Evidence of cross-country portfolio diversification benefits: The case of Saudi Arabia

Ali, Hakim and Masih, Mansur

INCEIF, Malaysia, INCEIF, Malaysia

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Evidence of cross-country portfolio diversification benefits: The case for Saudi Arabia

Hakim Ali[1] and Mansur Masih*

Abstract

Recent literature draw attention to the issue whether the time-varying correlation and the heterogeneity in investment horizons has an effect on investor's return. Earlier studies investigated the interdependence of Saudi Arabian Stock market with its major trading partners without taking care of the time-varying correlation and different investments horizons of the investors. We make the initial attempt to study the extent to which investors can benefit from portfolio diversification with the Shariah indices of the major trading partners (United States, China, Japan, Germane, India), using Saudi Arabia as a case study where investors recently suffered due to downward trend of oil price. In order to investigate that, the pertinent time-varying and time horizon techniques like, Multivariate GARCH-DCC, the continuous wavelet transform (CWT) and the maximal overlap discrete wavelet transform (MODWT) are applied. Our findings tend to indicate that the Saudi Arabian investors have portfolio diversification benefits with all major trading partners in the short investment horizon, However in the long run, all markets are correlated yielding minimum portfolio diversification benefits and more importantly Saudi Arabian Investors have portfolio diversification benefits with Indian Islamic equity market in almost all investment horizons.

Key words: portfolio diversification, Sharia (Islamic) indices, GARCH-DCC, Wavelets

JEL: C22, C58, G11

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[1] Graduate research student in Islamic finance at INCEIF, Lorong Universiti A, 59100 Kuala Lumpur, Malaysia.

*__Corresponding author__, Professor of Finance and Econometrics, INCEIF, Lorong Universiti A, 59100 Kuala Lumpur, Malaysia. Phone: +60173841464 Email: mansurmasih@inceif.org

Introduction: Motivation of the study

Recent oil price downward movement and subsequent crashes in financial markets across the world have induced investors contemplate the benefits from diversification in a globalized market and financial contagion. Increasing degrees of interaction among major financial markets result in transmission effects. The volatility of a portfolio includes variances and correlation coefficients of, and among, individual positions. Great losses may be yielded from holding this portfolio without a time-varying consideration of its variance and correlation parts simultaneously. If investors can sense the interacting dynamics among markets in advance, then adjusting and hedging activities will be implemented in time. Successful and profitable performances can therefore be made.

Studies on international portfolio diversification, however, have mostly focused on the diversification benefit of Saudi Arabian investors with GCC countries and MENA countries; no study has been done on the portfolio diversification benefit with the major trading partner of Saudi Arabia to the best of our knowledge. A limited number of studies in the literature have looked into the benefits of international portfolio diversification from the perspective of local investors. Driessen and Laeven (2007) examine 52 countries from a number of developed and developing markets and report substantial regional and global diversification benefits for domestic investors, particularly in developing countries however despite the fact that one of the leading oil exporting countries like Saudi Arabia possesses a large amount of cash, but suffer from limited investment opportunities domestically, the literature has not yet examined how major trading markets can be utilized by local investors in order to avoid domestic market risks.

We have been observing the widening and intensifying linkage between the countries. They are becoming more closely linked with the growing volume of cross-border transactions in case of goods, services and foreign direct investments. Over the time, the financial markets also have become increasingly global. Obviously the integration of the world's economies has accelerated with the gradual reduction of restrictions on capital flows and relaxation of exchange controls in many countries. The globalization of the world stock markets is one of the most important developments that have occurred during the last two decades. The stock market has gained its momentum because of its facility in raising capital and its movements. The advancement in information technology, telecommunication and the emergence of new international financial institutions offering financial services has accelerated the flows. At present, the investment opportunities are no longer limited to domestic markets. Investors can approach overseas market to seek profitable opportunities. The global markets have become more accessible. The knowledge of international stock market is significant for portfolio managers, investors as well as fund managers. Due to this fact, the researchers world-wide have been ardent in the performances of the cross-border stock markets and international portfolio diversification benefits. Thus the present study is an analysis of the Saudi stock market with its major trading partners and the portfolio diversification analysis, we have taken Saudi Arabia as a case study as one of the leading oil producing countries.

The contribution: this paper contributes to the literature on international diversification from a local shariah compliant investment perspective with a focus on the major trading partners.

Theoretical underpinning:

"…I thank my fortune for it,

My ventures are not in one bottom trusted,

Nor to one place; nor is my whole estate

Upon the fortune of this present year…"

In The Merchant of Venice, Act 1, Scene 1.

William Shakespeare (1596-1598).

The main underlying theory picked up by this study is the Markowitz's (1959) Modern Portfolio theory which assumes that the expected return of a portfolio in a given portfolio risk tends to be maximized or the portfolio risk in a given portfolio return teds to be minimized when selection of various securities in a portfolio made considering the way the price of each security changes compared to that of every other security in the portfolio instead of selecting securities individually. The theory says that each and every security has its own particular risk and a portfolio of diverse securities could be of lower risk than a single security portfolio. The model given below;

σ_p^2=($\sum W_i^2\,\sigma_i^2 + \sum W_i\,W_j Cov_{ij}$)

Where the sums are over all the securities in the portfolio, Wi is the proportion of the portfolio in security i, σi is the standard deviation of expected returns of security i, and Covij is the covariance of expected returns of securities of i and j. Assuming that the covariance is less than one (which is always true), this will be less than the weighted average of the standard deviation of the expected returns of the securities. This is why diversification reduces risk.

Going beyond national Grubel (1968) applied Modern portfolio theory to explore the potential benefits of holding long-term assets eventually Grubel came up with international portfolio diversification benefits between two countries A and B as follows;

$E(r_{a,b}) = W_a R_a + W_b R_b$

$$V(r_{a,b}) = W_a^2\sigma_a^2 + W_b^2\sigma_b^2 + 2W_a W_b Cov_{ab}$$

Where E ($r_{a,b}$) is the expected returns on portfolio invested in Country A and B with investment weights of $w_a$and $w_b$and V($r_{a,b}$) measures the variance on the portfolio. The crucial factor here is the Cov_{ab} and the lower the covariance between countries A and B, the greater would be

diversification benefits. Grubel found that if US investors allocate a part of capital to foreign stock markets, investors could achieve a significant reduction in portfolio risk and better portfolio return opportunities.

Islamic Stock portfolio Diversification:

There are few studies on Islamic stock market integration as the Islamic stock markets are infantile stage. However here are some findings of some studies Hakim and Rashidian (2004), examined the return performance of the Dow Jones Islamic Market Index (DJIM), Dow Jones World Index (DJW) and Dow Jones Sustainability World Index (DJS) by using CAPM model and came up with the result that DJIM has done relatively well compared to DJW. Hussein (2005), found that Islamic Indexes provides investors abnormal return across the entire bull period, but they underperform during the bear market compared to their counterparts non Islamic indexes.

Achsani et al. (2007), finds the interdependence of Islamic stock market is asymmetric across a wide geographical area, while there are strong correlations between the Islamic stock indices of Indonesia and Malaysia, The US and Canada, and Japan and Asia Pacific. In addition to that the study also finds Islamic stock market in the US has a strong influence on the other Islamic stock market, while the reverse is not the case. Aziz and Kurniawan (2007), finds that there are potential diversification benefits for Islamimc investors considering the Islamic stock market in Indonesia and Malaysia. Majid, M. S. A., & Kassim, S. H. (2010), finds that Portfolio diversification can be obtained in the Islamic stock markets across economic grouping such as that in the developed and developing countries, the study also finds that the limited benefits are available if investors diversify their investments within the same economic groupings using ARDL, VECM and GMM method. Some other studies Majid and Kassim (2012), Kamil et al (2012), find also mixed results in their analysis. Saiti, B., Bacha, O. I., & Masih, M. (2014), finds that the Islamic countries provide better diversification benefits compared to the Far East countries with strong policy implications for the domestic and international investors in their portfolio diversification for hedging against unforeseen risks. One of the recent studies by Najeeb, S. F., Bacha, O., & Masih, M. (2015), find that Effective portfolio diversification benefits exist among the sample countries and portfolio benefits varies over time as well using the recent technique MGARCH-DCC and Wavelet. As evident from the review above, there are inconsistencies in empirical literature analysis the portfolio diversification properties of the Islamic indices across the globe. While some studies find evidence of strong correlations across Islamic stock indices, others have found Islamic stocks to be weakly correlated allowing investors to gain international diversification opportunities.

Literature review:

The present time of globalization investors across the world tends to invest beyond their country in order to maximize their return and to take the benefit of fundamentals of other investing countries the literature on the origin of market portfolio traces back to theoretical postulates of Markowitz (1952), Sharpe (1964), Lintner (1965), Ross (1976), Fama and French (1996), etc.

The literature review shows that there is conflicting evidence on the issue of international stock market linkages.

One of the earliest studies by (Agmon 1972), finds no significant linkage among the common stocks of Germany, Japan, the United Kingdom and the United States and hence supports the idea of international portfolio diversification. On the contrary (Lessard 1973), (Solnik 1974), and (Jorion 1985), find the evidence of existing low correlation among cross border equity markets. (Businessfinance, 2016) show Saudi's market co integration with USA market using VECM also found that Saudi Arabia member of GCC market can be a venue for international portfolio diversification, particularly to the USA investors. (Abraham & Seyyed, 2006) investigated the flow of information among GCC equities, and found asymmetric spillover of volatility from the smaller to the larger but less accessible Saudi markets. The financial market deregulation contributed to the increase in linkage between GCC countries, by (Eun and Shim 1989), finds the substantial amount of interdependency among the national stock markets.

The objective of the study:

The theoretical and empirical literature studied previously provides contradicting results regarding international portfolio diversification benefits. The recently added dimension in literature highlights the importance of different holding periods for example 4 days, 8 days, 16 days, 32 days etc, in affecting the volatilities and correlation dynamics of market return. Only a handful studies have been done on this new dimension (to the best of our knowledge). As a result, we made a humble attempt to empirically explore the effects of different investment horizons on international portfolio diversification benefits using Saudi Arabia (known as one of the biggest oil producing countries) shariah equity market as a case study. The research questions are as follows;

1: Which market should Saudi Arabian investors invest in order to get international portfolio diversification benefits?

2: How would the international portfolio diversification benefits change over time given different investors' equity holding periods?

The result of this article would specially have significant implication for the investors, fund managers and portfolio managers. More importantly it would help address the new issue of whether portfolio diversification benefits change due to changes in equity holding periods.

Data and Methodology:

Time varying and time scale correlation:

A few studies have been done on time varying correlation (that are not constant and varies over time) across stock markets .(Paramati et Al 2012) finds correlation are time varying and significantly increased and go back to their initial levels after the crisis using AGDCC-MGARCH model. Another study by Ibrahim (2015) finds that foreign trade intensity does matter for stock market integration that might have implications for investors in their portfolio decision making process in order to obtain diversification benefits.

Another study by Celik (2012) explores contagion effects exists during the US subprime crisis for most of the developed and emerging countries but most of the effect directed towards the emerging countries, using MGRACH-DCC asymmetric version model. Narayan and Bannigidadmath (2016) discovers the level of correlation between the markets affect the portfolio diversification benefits. (Rahim & Masih, 2016) finds that portfolio diversification benefits vary across investors holding periods used methods were CWT and MODWT.

With regard to scale dependent correlation (Li, Kou, & Sun, 2015) uses the lagged DFA to analyze the scaling properties in 42 financial markets and finds that majority of the markets (both stock and foreign exchange markets) exhibits that the contrary behavior in the small and large time scale.

(Madaleno & Pinho, 2010) Finds that there are strong correlation among the stock market indices under wavelet coherence analysis, that at high scales (low frequencies) index returns show a strong and significant relation but not homogeneous across scales.

A clear concept of the way volatilities and correlation between assets return change over time along with their direction and size of great importance for both the domestic and international investors in order to diversify their portfolios for hedging against unforeseen risks as well as for dynamic option pricing.

In a multivariate GARCH model, conditional variance and covariance of each asset depend upon not only on its own past conditional variance and past squared innovation but also on the past squared innovations and past conditional variances of other assets(Bollerslev et al. 1994). The multivariate GARCH model can be used to estimate the dynamic conditional correlations (DCC) for a portfolio of composed asset returns. The dynamic conditional correlation gives a sense of whether the shocks to the volatilities in asset return are substitutes or complementing in terms of risk taking.

The main merits of dynamic conditional correlations(DCC) in relation to other time varying estimating methods (such as rolling regression and Kalman filters and their variants such as, Flexible Least squares) is that it accounts for changes in both the mean and variances of the time

series. In other words DCC allows for changes both in the first moment (mean) and the second moment (variance). Understanding how correlations and volatility change over time and when they would be strong and weak is a persuasive motivation for the use of DCC models particularly in the financial markets. The DCC modeling allows us to pinpoint changes (both when they occur and how) in the interdependence between time series variables.

Data:

The daily stock indices return are used for the period from 6/9/2008 to 3/11/2016 collected from DataStream,INCEIF. S&P SAUDI ARABIA SHARIAH - PRICE INDEX, FTSE SHARIAH INDIA - PRICE INDEX, FTSE SHARIAH CHINA - PRICE INDEX, FTSE SHARIAH JAPAN 100 - PRICE INDEX, FTSE SHARIAH USA $ - PRICE INDEX, S&P EUR 350 Shariah Germany $ - PRICE INDEX. The total of 2027 observations was used and the stock indices returns were calculated as difference of the logarithmic daily closing prices of indices $[\ln(p_t) - (\ln p_{t-1})]$ where p is an index value. In order to get stationarity in variance, the conversion is necessary Engle (2002).

Table 1: Selected Islamic Stock Indices for Study

SYMBOL	DEFINITION
SAUD	S&P SAUDI ARABIA SHARIAH - PRICE INDEX
IND	FTSE SHARIAH INDIA - PRICE INDEX
CHIN	FTSE SHARIAH CHINA - PRICE INDEX
JAPN	FTSE SHARIAH JAPAN 100 - PRICE INDEX
USA	FTSE SHARIAH USA $ - PRICE INDEX
GERM	S&P EUR 350 Shariah Germany $ - PRICE INDEX

Multivariate GARCH-DCC:

ARCH (Autoregressive conditional Hetroscedastic), one of the earliest volatilities model by Engle (1982) that captures time-varying conditional variance of time series based on lagged information. Thereupon the model was enhanced by Bollerslev (1986) to a Generalized ARCH (GARCH) which takes care not only the conditional variance but also its past error in its variance equation to avoid the problem of large estimated parameters (As parameter become large due to lagging periods increases in ARCH model). This model again extended by Bollerslev (1990) in way to propose Multivariate GARCH –Constant Conditional Correlation (MGARCH-DCC) model where Conditional Correlation among different variables assumed to be constant over time. So this MGARCH-DCC model allows the variances to be time-variant but correlation to be time-invariant. However due to the impertinent assumption to the reality that correlation

coefficient is constant (Longin and Solnik, 1995, 2001), Engle (2002) finally came out with MGARCH-DCC (Dynamic Conditional Correlation) whereby the conditional correlations among variables were allowed to be time-variant. Since then MGARCH-DCC model has been extensively used to identify portfolio diversification benefits, in this paper we make a humble effort to use this latest model in sorting out our research objective. This model can be stated as follows

r_t=$\beta_0 + \sum_{i=1}^{k} \beta_i r_{t-1} + u_t = \mu_t + u_t$

μ_t=E$\left[r_{t|\Omega_{t-1}|}\right]$

$u_t \mathrm{I} \Omega_{t-1} \backsim N(0, H_t)$

H_t=$G_t R_t G_t$

G_t=diag$\left\{\sqrt{h_{ii,t}}\right\}$

Z_t=$G_t^{-1} u_t$

Source:Ku (2008)

Where $h_{ii,t}$ is the estimated conditional variance from the individual univariate GARCH models, G_t is the diagonal matrix of conditional standard deviations, R_t is the time-varying conditional correlation coefficient matrix of returns, and z_t is the standardized residuals vector with mean zero and variance one. After the above basic construction, the dynamic correlation coefficient matrix of the DCC model can be specified further:

R_t=(diag$(Q_T))^{-1/2} Q_t (diag(Q_t))^{-1/2}$

$Q_t = (q_{ij,t})$

(diag$(Q_t))^{-1/2} = diag(\frac{1}{\sqrt{q_{11,t}}},,,,,\frac{1}{\sqrt{q_{nn,t}}})$

$q_{ij,t}$=$\bar{p}_{ij} = \alpha\left(Z_{i,t-1} Z_{j,t-1} - \bar{p}_{ij}\right) + \beta(q_{ij,t-1} - \bar{p}_{ij})$

Source:Ku (2008)

Where $\bar{p}_{ij}$is the unconditional correlation coefficient and the time-varying conditional correlation coefficient is $p_{i,j,t} = q_{i,j,t} \mathrm{I} \sqrt{q_{ii,t} q_{jj,t}}$ Meanwhile , the returns on financial assets have often been documented to be fat tailed where a normal distribution assumption is not appropriate . One possible remedy for such is to apply the Student-t distribution setting. That is the conditional distribution $u_t|\Omega_{t-1} \sim \mathrm{N}(0, \mathrm{H_t})$ is replaced by $u_t|\Omega_{t-1} \sim \int \mathrm{student} - \mathrm{t}(\mathrm{u_t}; \mathrm{v}), (0, \mathrm{H_t})$where v is the degree of freedom parameter. Continuous Wavelet Transformation (CWT):

There was time Fourier analysis used to be used to explore relation at different frequencies between interested variables, overtime time several shortcomings (the total loss of time information which make it difficult to discriminate empirical relations or to identify structural changes which is crucial for time series macro-economic variables for policy purposes, and also Fourier analysis is based on time series is stationary but in real world economic variables are rarely found stationary which makes the result less reliable) encourages Gabor (1946) to come up with new Fourier transform known as short term Fourier transform where the time series is broken into smaller sub-samples and the Fourier is used to each sub-sample. Later the technique was also criticized on the basis of its efficiency as it takes equal frequency resolution across all dissimilar frequencies (see, for details, Raihan et al., 2005)

Wavelet transform came into existence with three distinctive advantages; firstly wavelet analysis can decompose the data into several time scales instead of frequency domain which allows scrutinizing the behavior of a signal over various time scales. This is obviously very important because different investors have different investment horizon. Muller et al, (1997) and Lynch and Zumbach, (2003) found that time scale is important factor affecting trading behavior and also showed that many heterogeneous investors make decisions over different time scales and perform each movement on different time scales, market co-movement varies across different time scales according to the investment horizons of different investors. Secondly wavelet analysis enables windows to vary, it stretches into a long wavelet function to measure the low-frequency movements; and it compresses into a short wavelet function to measure the high-frequency movements" (Aguiar-Conraria and Soares, 2011, p. 646). Lastly, the ability to handle non-stationary data, and unlike others it is free of restrictive assumptions. In fact in the real world, 99% of financial data are non-stationary.

Basically, there are three types of wavelets that exist and are adopted in economics and finance: discrete wavelet transforms (DWT), maximal overlap discrete wavelet transforms (MODWT), and continuous wavelets transform (CWT). Recently some researchers such as Saiti (2012), Madaleno and Pinho (2012, Vacha and Barunik (2012) used wavelet analysis in economics and finance studies. The CWT maps the original time series, which is a function of just one variable time-separate into function of two different variables such as time and frequency. The number of wavelets time scales is not required to be defined in CWT since it will be generated automatically based on the data length. Other than that, the CWT maps the series correlations in a two-dimensional figure that allows us to easily identify and interpret patterns or hidden information. We use the Daubechies (1992) least asymmetric wavelet filter of length = 8 denoted by LA (8) based on eight non-zero coefficients. Earlier studies on high frequency data showed that a moderate length filter such as L=8 is adequate to deal with the characteristics features of the time series data (see Gencay et al., 2001, 2002, Ln and Kim 2013, etc.). In literature, it is debated that LA (8) filter generates more smooth wavelet coefficient than other filter such as Haar wavelet filter.

The continuous wavelet transform $w_{x(u,s)}$ is obtained by projecting a mother wavelet Ψ onto the examined time series $x(t) \in l^2$ (R), that is

$$W_x(u,s) = \int_{-\infty}^{\infty} x(t) \frac{1}{\sqrt{s}} \psi \left(\frac{t-u}{s}\right) dt$$

The position of the wavelet in the time domain is given by *u*, while its position in the frequency domain is given by *s*. Therefore, the wavelet transform, by mapping the original series into a function of *u* and *s*, gives us information simultaneously on time and frequency. In order to study the interaction between two time series, how closely X and Y are related by a linear transformation, we need to apply a bivariate framework which is called wavelet coherence. The wavelet coherence of two time series is defined as follows'

$$\mathrm{R}_{\mathrm{n}}^2(s) = \frac{\mathrm{I}\, S(s^{-1} W_n^{xy}(s))\mathrm{I}^2}{S\left(s^{-1}\mathrm{I} W_n^{x}(s)\right)\mathrm{I}^2 . S(s^{-1}\mathrm{I} W_n^{y}(s))\mathrm{I}^2}$$

While S is a smoothing operator, s is a wavelet scale, WWn^x(S) is the continuous transform of the time series X,Wn^y(S) is the continuous wavelet transform of the time series Y, Yn^{xy} (s) is a cross wavelet transform of the two time series X and Y (see Madaleno and Pinho, 2012). for more details see Gencay et al (2001; 2002) and In and Kim (2013)

Maximum Overlap Discrete Wavelet Transformation (MODWT):

The MODWT is a variant of the discrete wavelet transform (DWT) that unlike the classical DWT, is able to handle any sample size and not just those that are multiples of 2. The MODWT is highly redundant, non-orthogonal transform that enables alignment of the decomposed wavelet and scaling coefficient at each level with original time series and thus allowing a ready comparison between the series and its decompositions, which is missing in DWT. In comparison with DWT, MODWT variance estimator is also asymptotically more efficient. MODWT is generally known as stationary wavelet transform, shift or translation invariant DWT, time invariant DWT, and non-decimated DWT. The MODWT method can be described as follows;

Let X be an N-dimensional vector whose elements represent the real-valued time series {Xt: t= 0, …., N – 1}. For any positive integer, J0, the level J0 MODWT of X is a transform consisting of the J0 + 1 vectors $\widetilde{\mathrm{W}}_{1,,,,}\widetilde{\mathrm{W}}_{\mathrm{ij}}$ and $\widetilde{\mathrm{V}}_{\mathrm{j0}}$nd all of which have dimension N. The vector $\widetilde{\mathrm{W}}_{\mathrm{j}}$contains the MODWT wavelet coefficients associated with changes on scale $\mathrm{t_j} = 2^{\mathrm{j}-1}$(for j =1,…..J0) while $\widetilde{\mathrm{V}}_{\mathrm{j0}}$contains MODWT scaling coefficients association with averages on scale.$\lambda_{j0} = 2^{j0}$ Based on the definition of MODWT coefficients we can write (Percival and Walden,2000, p. 200):

$\widetilde{W}_j = \widetilde{W}_j X \; and \; \tilde{V}_j^0 = \tilde{V}_j^0 X$where $\widetilde{W}_j$and $\tilde{V}_{j0}$are N x N matrics Vectors are denoted by bold italics By definition, elements of $\widetilde{W}_j$and $\tilde{V}_{j0}$are outputs obtained by filtering X, namely

$$\widetilde{W}_{j,t} = \sum_{t=0}^{Li-1} \tilde{h}_{j,t} X_{t-1 \bmod N}$$

$$\tilde{V}_{j,t} = \sum_{t=0}^{Li-1} \tilde{g}_{j,t} X_{t-1 \bmod N}$$

For t =0,……., N-1, where $\tilde{h}_{j,t}$and $\tilde{g}_{j,t}$ are jth MODWT wavelet and scaling filters. The MODWT treats the series as if it were periodic , whereby the unobserved samples of the real valued time series $X_{-1}X_{-2}, \ldots\ldots X_{-n}$ are assigned the observed values at $X_{n-1}X_{n-2}, \ldots\ldots X_0$the MODWT coefficient are thus given by,

$$\widetilde{W}_{j,t} = \sum_{t=0}^{N-1} \tilde{h}^0_{j,t} X_{t-1 \bmod N}$$

And

$$\tilde{V}_{j,t} = \sum_{t=0}^{N-1} \tilde{g}^0_{j,t} X_{t-1 \bmod N} \{\text{for } t = 0, \ldots\ldots, N-1\}$$

$$\tilde{h}^0_{j,t} \text{and } \tilde{g}^0_{j,t} \text{ are periodization of } \tilde{h}^0_{j,t} \text{and } \tilde{g}^0_{j,t}$$

to circular filters of length N}

Wavelet variance is defined for stationary and non- stationary process with stationary backward differences, considering only the non-boundary wavelet coefficient, obtained by filtering stationary series with MODWT, the wavelet variance $V_x^2(\tau_j)$is defined as the expected value of,$\widetilde{W}_j^{\,2}$ in the case $V_x^2(\tau_j)$represents the contribution to the variance of $\{x_t\}$at the scale $\tau_i = 2^{j-1}$ and can be estimated by the unbiased estimator (see Percival and Walden, 2000, p. 306) for more details,

$\hat{V}_x^{\,2}(\tau_j) = \frac{1}{M_j} + \sum_{t=L_{i-1}}^{N-1} \widetilde{W}_{j,t}^{\,2}$Where $M_j = N - L_j + 1 > 0$is the number of non boundary coefficient at the jth level. The MODWT correlation estimator for scale $\tau_j$is obtained by making use of the wavelet covariance and the square root of wavelet variance

$\hat{\rho}X, Y(\tau_j) = \frac{\hat{V}_{X,Y}(\tau_j)}{\hat{V}_{X,}(\tau_j)\hat{V}_{,Y}(\tau_j)}$Where $|\hat{\rho}X.Y(\tau_j)| \leq 1$the Wavelet correlation is analogous to its Fourier equivalent, the complex coherency (see Gencay et al,. 2002, p. 258)

Empirical Results and discussions:

Initially we conducted MGARCH-DCC analysis on all indices returns (S&P Saudi Arabia Shariah - Price Index, FTSE Shariah India - Price Index, FTSE Shariah China - Price Index, FTSE Shariah Japan 100 - Price Index, FTSE Shariah Usa $ - Price Index, S&P Eur 350 Shariah Germany Price Index). Thereupon we look at the unconitonal volatilities and correlations given in table-1 below;

Table 2: Unconditonal volatility and correlation,

	SAUD	IND	CHIN	JAPN	USA	GERM
SAUD	0.01457	0.19209	0.23461	0.21436	0.17846	0.20309
IND	0.19209	0.017265	0.42211	0.29406	0.33228	0.38409
CHIN	0.23461	0.42211	0.023381	0.47325	0.23244	0.32461
JAPN	0.21436	0.29406	0.47325	0.01557	0.14425	0.28035
USA	0.17846	0.33228	0.23244	0.14425	0.013159	0.5692
GERM	0.20309	0.38409	0.32461	0.28035	0.5692	0.018997

On-diagonal element indicates the volatilities of the return while off-diagonal elements indicate the unconditional correlation between returns. The result shows that USA shariah index return has the lowest volatility (.013159) and China shariah index return has the highest volatility (.023381). When it comes to correlation, this result suggests that Saudi Arabic market has least correlations India's market and USA's market on the other hand with Chiana's market has the highest positive correlation. We have to bear in mind that this result is time-invariant, meaning to say that the stated correlations above are constants or static, which is not in the fact in financial markets. Therefore we proceed to examining the dynamic conditional correlations, which captures the time-varying properties in volatilities and correlations.

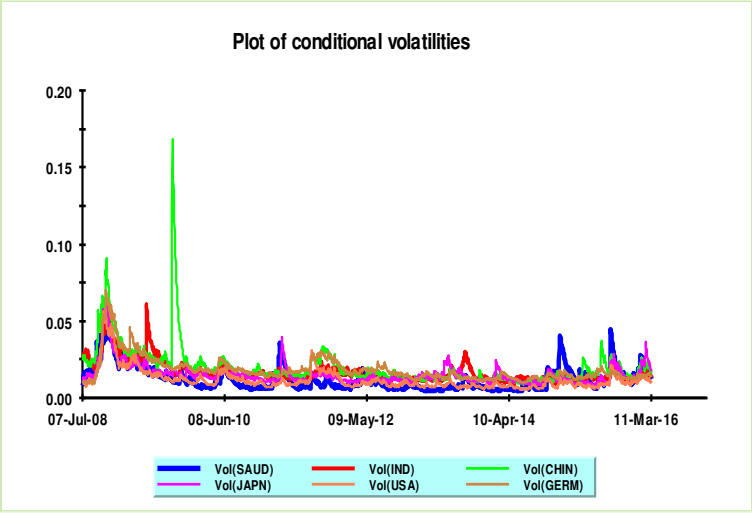
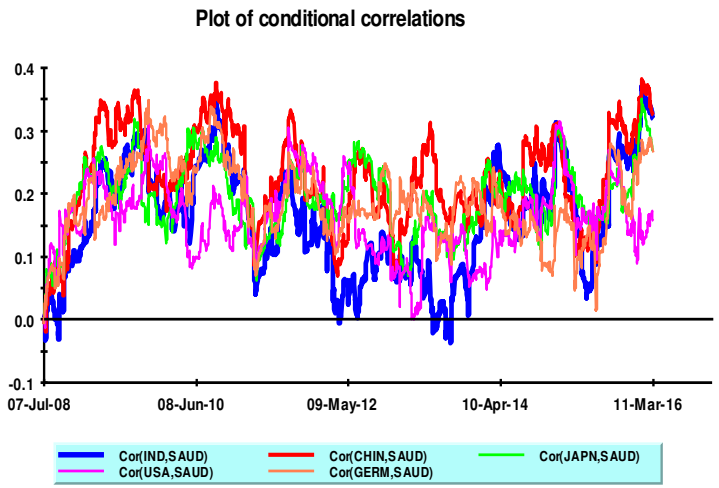


Figure 1 Volatility from July 08 to March16



Figures 2: Correlation from July 08 to March 16

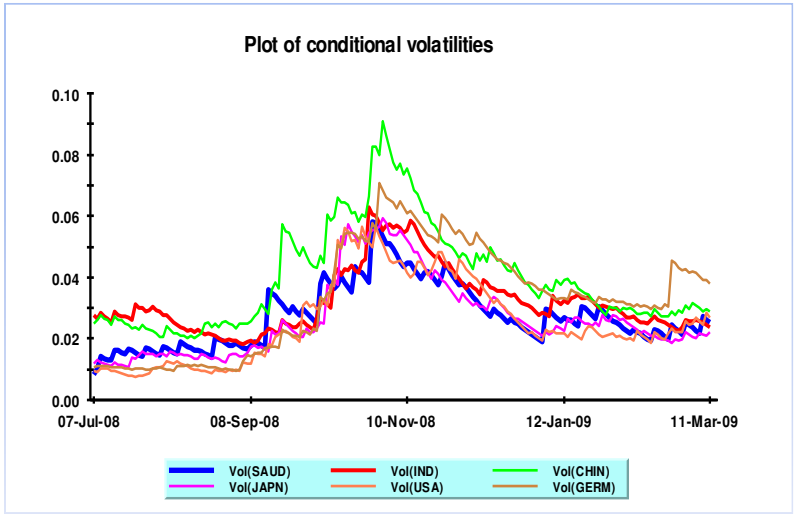


Figure 3: Volatilities during financial crisis

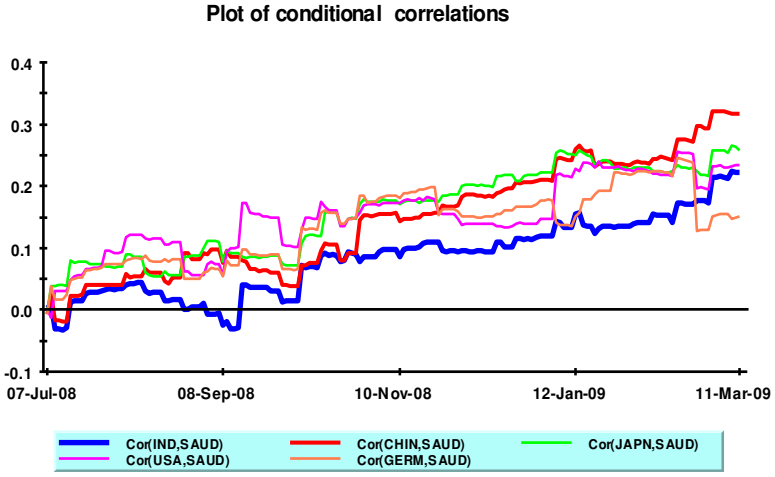


Figure 4: Correlations during global financial crisis

Here we carried out forecasting of the conditional volatilities and correlations of the Islamic Indices undertaken in this section of the study

Figure 1 and 2 shows the time varying volatilities and correlations among the sample countries from 07/07/2008 to 11/03/2016. The conditional volatilities of all Shariah indices returns move closely during the observation period except Shariah index return of China, which is in line with our previous result driven by unconditional volatility and correlation matrix table. The correlations among all the Shariah index here varies overtime, the India and The USA sharia equity indices has the lowest positive correlation sometime negative correlation Saudi sharia equity index with India shariah equity index. This suggests the portfolio diversification advantages for the investors. the deep look at the USA market shows there was a bit volatility during the 9/11 attack in 2011 but more positive correlation with Saudia sharia equity market, Interestingly the correlations of all shariah indices maintained lower than 0.60 which deemed to be not giving portfolio diversification benefits.

In 2014 we have seen there is a negative correlation with India Islamic equity market, one of the reason could be Narendra Modi's government came to the power which took economics reform to boost the equity markets.
In addition to that we have shown the volatilities and correlations among the all shaiah indices during the global financial crisis. The result suggests that all sample markets were volatile but the China was maintained the highest volatility. Volatilities was high during the peak period of financial crisis. On the other hand Saudi market return has the negative correlation with India market return meaning there was portfolio diversification benefits during the global financial crisis. Figure shows that correlations among all sharia equity indices tend to move upward after September of 2008. World Bank states that cross-country correlations increase during "crisis times" relative to correlations during "tranquil times." Furthermore, noted by Kim & Kim S. (2013), the important judgments about the contagion effect depend heavily on the time length and the extent of increased correlations

Earlier we have seen how the investors of Saudi Arabia can gain the portfolio diversification benefits with India and USA markets using MGARCH-DCC analysis. However the previous method ignores the different holding periods of investors which is the fact in the real world. In this section we use modern Wavelet transform to analyze the impact on portfolio diversification benefit given different investment horizons

Figure 4 Continuous Wavelet Transform-SAUDI and INDIA

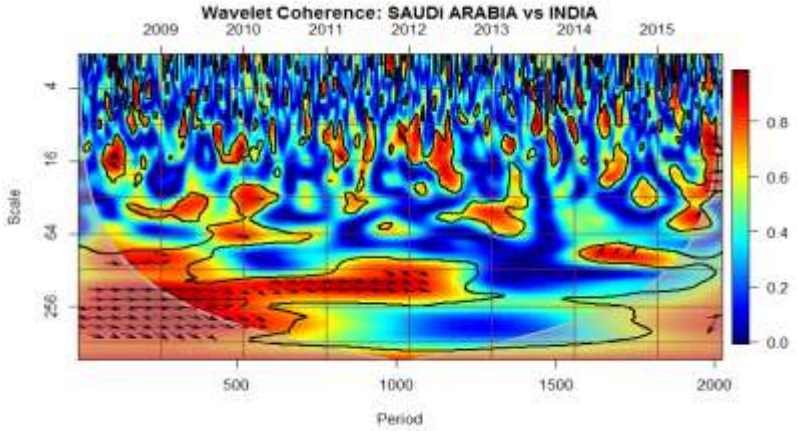


Figure 5 Continuous Wavelet Transform-SAUDI and USA

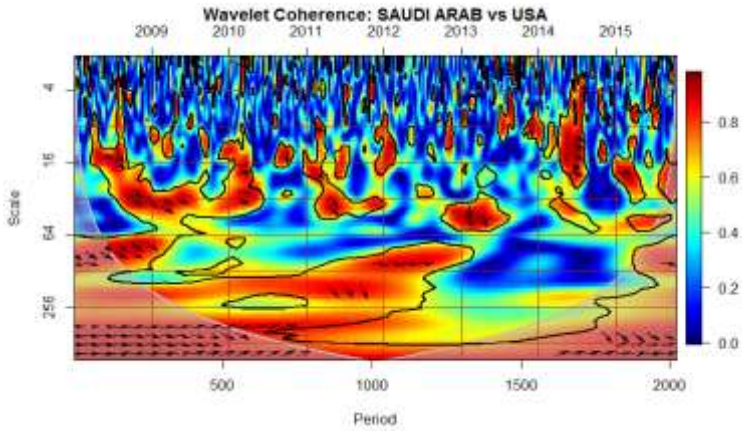


Figure:6 Continuous Wavelet Transform- SAUDI and CHINA

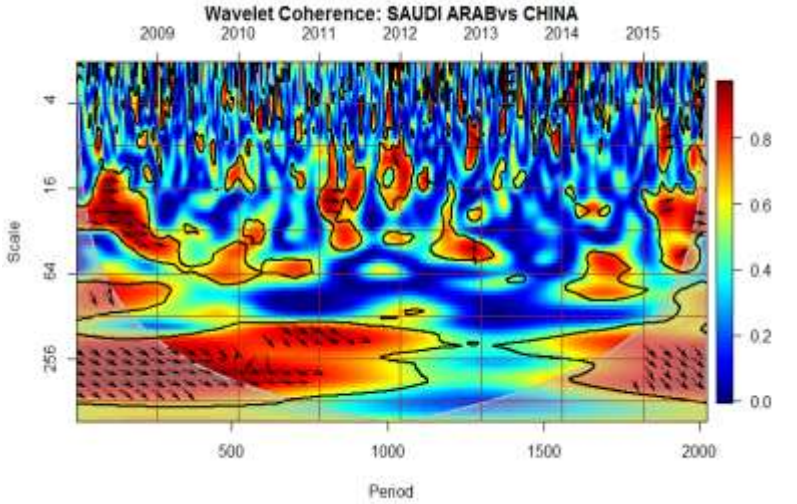


Figure: 7 Continuous Wavelet Transform- SAUDI and JAPAN

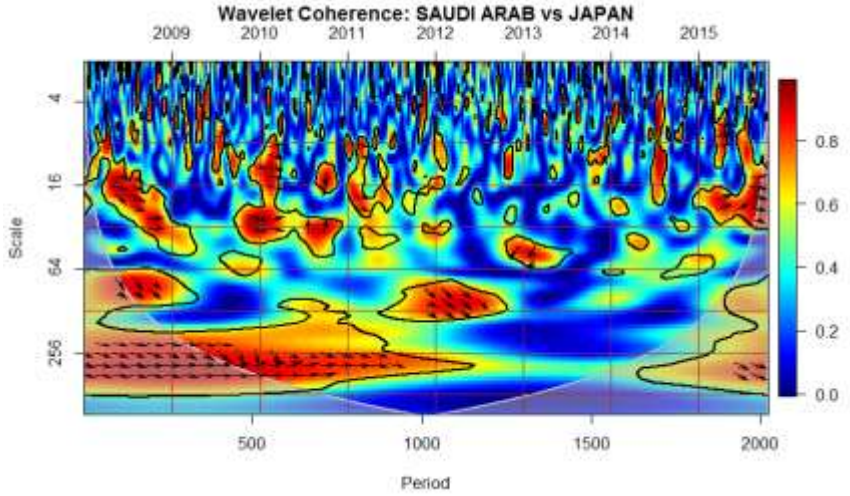
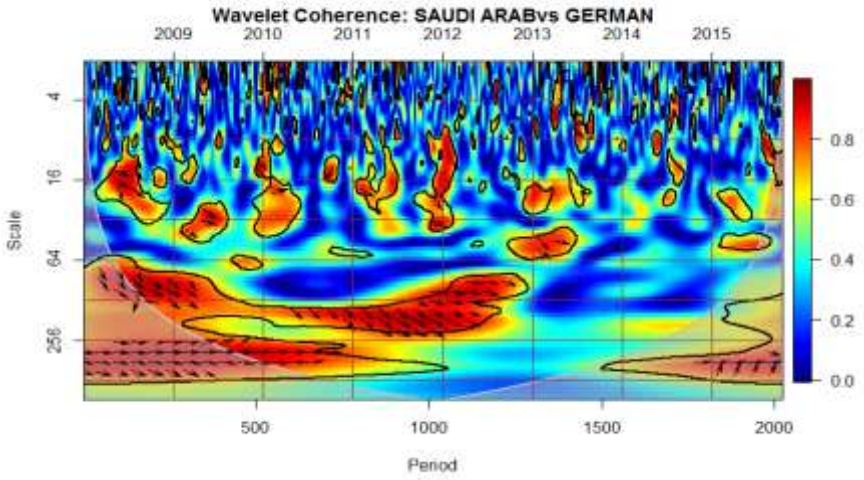


Figure:8 Continuous Wavelet Transform-SAUDI and GERMAN



In this section Continuous Wavelet analysis is done to analysis the impacts of portfolio diversification benefits given the different investment time horizons Time is shown in the horizontal axis in terms of number of trading days, while the vertical axis shows the investment horizon. The curved line below shows the 5% significance level which is estimated using Monte Carlo simulations. The figure follows a colour code as illustrated on the right with power ranges from blue (low correlations) to red (high correlations). The vectors pointing to the right mean that the indexes are in phase but if they point to the left means that the indices are out of phase. In the case that the vectors are pointing to the right and up would indicates that the first series is lagging, whereas when the vectors are pointing to the right down would indicates that the series is lagging. If the vectors are pointing to the left and up will mean that the first series is leading, whereas first series becomes lagging when the arrows are pointing towards the left and down (Madaleno & Pino, 2010).

As shows by figure 4 to figure 7 in the next page, it is found that for very short holding period, which are 4-16 and 16-32 and 32 to 64 days, it is found that the correlation between Saudi Arabian Sharia stock and its major trading partner are quiet low. If we compare it relatively, the returns of China Sharia stock indices tend to have stronger correlation relative to the correlation of the Sharia stock indices of other trading partners. The correlations of India's stock indices are also relative low during the global financial crisis, which occurs on the observation points of 16 days to 32 days. This would mean that speculators, who tends to invest in this holding period (very short period) would benefit from the investment, which is in line with our expectation. There is high correlation with China equity market during the global financial crisis at 16 days to 32 days holding period compared to other equity markets. In the case of investment horizons of 64-512 days, comparatively higher correlation are observed between all the Sharia stock index returns of Saudi Arab's major trading partners except less with India's Islamic equity index return. This will reduce or eliminate the potential portfolio diversification benefits.

Robustness test through estimation using the application of MODWT:

In order to test the robustness of the results that is obtained in CWT analysis, a Maximal Overlap Discrete Wavelet Transform (MODWT) was applied to our original data set consisting of return of series for all six Sharia indices. In MODWT,researcher is required to specify the time scales for the returns whereby the present paper use eight scales (1-2 days, 2-4 days, 4-8 days, 8-16 days, 16-32 days, 32-64days, 64-128 days, 128-256 days, and 256-512 days). The correlations between Saudi Arabia Shariah Index returns with the index returns of its major trading partners were examined using the generated MODWT returns series using R studio software and the result is shown by the table below:

Table 3: Correlation of Saudi Arabia Islamic equity market return vis a vis India, China, Japan, USA, German Islamic equity market returns

MODWT TRANSFORMATIONS					
MODWT SCALI	INDIA	CIHNA	JAPAN	USA	GERMANE
1-2 DAYS	0.1006657	0.15905072	0.1292999	0.10118996	0.1428343
2-4 DAYS	0.1558715	0.18155541	0.1474289	0.09644206	0.1291893
4-8 DAYS	0.3209354	0.36634673	0.4244505	0.35840675	0.3411678
8-16 DAYS	0.4060826	0.56486091	0.4424534	0.47376667	0.4372629
16-32 DAYS	0.3798621	0.45606845	0.3931458	0.5150828	0.4066196
32-64 DAYS	0.4078311	0.31146629	0.2274593	0.45078754	0.3806015
64-128 DAYS	0.6811541	0.58965165	0.5756927	0.67348701	0.6769713
128-256 DAYS	-0.1951082	0.3516383	0.7880871	0.96495913	0.3297222
256-512 DAYS	-0.2410731	0.04511409	0.8983471	0.88622056	0.8289613

The results are remarkably consistent with the results obtained from the earlier CWT analysis. Out of five markets, India Islamic equity markets offers portfolio diversification frequently However in the short investment holding periods almost all Islamic equity markets offer portfolio diversification benefits but in the long investment holding period Japan, USA, German Islamic equity markets don't offer portfolio diversification benefits, thus eliminating portfolio diversification benefits for the Saudi Islamic investors.

Table 4:The summary of all results obtained from three analysis

Analysis	India	China	Japan	USA	German
MGARCH	possible	possible	Possible	Possible	Possible
CWT	possible	Possible upto 2 months	Possible upto 2 months	Possible upto 2 months	Possible upto 2 months
MODWT	Possible all time	Possible all times horizons	Possible upto 2 months	Possible upto 2 months	Possible upto 2 months

Conclusions and Policy implications:

Our finding suggest that Portfolio diversification exists with all trading partners for Saudi Arabia but more specifically Indian Islamic equity markets offer portfolio diversification benefits across our observation period from 07/07/2008 to 11/03/2016. First step of our result shows that India and USA Islamic equity markets offer more portfolio diversification benefits than other Islamic equity markets. As the results are time-invariant, we proceed to explore the time-variant properties of dynamic conditional correlations and the volatilities. The results obtained from Continuous Wavelet Transform (CWT) suggest that Indian Islamic equity markets offer portfolio diversification benefits both in short run and long run investment holding periods. Apart from this, the other Islamic equity markets offer portfolio diversification benefits only up to two months investing holding periods. The result during the global financial crisis is intuitive, Indian Islamic equity market has negative correlation with Saudi Islamic equity market meaning that portfolio diversification benefits exists and also China showed lower correlation compared to normal times.

For robustness, Maximum Overlap Discrete Wavelet Transform (MODWT) is applied to our original return series, the result again suggests that only Indian Islamic equity market offers effective portfolio diversification benefits across the sample period considering both short run and long run investment horizons. However China, Japan, USA, German Islamic equity markets do not pose portfolio diversification benefits from 64 days to 512 days holding periods, suggesting that Saudi Islamic equity market investors need to invest in other global Islamic equity markets in order to get effective portfolio diversification benefits in those investment horizons. So the result could be beneficial for investors portfolio managers, hedge fund managers and institutional investors and also could be useful for policy makers for policy making decisions

Recommendations:

Other form of recent research methodology techniques such as panel techniques and nonlinear techniques involving the Markov switching process can be applied to provide more insightful and more robust results. Other than that, further studies can consider investigating the volatilities and correlations of sector wise market of Saudi Arabia with its major trading partners.

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