Pro-Consumer Price Ceilings under Regulatory Uncertainty

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Abstract

We examine optimal price ceilings when the regulator is uncertain about demand and supply conditions and maximizes expected consumer surplus. We consider both a perfectly competitive benchmark and imperfectly competitive settings where symmetric firms compete in supply functions. Our analysis indicates that regulatory uncertainty does not eliminate the scope for intervention with a price ceiling. Instead, sufficient uncertainty calls for softer intervention, with the price ceiling set at a relatively high level. We formalize the relationship between competitive pressure and the optimal price ceiling and show that, if uncertainty is great enough, the optimal price ceiling is increasing in the degree of competition, so that greater competitive pressure justifies less restrictive regulatory intervention. For the perfectly competitive case, we also explore how the optimal price ceiling is related to the level of rationing efficiency, pinning down a cut-off level of efficiency below which a price ceiling should not be used.

Keywords: price regulation, consumer surplus, uncertainty
JEL classification: L50, D80, D41, L13, D45

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1 Introduction

Although controversial, price ceilings have been used in a wide range of markets, including those for rental accommodation, gas, electricity, telecommunications, and pay-day lending. One rationale for introducing a price ceiling is to correct market inefficiency stemming from insufficient competition. Another is the protection of consumers, which may explain the use of rent controls in markets that are nearly competitive. Insofar as price regulation is designed to correct inefficiencies from market failures, it would aim to maximize total welfare. Nonetheless, empirical evidence on past intervention suggests that the realities of regulation are more complex, with lobby groups influencing regulatory intervention in their own interests.

Recent years have witnessed an increase in the focus of policy makers and regulators on consumer welfare. This has coincided with an increased role of consumer groups in policy debate. In the UK, following the 2002 Enterprise Act, approved bodies were designated 'super-complainants' at the Office of Fair Trading (now part of the Competition and Markets Authority). This was explicitly 'to strengthen the voice of consumers' and protect their interests.[1] These changes call for a better understanding of a consumer-surplus standard in economic analysis and a recent literature examines the impact of price ceilings in this context (e.g., Davis and Kilian, 2011, Bulow and Klemperer, 2012).[2]

In this paper, we explore how underlying market conditions affect the level of a price ceiling that maximizes expected consumer surplus in a setting where the regulator is uncertain about demand and supply conditions. Our framework allows for varying degrees of competition, so it fits a gamut of market structures. We first investigate the impact of regulatory uncertainty on the optimal ceiling and then explore the relationship between competitive pressure and the optimal level of intervention. Finally, we examine the effect of arbitrary levels of rationing inefficiency on the optimal ceiling.

In our model the regulator is imperfectly informed about demand and supply and is aware of this informational disadvantage.[3] However, private agents have all relevant information, so that uncertainty is entirely on the part of the regulator. This information structure also covers the possibility that the price ceiling is a long-term regulatory decision, whereas short-run market conditions may change.[4] We focus on a frictionless homogeneous product market where

[1] These changes may be due to the deregulation of utility markets, increased market complexity (e.g., banking and financial services), and growing evidence on consumer bounded rationality. Traditionally, industry lobbying (small supplier groups) has been more efficacious. Consumers (a large but fragmented group) have been less effective; see Viscusi et al. (2005, Ch.10) and Office of Fair Trading 511 (2003). The consumer organization *Which?* - which influenced tariff regulation in the UK energy sector in 2014 - was one of the first 'super-complainants'.

[2] This standard has been used in merger evaluations in Europe and the US. See Besanko and Spulber (1993) and Neven and Röller (2005) for related analyses, and Motta (2004) for a discussion including caveats of this standard.

[3] For example, a regulator may not have detailed knowledge of what innovations are in the pipeline or of the demand for new products.

[4] It has been widely employed in the theory of regulation (see Armstrong and Sappington, 2007) and was first

a binding price ceiling lowers the cost of purchase to the consumers, but results in a shortage.[5] For tractability, we assume quadratic cost and benefit functions, but we argue in section 3 that the qualitative results have more general applicability.[6]

In a preliminary analysis of a perfectly competitive benchmark with efficient rationing, we show that with no uncertainty the price ceiling that maximizes consumer surplus always binds. The reduction in price due to regulation increases the surplus of consumers who still purchase, whilst the resulting drop in quantity supplied decreases it; the price effect prevails and the net impact on consumer surplus is positive. This result is robust to the introduction of relatively low levels of demand or supply uncertainty. We then go on to introduce arbitrary levels of uncertainty, and characterize the optimal price ceiling.[7] We show that regulatory uncertainty does not eliminate the scope for intervention with a price ceiling. Instead, sufficient uncertainty calls for softer intervention, with the price ceiling set at a relatively high level. At this level the price ceiling may not bind ex post, and so may not benefit consumers. However, this higher ceiling offers protection against potentially high free market price levels.

We then develop an imperfectly competitive framework, which can be interpreted as the reduced form of a model where identical firms compete in linear supply functions. The analysis draws on the fact that in this setting aggregate supply in the imperfectly competitive equilibrium is a fraction of the perfectly competitive one. The reduced form model with an aggregate output restriction relative to the perfectly competitive benchmark can also represent a monopoly and so our results apply to this limiting case.

If there is no regulatory uncertainty the optimal price ceiling is the same for any degree of competition. However, with sufficient regulatory uncertainty, we show that the appropriate level of the ceiling depends on the degree of competition, as this determines the price in the unregulated market. The more concentrated the market, the greater is the expected consumer surplus loss if there is no regulation. This creates a stronger incentive to lower the price ceiling, triggering a positive relationship between the optimal ceiling and the degree of competition. Thus, with enough uncertainty, a higher price ceiling, i.e., less restrictive intervention, is justified in an environment with greater competitive pressure.[8]

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formalized by Weitzman (1974) and subsequently used in analyses of regulation starting with Baron and Myerson (1982) and Lewis and Sappington (1988).

[5] With imperfect competition and search frictions, Fershtman and Fishman (1994) show that a price ceiling reduces price dispersion and increases the average price. In a related model that allows for heterogeneity in consumer information costs, Armstrong, Vickers, and Zhou (2009) show that the policy stifles incentives to become informed about the prices and this lowers firms' incentive to undercut. See also Rauh (2004) for a model with frictions on both the demand and supply side.

[6] Our analysis focuses on the role of uncertainty in a static framework. However, from a normative perspective, the introduction of a price cap is likely to have dynamic consequences, most notably, by reducing firms' investment incentives. See, for instance, Dobbs (2004) for related discussion.

[7] Our benchmark model focuses on regulatory uncertainty regarding demand. In an online appendix we show that qualitatively similar results obtain in a model with supply uncertainty only. We also outline in this appendix the corresponding model with two-sided (demand and supply) uncertainty and we explore its solution numerically.

[8] A different stream of literature examines price ceilings in homogeneous-product oligopoly markets when firms face demand uncertainty. See, in particular, Earle, Schmedders, and Tatur (2007).

We also extend the benchmark model to explore the impact of rationing inefficiency on the optimal price ceiling. We determine a cut-off level of rationing efficiency below which a price ceiling should not be used, and we show that our previous findings are qualitatively robust when rationing efficiency exceeds this level.[9] At a binding price ceiling all consumers who buy the product enjoy a higher surplus than in the absence of regulation. But with inefficient rationing some of these consumers displace others who value the product more. If rationing is sufficiently inefficient, the loss in consumer surplus from such inefficient reallocations and lower supply fully offsets the benefits of the consumers (who buy) at the lower price.

By considering arbitrary levels of rationing inefficiency and regulatory uncertainty, our findings complement those of Bulow and Klemperer (2012). They focus on random rationing in a model with no uncertainty and general demand and supply. They show that if supply is locally more elastic than demand a price ceiling always hurts consumers when demand is convex. Price regulation and the welfare loss associated with inefficient rationing in competitive markets have also been explored by Glaeser and Luttmer (1997, 2003) and Davis and Kilian (2011).[10] Our analysis contributes to this emerging literature by exploring the role of uncertainty and its interplay with rationing inefficiency. By parameterizing the efficiency of rationing, we propose a flexible framework that allows for a range of outcomes and extends previous work.

Section 2 formulates the model. Section 3 presents a preliminary analysis of the benchmark model with little or no regulatory uncertainty, while section 4 introduces arbitrary levels of uncertainty. Section 5 then analyzes the imperfect competition case. Section 6 explores the implications of rationing inefficiency. Section 7 presents conclusions. All proofs missing from the text are relegated to appendices.

2 The Model

Consider a market in which the regulator may be uncertain about the demand and supply for a homogeneous product. There is no uncertainty on the part of private agents. On this basis, the regulator chooses a price ceiling that maximizes expected consumer surplus. The regulatory intervention is announced to all private agents (producers and consumers).

First we introduce a perfectly competitive benchmark model. Let consumers' gross benefit (or utility) from the consumption of q units of the product be given by $\mathfrak{B}(q, \eta) = (B+\eta)q - bq^2/2$ where $b > 0$, $B + \eta > 0$. From the regulator's point of view η is a random variable with zero mean ($E(\eta) = 0$) distributed according to a twice continuous and differentiable c.d.f. $\mathcal{F}(\eta)$ defined on a closed interval $[n_{\min}, n_{\max}]$. We assume that the hazard rate $\mathcal{F}'(\eta)/(1 - \mathcal{F}(\eta))$ is

[9] In the online appendix our model with only supply uncertainty allows for rationing inefficiency. The results we obtain are consistent with those with demand uncertainty only.

[10] Glaeser and Luttmer develop a model to examine empirically the misallocation due to rent control and show that under conservative assumptions, 20% of rented apartments in New York City were in the wrong hands. Davis and Kilian (2011) analyze empirically the US residential market for natural gas and find substantial allocative costs over the 1950-2000 period.

strictly increasing. The inverse demand is

$$P_d(q,\eta) = \partial\mathfrak{B}(q,\eta)/\partial q = B + \eta - bq \ .$$

The suppliers' cost of producing q units is given by $\mathfrak{C}(q,\theta) = (C+\theta)q + cq^2/2$ where $c > 0$, $C + \theta > 0$. From the regulator's point of view θ is a random variable with zero mean ($E(\theta) = 0$) distributed according to a twice continuous and differentiable c.d.f. $\mathcal{G}(\theta)$ defined on a closed interval $[t_{\min}, t_{\max}]$. We assume that the hazard rate $\mathcal{G}'(\theta)/(1-\mathcal{G}(\theta))$ is strictly increasing. The two shocks η and θ are assumed to be independent. The inverse supply is

$$P_s(q,\theta) = \partial\mathfrak{C}(q,\theta)/\partial q = C + \theta + cq \ .$$

Writing p for unit price, it follows that direct demand and supply are given, respectively, by

$$q^d(p) = \frac{B+\eta-p}{b} \text{ and } q^s(p) = \frac{p-C-\theta}{c} \ . \tag{1}$$

We assume that consumers can observe η, and that producers can observe η as well as θ, while the regulator can observe neither shock. This captures the informational advantage that producers may have over the regulator regarding demand.[11]

Denote by p^* the ex-post market-clearing price (where $q^d(p^*) = q^s(p^*)$) and by q^* the corresponding output level. Then,

$$p^* = \frac{c(B+\eta)+b(C+\theta)}{b+c} \text{ and } q^* = \frac{B+\eta-C-\theta}{b+c} \ . \tag{2}$$

We assume that $B + \eta > C + \theta$ for any η and θ. This guarantees a well-defined equilibrium output ex post. Note that ex ante (before the demand and supply shocks η and θ are realized), the regulator views $p^*(\eta,\theta)$ as a random variable with expected value

$$p_e^* = \frac{cB+bC}{b+c} \ . \tag{3}$$

We explore regulatory intervention that takes the form of a price ceiling $\bar{p}$, assuming that resale of rationed goods is not possible. A price ceiling stipulates a maximal trade price and only binds if the unregulated market price lies above the regulated level. If it lies at or below the ceiling, the outcome coincides with the unregulated market equilibrium; that is, for a given price ceiling $\bar{p}$, if $p^* \leq \bar{p}$, then $q(p) = q^*$ (as given by (2)) and if $p^* > \bar{p}$, then $q(p) = q^s(\bar{p})$ (as given by (1)). The c.d.f. of $p^*(\eta,\theta)$ is determined by the c.d.f.s of η and θ, that is, $\mathcal{F}(\eta)$ and $\mathcal{G}(\theta)$, respectively. Since both η and θ are defined on closed intervals, so is the c.d.f. of $p^*(\eta,\theta)$.

We then examine price ceilings in a model of imperfect competition, maintaining our assumptions regarding the demand side of the market and the quadratic cost presented above. We assume that there are N identical suppliers and each firm i's cost of producing q_i units is

[11] It may be that η is unknown when the regulator sets a price ceiling, but is revealed by the time the producers make supply decisions; or it may be that producers can adjust their behavior as information about η is revealed by the market.

given by $\mathfrak{C}(q_i, \theta) = (C + \theta)q_i + Ncq_i^2/2$.[12] We interpret our model as one where the firms compete by choosing linear supply functions. Building on Klemperer and Meyer (1989), each firm chooses a supply function $S_i(p - C - \theta) = d_i(p - C - \theta)$, where the slope $d_i > 0$.[13] In the linear supply function equilibrium, a firm's supply is only a fraction of its supply in the competitive benchmark (see Akgün, 2004 for related analysis). Thus, with imperfect competition, the firms restrict their output compared to that in the perfectly competitive market. It then follows that the aggregate quantity supplied in the symmetric supply function equilibrium at price p can be written as

$$q^s(p, \delta) = \delta \frac{p - C - \theta}{c} \ ,$$

where $\delta < 1$ captures the restriction in output below the competitive level. Note that δ increases in the degree of competition, captured by the number of firms, N. As the market becomes nearly competitive ($N \to \infty$), δ converges to 1. We show these results in the appendix, building on the supply function competition model. Alternatively, $q^s(p, \delta)$ can be regarded as an ad-hoc way of capturing the restriction in aggregate supply in a market where the firms have market power.

Using $q^s(p, \delta)$, we can derive the equilibrium outcome in the unregulated imperfectly competitive market. Denote by p^δ the ex-post unregulated market price (where $q^d(p^\delta) = q^s(p^\delta, \delta)$) and by q^δ the corresponding output level. Then,

$$p^\delta = \frac{c(B + \eta) + \delta b(C + \theta)}{\delta b + c} \text{ and } q^\delta = \frac{\delta\,(B + \eta - C - \theta)}{\delta b + c} \ .$$

As in the competitive framework, the regulator views the unregulated market price $p^\delta(\eta, \theta)$ with imperfect competition as a random variable.

In general, regulation that takes the form of a price ceiling may result in excess demand, in which case the scarce output will be rationed. Although rationing may in principle be efficient, with the supply of output allocated to the consumers whose valuations of the good are highest, in practice there may well be some degree of inefficiency. A recent literature (see, for instance, Davis and Kilian, 2011) has focused on the alternative of random rationing, whereby the available supply is allocated randomly amongst those consumers whose valuations of the product are at least as great as the sale price.

Intuitively, the scope for pro-consumer price regulation is limited by rationing inefficiency. To examine the impact of such inefficiency, we adapt our competitive benchmark by introducing a parameter $\alpha \in [0, 1]$ that captures the efficiency of rationing. We write the regulator's objective function as a linear combination of the expected consumer surplus for efficient rationing and for

[12] This individual cost function guarantees that when the total cost of producing q using N-plants in the industry is minimised, total cost equal is $\mathsf{C}(q, \theta) = (C + \theta)q + cq^2/2$, the same as in the perfectly competitive benchmark. Then, the marginal cost curve coincides with $P^s(q)$.

[13] Klemperer and Meyer (1989) provide a thorough analysis of supply function competition. In particular, they show that with unbounded demand uncertainty and a symmetric industry there is a unique equilibrium where firms choose linear supply functions of the form we consider. This result holds regardless of the distribution of uncertainty (even if it degenerates into a mass point), so long as the support is unbounded.

extremely inefficient rationing where the available supply is allocated to the consumers whose valuations are the lowest among those who are willing to buy.[14] In particular, rationing is efficient when $\alpha = 1$, extremely inefficient when $\alpha = 0$, and random when $\alpha = 1/2$.

3 A Preliminary Analysis

This section introduces some of our results in a 'textbook' framework of perfect competition and efficient rationing (i.e., we assume that $\delta = 1$ and $\alpha = 1$). We start with a situation where demand and supply are deterministic, and show that consumer surplus can be increased from the free-market benchmark by setting an appropriate binding price ceiling.[15] We then discuss why this result still holds when there is a limited amount of demand uncertainty. The next section analyzes formally the demand-uncertainty case, allowing for arbitrary degrees of uncertainty.

Using the model introduced in section 2, let us initially assume that $\eta = \theta = 0$. The demand and supply schedules become $q^d(p) = (B - p)/b$ and $q^s(p) = (p - C)/c$, and the equilibrium price in the unregulated market is $p^* = (cB + bC)/(b + c)$. For any price ceiling $\bar{p} \geq p^*$, the market price is p^* and consumer surplus is given by

$$CS(q^d(p^*)) = Bq^d(p^*) - \frac{1}{2}b\left(q^d(p^*)\right)^2 - p^* q^d(p^*) = \frac{b(B - C)^2}{2(b + c)^2} .$$

However, any price ceiling $\bar{p} < p^*$ is binding so that output is $\min[q^d(\bar{p}), q^s(\bar{p})] = q^s(\bar{p}) = (\bar{p} - C)/c$. With efficient rationing, consumer surplus is

$$CS(q^s(\bar{p})) = Bq^s(\bar{p}) - \frac{1}{2}b\left(q^s(\bar{p})\right)^2 - \bar{p}q^s(\bar{p}) = \frac{(\bar{p} - C)[2c(B - \bar{p}) - b(\bar{p} - C)]}{2c^2} \equiv CS_d^L . \qquad (4)$$

Using $CS(q^d(p^*))$ and CS_d^L we obtain the following result.

Lemma 1 *With no uncertainty, the price ceiling $\hat{p}$ that maximizes consumer surplus under perfect competition with efficient rationing is given by*

$$\hat{p} = \frac{cB + (b + c)C}{b + 2c} < p^* . \qquad (5)$$

Setting a price ceiling slightly lower than p^* has a negative effect on consumer surplus due to a reduction in supply and the resulting exclusion of some low-valuation consumers from the market. However, there is also a positive effect as the lower price makes consumers who still purchase enjoy a higher surplus. Panel A in Figure 1 illustrates the trade-off between these effects at the optimal price ceiling $\hat{p}$. The consumer surplus lost due to the output reduction is

[14] Alternatively, α may be interpreted as the probability, for the regulator, that rationing will be efficient. If this probability is independent of the other stochastic variables in the model, we obtain the solution for uncertainty of the rationing scheme. Another interpretation is that the available supply is split, with different portions being rationed with different degrees of efficiency.

[15] In this setting, the free-market outcome is also the welfare-maximizing one. However, our analysis explores pro-consumer price regulation, that is, intervention that aims to maximize consumer surplus rather than total welfare.

captured by the dotted triangle, while the gain in the surplus of consumers who still purchase is captured by the dotted rectangle. As illustrated in the figure, the latter gain more than offsets the loss due to undersupply. The optimal pro-consumer price ceiling balances this trade-off and is strictly lower than the free-market equilibrium price p^*, and so it binds irrespective of parameter values.

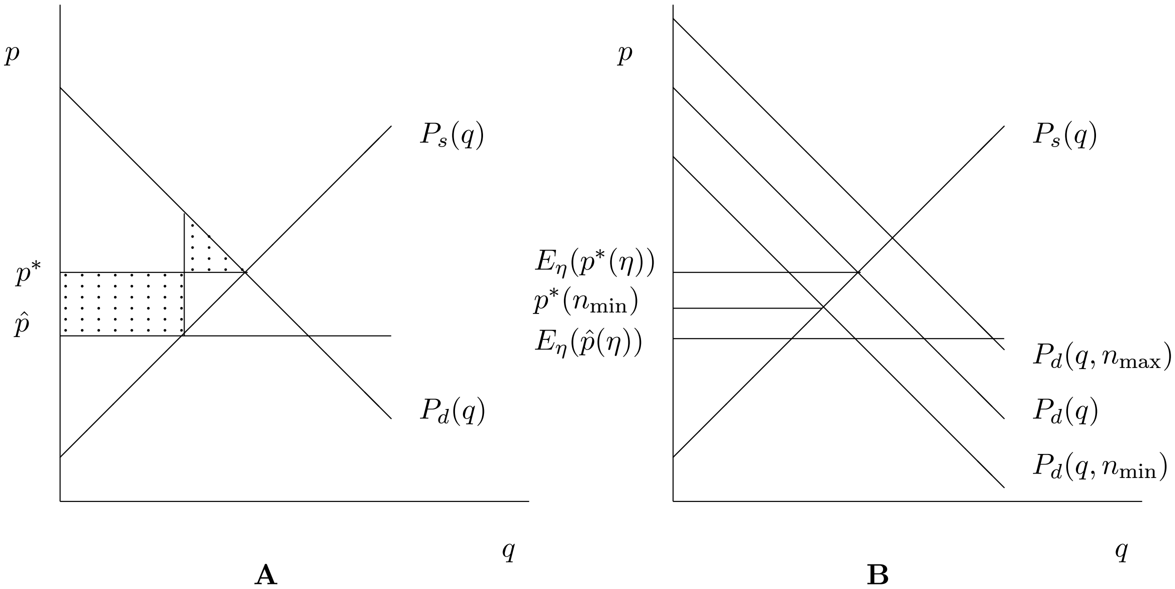

Figure 1: Optimal Pro-Consumer Price Ceiling with Efficient Rationing

Before introducing uncertainty, let us briefly discuss how the analysis above would be affected by nonlinearity of demand and supply. It can be seen that, for a given demand-supply intersection and given slopes of demand and supply at this intersection, the same qualitative conclusion holds if there is strict convexity or strict concavity of either curve. However, strict convexity results in a higher optimal ceiling, while strict concavity results in a lower one than in the linear case. Consider a strictly convex demand tangent to the straight line $P_d(q)$ in Figure 1A at the intersection with $P_s(q)$. The consumer surplus loss from the ceiling $\hat{p}$ is then larger than in the figure, but the gain is the same. Still, there are binding ceilings for which the net gain is positive (e.g., a ceiling marginally below p^*). A similar argument holds if supply is strictly convex. Then the loss is the same, and the gain is smaller than in the linear case. These conclusions are reversed for strict concavity.[16]

Let us now introduce a small amount of demand uncertainty into the model. We continue to assume that supply is certain. Panel B in Figure 1 illustrates the highest and lowest demand functions, $P_d(q, n_{\max})$ and $P_d(q, n_{\min})$, respectively. In this case, $P_d(q)$ is the expected demand and captures a situation where the realized value of η is equal to the expectation of η, $E(\eta) = 0$.

[16]Lemma 1 states that for a given demand curve with any slope $-b < 0$, and supply curve with any slope $c > 0$, the optimal price ceiling is below the market equilibrium price. It follows that if uncertainty is over slopes (but the market equilibrium point known), expected consumer surplus is then maximized with a price ceiling below the equilibrium price. We therefore conjecture that the general qualitative nature of our solutions would survive if slopes, rather than intercepts, were unknown.

For any realization of η, consumer surplus is maximized at a price ceiling

$$\hat{p}(\eta) = \frac{c(B+\eta) + (b+c)C}{b+2c} < p^*(\eta) \ .$$

This follows immediately from replacing B with $B + \eta$ in Lemma 1. The expected optimal ceiling is the same as the one presented in the proposition as $E_\eta(\hat{p}(\eta)) = \hat{p}$. This is because the objective function only depends on the expectation of η, $E(\eta) = 0$. This simple reasoning is correct so long as $\hat{p} < p^*(n_{\min})$ - which is the case if there is only relatively little uncertainty. Therefore, our result from the certainty benchmark carries over to a model with relatively little uncertainty; the price ceiling that maximizes expected consumer surplus binds regardless of the realization of demand. In contrast, if $\hat{p} > p^*(n_{\min})$, the analysis will be different: such a ceiling may bind for some realizations of demand but not for others.

4 The Benchmark Model

We now fully investigate the impact of arbitrary levels of demand uncertainty assuming supply is deterministic ($\theta = 0$). The analysis focuses on price regulation where the ceiling satisfies $\bar{p} > C$, so that supply is positive.

For a given price ceiling $\bar{p}$, we define $n^*(\bar{p})$, the specific value of η for which the market clears, i.e. $\bar{p} = p^*(\eta)$. Using (2) with $\theta = 0$, we obtain

$$n^*(\bar{p}) = \frac{(b+c)(\bar{p} - p_e^*)}{c} \ , \tag{6}$$

where p_e^*, the expectation of $p^*(\eta)$ is defined in (3).

In the parameter region where $\bar{p} < p^*(n_{\min})$ (i.e., for $\bar{p}$ so that $n^*(\bar{p}) < n_{\min}$) a price ceiling binds and results in excess demand for all values of η. In the region where $\bar{p} > p^*(n_{\max})$ (i.e., for $\bar{p}$ so that $n^*(\bar{p}) > n_{\max}$), a price ceiling does not bind and the free-market outcome prevails for all values of η. However, in the region where $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$ (i.e., for $\bar{p}$ so that $n^*(\bar{p}) \in [n_{\min}, n_{\max}]$), the effect of a price ceiling depends on the value of η. In particular, for low demand (when $\eta \in [n_{\min}, n^*(\bar{p})))$ the price ceiling does not bind, whereas for high demand (when $\eta \in [n^*(\bar{p}), n_{\max}]$) it results in excess demand. Thus, we identify three regions of price ceilings where the intervention has differing implications. We will examine each of these three regions as potential locations for the optimal value of $\bar{p}$.

- ▶ In the low price region, regardless of η, the intervention binds.
- ▶ In the middle region, the ceiling may bind or not depending on the realization of η.
- ▶ In the high price region, the intervention does not bind.

The low price region. In this case $\bar{p} < p^*(n_{\min})$ and, regardless of η, supply is a binding constraint, so output is $\min[q^d(\bar{p}), q^s(\bar{p})] = q^s(\bar{p}) = (\bar{p} - C)/c$. Consumer surplus is $CS(q^s(\bar{p}), \eta) = (\bar{p} - C)[2c(B + \eta - \bar{p}) - b(\bar{p} - C)]/2c^2$, and expected consumer surplus is $E(q^s(\bar{p}), \eta) = CS_d^L$

given in (4). The price ceiling that solves the f.o.c. of the optimization problem in this region is $\hat{p}$ in (5).

Before proceeding, we introduce some notation.

Definition 1 *Let* $n_0 = -\dfrac{c(B-C)}{b+2c}$.

If $n_{\min} \leq n_0$, CS_d^L is increasing for all $\bar{p} < p^*(n_{\min})$ and the critical value $\hat{p}$ is weakly larger than $p^*(n_{\min})$, which is inconsistent with the region $(C, p^*(n_{\min}))$. However, if $n_{\min} > n_0$, as $d^2CS_d^L/d\bar{p}^2 < 0$, (5) is a well-defined local maximum within the region $(C, p^*(n_{\min}))$. This proves the following result.

Lemma 2 *With demand uncertainty, if* $n_{\min} > n_0$*, then in the region where* $\bar{p} < p^*(n_{\min})$ *the price ceiling that maximizes expected consumer surplus under perfect competition is given by (5). If* $n_{\min} \leq n_0$*, an optimal price ceiling cannot be strictly lower than* $p^*(n_{\min})$*.*

This result shows that with large enough demand uncertainty, in the sense that $n_{\min} \leq n_0$, the optimal price ceiling must be at least as high as $p^*(n_{\min}) = \min_\eta p^*(\eta)$. Otherwise, Lemma 2 shows that there is a local maximum that is lower than $p^*(n_{\min})$. After analyzing the other possible levels of the price ceiling, we will explore the global optimality of this ceiling.

The high price region. Consider now a price ceiling in the region $\bar{p} > p^*(n_{\max})$. In this case, regardless of the realization of η, the ceiling does not bind (so the outcome is the same as with no intervention). For a given η, consumer surplus is $b(B+\eta-C)^2/2(b+c)^2$. So expected consumer surplus becomes

$$\frac{b[(B-C)^2 + E(\eta^2)]}{2(b+c)^2} \equiv CS_d^H \ , \tag{7}$$

and is the same as in the free-market equilibrium.

The middle price region. We explore price ceilings in the region $[p^*(n_{\min}),\, p^*(n_{\max})]$, so that the corresponding $n^*(\bar{p}) \in [n_{\min},\, n_{\max}]$. We make use of the following observation.

Remark 1 *From the definitions of* $p^*(\eta)$ *and* $n^*(\bar{p})$*, it follows directly that* $prob(p^* \leq \bar{p}) = prob(\eta \leq n^*(\bar{p})) = \mathcal{F}(n^*(\bar{p}))$*.*

For any price ceiling $\bar{p} \geq p^*(\eta)$ (or, equivalently, $\eta \leq n^*(\bar{p})$), the intervention does not bind, the market-clearing price $p^*(\eta)$ prevails and the quantity traded is $q^d(p^*, \eta) = q^s(p^*)$. Since

$$q^d(p^*, \eta) = \frac{B+\eta-p^*}{b} = \frac{B+\eta-C}{b+c} \equiv q_d^* \ ,$$

consumer surplus in this case is given by

$$CS(q_d^*) = \frac{b(B+\eta-C)^2}{2(b+c)^2} \ .$$

Using Remark 1, it follows that expected consumer surplus conditional on $\bar{p} \geq p^*(\eta)$ is $E(CS(q_d^*) \mid \eta \leq n^*(\bar{p})) = \left(\int_{n_{\min}}^{n^*(\bar{p})} CS(q_d^*) d\mathcal{F}(\eta)\right) / \mathcal{F}(n^*(\bar{p}))$. Substituting $CS(q_d^*)$, we obtain

$$\frac{b\left[(B-C)^2\mathcal{F}(n^*(\bar{p})) + 2(B-C)\epsilon_d^L(n^*(\bar{p})) + \Upsilon_d^L(n^*(\bar{p}))\right]}{2(b+c)^2\mathcal{F}(n^*(\bar{p}))} \equiv CS_d^* \ ,$$

where $\epsilon_d^L(n^*(\bar{p})) = \int_{n_{\min}}^{n^*(\bar{p})} \eta d\mathcal{F}(\eta)$ and $\Upsilon_d^L(n^*(\bar{p})) = \int_{n_{\min}}^{n^*(\bar{p})} \eta^2 d\mathcal{F}(\eta)$.

For $\bar{p} \leq p^*(\eta)$ (or, equivalently, $\eta \geq n^*(\bar{p})$), however, the intervention leads to excess demand. The realized consumer surplus from the $q^s(\bar{p}) = (\bar{p}-C)/c$ units produced is the same as in the low price region. Therefore, expected consumer surplus conditional on $\bar{p} \leq p^*(\eta)$ is $E_\eta(CS(q^s(\bar{p}), \eta) \mid \eta \geq n^*(\bar{p})) = \left(\int_{n^*(\bar{p})}^{n_{\max}} CS(q^s(\bar{p}), \eta) d\mathcal{F}(\eta)\right) / \left[1 - \mathcal{F}(n^*(\bar{p}))\right]$, which becomes

$$\frac{(\bar{p}-C)\left[2c(B-\bar{p}) - b(\bar{p}-C)\right]}{2c^2} + \frac{(\bar{p}-C)\,\epsilon_d^H(n^*(\bar{p}))}{c(1-\mathcal{F}(n^*(\bar{p})))} \equiv CS_d^S \ , \tag{8}$$

where $\epsilon_d^H(n^*(\bar{p})) = \int_{n^*(\bar{p})}^{n_{\max}} \eta d\mathcal{F}(\eta)$.

Using CS_d^* and CS_d^S, it follows that total expected consumer surplus for any given price ceiling $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$ is

$$E(CS(\bar{p})) = \mathcal{F}(n^*(\bar{p}))CS_d^* + (1 - \mathcal{F}(n^*(\bar{p}))CS_d^S \equiv CS_d \ . \tag{9}$$

Inspection of (4), (7) and (9) leads to the following result.

Lemma 3 *With demand uncertainty, expected consumer surplus under perfect competition is continuous and differentiable for all values of $\bar{p} > C$. Moreover, at any price ceiling $\bar{p} \geq p^*(n_{\max})$ expected consumer surplus is independent of $\bar{p}$.*

For $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$, the regulator sets the price ceiling to maximize CS_d. An interior optimal price ceiling in this region solves $dCS_d/d\bar{p} = 0$, where

$$\frac{dCS_d}{d\bar{p}} = -(1 - \mathcal{F}(n^*(\bar{p})))\frac{(b+2c)(\bar{p}-C) - c(B-C)}{c^2} - \mathcal{F}(n^*(\bar{p}))\frac{1}{c}\frac{\epsilon_d^L(n^*(\bar{p}))}{\mathcal{F}(n^*(\bar{p})} \ . \tag{10}$$

The two terms here are related to the fact that either supply or demand may bind with price ceiling $\bar{p}$, and they are weighted by their respective probabilities. The fraction $-[(b+2c)(\bar{p} - C) - c(B-C)]/c^2$ is the value of $dCS_d/d\bar{p}$ when there is no uncertainty and supply is the binding constraint, as in our preliminary analysis. The second term in (10) shows the impact of demand uncertainty. $\epsilon_d^L(n^*(\bar{p}))/\mathcal{F}(n^*(\bar{p}))$ (< 0) is the expected value of the demand shock η, conditional on demand binding, and a more negative value represents greater demand uncertainty. This term has a positive impact on $dCS_d/d\bar{p}$.

We then obtain the following proposition.

Proposition 1 *With demand uncertainty, if $n_{\min} \leq n_0$, the unique price ceiling that maximizes expected consumer surplus under perfect competition lies in the interval $[p^*(n_{\min}), p^*(n_{\max}))$. In addition, this ceiling may be greater or smaller than the expected market-clearing price. Specifically, CS_d is maximized at $\bar{p} \gtreqless p_e^*$ as $-(1-\mathcal{F}(0))c(B-C) - (b+c)\epsilon_d^L(0) \gtreqless 0$.*

Proposition 1 holds if the regulator faces enough uncertainty. This favours setting a price ceiling that is relatively high and, therefore, less likely to bind. If the price ceiling binds ex post so that supply is a constraint, variation of the demand curve has no effect on consumption. However, if ex post the price ceiling does not bind, greater demand uncertainty generates a larger expected consumer surplus. Consumer surplus for a given η equals $(b/2)\left[(B+\eta-C)/(b+c)\right]^2$ and so demand uncertainty ($\eta \neq 0$) raises the expected value of this expression. Since demand uncertainty has a positive effect on expected consumer surplus if the ceiling does not bind, and no effect otherwise, it supports a high price ceiling that is less likely to bind. This works against the incentive to set a relatively low price that we identified in our preliminary analysis and may even offset it.

Example 1 *Suppose η is uniformly distributed on $[-n, n]$ so that $\epsilon_d^L(0) = -n/4$ and $\mathcal{F}(0) = 1/2$. The condition $n_{\min} \leq n_0$ in Proposition 1 becomes $n \geq c(B-C)/(b+2c)$. It follows that CS_d is maximized at $\bar{p} \gtreqless p_e^*$ as $n \gtreqless 2c(B-C)/(b+c)$. Then, for $n \in [c(B-C)/(b+2c), 2c(B-C)/(b+c))$, the optimal price ceiling is lower than p_e^* and for $n > 2c(B-C)/(b+c)$, the optimal ceiling is higher than p_e^*.*

The result that with relatively little uncertainty CS_d is maximized at $\bar{p} < p_e^*$ is related to the effects discussed in section 3. There we saw that when demand and supply are certain, a price reduction below the market equilibrium raises consumer surplus per unit bought. In contrast, when demand is stochastic, it is not known ex ante whether realized supply will be a binding constraint on consumption and, as a result, we show that for large enough uncertainty CS_d is maximized at $\bar{p} > p_e^*$.

However, when there is small enough uncertainty, the result from the certainty case still holds so that CS_d is maximized at $\bar{p} < p_e^*$. Moreover, with very little uncertainty, that is, if $n_{\min} > n_0$, the analysis preceding Proposition 1 shows that an optimal price ceiling cannot lie in the interval $[p^*(n_{\min}), p^*(n_{\max})]$. Together with the findings in Lemma 3, this establishes the following result.

Proposition 2 *With demand uncertainty, if $n_{\min} > n_0$, the unique price ceiling that globally maximizes expected consumer surplus under perfect competition is given by (5) and always binds.*

With relatively little uncertainty ($n_{\min} > n_0$), the optimal pro-consumer price ceiling is strictly lower than $p^*(n_{\min})$ and, implicitly, than the expected market-clearing price. This result, along with Proposition 1, highlights the effects of the degree of uncertainty on optimal regulation. Proposition 2 generalizes the initial analysis illustrated in Figure 1B. In contrast, if there is enough uncertainty ($n_{\min} \leq n_0$), the optimal price ceiling is higher than $p^*(n_{\min})$ as shown in Proposition 1.

In this section we have focused on the benchmark case of demand uncertainty with deterministic supply. In the online appendix we present the corresponding analysis with supply uncertainty only (that is, with stochastic θ but deterministic η) and show that the qualitative results from the benchmark model carry over unchanged.

Drawing on these results, we also explore two-sided uncertainty in the online appendix, that is, we assume that η and θ are both stochastic and are independently distributed. Again we identify three candidate regions for the optimal price ceiling. However, in the 'middle' region the objective function depends on whether $p^*(n_{\min}, t_{\max}) - p^*(n_{\max}, t_{\min}) \gtrless 0$, and for each sign of this inequality this region divides into three sub-regions. Although a complete analysis of the model is then not tractable due to the complexity of the objective function, we show that the results of our benchmark model are qualitatively robust. We investigate further the joint impact of demand and supply uncertainty using numerical simulations, illustrating, for example, how an increase in uncertainty can change the optimal price ceiling from below above the expected market-clearing price.[17]

This section has focused on the role of regulatory uncertainty. However, the setting with no uncertainty is a special case of our analysis where η has a degenerate distribution so that $n_{\min} = n_{\max} = 0$, as we showed in section 3. Thus, the optimal price ceiling is $\hat{p}$ as given by (5) with no uncertainty and remains at this level when a limited amount of uncertainty (in the sense that $n_{\min} > n_0$) is introduced. But, with greater uncertainty (for $n_{\min} \leq n_0$) a price ceiling in the middle region, $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$, is optimal. In this case, the no-uncertainty optimal ceiling $\hat{p}$ belongs to the middle region. Evaluating (10) at $\hat{p}$, we obtain

$$\frac{dCS_d(\hat{p})}{d\bar{p}} = -\frac{1}{c}\epsilon_d^L(n^*(\hat{p})) \geq 0 \ ,$$

where the weak inequality follows from $\epsilon_d^L(n^*(\hat{p}) \leq 0$, and holds with equality only for $n_{\min} = n_0$ (or, $p^*(n_{\min}) = \hat{p}$). Therefore, if uncertainty is great enough ($n_{\min} \leq n_0$), the optimal price ceiling which lies in the middle region is strictly higher than $\hat{p}$, the optimal ceiling with smaller or no uncertainty.

5 Imperfect Competition

We now analyze the price ceiling that maximizes expected consumer surplus in the imperfectly competitive framework presented in section 2. We assume that the regulator only faces uncertainty regarding demand (η is stochastic and $\theta = 0$). We explore the robustness of our findings in the perfectly competitive benchmark to a more realistic setting where suppliers have (some) market power, and analyze how changes in the degree of competition affect the optimal pro-consumer ceiling. In this model, $\delta < 1$ measures competitive pressure, with a larger value corresponding to more intense competition. This analysis can be interpreted as the reduced form of a model where firms compete in supply functions. However, at the end of the section, we show that our analysis is informative also for monopoly markets.

[17] For both the one-sided supply uncertainty case and the two-sided uncertainty simulations we also allow for rationing inefficiency.

First note that in a model with no uncertainty, the optimal pro-consumer price ceiling from our preliminary analysis still obtains with imperfect competition because consumer surplus is maximized at the same price level. However, as we show below, this is no longer true with large uncertainty.

Consider a realization of demand η. Then the equilibrium price in the imperfectly competitive market is given by

$$p^{\delta}(\eta, \delta) = \frac{c(B + \eta) + \delta bC}{\delta b + c} > p^*(\eta)$$

where $p^*(\eta)$ is the perfectly competitive price (see Figure 2).

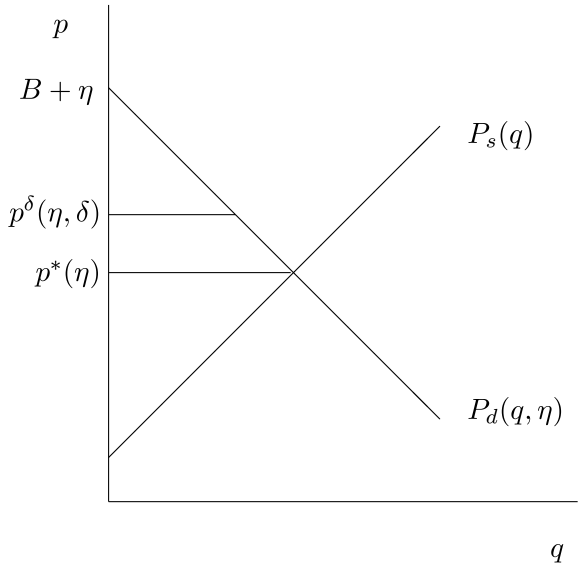

Figure 2: Imperfectly Competitive Price

A price ceiling $\bar{p}$ binds or not depending on its position relative to $p^{\delta}(\eta, \delta)$. If $\bar{p} \geq p^{\delta}(\eta, \delta)$, then the intervention does not bind and the imperfectly competitive market outcome prevails. If $\bar{p} < p^{\delta}(\eta, \delta)$, then the intervention binds and either demand or supply may be a constraint depending on the position of $\bar{p}$ relative to the perfectly competitive price $p^*(\eta)$. More specifically, if $\bar{p} < p^*(\eta)$, the quantity traded will be $q^s(\bar{p})$ and there is excess demand. If $p^*(\eta) < \bar{p} < p^{\delta}(\eta)$, the quantity traded will be $q^d(\bar{p})$ and there is excess supply.

Consider our supply function competition interpretation. If the market is unregulated, the firms compete by choosing price-quantity schedules. However, in the presence of a binding price ceiling, the firms can no longer adjust the price, so they can only choose optimally the output levels.[18] When the intervention binds, $\bar{p}$ becomes the (constant) marginal revenue. Therefore the firms will supply the quantity at which $\bar{p}$ equals marginal cost $P^s(q)$ unless demand is a binding constraint, in which case they supply the quantity demanded at $\bar{p}$. So, the quantity traded in the market at the regulated price is $\min\{q^d(\bar{p}), q^s(\bar{p})\}$, where $q^s(\bar{p})$ is the quantity

[18] In this sense, in a market where, before intervention, firms compete in supply functions, the introduction of a binding price ceiling leads to a change in competition as the firms become price-takers.

at which marginal cost equals $\bar{p}$.[19] This is different from the perfectly competitive benchmark where a binding ceiling could only result in excess demand.

As before, we examine the different price regions as potential locations for the optimal price ceiling. We first analyze the impact of a price ceiling in a situation where $p^\delta(n_{\min}, \delta) < p^*(n_{\max})$, which is arguably more general.[20] For a given δ, $p^\delta(n_{\min}, \delta)$ is the ex-post equilibrium price for the lowest demand realization, while $p^*(n_{\max})$ is the perfectly competitive price for the largest demand realization.

(i) First consider a price ceiling $\bar{p} > p^*(n_{\max})$. For values of η at which the ceiling binds, demand is a constraint, that is $\min\{q^d(\bar{p}), q^s(\bar{p})\} = q^d(\bar{p})$. As a result, a small decrease in $\bar{p}$ will lead to an increase in consumer surplus. At values of η where the ceiling does not bind, a decrease in the ceiling either has no effect on consumer surplus or, if it has, it leads to greater consumer surplus as the unregulated market price is above the competitive level for $\delta < 1$. This shows that an optimal price ceiling cannot be strictly larger than $p^*(n_{\max})$ as a slightly lower one increases consumer surplus.

(ii) If $\bar{p} < p^*(n_{\min})$, the price ceiling binds regardless of the realized demand and our corresponding analysis in section 4 carries over unchanged. It follows that, with demand uncertainty, if $p^\delta(n_{\min}, \delta) < p^*(n_{\max})$ and $n_{\min} > -c(B - C)/(b + 2c)$, in the region where $\bar{p} < p^*(n_{\min})$, the price ceiling that maximizes expected consumer surplus is given by (5). But, if $n_{\min} < -c(B-C)/(b+2c)$, an optimal price ceiling cannot be strictly lower than $p^*(n_{\min})$ as expected consumer surplus is increasing in $\bar{p}$ in this case.

(iii) Let us now focus on $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$. As $p^\delta(n_{\min}, \delta) \in [p^*(n_{\min}),\ p^*(n_{\max})]$, we can identify two sub-regions as potential locations for the optimal price ceiling, $[p^*(n_{\min}), p^\delta(n_{\min}, \delta)]$ and $(p^\delta(n_{\min}, \delta),\ p^*(n_{\max})]$.

a) Consider first $\bar{p} \in [p^*(n_{\min}), p^\delta(n_{\min}, \delta)]$. The price ceiling binds regardless of the value of η. However, depending on η, the intervention may lead to excess demand or excess supply. More precisely, for a given price ceiling $\bar{p}$, there exists a specific value of $\eta = n^*(\bar{p})$ for which $p^*(\eta) = \bar{p}$. This is the same $n^*(\bar{p})$ as the one presented in (6). If $\bar{p} \geq p^*(\eta)$ or, equivalently, if $\eta \leq n^*(\bar{p})$, then there is excess supply, so that consumer surplus is determined by the quantity demanded, $q^d(\bar{p})$. If $\bar{p} < p^*(\eta)$ or, equivalently, if $\eta > n^*(\bar{p})$, then there is excess demand and consumer surplus is determined by $q^s(\bar{p})$, the quantity at which marginal cost equals $\bar{p}$. Denote by CS_δ^L the expected consumer surplus in this case. Then,

$$\frac{dCS_\delta^L}{d\bar{p}} = -\frac{1}{b}(B-\bar{p}) - \frac{b+c}{bc}\epsilon_d^L(n^*(\bar{p})) + \frac{(b+c)\left[c(B-C) - (b+c)(\bar{p}-C)\right]}{bc^2}(1-\mathcal{F}(n^*(\bar{p}))), \quad (11)$$

[19] Note that $q^s(p)$ is the same as the perfectly competitive supply. In contrast, $q^s(p, \delta)$ is the aggregate supply in the unregulated market where the firms compete in supply functions and $q^s(p, \delta) = \delta q^s(p)$.

[20] Even if for some values of δ, $p^\delta(n_{\min}, \delta) \geq p^*(n_{\max})$, there exists a cut-off value δ_0, so that for $\delta \in [\delta_0, 1)$, $p^\delta(n_{\min}, \delta) < p^*(n_{\max})$. This is because $lim_{\delta\to 1} p^\delta(n_{\min}, \delta) = p^*(n_{\min}) < p^*(n_{\max})$. When $\delta \to 1$, the market becomes almost perfectly competitive. The cut-off value δ_0 is implicitly defined by $p^*(n_{\max}) = p^\delta(n_{\min}, \delta_0)$. Towards the end of this section, we will return to the case where $p^\delta(n_{\min}, \delta) \geq p^*(n_{\max})$.

where $\epsilon_d^L(n^*(\bar{p}))$ is defined in section 4.

b) For $\bar{p} \in (p^\delta(n_{\min}, \delta), p^*(n_{\max})]$ the price ceiling may bind or not depending on the value of η. More precisely, there exists a cut-off value of η, let it be $n^{**}(\bar{p}, \delta)$, implicitly defined by $p^\delta(n^{**}, \delta) = \bar{p}$ so that for $\eta < n^{**}(\bar{p}, \delta)$, the price ceiling does not bind and the quantity traded is $q^d(p^\delta(\eta, \delta))$. For $\eta \geq n^{**}(\bar{p}, \delta)$, the intervention may lead to excess demand or excess supply depending again on the value of η. Specifically, there exists another value of $\eta = n^*(\bar{p})$ for which $p^*(\eta) = \bar{p}$. As in part a) above, this is the same as the one presented in (6), and $n^{**}(\bar{p}, \delta) < n^*(\bar{p})$. If $n^{**}(\bar{p}, \delta) \leq \eta \leq n^*(\bar{p})$, then there is excess supply so that consumer surplus is determined by the quantity demanded, $q^d(\bar{p})$. If $\eta > n^*(\bar{p})$, then there is excess demand and consumer surplus is determined by $q^s(\bar{p})$, the quantity at which marginal cost equals $\bar{p}$. Let CS_δ^H be the expected consumer surplus in this case. Then,

$$\begin{aligned}\frac{dCS_\delta^H}{d\bar{p}} &= -\frac{1}{b}(B-\bar{p})(\mathcal{F}(n^*(\bar{p})) - \mathcal{F}(n^{**}(\bar{p},\delta))) - \frac{b+c}{bc}\epsilon_d^L(n^*(\bar{p})) + \frac{1}{b}\epsilon_d^L(n^{**}(\bar{p},\delta)) \\ &\quad + \frac{c(B-\bar{p})-(b+c)(\bar{p}-C)}{c^2}(1-\mathcal{F}(n^*(\bar{p}))) \ ,\end{aligned} \tag{12}$$

where $n^{**}(\bar{p}, \delta) = [(\delta b + c)p - cB - \delta bC]/c$ and $\epsilon_d^L(n^{**}(\bar{p})) = \int_{n_{\min}}^{n^{**}(\bar{p})} \eta d\mathcal{F}(\eta)$.

In the online appendix we show that for $n_{\min} > -c(B-C)/(b+2c)$, the globally optimal price ceiling is strictly lower than $p^*(n_{\min})$ and given by (5), whereas for $n_{\min} \leq -c(B-C)/(b+2c)$ it lies in the interval $[p^*(n_{\min}), p^*(n_{\max})]$ and, depending on the parameter values, it is given by either $dCS_\delta^H/d\bar{p} = 0$ or $dCS_\delta^L/d\bar{p} = 0$.[21,22]

Let us now briefly turn to the case where $p^\delta(n_{\min}, \delta) \geq p^*(n_{\max})$. As before, there are three sub-regions of prices where the optimal ceiling could lie. The analyses for $\bar{p} > p^*(n_{\max})$ and $\bar{p} < p^*(n_{\min})$ echo cases (i) and (ii) above and the corresponding results carry over unchanged. For $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$, as $p^\delta(n_{\min}, \delta) > p^*(n_{\max})$, the price ceiling binds regardless of the realization of η. The intervention may lead to excess demand or excess supply depending on the value of η. Intuitively, the analysis in part iiia) above applies now to the entire range. It is straightforward that the expected consumer surplus in this case is given by CS_δ^L.

We are now ready to state the following result.

Proposition 3 *For sufficiently large demand uncertainty ($n_{\min} > -c(B-C)/(b+2c)$), the optimal pro-consumer price ceiling with imperfect competition is increasing in the degree of competition. For smaller uncertainty ($n_{\min} < -c(B-C)/(b+2c)$), it is independent of the degree of competition.*

With no uncertainty, the optimal ceiling is independent of δ. But, with sufficient uncertainty, the optimal price ceiling is increasing in the degree of competition and, therefore, lower than the

[21] In the special case where $p^\delta(n_{\min}, \delta)$ is the optimal ceiling, both $dCS_\delta^H/d\bar{p} = 0$ and $dCS_\delta^L/d\bar{p} = 0$ hold.

[22] In section 4, we found that, with perfect competition, expected consumer surplus is single peaked. This single-peakedness result is robust with imperfect competition. We show this in the appendix by proving that the slope of the expected consumer surplus with imperfect competition is weakly smaller than that with perfect competition.

one in the perfectly competitive market. A decrease in the optimal ceiling generates significant gains in consumer surplus under high demand realizations. If the optimal ceiling is lower than $p^{\delta}(n_{\min}, \delta)$, so that it binds regardless of the realized demand, then it is independent of the degree of competition and is weakly lower than in the competitive case. If the optimal ceiling is higher than $p^{\delta}(n_{\min}, \delta)$, it may bind or not depending on the realization of demand and it is smoothly increasing in the degree of competition.

Suppose that the price ceiling that is optimal under perfect competition were set in an imperfectly competitive market. For demand realizations for which this ceiling does not bind under perfect competition, the price that obtains under imperfect competition[23] is higher than the perfectly competitive market-clearing price. It is therefore above the consumer-surplus maximizing price (see section 3), so that lowering the ceiling benefits consumers. This argument underlies the conclusion that the optimal ceiling is positively related to the extent of competition. In this sense, our analysis shows that, with sufficient regulatory uncertainty, competitive pressure weakens the scope for pro-consumer regulatory intervention.

In our imperfectly competitive market model, the aggregate output is a fraction δ of the competitive supply. This result builds on the supply function competition microfoundation and, in the appendix, we show that $\delta \geq 2c/(c + \sqrt{c(2b + c)})$ for $N \geq 2$. However, the reduced form model with an output restriction can also capture a monopoly market when $\delta = c/(b + c)$.[24] Therefore, the analysis in this section applies straightforwardly to a market with a monopoly supplier.

6 Inefficient Rationing

This section explores the implications of assuming inefficient rationing in our benchmark model. More precisely, we examine pro-consumer price ceilings in a perfectly competitive market with uncertain demand and deterministic supply allowing for inefficient rationing. As the analysis builds on section 4, we focus on differences stemming from rationing inefficiency and we highlight its interaction with the extent of demand uncertainty in the determination of the optimal price ceiling.[25] Provided rationing efficiency α is great enough, the results from the benchmark case still hold qualitatively, although the level of the optimal ceiling may depend on the value of α. However, we pin down a cut-off rationing efficiency below which a price ceiling should not be set.

We use the three candidate regions for the optimal price ceiling specified in section 4. Since a price ceiling in the high region, $\bar{p} > p^{*}(n_{\max})$, never binds, it can be seen immediately that

[23] This is either the imperfectly competitive equilibrium price or the ceiling, depending on the value of η.

[24] This value of δ can be pinned down by solving for the monopoly outcome but it also obtains in the supply function microfoundation if we allow $N \to 1$.

[25] Full details of the derivations are relegated to the online appendix. The parallel case with deterministic demand but uncertain supply is also analyzed there, allowing for inefficient rationing.

expected consumer surplus for a price ceiling in this region is unaffected by rationing inefficiency and so, as in section 4, expected consumer surplus is given by (7). However, in the middle region supply may be a binding constraint and in the low region it surely is, so for these regions inefficiency of rationing plays an important role.

When, for a given η, supply binds, output $q^s(\bar{p}) = (\bar{p} - C)/c$ is rationed. If this supply is allocated to the consumers whose valuations of the product are highest, consumer surplus is given by CS_d^L in (4). Alternatively, however, suppose $q^s(\bar{p})$ is purchased by the consumers with the lowest valuations amongst those willing to pay at least $\bar{p}$. Then the $q^s(\bar{p}) = (\bar{p} - C)/c$ units available are purchased by consumers along the lowest part of the demand curve at and above $\bar{p}$, i.e., from $q^d(\bar{p}) - q^s(\bar{p})$ to $q^d(\bar{p})$. At $q^d(\bar{p}) - q^s(\bar{p})$ consumer surplus per unit is $B + \eta - b\left(q^d(\bar{p}) - q^s(\bar{p})\right) - \bar{p}$, while at $q^d(\bar{p})$ it is zero. Taking the mean of $B + \eta - b\left(q^d(\bar{p}) - q^s(\bar{p})\right) - \bar{p}$ and zero, and multiplying by $q^s(\bar{p})$, consumer surplus is then $b(\bar{p} - C)^2/2c^2 \equiv CS_{d0}^L$. We can therefore write the consumer surplus for any η at which supply binds as

$$\alpha CS_d^L + (1 - \alpha) CS_{d0}^L = \frac{(\bar{p} - C)[2\alpha c(B - \bar{p}) + (1 - 2\alpha)b(\bar{p} - C)]}{2c^2} \equiv CS_d^L(\alpha) \ . \tag{13}$$

In the low region (where $\bar{p} < p^*(n_{\min})$) supply always binds and so $CS_d^L(\alpha)$ is the expected consumer surplus. If $\alpha \neq b/2(b + c)$, the price ceiling that solves the f.o.c. of the optimization problem in this region is

$$\hat{p}(\alpha) = \frac{\alpha c(B + C) - (1 - 2\alpha)bC}{2\alpha(b + c) - b} \ . \tag{14}$$

Note that, for $\alpha = 1$, $\hat{p}(\alpha)$ reduces to the value specified in (5). To be a well-defined local maximum, $\hat{p}(\alpha)$ must lie in the interval $(C, p^*(n_{\min}))$ and expected consumer surplus must be concave at this price. However, if $\alpha < b/2(b + c)$, $\hat{p}(\alpha) < C$ and gives a $CS_d^L(\alpha)$-minimum. $CS_d^L(\alpha)$ is increasing for all $\bar{p} \in (C, p^*(n_{\min}))$, and so an optimal ceiling cannot lie in this region.

Generalizing Definition 1, we introduce the following notation.

Definition 2 *Let* $n_0(\alpha) = -\dfrac{[\alpha(b + c) - b](B - C)}{2\alpha(b + c) - b}$.

We can now establish the conditions under which $\hat{p}(\alpha)$ is a local optimum and repeat the derivation in section 4, but using (13) to allow for rationing inefficiency. The following proposition generalizes Propositions 1 and 2 for arbitrary values of α.

Proposition 4 *Assume perfect competition and demand uncertainty. (a) For rationing efficiency* $\alpha > b/(b + c)$*, there is a unique price ceiling that globally maximizes expected consumer surplus. (i) If* $n_{\min} > n_0(\alpha)$*, this optimal ceiling is given by (14) and always binds; (ii) if* $n_{\min} \leq n_0(\alpha)$*, it lies in the interval* $[p^*(n_{\min}), p^*(n_{\max}))$ *and this optimal* $\bar{p} \gtreqless p_e^*$ *as* $-(1 - \mathcal{F}(0))\,[\alpha(b + c) - b]\,(B - C) - \alpha(b + c)\epsilon_d^L(0) \gtreqless 0$*. (b) For rationing efficiency* $\alpha \leq b/(b + c)$*, a price ceiling should not be set.*

This proposition highlights the combined effects of the efficiency of rationing and the degree of uncertainty on the optimal price ceiling. The analysis pins down a cut-off level of rationing efficiency $\alpha = b/(b+c)$ above which the same qualitative conclusions apply as in the benchmark case. Note, that the requirement that $\alpha > b/(b+c)$ is consistent with a wide range of inefficiency. If, for example, $b < c$, demand being less steep than supply, it is consistent with random rationing ($\alpha = 1/2$). Provided that $\alpha > b/(b+c)$, if uncertainty is relatively small ($n_{\min} > n_0(\alpha)$) a price ceiling in the low region, that is sure to bind, is optimal. A situation like this is illustrated in Figure 1B. Alternatively, if uncertainty is greater ($n_{\min} \leq n_0(\alpha)$), the optimal ceiling is in the middle region and may or may not bind. The critical level of uncertainty, as represented by $n_0(\alpha)$, is decreasing in α. Thus, the greater is rationing efficiency, the greater will be the maximum amount of uncertainty for which the optimal ceiling will lie in the low region, and bind for sure.

The condition for $\bar{p} \gtreqless p_e^*$ generalizes that in Proposition 1. When rationing efficiency α is smaller (but $\alpha > b/(b+c)$), the level of demand uncertainty required for $\bar{p}$ to exceed p_e^* is greater.

When instead $\alpha \leq b/(b+c)$ a price ceiling should not be set, and this result holds regardless of the amount of demand uncertainty. If rationing efficiency is low enough, price regulation has a large allocative cost which eliminates the scope for pro-consumer intervention. If $b > c$, this non-intervention result holds even for random rationing; but, if $b < c$, intervention can improve expected consumer surplus even with worse than random rationing.

Generalizing our result with efficient rationing, if uncertainty is great enough ($n_{\min} \leq n_0$), the optimal price ceiling lies in the middle region and is strictly higher than $\hat{p}(\alpha)$, which is the optimal ceiling with smaller or no uncertainty. Moreover, Proposition 4 allows us to characterize fully the relationship between the efficiency of rationing α and the optimal price ceiling.

Corollary 1 *With perfect competition and demand uncertainty, for $\alpha \geq b/(b+c)$, the optimal pro-consumer price ceiling is strictly decreasing in the efficiency of rationing α.*

Thus, whenever it is optimal to set a price ceiling that may bind, there is a negative relationship between the optimal level and the efficiency of rationing.

7 Conclusions

This paper explores the impact of underlying market conditions on optimal price regulation in a setting where the regulator's objective is to maximize expected consumer surplus, while being imperfectly informed about demand and/or supply in the market.

We first consider a perfectly competitive benchmark to show that regulatory uncertainty does not eliminate the rationale for intervention with a price ceiling. Instead, sufficient uncertainty calls for softer intervention, with the price ceiling set at a relatively high level. Then, we develop an imperfectly competitive setting where symmetric firms compete in supply functions and which accommodates arbitrary degrees of competitive pressure. We show that, if there is sufficient uncertainty, the optimal price ceiling is increasing in the degree of competition.

Thus, when the regulator maximizes expected consumer surplus, competitive pressure justifies less restrictive regulatory intervention if uncertainty is large enough. This result is broadly consistent, for example, with Oftel's decision in 2002 to increase the price ceiling imposed on British Telecom once new suppliers entered the telecommunications market.

We also analyze rationing inefficiency, which plays a role if the ceiling binds and causes undersupply. We identify a cut-off level of rationing inefficiency above a price ceiling should not be used. However, if rationing is efficient enough, the optimal ceiling depends on the degree of regulatory uncertainty and is decreasing in the efficiency of rationing, and our qualitative results from the benchmark model carry over unchanged.

In the main text we have focused on demand uncertainty, but our results are qualitatively robust in settings with supply uncertainty only, or where the regulator faces both demand and supply uncertainty, as we formally show in the online appendix. Furthermore, some of the basic intuition behind our results carries over to more general formulations of the cost and benefit functions, or of the regulatory uncertainty. Likewise, although our analysis explores the maximization of consumer surplus, the main insights would be relevant in settings where the regulatory objective is a weighted sum of consumer surplus and profit.

This study aims to shed light on how market characteristics impact on a price ceiling which maximizes expected consumer surplus. Our work formalizes the economic intuition in a static framework. A fuller analysis would need to investigate the dynamic effects, of both the instrument and the consumer-surplus standard, and to assess the relative performance of different regulatory interventions in this context.

8 Appendix

8.1 A Preliminary Analysis

Proof of Lemma 1. (i) At a price ceiling $\bar{p} \geq p^*$, $dCS(q^d(p^*))/d\bar{p} = 0$. (ii) When $\bar{p} < p^*$, differentiating CS_d^L w.r.t. $\bar{p}$, the f.o.c. gives the value in the proposition. The s.o.c. is always satisfied. It is easy to check that the optimal ceiling is well defined, i.e., lies in the interval (C, p^*). ■

8.2 The Benchmark Model

Proof of Lemma 3. It is straightforward to see from the definitions of CS_d^L, CS_d^H, and CS_d in (4), (7) and (9), respectively, that expected consumer surplus is piece-wise continuous and differentiable. Therefore this proof focuses on the continuity and differentiability of the expected consumer surplus at $p^*(n_{\min})$ and $p^*(n_{\max})$.

Continuity at $p^*(n_{\min}) = [c(B+n_{\min})+bC]/(b+c)$: $\lim_{\bar{p}\nearrow p^*(n_{\min})} CS_d^L(\bar{p}) = CS_d^S(p^*(n_{\min}))$ where CS_d^S is defined in (8) and the equality follows from the fact that $\mathcal{F}(n_{\min})) = \epsilon_d^H(n_{\min}) = 0$. Since $\Upsilon_d^L(n_{\min}) = \epsilon_d^L(n_{\min}) = 0$, this shows that $\lim_{\bar{p}\nearrow p^*(n_{\min})} CS_d^L(\bar{p}) = CS_d(n_{\min})$ so that expected consumer surplus is continuous at $p^*(n_{\min})$.

Continuity at $p^*(n_{\max})$: Given that $\mathcal{F}(n_{\max}) = 1$, $\epsilon_d^L(n_{\max}) = 0$ and $\Upsilon_d^L(n_{\max}) = E(\eta^2)$, it follows that $CS_d(n_{\max}) = CS_d^*(n_{\max}) = \lim_{\bar{p}\searrow p^*(n_{\max})} CS_d^H(\bar{p}) = CS_d^H$. So, expected consumer surplus is continuous at $p^*(n_{\max})$.

Differentiability at $p^*(n_{\min})$: In the low region,
$\lim_{\bar{p}\nearrow p^*(n_{\min})} \partial CS_d^L/\partial\bar{p} = [c(B-C) - (p^*(n_{\min}) - C)(b+2c)]/\ c^2$. In the middle region, as $\epsilon_d^L(n_{\min}) = \mathcal{F}(n_{\min}) = 0$, $\partial CS_d(p^*(n_{\min}))/\partial\bar{p} = \lim_{\bar{p}\nearrow p^*(n_{\min})} \partial CS_d^L/\partial\bar{p}$. So, expected consumer surplus is differentiable at $p^*(n_{\min})$.

Differentiability at $p^*(n_{\max})$: In the middle region, using $\partial CS_d/\partial\bar{p}$ given in (10), together with $\epsilon_d^L(n_{\max}) = 0$ and $\mathcal{F}(n_{\max}) = 1$, it follows that $\partial CS_d(p^*(n_{\max}))/\partial\bar{p} = 0$. As (7) is independent of $\bar{p}$, $\lim_{\bar{p}\searrow p^*(n_{\max})} \partial CS_d^H/\partial\bar{p} = 0$. So, expected consumer surplus is differentiable at $p^*(n_{\max})$.

The second part of the Lemma follows from the fact that expected consumer surplus is continuous at $p^*(n_{\max})$ and (7) is independent of $\bar{p}$. ■

Proof of Proposition 1. The second derivative with respect to $\bar{p}$ is

$$\frac{d^2CS_d}{d\bar{p}^2} = \frac{1}{c^2}[-(b+2c)(1-\mathcal{F}(n^*(\bar{p})) + (b+c)(\bar{p}-C)\mathcal{F}'(n^*(\bar{p}))]\ .$$

It is easy to see that, as $\epsilon_d^L(n_{\max}) = E(\eta) = 0$ and $\mathcal{F}(n_{\max}) = 1$, $p^*(n_{\max})$ satisfies the f.o.c. of the optimization problem in this region. The objective function is convex at this point, and so $p^*(n_{\max})$ is a local minimum. Hence, CS_d is decreasing in $\bar{p}$ as it approaches $p^*(n_{\max})$ from below. In addition, by Lemma 3, all $\bar{p} > p^*(n_{\max})$ result in the same levels of expected consumer surplus as $\bar{p} = p^*(n_{\max})$.

Using (6) and (10), we can see that CS_d is increasing at $p^*(n_{\min})$ iff $n_{\min} \le n_0$. Also, by Lemma 2, if $n_{\min} \le n_0$, there is no candidate optimal price ceiling strictly below $p^*(n_{\min})$. Therefore, if $n_{\min} \le n_0$, the global optimal price ceiling must belong to $[p^*(n_{\min}), p^*(n_{\max}))$ and is unique, and

$$sign\frac{d^2CS_d}{d\bar{p}^2} = sign\left(\frac{\mathcal{F}'(n^*(\bar{p}))}{1-\mathcal{F}(n^*(\bar{p}))} - \frac{b+2c}{c^2(b+c)}\frac{1}{(\bar{p}-C)}\right)\ ,$$

where the hazard rate $\mathcal{F}'(n^*(\bar{p}))/(1-\mathcal{F}(n^*(\bar{p})))$ is, by assumption, strictly increasing on $[p^*(n_{\min}), p^*(n_{\max})]$. As $1/(\bar{p}-C)$ is strictly decreasing in $\bar{p}$, it is straightforward to see that CS_d has a unique inflexion point, p_I^d. Recall also that at $p^*(n_{\max})$, $d^2CS_d/d\bar{p}^2 > 0$ so that the function is convex for all $\bar{p} \in (p_I^d, p^*(n_{\max}))$. However, if $\bar{p} < p_I^d$, $1/(\bar{p}-C)$ will take a higher value and the hazard rate a lower one, so that $d^2CS_d/d\bar{p}^2 < 0$ for all $\bar{p} \in (p^*(n_{\min}), p_I^d)$. It follows that, when $n_{\min} \le n_0$ there is a unique globally-optimal price ceiling in $[p^*(n_{\min}), p^*(n_{\max}))$. Finally, we evaluate $dCS_d/d\bar{p}$ at $\bar{p} = p_e^*$ as given in (3):

$$\frac{dCS_d(p_e^*)}{d\bar{p}} = -(1-\mathcal{F}(0))\frac{(B-C)}{(b+c)} - \frac{1}{c}\epsilon_d^L(0) \ .$$

The second term is non-positive, while the first is non-negative as $\epsilon_d^L(0) < 0$. ■

8.3 Imperfect Competition

Supply Function Competition Foundation

Suppose the market demand is given by

$$D(p) = \frac{B+\eta-p}{b}$$

and that aggregate competitive supply in this market is

$$S(p) = \frac{p-C-\theta}{c}.$$

Consider now an imperfectly competitive market. There are N identical firms in the market with a quadratic cost $\mathfrak{C}(q_i) = (C+\theta)q_i + Ncq_i^2/2$, where q_i is the output of firm i. This cost function guarantees that when the total cost of producing q using N-plants in the industry are minimized, total cost equal is $\mathfrak{C}(q) = (C+\theta)q + cq^2/2$ as in the competitive benchmark model.

The firms compete in supply functions and each submits a schedule

$$S_i(p-C-\theta) = d_i(p-C-\theta) = d_i\widetilde{p} = S_i(\widetilde{p}).$$

In the absence of regulation, each firm i is a monopolist against its residual demand given by q_i and maximizes

$$q_ip - q_i(C+\theta) - Ncq_i^2/2 = q_i\widetilde{p} - Ncq_i^2/2.$$

Firm i's residual demand is given by

$$q_i = \frac{B+\eta-p}{b} - \Sigma_{j\neq i}d_j(p-C-\theta) = \frac{B+\eta-C-\theta-\widetilde{p}}{b} - \widetilde{p}\Sigma_{j\neq i}d_j.$$

The f.o.c. of the maximization problem w.r.t. $\widetilde{p}$ requires that

$$\frac{dq_i}{d\widetilde{p}}\widetilde{p} + q_i - Ncq_i\frac{dq_i}{d\widetilde{p}} = 0 \Leftrightarrow$$
$$(-\frac{1}{b} - \Sigma_{j\neq i}d_i)\widetilde{p} + [1 - Nc(-\frac{1}{b} - \Sigma_{j\neq i}d_i)](\frac{B+\eta-C-\theta-\widetilde{p}}{b} - \widetilde{p}\Sigma_{j\neq i}d_i) = 0.$$

This defines the optimal price. It follows that the residual demand at this optimal price is given by

$$\frac{\frac{1}{b} + \Sigma_{j\neq i}d_i}{1 - Nc(-\frac{1}{b} - \Sigma_{j\neq i}d_i)}\widetilde{p}.$$

In equilibrium, firm i should choose its supply so that $S_i(\widetilde{p}) = d_i\widetilde{p}$ is equal to this residual demand at the optimal price. It then follows that

$$d_i = \frac{\frac{1}{b} + \Sigma_{j\neq i}d_i}{Nc(\frac{1}{Nc} + \frac{1}{b} + \Sigma_{j\neq i}d_i)} < \frac{1}{Nc}.$$

Using the fact that in a symmetric equilibrium $d_i = d$ for all i's, the optimal slope d solves

$$d = \frac{1}{Nc}\frac{\frac{1}{b} + (N-1)d}{\frac{1}{Nc} + \frac{1}{b} + (N-1)d} < \frac{1}{Nc}.$$

In equilibrium each firm supplies $S_i(p) = d_i(p - C - \theta) = d(p - C - \theta)$ and aggregate supply in the imperfectly competitive market is $\Sigma_i S_i(p) = Nd(p - C - \theta) < \frac{p-C-\theta}{c} = S(p)$ which is the supply in the corresponding perfectly competitive market. The inequality follows from

$$Nd = \frac{1}{c}\frac{\frac{1}{b} + (N-1)d}{\frac{1}{Nc} + \frac{1}{b} + (N-1)d} < \frac{1}{c}.$$

Therefore the supply in the imperfectly competitive market can be written as $\delta S(p)$ where δ is the 'abatement factor' (see Akgün, 2004) and is given by

$$\delta = Ncd < 1.$$

It is easy to see that $\delta \in [2c/[c + \sqrt{c^2 + 2bc}], 1)$ whenever $N \geq 2$ and $d\delta/dN > 0$.

Proof of Proposition 3. Suppose that $n_{\min} > -c(B - C)/(b + 2c)$, then the globally optimal price ceiling is given by (5) and independent of δ.
Suppose instead that $n_{\min} < -c(B - C)/(b + 2c)$.
Let $p^{\delta}(n_{\min}, \delta) < p^*(n_{\max})$. If the globally optimal ceiling lies in $[p^*(n_{\min}),\ p^{\delta}(n_{\min}, \delta))$, it solves $dCS^L_{\delta}/d\bar{p} = 0$ (see (11)). Evaluating (11) at the price ceiling that solves $dCS_d/d\bar{p} = 0$, i.e., the optimal ceiling under perfect competition (see (10)), we obtain $-[(B - \bar{p})\mathcal{F}(n^*(\bar{p}))+ \epsilon^L_d(n^*(\bar{p}))]/b = -[(B - \bar{p})+ \epsilon^L_d(n^*(\bar{p}))/\mathcal{F}(n^*(\bar{p}))]/b < 0$. The inequality follows from the fact that $\epsilon^L_d(n^*(\bar{p}))/\mathcal{F}(n^*(\bar{p})) = E(\eta \mid \eta < n^*(\bar{p})) \in [n_{\min}, n_{\max}]$. If the globally optimal ceiling lies in $[p^{\delta}(n_{\min}, \delta), p^*(n_{\max}))$, it solves $dCS^H_{\delta}/d\bar{p} = 0$ (see (12)). Whenever CS^H_{δ} has an interior maximum in the interval $(p^{\delta}(n_{\min}, \delta),\ p^*(n_{\max})]$, the optimal ceiling is increasing in δ. This follows from the local concavity of CS^H_{δ} at the optimal ceiling and from the fact that $d(CS^{\delta}_H)^2/d\bar{p}d\delta = \delta b(\bar{p} - C)^2\mathcal{F}'(n^{**}(\bar{p}, \delta))/c^2 > 0$. If the optimal price ceiling with imperfect competition lies in this sub-region, it is strictly lower than the optimal ceiling with perfect competition. Let $p^{\delta}(n_{\min}, \delta) \geq p^*(n_{\max})$. Then, the globally optimal ceiling lies in $[p^*(n_{\min}),\ p^*(n_{\max}))$ and it solves $dCS^L_{\delta}/d\bar{p} = 0$. The above comparison with the optimal ceiling under perfect competition applies and the result follows. ■

8.4 Inefficient Rationing

Proof of Proposition 4. Part (a). We first establish the conditions under which $\hat{p}(\alpha)$ is a local optimum. With $\alpha > b/2(b + c)$, if $n_{\min} \leq n_0(\alpha)$, $CS^L_d(\alpha)$ is increasing for all $\bar{p} < p^*(n_{\min})$

and the critical value $\hat{p}(\alpha)$ is weakly larger than $p^*(n_{\min})$, which is inconsistent with the region $(C, p^*(n_{\min}))$. Similarly, if $\alpha \in (b/2(b+c), b/(b+c))$ then, since $n_{\min} < 0$, obviously $n_{\min} < n_0(\alpha)$, and so the critical value exceeds $p^*(n_{\min})$. However, if $n_{\min} > n_0(\alpha)$ and $\alpha > b/(b+c)$, as $d^2CS_d^L(\alpha)/d\bar{p}^2 < 0$, (14) is a well-defined local maximum within the region $(C, p^*(n_{\min}))$. This generalizes Lemma 2.

In the middle region $\bar{p} \in [p^*(n_{\min}),\ p^*(n_{\max})]$. Following the derivation in section 4, but using (13) to allow for rationing inefficiency, we obtain

$$\frac{dCS_d}{d\bar{p}} = -(1 - \mathcal{F}(n^*(\bar{p})))\left(\frac{[2\alpha(b+c) - b]\,(\bar{p} - C) - \alpha c(B - C)}{c^2}\right) - \mathcal{F}(n^*(\bar{p}))\left(\frac{\alpha}{c}\frac{\epsilon_d^L(n^*(\bar{p}))}{\mathcal{F}(n^*(\bar{p})}\right) .$$

Repeating the derivation in the proof of Proposition 1, part (a) of the Proposition 4 follows.

Part (b). When $\alpha \leq b/(b+c)$, $dCS_d(p^*((n_{\max})))/d\bar{p} = 0$ and $d^2CS_d(p^*((n_{\max})))/d\bar{p}^2 < 0$, so that $p^*((n_{\max}))$ is a local maximum on $\bar{p} \in [p^*(n_{\min}), p^*(n_{\max})]$. Also, from (13), $dCS_d(\bar{p})/d\bar{p} > 0$ for all $\bar{p} < p^*(n_{\min})$. Finally, if a price ceiling is in the range $\bar{p} > p^*(n_{\max})$ it surely does not bind, and so the value of α is not relevant. Therefore Lemma 3 still applies when $\alpha \leq b/(b+c)$, and it follows that a price ceiling $p^*(n_{\max})$ is outcome-equivalent to any ceiling $\bar{p} > p^*(n_{\max})$. Hence, $p^*((n_{\max}))$ is a well-defined maximum in the region $[p^*(n_{\min}), p^*(n_{\max})]$, and any price ceiling higher than $p^*(n_{\max})$ yields the same outcome, whereas CS_d is lower for $\bar{p} < p^*(n_{\max})$. ■

Proof of Corollary 1. From Proposition 1, if $n_{\min} \leq n_0$ and $\alpha > b/(b+c)$, the optimal price ceiling solves the f.o.c. $dCS_d/d\bar{p} = 0$ and the objective function is locally concave at that price (see Proof of Proposition 1). It follows that $sign(d\bar{p}/d\alpha) = sign(dCS_d^2/d\bar{p}d\alpha)$. Using this, we obtain

$$\frac{d^2CS_d}{d\bar{p}d\alpha} = (1 - \mathcal{F}(n^*(\bar{p})))\frac{-2(b+c)(\bar{p} - C) + c(B - C)}{c^2} - \frac{\epsilon_d^L(n^*(\bar{p}))}{c} .$$

But the f.o.c. implies that

$$-\frac{1}{c}\epsilon_d^L(n^*(\bar{p})) = -(1 - \mathcal{F}(n^*(\bar{p})))\frac{[b - 2\alpha(b+c)]\,(\bar{p} - C) + \alpha c(B - C)}{\alpha c^2} .$$

Therefore $dCS_d^2/d\bar{p}d\alpha = -(1 - \mathcal{F}(n^*(\bar{p})))\frac{b}{c^2\alpha}\,(p - C) < 0$ which implies that $d\bar{p}/d\alpha < 0$.

From Proposition 4, if $n_{\min} > n_0$ and $\alpha > b/(b+c)$, the optimal price ceiling is given by $\hat{p}(\alpha)$ and it is straightforward to see that $d\hat{p}(\alpha)/d\alpha = -bc(B - C)/[2\alpha(b+c) - b]^2 < 0$. Also, if $\alpha \leq b/(b+c)$, the optimal ceiling is independent of α. ■

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