Inequality Causes Recessions: A Fallout from Ramsey's Conjecture

Belanger, Gilles

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Gilles Bélanger*

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Abstract

Ramsey's conjecture implies that a market economy tends toward a politically impossible form of extreme inequality (with "the thrifty enjoying bliss and the improvident at the subsistence level"). Because political actions are not systematic, but arbitrary or random, the combination of market and political forces leads to instability, sometimes full-fledged crisis. I show how the mechanisms of debt relief, redistribution and the uncertainty related to them could boil down to a discount rate shock for an aggregate representative agent. Furthermore, to make this shock possible into a simple Real Business Cycle (RBC) model, I propose a two-capital setup, which provides an improved solution to the interest-rate-inelasticity issue than the usual investment adjustment costs. Finally, I show the model's generality by adding wage rigidity and inflation. Considering the simplicity of the general equilibrium model, results provide rich narratives for recessions.

JEL: E32, E37, E22, (E21, E24).

Keywords: Inequality, Credit crises, business cycles, discount factor heterogeneity.

*Ministère des Finances du Québec; 12, rue Saint-Louis, Québec (Québec) Canada, G1R 5L3; e-mail: gilles.belanger@finances.gouv.qc.ca.

1 Introduction

There will be a revolution long before that appends! That was my reaction to Ramsey (1928) and the premise of this research.[1] This paper investigates credit crises as a consequence of this conflict between the free market and power of numbers. It yields a model of business cycles by using debt to model the effects of Ramsey's conjecture and a random debt-relief variable for the power of numbers. The contribution of this paper is twofold: It provides an intuitive interpretation of discount factor shocks, which I discuss next, and a parsimonious, yet realistic, model for downturns driven by these shocks, which I discuss after.

That the most patient agents end up owning everything is now textbook, and there are implications.[2] First, differences in time preferences are politically unattractive as they breed inequality past the effects and functions of talent, hard work or filling a particular need at the right time. Second, number is power, and as the multitudes sink into poverty, they will use this power to regain lost resources (they may say lost dignity). Therefore, the free market equilibrium from Ramsey's conjecture conflicts with political forces and short of a balance between the two, the economy moves from crisis to crisis.

Many papers tackle credit crunches, but the contribution of this paper is parsimony.[3] Parsimony results from reducing the problem to a discount factor shock within the confines of a representative agent model. The discount factor is a stationary random variable by virtue of the government canceling the inevitable trend on the aggregate discount factor that come from Ramsey's conjecture. Such discount factor yields a discount factor shock. Gollier and Zeckhauser (2005) showed the aggregate discount factor contains a drift that results from the more patient agents increasing their share of aggregate consumption. I correct for this

[1] Becker (1980) has a classic treatment of Ramsey's conjecture.

[2] There is a long literature on the effects of discount factor distribution on wealth distribution. A good representative is Krusell and Smith (1998), who use partially uninsurable idiosyncratic risks and dying agents to reach a steady state and bound the discount factor. Nardi (2015) and Krueger, Mitman and Perri (2016) discuss the literature.

[3] van Treeck (2014) and more recently Krueger, Mitman and Perri (2016) review the literature on the link between inequality, credit and financial crises.

drift with a random variable that represents the government debt-relief actions and threats of them to relieve inequality. Individual discount factors do not change, mind you, but it is an aggregate phenomenon. Figure 2 in Subsection 2.1.1 illustrates an intuitive description of the process with debt relief. For its part, the threat of debt relief acts more like a Minsky moment by freezing the debt market.

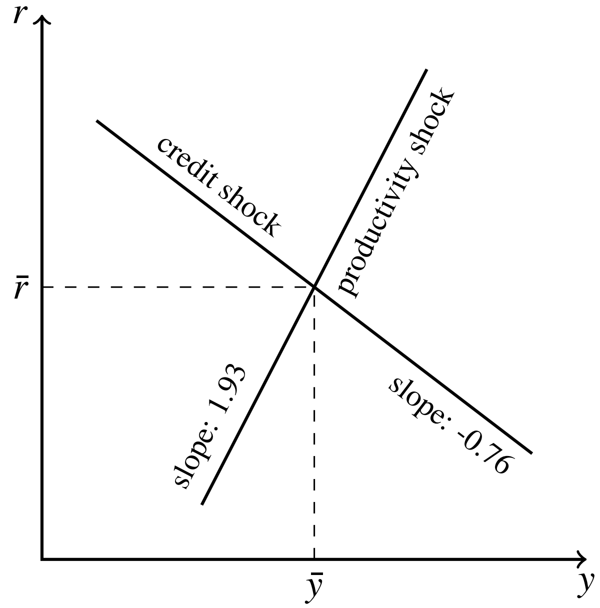
Note that wealth inequality differs from revenue inequality given the intense side effects from government alleviating a revenue inequality that may represent economically useful activities. Also, inequality is a complicated issue that may also result from market concentration, reduced unionization or other factors. This paper hinges on the inevitability of increasing inequality with a lax government, so Ramsey's conjecture is sufficient for my purpose. Treating the aggregate discount factor as a residue sidesteps these factors in the same way the TFP sidesteps the issues relating to the causes of productivity changes.

Next, I insert the discount factor in an RBC with two types of capital. The empirical relation between interest rates and capital has plagued the profession for a hundred years.[4] I avoid the issue by posing that some capital spending is imposed on the firm. One type of capital, I call structures, has low depreciation and is maintained exogenously at higher levels than may be optimal. The other, I call machinery, has high depreciation. The two-capital framework enables the model to behave intuitively when facing discount factor shocks, mostly by solving the issue relating to the marginal product of capital being rigid in the basic model.

The issue justifies capital or investment adjustment costs in DSGE models, but the two-capital framework may be more intuitive. First, it explains how some capital does not react to changes in interest rates. Second, high depreciating capital leads to intuitive responses to discount rate or credit shocks because the ratio of production to capital is then small enough to increase the volatility of the marginal product of capital. Third, this one ad hoc assumption replaces three. I use the two-capital setup instead of posing investment adjustment costs (of the change in investment spending), habit formation and Calvo prices.

[4] See Caballero (1999) for a survey.

Figure 1: First year impact of shocks on interest rate and log-production



Note: A negative credit shock is associated with a rise in interest rates, while a negative productivity shock is associated with a drop in interest rates. The intersection represents the steady state and the effects of the shocks are the first year effect from the log-linearized model.

As a bonus, changes in investments are restricted enough to push consumption in the same direction, thus consumption is procyclical when facing discount factor shocks. The wealth effect on consumption does not dominate the intertemporal substitution effect, but agents substitute consumption for leisure. It yields a mechanism that is observationally equivalent to the paradox of thrift.

The production function yields a downward sloping demand curve for capital while we usualy think of savings as upward sloping in interest rates as there are in the model. Credit shocks better explains the association of recession with tight credit than would a productivity shock. To see this, refer to Figure 1 that uses impulse response functions (IRFs) from productivity and discount factor shocks in the model to draw a pseudo supply and demand graph. The slopes of the curve are intuitive, negative for a preference shock (demand shock) and positive for a productivity shock (supply shock).[5] We can see how attractive a recession

[5]Look at a Coob-Douglas for intuition. By isolating out capital per worker, from the production function and the marginal product of capital, we have, using standard notation, $\ln(Y/L) = \ln(A) - (\alpha/(1-\alpha))(\ln(r+\delta) - \ln(\alpha)$. The equation states that unless there is a drop in productivity or capital destruction, neither of

from a discount rate shock is: the technology is intact, no capital was destroyed and the labor pool is still there, yet the economy is crashing. Such intuitive result necessitated high depreciation which other specification did not provide.[6] So if forcing firms to spend more on capital ends up pushing an issue aside, doing so is necessary to advance the discussion.

Finaly, Section 3 presents extensions to the model. The first extension accounts for wage rigidity by using prospective contracts as in Taylor (1979), but without the lags. The contracts generate persistence in average wages without affecting marginal wages. Since errors in predictions only lead to wealth transfers, they only affect the discount factor shocks' *meaning*, not the value of any variable in the model. Average wage is only an auxiliary variable that can be analyzed ex-post. The contracts are intuitive for a model that, unlike New Keynesian models, does not need wage rigidity to help the recession mechanism.

The second extension accounts for inflation by using the interest-rate-rigidity model. The model from Bélanger (2015) makes inflation dependent on the credit market. The initial justifications for the model comes from the number of stylized facts about inflation the model accounts for, but for our purpose, its low footprint and credit-condition driven inflation are the main justifications. Indeed, under an inflation-targeting central bank, results are robust to whether or not this model is included in the larger model. The inflation model yields only small differences from shocks compared to an inflation-less model. Furthermore, since recessions are not caused by demand here, but by credit constraint, it makes sense to use a model of inflation where deflation comes from a credit crunch, not lack of demand.

In fact, one strange aspect of this paper's model is how it advocates some form of non-Keynesian Keynesianism. Debt relief attenuates recessions, not government spending, but debt relief is a major side effect of government spending if it disproportionately helps the struggling class. In addition, as demand and tight credit are inversely proportional, both approaches predict deflation and inflation at about the same moments. In the end, the

which is evident in most recession, the recession must come with an increase in the cost of credit.

[6] Ireland (2004) discusses the tendency of estimated depreciation to be too high, which justifies its calibration in models. Furthemore, we can see from Long and Plosser (1983) how intuitive the results can get when we pose full depreciation.

model suggests that Keynes may have been right about everything... everything except for the reasons why he was right.

1.1 Historical evidence

Like Latin America going from failed land reform to failed land reform, or the Roman Plebes seceding to force debt cancellation, or the biblical Jubilee, the issue of debt relief has been a source of various crises since the earliest of recorded times. There is a periodicity to the issue. Land reforms, Plebs revolting on account of debt were almost cyclical events, so was the Jubilee by definition.

Think about what triggered the Great Depression. There is talk of the Smoot-Hawley tariff, but the size of the tariff and of the external sector only accounted for a small part of U.S. GDP. On the other hand, there was the German referendum on renouncing the whole of reparation debt coincident with the crash.[7] Because Allies owed about as much to the United States as German owed to the Allies, it was pretty much a debt Germans owed to the United States. And this time, sizes fit the story as reparation debt equalled 32 percent of the American GDP, or 25 percent if the Young Plan passed.[8] The threat of debt relief does sound like a good trigger for the Great Depression. Roosevelt getting off the Gold Standard, which erased a lot a debts, sounds like a good candidate for the cause of the recovery.

Can we analyze the Great Recession through the same lens? Housing threatens to push millions of Americans to put the keys of their house in the mailbox. Consumers and financial institutions are over-leveraged. George W. Bush is about to be replaced with someone likely to be less oligarchic.

[7] The dates: Black Thursday, October 24th; Black Monday, October 28th; Black Tuesday, October 29th; German nationalist amass enough signatures to force a referendum after a two-week campaign, October 29th; fall of Briand's government in France threatening the Young Plan on reparations, October 23th.

[8] The Young Plan, very much in the news at that time, proposed to reduce reparations to 112 billion gold marks (26.3 billion dollars) from 132 billion gold marks (33 billion dollars). The 1929 U.S. GDP was 103.6 billion dollars.

2 Model

The first subsection treats the consumer and discusses the intuition behind the discount rate shock. The second subsection treats the producer and discusses the two-capital setup. In a way, the model really starts at Subsection 2.1.3. If you are already convinced about the discount factor, you can skip to that subsection.

2.1 Consumers

When agents display heterogeneous discount factors, the representative agent displays a discount factor that increases with time.[9] This result derives from more-patient agents taking a bigger share of consumption at each period.

Pose that from time to time, the government enacts debt relief. The timing and size of debt relief are random, but restricted by a long-term political goal expressed as a steady state value of β for the discount factor of the representative agent. A steady state shares of consumption is embedded in the steady state value of β, and arises from an inequality level the government, or society, is comfortable with.[10]

Furthermore, the randomness of government actions completely drowns the deterministic movements of the aggregate discount factor Randomness coincides with counteracting the *natural* movements of the aggregate discount factor that arise from Ramsey's conjecture.

2.1.1 Illustration

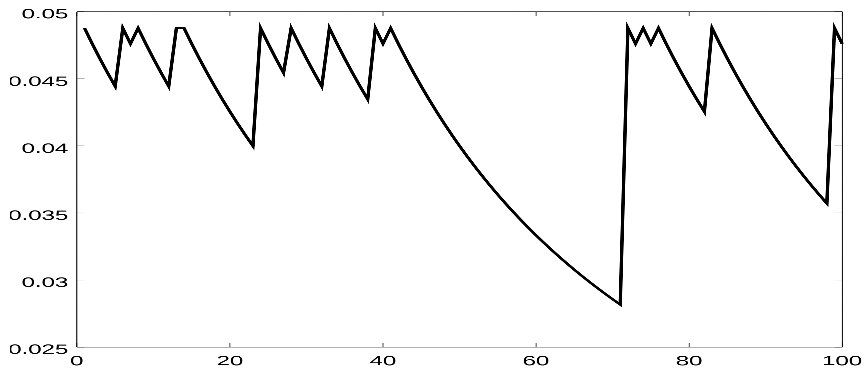
The probability of government intervention at each period is θ. Using an example from Gollier and Zeckhauser (2005), with $\mu = 0.05$, the mean rate of impatience in the population, $\gamma = 2$, relative risk aversion and $\eta = 1$, a distribution parameter, the decreasing $\beta(t)$,

$$\beta(t) = \frac{\mu\gamma^{-1} + t}{\mu\gamma^{-1} + t + \eta + \gamma},$$

[9]See Gollier and Zeckhauser (2005) for examples.

[10]Debt relief can take many forms other than actual debt relief. It can be redistribution programs that de facto reduce debt for the poorest people, spending programs that profits poor people more than rich people, or actions that create unexpected inflation.

Figure 2: Discount rate



Note: The curve represents the discount rate according to the CRRA case in Gollier and Zeckhauser (2005) with a 0.2 probability of resetting.

becomes

$$\mathrm{E}_{t-1}[\beta(t)] = (1-\theta)\frac{\mu\gamma^{-1}+t}{\mu\gamma^{-1}+t+\eta+\gamma} + \theta\nu(t).$$

for $\nu(t)$ which represents the size effect of the debt relief. In other words, the government enacts debt relief such that the long-term effects lead to a steady state of β. For $\mathrm{E}_{t-1}[\beta(t)] = \beta$, we have

$$\nu(t) = \frac{\beta}{\theta} - \frac{(1-\theta)(\mu\gamma^{-1}+t)}{\theta(\mu\gamma^{-1}+t+\eta+\gamma)}.$$

Figure 2 illustrates the example. However, government intervention would more be noisy given the rarity of let's-do-debt-relief-now moments, but instead a series of ad hoc actions taking all sorts of forms. Still, the figure illustrates well what happens. Here, discount rate shocks, leading to recessions, come back every now and then, but the longer the government lets the problem fester, the bigger they are. In this case, the rate decreases always in the same manner, but one can easily imagine times where regulations, or lack thereof, accelerates the inequality generating process making big recessions not a function of time, but of both time and regulations.

Unfortunately, modeling the process behind the aggregate discount rate is too difficult an undertaking. I leave this to future research though I have little hope. An empirical approach is best with the discount factor treated as a residue, like in the case for technology shocks.

2.1.2 The individual consumer

Here, we look at another effect of debt relief: the threat of debt relief. Pose an individual consumer j with the following utility function,

$$\sum_{s=0}^{\infty} \beta_j^s u(C_{j,t+s}, L_{j,t+s}),$$

where C and L represents consumption and labor, and β, the discount factor, and under the budget constraint,

$$B_{j,t} + X_{j,t} + C_{j,t} - W_t L_{j,t} + \Upsilon_t R_t B_{j,t-1} = 0,$$

where B are bonds and Υ accounts for the probability and size of debt relief. As a result, the probability affects the Euler equation,

$$\mathrm{E}_t \left[\beta_j \Upsilon_{t+1} \frac{u_c(C_{j,t+1}, L_{j,t+1})}{u_c(C_{j,t}, L_{j,t})} R_{t+1} \right],$$

creating the equivalent of a discount factor shock that affects every consumer symmetrically.

2.1.3 The representative consumer

Having discussed how debt relief, or the threat thereof, affects the aggregate discount factor, we will now simply pose the shock as is customary for productivity.

The representative consumer has a King-Plosser-Rebelo utility function in its most general form. The generality stems from robustness concerns as this form makes it easy to test the model's reaction to the different specialized forms of this utility function. Hence, the utility function is

$$\sum_{s=0}^{\infty} \beta^s \mathrm{e}^{b_{t+s}} \frac{(C_{t+s} v(L^* - L_{t+s}))^{1-\rho} - 1}{1-\rho},$$

where C and L represents consumption and labor. Time t discount factor, $\beta \mathrm{e}^{b_t}$, consist of the steady state discount factor augmented by the shock term b_t. Debt relief will affect the economy through b_t. Function v is defined as

$$v(L^* - L_t) = \exp\left(\frac{(L^* - L_t)^{1-\frac{1}{\eta}} - 1}{1 - \frac{1}{\eta}} \right).$$

Note that there is no scale parameter as it did not influence the results.

From optimization, we have the Euler equation,

$$\mathrm{E}_t\left[\beta\exp\{b_{t+1}\}\left(\frac{C_{t+1}}{C_t}\right)^{-\rho}\left(\frac{v(L^*-L_{t+1})}{v(L^*-L_t)}\right)^{1-\rho}R_{t+1}\right],$$

where b_t is the discount factor shock, and wages are

$$W_t = C_t(L^*-L_t)^{-\frac{1}{\eta}}.$$

2.2 Production

On top of the equations of Subsection 2.1.3, the production side in this subsection supplements the model.

I use the two-capital setup because the basic RBC does not yield predictable movements of interest rates to a preference shock. The goal of this assumption is to reduce the dimensionality of the model by replacing the many assumptions in Christiano, Eichenbaum and Evans (2005) type models with one. Investment or capital adjustment costs, sometimes with variable capital utilization, were initially used to correct the interest-rate-inelasticity issue. Investment or capital adjustment costs create a strong kink on real interest rates in the opposite direction at the first period. That needed habit formation in the consumer and Calvo prices to smooth things out. Yes, Cobb-Douglas is the problem, but habit formation with investment adjustment costs and other bells and whistles, may not be the best the solution.

Pose that firms over-invest in structures to satisfy regulations, like building codes, security codes and so on, and societal imperatives, like pretty furniture and whatnot. By treating the spending as a forced fix cost, we mimic the behavior seen in the economy. Long-term projects are not responsive to changes in interest rates, and this assumption yields that behavior, albeit in an artificial way.[11]

Goods consists of consumption and two types of investments produced with a Cobb-

[11] See Caballero (1999) for a survey.

Douglas production function, while each capital accumulates additively,

$$Y_t = M_{t-1}^{\gamma\alpha} S^{(1-\gamma)\alpha} (A_t L_t)^{1-\alpha},$$
$$M_t = (1-\delta_m) M_{t-1} + X_t - \delta_s S,$$
$$Y_t = C_t + X_t,$$

where Y represents total production, C, consumption, X, investments, S, structures, M machinery, A, labor augmenting technology and L, labor. Structures, S, are maintained at their steady state level at all time by investing $\delta_s S$ to keep them from depreciating. The rest of the investments, $X_t - \delta_s S$, goes into machinery.[12] In addition, we can retrieve total capital by defining variable K, as $K_t = M_t + S$.

We can now derive the first order conditions on the producer side which yield wages and real interest rates,

$$W_t = (1-\alpha)\frac{Y_t}{L_t},$$

and

$$R_t = \gamma\alpha\frac{Y_t}{M_t} + 1 - \delta_m.$$

[12]Incidentally, we could write $Y_t = M_{t-1}^{\gamma\alpha} S^{(1-\gamma)\alpha}(A_t L_t)^{1-\alpha}$ if $S_t \geq S$ and $Y_t = 0$ if $S_t < S$ to make S_t a choice.

3 Extensions

I chose these extensions because they have a low footprint on the model as they barely alter the results. The first yields an average wage that is different from the marginal wage. The second yields the nominal interest rate and inflation.

3.1 Extension 1: the labor market

The subsection argues that wage rigidity ought not to affect marginal behavior because new employees receive the market's marginal cost. Haefke, Sonntag, and Rens (2013) empirically support the argument, and find strong aggregate wage rigidity, but much flexibility in the wages of new workers. Gains or losses incurred by employers and employees coming from of wrong predictions during contract negotiations are simple rents or dead-weight losses that do not alter the macroeconomy by much. They are akin to simple wealth transfers between employers and employees, which only changes the interpretation of the discount factor. The discount factor is computed as a auxiliary statistic, so the model is otherwise left unchanged. An RBC model without it would yield the same results, but for the labor share and the average wage.

The Appendix compares such contracts with Calvo type staggered wage contracts. Subsection 5.2 comments on the principal results, while the Appendix presents them in full.

In the predetermined wage model, agents negotiate forward-looking wage contracts. Exponentially decreasing weights describe the proportion of contracts at time t that were negotiated at time $t-s$. Decisions stem from marginal cost and utility. The usual maximization yields marginal wages such that the market-clearing price equals supply (marginal utility) and demand (marginal product),

$$W_t^* = f_{n,t} = -\frac{u_{n,t}}{u_{c,t}}.$$

As for mean wages, they come from

$$W_t = \frac{1-\omega}{1-\omega^{\Omega+1}} \left(W_t^* + \sum_{i=1}^{\Omega} \omega^i \mathrm{E}_{t-i} \left[W_t^* \right] \right), \tag{1}$$

where $\omega \in [0, 1)$ and $\Omega \in \mathbf{N}^+$. For computational purposes, the length of time the contracts can go, Ω, is finite. Otherwise, the contracts have an exponential form akin to those of the Calvo setup.

Wage rigidity can explain persistence in wages as well as the actual existence of long-term contracts. But wage rigidity in our model does not need to create the kind of collateral damages new Keynesians need.

As mentioned before, since Taylor wages are not transfer neutral, they affect the aggregate discount factor. Thankfully, this endogeneity is irrelevant because the Taylor contract model is a satellite model in that it does not feedback into the larger model and because the discount factor is a residue. In other words, whatever happens with average wages will be correlated with the aggregate discount factor, but that will not matter.

For these wealth transfers to have real effects outside the discount factor, they would have to be through a bankruptcy channel from weaker firms when employers pay higher than equilibrium average wages. The effects of wage rigidity may combine, during a recession, with the effects of credit frictions. Both these effects put together, a labor and a financial accelerator, can better explain how small shocks can have large effects than any of them alone. This is quite outside the scope of this paper, though.

3.2 Extension 2: the financial market

The model uses the "Interest Rate Rigidity" model of inflation. Bélanger (2015) showed that in a world where recessions are associated with tight credit, interest rate rigidity is sufficient to explain a series of stylized facts about inflation. As for models with discount factor shocks, but within the New Keynesian framework, reader should read Smets and Wouters (2003), or more recent variants.

Interest rate rigidity is well documented.[13] It affects one market, whereas price rigidity affects a multitude of them. Markets that suffer from price rigidity may see its effects compensated by flexible price markets taking advantage of gains so created, and so nullifying rigidity in the aggregate. This will not happen in the market for interest rate. But it seems so obvious that the central bank sets nominal interest rates that we do not think about what they have to do to get the interest rate they want. They need to do market operations or threaten to do them, hence affecting real interest rates not nominal ones directly. With this model, we pose that the central bank directly affects only real rates even though they target the nominal ones.

In addition, this model of inflation only changes the results of a real model by mildly reducing the effect of shocks on real interest rates, then on real variables. The real version of the model shares the same properties, if not the same scale, thus keeping the propagation channels for real variables intact.

This is how the model works. The financial market adds two additional agents, a central bank and a private bank. The private bank sets nominal interest rates while the central bank influence real interest rates, pursuant to its inflation target. Switching from a sole private bank to a number of private banks acting in perfect competition leads to the same results.

First, the model uses the ex-post definition of real interest rates (in log form, so $r = log(R)$ where R is the rate plus one),

$$r_t = i_{t-1} - \pi_t. \tag{2}$$

The use of the ex-post Fisher equation supposes that the model pins down inflation by pinning down real interest rates. Pinning down real interest rates is done by using a production function homogeneous in capital and labor, like the production functions macroeconomists use anyway. The producers see the ex-post real interest rate from what they get after they

[13]There are two aspects of interest rate rigidity. Kobayashi (2008) discusses interest rate pass-through, which points to incomplete pass-through from the central bank's key rate to market rates. de Bondt, Mojon and Valla (2005) discuss bank rate rigidity, which points to incomplete pass-through from market rates to lending rates. In a macroeconomic context, most studies combine interest rate and price rigidity. See Teranishi (2015) for a recent example.

paid wages. The ex-ante one comes from their expectations of the ex-post ones. They price according to the inflation and expected inflation they know because, as usual, we suppose they know the full model.

Second, a central bank uses a Taylor Rule, but for real interest rates,

$$r_t = r_t^* + \mu \left(\pi_t - \pi_t^* \right), \tag{3}$$

where r^* comes from the model. Writing the Taylor rule in real terms avoids second order issues in the optimization.

Third, a private bank sets nominal interest rates under quadratic adjustment costs to nominal interest rates (Rotemberg pricing in nominal rates) and quadratic costs to real rates distortion,

$$\min_{i_t} \left\{ \mathrm{E}_t \left[\sum_{s=0}^{\infty} \beta^s \left(\left(r_{t+1+s} - r_{t+1+s}^* \right)^2 + \theta \left(i_{t+s} - i_{t-1+s} \right)^2 \right) \right] \right\}.$$

Optimization by the private bank leads to the following evolution equation for nominal interest rates (the proof is in Bélanger (2015)),

$$\Delta i_t = \beta \mathrm{E}_t \Delta i_{t+1} + \frac{1}{\theta} \left(\frac{\mu}{1+\mu} \right)^2 \left(\mathrm{E}_t \left[r_{t+1}^* + \pi_{t+1}^* \right] - i_t \right). \tag{4}$$

As shown in Subsection 5.3, removing the financial module from the model does not influence results much. It has a low footprint. Consequently, though it is a nominal rigidity model (in interest rates), it participates in the New Classical tradition with its small impact on an RBC model. Furthermore, monetary policy in the model only dampens variables reaction from its dampening of the real interest rate's reaction, which answers a common critique of DSGE model by making the channels of propagation easy to follow.

3.2.1 Inserting the model as a financial wedge

The distortion created by the central bank appears as a financial wedge inside the Euler equation. The marginal return to capital equation is unmodified, while the Euler equation becomes

$$\mathrm{E}_t \left[\beta \exp\{b_{t+1}\} \left(\frac{C_{t+1}}{C_t} \right)^{-\rho} \left(\frac{v(L^* - L_{t+1})}{v(L^* - L_t)} \right)^{1-\rho} R_{t+1}^* \right].$$

Table 1: Parameter values

Category	Description	Symbol	Value
Preference	Discount factor	β	0.995
Preference	Relative risk aversion	ρ	3
Preference	Frisch elasticity	η	0.5
Production	Share of capital in Cobb-Douglas	α	0.35
Production	Aggregate depreciation	δ	0.025
Production	Machinery depreciation	δ_m	0.25
Production	Capital type share	γ	0.5
Taylor wages	Wage decay	ω	0.8
Taylor wages	Wage decay periods	Ω	16
Monetary	Steady state inflation target	$\bar{\pi}^*$	0.005
Monetary	Steady state real interest rate	$\bar{r}^*$	0.005
Monetary	Monetary policy tightness	μ	0.5
Monetary	Adjustment cost	θ	1

4 Findings

This section presents results from the full model, including the two extensions. Except for average wages, nominal interest rates and inflation, differences between the model with and without the extensions are no bigger than small measurement errors.

Table 1 presents the parameter values used in simulations. They are all conventional except for machinery depreciation, which comes from NIPA numbers. Note that the depreciation parameter for structure is

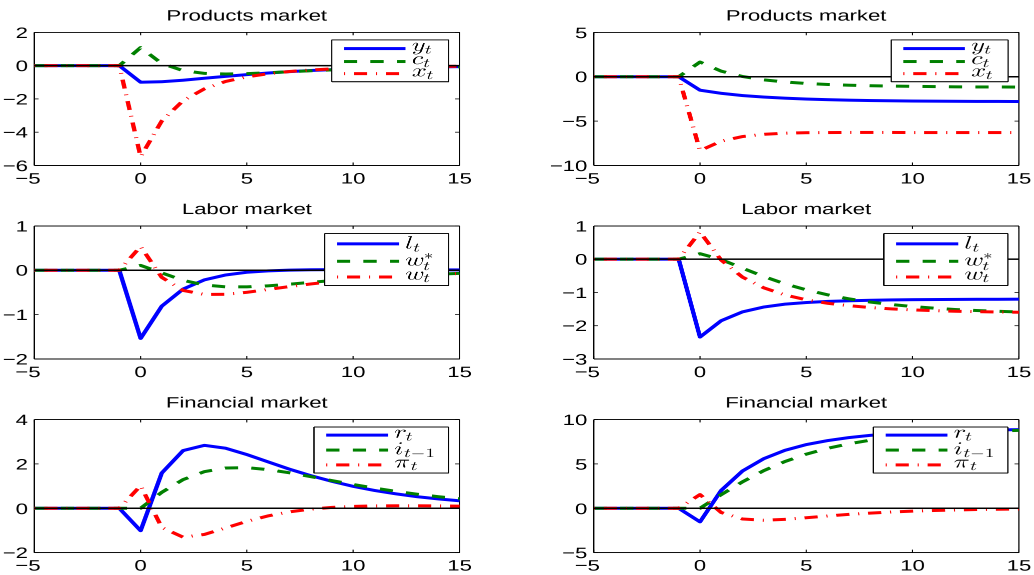
$$\delta_s = \frac{(\beta^{-1} - 1 + \delta)(1 - \gamma)}{1 - \frac{\gamma(\beta^{-1}-1+\delta)}{\beta^{-1}-1+\delta_m}} - \beta^{-1} + 1 \ \simeq 0.01,$$

which is close to NIPA numbers for structures.

You will find full results in the Appendix. In the Appendix, Figures A1 to A6 show the responses from each shock. They also figure as the pale lines on Figures A9 to A17, which compare the model with alternative specifications and present all shock variations in one page each.

Overall, the model suggests that cycles come from discount rate shocks while growth is

Figure 3: Surprise discount rate shock



Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

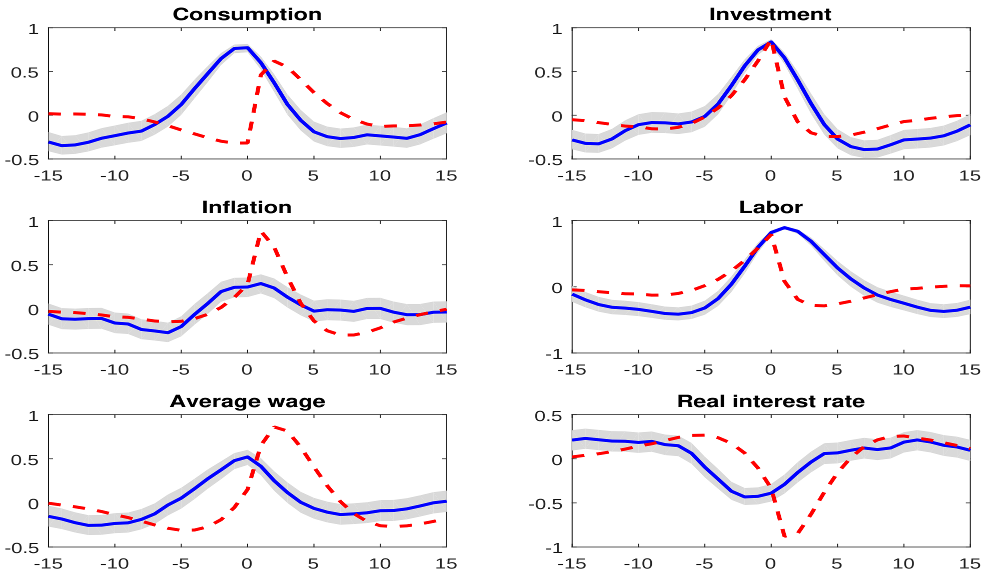
a productivity phenomenon.

4.1 Discount rate shocks

With the discount rate shocks on Figure 3, we see consumption increasing in the first period, but then dropping just as investments and production do. Inflation drops simultaneously with consumption. The drop in consumption with this type of shock is a feature of the two-capital setup as it comes from investment not dropping enough so consumption, being the difference between production and investment, has to drop too. Agents must decrease labor by more than otherwise to substitute consumption for leisure. By forcing agents to take on more leisure instead of increasing consumption, the two-capital setup makes a discount-factor recession look more like a regular recession than it would in other models. It also provides a justification for the paradox of thrift.

So labor drops by more with the two-capital setup. Because of that, and because we use

Figure 4: Cross-correlations with GDP with discount rate shocks only



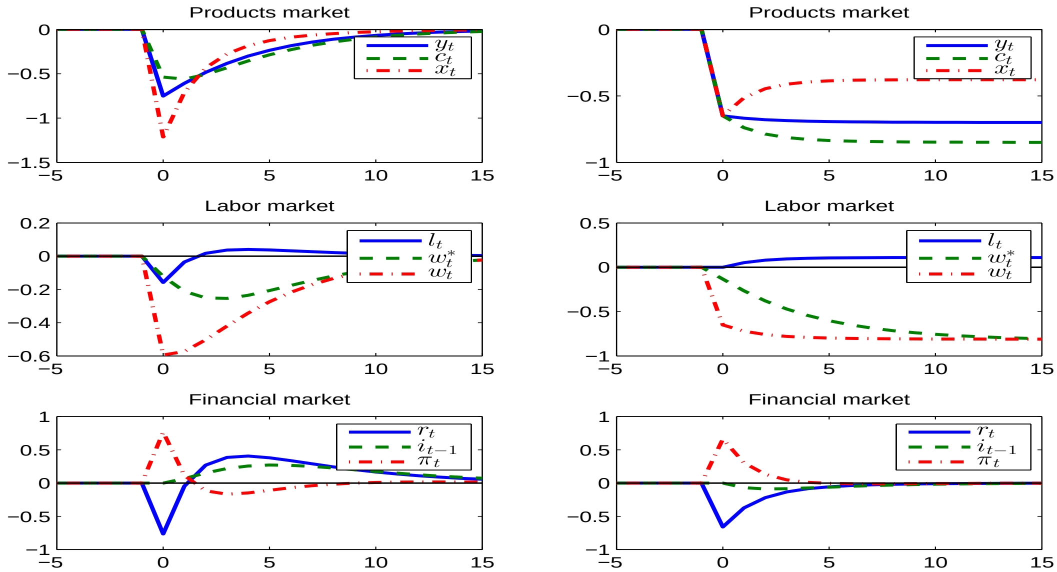
Note: the vertical correlation of the model (dashed line) and the Hodric-Prescott filtered data (full line) with a 95 percent confidence interval (shaded); the horizontal axes indicate quarters.

discount factor shocks for recessions, there was no need for high Frisch elasticity, which here stands at 0.5. The issue of Frisch elasticity has been a thorn of RBC's side since Kydland and Prescott (1982) put the elasticity at infinity. It justified some early attempts at separating the external and internal margins as in Hansen (1985). But there is no need for that here.

Figure 4 illustrates further by showing the cross-correlation between different variables and production.[14] It shows the timing weaknesses, as consumption, inflation and real interest rates react too late while labor reacts too early. In both cases, these weaknesses could be reduced by toying with the timing parameters, but the problem would still be there and less visible for our purpose. Furthermore, inflation and wages are too procyclical and real interest rates are too anticyclical. This means the Taylor wages does not do enough to solve wage issues, and the interest-rate-rigidity model should move to a ex-ante version (which is in the works).

[14]The data comes from the FRED database at the St-Louis Federal Reserve. Production is real GDP, GDPMC1, consumption, PCECC96, investment is private investment, GPDIC1, average wage is the labor share applied to real GDP per worker, GDPMC1*(A4102C1Q027SBEA/GDP)/PAYEMS, labor is the number of worker, PAYEMS, inflation is the growth of the GDP deflator, GDP/GDPMC1, and real interest rates is the ex-post quarterly real return of BAA graded long-term bonds, BAA.

Figure 5: Surprise productivity shock



Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

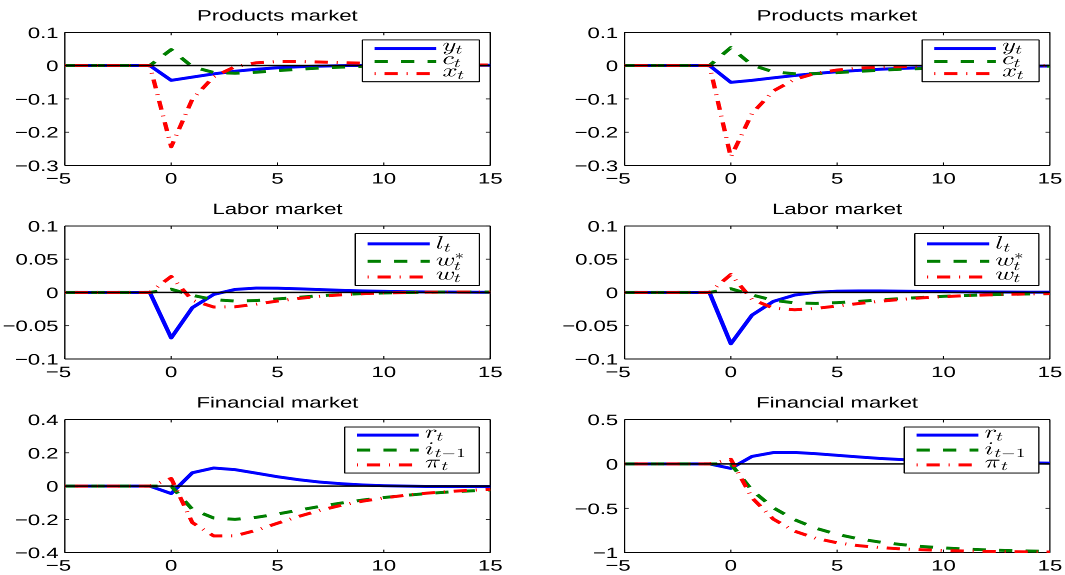
Figure A8, in the Appendix presents the auto-correlation. I used the auto-correlation results to calibrate the shock's persistence, at 0.75, and standard error, at 0.005, so they match the data closely except for consumption and labor that are not persistent enough.

4.2 Productivity shocks

The effects of a productivity shock, in Figure 5, on inflation are mixed for a surprise temporary shock, positive for a surprise permanent shock, though deflationary for anticipated shocks. The deflationary effects fit the too many dollars chasing too few goods narrative, though with a detour from real interest rates. Other responses are as they are in most models.

Note that the long-term effect of a permanent shock is different for production, consumption and investment because the shock broke the long-term relation between production and capital in structures that I set in the steady state. Had structures adjusted accordingly, the

Figure 6: Surprise inflation target shock



Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

percentage effect on production, consumption and investment would have been the same.

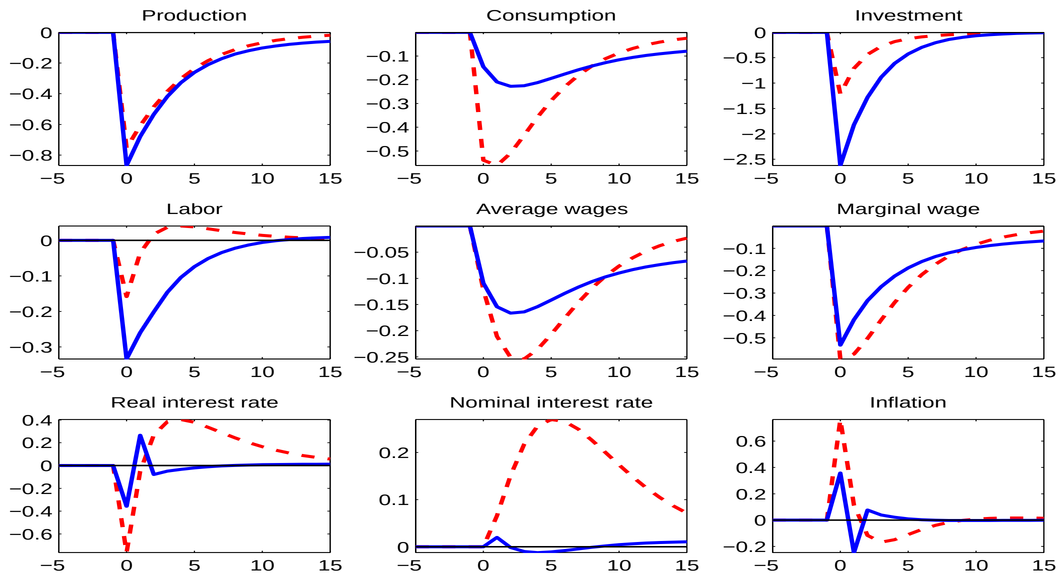
4.3 Inflation target shocks

The inflation target shock, in Figure 6, shows that the central banks wanting deflation will cost the economy in the short run. A permanent shock will have no real effect in the long run, but for changes in the level of inflation and nominal interest rates.

4.4 The secular decrease in real interest rates

There was a secular decrease in interest rates in the last few decades that fit with the secular increase in inequality witnessed in the same period. The phenomenon should be expressed as a permanent discount rate shock as in Figure 3. Unfortunately, the model fails to deliver the increased consumption share coincident in that period. The predicted increase in wages the model predicts may be explained by CEO pay, but consumption is a clear failure from

Figure 7: Temporary surprise discount rate shock with investment adjustment costs



Note: the vertical axes represent the model with investment adjustment costs and habit formation (full line) and the original model with the two-capital setup (dashed line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

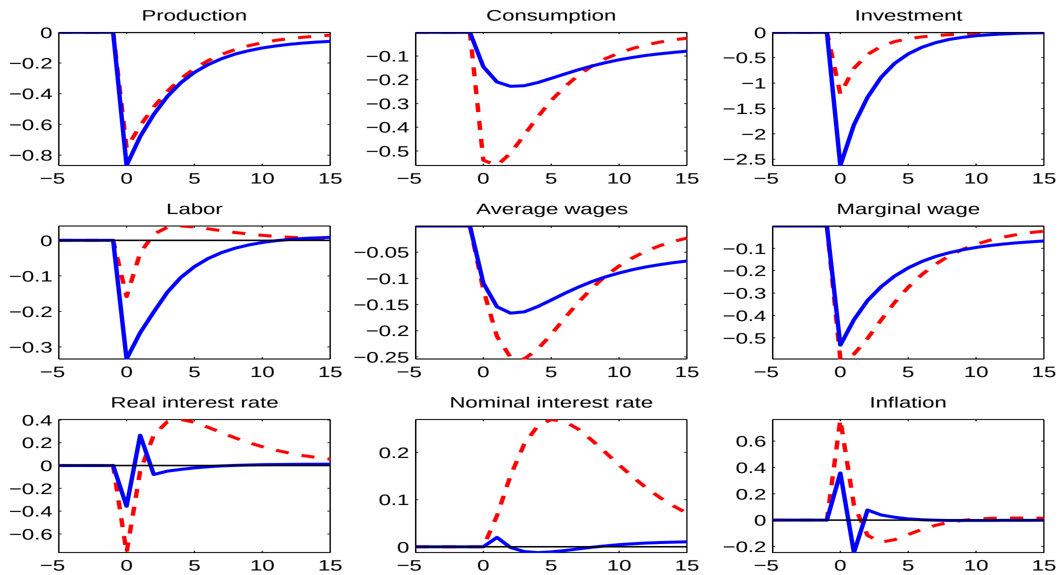
the model.

The share of consumption would have increased if I had made structures elastic to the interest rate in the long run as is the case in reality anyway. This is mostly a technical, though difficult, issue that I leave for future research. An ageing population, which is not part of the model, may also have played a part.

5 Alternative specifications

The section presents some robustness results. Subsection 5.1 compares the two-capital setup with investment adjustment costs and habit formation. Subsection 5.2 compares Taylor wages with Calvo wages. Subsection 5.3 presents the results without the inflation model.

Figure 8: Temporary surprise productivity shock with investment adjustment costs



Note: the vertical axes represent the model with investment adjustment costs and habit formation (full line) and the original model with the two-capital setup (dashed line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

5.1 Investment adjustment costs

Instead of the two-capital setup, we use one where capital accumulates according to

$$K_t = (1-\delta)K_{t-1} + \left(1 - \frac{\chi}{2}\left(\frac{X_t}{X_{t-1}} - \mu\right)^2\right) X_t,$$

where $\chi = 5$.

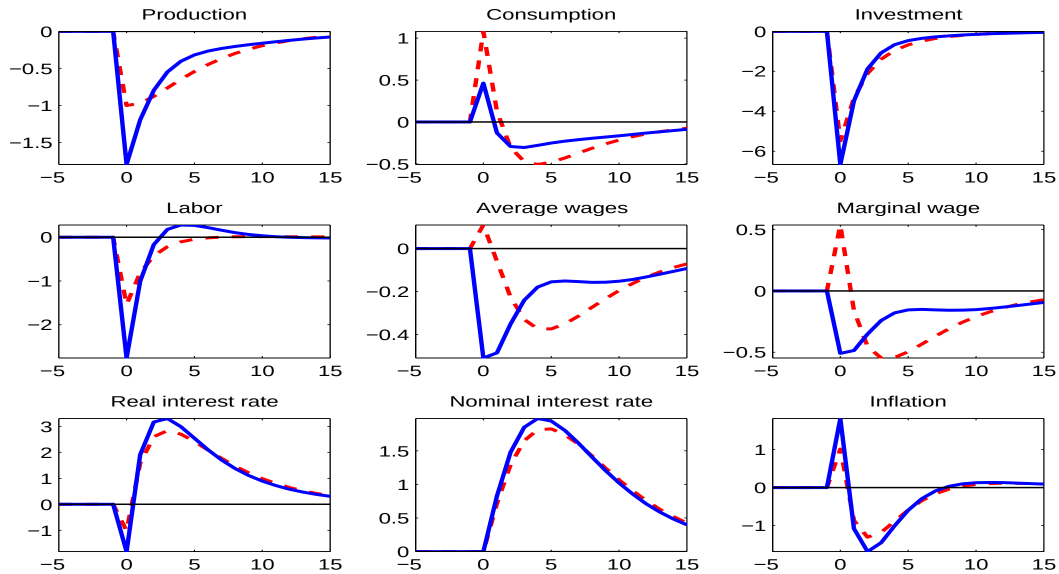
Using investment adjustment costs without habit formation is unfair as they are meant to be together. Hence, the utility function has an external habit added to it,

$$\sum_{i=0}^{\infty} \beta^i e^{b_{t+i}} \frac{\left((C_{t+i} - \upsilon \bar{C}_{t-1+i})v(L^* - L_{t+i})\right)^{1-\rho} - 1}{1-\rho},$$

where $\upsilon = 0.65$.

Replacing the two-capital setup with investment adjustment costs wreak havoc on financial variables. These will swing back and forth with any shocks. In fact, the responses on real interest rates often look like the EKG of a dying patient. The reason stems from the lack of a Phillips curve.

Figure 9: Temporary surprise discount rate shock with Calvo wages



Note: the vertical axes represent the model with Calvo wages (full line) and the original model with Taylor wages (dashed line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

The initial effects of a discount rate shock, in Figure 7 on investments are so strong that even with habit formation, consumption and wages increase with a recession. In Figure 8, the productivity shock displays the same problem with financial variables.

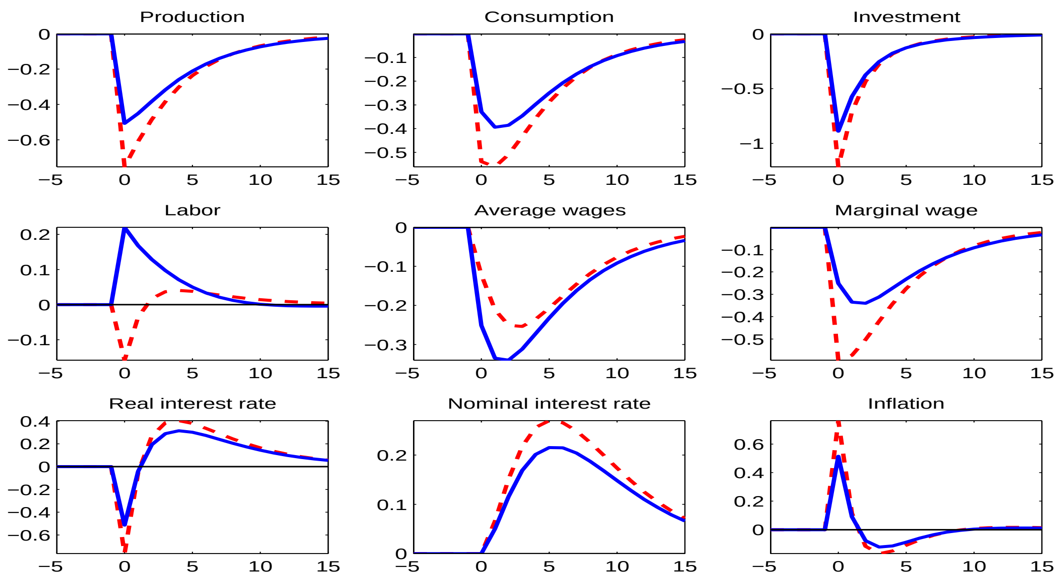
5.2 Calvo wage contracts

Calvo wage contracts are now textbook, so there is no need to discuss them at length. Calvo type staggered wage contracts yield the following equation,

$$\begin{aligned}\widehat{W}_t = \tfrac{1}{1+\beta}\widehat{W}_{t-1} + \tfrac{\beta}{1+\beta}\mathrm{E}_t\widehat{W}_{t+1} + \tfrac{\xi}{1+\beta}\pi_{t-1} - \tfrac{1+\xi\beta}{1+\beta}\pi_t + \tfrac{\beta}{1+\beta}\mathrm{E}_t\pi_{t+1} \\ + \tfrac{1-\kappa\beta}{1+\beta}\tfrac{1-\kappa}{\kappa}\left(\widehat{W}_t^m - \widehat{W}_t\right),\end{aligned} \tag{5}$$

where $\hat{w}$ and $\hat{\pi}$ represent the deviations of wage and inflation from their steady state, $\kappa = 0.7$, the share of workers who can reset their wages at each period, and ξ, the elasticity parameter that disappears with log-linearization. I take the discount factor as fix because wage distribution is not affected by debts, though in a surprising result, using the variable aggregate does not alter the response even when this discount factor is shocked.

Figure 10: Temporary surprise productivity shock with Calvo wages



Note: the vertical axes represent the model with Calvo wages (full line) and the original model with Taylor wages (dashed line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

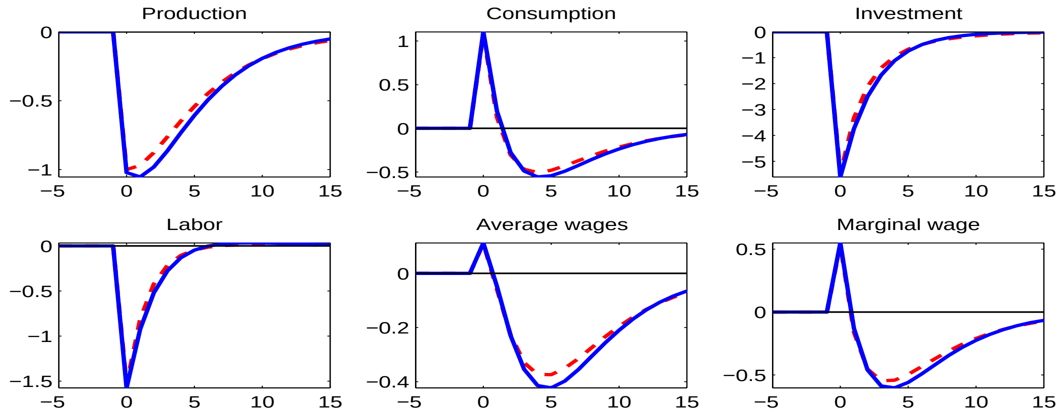
With Calvo contracts, average and marginal wages are the same. The results show that Calvo wages do not add much to the narrative, but just mess up the response functions and blur the channels.

For example, with the surprise discount rate shock of Figure 9, response functions are very close. Only marginal wage responses change, which does not matter because marginal wage is not an observed variable.

We see a second difference with the productivity shocks of Figure 10. Calvo wages reduce the effects of shocks by maybe a quarter and a third in the short run, but also change the response of labor. Labor moves from a mixed response to the shocks to a positive response.

With inflation target shocks, shown in the Appendix, both have small effect on real variables, but Calvo wages make variables swing back and forth.

Figure 11: Temporary surprise discount rate shock without monetary model



Note: the vertical axes represent the model without monetary policy ($\phi = 0$, full line) and the original model ($\phi = 0.5$, dashed line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

5.3 No monetary policy

Posing $\phi = 0$ is equivalent in this model to building a strictly real model; it sets nominal interest rates so as not to move. The financial part of the model does not affect the results because with nominal interest rates constant, the financial part does not feedback into the rest of the model.

Results show the difference between the model with and without monetary policy is small. For example, in the case of the goods market and the labor market, the differences amount to less than a measurement error. Results confirm the conclusion from Bélanger (2015) that inflation targeting reduces volatility compared to having no monetary policy, but the gains are quite small.

Because the difference with interest rate rigidity is small, interest rate rigidity is a great way to have inflation in an otherwise real model while keeping the results of the real model just as tractable.

Figure 11 displays a discount rate shock, but all shocks give similar response functions as shown in the Appendix.

6 Conclusion

> *To promote this disposition to exchange lands, which they have to spare and we want, for necessaries, which we have to spare and they want, we shall push our trading uses, and be glad to see the good and influential individuals among them run in debt, because we observe that when these debts get beyond what the individuals can pay, they become willing to lop them off by a cession of lands. (Jefferson (2000))*

As that quote from President Jefferson suggests, the relation between debts and inequality has a long history. The dire prediction from Ramsey's conjecture makes the reality of a slow walk to extreme inequality that more potent.

The paper does not argue that inequality causes crises directly. Crises result from government actions or threats of actions against inequality. Nevertheless, people's reaction to inequality may be inevitable and crises provoking, so governments ought to bear that in mind.

The paper has two contributions: an intuitive interpretation of discount factor shocks as the primary source of recessions, and a technical way of avoiding the interest-rate-inelasticity issue without using too many ad hoc assumptions. The main weaknesses stem from the recentness of the approach. First, the two-capital setup does not permit elasticity in the long run, which is a stylized fact. Because the way we model capital embeds resell value, there was no way I saw to restrict the movements of investment to only secular changes in interest rates. This is left for future research. Second, as this is theoretical paper, I did not take the model to the data beyond comparing cross-correlations between the model and the data. Third, it would be worth investigating increasing industrial concentration instead of Ramsey's conjecture as a mechanism for the secular increases in inequality. The model would be the same, with the threat of debt relief the central argument for the discount rate shock. The advantage would come from better data as industrial concentration is easier to measure than heterogeneity in discount factors.

The same way IS-LM serves to think about the more complicated New Keynesian models, there is one way to think about the approach or to approximate the model. That is to think about the costs that businesses support. Since interest rate payments constitute important business costs, we can explain the theory to a layman using business costs. Heck, we could say the price for businesses to buy oil triggered the seventies recessions. Though this proves an exaggeration, the narrative is attractive. To that end, Figure 1, which comes from the model, serves as a useful, though pseudo, supply and demand graph.

References

Becker, Robert A. (1980) "On the Long-Run Steady State in a Simple Dynamic Model of Equilibrium with Heterogeneous Households," *The Quarterly Journal of Economics*, **95**(2), pp. 375–382, URL http://dx.doi.org/10.2307/1885506.

Bélanger, Gilles (2015) "Interest Rate Rigidity and the Fisher Equation," *Available at SSRN*, URL http://dx.doi.org/10.2139/ssrn.2444280.

Caballero, Ricardo J. (1999) "Aggregate investment," in *Handbook of Macroeconomics*, eds. J. B. Taylor and M. Woodford, volume 1 of *Handbook of Macroeconomics*, chapter 12, pp. 813–862, Elsevier, URL http://dx.doi.org/10.1016/S1574-0048(99)10020-X.

Christiano, Lawrence J., Martin Eichenbaum and Charles L. Evans (2005) "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, **113**(1), pp. 1–45, URL http://dx.doi.org/10.1086/426038.

de Bondt, Gabe, Benoît Mojon and Natacha Valla (2005) "Term structure and the sluggishness of retail bank interest rates in euro area countries," *Working Paper Series*, **518**, pp. 1–49, URL http://www.ecb.europa.eu/pub/pdf/scpwps/ecbwp518.pdf.

Gollier, Christian and Richard Zeckhauser (2005) "Aggregation of Heterogeneous Time Preferences," *Journal of Political Economy*, **113**(4), pp. 878–896, URL http://dx.doi.org/10.1086/430853.

Hansen, Gary D. (1985) "Indivisible labor and the business cycle," *Journal of Monetary Economics*, **16**(3), pp. 309–327, URL http://dx.doi.org/10.1016/0304-3932(85)90039-X.

Ireland, Peter N. (2004) "A method for taking models to the data," *Journal of Economic Dynamics and Control*, **28**(6), pp. 1205–1226, URL http://dx.doi.org/10.1016/S0165-1889(03)00080-0.

Jefferson, Thomas (2000) "President Jefferson letter to Willian Henry Harrison, February 27, 1803," in *Documents of United States Indian policy*, ed. Francis Paul Prucha, p. 22, University of Nebraska Press.

Kobayashi, Teruyoshi (2008) "Incomplete Interest Rate Pass-Through and Optimal Monetary Policy," *International Journal of Central Banking*, **4**(3), pp. 77–118, URL http://www.ijcb.org/journal/ijcb08q3a4.pdf.

Krueger, Dirk, Kurt Mitman and Fabrizio Perri (2016) "Macroeconomics and Household Heterogeneity," Working Paper 22319, National Bureau of Economic Research, URL http://dx.doi.org/10.3386/w22319.

Krusell, Per and Anthony A. Jr. Smith (1998) "Income and Wealth Heterogeneity in the Macroeconomy," *Journal of Political Economy*, **106**(5), pp. 867–896, URL http://dx.doi.org/10.1086/250034.

Kydland, Finn E. and Edward C. Prescott (1982) "Time to Build and Aggregate Fluctuations," *Econometrica*, **50**(6), pp. 1345–70, URL http://dx.doi.org/10.2307/1913386.

Long, Jr, John B. and Charles I. Plosser (1983) "Real Business Cycles," *Journal of Political Economy*, **91**(1), pp. 39–69, URL http://dx.doi.org/10.1086/261128.

Nardi, Mariacristina De (2015) "Quantitative Models of Wealth Inequality: A Survey," NBER Working Papers 21106, National Bureau of Economic Research, Inc, URL http://dx.doi.org/10.3386/w21106.

Ramsey, Frank P. (1928) "A Mathematical Theory of Saving," *The Economic Journal*, **38(152)**, pp. 543–559, URL http://dx.doi.org/10.2307/2224098.

Taylor, John B. (1979) "Staggered Wage Setting in a Macro Model," *American Economic Review*, **69**(2), pp. 108–13, URL http://www.jstor.org/stable/1801626.

Teranishi, Yuki (2015) "Smoothed Interest Rate Setting by Central Banks and Staggered Loan Contracts," *The Economic Journal*, **125**, pp. 162–183, URL http://dx.doi.org/10.1111/ecoj.12092.

van Treeck, Till (2014) "Did Inequlity Cause The U.S. Financial Crisis?" *Journal of Economic Surveys*, **28**(3), pp. 421–448, URL http://dx.doi.org/10.1111/joes.12028.

A Appendix

A.1 Steady state

The shocks have a steady state of zero. Financial variables have a steady state in log form of $\pi = \pi^*, r = r^* = -\log(\beta)$ and $i = r + \pi$. Since the model is quarterly, these variables each have a display version for the figures that is multiplied by four. The other variables are as follow:

$$L^{1/\eta - 1} = \frac{\psi(1-\nu)(1 - \delta_m \gamma\alpha(1/\beta - 1 + \delta_m)^{-1} - \delta_s \sigma)}{(1/\upsilon - 1)^{1/\eta}(1-\alpha)},$$

$$Y = (\gamma\alpha(1/\beta - 1 + \delta_m)^{-1})^{\frac{\gamma\alpha}{1-\alpha}} \sigma^{\frac{(1-\gamma)\alpha}{1-\alpha}} L,$$

$$X = \left(\frac{\delta_m \gamma\alpha}{1/\beta - 1 + \delta_m} + \delta_s \sigma\right) Y,$$

$$C = \left(1 - \frac{\delta_m \gamma\alpha}{1/\beta - 1 + \delta_m} - \delta_s \sigma\right) Y,$$

$$W = (1-\alpha)\frac{Y}{L},$$

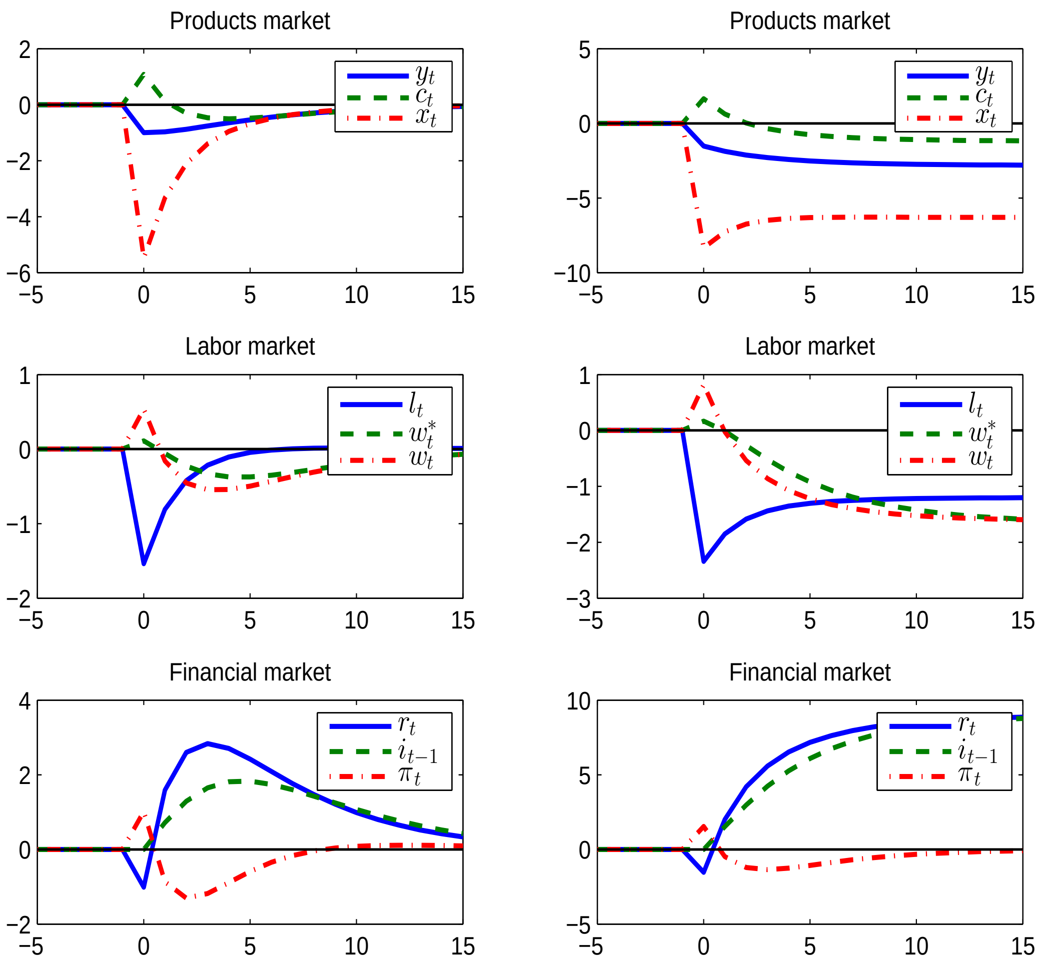
$$M = \frac{\gamma\alpha}{1/\beta - 1 + \delta_m} Y,$$

$$K = M + \sigma Y,$$

A.2 Figures

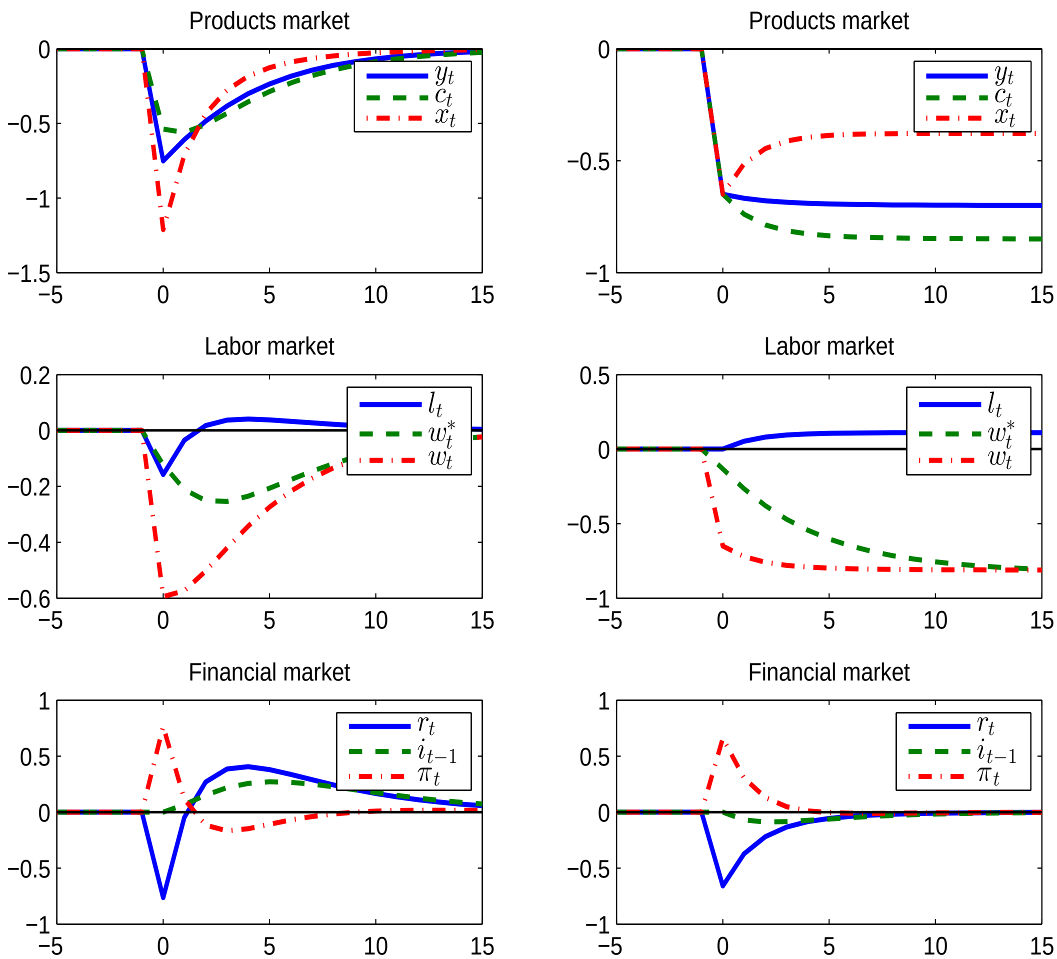
The following pages offer additional results. Figure A1 to Figure A6 present additional shocks from Section 4, while Figure A9 to Figure A17 present additional shocks from Section 5.

Figure A1: Surprise discount rate shock



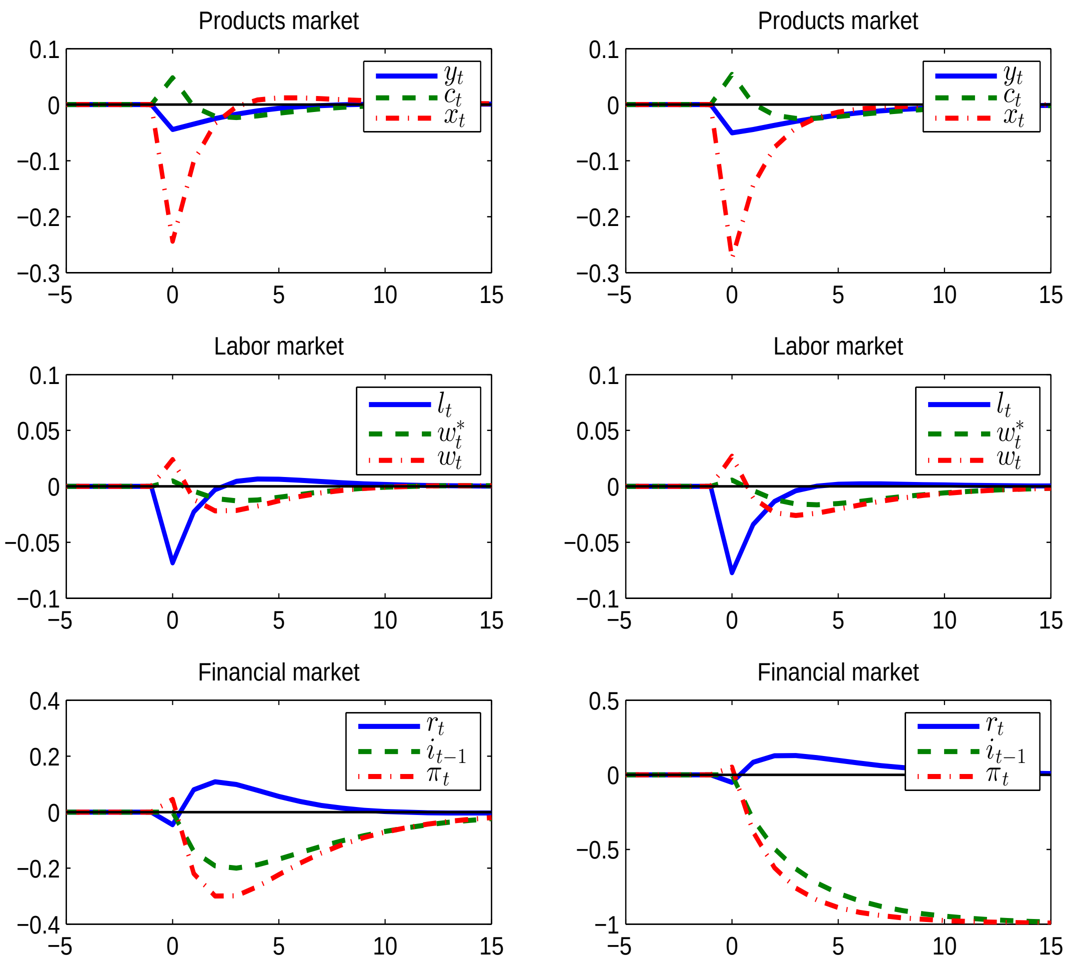
Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

Figure A2: Surprise productivity shock



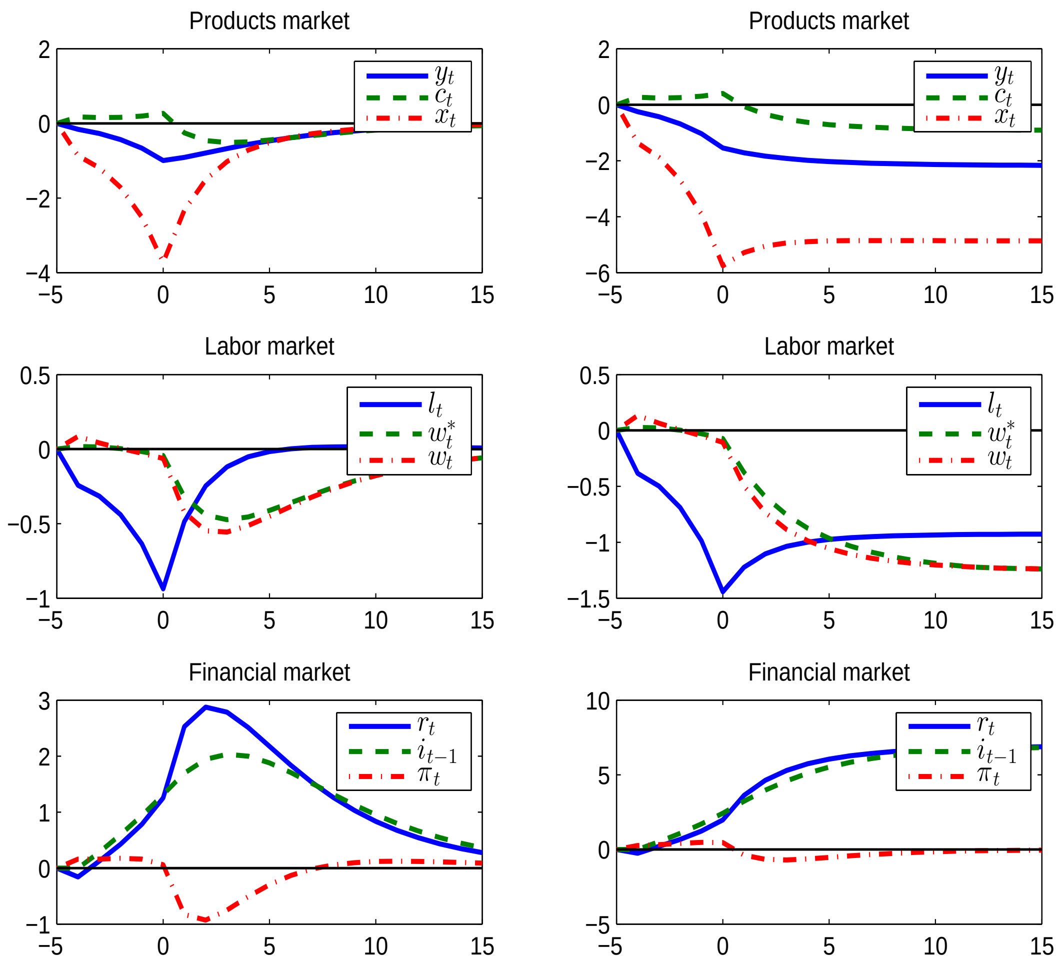
Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

Figure A3: Surprise inflation target shock



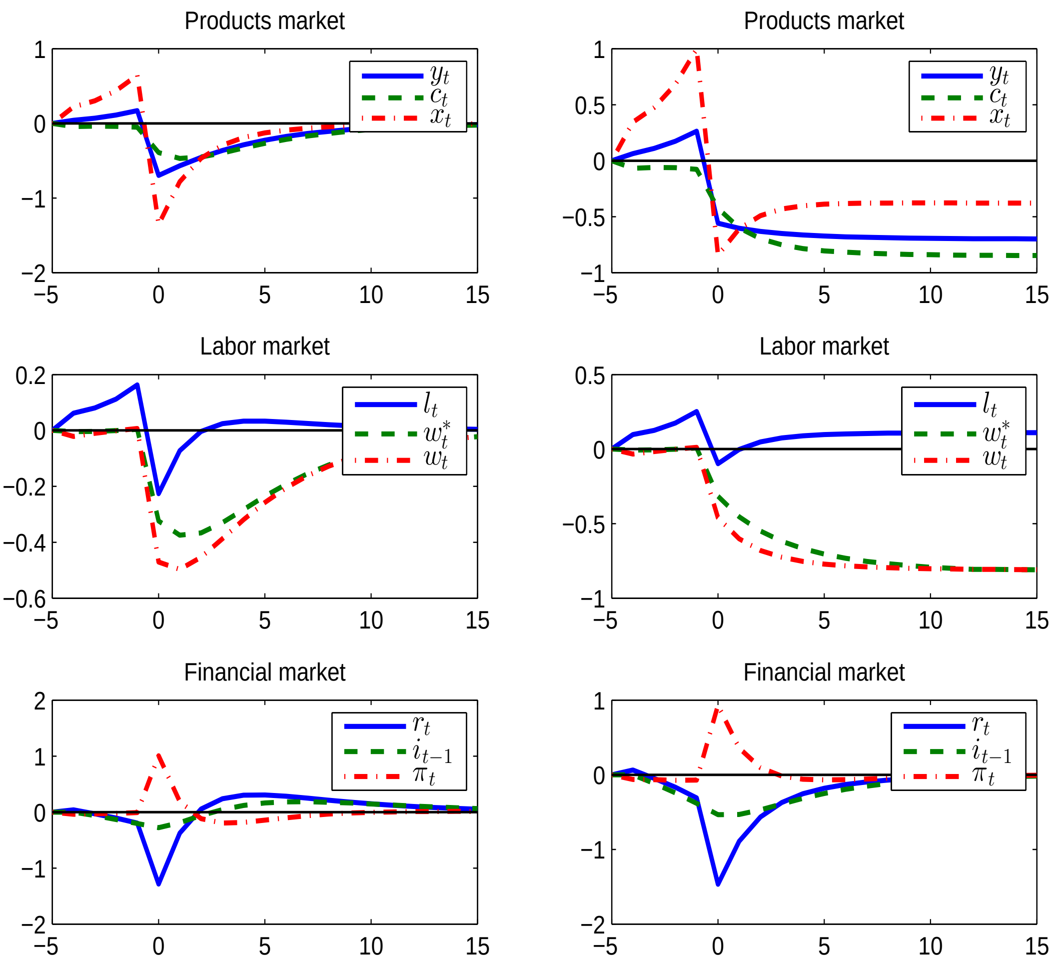
Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

Figure A4: Anticipated discount rate shock



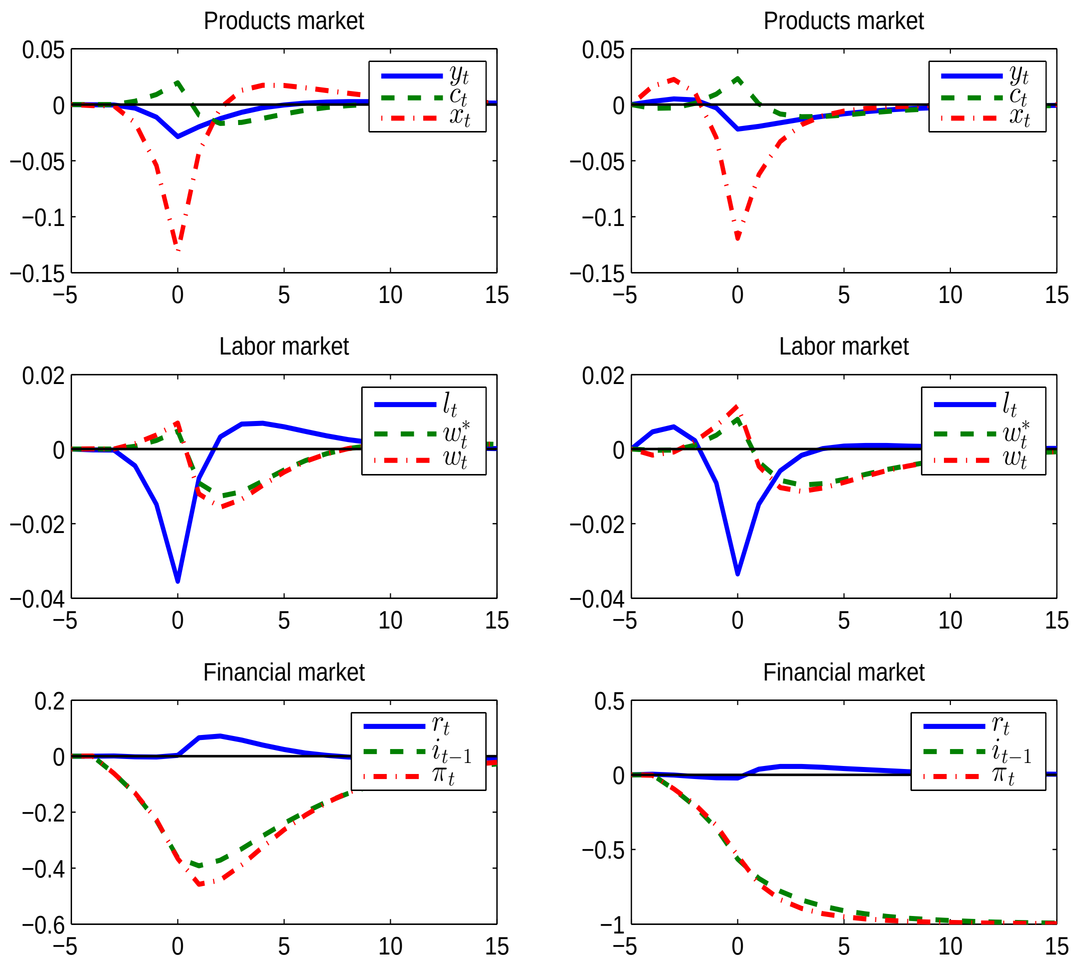
Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

Figure A5: Anticipated productivity shock



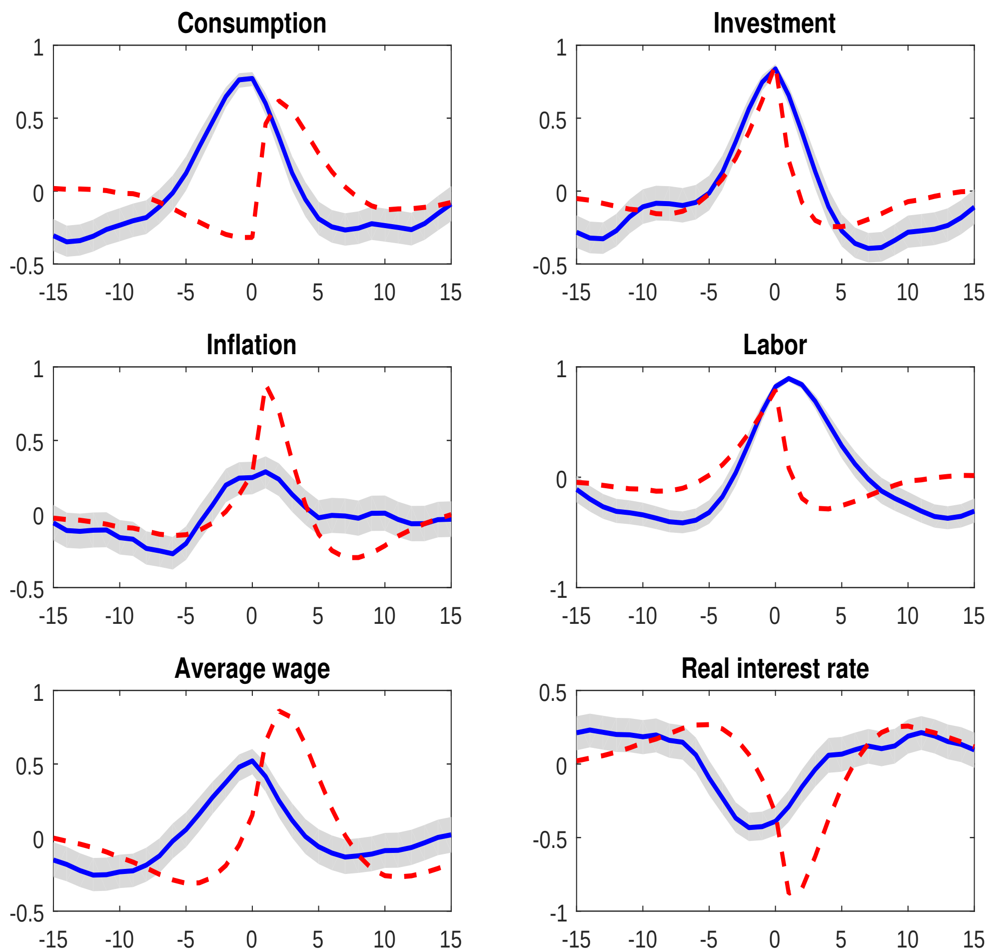
Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

Figure A6: Anticipated inflation target shock



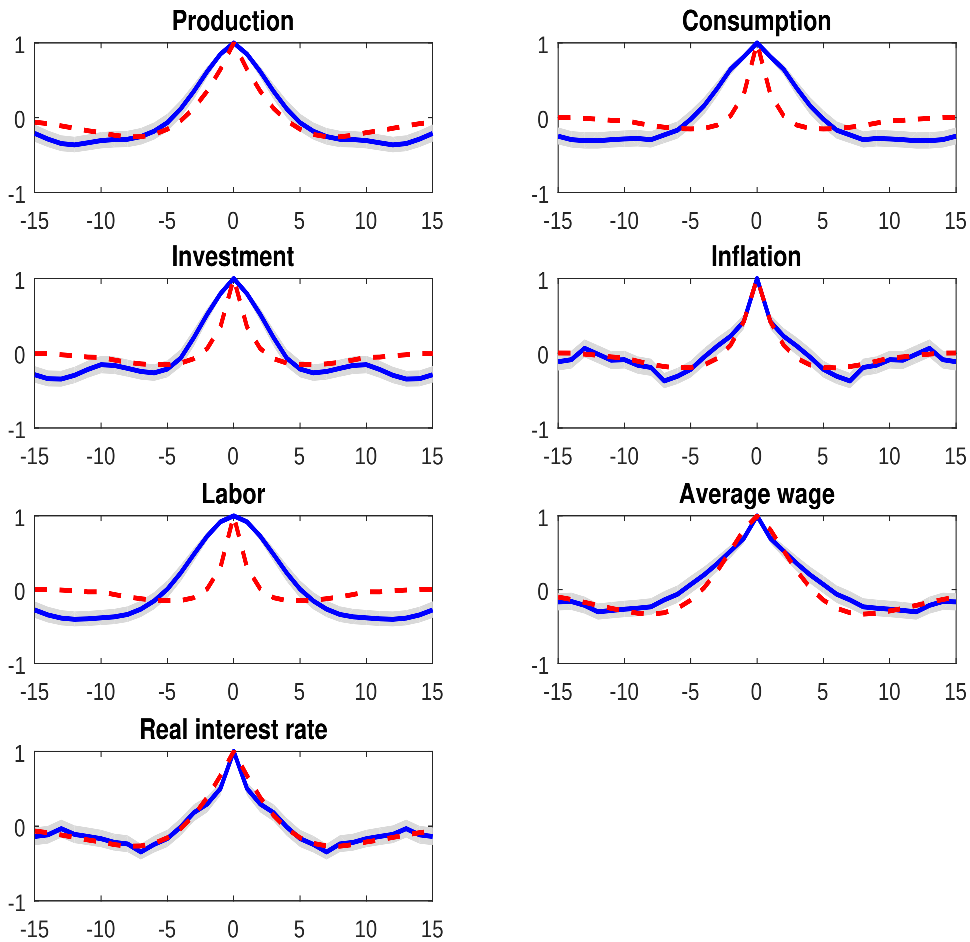
Note: the vertical axes represent production (y_t), consumption (c_t), investments (x_t), inflation (π_t), ex-post real interest (r_t), nominal interest (i_{t-1}), labor (l_t), average wages (w_t^*) and marginal wages (w_t) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock. Graphs on the left show responses to temporary shocks ($\rho = 0.75$), on the right to permanent shocks ($\rho = 0.9999$).

Figure A7: Cross-correlations with GDP with discount rate shocks only



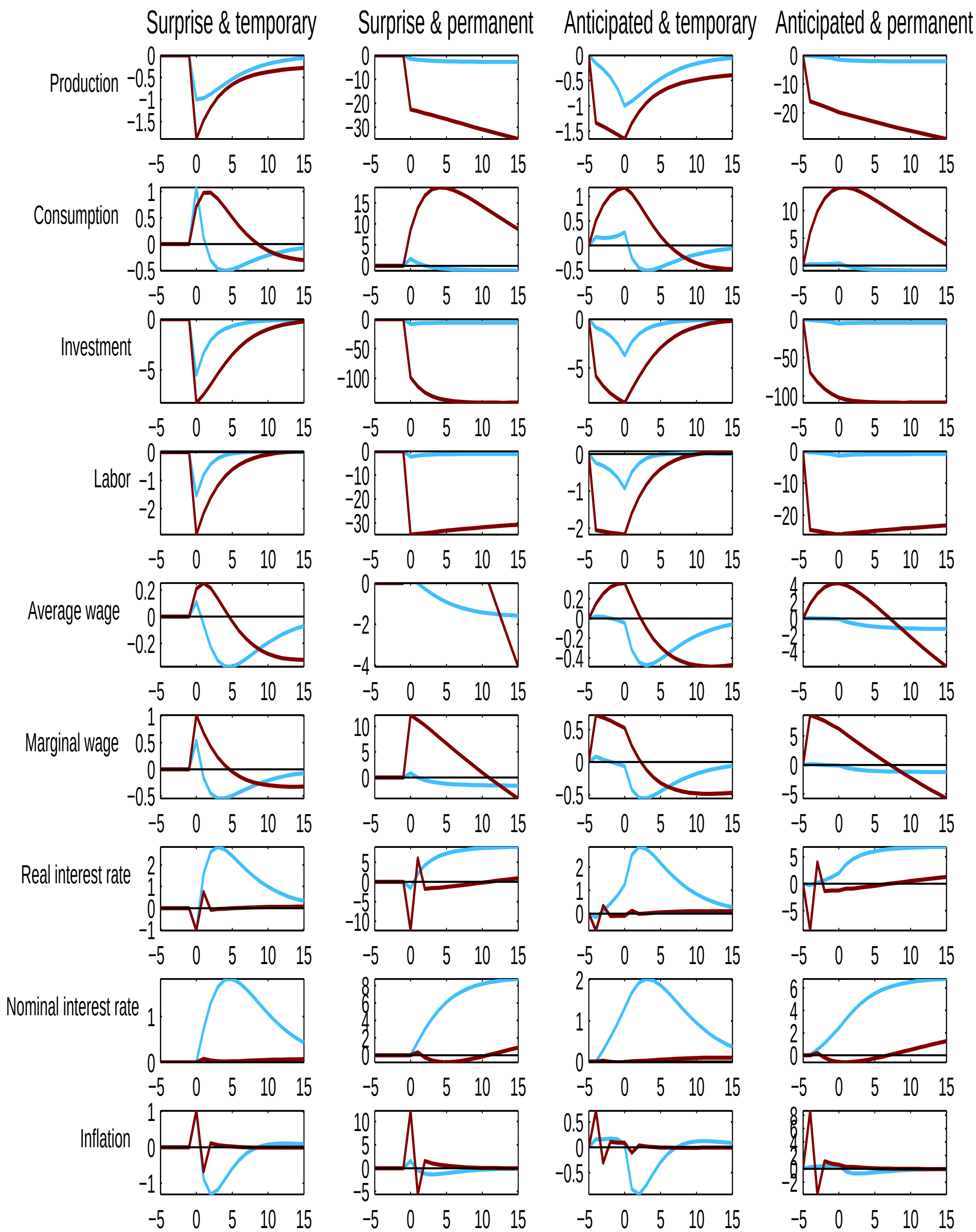
Note: the vertical correlation of the model (dashed line) and the Hodric-Prescott filtered data (full line) with a 95 percent confidence interval (shaded); the horizontal axes indicate quarters.

Figure A8: Auto-correlations with discount rate shocks only



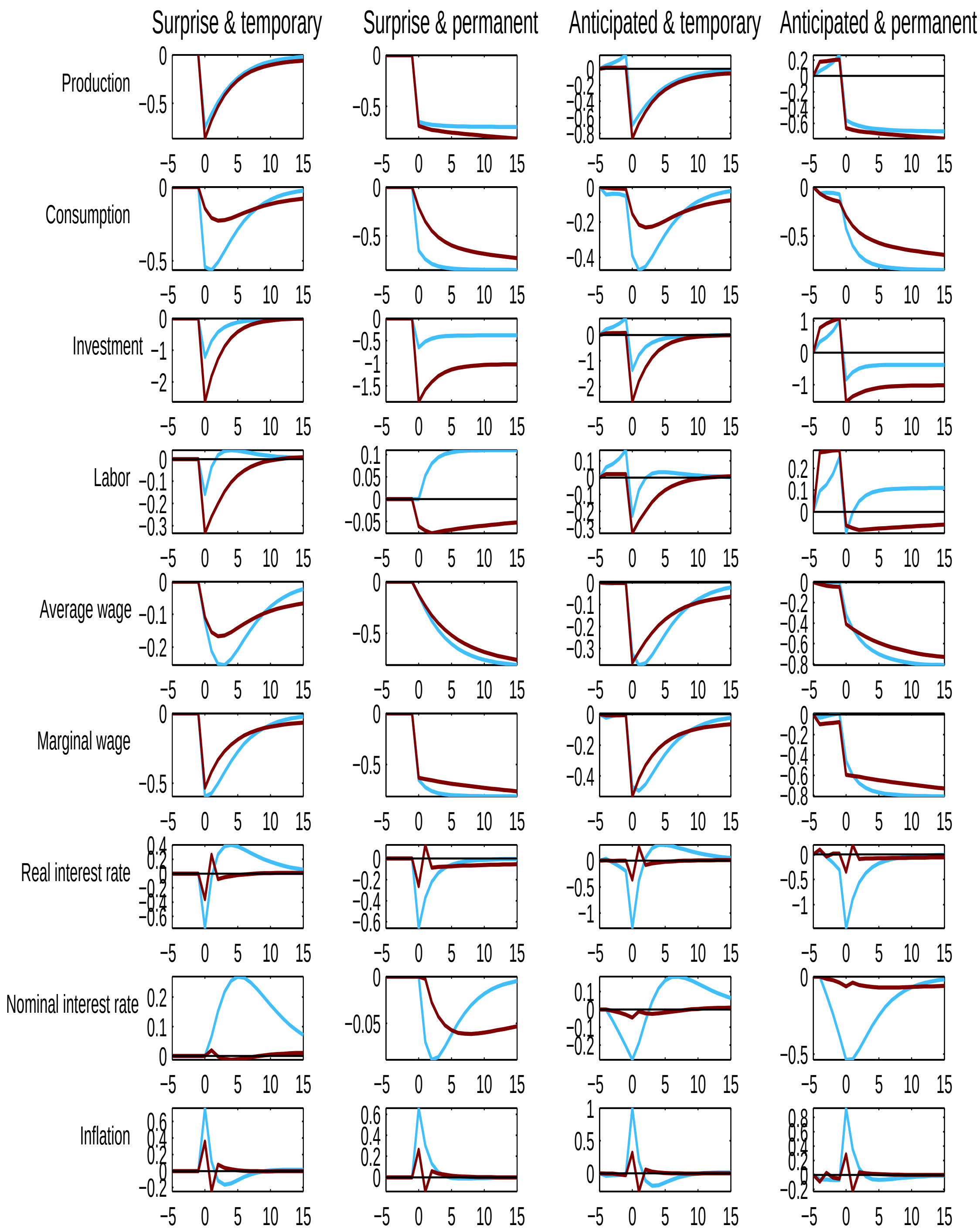
Note: the vertical correlation of the model (dashed line) and the Hodric-Prescott filtered data (full line) with a 95 percent confidence interval (shaded); the horizontal axes indicate quarters.

Figure A9: Discount rate shocks with investment adjustment costs



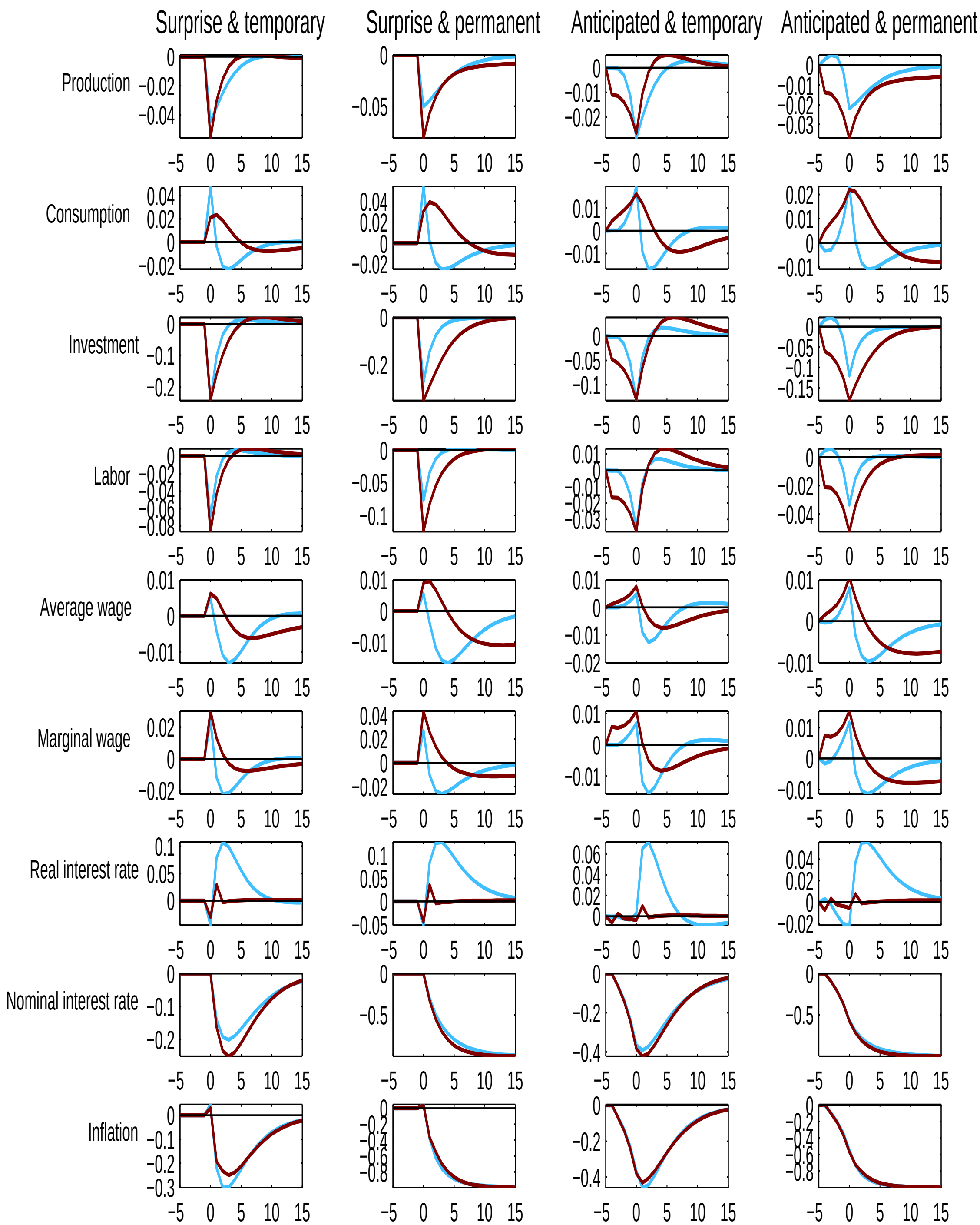
Note: the vertical axes represent the model with investment adjustment cost (dark red line) and the original model with the two-capital setup (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A10: Productivity shocks with investment adjustment costs



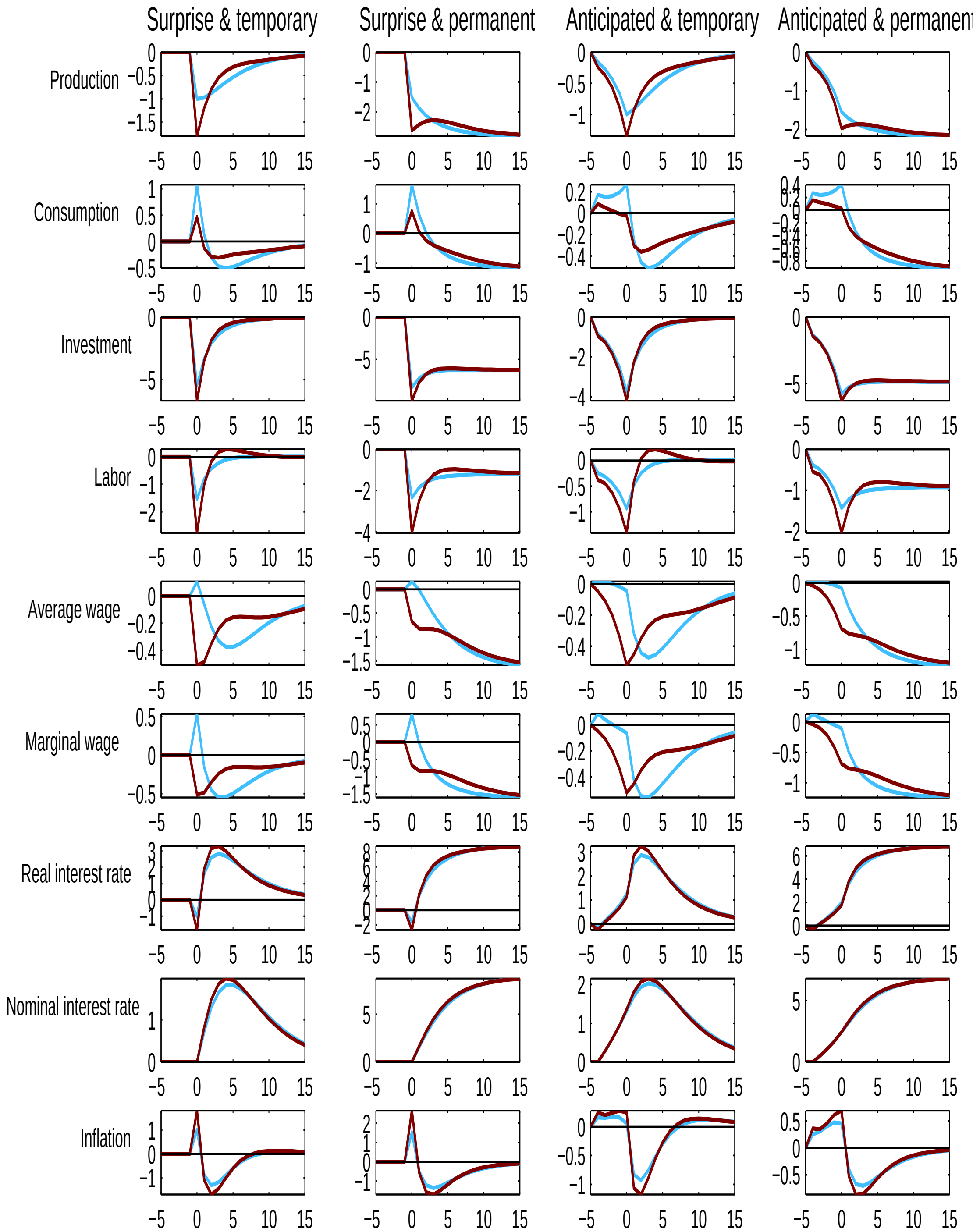
Note: the vertical axes represent the model with investment adjustment cost (dark red line) and the original model with the two-capital setup (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A11: Inflation target shocks with investment adjustment costs



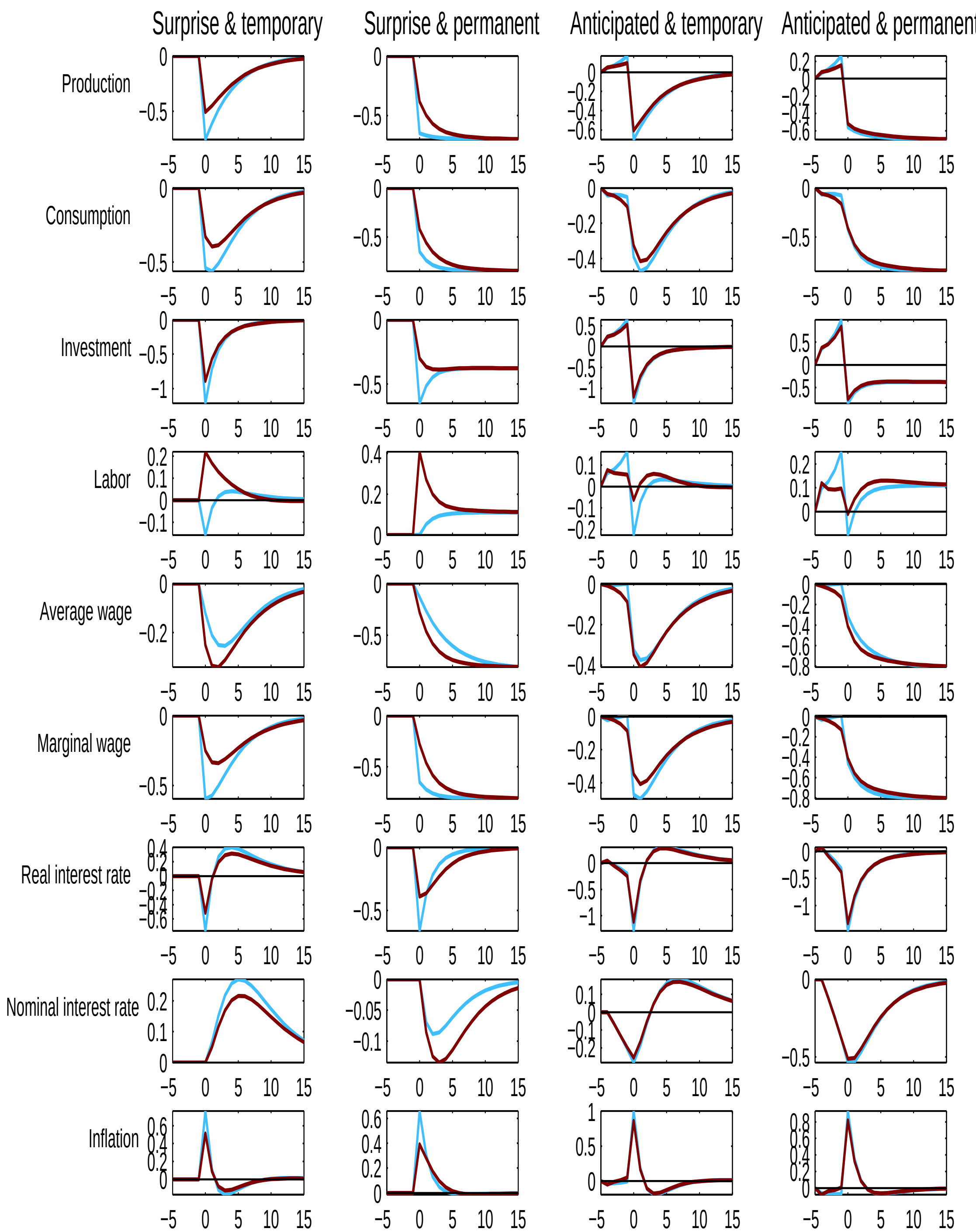
Note: the vertical axes represent the model with investment adjustment cost (dark red line) and the original model with the two-capital setup (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A12: Discount rate shocks with Calvo wage contracts



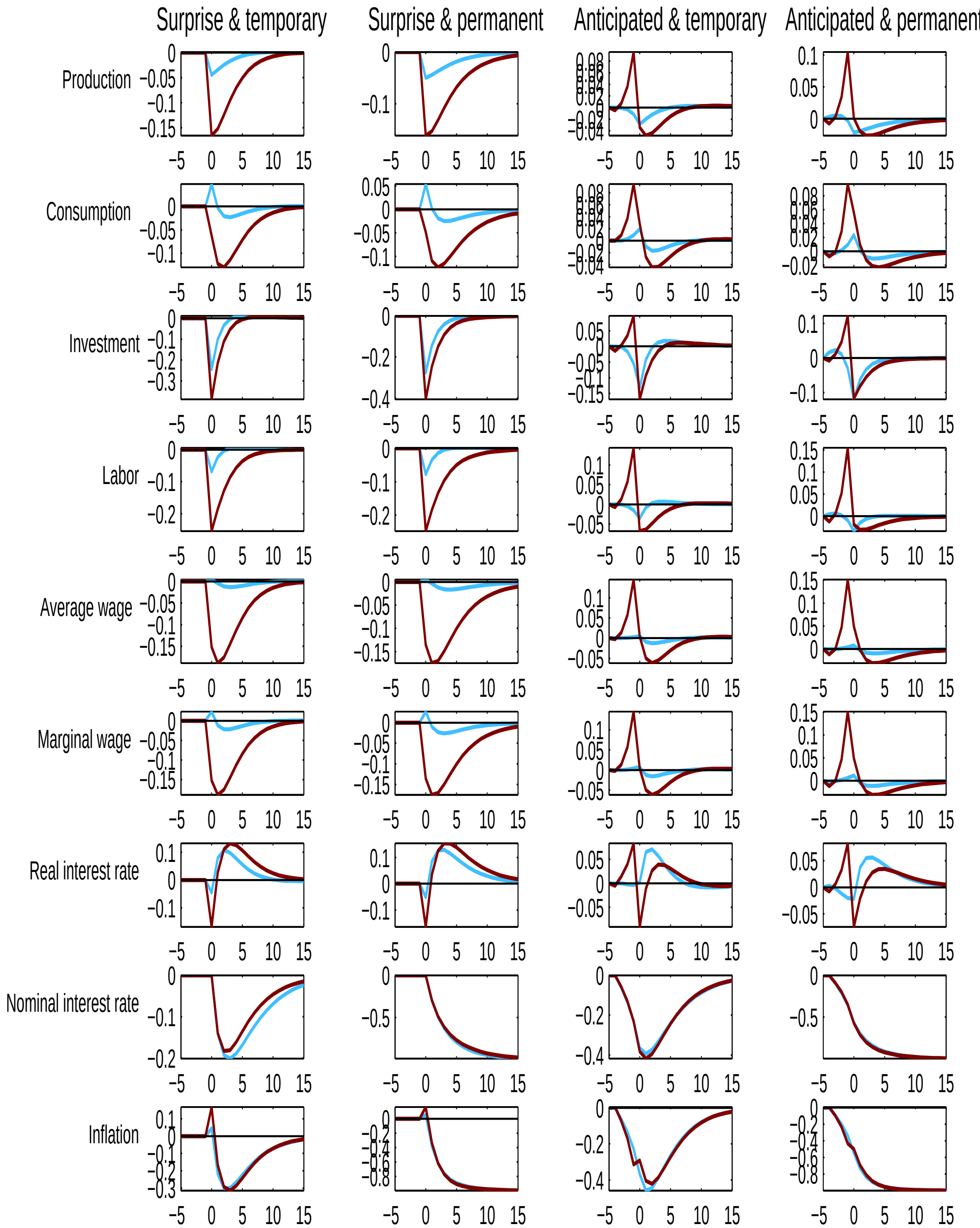
Note: the vertical axes represent the model with Calvo wages (dark red line) and the original model with Taylor wages (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A13: Productivity shocks with Calvo wage contracts



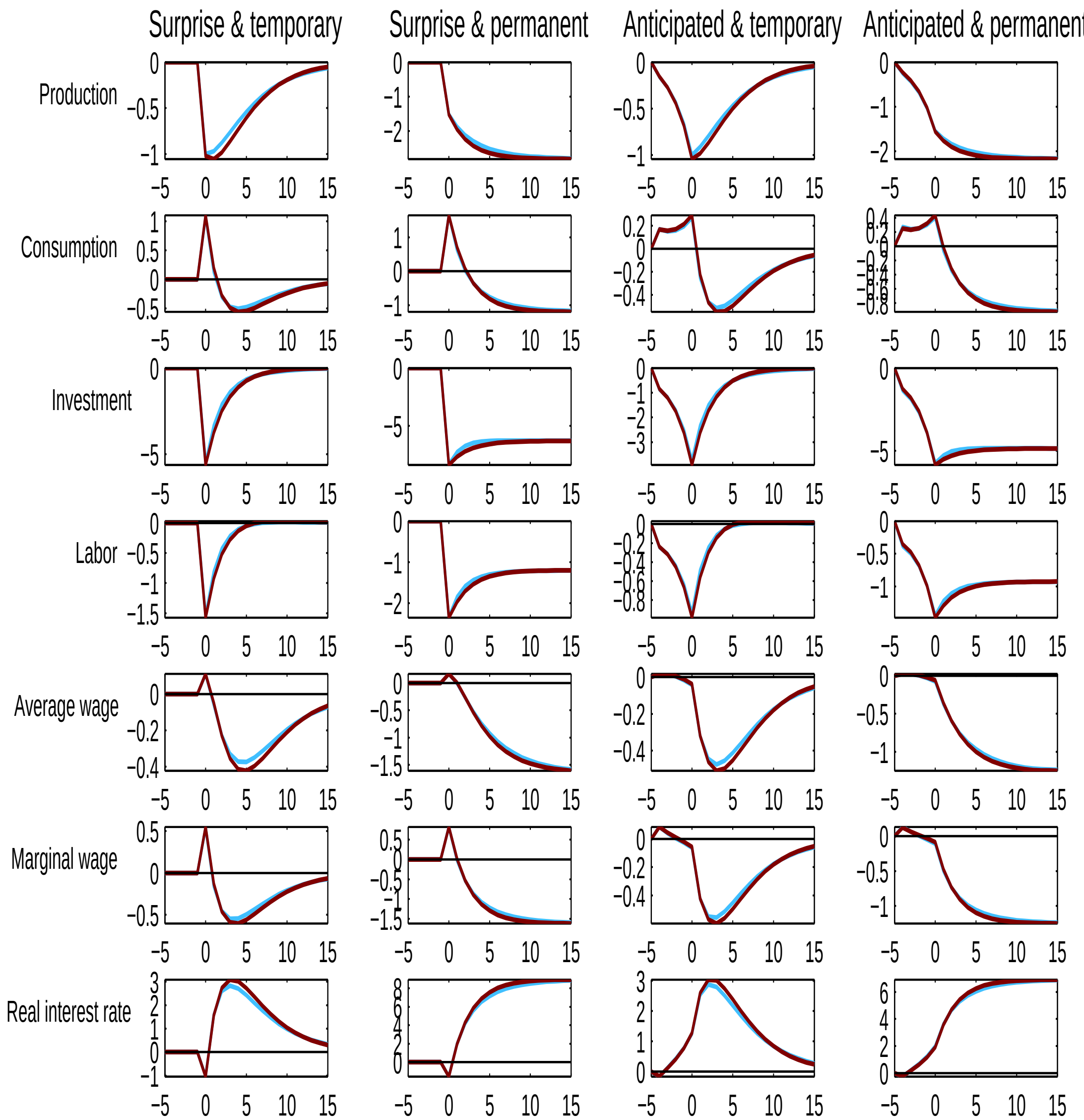
Note: the vertical axes represent the model with Calvo wages (dark red line) and the original model with Taylor wages (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A14: Inflation target shocks with Calvo wage contracts



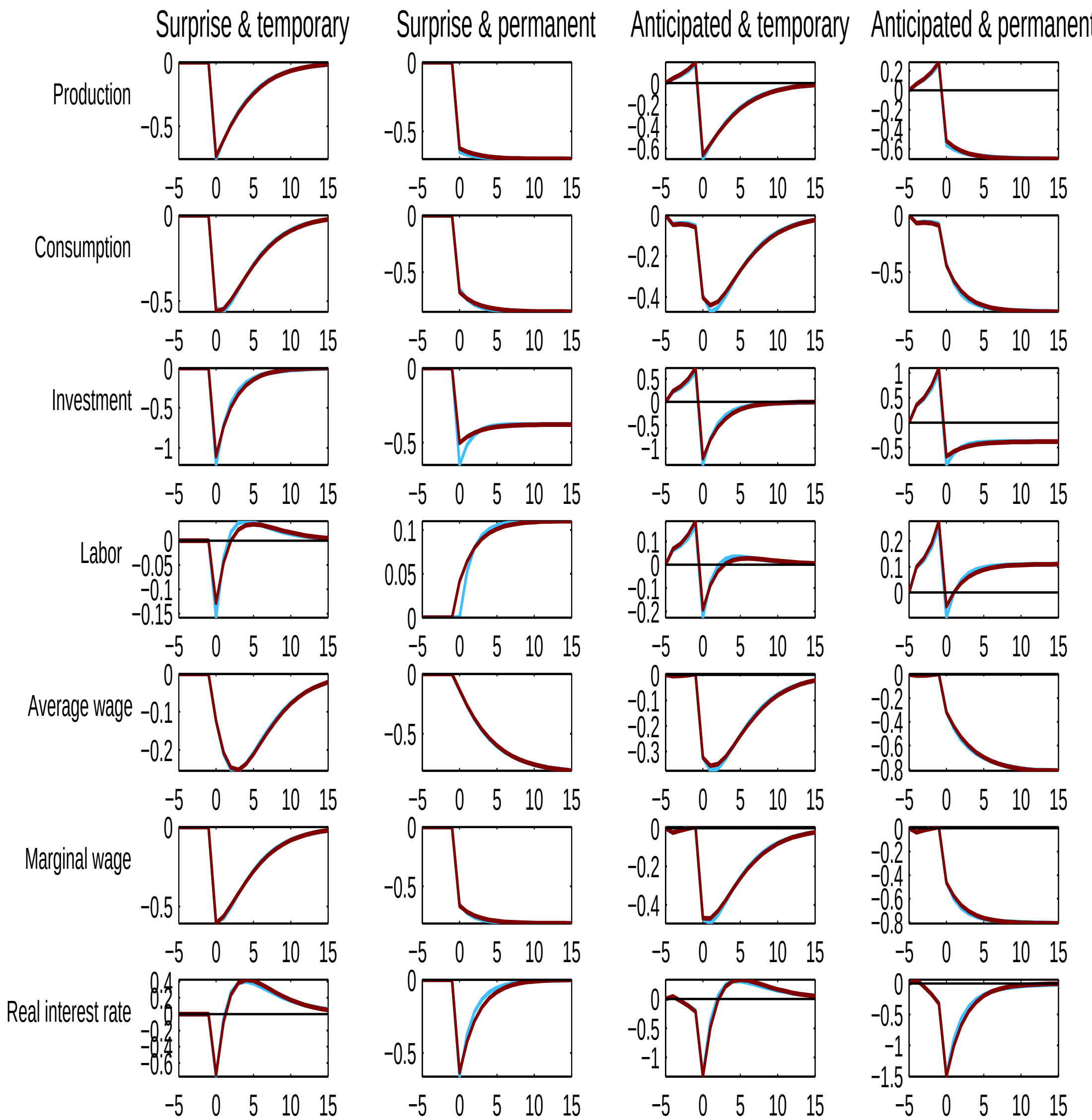
Note: the vertical axes represent the model with Calvo wages (dark red line) and the original model with Taylor wages (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A15: Discount rate shocks without monetary policy



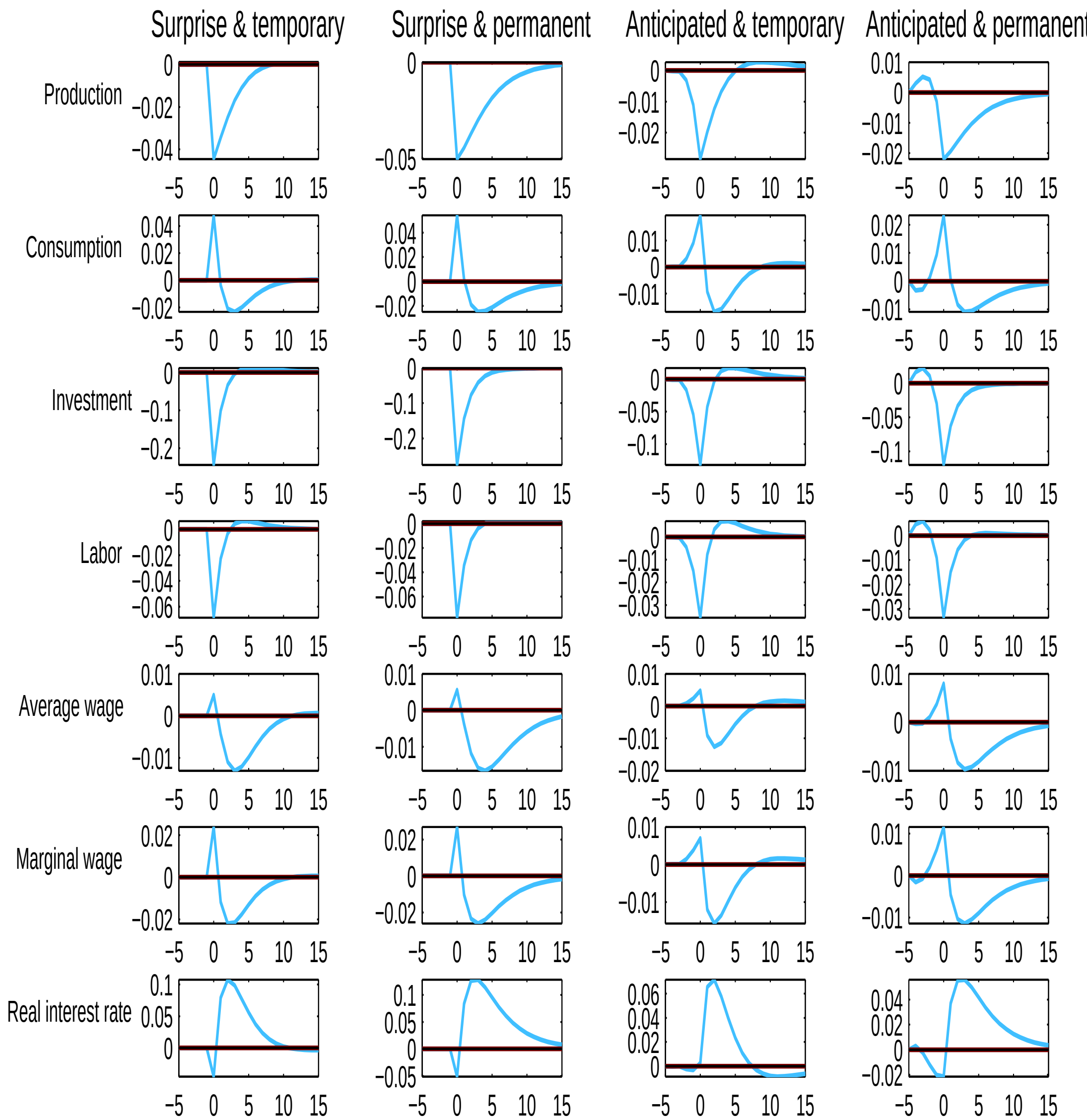
Note: the vertical axes represent the model without monetary policy ($\phi = 0$, dark red line) and the original model (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A16: Productivity shocks without monetary policy



Note: the vertical axes represent the model without monetary policy ($\phi = 0$, dark red line) and the original model (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.

Figure A17: Inflation target shocks without monetary policy



Note: the vertical axes represent the model without monetary policy ($\phi = 0$, dark red line) and the original model (light blue line) in percent deviation (or percentage point deviation for rates) from control state; the horizontal axes indicate quarters to or from the shock.