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Batabyal, Amitrajeet and Beladi, Hamid

Department of Economics, Rochester Institute of Technology,  
Department of Economics, University of Texas at San Antonio

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# **A Game Model of Competition for Market Share Between a New Good Producer and a Remanufacturer<sup>1</sup>**

by

**Amitrajeet A. Batabyal<sup>2</sup>**

and

**Hamid Beladi<sup>3</sup>**

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Department of Economics, Rochester Institute of Technology, 92 Lomb Memorial Drive, Rochester, NY 14623-5604, USA. Internet [aabgsh@rit.edu](mailto:aabgsh@rit.edu)

3

Department of Economics, University of Texas at San Antonio, One UTSA Circle, San Antonio, TX 78249-0631, USA. Internet [Hamid.Beladi@utsa.edu](mailto:Hamid.Beladi@utsa.edu)

# A Game Model of Competition for Market Share Between a New Good Producer and a Remanufacturer

## Abstract

We analyze the hitherto unstudied duopolistic interaction between a new good producer and a remanufacturer who compete for a *dominant share of the market* for a particular product. Each firm  $i$  spends  $d_i \geq 0$  on product development to sway consumers and this expenditure increases the likelihood that firm  $i$  captures a dominant market share. The revenue to each firm from obtaining a dominant market share is  $r > 0$ . Our analysis of this interaction leads to five results. First, given the two product development expenditures  $(d_1, d_2)$ , we specify the expected profit for each firm  $i$ . Second, we describe the function that characterizes each firm's best response function. Third, we compute the unique Nash equilibrium. Fourth, we show what happens to this Nash equilibrium when the revenue  $r$  increases. Finally, we study what happens to the Nash equilibrium when the remanufacturer's revenue from capturing a dominant market share is still  $r$  but the new good producer's revenue is  $\theta r$ , where  $\theta > 1$ .

**Keywords:** Duopoly, Market Share, Nash Equilibrium, New Good Producer, Remanufacturer

**JEL Codes:** L21, L13, D21

## 1. Introduction

### 1.1. Preliminaries

The term “remanufacturing” refers to an industrial process in which worn-out products are restored to like-new condition. As noted by Lund (1984), in remanufacturing, a series of industrial processes, often occurring in a factory environment, leads to the complete disassembly of a discarded product. Next, usable parts are cleaned, refurbished, and put into inventory. The product is then reassembled from the old parts—and sometimes with new parts as well—to produce a unit that is fully equivalent and sometimes superior in performance and expected lifetime to the original new product.

In the United States, remanufacturing has become important mainly because of two reasons. First, on the regulatory side, in an attempt to mitigate adverse environmental consequences, the Environmental Protection Agency (EPA) has taken some concrete steps. In this regard, it is worth highlighting the agency’s implementation in 1995 of the “Comprehensive Procurement Guideline.” *Inter alia*, this guideline sought to reduce waste and promote resource conservation by ensuring that materials collected in recycling programs would be used again to manufacture new products.<sup>4</sup> Second, there are the actual cost savings experienced by firms. In this regard, consider the following two examples from Mitra and Webster (2008). According to these researchers, in 1997, Ford avoided the disposal of more than 67,700 pounds of toner cartridges and hence saved \$180,000 in disposal costs. Similarly, in 1995, Union Carbide saved \$75,000 by avoiding disposal costs. Given the growing salience of remanufacturing from both an environmental and a practical perspective, a burgeoning literature has now begun to analyze the properties and the desirability of this industrial

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Go to <https://vsc.gsa.gov/green/files/CPG.pdf> for additional details. Accessed on December 10, 2015.

process from a variety of vantage points. We now briefly survey this literature.

### ***1.2 Review of the literature***

Lebreton and Tuma (2006) look at remanufacturing in the context of the disposal of 600,000 tons of used tires in Germany. On the basis of their analysis, these authors point to specific factors that are likely to raise remanufacturing rates in this nation. Ferrer and Swaminathan (2006) study the competition between an original equipment manufacturer (OEM) and an independent operator (IO). In a multi-period setting, the IO may intercept cores of products made by the OEM to sell remanufactured products in future time periods. These authors show that when the threat of competition increases, the OEM is more likely to completely utilize all available cores and offer the remanufactured product itself, at a lower price.

Mitra and Webster (2008) study the effects of government subsidies in a two-period model of competition in which a manufacturer makes and sells a new product in the first period but competes with a remanufacturer in the second period. They show that subsidy sharing creates incentives for the manufacturer to design a product that is more appropriate for remanufacturing and also to be more open to attempts to increase the return rate of end-of-life products. Atasu *et al.* (2008) demonstrate that in the presence of competition, remanufacturing can become a cogent marketing strategy in which a manufacturer can defend its market share via price discrimination.

Do remanufactured products cannibalize new product sales? Atasu *et al.* (2010) examine this question and point out that a product portfolio that includes both new and remanufactured products can make it possible for a firm to reach additional market segments and thereby block competition from new low-end products or third-party remanufacturers. Ferrer and Swaminathan (2010) characterize the optimal pricing and remanufacturing strategy of a monopolist that produces both

new and remanufactured goods.

New and remanufactured goods are often sold in the same market and therefore it makes sense to consider them together in the design of a product line. Aydin *et al.* (2015) adopt this perspective and propose a new methodology that enables them to compute the maximum profit and the market share associated with a product line. Finally, Shi *et al.* (2015) study the stability of the Nash equilibrium arising in the game between an OEM and a remanufacturer. They show that a higher willingness-to-pay (WTP) on the part of consumers can either strengthen or weaken the stability of the pertinent Nash equilibrium. Even so, a higher WTP always hurts the OEM and benefits the remanufacturer.

The various studies discussed in this section have certainly advanced our understanding of remanufacturing from both theoretical and empirical perspectives. Even so, to the best of our knowledge, there are *no* theoretical studies that have analyzed the competitive interaction between a new good producer<sup>5</sup> and a remanufacturer when the goal of both firms is to use expenditures on product development to capture a *dominant share of the market* in which they are operating.

Given this lacuna in the literature, in our paper, we analyze the duopolistic interaction between a new good producer and a remanufacturer who compete for a dominant share of the market for a particular product. Section 2 delineates the game model in which each firm  $i$  spends  $d_i \geq 0$  on product development to sway consumers and this expenditure increases the likelihood that firm  $i$  captures a dominant market share. In addition, the revenue accruing to each firm from capturing a dominant market share is  $r > 0$ . Next, given the two product development expenditures  $(d_1, d_2)$ ,

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The literature frequently uses the term “original equipment manufacturer” or OEM to refer to a new good producer.

section 3 states the expected profit for each firm  $i$ . Section 4 describes the function that characterizes each firm's best response function. Section 5 computes the unique Nash equilibrium of the game between the new good producer and the remanufacturer. Section 6 shows what happens to this Nash equilibrium when the revenue amount  $r$  increases. Section 7 examines what happens to the Nash equilibrium when the remanufacturer's revenue from capturing a dominant market share is still  $r$  but the new good producer's corresponding revenue is  $\theta r$ , where  $\theta > 1$ . Finally, section 8 concludes and then discusses two ways in which the research described in this paper might be extended.

## 2. The Theoretical Framework

Consider two firms—a new good producer and a remanufacturer—that are competing with each other for a dominant share of the market for a particular good such as a toner cartridge. In what follows, without loss of generality, the new good producer is firm 2 and the remanufacturer is firm 1. Each firm  $i$ ,  $i=1,2$ , spends  $d_i \geq 0$  on product development. The purpose of this expenditure is to influence consumers positively and to thereby make it more likely that this *ith* firm will end up capturing a dominant share of the underlying market. Note that we are using the notion of expenditures on product development broadly and hence this term includes expenditures on a number of things including, but not limited to, research and development (R&D), compliance with existing regulations, and advertising.

Given a pair of product development expenditure choices  $(d_1, d_2)$ , the probability that firm  $i$  captures a dominant share of the underlying market is given by  $d_i / (d_1 + d_2)$ . If neither firm incurs any expenditure on product development then we suppose that each firm captures a dominant market

share with probability  $1/2$ . The revenue accruing to each firm from obtaining a dominant market share is given by  $r > 0$  and the cost of spending  $d_i$  on product development is given simply by  $d_i$ .

We now specify the expected profit function for each firm  $i$ .

### 3. The Expected Profit Functions

Recall that the product development expenditure amounts  $(d_1, d_2)$  are given. Therefore, given the description of revenue and cost in the preceding paragraph, it is clear that firm  $i$ 's expected profit function  $\pi_i$  is given by

$$\pi_i(d_1, d_2) = \left\{ \frac{d_i}{d_1 + d_2} \right\} r - d_i, \quad i=1,2. \quad (1)$$

Our next task in this paper is to delineate the function that characterizes each firm's best response function.

### 4. The Best Response Functions

Firm 1 maximizes its expected profit function given in equation (1) above. Mathematically, it solves

$$\max_{\{d_1\}} \left[ \left\{ \frac{d_1}{d_1 + d_2} \right\} r - d_1 \right]. \quad (2)$$



The first order necessary condition for an optimum is

$$\frac{d_2 r}{(d_1 + d_2)^2} - 1 = 0. \quad (3)$$

Let  $d_1(d_2)$  denote firm 1's (remanufacturer's) best response function. Then, this best response function solves an equation derived from the optimality condition given in equation (3). Specifically, we get

$$\{d_1(d_2)\}^2 + 2d_1(d_2)d_2 + (d_2)^2 - d_2 r = 0. \quad (4)$$

Inspecting equation (4), it is clear that this is a quadratic equation and therefore it is not possible to write firm 1's best response function explicitly. However, since both firms are *symmetric* in our analysis, following a procedure similar to that we have followed thus far, it is straightforward to verify that firm 2's (new good producer's) best response function is given by

$$\{d_2(d_1)\}^2 + 2d_1 d_2(d_1) + (d_1)^2 - d_1 r = 0. \quad (5)$$

Let us now solve for the unique Nash equilibrium of the game between the new good producer and the remanufacturer.

## 5. The Nash Equilibrium

Inspecting equations (4) and (5) we see that the two best response functions are *symmetric* mirror images of each other. From this observation, it follows that these two best response functions must have a symmetric solution in which  $d_1 = d_2$  is the unique Nash equilibrium of the duopoly game

that we are analyzing. Given this finding, we can set  $d_1=d_2=d$  in either equation (4) or (5) and this substitution gives us

$$d^2+2d^2+d^2-dr=0. \tag{6}$$

The solution to equation (6) is  $d=r/4$  and hence the unique Nash equilibrium of the duopoly game between the new good producer and the remanufacturer has expenditures on product development given by

$$d_1^*=d_2^*=\frac{r}{4}. \tag{7}$$

In words, the above Nash equilibrium tells us that in the “capture dominant market share” game that we have been studying thus far, it is optimal for both the new good producer and the remanufacturer to spend one-quarter of the revenue obtained from capturing a dominant market share on product development. We now show what happens to this Nash equilibrium when the revenue amount  $r$  increases.

## 6. Impact of an Increase in Revenue

Inspecting equation (7), it is straightforward to confirm that as the revenue from the capture of dominant market share  $r$  rises, it makes sense for both the new good producer and the remanufacturer to spend *more* on product development. In other words, as the stakes of the prize (dominant market share) rise, it becomes more valuable for both firms to compete for it.

Thus far in our analysis, we have treated the new good producer and the remanufacturer as

symmetric players. Therefore, in the penultimate section of this paper, we consider the case in which the two firms under study are *asymmetric* players. Specifically, we now want to know what happens to the above Nash equilibrium when the remanufacturer's revenue from capturing a dominant market share is still  $r$  but the new good producer's corresponding revenue is  $\theta r$ , where  $\theta > 1$ .

## 7. The Asymmetric Nash Equilibrium

Because the revenue multiplicative factor for the new good producer or  $\theta > 1$ , the two best response functions that we shall now work with are asymmetric. In particular, firm 1's (remanufacturer's) best response function is still given by equation (4). However, firm 2's (new good producer's) best response function is now no longer given by equation (5) but instead by

$$\{d_2(d_1)\}^2 + 2d_1d_2(d_1) + (d_1)^2 - d_1\theta r = 0. \quad (8)$$

Let us subtract equation (8) from equation (4). This gives

$$d_1\theta = d_2. \quad (9)$$

Inspecting equation (9), it is clear that the Nash equilibrium of interest will now be asymmetric and, in addition, that we will have  $d_2 > d_1$  in this equilibrium. This last observation makes intuitive sense because the new good producer's revenue from capturing a dominant market share *exceeds* the corresponding revenue for the remanufacturer.

Using equation (9) to substitute for  $d_2$  in equation (4), we get

$$(d_1)^2 + 2(d_1)^2\theta + (d_1)^2\theta^2 - d_1\theta r = 0. \quad (10)$$

Simplifying equation (10), we see that the remanufacturer's optimal expenditure on product development satisfies

$$d_1^* = \frac{\theta r}{\theta^2 + 2\theta + 1} < \frac{r}{4}, \quad (11)$$

where the inequality follows from the fact that  $\theta > 1$ . Now, using equations (9) and (11), we see that the new good producer's optimal expenditure on product development satisfies

$$d_2^* = \frac{\theta^2 r}{\theta^2 + 2\theta + 1} > \frac{\theta^2 r}{\theta^2 + 2\theta^2 + \theta^2} = \frac{r}{4}, \quad (12)$$

where, once again, the inequalities arise because  $\theta > 1$ . Combining the results from equations (11) and (12), we see that in this asymmetric Nash equilibrium, the optimal product development expenditures for the new good producer (firm 2) and the remanufacturer (firm 1) satisfy

$$d_2^* > \frac{r}{4} > d_1^*. \quad (13)$$

In the symmetric Nash equilibrium of the “capture dominant market share” game studied in section 5, it was optimal for both the new good producer and the remanufacturer to spend one-quarter of the revenue obtained from capturing a dominant market share on product development. In contrast, when the new good producer's revenue from capturing a dominant market share *exceeds* the corresponding revenue for the remanufacturer, there is an asymmetric Nash equilibrium in which

the new good producer optimally spends *more* and the remanufacturer optimally spends *less* than the symmetric Nash equilibrium amount of one-quarter of the revenue or  $r/4$ . This completes our analysis of the competition for dominant market share between a new good producer and a remanufacturer.

## **8. Conclusions**

In this paper, we analyzed the game-theoretic interaction between a new good producer and a remanufacturer who competed for a dominant share of the market for a specific product. As a result of this analysis, we first specified the expected profit function for each firm, taking as given the two product development expenditures. Second, we described the function that characterized each firm's best response function. Third, we computed the unique Nash equilibrium. Fourth, we studied what happened to this Nash equilibrium when the revenue from the capture of a dominant market share increased. Finally, we examined an asymmetric Nash equilibrium with differential amounts for the revenue accruing to each firm from the capture of a dominant market share.

The analysis in this paper can be extended in a number of different directions. In what follows, we suggest two possible extensions. First, it would be useful to analyze a repeated game model of the competition between a new good producer and a remanufacturer in which the capture of a dominant market share is a dynamic phenomenon. Second, it would also be instructive to study a price leadership game in which a new good producer—and the price leader—interacts with multiple, heterogeneous remanufacturers who take the price leader's action as given. Studies that analyze these aspects of the underlying problem will provide additional insights into the industrial organization of markets with remanufacturing and the ways in which such markets ought to be viewed by policy makers seeking to promote the conservation of scarce resources.

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