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**Abstract**

In this note, we examine the effects of capital taxation on innovation and economic growth. We find that capital taxation has drastically different effects in the short run and in the long run. An increase in the capital income tax rate has both a *consumption* effect and a *tax-shifting* effect on the equilibrium growth rates of technology and output. In the long run, the tax-shifting effect dominates the consumption effect yielding an overall positive effect of capital taxation on steady-state economic growth. However, in the short run, the consumption effect becomes the dominant force causing an initial negative effect of capital taxation on the equilibrium growth rates. These contrasting effects of capital taxation at different time horizons may provide a plausible explanation for the mixed evidence in the empirical literature on capital taxation and economic growth.

**Keywords:** Capital taxation, economic growth, R&D, transition dynamics

**JEL classification:** H20, O30, O40

# 1 Introduction

In this note, we examine the effects of capital taxation on innovation and economic growth. In the literature of endogenous growth, one of the major issues is whether capital taxation stimulates or impedes growth. Earlier studies employing an AK-type endogenous growth model show that the impact of raising the capital tax rate on long-run economic growth is negative (Judd, 1985; Chamley, 1986; King and Rebelo, 1990; Rebelo, 1991; Jones *et al.*, 1993; Pecorino, 1993, 1994; Devereux and Love, 1994; Milesi-Ferretti and Roubini, 1998), although the quantitative magnitude could be negligibly small (Lucas, 1990; Stokey and Rebelo, 1995).[1] The intuition of this negative growth effect of capital taxation is that a higher capital tax rate discourages the accumulation of physical capital and is therefore detrimental to economic growth.

On the empirical side, the results are rather inconclusive. A number of empirical studies have found that capital taxation, such as corporate profit tax and capital gains tax, can be harmful to economic growth (see e.g., Lee and Gordon, 2005; Hungerford, 2010; Arnold *et al.*, 2011; Dahlby and Ferede, 2012), whereas other empirical studies have found a neutral or even positive effect of capital taxation on growth (see e.g., Mendoza *et al.*, 1997; Angelopoulos *et al.*, 2007; ten Kate and Milionis, 2015). Therefore, although the abovementioned theoretical prediction is consistent with some of the empirical studies, it seems to contrast other empirical findings in the literature.

While capital accumulation is undoubtedly an important engine of economic growth, technological progress driven by innovation and R&D also acts as an important driver for growth; see Aghion and Howitt (2009, p.109) for a discussion on data from OECD countries.[2] Recently, R&D-based growth models pioneered by Romer (1990) and Aghion and Howitt (1992) have been used to explore the interrelation between capital taxation, innovation and economic growth; see e.g., Lin and Russo (1999), Zeng and Zhang (2002), Haruyama and Itaya (2006) and Aghion *et al.* (2013). This note contributes to the literature by providing an analysis of both the short-run and long-run effects of capital taxation on innovation and economic growth within the seminal innovation-driven growth model in Romer (1990), which is a workhorse model in R&D-based growth theory. In our analysis, we consider different tax-shifting schemes. Specifically, we examine the growth effects of capital taxation with tax shifting from lump-sum tax and also labor income tax to capital income tax.

In the case of tax shifting from lump-sum tax to capital income tax, an increase in

[1] Other than focusing on the long-run growth effect, Frankel (1998) studies the dynamics of capital taxation during the transition process.

[2] Aghion and Howitt (2009, p.108) report that "TFP growth accounts for about two-thirds of economic growth in OECD countries, while capital deepening accounts for one third."

the capital tax rate leads to a *decrease* in the steady-state equilibrium growth rate via a *consumption* effect of capital taxation. Intuitively, a higher capital tax rate causes households to decrease their saving rate and increase their consumption rate, which in turn leads to an increase in leisure and a decrease in labor supply. Given that labor is a factor input for R&D, a smaller labor supply gives rise to a lower growth rate of technology, which in turn determines the long-run growth rates of output and capital.

In the case of tax shifting from labor income tax to capital income tax, an increase in the capital tax rate leads to an *increase* in the steady-state equilibrium growth rate via a *tax-shifting* effect of capital taxation. Intuitively, an increase in the capital income tax rate allows the labor income tax rate to decrease, which in turn leads to a decrease in leisure and an increase in labor supply. The larger labor supply gives rise to higher growth rates of technology, output and even capital despite the lower capital-investment rate caused by the higher capital tax rate. Although the previously mentioned consumption effect of capital taxation is also present, it is dominated by the tax-shifting effect in the long run. However, we find that the relative magnitude of these two effects becomes very different in the short run.

We calibrate the model to aggregate data in the US to provide a quantitative analysis on the dynamic effects of capital taxation on economic growth. We consider the case of tax shifting from labor income tax to capital income tax and find that an increase in the capital tax rate leads to a short-run *decrease* in the equilibrium growth rates of technology and output and a gradual convergence to the higher long-run growth rates of technology and output. The reason for these contrasting short-run and long-run effects is that the consumption effect of capital taxation is relatively strong in the short run. Intuitively, an increase in the capital income tax rate leads to a decrease in the steady-state equilibrium capital-technology ratio. Before the economy reaches this new steady-state capital-technology ratio, households drastically cut down their saving rate below its new steady-state level, which in turn increases their consumption rate substantially. This substantial increase in consumption leads to a substantial increase in leisure and a substantial decrease in labor supply, which in turn reduces temporarily the equilibrium growth rates of technology and output. In the long run, the effect of a lower wage-income tax rate becomes the dominant force and instead raises the supply of labor, which in turn increases the steady-state equilibrium growth rates of technology and output.

Our paper is most closely related to recent studies on taxation and economic growth in the R&D-based growth model. Zeng and Zhang (2002) show that the long-run growth rate is independent of labor income tax and consumption tax but decreasing in capital income tax. In contrast, Lin and Russo (1999) analyze how the taxation of different sources of capital income affects long-run growth and find that a higher capital income tax rate for innovative firms could be growth-enhancing if the tax system permits tax credits for R&D spending. Moreover, by focusing on the stability analysis of equilibria, Haruyama and Itaya (2006) also show that the growth effect of taxing capital income is positive when the economy exhibits indeterminacy. Although these two papers find that capital taxation and economic growth may exhibit a positive relationship, our paper departs from them in highlighting the contrasting dynamic effects of capital taxation on economic growth. More recently, Aghion *et al.* (2013) and Hong (2014) adopt a quality-ladder R&D-based growth model to investigate optimal capital taxation. Their primary focus, however, is on the normative analysis with respect to the Chamley-Judd (Chamley 1986; Judd 1985) result (i.e., the optimal capital tax is zero), while the present paper focuses on the positive analysis regarding the growth effect of capital taxation. Furthermore, their analysis does not deal with the case in which innovation is driven by R&D labor (e.g., scientists and engineers). When R&D uses labor as the factor input, we find that the effects of capital taxation are drastically different at different time horizons. This finding may provide a plausible explanation for the mixed evidence in the empirical literature on capital taxation and economic growth.

The remainder of this note is organized as follows. In Section 2, we describe the basic model structure. In Section 3, we investigate the growth effects of capital taxation. In Section 4, we calibrate the model to provide a quantitative analysis of capital taxation. Finally, some concluding remarks are discussed in Section 5.

# 2 The model

The model that we consider is an extension of the seminal workhorse R&D-based growth model from Romer (1990).[3] In the Romer model, R&D investment creates new varieties of intermediate goods. We extend the model by introducing endogenous labor supply and distortionary income taxes. In what follows, we describe the model structure in turn.

## 2.1 *Household*

The economy is inhabited by a representative household. Population is stationary and normalized to unity. The household has one unit of time that can be allocated between leisure

[3] In the case of extending the model into a scale-invariant semi-endogenous growth model as in Jones (1995), the long-run growth effect of capital taxation simply becomes a level effect. In other words, instead of increasing (decreasing) the growth rate of technology, capital taxation increases (decreases) the level of technology in the long run.

and production. The representative household's lifetime utility is given as:[4]

$$U = \int_0^\infty e^{-\rho t}[\ln C + \theta(1-L)]\, dt, \tag{1}$$

where the parameter $\rho > 0$ is the household's subjective discount rate and the parameter $\theta > 0$ determines the disutility of labor supply. The utility is increasing in consumption $C$ and decreasing in labor supply $L \in (0,1)$.

Two points regarding the utility function in equation (1) should be noted. First, to make our analysis tractable, the household is specified to have a quasi-linear utility function. In the quantitative analysis in Section 4, we will consider a more general utility function in order to examine the robustness of our results. Second, as pointed out by Hansen (1985) and Rogerson (1988), the linearity in work hours in the utility function can be justified as capturing indivisible labor.

The representative household maximizes its lifetime utility subject to the budget constraint:

$$\dot{K} + \dot{a} = ra + (1-\tau_K)r_K K + (1-\tau_L)wL - C - Z. \tag{2}$$

The variable $K$ denotes the stock of physical capital. The variable $a\,(= VA)$ denotes the value of equity shares of monopolistic firms, in which $A$ is the number of monopolistic firms and $V$ is the value of each firm. $w$ is the wage rate. $r$ is the real interest rate, whereas $r_K$ is the capital rental rate.[5] The rates of return on the two assets, physical capital and equity shares, must follow a no-arbitrage condition $r = (1-\tau_K)r_K$ in equilibrium. The policy instrument $Z$ is a lump-sum tax.[6] The other policy instruments $\{\tau_L, \tau_K\} < 1$ are respectively the labor and capital income tax rates.[7]

By solving the household's optimization problem, we can easily derive the typical Keynes-Ramsey rules:

$$\frac{\dot{C}}{C} = (1-\tau_K)r_K - \rho, \tag{3}$$

and also the optimality condition for labor supply, which is in the form of a horizontal labor supply curve given the quasi-linear utility function in (1):

$$w = \theta C/(1-\tau_L). \tag{4}$$

[4] For notational simplicity, we drop the time subscript.

[5] For simplicity, we assume zero capital depreciation rate.

[6] We allow for the presence of a lump tax simply to explore the implications of different tax-shifting schemes. Our main results focus on the more realistic case of $Z = 0$.

[7] In our analysis, we focus on the case in which $\tau_K > 0$; see for example Zeng and Zhang (2007) and Chu *et al.* (2016), who examine the effects of subsidy policies in the R&D-based growth model.

## 2.2 *Final goods*

There is a single final good $Y$, which is produced by combining labor and a continuum of intermediate goods, according to the following aggregator:

$$Y = L_Y^{1-\alpha} \int_0^A x_i^\alpha di, \tag{5}$$

where $L_Y$ is the labor input in final goods production, $x_i$ for $i \in [0, A]$ is the intermediate good of type $i$, and $A$ is the number of varieties of intermediate goods. The final good is treated as the *numeraire*, and hence in what follows its price is normalized to unity. We assume that the final goods sector is perfectly competitive. Profit maximization of the final goods firms yields the following conditional demand functions for labor input and intermediate goods:

$$L_Y = (1-\alpha)Y/w, \tag{6}$$

$$x_i = L_Y \left(\alpha/p_i\right)^{\frac{1}{1-\alpha}}, \tag{7}$$

where $p_i$ is the price of $x_i$ relative to final goods.

## 2.3 *Intermediate goods*

Each intermediate good is produced by a monopolist who owns a perpetually protected patent for that good. Following Romer (1990), capital is the factor input for producing intermediate goods, and the technology is simply a linear one-to-one function. That is, the production function is expressed as $x_i = k_i$, where $k_i$ is the capital input used by intermediate firm $i$. Accordingly, the profit of intermediate goods firm $i$ is:

$$\pi_i = p_i x_i - r_K k_i. \tag{8}$$

Profit maximization subject to the conditional demand function for intermediate goods firm $i$ yields the following markup-pricing rule:

$$p_i = \frac{r_K}{\alpha} > r_K. \tag{9}$$

Equation (9) implies that the level of price is the same across intermediate goods firms. Based on equation (7) and the production function $x_i = k_i$, we have a symmetric equilibrium among intermediate firms; i.e., $x_i = x$ and $k_i = k$. Then, we can obtain the following profit function of intermediate goods firms:

$$\pi_i = \pi = \frac{(1-\alpha)\alpha Y}{A}. \tag{10}$$

## 2.4 *R&D*

In the R&D sector, the familiar no-arbitrage condition for the value of a variety $V$ is:

$$rV = \pi + \dot{V}. \tag{11}$$

Equation (11) states that, for each variety, the rate of return on an invention must be equal to the sum of the monopolistic profit and capital gain (or loss) . As in Romer (1990), labor is the factor input of R&D. The innovation function of new varieties is given by:

$$\dot{A} = \phi A L_A, \tag{12}$$

where $\phi > 0$ is the R&D productivity parameter and $L_A$ denotes R&D labor. Given free entry into the R&D sector, the zero-profit condition of R&D is

$$\dot{A}V = wL_A \Leftrightarrow \phi AV = w. \tag{13}$$

## 2.5 *Government*

The government collects taxes, including capital income tax, labor income tax, and lump-sum tax, to finance its public spending. At any instant of time, the government budget constraint can be expressed as:

$$\tau_K r_K K + \tau_L wL + Z = G. \tag{14}$$

The variable $G$ denotes government spending, which is assumed to be a fixed proportion $\beta \in (0, 1)$ of final output such that

$$G = \beta Y. \tag{15}$$

## 2.6 *Aggregation*

Since the intermediate firms are symmetric, the total amount of capital is $K = Ak_i = Ak$. Given $x_i = k_i$, $x_i = x$, $k_i = k$, and $K = Ak$, the final output production function in equation (5) can then be expressed as:

$$Y = A^{1-\alpha} K^{\alpha} L_Y^{1-\alpha}. \tag{16}$$

After some calculations using equations (2), (6), (7), (11)-(14), and (16), we can derive the resource constraint in this economy:

$$\dot{K} = Y - C - G. \tag{17}$$

### 2.7 *Decentralized equilibrium and the balanced-growth path*

The decentralized equilibrium is a sequence of allocations $\{C, K, A, Y, L, L_Y, L_A, x, G\}_{t=0}^{\infty}$, prices $\{w, r, r_K, p_i, V\}_{t=0}^{\infty}$, and policies $\{\tau_K, \tau_L, Z\}$, such that at any instant of time:

a households maximize lifetime utility (1) taking prices and policies as given;

b competitive final goods firms choose $\{x, L_Y\}$ to maximize profit taking prices as given;

c monopolistic intermediate firms $i \in [0, A]$ choose $\{k_i, p_i\}$ to maximize profit taking $r_K$ as given;

d R&D firms choose $L_A$ to maximize profit taking $\{V, w\}$ as given;

e the market for final goods clears, i.e., $\dot{K} = Y - C - G$;

f the labor market clears, i.e., $L = L_A + L_Y$;

g the government budget constraint is balanced, i.e., $\tau_K r_K K + \tau_L w L + Z = G$.

The balanced growth path is characterized by a set of constant growth rates of all economic variables. Let $\gamma$ denote the growth rate of technology and a "~" over the variable denote its steady-state value. Along the balanced growth path, we have

$$\frac{\dot{K}}{K} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C} = \frac{\dot{w}}{w} = \frac{\dot{A}}{A} = \tilde{\gamma},\ \dot{L} = \dot{L}_Y = \dot{L}_A = 0.$$

## 3 Long-run growth effects of capital taxation

In this section we examine the growth effects of the capital tax rate. To maintain a constant proportion of government spending, raising the capital tax must be accompanied by a reduction in another tax. As revealed in equation (14), this can be either a reduction in the lump-sum tax (if it is available) or a reduction in the labor income tax (if the lump-sum tax is not available). In the analysis that follows, we deal with each of the two scenarios in turn.

## 3.1 *Tax shifting from lump-sum tax to capital income tax*

Equipped with the definition of the decentralized equilibrium in Section 2.7 and defining $\omega = w/A$, $c = C/A$ and $z = Z/A$, we can express the steady-state equilibrium conditions as follows:

$$\begin{aligned}
\tilde{\gamma} &= (1-\tau_K)\tilde{r}_K - \rho, && \text{(18a)}\\
\tilde{\omega} &= \theta\tilde{c}/(1-\tau_L), && \text{(18b)}\\
\tilde{L}_Y &= (1-\alpha)\tilde{x}^{\alpha}\tilde{L}_Y^{1-\alpha}/\tilde{\omega}, && \text{(18c)}\\
\tilde{x} &= \tilde{L}_Y(\alpha^2/\tilde{r}_K)^{1/(1-\alpha)}, && \text{(18d)}\\
\tilde{r} &= \phi\alpha\tilde{L}_Y, && \text{(18e)}\\
\tilde{r} &= (1-\tau_K)\tilde{r}_K, && \text{(18f)}\\
\tilde{\gamma} &= \phi\tilde{L}_A, && \text{(18g)}\\
\tilde{L} &= \tilde{L}_Y + \tilde{L}_A, && \text{(18h)}\\
\tilde{\gamma} &= (1-\beta)\tilde{x}^{\alpha-1}\tilde{L}_Y^{1-\alpha} - \tilde{c}/\tilde{x}, && \text{(18i)}\\
\tau_K\tilde{r}_K\tilde{x} + \tau_L\tilde{\omega}\tilde{L} + \tilde{z} &= \beta\tilde{x}^{\alpha}\tilde{L}_Y^{1-\alpha}, && \text{(18j)}
\end{aligned}$$

in which ten equations are used to solve ten unknowns $\tilde{\gamma}$, $\tilde{r}_K$, $\tilde{L}_Y$, $\tilde{L}_A$, $\tilde{L}$, $\tilde{\omega}$, $\tilde{c}$, $\tilde{x}$, $\tilde{r}$ and $\tilde{z}$. We briefly discuss how we obtain equations (18). (18a) is derived from the usual Keynes-Ramsey rule (3). (18b) is derived from the optimality condition for labor supply (4). (18c) and (18d) are respectively the demand functions for final-goods labor and intermediate goods, (6) and (7). (18e) is derived from inserting $\dot{V} = 0$ into the no-arbitrage condition in the R&D sector (11), and by using (6), (10) and (13). (18f) is the no-arbitrage condition of asset. (18g) is derived from the innovation function of varieties (12). (18h) is the labor-market clearing condition. (18i) is derived from dividing both sides of the resource constraint (17) by $A$ and using the condition $Ax = K$. (18j) is derived from dividing both sides of the government constraint (14) by $A$ and using the condition $G = \beta Y$.

We first use (18a), (18e), (18f)-(18h) to eliminate $\{\tilde{r}, \tilde{\gamma}, \tilde{r}_K\}$ and express $\{\tilde{L}_Y,\ \tilde{L}_A\}$ as functions of $\tilde{L}$ given by

$$\tilde{L}_Y = \frac{\tilde{L} + \rho/\phi}{1+\alpha},$$

$$\tilde{L}_A = \frac{\alpha\tilde{L} - \rho/\phi}{1+\alpha}.$$

These two equations indicate a positive relationship between $\{\tilde{L}_A, \tilde{L}_Y\}$ and $\tilde{L}$. Moreover, from the previous condition for $\tilde{L}_A$, we can derive the condition $\tilde{\gamma} = (\alpha\phi\tilde{L} - \rho)/(1+\alpha)$, which shows that the steady-state equilibrium growth rate of technology is increasing in $\tilde{L}$. Thus, we have

$$sgn\left(\frac{\partial\tilde{\gamma}}{\partial\tau_K}\right) = sgn\left(\frac{\partial\tilde{L}_A}{\partial\tau_K}\right) = sgn\left(\frac{\partial\tilde{L}}{\partial\tau_K}\right). \tag{19}$$

Accordingly, to investigate the growth effect of the capital tax rate, it is convenient to draw an inference from examining the effect of the capital tax rate on labor $\tilde{L}$.

We now derive an equilibrium expression of labor $\tilde{L}$. By using (8) and (9), we have $\pi = (\frac{1}{\alpha} - 1)\tilde{r}_K K/A$. This expression together with (10) implies that $r_K K = \alpha^2 Y$. Then, dividing both sides of (17) by $Y$ yields

$$\tilde{\gamma}\frac{K}{Y} = 1 - \beta - \frac{C}{Y}.$$

By inserting $C/Y = (1-\tau_L)(1-\alpha)/(\theta\tilde{L}_Y)$, which is derived from (4) and (6), and $r_K K = \alpha^2 Y$ into the above equation and using (18e), (18f) and (18g) along with the conditions for $\tilde{L}_Y$ and $\tilde{L}_A$, we can obtain the following equation with one unknown $\tilde{L}$:

$$\left[1 - \frac{\rho(1+\alpha)}{\alpha\phi(\tilde{L} + \rho/\phi)}\right]\alpha^2(1-\tau_K) = 1 - \beta - \frac{(1-\tau_L)(1-\alpha)(1+\alpha)}{\theta(\tilde{L} + \rho/\phi)}.$$

Simplifying this equation yields

$$\tilde{L} = \frac{1}{1-\Phi(\tau_K)}\left[\frac{1-\tau_L}{\theta} - \frac{\alpha(1-\tau_K)}{(1-\alpha)}\frac{\rho}{\phi}\right] - \frac{\rho}{\phi}, \tag{20}$$

where $\Phi(\tau_K) \equiv (\beta - \alpha^2\tau_K)/(1-\alpha^2)$ is a composite parameter and $\tau_L$ is an exogenous policy parameter. Then, from equation (20), we can obtain the following relationship:

$$\frac{\partial\tilde{L}}{\partial\tau_K} = -\frac{\alpha^2}{(1-\alpha^2)[1-\Phi(\tau_K)]^2}\left[\frac{1-\tau_L}{\theta} - \frac{\alpha(1-\tau_K)}{(1-\alpha)}\frac{\rho}{\phi} - [1-\Phi(\tau_K)]\frac{1+\alpha}{\alpha}\frac{\rho}{\phi}\right],$$

which can be further simplified to[8]

$$\frac{\partial\tilde{L}}{\partial\tau_K} = -\frac{\alpha[(1+\alpha)\tilde{L}_A + 2\alpha\rho/\phi]}{(1-\alpha^2)[1-\Phi(\tau_K)]} < 0. \tag{21}$$

From (19) and (21), we have established the following proposition:

[8] The following reasoning ensures that $1-\Phi(\tau_K) = [1-\beta-\alpha^2(1-\tau_K)]/(1-\alpha^2) > 0$. The steady-state consumption-output ratio is $C/Y = 1-\beta-\alpha^2(1-\tau_K) + \alpha^2(1-\tau_K)\rho/(\tilde{\gamma}+\rho)$. Therefore, $\lim_{\rho\to 0} C/Y = 1-\beta-\alpha^2(1-\tau_K)$. In other words, one can restrict $1-\Phi(\tau_K) > 0$ by appealing to the fact that $C/Y > 0$ for all values of $\rho$.

**Proposition 1** *In the case of tax shifting from lump-sum tax to capital income tax, raising the capital income tax rate reduces the steady-state equilibrium growth rate.*

Equation (19) is the key to understanding Proposition 1. It essentially says that the effect of the capital tax rate on long-run growth hinges on its effect on labor $\tilde{L}$. When the capital tax rate is higher, households tend to reduce their investment rate and increase their consumption rate. The increase in consumption raises leisure and reduces labor supply (by shifting up the horizontal labor supply curve). Therefore, a higher capital tax rate reduces the equilibrium levels of labor input, R&D labor and economic growth.

## 3.2 *Tax shifting from labor income tax to capital income tax*

A lump-sum tax is not a realistic description in most economies. In this subsection, we therefore set aside the possibility of a lump-sum tax and deal with the more realistic case in which a rise in the capital tax rate is coupled with a reduction in another distortionary tax. This kind of tax shifting has been extensively investigated in the literature on factor taxation; see e.g., Judd (1985), Chamley (1986), Niepelt (2004), Aghion *et al.* (2013) and Chen and Lu (2013). Under such a situation we drop $\tilde{z}$ from the model in this subsection. Thus, equation (18j) is rewritten as:

$$\tau_K \tilde{r}_K \tilde{x} + \tilde{\tau}_L \tilde{\omega} \tilde{L} = \beta \tilde{x}^{\alpha} \tilde{L}_Y^{1-\alpha}. \tag{22}$$

It is useful to note that in (22) the labor income tax rate $\tilde{\tau}_L$ becomes an endogenous variable because it needs to adjust in response to a change in the capital tax rate.

The macroeconomy is now described by (18a)-(18i) and (22) from which we solve for ten unknowns $\tilde{\gamma}$, $\tilde{r}_K$, $\tilde{L}_Y$, $\tilde{L}_A$, $\tilde{L}$, $\tilde{\omega}$, $\tilde{c}$, $\tilde{x}$, $\tilde{r}$ and $\tilde{\tau}_L$. By arranging (22) with (6), (16), (18c) and the condition $r_K K = \alpha^2 Y$, we can obtain

$$\tilde{\tau}_L = \frac{(\beta - \alpha^2 \tau_K)}{1-\alpha} \frac{\tilde{L}_Y}{\tilde{L}} = \left(1 + \frac{\rho}{\phi \tilde{L}}\right) \Phi(\tau_K),$$

where the second equality uses $\tilde{L}_Y = (\tilde{L} + \rho/\phi)/(1+\alpha)$. Using the above condition and (20), we can solve the two unknowns $\{\tilde{L}, \tilde{\tau}_L\}$ and obtain the following quadratic equation:

$$\frac{\phi}{\rho}\tilde{L}^2 - \left[\frac{\phi}{\rho\theta} - 1 - \frac{\alpha(1-\tau_K)}{(1-\alpha)[1-\Phi(\tau_K)]}\right]\tilde{L} + \frac{\Phi(\tau_K)}{[1-\Phi(\tau_K)]\theta} = 0.$$

This quadratic equation has two solutions, denoted as $\tilde{L}_1$ and $\tilde{L}_2$, which are given by:

$$\tilde{L}_1 = \frac{B(\tau_K) + \sqrt{B(\tau_K)^2 - 4\Phi(\tau_K)\phi/\{[1-\Phi(\tau_K)]\rho\theta\}}}{2\phi/\rho}, \tag{23a}$$

$$\tilde{L}_2 = \frac{B(\tau_K) - \sqrt{B(\tau_K)^2 - 4\Phi(\tau_K)\phi/\{[1-\Phi(\tau_K)]\rho\theta\}}}{2\phi/\rho}, \tag{23b}$$

where $B(\tau_K) \equiv \phi/(\rho\theta) - 1 - \alpha(1-\tau_K)/\{(1-\alpha)[1-\Phi(\tau_K)]\}$ is a composite parameter.[9]

To ensure that $\tilde{L}$ is positive, we assume that the set of parameters jointly satisfies the condition $B > \sqrt{4\Phi\phi/[(1-\Phi)\rho\theta]}$. Moreover, we restrict our analysis to the case where an increase in the capital tax rate is coupled with a decrease in the labor tax rate. By doing so, we can show that $\tilde{L}_1$ is the only possible solution to this system.[10] From (23a), we can derive the relationship:

$$\frac{\partial \tilde{L}_1}{\partial \tau_K} = \frac{\rho}{2\phi}\left\{\frac{\partial B}{\partial \tau_K} + \frac{B\partial B/\partial\tau_K + 2\phi\alpha^2/[(1-\alpha^2)(1-\Phi)^2\rho\theta]}{\sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}}\right\} > 0 \tag{24}$$

where $\partial B/\partial \tau_K = \alpha\left[1-\Phi+\alpha^2(1-\tau_K)/(1-\alpha^2)\right]/\{(1-\alpha)(1-\Phi)^2\} > 0$. The result in equation (24) leads us to establish the following proposition:

**Proposition 2** *In the case of tax shifting from labor income tax to capital income tax, raising the capital income tax rate increases the steady-state equilibrium growth rate.*

It would not be difficult to understand the intuition underlying the positive growth effect given that we have already shown the importance of equilibrium labor input on economic growth from previous discussion. In the present case, there are two conflicting effects on labor supply. The first is the consumption effect that we discussed in Proposition 1; i.e., raising the capital tax rate induces the households to lower the investment rate and increase the consumption rate, which in turn reduces labor supply. The second effect emerges from the channel of shifting taxes from labor income to capital income. A rise in the capital income tax rate leads to a reduction in the labor income tax rate, which tends to boost labor supply. In particular, this latter tax-shifting effect has a more powerful direct impact on the labor market so that it dominates the former one. As a result, the net effect is positive such that a higher capital income tax rate stimulates economic growth in the long run.

[9] For notational simplicity, we suppress the argument of $\Phi(\tau_K)$ and $B(\tau_K)$ in the following equations.

[10] Based on the definition of tax shifting, an increase in one tax rate should be coupled with a fall in another tax rate. In Appendix A, we will show that when $L = \tilde{L}_2$, to hold a constant proportion of the government spending, the labor tax rate actually increases in response to an increase in the capital tax rate. In this paper, we rule out this unrealistic case and only focus on the solution $L = \tilde{L}_1$.

# 4 Quantitative analysis

To examine the robustness of our results, we generalize the utility function as follows:

$$U = \int_0^\infty \left[ \ln C + \theta \frac{(1-L)^{1-\eta}}{1-\eta} \right] e^{-\rho t} dt, \tag{25}$$

where $\eta \geq 0$ determines the Frisch elasticity of labor supply. Equation (25) nests equation (1) as a special case when $\eta = 0$. The model features 7 parameters: $\{\rho, \alpha, \eta, \beta, \theta, \phi, \tau_K\}$. We consider the following standard parameter values or empirical moments in the literature. First, we set the discount rate to $\rho = 0.04$ and the capital share to $\alpha = 0.30$. Second, we set $\eta = 1.67$, which implies a Frisch elasticity of 1.2; see Chetty *et al.* (2011). Third, in line with Belo *et al.* (2013), the government spending ratio is set to $\beta = 0.20$. Fourth, to obtain a leisure time of two-thirds (i.e., $L = 1/3$), we set $\theta$= 1.17. Fifth, to generate a steady-state output growth rate of 1.92%, which is the per capita long-run growth rate of the US economy, we set $\phi$= 0.65. Finally, the benchmark value of the capital tax rate is set to $\tau_K = 0.36$; see for example Lucas (1990). The parameter values are summarized below.

Table 1: Calibrated parameter values

| $\rho$ | $\theta$ | $\eta$ | $\alpha$ | $\phi$ | $\beta$ | $\tau_K$ |
|---|---|---|---|---|---|---|
| 0.04 | 1.17 | 1.67 | 0.30 | 0.65 | 0.20 | 0.36 |

Figure 1 presents the growth effects of varying the capital income tax rate from 0 to 0.6. We can clearly see that, as the capital tax rate increases, the steady-state equilibrium growth rate increases. From this illustrative numerical exercise, we find that if the government raises the capital tax rate from the benchmark value of 36% to a hypothetical value of 50%, the steady-state equilibrium growth rate increases from 1.92% to 2.02%. The intuition can be explained as follows. Although an increase in the capital tax rate exerts a negative effect on economic growth by depressing capital accumulation, it also causes a fall in the labor income tax rate, which boosts labor supply and thus is beneficial to R&D and economic growth. In the long run, the latter effect dominates. Consequently, the steady-state equilibrium growth rate increases in response to a rise in the capital income tax rate.

In the rest of this section, we simulate the transition dynamics of an increase in the capital income tax rate. The dynamic system is presented in Appendix B. We consider the case of an increase in the capital income tax rate by one percentage point (i.e., from 36% to 37%).[11] First of all, the higher rate of capital taxation leads to a decrease in the investment

[11] In the case of a larger increase in the capital income tax rate, the qualitative pattern of the transitional paths of variables remains the same. Results are available upon request.

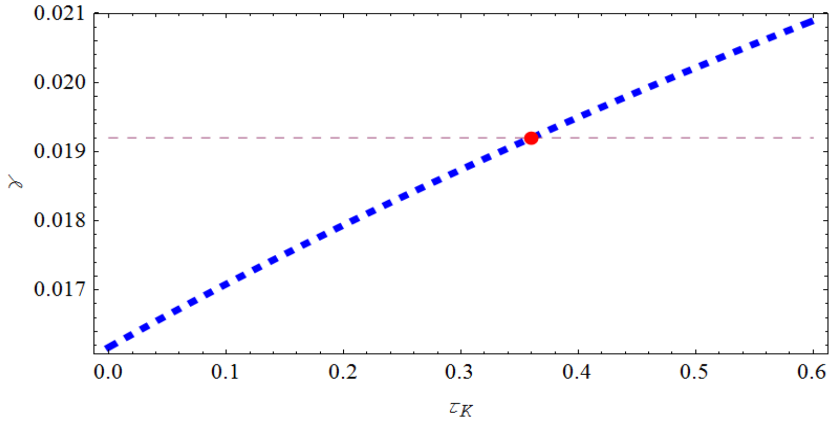


Figure 1: Long-run growth effect of capital taxation

rate and an increase in the consumption rate as shown in Figures 2 and 3, where investment $I = \dot{K}$.

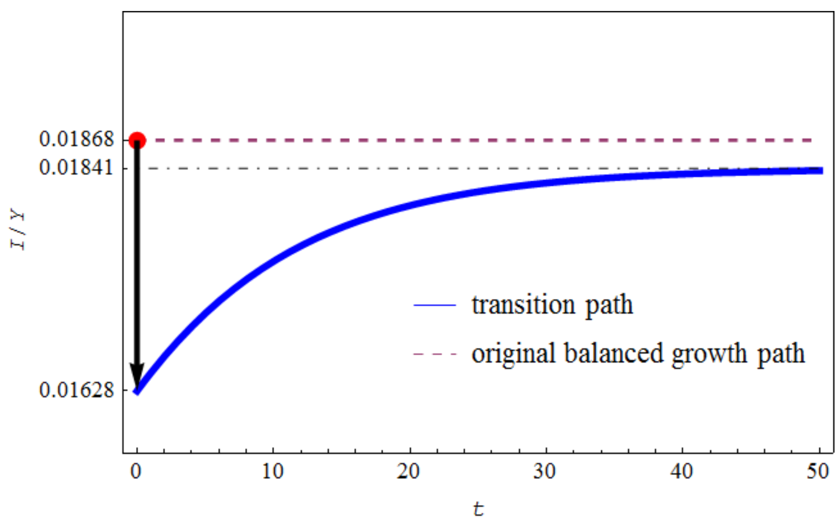


Figure 2: Transition path of the investment rate

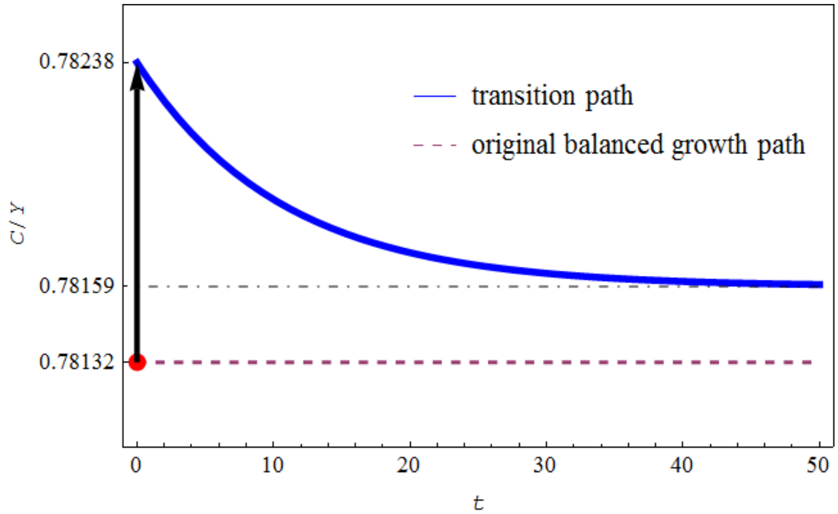


Figure 3: Transition path of the consumption rate

The lower capital-investment rate gives rise to an initial fall in the capital growth rate as shown in Figure 4, which contributes to an initial fall in the output growth rate as we will show later. The rise in the consumption rate increases leisure and decreases labor supply as shown in Figure 5. This decrease in labor supply reduces the amount of factor input available for R&D. As a result, the growth rate of technology also decreases initially as shown in Figure 6.

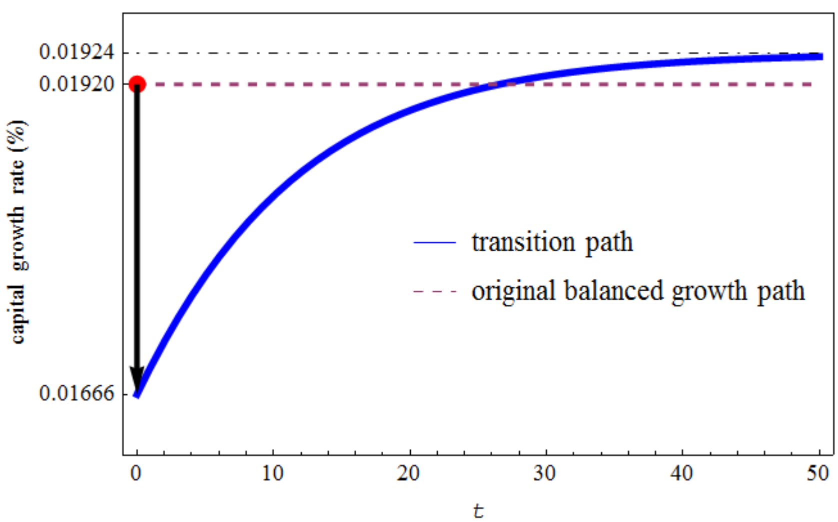


Figure 4: Transition path of the capital growth rate

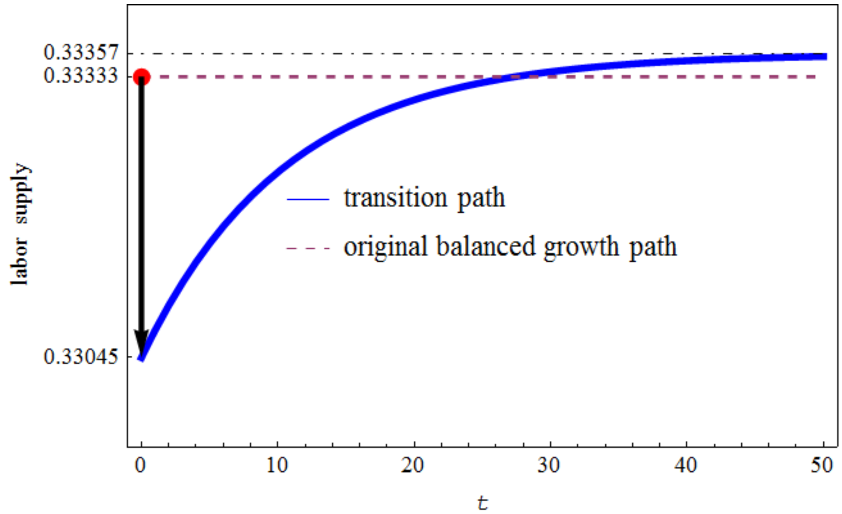


Figure 5: Transition path of labor supply

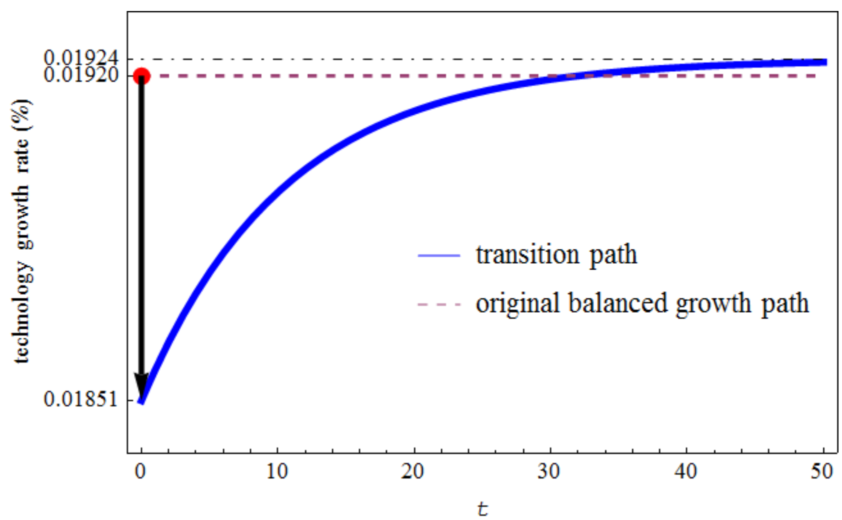


Figure 6: Transition path of the technology growth rate

Although tax shifting resulting from a higher capital income tax rate gives rise to a lower labor income tax rate, this effect is weak in the short run. However, it becomes a stronger force in the long run as shown in Figure 7. As a result, labor supply eventually rises above the original level, which in turn leads to a higher steady-state equilibrium growth rate of technology. Therefore, the initial drop in the growth rates of output and capital is followed by a subsequent increase. In the long run, the steady-state equilibrium growth rate of output is higher than the initial steady-state equilibrium growth rate as shown in Figure 8. To sum up, the reason for the contrasting short-run and long-run effects of capital taxation on economic growth is that the consumption effect is stronger (weaker) than the tax-shifting effect in the short (long) run.

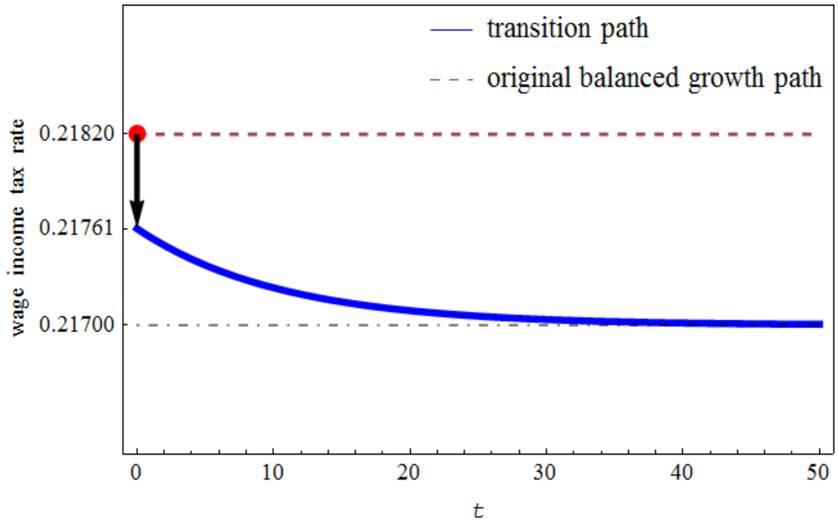


Figure 7: Transition path of the labor tax rate

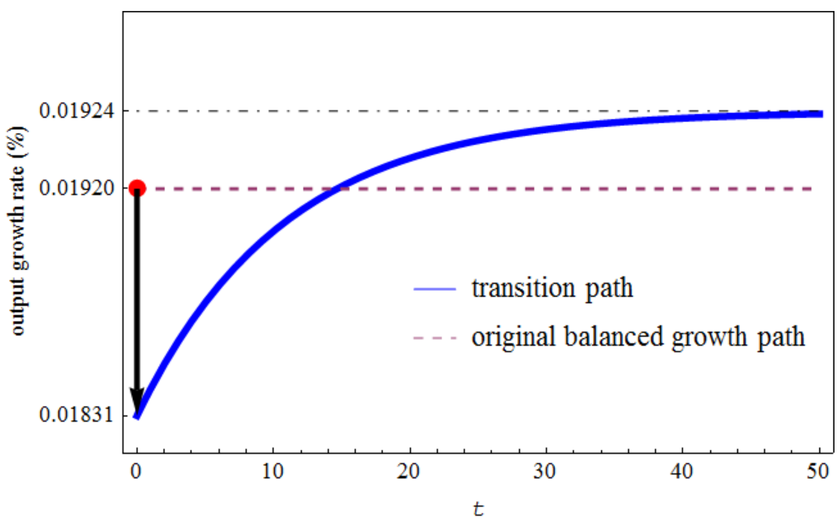


Figure 8: Transition path of the output growth rate

# 5 Conclusion

In this note, we have explored the short-run and long-run effects of capital taxation on innovation and economic growth. Our results can be summarized as follows. An increase in the capital income tax rate has both a positive tax-shifting effect and a negative consumption effect on innovation and economic growth. In the long run, increasing the capital tax rate has an unambiguously positive effect on the steady-state equilibrium growth rate because the positive tax-shifting effect strictly dominates the negative consumption effect. However, along the transitional path, increasing the capital tax rate first decreases the equilibrium growth rates of technology and output before these growth rates converge to a higher steady-state equilibrium level. These contrasting implications of capital taxation on economic growth suggest that a complete empirical analysis of capital taxation and economic growth needs to take into consideration the possibility that the effects of capital taxation change sign at different time horizons.

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## Appendix A

The system has ten equations, (18a)-(18i), and (22). After some calculations, we can derive the following expressions for $\tilde{L}$ and $\tilde{\tau}_L$:

$$\frac{\phi}{\rho}\tilde{L}^2 - \left[\frac{\phi}{\rho\theta} - 1 - \frac{\alpha(1-\tau_K)}{(1-\alpha)(1-\Phi)}\right]\tilde{L} + \frac{\Phi}{(1-\Phi)\theta} = 0, \tag{A1}$$

$$\tilde{\tau}_L = [1 + \rho/(\phi\tilde{L})]\Phi, \tag{A2}$$

where $\Phi \equiv (\beta - \alpha^2\tau_K)/(1-\alpha^2)$. Equation (A1) gives the two solutions for $\tilde{L}$:

$$\tilde{L}_1 = \frac{\rho\left(B + \sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}\right)}{2\phi}, \tag{A3}$$

$$\tilde{L}_2 = \frac{\rho\left(B - \sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}\right)}{2\phi}, \tag{A4}$$

where $B \equiv \phi/(\rho\theta) - 1 - \alpha(1-\tau_K)/[(1-\alpha)(1-\Phi)]$.

As mentioned in the main text, our analysis focuses on the case where the notion of tax shifting is sustained. That is, we impose the condition $\partial\tilde{\tau}_L/\partial\tau_K < 0$. We can then show that the condition $\partial\tilde{\tau}_L/\partial\tau_K < 0$ does not hold if $L = \tilde{L}_2$.

By plugging $\tilde{L}_2$ into (A2) and differentiating it with respect to $\tau_K$ yields:

$$\left.\frac{\partial\tilde{\tau}_L}{\partial\tau_K}\right|_{L=\tilde{L}_2} = \left[\frac{\alpha(1-\alpha+\alpha(1-\rho\theta/\phi))}{1-\alpha^2} + \frac{\partial\Lambda}{\partial\tau_K}\right]/2. \tag{A5}$$

where $\Lambda \equiv \sqrt{B^2 - 4\Phi\phi/[(1-\Phi)\rho\theta]}$ and

$$\frac{\partial\Lambda}{\partial\tau_K} = \frac{1}{\Lambda}\left(B\frac{\partial B}{\partial\tau_K} + \frac{2\alpha^2\phi}{(1-\alpha^2)(1-\Phi)^2\rho\theta}\right) > 0, \tag{A6}$$

where $\partial B/\partial\tau_K > 0$. It is clear from equation (A5) that $\partial\tilde{\tau}_L/\partial\tau_K|_{L=\tilde{L}_2}$ is positive, which contradicts the assumption of tax shifting. Therefore, we should rule out the possibility $\tilde{L} = \tilde{L}_2$.

## Appendix B (not for publication)

This appendix solves the dynamic system of the model under *tax shifting from labor income taxes to capital income taxes* ($Z = 0$). The set of equations under the model is expressed by:

$$\begin{aligned}
1/C &= \lambda && \text{(B1)}\\
\theta(1-L)^{-\eta} &= \lambda(1-\tau_L)w && \text{(B2)}\\
r &= (1-\tau_K)r_K && \text{(B3)}\\
\dot{C}/C &= (1-\tau_K)r_K - \rho && \text{(B4)}\\
wL_Y &= (1-\alpha)Y && \text{(B5)}\\
x &= L_Y(\alpha^2/r_K)^{1/(1-\alpha)} && \text{(B6)}\\
r_K K &= \alpha^2 Y && \text{(B7)}\\
A\pi &= \alpha(1-\alpha)Y && \text{(B8)}\\
r &= \frac{\pi}{V} + \frac{\dot{V}}{V} && \text{(B9)}\\
G &= \beta Y && \text{(B10)}\\
\beta Y &= \tau_K r_K K + \tau_L w L && \text{(B11)}\\
Y &= K^{\alpha}(AL_Y)^{1-\alpha} && \text{(B12)}\\
\dot{K} &= Y - C - G && \text{(B13)}\\
\dot{A} &= \phi A L_A && \text{(B14)}\\
V &= \frac{w}{\phi A} && \text{(B15)}\\
L &= L_Y + L_A && \text{(B16)}
\end{aligned}$$

in which 16 equations are used to solve 16 unknowns endogenous variables $\{C, L, A, K, L_Y, x, r_K, \pi, r, G, \tau_L, Y, \lambda, L_A, V, w\}$, where $\lambda$ denotes the Hamiltonian multiplier. Based on $K = Ax$, (B1), (B2), (B5) and (B12) and let $f = C/K$ be the ratio between consumption and capital, we can obtain:

$$L = 1 - [(1/(\theta f))(1-\tau_L)(1-\alpha)(1/L_Y)(L_Y/x)^{1-\alpha}]^{-1/\eta}. \tag{B17}$$

Based on (B5), (B7) and (B11), we have:

$$\tau_L = \frac{\beta - \alpha^2\tau_K}{1-\alpha}(\frac{L_Y}{L}). \tag{B18}$$

We now turn to deal with the transitional dynamics of the model. By using $x = K/A$, (B16), (B17) and (B18), we can infer the following expression:

$$L = L(x, f, L_Y; \tau_K), \tag{B19}$$

where

$$\frac{\partial L}{\partial x} = \frac{(1-\alpha)}{x(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}, \tag{B20a}$$

$$\frac{\partial L}{\partial f} = \frac{1}{f(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}, \tag{B20b}$$

$$\frac{\partial L}{\partial L_Y} = \frac{(\frac{\beta-\alpha^2\tau_K}{1-\alpha}\frac{L_Y}{L})/(1-\tau_L)+\alpha}{L_Y(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}, \tag{B20c}$$

$$\frac{\partial L}{\partial \tau_K} = \frac{(\alpha^2\frac{L_Y}{L})/((1-\alpha)(1-\tau_L))}{(-\frac{\eta}{1-L} + \frac{\tau_L}{(1-\tau_L)L})}. \tag{B20d}$$

Based on (B14) and (B15), we have:

$$\frac{\dot{V}}{V} = \frac{\dot{w}}{w} - \frac{\dot{A}}{A}. \tag{B20e}$$

From $K = Ax$, (B5) and (B12), we can further obtain:

$$\dot{w}/w = \dot{A}/A - \alpha\dot{L}_Y/L_Y + \alpha\dot{x}/x. \tag{B20f}$$

Additionally, substituting (B20f) into (B20e) yields:

$$\frac{\dot{V}}{V} = \alpha(\dot{x}/x - \dot{L}_Y/L_Y). \tag{B20g}$$

Combining (B3), (B5), (B7), (B8), (B9), (B12) and (B15), we can obtain:

$$(1-\tau_K)\alpha^2(\frac{L_Y}{x})^{1-\alpha} = \alpha\phi L_Y + \frac{\dot{V}}{V}. \tag{B21a}$$

Substituting (20g) into (B21a), (B21a) can be rearranged as:

$$\dot{L}_Y/L_Y = \phi L_Y + \dot{x}/x - (1-\tau_K)\alpha(\frac{L_Y}{x})^{1-\alpha}. \tag{B21b}$$

Based on $x = K/A$, we have the result:

$$\dot{x}/x = \dot{K}/K - \dot{A}/A. \tag{B21c}$$

Substituting $f = C/K$, (B10), (B13), (B14) and (B16) into (B21c), we have:

$$\dot{x}/x = (1-\beta)(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L - L_Y). \tag{B21d}$$

From (B21b) and (B21d), we can obtain:

$$\dot{L}_Y/L_Y = (1-\beta-\alpha(1-\tau_K))(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L - 2L_Y). \tag{B21e}$$

Moreover, from (B3), (B4), (B5), (B7) and (B12) we can obtain:

$$\dot{C}/C = (1-\tau_K)\alpha^2(\frac{L_Y}{x})^{1-\alpha} - \rho. \tag{22a}$$

Based on $f = C/K$, we have the following expression:

$$\dot{f}/f = \dot{C}/C - \dot{K}/K. \tag{B22b}$$

Substituting (B10), (B12) (B13) and (B22a) into (B22b), we can derive:

$$\dot{f}/f = ((1-\tau_K)\alpha^2 - (1-\beta))(\frac{L_Y}{x})^{1-\alpha} - \rho + f. \tag{B22c}$$

Based on (B19), (B21d), (B21e), and (B22c), the dynamic system can be expressed as:

$$\begin{aligned}
\dot{x}/x &= (1-\beta)(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L(x,f,L_Y;\tau_K) - L_Y), && \text{(B23a)}\\
\dot{f}/f &= ((1-\tau_K)\alpha^2 - (1-\beta))(\frac{L_Y}{x})^{1-\alpha} - \rho + f, && \text{(B23b)}\\
\dot{L}_Y/L_Y &= (1-\beta-\alpha(1-\tau_K))(\frac{L_Y}{x})^{1-\alpha} - f - \phi(L(x,f,L_Y;\tau_K) - 2L_Y). && \text{(B23c)}
\end{aligned}$$

Linearizing (B23a), (B23b) and (B23c) around the steady-state equilibrium yields:

$$\begin{pmatrix} \dot{x} \\ \dot{f} \\ \dot{L}_Y \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} x - \tilde{x} \\ f - \tilde{f} \\ L_Y - \tilde{L}_Y \end{pmatrix} + \begin{pmatrix} b_{14} \\ b_{24} \\ b_{34} \end{pmatrix} d\tau_K, \tag{B24}$$

where

$b_{11} = -(1-\alpha)(1-\beta)(\frac{L_Y}{x})^{1-\alpha} - \phi x \frac{\partial L(x,f,L_Y;\tau_K)}{\partial x}$,
$b_{12} = x(-1 - \phi(\frac{\partial L(x,f,L_Y;\tau_K)}{\partial f}))$,
$b_{13} = (1-\alpha)(1-\beta)(\frac{L_Y}{x})^{-\alpha} - \phi x(\frac{\partial L(x,f,L_Y;\tau_K)}{\partial L_Y} - 1)$,
$b_{14} = -\phi(\frac{\partial L(x,f,L_Y;\tau_K)}{\partial \tau_K})$,
$b_{21} = -(1-\alpha)((1-\tau_K)\alpha^2 - (1-\beta))(\frac{L_Y}{x})^{1-\alpha}(\frac{f}{x})$,

$b_{22} = f,$
$b_{23} = (1-\alpha)((1-\tau_K)\alpha^2 - (1-\beta))(\frac{L_Y}{x})^{-\alpha}(\frac{f}{x}),$
$b_{24} = -\alpha^2(\frac{L_Y}{x})^{1-\alpha} f,$
$b_{31} = -(1-\alpha)(1-\beta-\alpha(1-\tau_K))(\frac{L_Y}{x})^{2-\alpha} - \phi\frac{\partial L(x,f,L_Y;\tau_K)}{\partial x}L_Y,$
$b_{32} = -L_Y - \phi\frac{\partial L(x,f,L_Y;\tau_K)}{\partial f}L_Y,$
$b_{33} = (1-\alpha)(1-\beta-\alpha(1-\tau_K))(\frac{L_Y}{x})^{1-\alpha} - \phi(\frac{\partial L(x,f,L_Y;\tau_K)}{\partial L_Y}L_Y - 2L_Y),$
$b_{34} = \alpha L_Y(\frac{L_Y}{x})^{1-\alpha} - \phi(\frac{\partial L(x,f,L_Y;\tau_K)}{\partial \tau_K}L_Y).$

Let $\ell_1$, $\ell_2$ and $\ell_3$ be the three characteristic roots of the dynamic system. We do not analytically prove the saddle-path stability of the dynamic system; instead, we show that the dynamic system features two positive and one negative characteristic roots numerically. For expository convenience, in what follows let $\ell_1$ be the negative root and $\ell_2$, and $\ell_3$ be the positive roots. From (B24), the general solutions for $x_t$, $f_t$ and $L_{Y,t}$, can be described by:

$$\begin{aligned} x_t &= \tilde{x} + D_1 e^{\ell_1 t} + D_2 e^{\ell_2 t} + D_3 e^{\ell_3 t}, && \text{(B25a)} \\ f_t &= \tilde{f} + h_1 D_1 e^{\ell_1 t} + h_2 D_2 e^{\ell_2 t} + h_3 D_3 e^{\ell_3 t}, && \text{(B25b)} \\ L_{Y,t} &= \tilde{L}_Y + \frac{\ell_1 - b_{11} - b_{12}h_1}{b_{13}} D_1 e^{\ell_1 t} + \frac{\ell_2 - b_{11} - b_{12}h_2}{b_{13}} D_2 e^{\ell_2 t} + \frac{\ell_3 - b_{11} - b_{12}h_3}{b_{13}} D_3 e^{\ell_3 t}. && \text{(B25c)} \end{aligned}$$

where $h_1 = [(\ell_1 - b_{33})(\ell_1 - b_{11}) - b_{31}b_{13}]/[b_{32}b_{13} + b_{12}(\ell_1 - b_{33})]$, $h_2 = [(\ell_2 - b_{33})(\ell_2 - b_{11}) - b_{31}b_{13}]/[b_{32}b_{13} + b_{12}(\ell_2 - b_{33})]$, $h_3 = [(\ell_3 - b_{33})(\ell_3 - b_{11}) - b_{31}b_{13}]/[b_{32}b_{13} + b_{12}(\ell_3 - b_{33})]$, and $D_1$, $D_2$ and $D_3$ are undetermined coefficients.

The government changes the capital tax rate $\tau_K$ from $\tau_{K0}$ to $\tau_{K1}$ at $t$=0, based on (B25a)-(B25c), we employ the following equations to capture the dynamic adjustment of $x_t$, $f_t$, and $L_{Y,t}$:

$$x_t = \begin{cases} \tilde{x}(\tau_{K0}); & t = 0^- \\ \tilde{x}(\tau_{K1}) + D_1 e^{\ell_1 t} + D_2 e^{\ell_2 t} + D_3 e^{\ell_3 t}; & t \geq 0^+ \end{cases} \quad \text{(B26a)}$$

$$f_t = \begin{cases} \tilde{f}(\tau_{K0}); & t = 0^- \\ \tilde{f}(\tau_{K1}) + h_1 D_1 e^{\ell_1 t} + h_2 D_2 e^{\ell_2 t} + h_3 D_3 e^{\ell_3 t}; & t \geq 0^+ \end{cases} \quad \text{(B26b)}$$

$$L_{Y,t} = \begin{cases} \tilde{L}_Y(\tau_{K0}); & t = 0^- \\ \tilde{L}_Y(\tau_{K1}) + \frac{\ell_1 - b_{11} - b_{12}h_1}{b_{13}} D_1 e^{\ell_1 t} + \frac{\ell_2 - b_{11} - b_{12}h_2}{b_{13}} D_2 e^{\ell_2 t} + \frac{\ell_3 - b_{11} - b_{12}h_3}{b_{13}} D_3 e^{\ell_3 t}; & t \geq 0^+ \end{cases} \quad \text{(B26c)}$$

where $0^-$ and $0^+$ denote the instant before and after the policy implementation, respectively.

The values for $D_1$, $D_2$, and $D_3$ are determined by:

$$
\begin{aligned}
x_{0^-} &= x_{0^+}, && \text{(B27a)} \\
D_2 &= D_3 = 0. && \text{(B27b)}
\end{aligned}
$$

Equation (B27a) indicates that the level of intermediate goods remains unchanged at the instant of the policy implementation. Equation (B27b) is the stability condition which ensures that all $x_t$, $f_t$, and $L_{Y,t}$ converge to their new steady-state equilibrium. By using (B27a) and (B27b), we can obtain:

$$
D_1 = \tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}). \tag{B28}
$$

Inserting (B27b) and (B28) into (B26a)-(B26c) yields:

$$
x_t = \begin{cases} \tilde{x}(\tau_{K0}); & t = 0^- \\ \tilde{x}(\tau_{K1}) + (\tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}))e^{\ell_1 t}; & t \geq 0^+ \end{cases} \tag{B29a}
$$

$$
f_t = \begin{cases} \tilde{f}(\tau_{K0}); & t = 0^- \\ \tilde{f}(\tau_{K1}) + h_1(\tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}))e^{\ell_1 t}; & t \geq 0^+ \end{cases} \tag{B29b}
$$

$$
L_{Y,t} = \begin{cases} \tilde{L}_Y(\tau_{K0}); & t = 0^- \\ \tilde{L}_Y(\tau_{K1}) + \frac{\ell_1 - b_{11} - b_{12}h_1}{b_{13}}(\tilde{x}(\tau_{K0}) - \tilde{x}(\tau_{K1}))e^{\ell_1 t}; & t \geq 0^+ \end{cases} \tag{B29c}
$$