Equilibrium to Equilibrium Dynamics in a Climate Change Economy

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Abstract

Different from past economic research, I incorporate recent theories by climatologists that burning fossil fuels increases the equilibrium level of carbon dioxide, CO_2, in the atmosphere into a macroeconomic growth model. In the model, both production and consumption produces CO_2 emissions. I also assume that Anthropogenic Global Warming, AGW, not only damages production, but also capital stock and utility. In addition, a boundary condition similar to the Simpson-Kombayashi-Ingersoll (SKI) Limit holds, that there exists a critical temperature which leads to runaway greenhouse warming. Under these assumptions with no alternative fuel sources, and unlimited fossil fuel supply, the economy eventually flat lines (dies). Abatement only delays the inevitable. Using discount factors of .95 and .99, modest abatement policy can increase world welfare but do not increase life expectancy. When the discount factor is zero, conclusions of which policy is best is sensitive to the time horizon policy makers use. If a 100 year time horizon is used, modest consumption abatement may be the best policy, which may actually decrease life expectancy. However, a theoretical infinite time horizon may imply near zero emissions as the growth of damages is reduced and the probability of survival increases. Unfortunately, because humans have finite lives, this creates an ethical dilemma. In order for society to be better off, the current living must sacrifice by accepting policies that lead them to experience near zero output, consumption and near zero lifetime utility and welfare.

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1 Introduction

Early in the twentieth century, climate change or global warming can be considered as the mother of all externalities.[1] The great majority of research scientists studying climate change endorse anthropogenic global warming (AGW), the theory that man-made emissions of greenhouse gasses is causing climate change and increasing average global temperatures. In fact, in a study conducted by Cook et al. (2013), p.4, Table 3, in a sample of 10,356 authors of published research papers on climate change, 98.4% endorse AGW. Only 1.2% of authors rejected AGW, while 0.4% were undecided.

Furthermore, many research scientists believe that the consequences of AGW will be catastrophic unless we take action to stop climate change. Consider this quote from Hansen et al. (2013):

> Our calculated global warming in this case is 16 C, with warming at the poles approximately 30 C. Calculated warming over land areas averages approximately 20 C. Such temperatures would eliminate grain production in almost all agricultural regions in the world. Increased stratospheric water vapour would diminish the stratospheric ozone layer. More ominously, global warming of that magnitude would make most of the planet uninhabitable by humans.

The scientists warn that if climate change is not stopped, walking on most of the earth's surface will result in lethal hyperthermia. They claim that enough global heating has occurred to damage health and worker productivity, hitting middle income and lower income countries located at low latitudes the hardest.

The worst case scenario occurs when the greenhouse effect reaches a critical point called the Simpson-Kombayashi-Ingersoll Limit (SKI Limit), AKA

[1] See Tol (2009), p. 29.

Kombayashi-Ingersoll Limit.[2] In planetary science, the SKI Limit is the maximum rate of energy loss a planet's atmosphere by infrared radiation or the maximum solar flux a planet can handle without a runaway greenhouse effect occurring. When this limit is reached, the temperature of a planet's surface increases until all the planet's surface water evaporates. The water then disassociates into hydrogen and oxygen molecules. The hydrogen molecules escapes the earth's atmosphere, and the planet becomes a furnace like Venus. The temperature on the surface of Venus is so hot, that lead melts. Obviously, earth would not be habitable for humans if the SKI Limit is reached.[3]

Although most economists studying climate change believe that AGW is occurring, they are skeptical that dramatic action is needed to curb greenhouse gasses. The main tool used to study the economics of climate change is the Dynamic Integrated Climate-Economy model (DICE) developed by William Nordhaus.[4] The DICE model modifies the Ramsey Growth model by incorporating carbon dioxide emissions and other greenhouse gasses (GHGs). Climate-emissions-damage equations are then used to link the economy with economic change. Nordhaus also developed Regional Integrated Climate-Economy model (RICE). The RICE model modifies the DICE model by breaking down the economy or world into 12 separate regions. Using the RICE model, economists can study the economic effects of climate change on separate regions of the world, and answer questions regarding whether or not climate change impacts poor and rich equally. Nordhaus concludes

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[2] When the SKI Limit is reached, the earth becomes a furnace like Venus. This is the worst case scenario that anthropogenic CO_2 probably cannot cause. However, there are other lower ended runaway greenhouse effect temperatures that are not technically classified as a SKI Limit. In this paper, I also call these lower ended instability points as SKI Limits.

[3] For an explanation of the SKI Limit see Van Delden (2015), pp. 228-29.

[4] See Nordhaus (1992).

that at least in the early stages of climate change, only modest economic measures are needed to control GHGs.

Nicholas Stern criticizes the results of these DICE model predictions, Stern (2006). According to Stern, traditional economic climate change models underestimate climate sensitivity, damages to health and natural capital, positive feedback mechanisms and low probability but catastrophic outcomes. In this paper, I attempt to explain why the disconnect between science and economics by addressing some of Stern's concerns. I believe the possibility of reaching the SKI Limit, however small, is one reason for this disconnect. Another possibility is viewing greenhouse gasses as stock quantities that decay over time, rather than quantities that are transitioning from equilibrium to equilibrium.[5]

In this paper I revise the DICE framework to make more compatible to scientific theory. Modeling low probability but high costs outcomes will be reported. My ultimate goal, perhaps in another paper, is to see how AGW and public policies regarding AGW effects the GDP, business cycles and the distribution of wealth and income. This is an ambitious goal, but these issues are some of the world's biggest challenges for centuries to come.

The organization of this paper is as follows: A scientific background of climate change is discussed in Section 2. I propose a climate change model in Section 3. I conduct simulations of the climate change model in Section 4. I make concluding thoughts and propose future research in Section 5.

[5] According to Archer (2005), "The mean lifetime of anthropogenic CO_2 is dominated by the long tail, resulting in a range of 30–35 kyr." In the past, the lifetime of fossil fuel CO_2 was assumed to be 300 years.

2 Scientific Background of Anthropogenic Global Warming

Before we model the economics of AGW, we must know the scientific basis for climate change. Without a scientific model of climate change, we cannot have a model that approximates economic reality. First, we must know how do scientists know that GHGs, in particular carbon dioxide CO_2, cause climate change. We must ask what is the relationship of atmospheric carbon dioxide concentration to global temperatures. What percentage of carbon dioxide in the atmosphere is anthropogenic? When a lot of carbon dioxide is released into the atmosphere what is its residence time in the atmosphere? Will all the carbon dioxide released into the atmosphere eventually be absorbed by the oceans or the biosphere or does the atmosphere transition to a permanently higher carbon dioxide concentration equilibrium level? Are there positive and/or negative feedback loops? Is it possible that there exist a critical concentration level that will cause a runaway greenhouse effect that makes the world uninhabitable for humans?

2.1 The Relationship Between Atmospheric Carbon Dioxide and Average Global Temperatures

Like in economics, we like to explain certain behavior both on a macro and micro level, hence, the New Classical revolution in macroeconomics. Evidence that greenhouse gasses such as carbon dioxide causes global warming comes from both the molecular and macro level. At the molecular level, molecules that display a greenhouse effect on earth have dipole moments or as they vibrate produce temporary dipole moments. A dipole moment occurs when a molecule display local charges. Overall, these molecules have a neutral charge, but the molecule is not uniformly neutrally charged. Water is an example of a molecule with a dipole moment. Because some parts of a water molecule has positive charge and some parts are negatively charged, water molecules have a tendency to stick together. This feature also causes water vapor molecules to trap infrared radiation. Carbon dioxide is a good example of a molecule with a dipole moment.

However, carbon dioxide and water molecules are not greenhouse gasses on planets with a dense atmosphere where the molecules undergo frequent collisions. On planets or moons with dense atmospheres such as gas giant planets or moons such a Titan, diatomic molecules such as nitrogen and hydrogen are greenhouse gasses.[6]

Simple experiments also show that carbon dioxide is a greenhouse gas. A common experiment is to fill half of two identical bottles with water. Fill one bottle with air, and the other fill it with carbon dioxide. Filling a bottle with carbon dioxide can be as simple as dissolving alka seltzer in the water. Enclose the bottles with a stopper and stick a thermometer through the stopper. Shine a light or leave the two bottles in the sunlight. After one hour, the bottle with carbon dioxide is nine degrees Celsius greater than the bottle with just air.[7]

At the macro level, scientists use various means to measure the concentration of carbon dioxide in the atmosphere and compare that to the global average surface temperature. The first question is whether if anthropogenic carbon dioxide is increasing in the atmosphere. The answer is yes. The laboratory observatory in Muana Loa, Hawaii has been measuring atmospheric carbon dioxide concentrations since March, 1958. In their first measurement, the concentration of atmospheric

[6] See Colose (2008) for a general discussion on the Physics or Chemistry of greenhouse gasses.
[7] See Christensen (2015)

carbon dioxide was found to be 315.71 parts per million volume (ppmv). Concentration levels have trended upwards and in April, 2015 reached 403.26 ppmv.[8]

For atmospheric carbon dioxide concentrations before 1958, ice core from Siple Station, Antarctica has been analyzed. Friedli et al. (1986) estimates that atmospheric carbon dioxide concentration was 276.8 ppmv in 1744. Concentration levels steadily rose to 312.7 ppmv in 1953. The trend in carbon dioxide build up is illustrated in Figure 1.

If we assume that in 1744 the world was pre industrial revolution, and that a concentration of 276.8 ppmv is a good approximation of the equilibrium concentration of carbon dioxide CO_2 in the atmosphere with zero man-made carbon dioxide emissions, and the rise of carbon dioxide concentration levels represent anthropogenic carbon dioxide. Then man-made carbon dioxide represents 31.4% of the total atmospheric carbon dioxide.

The National Aeronautics and Space Administration (NASA) has conducted surface temperature studies of average monthly global, and regional temperatures from the years 1880-current, GISTEMPTeam (2015a). In 1880, the average global temperature was 0.38 degrees Celsius lower than the average mean global temperature between 1951-1980. In 2015, the average global temperature of 0.86 degrees Celsius higher than the average global temperature between 1951-1980. This means that the world was 1.24 degrees Celsius warmer in 2015, then in 1880. From Figure 2, one can observe that average global temperatures have been on the uptrend since 1880.[9]

[8] See Tans and Keeling (2015) for the full monthly data table.

[9] For a more detailed explanation on temperature measurement methodology see Hansen et al. (2010).

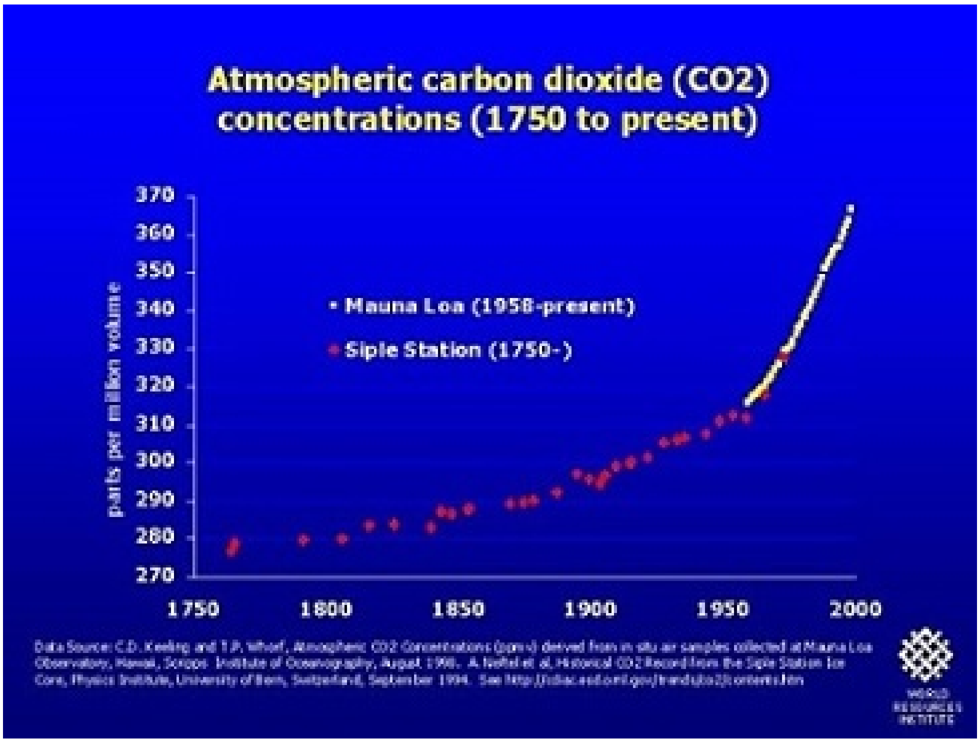

Figure 1: Historical Carbon Dioxide levels using data obtained from the Mauna Loa Observatory and Siple Station, Antarctica.

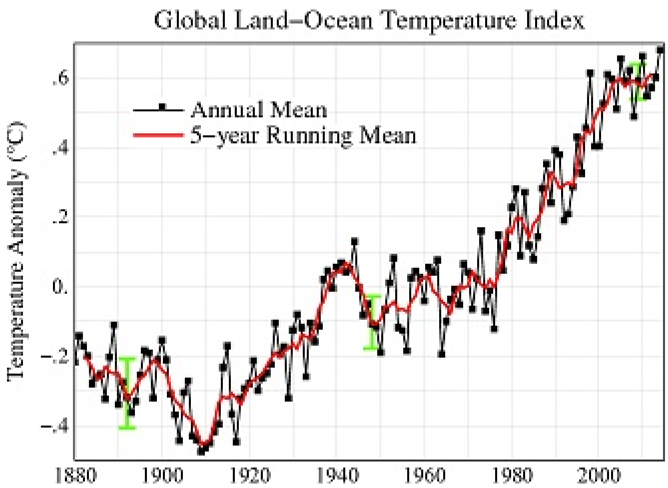

Figure 2: The average global temperature has trended upwards since 1880. Note, that the temperature is in terms of deviation from the 1951-1980 mean temperature. Graph retrieved from GISTEMPTeam (2015b).

Mann and Bradley (1994) extends the analysis to between 1000 AD to the current year using tree ring data. He finds that temperatures actually fell in the early part of the second millennial was near the 1902-1980 global mean temperature. The temperature gradually cooled down most of the second millennial, but since the Industrial Revolution, global mean temperatures have soared.

Once there appears to be a cause and effect relationship between atmospheric carbon dioxide concentrations and temperature, or temperature appears to be "climate sensitive" to carbon dioxide, a mathematical relationship between the two variables is theorized. To calculate climate sensitivity, radioactive forcing F is related to a greenhouse gas. Let λ be a climate sensitivity parameter, ΔT_s the change in surface temperature, and ΔF the change in radioactive forcing, the general equation is:

$$\Delta T_s = \lambda \Delta F \tag{1}$$

Each GHG has a different expression for its radiative forcing. Radioactive forcing from carbon dioxide actually is expressed by several different alternative equations. Let, α be a constant, C_0 denote initial carbon dioxide concentration, c_f final concentration, a common relationship used is:

$$\begin{aligned}\Delta F = 3.35\big[&ln(1 + 1.2C_f + .005C_f^2 + 1.4x10^{-6}C_f^3) - \\ &- ln(1 + 1.2C_0 + .005C_0^2 + 1.4x10^{-6}C_0^3)\big]\end{aligned} \tag{2}$$

For a more detailed discussion on radioactive forcing and climate change see IPPC (2001).

Most scientists follow Hansen et al. (1988) and state the relationship in terms of temperature increase per doubling of atmospheric carbon dioxide. For example, Hansen states that his model has an equilibrium climate sensitivity, ESC, of 4.2 degrees Celsius for every doubling of atmospheric carbon dioxide concentration. Combining equations (1) and (2), then:

$$\begin{aligned}\Delta T_s = 3.35\lambda \left[ln(1+1.2C_f+.005C_f^2+1.4x10^{-6}C_f^3)- \right.\\ \left. -ln(1+1.2C_0+.005C_0^2+1.4x10^{-6}C_0^3) \right]\end{aligned} \tag{3}$$

Using Hansen's numbers and $C_0 = 278.6$, λ is approximately 1.06. Notice, that λ is increasing in climate sensitivity. Also, if climate sensitivity is constant every time that atmospheric CO_2 concentration doubles, then there is a simple logarithmic relationship between temperature change in CO_2 concentration in the form:

$$b_1 Ln\left(\frac{C_f}{C_0}\right) = \Delta T_s \tag{4}$$

where b_1 is a climate sensitivity parameter. For the current atmospheric CO_2 concentration, equation 4 is a good approximation to equation 3.[10] Thus, to simplify proofs, I will use equation 4 instead of equation 3.

Many models have been constructed that have estimated an ESC. In a report by the Intergovernmental Panel of Climate Change (IPCC), Flato and Marotzke (2013), estimations of ECS from 23 climate change models were reported. The

[10] Equation 3 is a third order Taylor Series approximation of a more complex equation, while equation 4 is the first order approximation. To simplify proofs, I will use equation 4 instead of equation 3.

values ranged from 2.1 to 4.6 degrees Celsius, with a mean of 3.2 degrees Celsius. The mean ESC corresponds to a λ of .807. Let $b_1 = 3.35\lambda$, then $b_1 = 2.705$.

2.2 Transitional Dynamics to New Equilibrium Atmospheric Carbon Dioxide Levels

So far we have described the historical relationship of atmospheric carbon dioxide to global surface temperatures. However, what we want to know is what happens with increasing anthropogenic carbon dioxide emissions. One temptation is to think about what happens when 100 molecules of carbon dioxide is released into the air. We think of the release as increasing the carbon dioxide stock in the atmosphere by 100 molecules. We are tempted to want to know what the half life or average residence time is for a carbon dioxide in the atmosphere, before it is absorbed by the biosphere or ocean. If the average residence time for a carbon dioxide molecule is 30 years, then in 30 years we expect 50 molecules of carbon dioxide to remain in the atmosphere. We would expect all of the molecules to be gone from the atmosphere in approximately 200 years. Unfortunately, this is not the appropriate story to tell. Rather, when a substantial amount of carbon dioxide is released into the atmosphere, some of the molecules will remain in the atmosphere for thousands of years. This occurs because a substantial emission of carbon dioxide will increase the equilibrium level of carbon dioxide molecules in the atmosphere.

Some researchers will assume that the deep oceans will take up more carbon dioxide and act like a sinkhole, thereby allowing an emission of carbon dioxide to slowly diffuse out of the atmosphere. However, during the equilibrium process, the oceans will warm, decreasing its efficiency in absorbing carbon dioxide. Moreover, the ocean will become more acidic, shrinking the biosphere and slowing down the absorption rate of the oceans. In the end, increasing carbon dioxide emissions into the atmosphere will lead to more carbon dioxide in the atmosphere, biosphere and oceans.

In fact, dumping carbon dioxide into the biosphere or ocean will eventually increase atmospheric carbon dioxide. Some sources of emissions into the sea are submarines, oil spills and fuel leaks. According to the Second Law of Thermodynamics, the greenhouse gas will disperse throughout the carbon cycle system, explained below.

We do have data of carbon dioxide emissions data since the pre-Industrial Revolution. Boden et al. (2008) of the Carbon Dioxide Information Analysis Center (CDIAC), reports global emissions from 1751-2008. Global emissions have steadily increased since 1751 when anthropogenic carbon dioxide emissions were approximately three million metric tons. In 2008, global emissions were approximately 8.75 billion metric tons. This represents nearly a 3,000 times increase of carbon dioxide release into the atmosphere. Figure 3, will show you a graph of the CDIAC data.

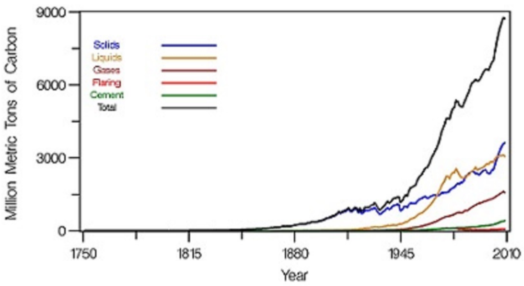

Figure 3: Global of anthropogenic carbon dioxide emissions have exploded since the pre-Industrial Revolution period.

The figure breaks down carbon dioxide emissions from solids, liquids, gasses, flaring and cement. Emissions from solids, liquids and gasses have especially been large.

The anthropogenic carbon dioxide emission explosion corresponds to the increase of carbon dioxide concentration levels in the atmosphere. This is an indication that indeed that man-made carbon dioxide emissions are significant in the increasing carbon dioxide levels in the atmosphere. The question becomes, what is the relationship of future carbon dioxide emissions with carbon dioxide concentration levels in the atmosphere, and then subsequently with average global temperatures.

Economists might be tempted to write the carbon dioxide or GHG concentration in the atmosphere in the form of:

$$M(t) = \varphi m(t) + (1 - \delta_m)M(t-1) \tag{5}$$

where $M(t)$ is the pollution or greenhouse gas level in the atmosphere at time or year t, φ is the retention level of the current amount of emissions of pollution $m(t)$, and $M(t-1)$ is the pollution level in the previous time period. In fact, this is the form Nordhaus (1992) and Nordhaus and Sztorc (2013) uses in his DICE model. In the DICE model, the deep ocean acts like a sinkhole so emissions in the atmosphere will slowly decay throughout time. As mentioned before, this equation does not take into account that substantial emissions of GHGs (or carbon dioxide in this example) into the atmosphere increases the equilibrium concentration levels of the greenhouse gasses. This occurs because the Second Law of Thermodynamics states that the molecules must disperse from greater order to less order. In other words, entropy is always increasing. This relationship will underestimate the amount of build up of atmospheric greenhouse gasses.

To understand the equilibrium concentration level of atmospheric carbon dioxide, one must understand how carbon dioxide moves along in the carbon cycle and how fossil fuels are made. Fossil fuels such as coal and oil are formed over millions of years by once living organisms. When fossil fuels are burned for its energy, hydrocarbons denoted as $C_xH_y$reacts with z molecules of oxygen O_2. This forms x molecules of carbon dioxide CO_2 and $y/2$ molecules of water H_2O. Formally, the chemical reaction is written as:

$$C_xH_y + zO_2 \rightarrow xCO_2 + \frac{y}{2}H_2O \quad (6)$$

The xCO_2 molecules are released into the atmosphere. Overtime, some of these molecules are absorbed by the biosphere and by the ocean. The carbon dioxide enters the biosphere through photosynthesis by plants. Plants use sunlight denoted as ν, CO_2 and H_2O to produce a carbohydrate CH_2O and O_2 or:

$$CO_2 + H_2O + \nu \rightarrow CH_2O + O_2 \quad (7)$$

Some of the carbon dioxide are brought back into the atmosphere through a process caused respiration. In respiration, some living organisms breath in O_2. The oxygen reacts with the carbohydrate, which produces energy ε, H_2O and CO_2 or:

$$CH_2O + O_2 \rightarrow CO_2 + H_2O + \varepsilon \tag{8}$$

Some of carbon dioxide in both the atmosphere will be absorbed by the ocean. The CO_2 combines with H_2O to form carbonic acid H_2CO_3 or:

$$CO_2 + H_2O \rightarrow H_2CO_3 \tag{9}$$

The carbon cycle is completed when two carbonic acid molecules interact with three water molecules and double positive charged calcium cations Ca^{++} to form calcium carbonate $CaCO_3$, CO_2 and four molecules of water or:

$$2H_2CO_3 + 3H_2O + Ca^{++} \rightarrow CaCO_3 + CO_2 + 4H_2O \tag{10}$$

Because it takes million of years for fossil fuels to form, within a short time frame of thousands of years, the carbon locked in these fossil fuels can be considered to be outside of the carbon cycle. Thus, introducing thousands of tons of carbon dioxide into the carbon cycle system will increase the equilibrium level of carbon dioxide in the whole system of the atmosphere, biosphere and ocean. Some of the carbon dioxide molecules will diffuse into the biosphere, others into the ocean, but others will remain in the atmosphere (or at least the molecules that leave the atmosphere are replaced by molecules that enter the atmosphere). Thus, the atmospheric carbon dioxide concentration written in equation 5 underestimates the long run concentration of atmospheric carbon dioxide.

A better equation would take into account that a fraction of a period $t+1$ emissions quantity, ϖ is part of the long run equilibrium amount that will remain or be replaced by CO_2 molecules from the biosphere or ocean. The remaining fraction $(1-\varpi)$ will slowly diffuse into the ocean or biosphere. This quantity will leave the atmosphere at a rate δ_m per period. The pre-Industrial Revolution quantity of carbon dioxide $\dot{M}_t^e$ will always remain in the atmosphere as long as net cumulative emissions is nonzero, and the pre-Industrial Revolution concentration approximates an equilibrium level, hence the superscript e. The dot over the letter indicates that the variable is a net quantity or flow variable. The atmospheric GHG or carbon dioxide concentration in period $t+N$ is then:

$$\dot{M}(t+n) = \varpi\varphi\sum_{i=1}^{N}\dot{m}(t+i) + (1-\varpi)\varphi\sum_{i=0}^{N}(1-\delta_m)^i\dot{m}(t+N-i) + \dot{M}^e(t) \quad (11)$$

where $0 \geq \varpi \leq 1$.

The term $(1-\varpi)\varphi\sum_{i=0}^{N}(1-\delta_m)^i\dot{m}(t+N-i)$ are the net emissions that will eventually diffuse back into the biosphere and oceans. The term $\varpi\varphi\sum_{i=1}^{N}\dot{m}(t+1)$ is the net emissions that will always stay in the atmosphere, and with constant emissions diverges. Under constant emissions, if there is an infinite supply of fossil fuels outside of the carbon cycle system, eventually either the earth's atmosphere will have an infinite number of carbon dioxide molecules, or the earth ends up as a ball of fire. This occurs even if though equation 11 underestimates the atmospheric CO_2 concentration level by assuming that ϖ is constant. In reality, the biosphere is contracting. Since the biosphere is contracting due to deforestation and urban spread, ϖ is really rising over time.

We can write the mass balance equation above in terms of long term equilibrium concentration levels $\dot{M}^e_{t+N}$. The long run equilibrium concentration of GHGs is:

$$\dot{M}^e_{t+N} = \varpi \sum_{i=1}^{N} \dot{m}(t+i) + \dot{M}^e(t) \tag{12}$$

Equation 12 implies that the long term equilibrium concentration level of greenhouse gasses increases when fossil fuels obtained outside of the carbon cycle system is introduced into the system. This is why AGW is potentially a catastrophic event.

But the story does not end here. Carbon and other GHG emissions occur in the biosphere and oceans, denoted as $\dot{M}^x_{t+1}$. In period $t+2$, a fraction of the $t+1$ emissions δ_{xm} transfers into the atmosphere. In period $t+3$ another $\delta_{xm}(1-\delta_{xm})$ of period t+1 emissions, δ_{xm} of period t+2 emissions will transfer to the atmosphere. In period $t+4$, $\delta_{xm}(1-\delta_{xm})^2$ of period t+1 emissions, $\delta_{xm}(1-\delta_{xm})$ of period $t+2$, and δ_{xm} of $t+3$ emissions will transfer into the atmosphere. This reasoning can be applied up to period $t+N$. Thus, letting ϖ_x denote the equilibrium fraction of CO_2 that remain outside of the atmosphere, φ_x is the composite retention rate of emissions in the sea and biosphere, equation 11 can be rewritten as:

$$\begin{aligned}\dot{M}(t+n) = \varpi\varphi \sum_{i=1}^{N} \dot{m}(t+i) + (1-\varpi)\varphi \sum_{i=0}^{N} (1-\delta_m)^i \dot{m}(t+N-i) + \\ +(1-\varpi_x)(1-\varphi_x)\delta_{xm}\left(\sum_{j=1}^{N-1}\sum_{i=1}^{j-1}(1-\delta_{xm})\dot{m^x}_{t+N-j}\right) + \dot{M}^e(t)\end{aligned} \tag{13}$$

Equation 13 is the general mass balance equation for GHG accumulation in the atmosphere. Unfortunately, it implies that AGW is even worse than what equations (11) and (12) suggest. In practice, the atmospheric CO_2 buildup due to emissions into the biosphere and oceans term may be negligible, and computer modelers can just use equation 11.

The good news from equation 13 is that there is not an infinite supply of fossil fuels buried beneath the earth. Eventually, we will run out of fossil fuels before the earth sizzles and have to switch to alternative energy sources. The bad news is that a minority of climatologists believe that there is enough fossil fuels for the earth to reach the Simpson-Kombayashi-Ingersoll (SKI) Limit.

We can think of the SKI Limit as an existence of discontinuity in the carbon dioxide build up equation. As mentioned in the introduction, the SKI Limit is the maximum rate of energy loss a planet's atmosphere by infrared radiation or the maximum solar flux a planet can handle without a runaway greenhouse effect occurring. When this occurs, the earth reaches a critical temperature where ocean water begins to evaporate at a rate where water vapor, another GHG, is rapid enough that when it causes the temperature rise, the evaporation rate of water increases, causing a vicious cycle or a positive feedback loop. Other phenomenon such as increasing cloud cover will tend to cool the earth. But in the worst case scenario, the negative feedback cloud effect is overwhelmed by the positive feedback effects. Although water is considered to be a greenhouse gas, its main effect is to augment the effects of CO_2 on temperature. Let b_1 be the climate sensitivity with respect to carbon dioxide, b_2 an added component to climate sensitivity caused by water vapor, τ_f the final temperature, τ_0 the pre-Industrial Revolution temperature, and $S_t = 0, 1$ a switching or dummy variable, then, the climate sensitivity equation is:

$$\begin{aligned} 3.35\lambda \left[ln(1 + 1.2C_f + .005C_f^2 + 1.4x10^{-6}C_f^3) - \right. \\ \left. -ln(1 + 1.2C_0 + .005C_0^2 + 1.4x10^{-6}C_0^3)\right] + b_2 S_t \frac{[C_f]}{[C_0]} = \tau_f - \tau_0 = \Delta\tau_s \end{aligned} \quad (14)$$

The dummy variable switches on when the earth's average surface temperature reaches a critical temperature. Thus, letting τ_c denote the critical temperature, $S_t = 0$ for $\tau_f < \tau_c$ and $S_t = 1$ for $\tau_f \geq \tau_c$. Once the critical temperature reaches the critical temperature, average global temperatures accelerate. Beyond this temperature, there is no point of return, earth sizzles.

2.3 Work, Energy and Damages

The economy runs on energy, specifically on work. Work is the portion of energy that does what we want to do. For example, if we want to drive 100 miles, work is the force that acts upon the car that makes it drive 100 miles. The rest of the energy dissipates as heat, often measured as the change in enthalpy. Unfortunately, in our need to use work, carbon dioxide is emitted into the atmosphere causing damages.

Equation 6 is a representation of the mass balance of a combustion reaction of fossil fuels that emits carbon dioxide into the atmosphere. Associated with the mass balance is an energy balance. The energy balance of a car trip, assuming no stops at a gas station is as follows: Let E_o be the energy level of a tank of gas before the trip, E_f the energy level of the tank of gas after the trip, W the work done during the trip, and Q the heat created, then equation 6 under went the following energy transformation:

$$E_o \rightarrow E_f + W + Q \tag{15}$$

Thus,

$$W = \Delta E + Q \tag{16}$$

From equation 16, one way to reduce carbon dioxide emissions is to make an engine more efficient by making it create more work and less heat. Nevertheless, when most people pay their electric bill, they pay for total energy usage in Kilowatt-hours. Thus, in my economic models I will use energy rather than work as one of my factors of production.

The question is: How much damage does consuming a kilowatt-hour of energy cost the economy? This is a difficult question, for what is damage is often a matter of perspective. This is why damage functions across economic models vary. However, most economic climate change models assume that damages are proportional to average world surface temperatures. For example, DICE models assumes that AGW damages a fraction of the global output Y, Nordhaus and Sztorc (2013). Let T denote average global surface temperatures above the average global surface temperature pre-Industrial Revolution, π_1 and π_2 be coefficients determined by curve fitting techniques, and $\hat{\Omega} \in [0,1]$ be the fractional damages to world output, fractional damages to world output is:

$$\hat{\Omega} = 1 - 1/(1 + \pi_1 \tau + \pi_2 \tau^2) \tag{17}$$

Since the coefficients are positive, AGW cannot cause more damages than world output.[11]

Other researchers such as Tol (2002) believe that damages must be disaggregated among sectors of the economy and regions of the world. Tol concludes that AGW will result in winners and losers. Some regions of the world will benefit from global warming and other regions will be harmed. Depending on the aggregate rule for global estimates, AGW may overall harm or benefit the world. This Climate Framework for Uncertainty, Negotiation and Distribution (FUND) model has been updated and described in Waldhoff et al. (2014). In the updated version, other non-carbon dioxide GHGs are analyzed. Nevertheless, similar conclusions are reached.

Nicholas Stern and Simon Dietz in Dietz and Stern (2014) believe that the economic profession in general has underestimated economic damages due to AGW. First, discount rates used have been too high. Second, damages can occur to the natural capital stock as well as to global output. Thus, it is possible that AGW in a given year can cause more damage than global output. Finally, damages are convex, and they propose a damage equation in the form:

$$\hat{\Omega} = 1 - 1/(1 + \pi_1 \tau + \pi_2 \tau^2 + \pi_3 \tau^{6.754}) \tag{18}$$

[11] Note for convenience I use $\hat{\Omega}$ where as Nordhaus uses just Ω. The relationship between these two variables is $\hat{\Omega} = 1/(1+\Omega)$.

where at $\tau = 6$, the coefficients π_1, π_2, and pi_3 are fixed such that $\hat{\Omega} = 0.5$.

In my view, Stern is closer to the truth than Tol and Nordhaus. I will use Nordhaus's formulation, but assume that AGW can damage capital and human capital stock. Moreover, I assume that CO_2 emissions whether in the atmosphere, biosphere or oceans increases equilibrium carbon dioxide concentrations throughout the carbon cycle system. Modeling the possibility of reaching the SKI Limit, raises the possibility of even greater catastrophe.

In most models, $\hat{\Omega}$ converges to one, only if average global temperatures converges to infinity. However, common sense tells us that damages on earth are complete when average global temperature difference from the pre-Industrial temperature reaches a critical temperature difference τ_c. Thus, letting $S = 0$ for $\tau < \tau_c$ and $S = 1$ for $\tau \geq \tau_C$, capital stock K, I use the fractional damage function to capital stock $\hat{\Omega}^k = 1 \in [0,1]$ as

$$\hat{\Omega}^k = 1 - (1-S)/(1 + \pi_1^k \tau + \pi_2^k \tau^2) \tag{19}$$

Equation 19 implies that all of the earth's capital stock is destroyed when the earth's temperature reaches the critical temperature. This form of damages also applies to global output Y, and human capital H.

I can now state the main theory of this research:

Theorem 1. *If the volumes of the atmosphere, biosphere and oceans are constant, the supply of fossil fuels is unlimited and CO_2, M, emissions are always positive and decreasing but never converging to zero, then the economy eventually suffers* 100% *damage or* $\hat{\Omega} \to 1$ *as time* $t \to \infty$.

Proof. Assuming no emissions into the biosphere or oceans, the concentration of CO_2 at $t = t + m$ is given by equation 11 or $\dot{M}(t+n) = \varpi\varphi \sum_{i=1}^{N} \dot{m}(t+i) + (1 - \varpi)\varphi \sum_{i=0}^{N} (1-\delta_m)^i \dot{m}(t+N-i) + \dot{M}^e(t)$. The term $(1-\varpi)\varphi \sum_{i=0}^{N}(1-\delta_m)^i \dot{m}(t+N-i)$ although decreasing is always positive. The term $\dot{M}^e(t)$ is constant and positive. However, the term $\varpi\varphi \sum_{i=1}^{N} \dot{m}(t+i)$ is always increasing, and if emissions decrease but do not converge to zero, then the emissions converge to a positive constant rate. If the emission rate is a positive constant, then $\varpi\varphi \sum_{i=1}^{N} \dot{m}(t+i) \to \infty \therefore \lim_{t\to\infty} M(t+N) = \infty$.

Assume that there is no critical temperature τ_c, and τ be $\Delta\tau_s$, $[\dot{M}(0)]$ the pre-Industrial Revolution CO_2 concentration and redefine $\dot{M}(t+N)$ as $\dot{M}(t)$ then $b_1 \ln \frac{[\dot{M}_t]}{[\dot{M}_0]} = \tau$. Since $b_1 > 0$, $\lim_{t\to\infty} b_1 \ln \frac{[\dot{M}(t)]}{[\dot{M}(0)]} = \infty$. $\therefore \tau \to \infty$, but damages $\hat{\Omega} = 1 - 1/(1 + \pi_1\tau + \pi_2\tau^2)$, $\lim_{t\to\infty} 1/(1+\pi_1\tau+\pi_2\tau^2) = 1 \therefore \hat{\Omega} \to 1$ □

The good news from Theorem 1 is that extinction and complete devastation never occurs in finite time. However, if T_c is finite, then sometime in finite time, the switching variable S turns on. At this time, the human race is completely destroyed. I can now state the following theorem:

Theorem 2. *If theorem 1 is valid except that τ_c is finite, then the economy suffers 100% damage in finite time or $\hat{\Omega} \to 1$ in finite time.*

Proof. From Theorem 1, $\lim_{t\to\infty} \tau = \infty$. Let $S(t) = 0$ for $\tau < \tau_c$, and $S(t) = 1$ for $\tau \geq \tau_c$. Since $\tau \to \infty$, in finite time it will reach an arbitrary finite temperature. Since T_c is finite, $S(t) = 1$ sometime in finite time. Since $\hat{\Omega} = 1 - (1-S)/(1+\pi_1\tau+\pi_2\tau^2)$, $\hat{\Omega} \to 1$ in finite time. □

The moral of the story is that the existence of a SKI Limit limits our time on earth if fossil fuels are unlimited, removal technology is not possible, and we do not move toward zero emissions.

3 A Simple Economic Model

The purpose of this economic model is expository and not to accurately predict the economic future. Rather, I want to explain what may happen if we do not move toward zero emissions. Thus, for the purpose of this paper, I use a simple neoclassical model. We can introduce more complicated neoclassical features in future papers.

3.1 The General Setup

We start with Robert Solow's growth model[12], but also postulate that output is also related to human capital. As in Dietz and Stern (2014), AGW can damage capital stock K_t but also human capital H_t, and labor L_t. Denote energy for production E_t^y. Abatement and removal costs at time t is denoted as $\wedge_t^y \in [0,1]$. Damages due output Y_t, K_t, H_t and L_t are denoted Ω_t^Y, Ω_t^K, Ω_t^H and Ω_t^L, respectively, each $\in [0,1]$. I assume that the resource constraint of the aggregate economy is:

$$(1-\Omega_t^Y)(1-\wedge_t^y)A_t F\left[K_t(\Omega_t^K), H_t(\Omega_t^H), L_t(\Omega_t^L), E_t^y\right] = Y_t \tag{20}$$

where A_t represents a technical growth index, F is a concave production function, which is increasing in K_t, H_t and labor L_t, but decreasing in all damage functions.

[12] See Solow (1957)

For the purpose of this paper A_t is assumed to grow at a constant rate. In the long run, technical change drive economic growth when the period of explosive capital and labor growth is over. Abatement strategies differ in removal costs as abatement costs prevent CO_2 emissions while removal strategies actually remove CO_2 from the atmosphere.

One novelty of this model is that AGW can cause direct damage to utility. Direct damages to utility means that the damage can be unrelated to traditional economic variables such as production, wealth and consumption. Although damages to utility is subjective, our happiness and well-being are important, too. While it is difficult to put a price on the value lost when a species go extinct, not to consider this possibility will result in the underestimation of damages. Fractional damages to utility is denoted as $\Omega_t^u \in [0,1]$. Total fractional damages to the economy is the theoretical best measure of damages because according to this economic model, utility maximization is the name of the game. If we observe the utility maximization problem, we notice that the resource constraint can be substituted in for consumption. We know that a composite loss of momentary utility will have two components; a direct loss from utility, and a drop in utility due to a drop in production. In addition, damages to human and capital stock will reduce production, too. Dropping subscript t, damages from loss of production will be in the form of $1-(1-\Omega^Y)f\big((1-\Omega_K)(1-\Omega_H)\big)$. That is, the fractional loss is multiplicative in output and a function of capital stock damages. This means that composite damages to utility will be in the form:

$$\Omega = \Omega_U + (1-\Omega_U)U\Big[1-(1-\Omega_Y)f\big((1-\Omega_K)(1-\Omega_H)\big)\Big] \tag{21}$$

where $\Omega \in [0,1]$. Equation 21 states that total fractional damages is the sum of the direct fractional damages to utility and the indirect fractional damages to utility caused by a reduction in production.

In this simple model, all consumers are identical and the representative agent maximizes her welfare W, which is a sum of their discounted momentary utilities. Although there are many consumption goods in the economy, some like travel resulting in high emissions of CO_2, and others that result in no emissions, for simplicity, I will just assume one aggregate composite consumption good C_t. People receive utility from consumption, and disutility from labor L_t. Energy usage is divided between production and household use E^c. Although we care about the total amount of CO_2 emissions, policymakers would like to know the relative emission amounts between production and consumption. If a substantial amounts of CO_2 emission sources are from households, then policies only dealing with production may be ineffective.

Consumers are also burdened with the possibility that a catastrophic event such as the SKI Limit being reached can happen. When the critical temperature is reached, the switching variable is turned on, and momentary utility U_t goes to zero. This also applies to production, where Y also goes to zero. The probability of survival is a function of global average temperature. Because the critical temperature is unknown, it is also a function of the lowest temperature where it is certain that the runaway greenhouse effect will take place and destroy the whole economy, denoted as $\tau_{c,max}$. After any given period, the probability of survival is $\mathcal{S}(\tau, \tau_{c,max})$. Letting β be the discount factor, aggregate welfare is given as:

$$W = \mathcal{S}(\tau, \tau_{c,max}) \sum_{t=1}^{Tmax} (1 - \Omega_t^u) \beta^{t-1} U_t \Big(C_t(E_t^c), L_t \Big) \tag{22}$$

where the maximum time period $Tmax$ is either infinite or when total fractional damages equals one.

Let C_t be consumption spending, I_{kt} be aggregate investment, I_{ht} be aggregate investment in human capital, and G_t be government expenditures. Aggregate demand is just given as the accounting identity of:

$$Y_t = C_t + I_{kt} + I_{ht} + G_t \tag{23}$$

If we assume the equilibrium condition of aggregate supply equaling aggregate demand, then:

$$\mathcal{S}(\tau, \tau_{c,max})(1-\Omega_t^Y)(1-\wedge_t^y)A_t F\left[K_t(\Omega_t^K), H_t(\Omega_t^H), L_t(\Omega_t^L), E_t^y\right] = C_t + I_{kt} + I_{ht} + G_t \tag{24}$$

Investment and capital stock follows the law of motion of:

$$I_t = K_{t+1} + (1-\delta_k)K_t \tag{25}$$

where K_{t+1} represents the capital stock in period $t+1$, K_t is the capital stock in period t, and δ_k is the depreciation rate. Equation 4 implies that investment is equal to the increase in capital stock when there is no depreciation.

I follow Schultz (1961) definition of human capital. According to Schultz, human capital includes health, formal education and job training. The production function now depends on period t human capital H_t. Human capital evolves over time just like capital:

$$H = H_{t+1} + (1 - \delta_h)H_t \tag{26}$$

where δ_h is the depreciation rate of human capital.

We can substitute Equation 25 and 26 into Equation 24 to get:

$$\begin{aligned} \mathcal{S}(\tau, \tau_{c,max})(1 - \Omega_t^Y)(1 - \wedge_t^y)A_t F\left[K_t(\Omega_t^K), H_t(\Omega_t^H), L_t(\Omega_t^L), E_t^y\right] = \\ = C_t(E_t^c) + K_{t+1} + (1 - \delta)K_t + H_{t+1} + (1 - \delta_h)H_t \end{aligned} \tag{27}$$

Equation 27 states that aggregate output is the sum of all individual economic activity in the economy, tempered by the effects of GHG emissions and measures to abate or remove the emissions. Note that government does not use human capital as a component of Gross Domestic Product GDP. Therefore, I will ignore this term in the model.

Ordinarily, $E_t^y = \sum_i^N E_{it}^y$ and $E_t^c = \sum_i^N E_{it}^c$, where $E_{1t}, E_{2t}...E_{Nt}$ are the different types of energy sources such as coal, oil, natural gas, solar, wind, hydroelectric, and nuclear power. Each type of energy source emits a different quantity of greenhouse gas emissions. Moreover, the conservation of energy states total energy E is equal to the sum of all energy sources:

$$E_t = E_t^y + E_t^c = \sum_i^N E_{it}^y + \sum_i^N E_{it}^c \tag{28}$$

Each energy source is also associated with a quantity of CO_2 emissions $E(\dot{m})$

Since consumption also produces and emits carbon dioxide into the atmosphere, the government can regulate consumption. Let $\wedge_t^c$ be consumption abatement, then the proportion of consumption is actually only $(1-\wedge_t^c)$. Regulation of consumption can come in a form of a sales tax or prohibitions on consumption of high carbon content goods. Damages to capital stock acts as a depreciation beyond normal a tear. Thus, capital stock follows the law of motion of:

$$K_{t+1} = I_t + (1-\Omega_t^K)(1-\delta)K_t \tag{29}$$

Assuming that L is fixed, each person maximizes their welfare by choosing an optimal level mix of consumption and leisure (labor) subject to their resource constraints and mass balance of atmospheric CO_2 . Letting there be T max time periods, and N energy sources, the final problem to be solved is to maximize utility:

$$\underset{C_t,}{\text{maximize}}\ W = \sum_{t=1}^{Tmax} \mathcal{S}(\tau, \tau_{c,max})(1-\Omega_t^u)\beta^{t-1}U_t\left(C_t\left(\sum_i^N E^c(\dot{m}_{it})\right), L_t\right)$$

subject to the resource constraint

$$\mathcal{S}(\tau, \tau_{c,max})(1-\Omega_t^Y)(1-\wedge_t)A_tF\left[K_t(\Omega_t^K), H_t(\Omega_t^H), L_t(\Omega_t^L), \sum_i^N E^c(\dot{m}_{it})\right] =$$
$$= C_t\left(\sum_i^N E^y(\dot{m}_{it})\right) + K_{t+1} + (1-\Omega^k)(1-\delta)K_t + H_{t+1} + (1-\delta_h)H_t + G_t$$

subject to your carbon balance constraint

$$\dot{M}(T) = \varpi\varphi \sum_{i=1}^{T} \dot{m}(i) + (1-\varpi)\varphi \sum_{i=0}^{T} (1-\delta_m)^i \dot{m}(T-i) +$$
$$+ (1-\varpi_x)(1-\varphi_x)\delta_{xm} \left(\sum_{j=1}^{T-1} \sum_{i=1}^{j-1} (1-\delta_{xm}) \dot{m^x}_{T-j} \right) + \dot{M}^e(1)$$

subject to your carbon dioxide-temperature relationship

$$\tau = b_1 \left[ln(1 + 1.2C_f + .005C_f^2 + 1.4x10^{-6}C_f^3) - \right.$$
$$\left. - ln(1 + 1.2C_0 + .005C_0^2 + 1.4x10^{-6}C_0^3) \right] + b_2 S_t \exp \frac{[\dot{M}_t]}{[\dot{M}_1]}$$

subject to the damage conditions:

$$\begin{aligned} \hat{\Omega}^y &= 1 - (1-S)/(1 + \pi_1^y \tau + \pi_2^y \tau^2) \\ \hat{\Omega}^k &= 1 - (1-S)/(1 + \pi_1^k \tau + \pi_2^k \tau^2) \\ \hat{\Omega}^h &= 1 - (1-S)/(1 + \pi_1^h \tau + \pi_2^h \tau^h) \\ \hat{\Omega}^u &= 1 - (1-S)/(1 + \pi_1^u \tau + \pi_2^u \tau^u) \\ \Omega_t = \Omega_{Ut} + (1-\Omega_{Ut})U &\Big[1 - (1-\Omega_{Yt}) f\big((1-\Omega_{Kt})(1-\Omega_{Ht})\big) \Big] \end{aligned} \tag{30}$$

From Equation 30, as carbon dioxide builds up in the atmosphere, damages to utility and the probability of doom increases. This effect will raise the relative price of consumption to production. However, damages to production also increases. This will raise the relative price of production. This effect will lower investment spending and increase consumption spending.

3.2 Simplifying and Specifying the Model

With all agents being identical, there is no borrowing and lending. What is produced in a given period must be consumed, invested in capital or invested in human capital. We know that as the world's capital stock becomes large, economic variables converge to a steady state growth. However, we will see that as damages and probability of doom increases, people substitute investment for consumption. Thus, capital stock may sometimes be convex. Moreover, we drop human capital from the analysis. However, we must keep in mind that increased chronic medical conditions does damage human capital.

The mass balance equations can be simplified by assuming perfect mixing of emissions into the atmosphere. The CO_2 level is broken down into two parts, the equilibrium and transitory levels. This means that the current CO_2 level is just the current retained emissions plus last period's equilibrium level and last years transitory level of CO_2 multiplied by the percentage of net CO_2 that does not transfer to the biosphere or oceans. Retained emissions is the quantity of CO_2 that is emitted into the atmosphere and not to the biosphere or sea and is given by the relationship $\dot{m}^r(t) = (1-\varpi)\varphi \sum_{i=0}^{N}(1-\delta_m)^i \dot{m}(t)$. Let $\dot{M}^t(t)$ be the transitory quantity of atmospheric CO_2, then the mass balance in Equation 30 is just:

$$\dot{M}(t) = \dot{m}(t) + \dot{M}^e(t-1) + (1-\delta_m)\dot{M}^t(t-1) \tag{31}$$

We assume that fossil fuel is the only energy source, and the quantity of emissions from consumption E^c is proportional to total consumption. Let α be a proportionality constant. Then

$$E^c = \sigma C \tag{32}$$

We will also assume that the ratio of E^y to E^c, ξ be a constant, then:

$$\frac{E^c}{E^y} = \sigma \tag{33}$$

where according to my calculations $\xi = .958$. Thus, carbon dioxide emissions per unit usage of energy is approximately equal for production and consumption. Assuming that emissions to the biosphere and sea are insignificant, cumulative atmospheric $C0_2$ emissions is given by Equation 11 where 276.8 ppmv is the initial equilibrium atmospheric carbon dioxide level. In 2014 according to NOAA (2015) the carbon dioxide concentration level averaged 396.48 ppmv.

Once total energy consumption is determined, we calculate total emissions of CO_2 by knowing that fossil fuels emit approximately 1.72 pounds of CO_2 per Kilo-Watt-Hour (KWH) of energy produced. This figure is approximated by taking a weighted average by consumption of the pounds of CO_2 can then be converted into units of ppmv,[13]

Cumulative atmospheric CO_2 concentration levels is given by Equation 31, with 276.8 ppmv as the initial equilibrium level of atmospheric carbon dioxide. The equilibrium level of CO_2 in 2014, is the initial equilibrium level plus the difference between pre-industrial CO_2 levels with 2014 levels, multiplied by the long run equilibrium fraction. The transitory level will just be the CO_2 level minus the

[13] See EIA (2015).

equilibrium level. From Nordhaus (1992) the transfer rate δ_m is .0083. According to CO2Now (2015), the net emission rate into the atmosphere between 2005-2014 was 2.11 ppmv per year. Assuming a retention rate of .64, as reported by Nordhaus (1992), total emissions into the carbon cycle system has averaged 3.3 ppmv per year.

Temperature deviation from the pre-Industrial Revolution average global temperature is:

$$\begin{aligned} \tau = b_1 \left[ln(1 + 1.2\dot{M}_t + .005\dot{M}_t^2 + 1.4x10^{-6}\dot{M}_t^3) - \right. \\ \left. - ln(1 + 1.2C_0 + .005(276.8)^2 + 1.4x10^{-6}276.8^3 \right] + b_2 S_t \frac{[\dot{M}_t]}{276.8} \end{aligned} \tag{34}$$

The critical temperature τ_c that triggers when the switching variable S_t is turned on, is assumed to be a temperature deviation of 35 degrees C, but can be varied during the simulation runs. I tried to use an exponential function on the switching variable term, but the computer was not able to handle such large number. Thus, I use a linear relationship. The initial predicted temperature is $\tau = 1.52$. This figure is much higher than the general consensus measurements of $\tau \approx .8$[14]

The damage function used is:

$$\hat{\Omega} = 1 - (1 - S)/(1 + \pi_1 \tau + \pi_2 \tau^2) \tag{35}$$

I will initially assume temperature increases damages output, capital, and utility in equal proportions. Thus, $\pi_1 = \pi_1^k = \pi_1^u$ and $\pi_2 = \pi_2^k = \pi_2^u$. Estimating damage functions is tricky and is probably the weakest link in AGW economic modeling.

[14] For an example, see Nordhaus (2013a).

According to the Energy Information Agency, EIA (2004), American households contributed about 41% of US carbon dioxide emissions in 2003. According to the Stockholm Environment Institute, Alistair et al. (2006), in 2006, CO_2 from production was 10.16 tonnes per capital, while CO_2 emissions from consumption was 11.01 per capital. Thus, in the United Kingdom, E^c accounted for 50.4% of emissions, and E^y was 49.6%. Dietz et al. (2009) reports that in 2005, American households contributed about 38% of US carbon dioxide emissions in 2005. Thus, 40% is a good estimate of the consumption contribution is to CO_2 emissions, while production contributes to 60% of emissions. From the data, I calculate E^c and E^y worldwide to be 48.9% and 51.1%, respectively. According to Nordhaus (1992), the retention rate $\varphi = .64$.

According to Archer (2005), 17 - 33% of fossil fuel CO_2 remains in the atmosphere after 1,000 years, the approximate time horizon of this research's simulations. Moreover, the atmospheric fraction of the CO_2 reservoir is 18%[15]. Thus, for the purpose of this research I assume that 18% of carbon dioxide emitted into the atmosphere is permanent. Therefore, I set $\varpi = .18$. I follow Knutti and Hegerl (2008) who finds in a survey of observations from various researchers a climate sensitivity of about 3.2. Coefficients on the damage terms follow Nordhaus (2013a) most recent updates, $\pi_1 = 0$ and $\pi_2 = .003$.

The functional form of the survival probability function must ensure that its value is always between zero and one. In this model, should τ dip equal to or below the pre-Industrial Revolution global average temperatures, then $\mathcal{S}(\tau, \tau_{c,max}) = 0 \; \forall \; \tau \leq \; 0$. Should the temperature reach the critical temperature, then $\mathcal{S}(\tau, \tau_{c,max}) = 0 \; \forall \; \tau \geq \; \tau_c$. When the temperature is between the pre-industrial

[15] The total reservoir size includes the reservoirs from the oceans and atmosphere.

and minimum maximum temperature required to reach the SKI Limit, the probability of a discontinuous catastrophe increases with rising temperatures. According to Goldblatt et al. (2013), the SKI Limit is reached when atmospheric CO_2 concentrations reach about 30,000 ppmv[16] or a temperature deviation from the pre-Industrial Revolution average global temperatures reaches about 22.1 degrees Celsius. Their claim that when the average global temperatures reaches 34.1 degrees Celsius, or 93.4 degrees Fahrenheit, evaporation of the oceans become irreversible. The earth's temperature soars to an unknown temperature in which I arbitrary set to 127 degrees Celsius or 260.2 degrees Fahrenheit in my trial runs.[17] For all practical purposes, earth becomes uninhabitable. Wikipedia (2015b) offers a temperature of 47 degrees Celsius or a temperature differential of 35 degrees Celsius, which I use as the actual critical temperature. I surmise a 37 degree figure is about where there is an equal probability of catastrophe. Thus, I specify a function where if the earth is 28 degrees above its pre-Industrial Revolution temperature of 12 degrees Celsius (98.6 degrees F), there is a 50 percent chance of the end of civilization. A commonsense temperature of a 100 percent chance of catastrophe is a critical temperature of 60 degrees C (140 degrees F) or $\tau_{c,max} = 48$.

Using the simpler radioactive forcing Equation 4, a $\tau_{c,max} = 48$ implies that doom is certain to occur when the atmospheric CO_2 concentration is 32,768 times the pre-industrial level of 276.8. Thus, the switching variable is sure to switch

[16] Since the current CO_2 atmospheric concentration is about 400 ppmv, the much larger 30,000 ppmv figure gives climatologists a sigh of relief. Although a growth rate of 3% seems small. This means output doubles approximately every 23 years. This is why the economy flat lines in 234 years. In 234 years, output and CO_2 emissions doubles more than ten times. This means that output and emissions are more than a thousand times greater than its initial year 0 value.

[17] Goldblatt et al. (2013) methodology to determine CO_2 and surface temperatures is different from the method that I use. Thus, 30,000 ppmv corresponds to a much higher temperature. They don't see a runaway greenhouse effect until temperatures are well above 300 degrees Celsius.

on when $\dot{M}(t) \approx 83,000$ ppmv. Although, we do not know the actual critical temperature and b_2, we know the lowest maximum b_2 or $b_{2,max}$. In this case $b_{2,max} = .002$.

Given these critical temperatures, we know that the probability of doom $1 - \mathcal{S}(\tau, \tau_{c,max})$ is a convex function. Thus, I assume a functional form of:

$$1 - \mathcal{S}(\tau^2, \tau_{c,max}^2) = \frac{\tau^2}{\tau_c^2 + \gamma(\tau_c - \tau)^2} \tag{36}$$

Equation 36 implies $\gamma = 1.96$. Substituting into all the parameter values, Equation 38 becomes:

$$1 - \mathcal{S}(\tau, \tau_{c,max}) = \frac{\tau^2}{\tau_c^2 + 1.96(48 - \tau)^2} \tag{37}$$

Given the parameters of the doomsday function, today with $\tau \approx 1$ we have about a 1 in 5,000 chance of total catastrophe. A global average deviation of $\tau = 20$ a temperature four degrees higher than any time in the last 500 million years,[18] would yield a 20.65 percent chance of doom. So this model predicts we have a long way to go until doom. Nevertheless, we will see below, a 20 degree rise in temperatures would cause severe damage to the earth.

I follow Nordhaus's DICE model and assume a functional form for momentary utility:

$$U\big(C(t)\big) = \frac{C(t)^{1-\alpha}}{1-\alpha} \tag{38}$$

[18] See Wikipedia (2015a).

where $L(t)$ has been normalized to equal one. I follow Evans (2005) who finds that the marginal elasticity of marginal utility of consumption $-\alpha = 1.4$.

The reduction rate of production and consumption is the percentage of production and consumption a policymaker imposes in order to reduce CO_2 emissions. Letting θ_1 be the proportional cost coefficient on the reduction rate μ and θ_2 the exponent coefficient, using the DICE formulation for abatement costs:

$$\wedge_t^y = \theta_1 \mu^{\theta_2}$$

and

$$\wedge_t^c = \theta_1 \mu^{\theta_2} \tag{39}$$

The DICE model assumes $\theta_1 = 1$ and $\theta_2 = 2.8$. Thus, the abatement cost function is highly convex.

The production function excluding emissions, cost and damage functions is Cobb-Douglas in the form:

$$A_t f(K_t, L_t) = A_t K_t^{\gamma} L_t^{1-\gamma} \tag{40}$$

where the capital share γ is set to .3 in the DICE model. However, I break the growth rate in A_t into two components. Following Piketty (2014) world economic growth rate is set at 3.0%. The three percent is broken down into two components, per capita output 1.6% and population growth 1.4%. Nordhaus (2013b) sets productivity factor at .03. Global economic data such as GDP, investment spending, consumption spending and government spending can be found in the CIA World

Factbook, CIA (2015). Most other parameter and initial variable values can be found in the vanilla version of the Dice model, Nordhaus (2013b).

Using these parameters the composite damage function to utility reduces to:

$$\Omega_t = \Omega_{Ut} + (1-\Omega_{Ut})\left[1-(1-\Omega_{Yt})^{1-\alpha}(1-\Omega_{Kt})^{\gamma(1-\alpha)}\right] \tag{41}$$

3.3 Solving the Model

From Equation 30, we see the problem with solving this model is a simultaneity problem between damages and output. Output causes damages, but damages also lowers output. An iteration process can be used to solve this model. However, I will use a procedure that uses last period's damage quantities multiplied by a small factor of 1.00001, as proxies for current damages.

Because data exists, computing the initial damage and other initial variable values is straight forward. To simplify the calculations, I assume, under the constructive fiction of no global warming, constant share between consumption, investment and government spending. This assumption simplifies the problem and may be the best guess when forecasting the future, and when analyzing the effects of consumption abatement or a carbon tax on consumption.

Looking at Equation 30, differentiating with respect to consumption, and comparing a no abatement versus an abatement case, letting C_A denote consumption with consumption abatement and C_{NA} is consumption with no abatement, then

$$C_A = (1-\wedge^c)^{-1/\alpha} C_{NA} \tag{42}$$

If abatement is in the form of a carbon tax, then the revenues may go to increase government spending. If abatement is in a form of a regulation, then some of the loss in consumption may become deadweight loss. Consumers will also shift some of their consumption spending into investment spending. Under the assumptions I make, the quantity of consumption shifted to investment will be $[1-(1-\wedge^{c})^{-1/\alpha}]C_{NA}$ until investment shares fall to zero.

Looking at the first order conditions, consumption is going to evolve according to the first order condition

$$C_s = \bar{C}_s \left[\frac{\mathcal{S}_t(1-\Omega_{Ut})}{\mathcal{S}_{t+1}(1-\Omega_{U(t+1)})} \right]^{-1/\alpha} \tag{43}$$

Equation 43 implies that consumption shares fall as economic damages rises and the probability of survival falls.

Once we estimate the initial variable values, we assume that under no greenhouse gasses, that in steady state all growth variables increase by 3% per year. Because we are not solving the social planner's problem, we do not have to worry about maximizing the damage and doomsday probability function with respect to consumption and income because agents, greenhouse gasses are externalities, agents do not consider these functions when solving their maximization problems. All we have to do is solve for how much these functions cause production and consumption to deviate from its steady state path.

We can further simplify the equation by noting that the ratio of future damages and doomsday probability to next period's damages and doomsday probability probably grows slowly and smoothly over time. These vices probably grow slow enough such that most people do not see them grow over a ten year period. Moreover, there is no simultaneity problem with output and past damages. Thus, for sufficiently short period lengths, say ten or less years, we can use the previous year's ratios as instrumental variables when solving for consumption shares.

Because the probability of survival and damages do not grow at a steady rate, output will not grow at a constant rate. Rather, whether it grows or contracts will depend on whether population and productivity grows faster than the damage and cost variables. Decreasing survivability coupled with increasing costs and damages will slow economic growth.

Solving the first order conditions with respect to C_t and C_{t+1} yields:

$$\frac{C_{t+1}}{C_t} = \left(\frac{\lambda_{t+1} S_t(\tau, \tau_{c,max})(1-\wedge^c_{t+1})(1-\Omega^u_t)}{\beta \lambda_t S_{t+1}(\tau, \tau_{c,max})(1-\wedge^c_t)(1-\Omega^u_{t+1})} \right)^{2.5} \tag{44}$$

Note, if there were no damages and no abatement Equation 42 would be just be

$$\frac{C_{t+1}}{C_t} = \left(\frac{\lambda_{t+1}}{\beta \lambda_t} \right)^{2.5} \tag{45}$$

Equation 44 implies that AGW causes the path of consumption to deviate from steady state growth by

$$\left(\frac{S_t(\tau, \tau_{c,max})(1-\wedge^c_{t+1})(1-\Omega^c_t)}{S_{t+1}(\tau, \tau_{c,max})(1-\wedge^c_t)(1-\Omega^u_{t+1})} \right)^{2.5}$$

Equation 44 implies that consumption grows as utility damages rise and survivability falls. At first glance, consumption growing with increasing doom may seem counter-intuitive. However, if we are going to all be dead tomorrow, optimality requires that we consume all of our wealth before our deaths. We now have the tools to run simulations.

4 Simulation Results

I looked at the simulations using time horizons of 100 and 1000+ years. In models with large time periods and large deviations from pre-Industrial Revolution temperatures and CO_2 concentration levels, we must be cautious about the results, because forcing equations are third order Taylor Series approximations of complicated equations. Thus, these equations become less reliable as the time horizon increases. On the other hand, this could mean that AGW is more damaging to the economy than previously assumed. The length of time horizon determines which policy is best and which are worse.

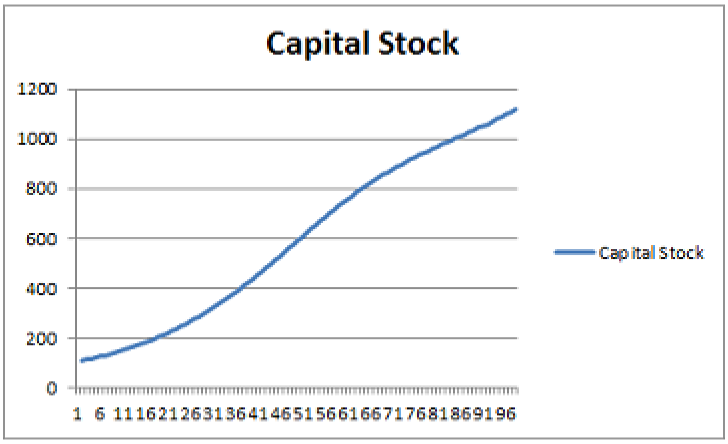

Figure 4: Capital Stock growth over 100 years with no abatement.

In Figure 4, we see that the value of the world's capital stock is somewhat concave. However, the path seems to have convex portions. Its value rises from about 109 trillion dollars (109 T), to about 1141 T from year zero to year 100. The resulting effect as illustrated in Figures 5 is that economic variables tend to grow following a convex path, with energy consumption, CO_2 emissions and temperature also rising along a convex path.

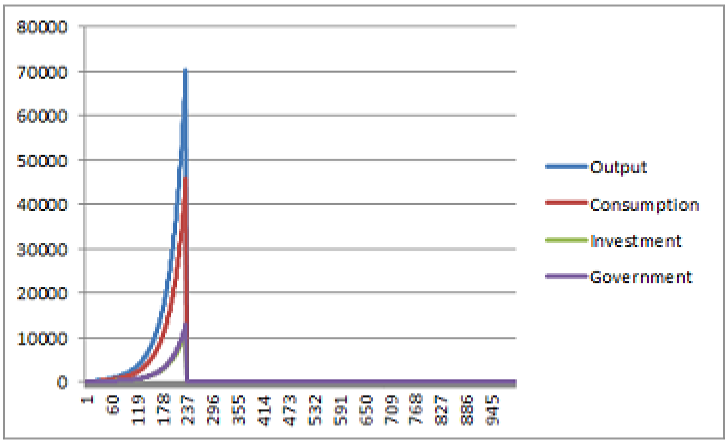

Figure 5: Economic variables such as output, consumption, government and investment spending grow over time. Government and investment spending follow close to the same path in this example.

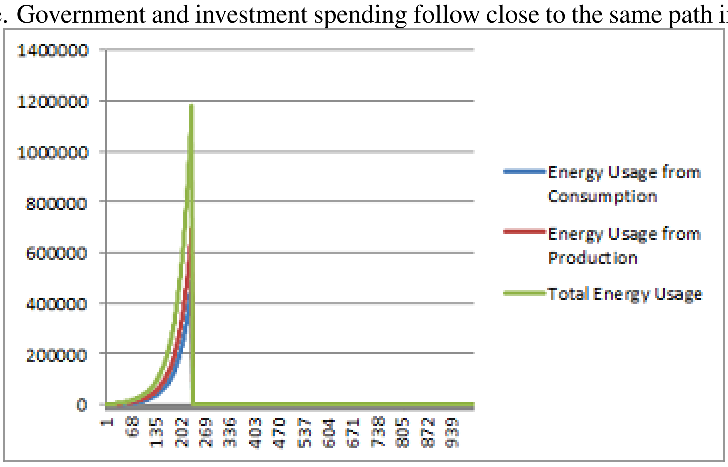

Figure 6: Higher economic growth leads to higher energy usage.

Figure 5 illustrates that energy usage rises with increasing output. Note that both consumption and income require energy usage. Watching TV, going to the movies, playing video games, and toasting and eating bread all require higher energy usage.

The end result as illustrated in Figure 6, is that AGW is destroying 27.3% of the world capital stock per year. The combined direct damage of effects on production and the indirect effects on capital stock reduces output by over 33.9% per year.

Nevertheless, the human species is safe from extinction with a 97.5% probability of no runaway greenhouse effect. We will see that the shape of the capital stock growth path is irrelevant of whether the human species survive or not. This is because over hundreds of years capital stock becomes so productive in creating output, that minute levels of capital stock is capable of creating strong economic growth.

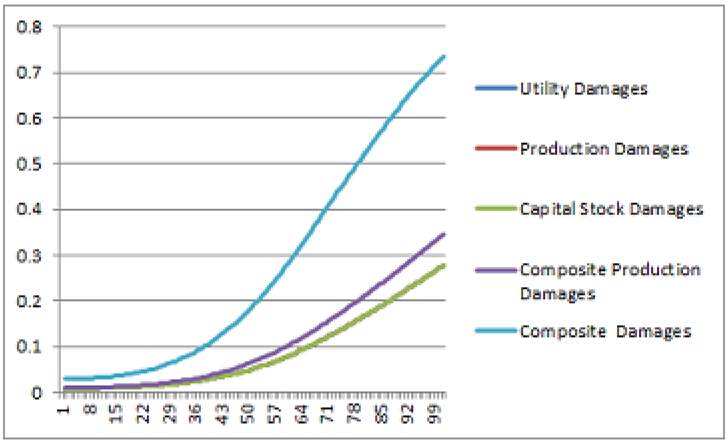

Figure 7: Damages rises over time. At the end of 100 years, damages to output, utility and production hover around 27 percent.

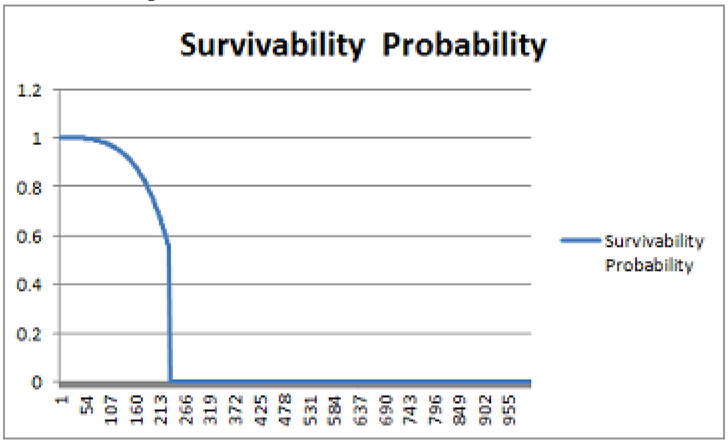

Figure 8: The probability of survivability drops, but the human race is relatively safe with a 97.5 percent chance they will not become extinct.

On Table 1, Appendix, we see that attempts to abate CO_2 emissions when the discount factor is .95 are counterproductive. Of the remaining 21 policy regimes tested with various consumption and production controls only two policy regimes resulted with a higher welfare level than no abatement at all. Most of these policy regimes devastated welfare, some models saw welfare totally wiped out. A fifty percent reduction output control destroyed welfare by 97.1%! A 10% reduction rate of consumption policy resulted in a paltry .12% welfare gain. Consumption reduction rates of 5% and 25% resulted in welfare gain of .02% and loss of 8.6%, respectively, suggesting an optimal reduction rate between 5 and 10%, though the gains would be small. The results of looking 1,000 years ahead also produces similar results, with a modest 5% reduction rate policy on production yielding a small .94% welfare gain. See Table 2, Appendix.

Some economists find that low discount rates or high discount factors will raise welfare. But from Table 3, with a 100 year horizon we see this is not true with a discount factor of .99, or a discount rate of 1.01%. Most models show abatement policies counterproductive. We have a tiny .08% welfare gain with a 10% reduction rate for consumption. But what if the policymakers look 1000 years into the future? Will this change their conclusions? The answer is a resounding yes. In a 1000 year horizon, Table 4, a large welfare gain of about 154% occurs with a 75% production reduction rate. Life expectancy increases by 110 years from 234 to 344 years.

According to this model, if policymakers plan 100 years ahead, they will conclude that most climate change policies are counterproductive even when the discount factor is 100%. A 10% reduction rate policy on consumption will increase welfare by .08%. The social optimal policy is probably some type of very modest consumption tax and the climate change skeptics are essentially right even though AGW is a real and dangerous threat. If we look 1000, years in the future, the opposite conclusion will occur. A 75% reduction rate in production will produce an increase in welfare by 676%. A global maximum appears to occur with a production rate between 50-75%. Even more interesting is that there are two local maximums with a consumption reduction rate of 75% and production reduction rate over 99.9999%. With a 99.9999% reduction rate in production, welfare increases by 601 percent and life expectancy increases to 667 years. When emissions become even closer to zero a 99.99999% production reduction rate, welfare increased by 32.06% and life expectancy increases to 1258 years.

Without abatement, over the decades, temperature rises lowering the probability of human specie survival. Then in the year 234, the temperature deviation hits a critical level and it soars to 115 degrees Celsius, and the oceans boil away. See Figure 9.

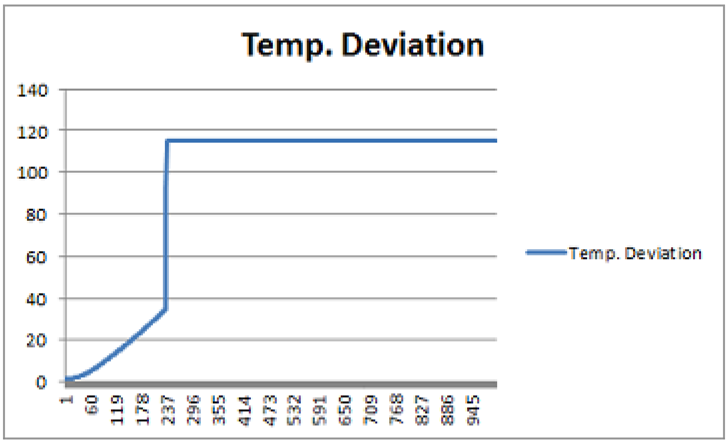

Figure 9: Temperature makes a discontinuous jump upwards when the critical temperature deviation is hit.

From Figure 10, we see that capital stock rises until year 234. Then when the temperature deviation hits the critical temperature capital stock flat lines. The whole world's capital stock is destroyed. In the year 234, Armageddon arrives.

Economic variables flat line, damages soar to one, and the probability of survival goes to zero.

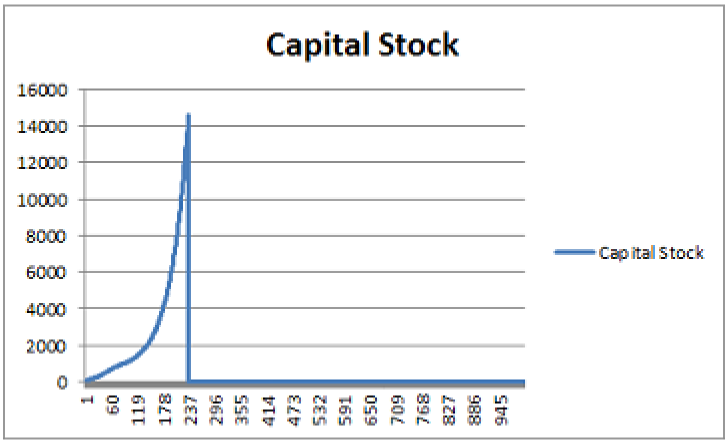

Figure 10: Capital stock rises until the year 234. Then in year 235 it flat lines as the SKI Limit is reached.

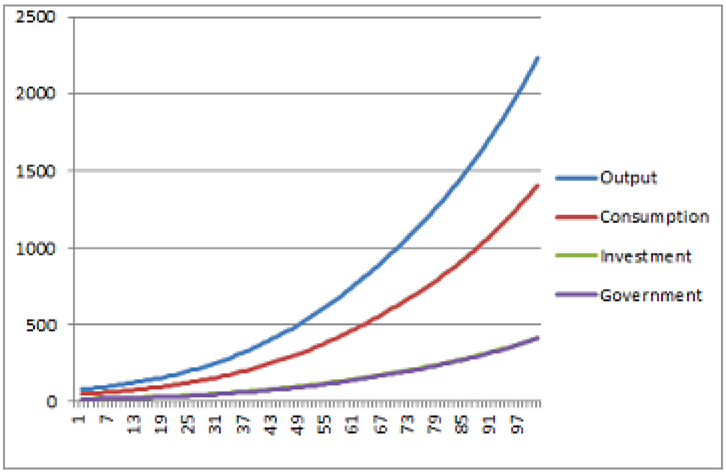

Figure 11: Economic variables rises until the year 234. Then in year 235 it flat lines as the SKI Limit is reached.

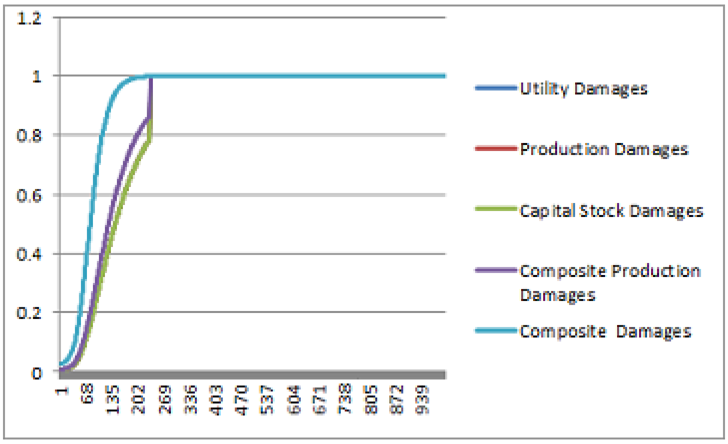

Figure 12: Capital stock, output and utility are completely destroyed. The damages for utility and production follow the same path as capital stock damages. Composite production damages follow a higher path than capital stock damages, with composite damages following the highest path.

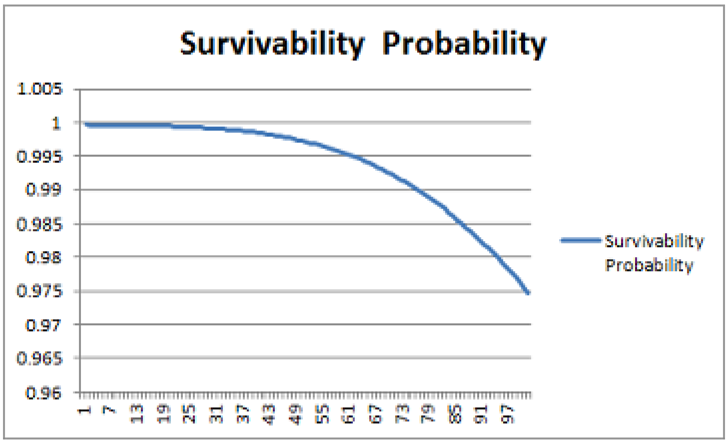

Figure 13: When the critical temperature is reached, survivability plunges to zero.

If there is any good news, the atmosphere somewhat heals after humans become extinct. However, 18% of emissions are permanent. This still means that 82% of anthropogenic CO_2 net emissions are transient and will eventually enter the oceans and biosphere.

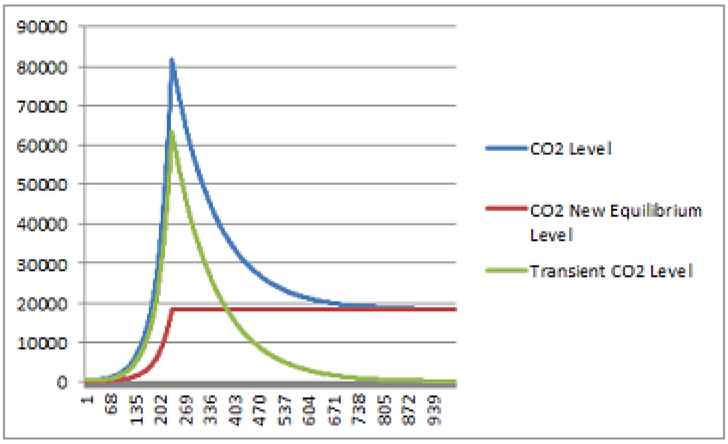

Figure 14: Atmospheric CO_2 soars as increase economic activity increases emissions. However, after humans become extinct, CO_2 levels decay back to its new higher equilibrium levels.

Even in a model with a discount factor of 1.0 or a flat zero discount rate, did abatement do little good in a 100 year horizon model. Of the policy regimes tested, the policy regime of a 10% consumption reduction rate did best but only .08% better than no abatement at all, see Table 5. From Table 6, we see that abatement policies now result in significant welfare gains. Pure production abatement with no consumption abatement is the best strategy. Looking at the relationship between production abatement and welfare, there appears to be two maximums that yields over 600% welfare gains. There is a maximum between a production reduction rate between 50-75%, and another not quite as big local maximum around 99.9999%. I Looking at the 99.9999% production reduction rate, we see that implementation of the strategy results in capital stock plunging down to near zero, but it explodes in the years 608, 635, 647 and 659, the last three spikes producing a puzzling 12-year cycle, Figure 15. In a 99.9999% reduction economy, output and other economic

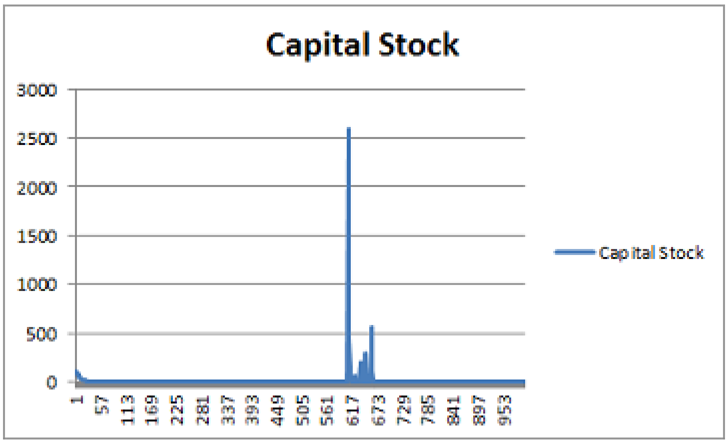

Figure 15: In an economy with 99.9999 percent production rate reduction, capital stock plunges to near zero, then explodes.

variables plunge to near zero. But over time population and productivity increases. For approximately 604 years, people live spartan lives. But then productivity and population become large enough to increase output. Growth for economic variables accelerate, and the people live happy lives until year 667, when the critical deviation temperature is hit and the economy flat lines, see Figure 16. Energy

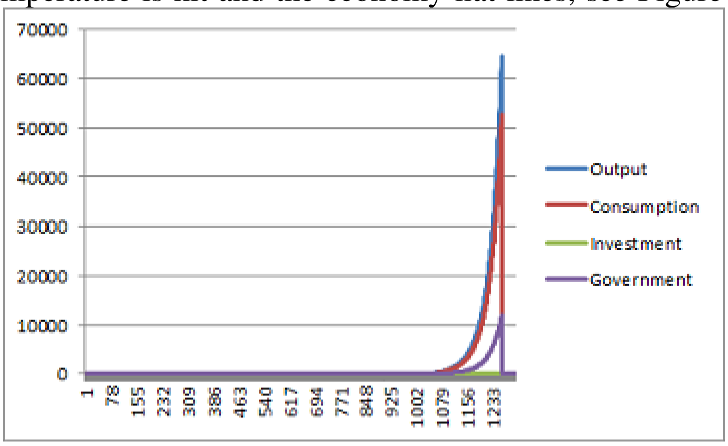

Figure 16: In an economy with 99.9999 percent production rate reduction, output and other economic variables stay near zero for centuries before finally rising.

usage, emissions, and damages follow economic output, while survival probability follows an inverse path. Welfare is sacrificed for many years, and then explodes near the end of the human species lifetime, See Figure 17.

The policymakers can impose even greater restrictions than a 99.999% rate. This will delay the output explosion and increase human species longevity. The people sacrifice and have nearly zero welfare gains for 950 years. Welfare then explodes until the year 1260, when the economy flat lines. The question becomes whether it is better to let generations of people live spartan lives for about a thousand years, which allows for later generations to experience life, and a materialistic one, or is it better for a few generations to live happy materialistic lives in exchange for quick human extinction.

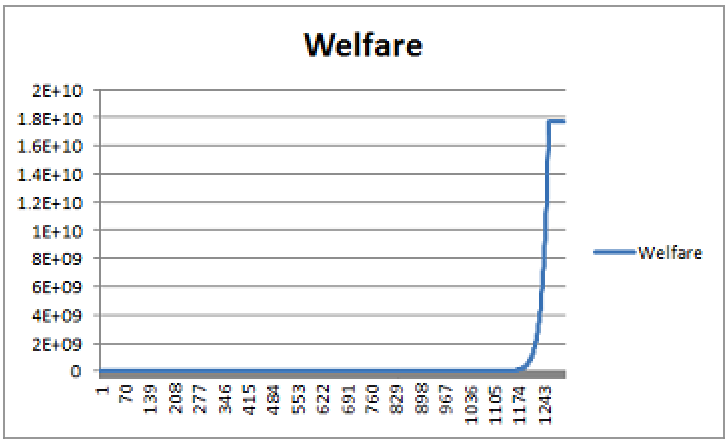

Figure 17: Under a near zero emission policy, for about 980 years there is virtually no welfare gains. Then in the last 280 years, the human species enjoy great welfare gains until the economy terminates in the year 1260.

5 Discussion and Conclusions

Because carbon dioxide emissions does not decay like Carbon-14, but instead increases the CO_2 equilibrium level in the atmosphere, a continuous increase in emissions eventually will flat line the economy. The good news is that this scenario is unlikely because fossil fuel resources are finite and alternative sources of energy like solar power will probably replace fossil fuels. But what this study shows that in the long run, abatement strategies are not enough in preventing catastrophe and severe economic damages. In order to survive, humans will need to develop zero emission power sources or CO_2 removal techniques and/or technology. Carbon dioxide removal techniques need not be difficult or complex, a simple solution is simply to expand the biosphere by preserving forests, planting more trees and reviving topsoil, especially in agricultural lands.

In many ways, this model is similar to human health and life expectancy. The longer a person lives, the greater build up of oxidase and other bad substances in his or her body. The human can only delay the inevitable through weight control and eating less bad food. Thus, she can enjoy life by eating her favorite foods, but die at a relatively young age. On the other hand, the human can enjoy a longer life by foregoing good tasty processed foods. The human species can survive a much longer time by living spartan lives, or they can live it up and burn fossil fuels without constraint.

There is no surprise that the best solutions depend on the discount factor or rate. At low discount factors or high discount rates, abatement strategies are less beneficial. The average person cares little what happens three centuries from now. Not only will she be dead, but probably her children, grandchildren and great grandchildren as well. But the big question is, why do we value the current generation more than later generations? Is it fair to live up now and leave future generations where the earth is severely damage? On the other hand, why should the current generation have to live spartan lives so that people they never know can live good lives? Do we punish people for being single and denying their potential children life?

As we have seen, imposing or easing abatement controls can decrease or increase output. This could be true with regulations in general. Should business cycle theorists be studying the effects of regulations on the ups and downs of the economy? Other research topics could be how AGW and policies to neutralize it effects the distribution of income and wealth in society. While Nordhaus and Sztorc (2013) have studied the relative effects of AGW and its abatement policies on poor and rich countries, what effects do they have on the poor and rich in the same country? This leads to another extension of this research of studying the effects on human capital. Not only does AGW damage capital stock but human capital stock as well. Increase rates in asthma and other chronic conditions in children will effect their health and education level they obtain, and thus, the over all economy as well. As mentioned before, taking into account of expanding or contracting the biosphere should also be considered.

AGW may be with us for centuries to come. Thus, ideas that sound like Star Trek or Star Wars will have to be considered. Will humans create habitats that are substitutes for habitats on earth? Earth has a finite resource of fossil fuels, but Titan, a satellite of Saturn, may have lakes and oceans of methane and other hydrocarbons.

References

Alistair, P., Welch, A., Barret, J., and Ravetz, J. (2006). Counting Consumption: CO2 Emissions, Material Flows and Ecological Footprint of the North East. Discussion paper, WWF-UK.

Archer, D. (2005). Fate of fossil fuel CO2 in geologic time. *Journal of Geophysical Research*, 110(C9): C09S05.

Boden, T., Marland, G., and Andres, B. (2008). Global CO2 Emissions from Fossil-Fuel Burning, Global CO2 Emissions from Fossil-Fuel Burning. Carbon Dioxide Information Analysis Center web site. URL cdiac.ornl.gov/ftp/ndp030/global.1751_2008.ems.

Christensen, E. (2015). Greenhouse Gas in a Bottle Demonstration. URL http://www.cleanet.org/clean/community/activities/c2.html/.

CIA (2015). The World Factbook. Online Publication, Central Intelligence Agency Library Page. URL https://www.cia.gov/library/publications/resources/the-world-factbook/geos/xx.html. Retrieved on 6-11-15.

CO2Now (2015). Accelerating Rise of Atmospheric CO2. Online publication: CO2Now webpage. URL http://co2now.org/Current-CO2/CO2-Trend/acceleration-of-atmospheric-co2.html.

Colose, C. (2008). Physics of the Greenhouse Effect Pt 2. URL https://chriscolose.wordpress.com/2008/03/10/physics-of-the-greenhouse-effect-pt-2/.

Cook, J. C., Nuccitelli, D., Green, S. A., Richardson, M., Winkler, B., Painting, R., Way, R., Jacobs, P., and Skuce, A. (2013). Quantifying the consensus on anthropogenic global warming in the Scientific literature. *Environmental Research Letters*, 8: 1–7.

Dietz, S., Gardner, G. T., Gilligan, J., Stern, P. C., and Vandenbergh, M. P. (2009). Household actions can provide a behavioral wedge to rapidly reduce U.S. carbon emissions. *PNAS*, 106(44): 18452—-18456.

Dietz, S., and Stern, N. (2014). Endogenous growth, convexity of damages and climate risk: how Nordhaus' framework supports deep cuts in carbon emissions, Centre for Climate Change Economics and Policy Working Paper No. 180.

EIA (2004). How Americans Produce Emissions. Energy Information Agency (EIA) online publication: EIA Annual Energy Review 2004. URL http://www.eia.doe.gov/kids/energyfacts/uses/residence.html.

EIA (2015). Frequently Asked Questions: How much carbon dioxide is produced per kilowatthour when generating electricity with fossil fuels? Energy Information Agency (EIA) online publication. URL http://www.eia.gov/tools/faqs/faq.cfm?id=74&t=11.

Evans, D. J. (2005). The Elasticity of Marginal Utility of Consumption: Estimates for 20 OECD Countries. *Fiscal Studies*, 26(2): 197–224.

Flato, G., and Marotzke, J. (2013). Chapter 9: Evaluation of Climate Models. In *The IPCC Fifth Assessment Report Cllimate Change 2013*, pages 9–1–9–205. Intergovernmental Panel on Climate Change.

Friedli, J., Lotscher, H., Oeschger, H., Siegenthaler, U., and Stauffer, B. (1986). Ice core record of the 13C/12C ratio of atmospheric CO2 in the past two centuries. *Nature*, 324: 237–238.

GISTEMPTeam (2015a). GISS Surface Temperature Analysis. URL http://data.giss.nasa.gov/gistemp/tabledata_v3/GLB.Ts.txt.

GISTEMPTeam (2015b). GISS Surface Temperature Analysis. URL http://data.giss.nasa.gov/gistemp/graphs_v3/.

Goldblatt, C., Robinson, K. J., Tyler D.and Zahnle, and Crisp, D. (2013). Low simulated radiation limit for runaway greenhouse climates. On Web Page: Nature Geoscience. URL www.nature.com/naturegeoscience.

Hansen, J., Fung, I., Rind, D., Lebedeff, S., Ruedy, R., and Russell, G. (1988). Global Climate Change as Forecast by Goddard Institute for Space Studies Three-Dimensional Model. *Journal of Geophysical Research*, 93(D8): 9341–9361.

Hansen, J., Ruedy, R., Sato, M., and Lo, K. (2010). Global Surface Temperature Change. *Reviews of Geophysics*, 48: RG4004.

Hansen, J. C., Sato, M., Russel, G., and Kharecha, P. (2013). Climate Sensitivity, Sea Level and Atmospheric Carbon Dioxide. Discussion paper, Philosophical Transactions of the Royal Society.

IPPC (2001). IPCC Third Assessment Report - Climate Change 2001. Discussion paper, Intergovernmental Panel on Climate Change, Geneva, Switzerland.

Knutti, R., and Hegerl, G. C. (2008). The equilibrium sensitivity of the Earth's temperature to radiation changes. *Nature Geoscience*, 1(6): 735–743.

Mann, M. E., and Bradley, R. W. (1994). Northern Hemisphere Temperatures During the Past Millennium: Inferences, Uncertainties, and Limitations. *AGU GRL galley style*, 3.1(14): 1–13.

NOAA (2015). Trends in Atmospheric Carbon Dioxide. Online Publication by Earth System Research Laboratory of the NOAA. URL http://qvmgroup.com/invest/2012/04/02/. Retrieved on 6-12-15.

Nordhaus, W. D. (1992). An Optimal Transition Path for Controlling Greenhouse Gases. *Science*, 258: 1315–1319.

Nordhaus, W. D. (2013a). DICE-2013R Model as of November 15, 2013. Online Publication, Nordhaus Yale University Homepage. URL http://www.econ.yale.edu/~nordhaus/homepage/Web-DICE-2013-April.htm. Retrieved on 6-5-15.

Nordhaus, W. D. (2013b). DICE-2013R Model as of November 15, 2013: Vinilla Version. Online Publication, Nordhaus Yale University Homepage. URL http://www.econ.yale.edu/~nordhaus/homepage/DICE2013R_110513_vanilla.gms. Retrieved on 6-8-15.

Nordhaus, W. D., and Sztorc, P. (2013). DICE 2013R: Introduction and User's Manual, Second Edition. Discussion paper, Yale University, New Haven, CT.

Piketty, T. (2014). *Capital in the Twenty-First Century*. Campbridge, MA.: The Belknap Press of Harvard University Press.

Schultz, T. (1961). Investment in Human Capital. *American Economic Review*, 51(1): 1–17.

Solow, R. M. (1957). Technical Change and the Aggregate Production Function. *Economics and Statistics*, 39(3): 312–320.

Stern, N. (2006). Stern Review: The Economics of Climate Change.

Tans, P., and Keeling, R. (2015). Mauna Loa CO2 monthly mean data. URL ftp://aftp.cmdl.noaa.gov/products/trends/co2/co2_mm_mlo.txt.

Tol, R. S. (2002). Estimates of the Damage Costs of Climate Change. *Environmental and Resource Economics*, 21: 47–73.

Tol, R. S. (2009). The Economic Effects of Climate Change. *Journal of Economic Perspectives*, 23(2): 29–51.

Van Delden, A. J. (2015). Chapter 2 of the lecture notes on Atmospheric Dynamics. URL http://www.staff.science.uu.nl/~delde102/AtmosphericDynamics2015aCh2.pdf.

Waldhoff, S., Anthoff, D., Rose, S., and Tol, R. S. J. (2014). Non-Self-Averaging in Macroeconomic Models: A Criticism of Modern Micro-founded Macroeconomics: Economics, The Open-Access, Open-Assessment E-Journal, Discussion Paper 2007-49. URL http://dx.doi.org/10.5018/economics-ejournal.ja.2014-31.

Wikipedia (2015a). Geologic temperature record. Online Publication: Wikipedia. URL http://en.wikipedia.org/wiki/Geologic_temperature_record.

Wikipedia (2015b). Runaway Greenhouse Effect. Online Publication: Wikipedia. URL Runawaygreenhouseeffect.

Appendices

In the following tables, different policy regimes are compared. Each table represents policies under a specific discount factor. In the first table, the discount factor is .95, which corresponds to a discount rate of approximately 5.26%. Percent abatement is the reduction rate of production or consumption. This is not the same as the cost of production or consumption. Rather reduction rates and costs are related by Equation 39.

Table 1: Comparing policy regimes over a 100 year horizon. Discount factor equals .95.

Model	Consumption Abatement (%)	Production Abatement (%)	Output (100th Year)	Capital Stock (100th Year)	Welfare Gain with Abatement (%)	Survival Probability
Base	0	0	2220.20	1141.10	0.00%	97.4833%
1	0	5	2218.22	1137.89	-0.07%	97.4853%
2	0	10	2206.36	1118.78	-0.52%	97.4969%
3	0	25	2028.37	884.06	-8.59%	97.6786%
4	0	50	53.38	0.00	-97.11%	99.9582%
5	0	75	45.41	0.01	-98.88%	99.9819%
6	0	90	23.87	0.01	-99.73%	99.9994%
7	5	0	2918.73	1144.22	0.02%	97.4820%
8	10	0	2230.92	1162.82	0.12%	97.4744%
9	25	0	2351.36	1426.25	-8.59%	97.6786%
10	50	0	53.38	3174.26	-97.11%	97.1143%
11	75	0	45.10	0.01	-94.32%	97.1529%
12	90	0	3986.64	8636.20	-99.86%	97.2453%
13	5	5	2219.77	1141.01	-31.42%	97.4840%
14	10	10	2217.21	1140.48	-0.38%	97.4877%
15	25	25	2181.29	1133.10	-4.91%	97.5397%
16	50	50	1946.11	1085.09	-31.42%	97.8817%
17	75	75	1365.35	965.26	-75.69%	98.7252%
18	90	90	625.39	685.67	-97.34%	99.6592%
19	0	99	0.00	0.00	-99.89%	99.9985%
20	0	99.9999	0.00	0.00	-99.89%	99.9922%
21	Near Zero	Near Zero	0.00	0.00	-99.89%	99.9887%

In Table 2, we compare the same policy regimes using a 1000 year time horizon.[19] Even though the discount factor is unchanged from the hundred year time horizon, the optimal policy regimes are somewhat different.

[19] Model 21, the Near Zero Emission Model is extended to 1258 years, when the economy flat lines.

Table 2: Comparing policy regimes over a 1000 year horizon. Discount factor equals .95.

Model	Consumption Abatement (%)	Production Abatement (%)	Output (Last Living Year)	Capital Stock (Last Living Year)	Flat Line Date (Years from Now)	Welfare Gain with Abatement (%)
Base	0	0	69867.24	13591.72	234	0.00%
1	0	5	69793.69	13543.17	234	-0.98%
2	0	10	69352.80	13254.06	234	-1.36%
3	0	25	69352.80	66176.97	235	-6.84%
4	0	50	86703.32	14.76	320	-97.20%
5	0	75	102820.96	2.40	344	-98.52%
6	0	90	63015.37	0.00	639	-99.95%
7	5	0	69925.98	13637.92	234	0.94%
8	10	0	68378.59	13601.06	233	-0.83%
9	25	0	70804.15	0.00	232	-1.16%
10	50	0	82991.94	39646.32	229	-36.41%
11	75	0	105952.74	88719.59	232	98.64%
12	90	0	117464.85	113147.78	230	-99.95%
13	5	5	69852.54	13589.34	234	-0.05%
14	10	10	69764.85	13575.20	234	-0.32%
15	25	25	68534.41	13376.19	234	-4.16%
16	50	50	69402.83	13516.73	239	-23.69%
17	75	75	71542.26	13861.08	253	-62.46%
18	90	90	69548.20	13540.17	280	-90.57%
19	0	99	69352.73	13508.56	355	-99.78%
20	0	99.9999	31006.34	0.12	667	-99.98%
21	Near Zero	Near Zero	64466.24	0.00	1258	-99.98%

In Tables 3, and 4 we report the same tests, except that a discount factor of .99 is used. Finally, in Tables 5, and 6 a discount factor of one is used. A discount factor of one implies that future periods are given equal weights to future periods. People born centuries for now, will be treated as being just as important as people who are alive today.

Table 3: Comparing policy regimes with a one hundred year time horizon and discount factor of .99

Model	Consumption Abatement (%)	Production Abatement (%)	Output (100th Year)	Capital Stock (100th Year)	Welfare Gain with Abatement (%)	Survival Probability
Base	0	0	2220.20	1141.10	0.00%	97.4833%
1	0	5	2218.22	1137.89	-0.06%	97.4853%
2	0	10	2206.36	1118.78	-0.45%	97.4969%
3	0	25	2028.37	884.06	-7.67%	97.6786%
4	0	50	53.38	0.00	-99.69%	99.9582%
5	0	75	45.41	0.01	-99.87%	99.9819%
6	0	90	23.87	0.01	-99.97%	99.9994%
7	5	0	2918.73	1144.22	0.01%	97.4820%
8	10	0	2230.92	1162.82	0.08%	97.4744%
9	25	0	2351.36	1426.25	-7.67%	97.6786%
10	50	0	53.38	3174.26	-99.69%	97.1143%
11	75	0	45.10	0.01	-94.44%	97.1529%
12	90	0	3986.64	8636.20	-99.96%	97.2453%
13	5	5	2219.77	1141.01	-28.89%	97.4840%
14	10	10	2217.21	1140.48	-0.35%	97.4877%
15	25	25	2181.29	1133.10	-4.44%	97.5397%
16	50	50	1946.11	1085.09	-28.89%	97.8817%
17	75	75	1333.49	957.91	-74.04%	98.7700%
18	90	90	625.39	685.67	-96.67%	99.6592%
19	0	99	0.00	0.00	-99.99%	99.9985%
20	0	99.9999	0.00	0.00	-100.00%	99.9922%
21	Near Zero	Near Zero	0.00	0.00	-100.00%	99.9887%

Table 4: Comparing policy regimes over a 1000 year horizon. Discount factor equals .95.

Model	Consumption Abatement (%)	Production Abatement (%)	Output (Last Living Year)	Capital Stock (Last Living Year)	Flat Line Date (Years from Now)	Welfare Gain with Abatement (%)
Base	0	0	69867.24	13591.72	234	0.00%
1	0	5	69793.69	13543.17	234	-4.60%
2	0	10	69352.80	13254.06	234	-4.97%
3	0	25	69352.80	66176.97	235	-3.47%
4	0	50	86703.32	14.76	320	68.22%
5	0	75	102820.96	2.40	344	154.38%
6	0	90	63015.37	0.00	639	-97.73%
7	5	0	69925.98	13637.92	234	4.78%
8	10	0	68378.59	13601.06	233	-4.44%
9	25	0	70804.15	0.00	232	-4.57%
10	50	0	82991.94	39646.32	229	-46.87%
11	75	0	105952.74	88719.59	232	95.03%
12	90	0	117464.85	113147.78	230	-99.97%
13	5	5	69852.54	13589.34	234	-0.04%
14	10	10	69764.85	13575.20	234	-0.30%
15	25	25	68534.41	13376.19	234	-3.90%
16	50	50	69402.83	13516.73	239	-6.20%
17	75	75	69803.79	13581.46	253	-18.37%
18	90	90	69548.20	13540.17	280	-37.62%
19	0	99	69352.73	13508.56	355	-70.81%
20	0	99.9999	236233.34	110.83	667	-89.27%
21	Near Zero	Near Zero	62850.73	0.00	1258	-100.00%

Table 5: Comparing policy regimes with a one hundred year time horizon and discount factor of 1.0.

Model	Consumption Abatement (%)	Production Abatement (%)	Output (Last Living Year)	Capital Stock (Last Living Year)	Flat Line Date (Years from Now)	Welfare Gain with Abatement (%)
Base	0	0	69867.24	13591.72	234	0.00%
1	0	5	69793.69	13543.17	234	-4.60%
2	0	10	69352.80	13254.06	234	-4.97%
3	0	25	69352.80	66176.97	235	-3.47%
4	0	50	86703.32	14.76	320	68.22%
5	0	75	102820.96	2.40	344	154.38%
6	0	90	63015.37	0.00	639	-97.73%
7	5	0	69925.98	13637.92	234	4.78%
8	10	0	68378.59	13601.06	233	-4.44%
9	25	0	70804.15	0.00	232	-4.57%
10	50	0	82991.94	39646.32	229	-46.87%
11	75	0	105952.74	88719.59	232	95.03%
12	90	0	117464.85	113147.78	230	-99.97%
13	5	5	69852.54	13589.34	234	-0.04%
14	10	10	69764.85	13575.20	234	-0.30%
15	25	25	68534.41	13376.19	234	-3.90%
16	50	50	69402.83	13516.73	239	-6.20%
17	75	75	69803.79	13581.46	253	-18.37%
18	90	90	69548.20	13540.17	280	-37.62%
19	0	99	69352.73	13508.56	355	-70.81%
20	0	99.9999	236233.34	110.83	667	-89.27%
21	Near Zero	Near Zero	62850.73	0.00	1258	-100.00%

Table 6: Comparing policy regimes over a 1000 year horizon. Discount factor equals .1.0.

Model	Consumption Abatement (%)	Production Abatement (%)	Output (Last Living Year)	Capital Stock (Last Living Year)	Flat Line Date (Years from Now)	Welfare Gain with Abatement (%)
Base	0	0	69867.24	13591.72	234	0.00%
1	0	5	69793.69	13543.17	234	-5.53%
2	0	10	69352.80	13254.06	234	-5.89%
3	0	25	69352.80	66176.97	235	-2.55%
4	0	50	14523.34	0.01	320	252.31%
5	0	75	102820.96	2.40	344	675.66%
6	0	90	63015.37	0.00	639	32.84%
7	5	0	69925.98	13637.92	234	5.81%
8	10	0	68378.59	13601.06	233	-5.37%
9	25	0	70804.15	0.00	232	-5.47%
10	50	0	82991.94	39646.32	229	-49.36%
11	75	0	105952.74	88719.59	232	94.11%
12	90	0	117464.85	113147.78	230	-99.97%
13	5	5	69852.54	13589.34	234	-0.04%
14	10	10	69764.85	13575.20	234	-0.30%
15	25	25	68534.41	13376.19	234	-3.89%
16	50	50	69402.83	13516.73	239	-1.36%
17	75	75	69803.79	13581.46	253	-0.19%
18	90	90	69548.20	13540.17	280	-0.95%
19	0	99	69352.73	13508.56	355	-1.52%
20	0	99.9999	236233.34	110.83	667	601.34%
21	Near Zero	Near Zero	62850.73	0.00	1258	32.06%