, which can yield the same equilibrium allocation (see Mares and Swinkels, 2014).

[17] Namely whenever the welfare loss due to delivering an inadequate design outweighs the higher cost of production that the alternative design would entail.

procurement contracts in the US construction industry.[18] Its distinguishing feature is that the design and the building tasks are carried out by two different entities. First, the buyer engages architects and engineers to prepare the desired specification of the project which is then put out for tender to interested general contractors. The insights of our model may prove useful in all those procurement environments where there are alternative designs available to produce the same good, but ex-ante there is uncertainty about which specification is the most appropriate. Consider, for instance, the Souterrain tram tunnel project in The Hague. There, the contractor won the tender with a very low bid and questioned a part of the design. It was eventually allowed to finish the project using its own design and technology (see Leijten, 2009).

While the change in the design considered in this paper occurs shortly after the sign of the contract, in other circumstances the buyer may grow unsatisfied with the design only after a relevant period of time. For instance, this happens when flaws in the specification of the good are discovered once production is already underway or it is already completed. Then fixing the faulty design might be extremely costly and the buyer may strive to minimize the ex-ante probability of a design failure. This is a topic which requires further investigation.

We have considered a simplified setting in which there are two bidders and two projects. The results would carry over to an environment in which there are several bidders specialized in either delivering A or B.

[18]A recent study by RSMeans Reed Construction Data Market Intelligence for The Design-Build Institute of America (DBIA) shows that more than 50% of non-residential constructions in the US were procured through the DBB system in 2010.

Appendix

Lemma 1

The bidders bid accordingly to a weakly monotonic bidding function. That is, if $t_i = \gamma_i(\theta_{iJ})$, then $\gamma_i(\theta'_{iJ}) \geq \gamma_i(\hat{\theta}_{iJ})$ if $\theta'_{iJ} > \hat{\theta}_{iJ}$, for $i = a, b$ and $J = A, B$.

Proof of Lemma 1. Assumption 1 is required to prove this lemma. Its three conditions can be rewritten as follows.

(a) Bidders' pseudo-types are drawn independently. Formally:

$$\begin{cases} \phi_{aJ}(\theta_{aJ}|\theta_{bJ}) = \phi_{aJ}(\theta_{aJ}) \\ \phi_{bJ}(\theta_{bj}|\theta_{aJ}) = \phi_{bJ}(\theta_{bJ}) \end{cases}$$

for $J = A, B$.

(b) Bidders' expected profits must decrease monotonically in their own types: $\frac{\partial E\pi_i(J)}{\partial \theta_{iJ}} < 0$ for $i = a, b$ and $J = A, B$.

(c) Firms' expected profits must be weakly supermodular: $\frac{\partial^2 E\pi_i(J)}{\partial t \partial \theta_{iJ}} \geq 0$ for $i = a, b$ and $J = A, B$.

In our model, the expected profit function of bidder i who competes with bidder l to be awarded the contract for project $J = A, B$ takes the following form:

$$E_{\theta_{lJ}}\pi_i(t_i, \theta_{iJ}) = \int_{Pr(t_i \leq \gamma_l(\theta_{lJ}))} (t_i - \theta_{iJ})\phi_{lJ}(\theta_{lJ})d\theta_{lJ}$$

where $\gamma_l(\theta_{lJ})$ is firm l's bid function which solely depends on his own pseudo-type. We can define $P_i(t_i)$ as the conditional probability of i's winning the procurement auction with a bid equal to t_i. Formally:

$$P_i(t_i) = \int_{Pr(t_i \leq \gamma_l(\theta_{lJ}))} \phi_{lJ}(\theta_{lJ})d\theta_{lJ}$$

which is a weakly decreasing function of t_i, because of assumptions (b) and (c).

Now, suppose that $\hat{t}_i$ and t'_i are the best responses of player i when his type is $\hat{\theta}_{iJ}$ and θ'_{iJ}, respectively. If so, for any $\hat{t}_i$ and t'_i it must hold that:

$$E_{\theta_{lJ}}\pi_i(\hat{t}_i, \hat{\theta}_{iJ}) = (\hat{t}_i - \hat{\theta}_{iJ})P_i(\hat{t}_i) \geq (t'_i - \hat{\theta}_{iJ})P_i(t'_i) \tag{4}$$

by definition of best response. Note that the right-hand side of 4 can be written as:

$$t'_i P_i(t'_i) - \theta'_{iJ}P_i(t'_i) + \theta'_{iJ}P_i(t'_i) - \hat{\theta}_{iJ}P_i(t'_i) = (t'_i - \theta'_{iJ})P_i(t'_i) + (\theta'_{iJ} - \hat{\theta}_{iJ})P_i(t'_i)$$

Therefore, 4 can be rewritten as

$$E_{\theta_{lJ}}\pi_i(\hat{t}_i, \hat{\theta}_{iJ}) \geq E_{\theta_{lJ}}\pi_i(t'_i, \theta'_{iJ}) + (\theta'_{iJ} - \hat{\theta}_{iJ})P_i(t'_i) \tag{5}$$

And, if $\theta'_{iJ} > \hat{\theta}_{iJ}$, we attain that:

$$P_i(\hat{t}_i) \geq \frac{E_{\theta_{lJ}}\pi_i(\hat{t}_i, \hat{\theta}_{iJ}) - E_{\theta_{lJ}}\pi_i(t'_i, \theta'_{iJ})}{\theta'_{iJ} - \hat{\theta}_{iJ}} \geq P_i(t'_i) \tag{6}$$

The numerator is always positive due to assumption (b). Furthermore, if we let $\theta'_{iJ} \to \hat{\theta}_{iJ}$ we have that

$$\frac{\partial E_{\theta_{lJ}}\pi_i(t_i, \theta_{iJ})}{\partial \theta_{iJ}} = -P_i(t_i)$$

Since the probability that i wins the auction when his pseudo-type rises does not increase and the fact that the function P_i in weakly decreasing in t_i it cannot be that $\theta'_{iJ} > \hat{\theta}_{iJ}$ and $t'_i < \hat{t}_i$.

□

Proof of Proposition 1

Proof. Condition (i) ensures that a profitable renegotiation is always successful as the informed party, the contractor, is entitled to make a TIOLI offer to which he can fully commit.

Because of condition (iii), it is always socially optimal to change a flawed design: the maximum cost differential is at most as high as h. Stated differently, there are always gains from renegotiation.

The above has two implications. First, given the design auctioned off, the extent to which both costs' distributions are left-shifted by the expected value of the renegotiation rent is the same. Therefore, at the tender stage the bidders entirely discount the renegotiation rents they expect to receive if they win the auction, which is βh if the initial design is A and $(1-\beta)h$ if the initial design is B. As a result, the renegotiation rent is not an issue for the buyer.

The second implication is that a bidder's expected cost of production is unaffected by which design is auctioned off. Namely, $E[c_a(A)] = E[c_a(B)]$ and $E[c_b(A)] = E[c_b(B)]$. The outcome of the bidding process is unaffected by the probability of renegotiating the design ex-post. Because of condition (ii), bidders whose pseudo-type is higher submit weakly higher bids (see Lemma 1). The buyer is then indifferent between auctioning off either of the two specifications of the project because the choice of the initial design does not affect the relative strength of the bidders. □

Lemma 3

Proof of Lemma 3. (i) The distribution of the equilibrium bids of the weak firm first order stochastically dominates that of the strong firm, if hazard rate dominance is fulfilled. That is, if $\frac{\phi_w(\theta)}{1-\Phi_w(\theta)} < \frac{\phi_s(\theta)}{1-\Phi_s(\theta)} \forall\theta \in [\underline{\theta_w}; \overline{\theta_s}]$, then $P_w(t) < P_s(t)\ \forall t \in (\underline{t}, \bar{t})$.

Define the equilibrium bidding strategy for player i as $t_i = \gamma_i(\theta_i)$. Since the distribution of winning bids in equilibrium is continuous (see Lemma 2), it follows that $\gamma_i(\theta_i)$ is strictly increasing at all θ_i for which $\gamma_i(\theta_i) \geq \underline{t}$. As a result the equilibrium inverse bid function, $g_i(t)$, is well defined at all $t \geq \underline{t}$ for which there exists θ_i such that t belongs to the support of $\gamma_i(\theta_i)$. Bidder i chooses his bid so as to maximize his expected profit:

$$\max_t (t - \theta_i)[1 - \Phi_j(g_j(t))]$$

The first order condition is
$$\frac{1}{t-\theta_i} = g_j'(t)\frac{\phi_j(g_j(t))}{1-\Phi_j(g_j(t))}$$
The equilibrium inverse bid functions can be found as a solution to a system of first order differential equations:
$$\begin{cases} \frac{1}{t-g_w(t)} = g_s'(t)\frac{\phi_s(g_s(t))}{1-\Phi_s(g_s(t))} \\ \frac{1}{t-g_s(t)} = g_w'(t)\frac{\phi_w(g_w(t))}{1-\Phi_w(g_w(t))} \end{cases} \tag{7}$$
with the following boundary conditions:

(i) $\Phi_i(g_i(\underline{t})) = 0$ for $i = s, w$;

(ii) if $\bar{\theta}_s = \bar{\theta}_w = \bar{\theta}$, then $\bar{t} = \bar{\theta}$;

(iii) if $\bar{\theta}_s < \bar{\theta}_w$, then $\bar{t} = \min\{\arg\max_t (t-\bar{\theta}_s)[1-\Phi_w(g_w(t))]\}$.

The second and third boundary conditions imply that $\bar{t} \in [\bar{\theta}_s, \bar{\theta}_w]$. Note first that it cannot be that $\bar{t} < \bar{\theta}_s$, or else either the strong pseudo-type with realization $\bar{\theta}_s$ or the weak pseudo-type with realization $\theta_w \in [\bar{\theta}_s, \bar{\theta}_w]$ would be incurring a loss when winning the contract. Second, it cannot be that $\bar{t} > \bar{\theta}_w$: since the distribution of the winning bids is continuous, $\bar{\theta}_s$ could slightly undercut his bid increasing his probability of winning in such a way that the overall effect on his expected profit is positive. Thus, when the upper bound of the distributions of the weak's and strong's pseudo-types is the same and equal to $\bar{\theta}$, then $\bar{t} = \bar{\theta}$. Conversely, if $\bar{\theta}_s < \bar{\theta}_w$, $\bar{t} < \bar{\theta}_w$ which increases $\bar{\theta}_s$'s probability of winning discontinuously and is given by the minimum bid which satisfies his maximization problem.

Now, let $P_i(t) = \Phi_i(g_i(t))$, i.e. $P_i(t)$ is the equilibrium bid distribution of bidder i. Let $H_i(P_i) = \Phi_i^{-1}(P_i(t)) = g_i(t)$. The system of equations 7 becomes:
$$\begin{cases} \frac{1}{t-H_w(P_w(t))} = \frac{P_s'(t)}{1-P_s(t)} \\ \frac{1}{t-H_s(P_s(t))} = \frac{P_w'(t)}{1-P_w(t)} \end{cases} \tag{8}$$
Suppose there exists $t' \in [\underline{t}, \bar{t}]$ such that $P_s(t') = P_w(t')$. Since hazard rate dominance implies first order stochastic dominance, $\Phi_w(\theta) < \Phi_s(\theta)$ and as a result $H_s(P) < H_w(P)$, $\forall P \in (0,1)$. Therefore, from 8 we obtain[19]:
$$\frac{P_w'(t')}{1-P_w(t')} = \frac{1}{t'-H_s(P_s(t'))} < \frac{1}{t'-H_w(P_w(t'))} = \frac{P_s'(t')}{1-P_s(t')} \tag{9}$$
From the above it follows that the ratio $\frac{P_w}{P_s}$ is decreasing at t'. The same would occur for any t for which $\frac{P_w(t)}{P_s(t)} = 1$. But this implies that the distributions of the equilibrium bids cannot cross more than once. Since they cross at $t = \bar{t}$, it follows that $P_w(t) < P_s(t)$ for all $t \in (\underline{t}, \bar{t})$.

(ii) If Assumption 1 and 2 hold, the weak bidder will bid more aggressively than the strong bidder for any bid in the interior of their common support, that is $\forall t \in (\underline{t}, \bar{t})$ it holds that $g_w(t) > g_s(t)$.

[19] Note that the first part of Lemma 3 just requires first order stochastic dominance.

Consider that if $\bar{\theta}_s < \bar{\theta}_w$, then at $t = \bar{t}$, $g_w(\bar{t}) > g_s(\bar{t})$. Instead if $\bar{\theta}_s = \bar{\theta}_w$, then $g_w(\bar{t}) = g_s(\bar{t})$, and because of HRD, it must be that $g'_w(t) > g'_s(t)$ in a neighborhood of $t = \bar{t}$:

$$\frac{1}{\bar{t} - g_s(\bar{t})} = g'_w(\bar{t}) \frac{\phi_w(g_w(\bar{t}))}{1 - \Phi_w(g_w(\bar{t}))} = g'_s(\bar{t}) \frac{\phi_s(g_s(\bar{t}))}{1 - \Phi_s(g_s(\bar{t}))} = \frac{1}{\bar{t} - g_w(\bar{t})}$$

But if so, for $t \in (\underline{t}, \bar{t})$, it cannot be that $g_s(t) > g_w(t)$, otherwise:

$$\frac{1}{t - g_s(t)} = g'_w(t) \frac{\phi_w(g_w(t))}{1 - \Phi_w(g_w(t))} > g'_s(t) \frac{\phi_s(g_s(t))}{1 - \Phi_s(g_s(t))} = \frac{1}{t - g_w(t)}$$

Because of HRD, that condition would imply:

$$\frac{P'_w(t)}{1 - P_w(t)} > \frac{P'_s(t)}{1 - P_s(t)},$$

which contradicts our previous finding. Therefore, $g_w(t) > g_s(t)$, namely, the weak bidder bids consistently more aggressively than the strong bidder, or, to put it differently, the degree of bid shading of the weak bidder is lower than that of the strong bidder:

$$\frac{1}{t - g_s(t)} = g'_w(t) \frac{\phi_w(g_w(t))}{1 - \Phi_w(g_w(t))} < g'_s(t) \frac{\phi_s(g_s(t))}{1 - \Phi_s(g_s(t))} = \frac{1}{t - g_w(t)} \forall t \in (\underline{t}, \bar{t}) \quad (10)$$

□

Proof of Proposition 2

Proof. Suppose that $\beta > \frac{1}{2}$ so that the wrong project is A. Let us first show that, by choosing the wrong design, the distributions of both the weak and the strong bidders' pseudo-types are shifted to the left with respect to the case in which the right design is adopted (only weakly for the strong bidder). In particular, $\Phi_{wA} > \Phi_{wB}$ for all $\theta \in (\underline{\theta}_{wA}, \bar{\theta}_{wB})$ and $\Phi_{sA} \geq \Phi_{sB}$ for all $\theta \in (\underline{\theta}_{sA}, \bar{\theta}_{sB})$.

For the weak stochastic dominance of the strong bidder under B consider the following:

- b is always the strong bidder when design B is auctioned off since he has a strictly lower probability of having a high cost of production than a.

- For design A, the strong bidder can be either b or a. Suppose that a is the strong bidder. If so, for both the wrong (A) and the right (B) project, the strong bidder is the one specialized in the initial design. This means that the initial cost of production is low and renegotiation succeeds only if h is large enough so that it more than offsets the higher cost of production that the alternative design entails. Note that the probability that renegotiation takes place is higher under A than B (β rather than $1 - \beta$). Hence $\Phi_{aA} \geq \Phi_{bB}$, with strict inequality for h sufficiently high. If h is small, renegotiation never succeeds and the two distributions coincide.

- Suppose conversely that b is the strong bidder for both designs A and B. This implies that $\Phi_{bA} > \Phi_{aA}$ and, since $\Phi_{aA} \geq \Phi_{bB}$, $\Phi_{bA} > \Phi_{bB}$.

For the strict stochastic dominance of the weak bidder under B consider the following:

- Since the strong bidder when B is auctioned off is b, the weak bidder for B is always a.
- When design A is auctioned off, the weak bidder can be either b or a. Suppose that b is the weak bidder. Note that b's pseudo-type distribution under A is left-shifted more than that of a under B as the former has a higher probability of enjoying a low cost and getting the renegotiation rent (β rather than $1-\beta$). Hence, $\Phi_{bA} > \Phi_{aB}$.
- Suppose conversely that a is the weak bidder also when design A is auctioned off. Since $\Phi_{aA} \geq \Phi_{bB}$ and $\Phi_{bB} > \Phi_{aB}$, it must be that $\Phi_{aA} > \Phi_{aB}$.

Therefore, holding an auction for the wrong project weakly strengthens the strong bidder and strictly strengthens the weak bidder[20].

Auctioning off the wrong project induces the strong bidder to submit consistently more aggressive bids only if the weak bidder's pseudo-type distribution is reinforced more than the strong bidder's pseudo-type distribution:

$$\frac{\Phi_{sB}}{\Phi_{wB}} > \frac{\Phi_{sA}}{\Phi_{wA}} > 1, \forall \theta \in (\underline{\theta}_{sA}, \overline{\theta}_{wB}) \tag{11}$$

The above ratio measures the relative strength of two bidders and has been introduced by Kirkegaard (2009) who shows that first-order stochastic dominance (the ratio being above 1) is necessary for the more aggressive bidding behavior of the weak bidder. A stronger condition is required for the result of Proposition 2 to hold, which concerns the ratio of the hazard rate functions:

$$\frac{\frac{\phi_{sA}}{1-\Phi_{sA}}}{\frac{\phi_{wA}}{1-\Phi_{wA}}} < \frac{\frac{\phi_{sB}}{1-\Phi_{sB}}}{\frac{\phi_{wB}}{1-\Phi_{wB}}}, \forall \theta \in (\underline{\theta}_{sA}, \overline{\theta}_{wB}) \tag{12}$$

When the above holds, bidders are made less asymmetric when an auction for the wrong project is held. As shown by Maskin and Riley (2000a) (Proposition 3.5), when a strong bidder faces a relatively less weak bidder, he reacts by bidding consistently more aggressively. Therefore, if (12) is fulfilled, the strong bidder faces a relatively less weak bidder in the auction for A than for B and he will bid consistently more aggressively in the wrong auction. This implies:

$$\frac{1}{t - g_{sA}(t)} > \frac{1}{t - g_{sB}(t)} \tag{13}$$

$\forall\ t \in (\underline{t}_A, \overline{t}_A)$, which is the interior of the support of the distribution of the equilibrium bids when the project design auctioned off is A. From above $g_{sA}(t) > g_{sB}(t)$. By holding an auction for the wrong project the buyer manages to stiffen competition as she makes the strong bidder bid consistently more aggressively and the strong bidder is the bidder more likely to win the auction, as shown in Lemma 3: $P_s(t) = \Phi_s(g_s(t)) > \Phi_w(g_w(t)) = P_w(t)$.

□

[20] In investigating firms' incentives to invest in cost reduction in a procurement auction, Arozamena and Cantillon (2004) model the investment as a reduction of the ex-ante distribution of costs (a distributional upgrade) which is similar to the effects that the auction of a wrong project design brings about in our paper.

Proof of Proposition 3

Proof. Assume that $h \geq \bar{c}_h - \underline{c}_l$ so that renegotiation is always successful. In a second-price auction it is a dominant strategy to bid one's own expected cost net of the expected renegotiation rent.

Thus, if the buyer auctions off design A, $t_{iA} = E[c_i(A)] - \beta h$ for $i = a, b$. The buyer's expected utility is then[21]:

$$EU(A) = v - \max\{t_{aA}, t_{bA}\} - \beta h = v - \max\{E[c_a(A)], E[c_b(A)]\} + \beta h - \beta h = v - \max\{E[c_a(A)], E[c_b(A)]\}.$$

If the buyer auctions off design B, $t_{iB} = E[c_i(B)] - (1-\beta)h$ for $i = a, b$. The buyer's expected utility is then:

$$\begin{aligned} EU(B) &= v - \max\{t_{aB}, t_{bB}\} - (1-\beta)h = v - \max\{E[c_a(B)], E[c_b(B)]\} + (1-\beta)h - (1-\beta)h \\ &= v - \max\{E[c_a(B)], E[c_b(B)]\}. \end{aligned}$$

The renegotiation rent is not a concern for the buyer since it is fully competed away at the bidding stage. Moreover, the buyer is indifferent between auctioning off design A or B as $E[c_a(A)] = E[c_a(B)]$ and $E[c_b(A)] = E[c_b(B)]$. That is, when renegotiation is always successful $EU(A) = EU(B)$.

Note that when A is auctioned off, the distributions of the pseudo-types are left-shifted with respect to the case in which design B is auctioned off. The magnitude of the relative shift to the left is the same for both bidders and equals $(2\beta - 1)h$. Hence, the expected winning bid is lower under the wrong auction by the amount $(2\beta - 1)h$. However, this exactly offsets the increase in the cost of renegotiating the project that the buyer expects to incur when she selects design A rather than B. □

Proof of Lemma 4

Proof. Assume that a renegotiation does not always succeed. Given a configuration (Φ_{aJ}, Φ_{bJ}), in a second-price auction the distribution of the buyer's payment coincides with the cumulative distribution function of the second-order statistics of configuration (Φ_{aJ}, Φ_{bJ}).

Suppose that $\beta > 1/2$ so that the right design is B and compare S_A and S_B (arguments are dropped):

$$\begin{aligned} S_A &= \Phi_{aA} + \Phi_{bA} - \Phi_{aA}\Phi_{bA} \\ S_B &= \Phi_{aB} + \Phi_{bB} - \Phi_{aB}\Phi_{bB} \end{aligned}$$

In the proof of Proposition 2 it is shown that $\Phi_{aA} \geq \Phi_{bB}$ and $\Phi_{bA} > \Phi_{aB}$. Suppose first that $\Phi_{aA} = \Phi_{bB}$ and consider the difference between S_A and S_B:

$$S_A - S_B = \Phi_{aA} + \Phi_{bA} - \Phi_{aA}\Phi_{bA} - (\Phi_{aB} + \Phi_{bB} - \Phi_{aB}\Phi_{bB})$$

This is equivalent to:

$$(\Phi_{bA} - \Phi_{aB})(1 - \Phi_{bB}) \geq 0$$

[21] While the lowest bidder is awarded the contract, the price he receives equals the second-lowest bid.

with strict inequality for some θ. Suppose instead that $\Phi_{aA} > \Phi_{bB}$ and in particular $\Phi_{aA} = \Phi_{bB} + \eta$, where also η is a function of θ. Then:

$$S_A - S_B = (\Phi_{bA} - \Phi_{aB})(1 - \Phi_{bB}) + \eta(1 - \Phi_{bA}) \geq 0$$

where the strict inequality holds for some θ. Hence, the expected payment to the winning bidder is lower when the wrong project is auctioned off.

□

Proof of Lemma 5

Proof. Suppose that $\beta > 1/2$ so that the wrong project design is A. Suppose further that renegotiation does not always succeed and the identities of the weak and the strong bidders are affected by which design is initially chosen by the buyer. This means that, as seen in the proof of Proposition 2, the weak (respectively, strong) bidder is b (a) when the initial design is A and is a (b) when the initial design is B. This is more likely to occur when β is not too high.

Note that in this case the weak bidder is always willing to accept a change in the design. Comparing the distributions of the weak bidder's pseudo-type when the design auctioned off is A and B, it is possible to note that the former is shifted to the left with respect to the latter by two different components. The first component pertains to the increase in the expected renegotiation gain which is equal to $(2\beta - 1)h$. The second component pertains to the reduced expected cost of production: under the wrong auction the weak bidder has a higher probability of incurring a low production cost. In expectation this second component amounts to $(2\beta - 1)(E\widetilde{c}_h - E\widetilde{c}_l) > 0$. Thus, distribution of the weak bidder's pseudo-type is shifted to the left by more than the higher renegotiation cost that the buyer incurs by selecting the wrong design - which is just $(2\beta - 1)h$.

□

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