Measuring spot variance spillovers when (co)variances are time-varying - the case of multivariate GARCH models

Fengler, Matthias R. and Herwartz, Helmut

School of Economics and Political Science, University of St. Gallen, Department of Economics, Georg-August-University Göttingen

17 March 2015

Measuring spot variance spillovers when (co)variances are time-varying – the case of multivariate GARCH models*

Matthias R. Fengler[1] Helmut Herwartz[2]

First version: March 17, 2015

This version: June 10, 2016

Abstract

We propose global and disaggregated spillover indices that allow us to assess variance and covariance spillovers, locally in time and conditionally on time-t information. Key to our approach is the vector moving average representation of the half-vectorized 'squared' multivariate GARCH process of the popular BEKK model. In an empirical application to a four-dimensional system of broad asset classes (equity, fixed income, foreign exchange and commodities), we illustrate the new spillover indices at various levels of (dis)aggregation. Moreover, we demonstrate that they are informative of the value-at-risk violations of portfolios composed of the considered asset classes.

Keywords: BEKK model, forecast error variance decomposition, multivariate GARCH, spillover index, value-at-risk, variance spillovers,

JEL Classification: C32, C58, F3, G1

---

*The authors gratefully acknowledge helpful comments from Jörg Breitung, Katja Gisler, Roman Liesenfeld, Ostap Okhrin, Kamil Yılmaz, as well as from seminar participants at the Universität zu Köln, TU Dresden, and the CFE 2015, London.

[1]School of Economics and Political Science, University of St. Gallen, Bodanstrasse 6, CH–9000 St. Gallen, Switzerland, `matthias.fengler@unisg.ch`. Financial support by the Swiss National Science Foundation is gratefully acknowledged (Grant No. 144033).

[2]Department of Economics, Georg-August-University Göttingen, Platz der Göttinger Sieben 5,

1 Introduction

In highly integrated markets, shocks spread at a fast pace and bedevil risk management and optimal asset allocation because of disappearing diversification benefits and cascade effects. Awareness of this fact has risen especially during the financial crisis of 2008 and over the subsequent years of economic fragility. Consequently, much effort has been devoted to developing quantitative measures of economic interdependence. Examples include the systemic expected shortfall of Acharya et al. (2010), the conditional value-at-risk of Adrian and Brunnermeier (2016), and the spillover indices of Diebold and Yilmaz (2009, 2012, 2014).

Among these, the spillover indices of Diebold and Yilmaz (2009, 2012, 2014) have garnered much attention, because in contrast to other measures, they allow one to track the associations between individual variables and the system as a whole at all levels, from pairwise to system-wide, in a mutually consistent way. The notion of a spillover is that of a forecast error variance share derived from the forecast error variance decomposition of an underlying vector autoregressive model (VAR). For example, to study variance spillovers, one simply estimates a VAR on measures of realized variance – see Yilmaz (2013), Fengler and Gisler (2015), Baruník et al. (2016).

Because the indices are based on the forecast error variance decomposition of a single VAR, they produce static and average spillover information. While this is undoubtedly valuable, it would be of greater use to have up-to-date spillover information, especially when thinking of variance spillovers. There is ample evidence that conditional variances are time-varying, and it is natural to expect that spillovers are as well. Diebold and Yilmaz (2009, 2012) therefore suggest computing the indices from VAR models that are estimated on rolling subsamples. In this way, one obtains an impression of the time-varying patterns of spillovers. But as with all rolling window approaches, the estimates reflect only the average information of the respective

D–37073 Göttingen, Germany, hherwartz@uni-goettingen.de. Helmut Herwartz gratefully acknowledges financial support by the Deutsche Forschungsgemeinschaft (Grant No. HE 2188/8-1).

estimation window. Because the subsamples must be of sufficient length to provide reasonably accurate parameter estimates, the rolling window indices are probably more useful for a retrospective analysis than for the timely monitoring of spillovers. For this purpose, one would need a time-t conditional spillover index.

In this paper, we propose such an approach. We adopt the ideas of Diebold and Yilmaz (2009, 2012) and construct variance spillover indices that are updated with time-t information. To this end, we build on multivariate GARCH (MGARCH) models of the BEKK-type (Baba et al., 1990; Engle and Kroner, 1995) and calculate the indices from the forecast error variance decomposition that is derived from the vector moving average (VMA) representation of the 'squared' and vectorized return process. This process is driven by serially uncorrelated heteroskedastic innovations and, as we show here, its conditional covariance matrix can be derived analytically. This allows us to absorb the regime dependence into the parameters of the VMA representation. The variance spillover indices that are based on this time-varying VMA representation therefore take full advantage of the time-t conditional information of the prevailing variance regime. In contrast to rolling window estimates, they convey on-the-spot variance spillover information. In our empirical applications, we show not only that the time-t conditional variance spillover indices allow a study of the prompt impact of major economic or political events, but also that they are informative of the likelihood of value-at-risk violations.

Aside from the value of well-timed spillover information, our approach differs in methodological terms from the extant literature in that we derive the spillover indices from a full-fledged model of variance and covariance dynamics. This has advantages that are more than conceptual. First and most importantly, the variance spillover indices take full advantage of the informational content embedded in covariances. It appears common sense to expect covariance dynamics to play a decisive role in the mechanisms of variance spillovers. In Fengler and Gisler (2015), a first step toward incorporating covariance information into variance spillover indices is made, but the authors follow the traditional route of applying a VAR to vectorized realized covariance matrices estimated from intra-day data. Thus, they cannot ensure positive definiteness of the dynamic covariance matrices. While the realized variance literature proposes such models, it does so at the expense of transforming the covariance matrices in a nonlinear way – see, e.g., Bauer and Vorkink (2011) and Golosnoy et al. (2012). This makes attributing the shocks to specific variables difficult if not impossible. The most attractive property of the indices of Diebold and Yilmaz (2009, 2012) would thereby be lost. With the MGARCH model, we circumvent this difficulty, because the VMA representation of the vector collecting the squared observations and the crossproducts of the observations remains linear in a serially uncorrelated vector innovation process.

Our work is related to the broad strand of the contagion and transmission literature – see, among others, Engle, Ito and Lin (1990), Hamao et al. (1990), Forbes and Rigobon (2002), and Bali and Hovakimian (2009). These studies investigate whether transmission channels for shock spillovers between returns and variances of different markets exist, and whether these channels emerge or disappear, for instance, in times of crisis. For this purpose, one is often interested in strategies to test certain parameter restrictions, as thoroughly studied by Nakatani and Teräsvirta (2009), Billio et al. (2012), and Woźniak (2015), or one examines variance impulse responses as proposed by Hafner and Herwartz (2006). By contrast, here we focus on the measurement of the time-varying magnitude of variance spillovers, conditional on the diagnosed transmission channels. Our primary interest is a quantitative assessment of the contemporaneous variance spillover activity. In this sense, the advances of constructing spillover indices and the classical contagion and transmission literature complement each other perfectly.

Section 2 provides a brief sketch of the BEKK model and its translation into a vectorized representation of the 'squared' MGARCH process. This representation is employed in Section 3 to define indices of variance spillover. An empirical analysis of returns of four major US asset classes is provided in Section 4. Section 5 concludes.

2 Multivariate GARCH

In this section, we first discuss the MGARCH model of the BEKK type. Second, we make explicit the translation of the BEKK model into its half-vectorized (vech) representation. Third, we discuss in detail the conditional covariance matrix of the half-vectorized 'squared' MGARCH process, which is a key input parameter for implementing the variance spillover indices developed in Section 3.

2.1 The BEKK model

We consider an N-dimensional vector of returns (first differences of log asset prices) $\{r_t\}$ such that $E[r_t] = 0$. Denote by $\mathcal{F}_t$ the filtration generated by $\{r_t\}$ up to and including time t. The return process is assumed to follow

$$r_t = \varepsilon_t = H_t^{1/2}\xi_t, \quad \xi_t \overset{\text{iid}}{\sim} N(0, I_N),\, t = 1, 2, 3, \ldots, T\ , \tag{1}$$

where the conditional covariance H_t is measurable with respect to $\mathcal{F}_{t-1}$. In (1), $H_t^{1/2}$ denotes the symmetric matrix square root of H_t.[1] The innovation vector $\{\xi_t\}$ is assumed to be independently and identically (iid) mean zero normally distributed with a covariance matrix equal to the N-dimensional unit matrix, which is denoted by I_N. The assumption of conditional normality of ξ_t is commonly adopted to implement (Quasi) Maximum Likelihood (QML) estimation, but it is not essential for the subsequent discussions. We do not specify a conditional mean process for $\{r_t\}$ because our interest is in measuring variance spillovers in daily asset price returns. Moreover, one would typically specify the conditional mean as a function of past values of r_t, in which case it is immaterial to the following discussion.

In the literature, a considerable number of alternative specifications of MGARCH models have been proposed – see Bauwens et al. (2006) for a comprehensive review. Here we adopt the so-called BEKK model for two main reasons. First, it has the

[1] The square root of a symmetric positive definite matrix Z is defined as $Z^{1/2} = \Gamma\Lambda^{1/2}\Gamma'$, where the columns of Γ contain the eigenvectors of Z, and $\Lambda^{1/2}$ is diagonal with the positive square roots of the eigenvalues on its diagonal.

attractive feature that under mild restrictions applying to the initial conditions, the process of conditional covariances H_t is positive definite by construction (Engle and Kroner, 1995). Second, the BEKK model is among the most general MGARCH specifications and also embeds the general vec MGARCH model of Bollerslev et al. (1988), except for degenerate covariance processes – see Stelzer (2008) for more details. Therefore, while relying on BEKK is hardly costly in terms of model flexibility, other MGARCH variants are special cases of the BEKK specification, for example, the factor model of Engle, Ng and Rothschild (1990), the orthogonal GARCH model of Alexander (2001, pp. 21–38), its generalization introduced by van der Weide (2002) and the Cholesky GARCH of Dellaportas and Pourahmadi (2012).

In its most flexible form, the BEKK(p, q, K) model of the conditional covariance $\text{Cov}_{t-1}[\varepsilon_t] = E_{t-1}[\varepsilon_t \varepsilon_t'] = H_t$ is given by

$$H_t = CC' + \sum_{k=1}^{K} \sum_{i=1}^{q} F_{ki}' \varepsilon_{t-i} \varepsilon_{t-i}' F_{ki} + \sum_{k=1}^{K} \sum_{i=1}^{p} G_{ki}' H_{t-i} G_{ki} \ , \tag{2}$$

where C is a lower triangular matrix and F_{ki} and G_{ki} are $N \times N$ parameter matrices.

The BEKK model is well understood. Boussama et al. (2011, Theorem 2.4) establish that under weak regularity conditions on the law of the iid process $\{\xi_t\}$, the MGARCH process $\{\varepsilon_t\}$ is ergodic and both strictly and weakly stationary if

$$\rho \left(\sum_{k=1}^{K} \sum_{i=1}^{q} F_{ki} \otimes F_{ki} + \sum_{k=1}^{K} \sum_{i=1}^{p} G_{ki} \otimes G_{ki} \right) < 1 \ , \tag{3}$$

where $\rho(Z)$ denotes the spectral radius of a square matrix Z and $\otimes$ is the Kronecker matrix product.

Regarding parameter estimation, Jeantheau (1998) provides regularity conditions to establish consistency of QML estimators. Focussing on the BEKK specification, Comte and Lieberman (2003) show consistency and asymptotic normality of the QML estimator under the particular assumptions of, respectively, finite second and eighth-order moments of the MGARCH process. Hafner and Preminger (2009) show consistency of QML estimation under the weaker condition of finite second-order moments of MGARCH innovations $\{\xi_t\}$, thereby allowing for integrated

MGARCH. Moreover, they establish asymptotic normality of QML estimators under the weakened condition that the sixth-order moments of the MGARCH process be finite.

The general BEKK(p, q, K) model requires the estimation of $(p+q)KN^2+N(N+1)/2$ parameters and therefore is computationally demanding. For this reason, it has become standard in applied work to use the more parsimonious BEKK(1,1,1) model

$$H_t = CC' + F'\varepsilon_{t-1}\varepsilon_{t-1}'F + G'H_{t-1}G, \tag{4}$$

where the subscripts of the BEKK parameter matrices are suppressed for notational convenience. In that case, for $N = 2, 3, 4$, the estimation problem involves 11, 24, and 42 parameters, respectively. We will follow this practice but emphasize that the derivation of our variance spillover measures only requires the vech form of the BEKK model, which exists for any order.

2.2 The BEKK model in vech and VMA form

Encompassing all linear covariance specifications, the vech representation provides a general framework to compare the dynamic features implied by alternative covariance models, such as the underlying VMA representation. For the derivation of the vech representation of the BEKK model, some elementary matrices turn out to be useful, namely the elimination matrix L_N, the duplication matrix D_N, and its generalized inverse D_N^+.[2] Let $\eta_t = \text{vech}(\varepsilon_t\varepsilon_t')$ and $h_t = \text{vech}(H_t)$. Then, the vech representation of the BEKK model in (4) is given by

$$h_t = \upsilon + A\eta_{t-1} + Bh_{t-1}, \tag{5}$$

[2] Denote by vec(Z), the operator that stacks the columns of a matrix Z into a vector. Similarly, but for a square symmetric matrix, the vech-operator stacks the elements on and below the diagonal into a vector. Let $N^* = N(N+1)/2$. With reference to a symmetric square $N \times N$ matrix Z, the $N^* \times N^2$ elimination matrix L_N is defined by the property $\text{vech}(Z) = L_N \,\text{vec}(Z)$. Conversely, the $(N^2 \times N^*)$ dimensional duplication matrix D_N is defined by $\text{vec}(Z) = D_N \,\text{vech}(Z)$. Because $D_N'D_N$ is nonsingular, the Moore-Penrose inverse or generalized inverse of D_N is $D_N^+ = (D_N'D_N)^{-1}D_N'$. See Lütkepohl (1996).

where $\upsilon = \text{vech}(CC')$, $A = D_N^+(F \otimes F)' D_N$ and $B = D_N^+(G \otimes G)' D_N$.

Now consider the $N^* = N(N+1)/2$ dimensional vector of mean zero random variables

$$u_t = \eta_t - h_t \ . \tag{6}$$

It is not difficult to see that $\{u_t\}$ is a mean zero, serially uncorrelated, but conditionally heteroskedastic process, i.e., $\text{Cov}_{t-1}[u_t] = E_{t-1}[u_t u_t'] = \Sigma_t$. Moreover, if in addition to the stationarity conditions of Boussama et al. (2011) one assumes finite fourth-order moments of $\{\varepsilon_t\}$, $\{u_t\}$ is a white noise process (Hafner, 2008, Prop. 2).

For our subsequent discussions, we require condition (3) to hold, which for the vech-form translates into $\rho(\mathcal{A}) < 1$, where $\mathcal{A} = A + B$. Moreover, let L denote the lag operator such that $L\eta_t = \eta_{t-1}$. Using (6) to replace h_t in (5), we obtain

$$\begin{aligned} \eta_t &= \upsilon + A\eta_{t-1} + B(\eta_{t-1} - u_{t-1}) + u_t & (7) \\ \Leftrightarrow (I_{N^*} - \mathcal{A}L)\eta_t &= \upsilon + (I_{N^*} - BL)u_t \\ \Leftrightarrow \eta_t &= (I_{N^*} - \mathcal{A}L)^{-1}\upsilon + (I_{N^*} - \mathcal{A}L)^{-1}(I_{N^*} - BL)u_t \\ &= \tilde{\upsilon} + \Phi(L)(I_{N^*} - BL)u_t, & (8) \\ &= \tilde{\upsilon} + \Theta(L)u_t, & (9) \end{aligned}$$

with $\tilde{\upsilon} = (I_{N^*} - \mathcal{A})^{-1}\upsilon$ and $\Phi(L) = (I_{N^*} - \mathcal{A}L)^{-1}$, which exist if $\rho(\mathcal{A}) < 1$. In (8), the parameter matrices specifying the operator $\Phi(L) = (I_{N^*} - \mathcal{A}L)^{-1} = I_{N^*} + \Phi_1 L + \Phi_2 L^2 + \Phi_3 L^3 + \ldots$ are given by

$$\Phi_0 = I_{N^*},\ \Phi_k = \mathcal{A}\Phi_{k-1},\ k = 1, 2, 3, 4, \ldots \ .$$

Summarizing the autoregressive and moving average part of the vech representation, the operator $\Theta(L)$ in (9) conforms with the parameterization

$$\Theta_0 = I_{N^*},\ \Theta_1 = \mathcal{A} - B = A,\ \Theta_k = \mathcal{A}\Theta_{k-1},\ k = 2, 3, \ldots .$$

Because $\text{Cov}_{t-1}[u_t] = \Sigma_t$ is typically not diagonal, the elements of u_t are simultaneously correlated. As an implication, the coefficient matrices Θ_k, $k = 0, 1, 2, \ldots$ are not suitable for tracing how isolated shocks contribute to forecast uncertainties attached to particular variables in η_t. To cope with cross-equation correlation, it has become conventional to derive forecast error variance decompositions from VMA representations under the presumption of orthogonalized shocks. For this purpose, (9) is rephrased as

$$\eta_t = \tilde{\upsilon} + \Theta(L)\Sigma_t^{1/2}\Sigma_t^{-1/2}u_t \tag{10}$$

$$= \tilde{\upsilon} + \Psi_t(L)\nu_t \ , \tag{11}$$

where $\nu_t = \Sigma_t^{-1/2}u_t$, $\Psi_t(L) = \Theta(L)\Sigma_t^{1/2}$. Unlike standard VMA representations of homoskedastic VARs, however, the operator $\Psi_t(L)$ depends on Σ_t, and hence is time-varying. Specifically, we have

$$\Psi_t(L) = \Psi_{t,0} + \Psi_{t,1}L + \Psi_{t,2}L^2 + \Psi_{t,3}L^3 + \ldots \tag{12}$$

where $\Psi_{t,0} = \Sigma_t^{1/2}$, $\Psi_{t,k} = \Theta_k\Sigma_{t-k}^{1/2}$. As in the usual VMA representation of stationary VAR models, the coefficients in $\Psi_{t,k}$, $k = 0, 1, 2, \ldots$ describe how (unit) shocks in the elements of ν_t impact the variables in $\eta_t = \mathrm{vech}(\varepsilon_t\varepsilon_t')$ simultaneously $(k = 0)$ and over time $(k = 1, 2, \ldots)$.

2.3 The conditional covariances of the VMA representation

The implementation of the VMA coefficient matrices in (12) requires an analytic expression of the conditional covariance of Σ_t. Define the elimination matrix L_N and the duplication matrix D_N as in footnote 2. We have the following:[3]

Proposition 1 *Let u_t defined in* (6) *be the innovation process of the half-vectorized 'squared' MGARCH process $\eta_t = \mathrm{vech}(\varepsilon_t\varepsilon_t')$ and let $\Sigma_t = \mathrm{Cov}_{t-1}[u_t]$. Then*

$$\Sigma_t = L_N\mathcal{H}_t\widetilde{\Omega}\mathcal{H}_tL_N' - h_th_t' \ , \tag{13}$$

where $\mathcal{H}_t = H_t^{1/2} \otimes H_t^{1/2}$ and $\widetilde{\Omega} = E[(\xi_t\xi_t') \otimes (\xi_t\xi_t')']$.

[3] We thank an anonymous referee for helpful suggestions on how to simplify both a former representation of our proposition and its proof.

Proof: Using results in Lütkepohl (1996, Ch. 2.4 and 7.2), we notice that

$$\eta_t = \operatorname{vech}(\varepsilon_t\varepsilon_t') = L_N \operatorname{vec}(\varepsilon_t\varepsilon_t') = L_N \mathcal{H}_t \operatorname{vec}(\xi_t\xi_t') \ .$$

Then, because h_t is $\mathcal{F}_{t-1}$-measurable, we get

$$\begin{aligned}
\Sigma_t &= \operatorname{Cov}_{t-1}[u_t] = \operatorname{Cov}_{t-1}[\eta_t] \\
&= E_{t-1}[\eta_t\eta_t'] - E_{t-1}[\eta_t]E_{t-1}[\eta_t'] \\
&= E_{t-1}[\operatorname{vech}(\varepsilon_t\varepsilon_t')\operatorname{vech}(\varepsilon_t\varepsilon_t')'] - h_t h_t' \\
&= E_{t-1}\left[L_N\mathcal{H}_t \operatorname{vec}(\xi_t\xi_t')\operatorname{vec}(\xi_t\xi_t')'\mathcal{H}_t L_N'\right] - h_t h_t' \\
&= L_N\mathcal{H}_t\widetilde{\Omega}\mathcal{H}_t L_N' - h_t h_t' \ .
\end{aligned}$$

The last line uses the notation

$$\widetilde{\Omega} = E[\operatorname{vec}(\xi_t\xi_t')\operatorname{vec}(\xi_t\xi_t')'] = E[(\xi_t\xi_t') \otimes (\xi_t\xi_t')'] \ , \tag{14}$$

which holds thanks to $\operatorname{vec}(\xi_t\xi_t') = \xi_t \otimes \xi_t$ and due to the fact that $\{\xi_t\}$ is iid. $\Box$

The matrix $\widetilde{\Omega}$ in (14) collects the fourth-order moments of ξ_t and is of dimension $N^2 \times N^2$ with $N \times N$ dimensional blocks Ω_{ij}, $i, j = 1, 2, \ldots, N$. Let $\omega_{kl}^{(ij)}$ denote a typical element of block Ω_{ij}. Specifically, along the diagonal, the block matrices Ω_{ii} have elements

$$\omega_{ii}^{(ii)} = \kappa_i,\ \omega_{jj}^{(ii)} = 1, j \neq i \text{ and } \omega_{ij}^{(ii)} = 0,\ i \neq j \ , \tag{15}$$

where κ_i is the fourth-order moment of the i-th element in ξ_t. Under the Gaussian assumption, we have $\kappa_i = 3$, $\forall i$. The off-diagonal blocks Ω_{ij}, $i \neq j$, are such that

$$\omega_{ij}^{(ij)} = 1, \text{ and } \omega_{kl}^{(ij)} = 0 \text{ for } (k,l) \neq (i,j).$$

As we discuss below, instead of assuming Gaussianity, one may alternatively use the empirical fourth-order moments to parameterize $\widetilde{\Omega}$ and thereby Σ_t in (13).

3 Measuring variance spillovers

3.1 A time-varying forecast error variance decomposition of the half-vectorized 'squared' MGARCH process

The spillover measures to be presented in Section 3.2 build on the conditional forecast error variance decomposition, which we obtain from the time-varying VMA representation (11). Denote by $\hat{\eta}_{t+M|t} = E_t[\eta_{t+M}]$ the time-t conditional M-step ahead forecast of the half-vectorized 'squared' MGARCH process. Then the error of predicting η_{t+M} reads as

$$\eta_{t+M} - \hat{\eta}_{t+M|t} = \Psi_{t+M,0}\nu_{t+M} + \Psi_{t+M,1}\nu_{t+M-1} + \ldots + \Psi_{t+M,M-1}\nu_{t+1} \, , \tag{16}$$

where the parameter matrices are defined in (12). The elements in the i-th rows of $\Psi_{t+M,0}, \Psi_{t+M,1}, \ldots, \Psi_{t+M,M-1}$ describe how the innovations in $\nu_{t+M}, \ldots, \nu_{t+1}$ contribute to the forecast errors of variable i at horizon M. Because (16) has time-varying parameter matrices, we study the conditional forecast error variance given by

$$E_t\left[(\eta_{t+M} - \hat{\eta}_{t+M|t})(\eta_{t+M} - \hat{\eta}_{t+M|t})'\right] \tag{17}$$

$$= \hat{\Sigma}_{t+M|t} + \Theta_1\hat{\Sigma}_{t+M-1|t}\Theta_1' + \ldots + \Theta_{M-1}\hat{\Sigma}_{t+1|t}\Theta_{M-1}' \tag{18}$$

$$= \hat{\Psi}_{t+M,0|t}\hat{\Psi}_{t+M,0|t}' + \hat{\Psi}_{t+M,1|t}\hat{\Psi}_{t+M,1|t}' + \ldots + \hat{\Psi}_{t+M,M-1|t}\hat{\Psi}_{t+M,M-1|t}' \, , \tag{19}$$

where we set

$$\begin{aligned} \hat{\Psi}_{t+M,0|t} &= (\hat{\Sigma}_{t+M|t})^{1/2} \\ \hat{\Psi}_{t+M,1|t} &= \Theta_1(\hat{\Sigma}_{t+M-1|t})^{1/2} \\ &\vdots \\ \hat{\Psi}_{t+M,M-1|t} &= \Theta_{M-1}\Sigma_{t+1}^{1/2} \, , \end{aligned} \tag{20}$$

with $\hat{\Sigma}_{t+m|t} = E_t[\Sigma_{t+m}]$, $m = 2, \ldots, M$. We emphasize that $\hat{\Psi}_{t+M,m|t}$, $m = 0, 1, \ldots, M-1$, is introduced for notational reasons and is not meant to imply that $\hat{\Psi}_{t+M,m|t} = E_t[\Psi_{t+M,m}]$, which is neither needed for the following nor does it hold in general, except trivially for $m = M-1$.

Let $\hat{\psi}_{ij}^{(t+M,m)}$ denote a typical element of $\hat{\Psi}_{t+M,m|t}$, $m = 0, 1, \dots, M-1$. In accordance with the VAR literature (Lütkepohl, 2007, p. 63-64), the proportion of the M-step ahead forecast error variance of variable i, accounted for by innovations in variable j, is given by

$$\lambda_{t,ij}^{(M)} = \frac{\sum_{m=0}^{M-1} \left(\hat{\psi}_{ij}^{(t+M,m)}\right)^2}{\sum_{m=0}^{M-1} \sum_{j=1}^{N^*} \left(\hat{\psi}_{ij}^{(t+M,m)}\right)^2} \,, \tag{21}$$

where $N^* = N(N+1)/2$, i.e., the dimension of the half-vectorized 'squared' MGARCH process. In light of our applications in Section 4.3, this quantity has telling interpretations. For example, $\lambda_{t,ij}^{(M)}$ says, conditionally on time t, which proportion of the forecast error variance of, for instance, squared equity returns can be ascribed to shocks to squared bond returns, squared commodity returns, or cross-products of bond and commodity returns. Thus, $\lambda_{t,ij}^{(M)}$, $i \neq j$, measures a *cross (forecast error) variance share*, i.e., a *spillover* in the sense of Diebold and Yilmaz (2009, p. 159). Because conditional predictions of the squared GARCH process or cross-products of two GARCH processes are equivalent to predicting the conditional variance and covariance of the process, $\lambda_{t,ij}^{(M)}$ serves as the basis for the definitions of the time-varying variance spillover statistics of Section 3.2.

We conclude this section with two comments on the actual implementation of (21). First, note that while Σ_{t+1} is time t-measurable, for $M \geq 2$, the forecast error variance decomposition depends on the time-t predictions of future covariances $\Sigma_{t+2}, \Sigma_{t+3}, \dots, \Sigma_{t+M}$. Although $\Sigma_t = \Sigma_t(H_t)$ is an explicit function of H_t, using the variance dynamics in (4) to evaluate these predictions is involved. We therefore suggest employing approximations $\hat{\Sigma}_{t+m|t} \approx \Sigma_{t+m}(\hat{H}_{t+m|t})$, $m = 1, \dots, M$, which can be obtained readily from the recursive one-step ahead predictors of the conditional covariance process:

$$\begin{aligned} \widehat{H}_{t+1|t} = H_{t+1} &= CC' + F'\varepsilon_t\varepsilon_t'F + G'H_tG \\ \text{and } \widehat{H}_{t+m|t} &= CC' + F'\widehat{H}_{t+m-1|t}'F + G'\widehat{H}_{t+m-1|t}G, \; m = 2, 3, \dots \end{aligned} \tag{22}$$

Second, to evaluate Σ_t, the fourth-order moment matrix $\widetilde{\Omega}$ is needed. We will assume $\omega_{ii}^{(ii)} = E[\xi_{i,t}^4] = 3$, $i = 1, \ldots, N$, in (15). While this may seem a rather strong assumption, from the definition of $\lambda_{t,ij}^{(M)}$ as a ratio, one may imagine that the approximation error implied by this normality assumption is minor despite the actual excess kurtosis of return innovations.[4]

3.2 Variance spillover indices

Diebold and Yilmaz (2009) motivate the use of statistics of the form in (21) to define spillover indices. It is, however, important to observe that $\lambda_{t,ij}^{(M)}$ depends on the construction of underlying shocks ν_t and the determination of $\Sigma_t^{1/2}$. Both ν_t and $\Sigma_t^{1/2}$ lack invariance under rotation, or, put differently, rival definitions are observationally equivalent in η_t. Specifically, consider a counterpart of (10)

$$\eta_t = \tilde{\upsilon} + \Theta(L)\Sigma_t^{1/2} Q Q' \Sigma_t^{-1/2} u_t,\ QQ' = I_{N^*},\ Q \neq I_{N^*}, \tag{23}$$

where Q is a rotation matrix. In the literature on structural VARs, the identification of $\Sigma_t^{1/2} Q$ has attracted great interest (Amisano and Giannini, 1997). Typically, external information, for instance, derived from economic theory, is employed to address model identification. Recently, sign restrictions have become a prominent identification approach (Faust, 1998; Uhlig, 2005). Because economic theory concerning the contemporaneous relations among daily financial data is scarce, the decomposition set out in (10) can only be justified in light of its economic content and the plausibility of the statistical functionals derived from the definitions in (10) or (21). In discussing the empirical implications of our model, we will justify the identifying content of the symmetric square root matrix $\Sigma_t^{1/2}$ in light of the detected patterns of aggregate total variance spillovers and disaggregate asset-specific net variance spillovers.[5]

[4] Indeed, when we replace $\omega_{ii}^{(ii)} = 3$ with the empirical fourth-order moments of the leptokurtic innovations $\xi_{i,t}$, $i = 1, \ldots, N$, the mean (standard deviation) of the absolute differences between the respective indices of total variance spillovers shown in Section 4 is 0.007 (0.003).

[5] As an alternative to the symmetric matrix square root, Diebold and Yilmaz (2014) build on the Cholesky factorization for identification. Because the Cholesky factorization is dependent on the

For the construction of the spillover index, note that we have $\sum_{j}^{N^*} \lambda_{t,ij}^{(M)} = 1$ and $\sum_{i,j}^{N^*} \lambda_{t,ij}^{(M)} = N^*$. Let $\eta_{i,t}$, $i = 1, \dots, N^*$, denote an element of η_t. Then a quantitative measure of total spillovers can be defined as

$$\mathcal{S}_t^{(M)} = \frac{1}{N^*} \sum_{i,j=1, i \neq j}^{N^*} \lambda_{t,ij}^{(M)} \ . \tag{24}$$

$\mathcal{S}_t^{(M)}$ measures the total forecast error variance share of the variables $\eta_{i,t}$, $i = 1, \dots, N^*$, that is attributable to shocks in all other variables $\eta_{j,t}$, $j = 1, \dots, N^*, j \neq i$. It thus is a normalized aggregate of all spillovers in the system, justifying the term 'spillover index.' Since it is built from information conditional on time-t, it is a spot measure of variance spillovers.

Besides the total index (24), many other spot variance indices are possible. Following Diebold and Yilmaz (2012), we can define directional spillovers between all variables involved. They provide decompositions of the spillover index into spillovers coming from (or to) a specific source. The directional spillovers received by variable i from all other variables j are defined as

$$\mathcal{R}_{t,i}^{(M)} = \frac{\sum_{j=1, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} \ , \tag{25}$$

whereas the directional spillovers transmitted by variable i to all other variables j are measured by

$$\mathcal{T}_{t,i}^{(M)} = \frac{\sum_{j=1, i \neq j}^{N^*} \lambda_{t,ji}^{(M)}}{N^*} \ . \tag{26}$$

Furthermore, it is meaningful to compute their difference

$$\mathcal{N}_{t,i}^{(M)} = \mathcal{T}_{t,i}^{(M)} - \mathcal{R}_{t,i}^{(M)} \ , \tag{27}$$

because it can be interpreted as the net contribution of variable i to the entire transmission process.

ordering of the variables, they justify this choice by showing that they recover very similar spillover patterns for other randomized orderings. In light of their Figure 5, it seems likely that the spillover index obtained by averaging all indices of the randomized orderings will be close to the index based on the symmetric matrix square root.

Since the elements in $\eta_t = \text{vech}(\varepsilon_t \varepsilon_t')$ correspond to patterns of variation and covariation, it is of interest to further distinguish between these two groups. Let J_{cov} and I_{cov} be the sets of all $i, j = 1, \ldots, N^*$ that index a covariance, and define J_{var} and I_{var} accordingly. As suggested in Fengler and Gisler (2015), we define

$$\mathcal{R}_{t,i}^{(M,cov)} = \frac{\sum_{j \in J_{cov}, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} , \qquad \mathcal{T}_{t,i}^{(M,cov)} = \frac{\sum_{j \in J_{cov}, j \neq i}^{N^*} \lambda_{t,ji}^{(M)}}{N^*} , \tag{28}$$

$$\mathcal{R}_{t,i}^{(M,var)} = \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} , \qquad \mathcal{T}_{t,i}^{(M,var)} = \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ji}^{(M)}}{N^*} , \tag{29}$$

which can be interpreted as the directional spillovers received by variable i from all covariances j (left-hand side of (28)) or transmitted by variable i to all covariances j (right-hand side of (28)); and likewise for the variances in (29). As discussed above, for each i, the differences among these indices, e.g., $\mathcal{N}_{t,i}^{(M,cov)} = \mathcal{T}_{t,i}^{(M,cov)} - \mathcal{R}_{t,i}^{(M,cov)}$, provide insights into the net spillovers between covariances and variances.

Based on this, we define the following total covariance and total variance spillover indices. An index of total *own (co)variance spillovers*, which measures the spillovers between covariances (between variances, right-hand side), is given by

$$\mathcal{S}_t^{(M,ocov)} = \sum_{i \in I_{cov}} \frac{\sum_{j \in J_{cov}, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} , \qquad \mathcal{S}_t^{(M,ovar)} = \sum_{i \in I_{var}} \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} . \tag{30}$$

Moreover, the total *cross (co)variance spillovers*, which are spillovers from covariances to variances (variances to covariances, right-hand side), are defined by

$$\mathcal{S}_t^{(M,ccov)} = \sum_{i \in I_{var}} \frac{\sum_{j \in J_{cov}, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} , \qquad \mathcal{S}_t^{(M,cvar)} = \sum_{i \in I_{cov}} \frac{\sum_{j \in J_{var}, j \neq i}^{N^*} \lambda_{t,ij}^{(M)}}{N^*} . \tag{31}$$

It holds that $\mathcal{S}_t^{(M)} = \mathcal{S}_t^{(M,ocov)} + \mathcal{S}_t^{(M,ovar)} + \mathcal{S}_t^{(M,ccov)} + \mathcal{S}_t^{(M,cvar)}$. The indices (30) and (31) therefore shed light on the relative contribution of variance and covariance spillovers to the total index. It is also useful to study the net cross spillover index between variances and covariances, which decodes the total net exposure of all covariances vis-à-vis variance spillovers. It is given by

$$\begin{aligned} \mathcal{N}_t^{(M,cross)} &= \sum_{i \in I_{cov}} (\mathcal{T}_{t,i}^{(M,var)} - \mathcal{R}_{t,i}^{(M,var)}) \\ &= - \sum_{i \in I_{var}} (\mathcal{T}_{t,i}^{(M,cov)} - \mathcal{R}_{t,i}^{(M,cov)}) = \mathcal{S}_t^{(M,ccov)} - \mathcal{S}_t^{(M,cvar)} . \end{aligned} \tag{32}$$

4 Empirics

4.1 Data

For the application, we consider the same set of four key US asset classes as in Diebold and Yilmaz (2012): equity, fixed income, foreign exchange and commodities. We study returns of the S&P 500 index, the 10-year treasury bond yields, the New York Board of Trade US dollar index futures, and the Dow-Jones/UBS commodity index. The data are obtained from Thomson Reuters Datastream and the sample period is from March 1, 1995, to December 31, 2014, with 5176 daily returns altogether. See Figure 1 for an overview of the data.

Figure 1 about here

4.2 Estimation of the BEKK model

To estimate the four-dimensional variance specification for the vector of asset returns, we use a modified version of the module 'arch_mg.src' that is part of the software *JMulti* (Lütkepohl and Krätzig, 2004, *http://www.jmulti.de/*). We verify that the estimated parameters correspond to a maximum of the log-likelihood function by multiplying each parameter estimate by 0.995 and 1.005 and checking the reductions of the log-likelihood. For inferential purposes, we use the estimates of the analytical QML covariance matrices as provided in Hafner and Herwartz (2008).

Table 1 about here

Table 1 documents the coefficient estimates along with their QML t-statistics. Parameterized by the off-diagonal elements of $\hat{F}$ and $\hat{G}$, some cross-equation dynamics are significant at conventional levels. We also consider a QML-based Wald test on the joint insignificance of all 24 off-diagonal model parameters. Asymptotically, the test statistic has a $\chi^2(24)$ distribution under the null hypothesis of zero off-diagonals. The statistic is 39.96, corresponding to a p-value of 2.16%. This supports the presence of nonlinear and complex off-diagonal dynamics in the system of (co)variance equations.

Figure 2 about here

Figure 2 displays the estimated conditional standard deviations of the four asset classes (upper panel), and the six BEKK implied pairwise correlations (lower panel). The estimated conditional standard deviations reflect the typical features of volatility clustering. Starting with the subprime crises at the end of 2007, conditional standard deviations have accelerated over all asset classes (except foreign exchange). While conditional second-order moments of equity and fixed income indices are of similar magnitude until 2011, for the most recent part of the sample, fixed income risk turns out to be more pronounced in comparison with stock market volatility. Pairwise correlations in the lower panel of Figure 2 show that the comovements of almost all asset classes exhibit strong time variation. With regard to the two asset classes with the highest volatility on average, it turns out that the correlation between equity and fixed income markets is markedly negative (positive) in the beginning (at the end) of the sample period. The periods of turmoil starting in 2008 are characterized by strongly negative correlations among foreign exchange markets and the remaining asset classes.

4.3 Descriptive conditional spillover analysis

4.3.1 The time-varying total variance spillover index

We start the analysis by considering the time evolution of the total spillover index (24) displayed in Figure 3. The index (black line) is plotted along with a number of major political and economic events – see Table 2 for a compilation. This is as in Diebold and Yilmaz (2009, 2012), but because our modeling approach allows us to compute the index at the daily frequency, we can exactly spot the events and analyze their impact to a degree of detail that is not feasible in rolling window applications. At the same time, we can study the long-term cyclical trends of variance spillovers in the 20 years of our sample. For the forecast horizon, we consider about one week, i.e., $M = 5$.

Table 2 and Figure 3 about here

According to Figure 3, the spillovers are very moderate between 1995 and 2001, hovering at and often below 10%. Although events like (1) the Thai Bhat devaluation, which is seen as the starting point of the Asian crisis, (2) the Russian crisis, (3) the first market disruptures at the beginning of the dot-com crisis, such as the April 14, 2000 NASDAQ crash, and (4) the 09/11 twin-tower attacks make the spillover index soar, they are rather short-lived.

A first major period of increased variance spillovers can be detected in the forefront of the geopolitical tensions surrounding the impending US-led war in Iraq (5). At the outbreak of the war, the index spikes to unprecedented levels of 40%, after which it returns to previous levels. The most important period of increased interdependence and variance spillovers by far, however, is the crisis complex of the subprime mortgage crunch, the banking crisis, and the US recession from December 2007 to June 2009, all of which are accompanied by extraordinary US central bank measures and by the political controversies over the impending US debt limits in 2011 and 2013.[6] Several incidents can be clearly distinguished: (6) Freddie Mac's announcement that it would no longer take the worst subprime risks; (7) the Northern Rock crisis; (8) the Carlyle Capital Corporation's press release on failing to meet margin calls on one of its mortgage bond funds; (9) the Lehman default. Between 2006 and the end of 2008, the index rises continuously from about 5% to about 25% and remains at about these levels till the end of 2012. The overall climax of the index is reached in November 2011 with about 60%. Over this crisis, we also document the announcements of the major monetary policy measures of the Fed, later known as 'quantitative easing': (10) the first program to purchase the direct obligations

[6]For these dates, we borrow from a timeline of events published on the website of the Federal Reserve Bank of St. Louis at https://www.stlouisfed.org/financial-crisis/full-timeline and the press releases linked to this site.

of housing-related, government-sponsored enterprises announced in November 2008; (11) the expansion of the program to buy long-term Treasury securities of November 2010; (13) the operation 'twist' to influence the term structure of interest rates; (14) the open-ended bond purchasing program of agency mortgage-backed securities of September 2012. It is remarkable that despite their exceptional nature, none of these policy announcements has any visible, ameliorating impact on variance spillovers. It is only at the end of 2012 that the spillover index levels start to retreat. Interestingly, the political debates about the US fiscal cliffs in 2011 (12) and 2013 (15) are also hardly detectable in the graph.

As we have argued above, using the symmetric eigenvalue decomposition of the contemporaneous covariance Σ_t for identification deserves further economic underpinnings. As a first justification of our identification scheme, consider the lower part of Figure 3, which displays a 20-day moving average of the daily US Economic Policy Uncertainty Index (EPUI) of S. R. Baker, N. Bloom, and S. J. Davis (red line). This index measures policy-related economic uncertainty according to newspaper coverage.[7] For the purpose of illustration, the index is scaled so as to have the same standard deviation as the total spillover index, and it is reflected along the horizontal axis.

The similarity between the graphs is striking. The total spillover index moves almost in a one-to-one fashion with the moving average of the EPUI. While the amplitudes may differ in detail, both indices exhibit the same long-term trends as well as very similar reactions to the events singled out and discussed above. The correlation among both indices is high: 57%. Hence, the symmetric eigenvalue decomposition of Σ_t supports the detection of an economically well-founded index of variance spillovers.

Despite the one week ahead forecasts, the graph of the spillover index in Figure 3 is 'in-sample' because the underlying parameter estimates are obtained from the full sample. For real-time applications, it is natural to ask how much the spillover graph

[7] Data and methodological details can be found on *http://www.policyuncertainty.com/index.html*.

would change if one worked in a framework that was entirely ex ante. To explore this question, we employ rolling subsamples that comprise 1500 return observations each to estimate the BEKK model as described in Section 4.2. To economize on computational time, the windows are shifted only every 250 observations after each estimation. For given parameter estimates, the covariance paths H_t, $t = 1501, 1502, \ldots, T$, are determined by updating the covariance dynamics with the observed time series innovations ε_t.

Figure 4 about here

In Figure 4, we superimpose the previous graph of Figure 3 onto such a fully ex ante spillover plot. Due to parameter variations, we find moderate deviations between the two indices, in particular between 2006 and 2007 and in 2014. Overall, however, there is strong agreement between the two indices.

4.3.2 Further decompositions of the total spillover index

What drives the total variance index? This and related questions can be answered by studying the subindices of Section 3.2. In Figure 5, we decompose the total index into own and cross (co)variance spillovers.

Figure 5 about here

Two observations are evident. First, the major features of the total spillover index are traced out by the own covariance ($\mathcal{S}_t^{(M,ocov)}$), the cross covariance ($\mathcal{S}_t^{(M,ccov)}$) and the cross variance spillover graphs ($\mathcal{S}_t^{(M,cvar)}$). All three are approximately of equal size, their fluctuations are highly correlated and their paths are very much akin to the total index. Therefore, each of them reflects very similar information as the EPUI. Second, the own variance spillover index ($\mathcal{S}_t^{(M,ovar)}$) is markedly different from the other three series. This is remarkable because the own variance spillover index corresponds to what the standard variance spillover literature, which ignores covariances when computing the total index, would report as the total variance spillover index.[8] While similar observations are also made in Fengler and Gisler (2015), in our BEKK model with fully specified covariance dynamics, this discrepancy is even more eye-catching. It suggests that most of the systemic interdependence is propagated through the joint variance-covariance dynamics rather than the variance dynamics alone. This interpretation is also confirmed by comparing the reactions of the various indices to the selected events discussed in the previous section. Moreover, Figure 5 reveals that the net exposure of all covariances vis-à-vis the variances (recall that $\mathcal{N}_t^{(M,cross)} = \mathcal{S}_t^{(M,ccov)} - \mathcal{S}_t^{(M,cvar)}$) is negative on average; thus, overall, the covariances receive more spillovers from the variances than they transmit back.

Figure 6 about here

As a more disaggregated decomposition, we present in Figure 6 the asset-specific net exposures of variances $\mathcal{N}_{t,i}^{(M,var)}$ and covariances $\mathcal{N}_{t,i}^{(M,cov)}$. The top left panel reveals that, generally, stock markets as well as bonds are transmitters of variance spillovers. Whereas they are of about equal size in the first half of the sample, the bond net variance spillovers dominate since 2003. They are particularly strong from 2010 to 2012, in which period the stock markets even become net receivers of variance spillovers. This period coincides with the times when the Fed adopted extraordinary policy measures to influence the bond markets – see Table 2. It therefore appears natural that bond markets are positive net transmitters of variance spillovers. Referring to our discussions in Section 3.2, we read these characteristics as supportive evidence for the identification scheme based on the symmetric eigenvalue decomposition.

In contrast to stocks and bonds, over the entire sample, the commodity market is a net transmitter and the foreign exchange market a net receiver of variance spillovers (top right element in Figure 6). Moreover, both net variance spillovers exhibit pronounced trends from 1995 to about 2010/2012, which reflects their increasing

[8] Note, however, that the absolute scales are different, because in a spillover analysis with N assets without covariances, one has $N^* = N$ instead of $N^* = N(N+1)/2$ as the scaling constant.

importance for investors as asset classes. The net receiver position of the foreign exchange market becomes particularly pronounced from 2008 onward.

Finally, the lower panel of Figure 6 shows the net covariance spillovers $\mathcal{N}_{t,i}^{(M,cov)}$. Overall, they fluctuate around zero, but with deviations of about two percent around zero, they are of smaller size than the variance net spillovers $\mathcal{N}_{t,i}^{(M,var)}$. This implies, interestingly, that covariance spillovers – in contrast to the variance spillovers – are generally less asymmetric among the different asset classes.

4.4 Informational content of time-t conditional variance spillovers: the case of value-at-risk predictions

We now investigate whether our proposed variance indices have any value beyond descriptive analysis. For this purpose, we study whether they can help assess the informational efficiency of a portfolio value-at-risk.

4.4.1 Value-at-risk

In risk management, GARCH models have become a standard econometric tool to evaluate risk measures such as the value-at-risk (VaR) – see Andersen et al. (2013) for a discussion of GARCH-based approaches to quantify the VaR. For a portfolio with shares w_t and conditional on time $t-1$, the VaR at level α is the (negative) return quantile

$$\mathrm{VaR}_t^{(\alpha)} = -q_\epsilon(\alpha) s_t \;, \tag{33}$$

where s_t is the conditional standard deviation, $s_t = \sqrt{w_t' H_t w_t}$, and $q_\epsilon(\alpha)$ is the empirical α-quantile of standardized portfolio returns $\{\epsilon_t = w_t' r_t / s_t\}_{t=1}^{T}$. We consider three portfolios with time-invariant composition $w_t = w$, namely (i) a portfolio assigning equal weight to all asset classes (denoted by ewp); (ii) a portfolio consisting only of equity (eqp); and (iii) a portfolio assigning equal weight to all asset classes except equity (noeq). In addition, (iv) we consider the minimum variance portfolio (mvp) with portfolio weights $w_t = H_t^{-1}\mathbf{1}/c$, where $c = \mathbf{1}' H_t^{-1}\mathbf{1}$ and $\mathbf{1}$ is a four-dimensional vector of ones (Campbell et al., 1997, Chap. 5).

4.4.2 Value-at-risk diagnosis

Backtesting the VaR relies on a series of binary auxiliary variables, the VaR hits,

$$\tilde{y}_{t,\alpha} = \mathbb{1}(w_t' r_t \leq -\mathrm{VaR}_t^{(\alpha)}) \,, \tag{34}$$

where $\mathbb{1}(B)$ denotes the indicator function of the set B. An unconditionally valid risk assessment requires the mean of $\tilde{y}_{t,\alpha}$ to be equal to α. For an informationally efficient risk assessment, one demands that conditional on time-$(t-1)$ information, the deviations of $\tilde{y}_{t,\alpha}$ from its unconditional expectation be first-order unpredictable.

To test our risk model, we apply the dynamic quantile (DQ) test introduced in Engle and Manganelli (2004), because informational efficiency can be tested within this framework in a straightforward manner. Under the null hypothesis of the DQ test, the VaR model is (conditionally and unconditionally) well specified. Specifically, it is tested whether the centered hits, $y_{t,\alpha} = \tilde{y}_{t,\alpha} - \alpha$, follow a martingale difference sequence conditional on information that is available in time $t-1$. Established indicators of informational inefficiency comprise the history of the VaR hit process. In the present framework, it is natural to regard (lagged) indices of variance transmission as further indicators of informational inefficiency of the VaR. In summary, we consider the DQ regression model

$$y_{t,\alpha} \quad = \quad \beta_0 + \sum_{k=1}^{5} \beta_k y_{t-k,\alpha} + x_{t-1}' \delta + e_t \,, \tag{35}$$

where the vector x_{t-1} comprises predetermined measures of risk transmission as introduced in Section 3 with the corresponding regression parameters collected in δ.[9] The null hypothesis of correct conditional and unconditional coverage of the model

[9]While DQ regressions have turned out to dominate rival VaR diagnostics in terms of power against misspecified VaRs (Berkowitz et al., 2011), their implementation relies on the binary hit processes $\tilde{y}_{t,\alpha}$. As is visible from (34), their determination comes along with a substantial loss of information. Against this background, the VaR diagnostic introduced by Gaglianone et al. (2011) promises further power improvements in comparison with the DQ-test, since it directly addresses the conditional validity of the quantile $\mathrm{VaR}_t^{(\alpha)}$. Our empirical results, however, suggest that the DQ test based on the spillover indices is sufficiently powerful as a diagnostic check of the VaR.

reads as $H_0 : \beta_k = 0, \forall\, k = 0, 1, \ldots, 5,$ and $\delta = 0$. Because the regression is specified for centered binary variables, the significance of $\hat{\beta}_0$ indicates in a separate test that the VaR model violates the unconditional coverage criterion. We assess the martingale property for the hit processes derived from two levels of VaR coverage, namely $\alpha = 0.01$ and $\alpha = 0.05$.

Table 3 documents some diagnostic results for the standardized portfolio returns (innovations) $\epsilon_t = w_t' r_t / \sqrt{w_t' H_t w_t}$. In case of a valid specification of the dynamic covariance process, these innovations should have mean zero and unit variance and should not indicate any kind of non-modeled or remaining conditional heteroskedasticity. The documented moments of $\{\epsilon_t\}$ show that for almost all portfolios, the standard error of portfolio innovations is close to unity. As the only exception, the second-order moments of the equity portfolio have a standard error of 0.95. Higher-order moments reveal some negative skewness of portfolio innovations, and the fourth-order moments are between four and five for all considered portfolios.

Table 3 about here

For some portfolios, we diagnose patterns of remaining heteroskedasticity in return innovations. In particular, the portfolios including equity show significant ARCH-LM diagnostics at order five, while low-order diagnostics do not indicate departures from an iid distribution. This may reflect the presence of outliers or the fact that the BEKK model is symmetric, i.e., positive and negative shocks impact symmetrically on the conditional second-order moments. In summary, both in-sample and ex ante portfolio innovations indicate that the employed four-dimensional BEKK model is well suited for extracting second-order characteristics of portfolio returns.

Table 4 and Table 5 about here

DQ diagnostics are shown in Tables 4 and 5 for nominal VaR levels of $\alpha = 0.01$ and $\alpha = 0.05$, respectively. Subjecting the model-implied VaR estimates to purely autoregressive DQ regressions of order five shows for most portfolios that the process of VaR hits does not exhibit significant serial correlation patterns. Moreover, the mean hit frequencies are in line with the nominal VaR levels.[10] The purely autoregressive DQ test regression indicates with 5% significance some misspecification of the risk model applied to in-sample portfolios that include equity components. However, since the full sample covers a period of almost 20 years, the significance of the DQ statistics might be due to violations of model stability.

In contrast, the indices of (co)variance spillovers carry predictive content for the dynamic patterns of centered VaR hits. For the smallest coverage level $\alpha = 0.01$, spillover measures contribute significantly to the explanation of the occurrence of overly negative returns. By means of these indicators, misspecifications of the risk model can be diagnosed with 5% significance for all portfolios in-sample, and ex ante for all portfolios, except mvp (Table 4, upper left and upper right panel, respectively). For the VaR coverage of $\alpha = 0.05$ (Table 5), measures of (co)variance spillovers are less indicative of risk misspecification. In particular, however, for monitoring the risk of the equal weight portfolios ex ante, augmenting DQ regressions with indices of (co)variance spillovers is significantly more informative than using purely autoregressive diagnostic models.

The DQ diagnostics show that the VaR estimates may suffer from violations of informational efficiency, but the evidence is only mildly specific on the particular indices of variance transmission that are most informative for the process of VaR hits. In this context, it is worthwhile to recall that the DQ tests indicate joint significance (or insignificance) of both the autoregressive patterns and the spillover statistics. To shed light on the marginal explanatory content of the autoregressive parameters on the one hand, and the indices of variance transmission on the other hand, the lower panels of Tables 4 and 5 document statistics that are derived from the marginal degrees of explanation in the DQ regressions. Specifically, we provide $\sqrt{R^2}$

[10] All t-statistics of DQ intercept estimates $\hat{\beta}_0$ lack significance at conventional levels. Detailed results on testing for unconditional coverage are available upon request.

statistics for regressions such that VaR hits are either explained by autoregressive patterns or by the lagged (co)variance spillover indices. The largest statistics are printed in bold. For 15 out of 20 diagnostic regressions, the highest marginal degrees of explanation are documented for some index of (co)variance spillovers.[11]

Distinguishing the particular indices with highest explanatory content, we find that generally covariance spillovers (in comparison with variance spillovers) have some lead in explaining the VaR hit processes. From an overall perspective, the measure of net covariance transmission ($\mathcal{N}_t^{(M,cov)}$) appears to have the highest explanatory content. In 9 out of 20 diagnostic regressions, this set of indicators obtains the highest marginal degree of explanation in the DQ regressions.

As is visible for the case of the minimum variance portfolios, the horizon M to which the spillover statistics refer has only negligible impact on the DQ diagnosis. For the alternative choices $M = 1$ and $M = 5$, the inferential outcomes and the degrees of explanation documented in Tables 4 and 5 are very similar.

5 Concluding remarks

We have proposed (co)variance spillover indices which are derived from the forecast error variance decomposition of the 'squared' returns process of a multivariate GARCH model of the BEKK class. On this basis, the proposed indices are time-t conditional and take full advantage of time-varying covariance information.

Empirically, we study a system of four major US asset classes. In contrast to extant (co)variance spillover indices, our approach allows one to simultaneously study the impact of specific events at the daily frequency along with the implications of lingering times of political and economic uncertainty over weeks and months. This is because the new indices are highly responsive to the news innovation process while at the same time featuring long-term secular trends of market interdependence. In

[11] Relating the TR^2 with the critical values from χ^2 distributions of either four or six degrees of freedom, we find that about one-third out of the $7 \times 5 \times 2 \times 2 = 140$ diagnostic results are significant at the 5% level.

an application to risk management, we demonstrate that the indices are informative of the likelihood of value-at-risk violations. Hence, they are of interest not only in a descriptive-analytic sense, but also for predictive purposes in a standard problem of empirical finance.

Analytically, our approach is attractive because we rely on the very general vech representation of the multivariate GARCH model. It is therefore not limited to the BEKK class: any multivariate GARCH model having a vech representation could be treated in this way. As a potential shortcoming, the curse of dimensionality could be cited as a problem known to afflict such models. Using covariance targeting strategies and suitable parameter restrictions, one might push the limits further than we do. We leave this for future research.

References

Acharya, V. V., Pedersen, L. H., Philippon, T. and Richardson, M. P. (2010). Measuring systemic risk, *Technical report*, New York University.

Adrian, T. and Brunnermeier, M. K. (2016). CoVar, *American Economic Review* . In press.

Alexander, C. (2001). *Mastering Risk*, Vol. II, Prentice Hall.

Amisano, G. and Giannini, C. (1997). *Topics in structural VAR econometrics*, 2nd edn, Springer-Verlag, Berlin.

Andersen, T. G., Bollerslev, T., Christoffersen, P. and Diebold, F. X. (2013). Financial risk measurement for financial risk management, *Handbook of the Economics of Finance*, Vol. 2B, Elsevier, chapter 17, pp. 1127–1220.

Baba, Y., Engle, R. F., Kraft, D. F. and Kroner, K. F. (1990). Multivariate simultaneous generalized ARCH models, *Technical report*, Department of Economics, University of California, San Diego.

Bali, T. G. and Hovakimian, A. (2009). Volatility spreads and expected stock returns, *Management Science* **55**(11): 1797–1812.

Baruník, J., Kočenda, E. and Vácha (2016). Asymmetric connectedness on the U.S. stock market: Bad and good volatility spillovers, *Journal of Financial Markets* **27**: 55–78.

Bauer, G. H. and Vorkink, K. (2011). Forecasting multivariate realized stock market volatility, *Journal of Econometrics* **160**(1): 93–101.

Bauwens, L., Laurent, S. and Rombouts, J. V. K. (2006). Multivariate GARCH models: A survey, *Journal of Applied Econometrics* **31**: 79–109.

Berkowitz, J., Christoffersen, P. and Pelletier, D. (2011). Evaluating Value-at-Risk models with desk-level data, *Management Science* **57**(12): 2213–2227.

Billio, M., Getmansky, M., Lo, A. W. and Pelizzon (2012). Econometric measures of connectedness and systemic risk in the finance and insurance sectors, *Journal of Financial Economics* **104**(3): 535–559.

Bollerslev, T., Engle, R. F. and Wooldridge, J. M. (1988). A capital asset pricing model with time varying covariances, *Journal of Political Economy* **96**: 116–131.

Boussama, F., Fuchs, F. and Stelzer, R. (2011). Stationarity and geometric ergodicity of BEKK multivariate GARCH models, *Stochastic Processes and their Applications* **121**: 2331–2360.

Campbell, J. Y., Lo, A. and MacKinley, C. (1997). *The econometrics of financial markets.*, Princeton University Press, Princeton University Press, Princeton.

Comte, F. and Lieberman, O. (2003). Asymptotic theory for multivariate GARCH processes, *Journal of Multivariate Analysis* **84**: 61–84.

Dellaportas, P. and Pourahmadi, M. (2012). Cholesky-GARCH models with applications to finance, *Statistics and Computing* **22**(4): 849–855.

Diebold, F. X. and Yilmaz, K. (2009). Measuring financial asset return and volatility spillovers, with application to global equity markets, *Economic Journal* **119**: 1–14.

Diebold, F. X. and Yilmaz, K. (2012). Better to give than to receive: Predictive directional measurement of volatility spillovers, *International Journal of Forecasting* **28**: 57–66.

Diebold, F. X. and Yilmaz, K. (2014). On the network topology of variance decompositions: Measuring the connectedness of financial firms, *Journal of Econometrics* **182**(1): 119–134.

Engle, R. F., Ito, T. and Lin, W. L. (1990). Meteor showers or heat waves? Heteroskedastic intra-daily volatility in the foreign exchange market, *Econometrica* **58**(3): 525–542.

Engle, R. F. and Kroner, K. F. (1995). Multivariate simultaneous generalized ARCH, *Econometric Theory* **11**: 122–150.

Engle, R. F. and Manganelli, S. (2004). CAViaR: Conditional autoregressive value at risk by regression quantiles, *Journal of Business & Economic Statistics* **22**(4): 367–381.

Engle, R. F., Ng, V. M. and Rothschild, M. (1990). Asset pricing with a factor-ARCH structure: Empirical estimates for treasury bills, *Journal of Econometrics* **45**: 213–237.

Faust, J. (1998). The robustness of identified VAR conclusions about money, *Carnegie-Rochester Conference Series on Public Policy* **49**(1): 207–244.

Fengler, M. R. and Gisler, K. I. M. (2015). A variance spillover analysis without covariances: What do we miss?, *Journal of International Money and Finance* **51**: 174–195.

Forbes, K. J. and Rigobon, R. (2002). No contagion, only interdependence: Measuring stock market comovements, *Journal of Finance* **57**(5): 2223–2261.

Gaglianone, P. W., Lima, L. R., Linton, O. B. and Smith, D. R. (2011). Evaluating Value-at-Risk models via quantile regression, *Journal of Business & Economic Statistics* **29**(1): 150–160.

Golosnoy, V., Gribisch, B. and Liesenfeld, R. (2012). The conditional autoregressive Wishart model for multivariate stock market volatility, *Journal of Econometrics* **167**: 211–223.

Hafner, C. M. (2008). Temporal aggregation of multivariate GARCH processes, *Journal of Econometrics* **142**: 467–483.

Hafner, C. M. and Herwartz, H. (2006). Volatility impulse responses for multivariate GARCH models: An exchange rate illustration, *Journal of International Money and Finance* **25**: 719–740.

Hafner, C. M. and Herwartz, H. (2008). Analytical quasi maximum likelihood inference in multivariate volatility models, *Metrika* **67**: 219–239.

Hafner, C. M. and Preminger, A. (2009). On asymptotic theory for multivariate GARCH models, *Journal of Multivariate Analysis* **100**: 2044–2054.

Hamao, Y., Masulis, R. and Ng, V. K. (1990). Correlations in price changes and volatility across international stock markets, *Review of Financial Studies* **3**: 281–307.

Jeantheau, T. (1998). Strong consistency of estimators for multivariate ARCH models, *Econometric Theory* **14**(1): 70–86.

Lütkepohl, H. (1996). *Handbook of Matrices*, Wiley & Sons, New York.

Lütkepohl, H. (2007). *New Introduction to Multiple Time Series Analysis*, Springer-Verlag, Berlin, Heidelberg. Corr. 2nd printing.

Lütkepohl, H. and Krätzig, M. (2004). *Applied Time Series Econometrics*, Cambridge University Press, Cambridge.

Nakatani, T. and Teräsvirta, T. (2009). Testing for volatility interactions in the constant conditional correlation garch model, *Econometrics Journal* **12**: 147–163.

Stelzer, R. (2008). On the relation between the Vec and BEKK multivariate GARCH models, *Econometric Theory* **24**(4): 1131–1136.

Uhlig, H. (2005). What are the effects of monetary policy on output? Results from an agnostic identification procedure, *Journal of Monetary Economics* **52**(2): 381–419.

van der Weide, R. (2002). GO-GARCH: A multivariate generalized orthogonal GARCH model, *Journal of Applied Econometrics* **17**: 549–564.

Woźniak, T. (2015). Testing causality between two vectors in multivariate GARCH models, *International Journal of Forecasting* **31**: 876–894.

Yilmaz, K. (2013). Return and volatility spillovers among the East Asian equity markets, *Journal of Asian Economics* **21**: 304–313.

Tables and figures

$\hat{C}$				$\hat{F}$				$\hat{G}$			
1.16E[-03]	-	-	-	.2685	.0325	$-.0115$	.0045	.9562	$-.0031$	.0048	$-.0008$
(7.73)				(13.4)	(1.40)	(-1.94)	(0.40)	(144.8)	(-0.42)	(2.78)	(-0.25)
-5.28E[-04]	1.04E[-03]	-	-	.0063	.2332	.0034	.0004	.0011	.9699	$-.0013$	.0006
(-1.93)	(3.84)			(0.81)	(9.82)	(0.86)	(0.05)	(0.52)	(147.2)	(-1.31)	(0.36)
-1.13E[-04]	2.31E[-04]	2.78E[-04]	-	.0519	$-.0230$	.1406	$-.0309$	$-.0050$	$-.0023$	.9856	.0067
(-1.25)	(2.08)	(3.52)		(1.63)	(-0.56)	(13.26)	(-1.51)	(-0.69)	(-0.22)	(394.0)	(1.89)
-1.70E[-04]	-1.70E[-05]	1.05E[-04]	5.29E[-04]	$-.0067$	$-.0040$	$-.0083$	.1408	.0057	$-.0018$	$-.0012$	.9884
(-0.94)	(-0.07)	(0.58)	(3.16)	(-0.35)	(-0.12)	(-0.93)	(10.7)	(1.18)	(-0.27)	(-0.66)	(353.0)

Table 1: QML parameter estimates of the unrestricted BEKK model. Entries in parentheses are the QML t-ratios determined by means of the analytical derivatives as provided by Hafner and Herwartz (2008). See also the module '*arch_mg.src*' implemented in *JMulTi* (Lütkepohl and Krätzig, 2004).

Event	Date	Description
(1)	Jul 02, 1997	Thai Bhat devaluation, start of the 1997 Asian crisis
(2)	Aug 17, 1998	Russia defaults on domestic debt, start of Russian crisis
(3)	Apr 14, 2000	NASDAQ crash, dot-com crisis till March 2003
(4)	Sep 11, 2001	09/11 attacks
(5)	Mar 20, 2003	US-led war in Iraq
(6)	Feb 27, 2007	Freddie Mac refuses to take worst subprime risks
(7)	Sep 14, 2007	Northern Rock crisis
(8)	Mar 05, 2008	Carlyle Capital Corp. fails to meet margin calls on a mortgage bond fund
(9)	Sep 15, 2008	Lehman Brothers default
(10)	Nov 25, 2008	Quantitative easing 1: Fed buys mortgage-backed securities
(11)	Nov 03, 2010	Quantitative easing 2: Fed buys long-term Treasury bonds
(12)	Aug 01, 2011	Fiscal cliff 2011: House passes 2011 debt ceiling bill
(13)	Sep 21, 2011	Fed announces operation Twist
(14)	Sep 13, 2012	Quantitative easing 3: open-ended bond purchasing program of agency mortgage-backed securities
(15)	Jul 22, 2013	Fiscal cliff 2013: House passes 2013 Continuing Appropriations Act

Table 2: Calendar of political and economic events highlighted in Figs. 3, 4, and 5.

	In-sample BEKK estimates				Ex ante BEKK estimates			
	ewp	eq	noeq	mvp	ewp	eq	noeq	mvp
M	1	1	1	1	1	1	1	1
std(ϵ)	0.996	0.993	0.993	1.011	0.994	0.950	1.009	0.989
$\overline{\epsilon^3}$	-0.214	-0.457	-0.018	-0.202	-0.185	-0.403	0.004	-0.114
$\overline{\epsilon^4}$	4.217	4.692	4.045	4.294	4.273	4.112	4.613	4.005
LM(1)	0.213	0.112	0.283	0.015	0.276	0.245	0.255	0.040
LM(5)	0.001	0.002	0.274	0.089	0.007	0.012	0.289	0.129

Table 3: Descriptive statistics of the standardized portfolio returns ϵ (standard deviation, third and fourth-order moment, ARCH-LM tests of order 1 and 5). Four portfolio compositions are considered: equal weight (ewp), only equity (eqp), equal weight without equity (noeq), minimum variance portfolio (mvp). Second-order characteristics of portfolio returns are evaluated by means of full sample information (left-hand side), or using the ex ante BEKK parameters determined by means of rolling sample windows.

	In-sample BEKK estimates					Ex ante BEKK estimates				
	ewp	eq	noeq	mvp		ewp	eq	noeq	mvp	
M	1	1	1	1	5	1	1	1	1	5
	Dynamic Quantile (DQ) diagnostics (p-values times 100)									
AR	0.018	0.614	37.38	7.812		10.43	5.857	15.95	49.32	
$\mathcal{R}_t^{(M,var)}$	0.122	2.461	32.25	3.253	3.272	28.50	10.40	12.17	47.70	45.70
$\mathcal{R}_t^{(M,cov)}$	0.007	0.199	9.082	9.593	9.855	14.90	1.869	11.75	48.39	47.29
$\mathcal{T}_t^{(M,var)}$	0.013	1.233	4.958	7.746	8.091	8.704	21.93	8.489	41.33	50.91
$\mathcal{T}_t^{(M,cov)}$	0.006	0.361	1.296	6.647	6.451	2.738	5.658	2.614	50.26	57.79
$\mathcal{N}_t^{(M,var)}$	0.034	0.828	15.21	3.570	3.986	6.588	14.76	7.640	39.00	47.12
$\mathcal{N}_t^{(M,cov)}$	0.029	1.075	1.691	2.484	3.374	18.75	2.893	10.89	36.97	35.51
$\mathcal{S}_t^{(M)}$	0.040	0.806	42.10	12.39	12.40	16.06	8.564	18.87	60.28	60.99
	Marginal $\sqrt{R^2}$ (times 100)									
AR	**7.276**	**5.996**	3.578	4.781		5.507	5.927	5.002	3.900	
$\mathcal{R}_t^{(M,var)}$	2.298	1.980	3.243	4.565	4.560	2.503	3.104	4.170	3.718	3.827
$\mathcal{R}_t^{(M,cov)}$	5.267	5.067	4.991	4.222	4.190	4.409	**5.978**	4.971	4.450	4.518
$\mathcal{T}_t^{(M,var)}$	4.159	2.969	4.868	3.725	3.682	4.193	2.068	4.289	4.011	3.583
$\mathcal{T}_t^{(M,cov)}$	5.383	4.718	**6.148**	4.580	4.601	**6.004**	5.033	**6.137**	4.384	4.058
$\mathcal{N}_t^{(M,var)}$	3.619	3.275	4.017	4.463	4.371	4.682	2.713	4.510	4.113	3.767
$\mathcal{N}_t^{(M,cov)}$	4.705	3.989	6.015	**5.381**	**5.152**	4.334	5.642	4.910	**4.962**	**5.050**
$\mathcal{S}_t^{(M)}$	0.107	1.224	1.224	0.032	0.019	0.387	1.538	1.130	0.439	0.123

Table 4: Inferential (p-values) and descriptive (marginal R^2) statistics from DQ regressions for VaR estimates with nominal coverage $\alpha = 1\%$. All DQ regressions include autoregressive dynamics up to order 5. Additional joint misspecification indicators included in enhanced DQ regressions are listed rowwise. The degrees of freedom for testing the null hypothesis of a conditionally valid VaR model are 6 for the purely autoregressive design ('AR'), and 10 and 12, respectively, for enhanced DQ regressions comprising either (four) variance or (six) covariance spillovers. The lower panel documents the degrees of explanation for the autoregressive DQ model and the marginal degrees of explanation for the group of spillover indices achieved within enhanced DQ regressions. Entries in bold face indicate the group of DQ misspecification indicators with highest marginal explanatory content. M is the forecast horizon of variance spillovers. DQ regressions for in-sample and ex ante analysis comprise 5176 and 3676 observations, respectively. For further annotations, see Table 3.

	In-sample BEKK estimates					Ex ante BEKK estimates				
	ewp	eq	noeq	mvp		ewp	eq	noeq	mvp	
M	1	1	1	1	5	1	1	1	1	5
	Dynamic Quantile diagnostics (p-values times 100)									
AR	0.026	0.006	90.28	31.17		19.24	15.19	63.07	70.23	
$\mathcal{R}_t^{(M,var)}$	0.048	0.037	80.29	13.75	13.90	0.467	8.438	11.30	22.76	12.26
$\mathcal{R}_t^{(M,cov)}$	0.000	0.058	55.38	6.678	7.310	0.031	16.23	56.03	63.28	48.50
$\mathcal{T}_t^{(M,var)}$	0.004	0.011	14.86	24.27	25.64	10.32	40.70	13.33	29.00	33.94
$\mathcal{T}_t^{(M,cov)}$	0.001	0.075	8.177	7.936	6.249	1.336	19.16	25.70	47.30	26.63
$\mathcal{N}_t^{(M,var)}$	0.003	0.045	34.14	21.00	21.89	2.443	23.21	8.391	37.76	58.45
$\mathcal{N}_t^{(M,cov)}$	0.001	0.222	30.75	3.732	5.611	0.005	19.57	12.83	35.79	37.04
$\mathcal{S}_t^{(M)}$	0.049	0.015	93.31	41.40	41.33	10.46	19.40	62.10	75.37	76.68
	Marginal $\sqrt{R^2}$ (times 100)									
AR	**7.056**	**7.493**	2.053	3.714		4.801	**5.041**	3.373	3.209	
$\mathcal{R}_t^{(M,var)}$	3.576	2.483	2.888	4.205	4.194	6.697	4.692	5.494	5.207	**5.775**
$\mathcal{R}_t^{(M,cov)}$	6.776	3.267	4.162	5.304	5.240	8.459	4.633	4.102	4.226	4.770
$\mathcal{T}_t^{(M,var)}$	4.996	3.502	4.969	3.524	3.460	4.179	2.048	5.539	4.871	4.697
$\mathcal{T}_t^{(M,cov)}$	6.186	2.948	**5.850**	5.166	5.337	6.339	4.221	5.528	4.767	5.585
$\mathcal{N}_t^{(M,var)}$	5.269	2.302	4.287	3.731	3.682	5.569	3.470	5.933	4.563	3.734
$\mathcal{N}_t^{(M,cov)}$	6.556	1.967	4.886	**5.704**	**5.438**	**9.280**	4.486	**6.092**	**5.264**	5.201
$\mathcal{S}_t^{(M)}$	0.607	0.074	0.780	0.322	0.348	2.474	1.568	1.832	1.124	0.964

Table 5: Inferential (p-values) and descriptive (marginal R^2) of the dynamic quantile regressions for the hit processes of VaR estimates with nominal coverage α =5.0%. For further annotations, see Table 4.

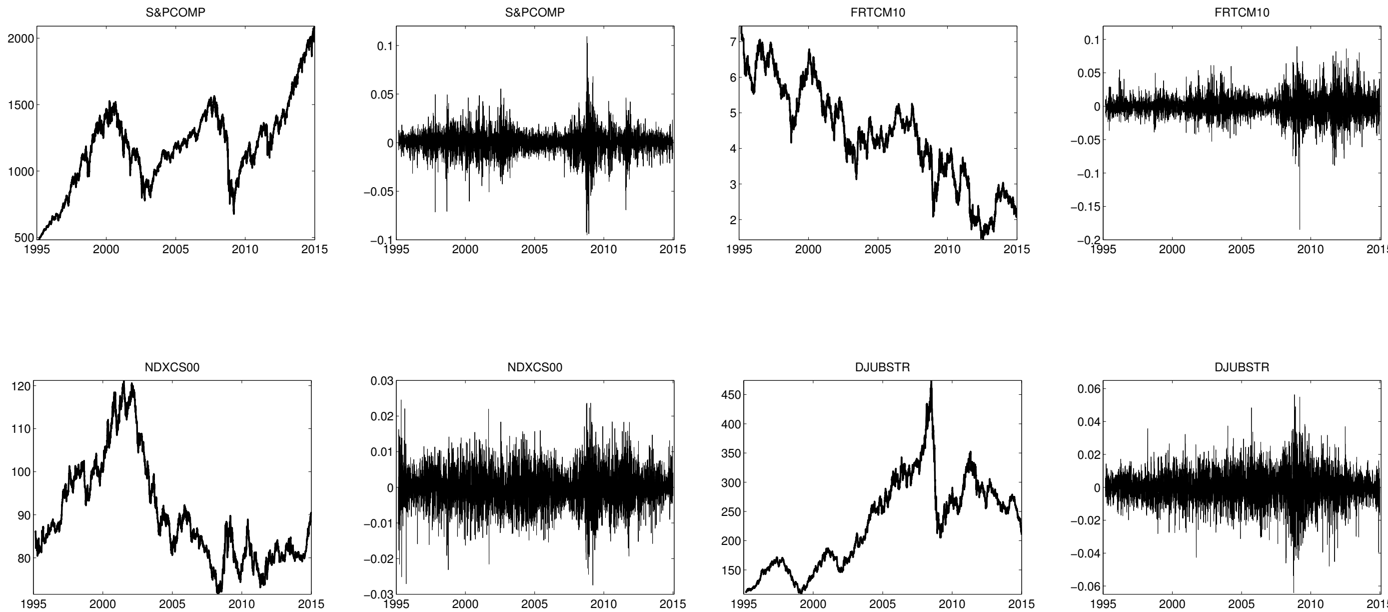
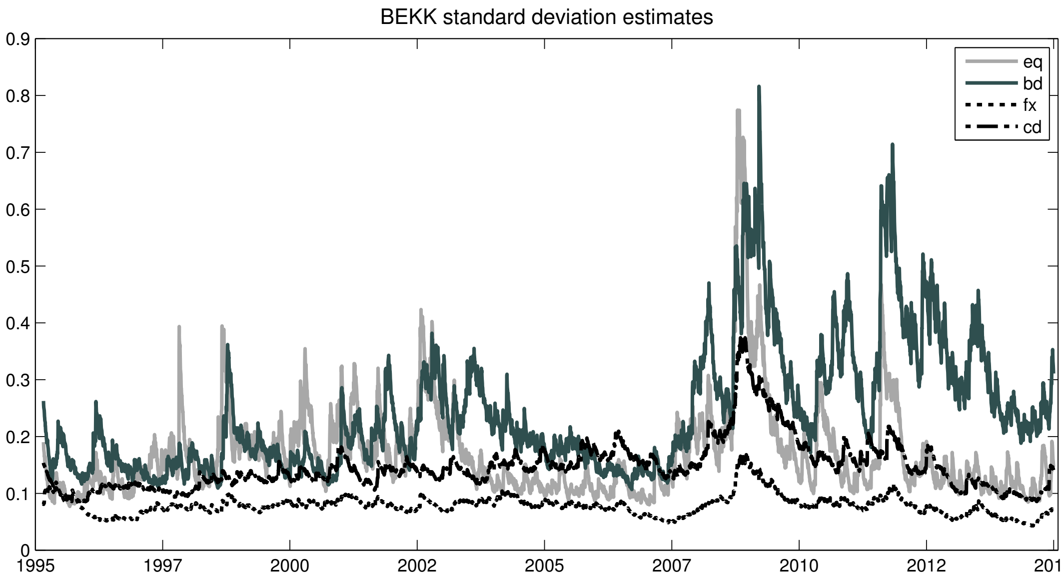
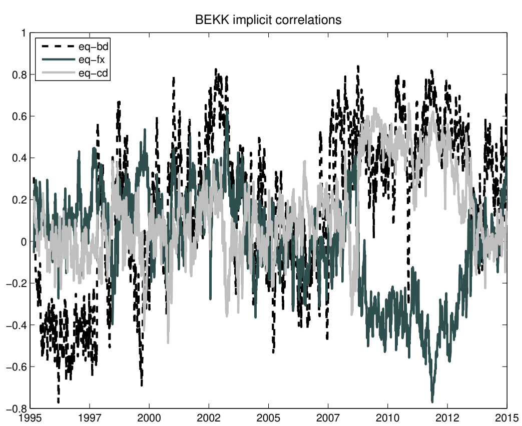


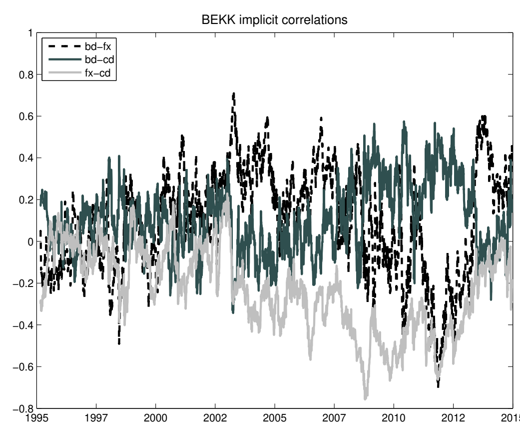
Figure 1: Price levels and log returns of the S&P 500 index (top left panel), the 10-year treasury bond yields (top right panel), the New York Board of Trade US dollar index futures (lower left panel) and the Dow-Jones/UBS commodity index (lower right panel). Sample from March 1, 1995, to December 31, 2014.



(a)



(b)



(c)

Figure 2: Estimated BEKK implicit correlations of the four asset classes: equity (eq), bonds (bd), foreign exchange (fx), and commodities (cd). Sample from March 1, 1995, to December 31, 2014.

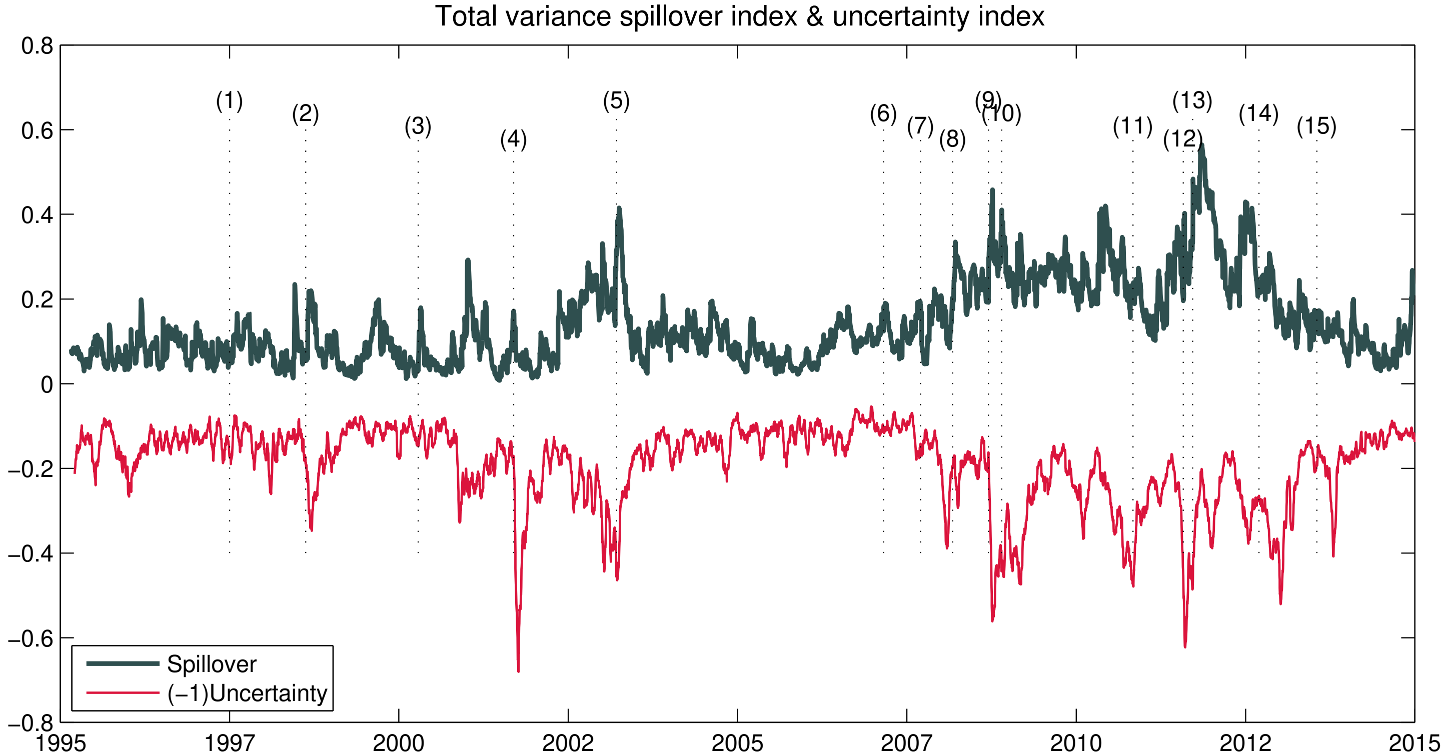


Figure 3: Total variance spillover index $\mathcal{S}_t^{(5)}$ and the 20-days moving average of the daily US Economic Policy Uncertainty Index (EPUI) of S. R. Baker, N. Bloom, and S. J. Davis. The EPUI is scaled (has the same standard deviation as the total variance spillover index) and reflected along the horizontal axis. See Table 2 for a listing of the events tagged in the plot.

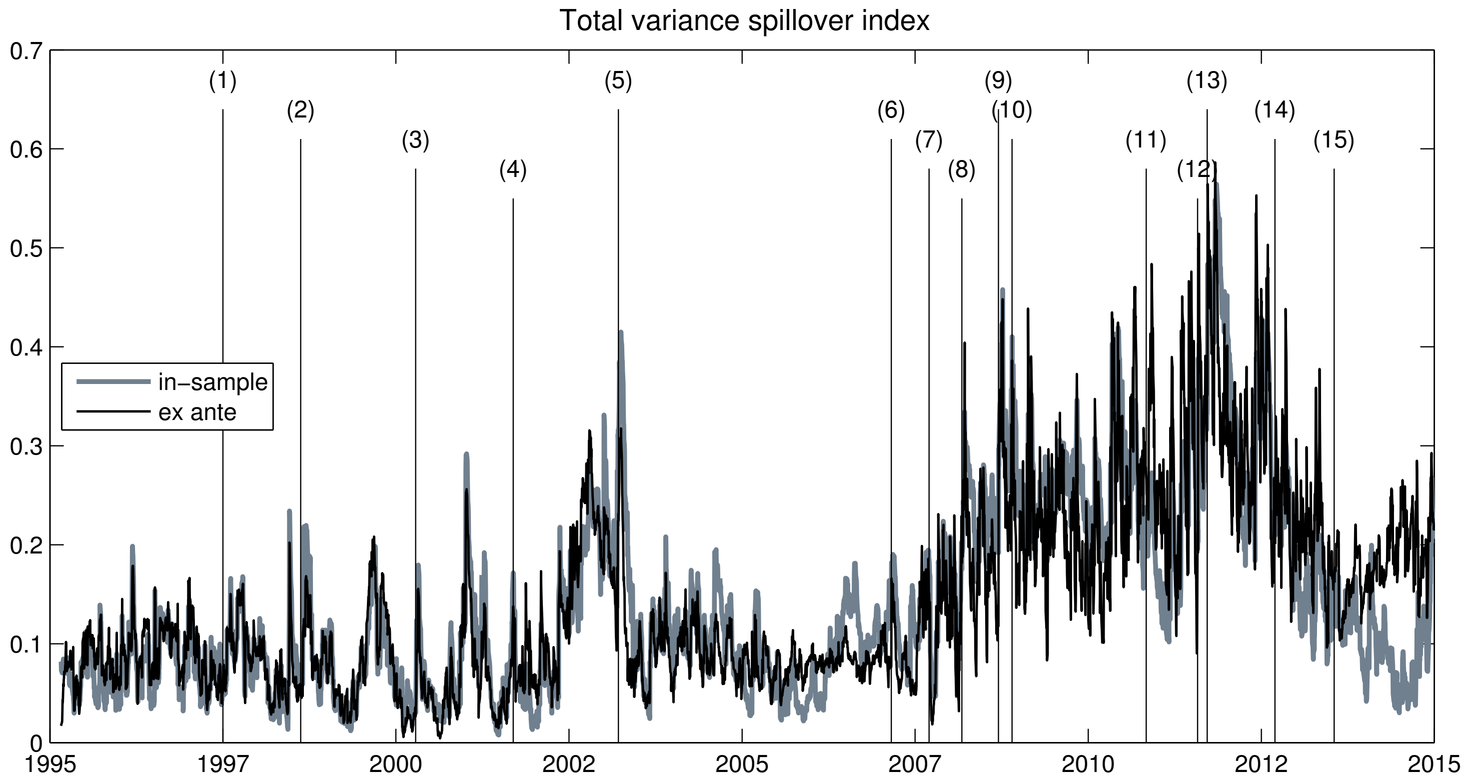


Figure 4: Total variance spillover index $\mathcal{S}_t^{(5)}$ of Figure 3 (light grey) and a fully ex ante total variance spillover index based on rolling window estimates (black line). See Table 2 for a listing of the events tagged in the plot.

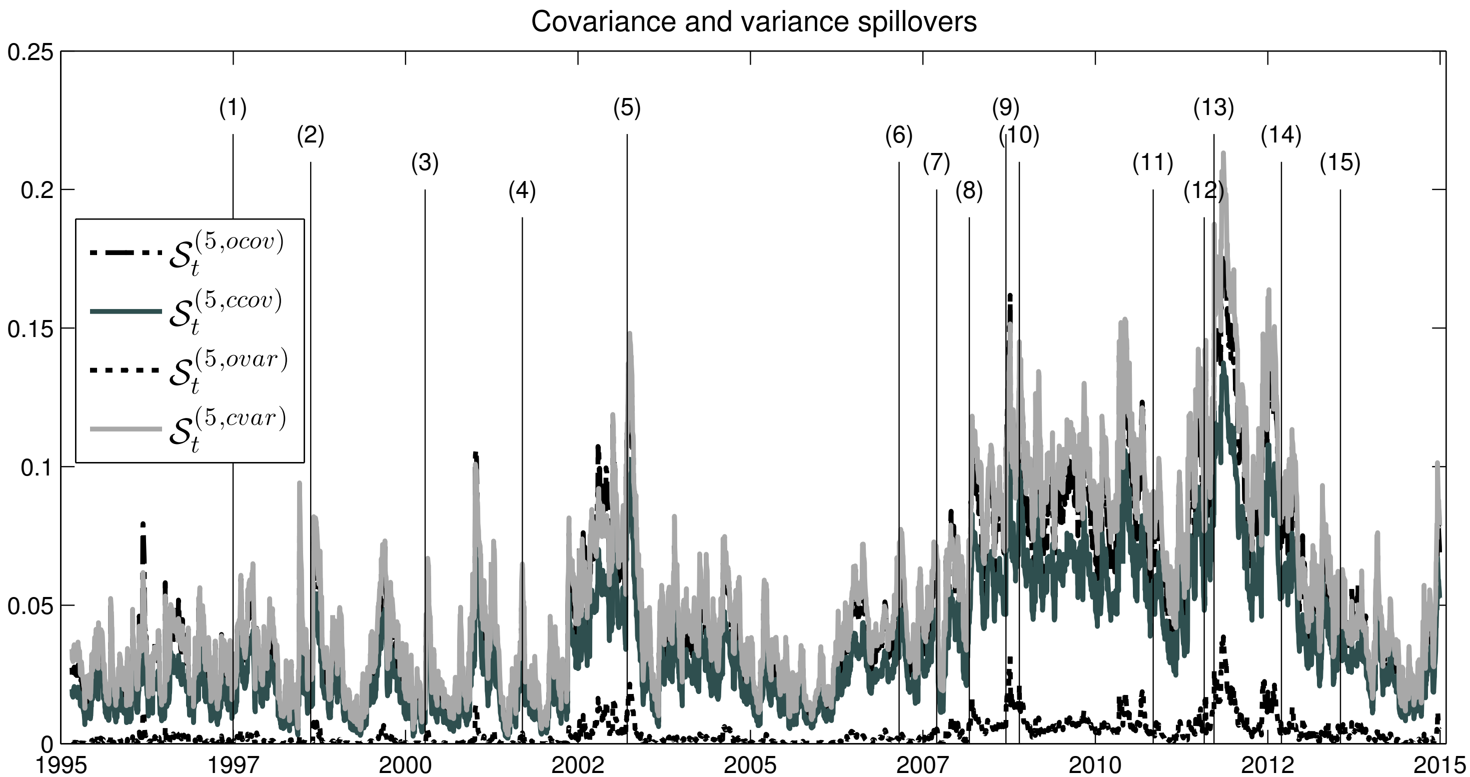


Figure 5: Own and cross variance and covariance spillovers. See Table 2 for a listing of the events tagged in the plot.

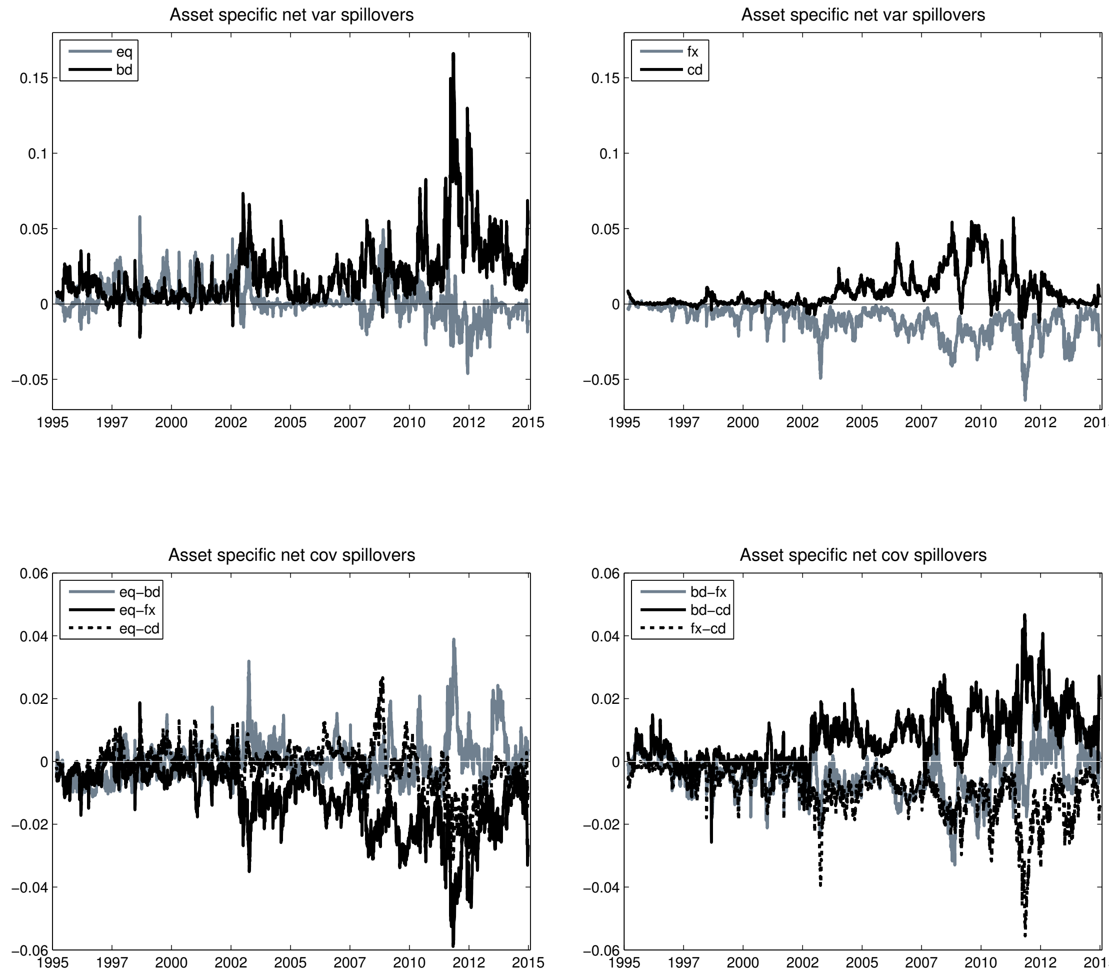


Figure 6: Net variance $\mathcal{N}_{t,i}^{(5,var)}$ (top panel) and net covariance $\mathcal{N}_{t,i}^{(5,cov)}$ (lower panel) spillovers aggregated per asset class: equity (eq), bonds (bd), foreign exchange (fx), and commodities (cd). Sample from March 1, 1995, to December 31, 2014.