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February 2016

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Abstract

This paper is an econometric investigation, an analysis on the difficulty of modeling the south african exchange rate. The aim of our paper is to examine the nature of the existing relationship between the real exchange rate of the Rand, real prices of gold, platinum and the real interest rate differential through different empirical models, and this over the period going from January 2000 *to September* 2014*. Our analysis shows that, over the same period, with different empirical methods, the variables used in the paper can be the determinants of the real value of the south african Rand, but at different horizons. To achieve our goals, long run (Engle and Granger (1987) ; Johansen (1988)) and short run (VAR process (Sims (1980))) analysis have been performed. We come to the conclusion that, the determinants of the Rand change according to the methods used and these do not therefore allow us to have robust results. The long run analysis performed by Engle and Granger approach result to a lack of long run relationship among our variables. To have an robust idea on the lack of long run relationship, we have performed another long run analysis: the vector error correction model (VECM approach) of Johansen which results on the existing of one co-integrating relation among real value of te Rand and their determinants. However, because of the lack of long run relationship resulting of the Engle and Granger approach, we have performed a short run analysis with the vector autoregressive process. We find that only the real platinum price in our study is a short term determinant of the real value of the Rand. The real impact is effective only at the end of the first quarter with a real appreciation of the Rand. The main surprise is the absence of impact of real price of gold shock on the real value of the Rand. Analyze the south african exchange rate through one empirical method/model to find theirs determinants can be biased.*

Keywords: Exchange rate, raw materials, Vector Auto-Regressive, Co-integration.

1 Introduction

The collapse of the Bretton Woods system and the adoption of floating exchange rate have generated a growing interest in the study of the relationship between the real value of a country's currency and the macroeconomic fundamentals such as global demand, interest rates, economic growth, commodity prices, etc. The exchange rate is a very important economic variable in international trade because it determines the terms of trade, especially for large exporters of raw materials. For most of those countries, global commodities prices appear to have a systematic effect on the actual value of their currencies (Cashin et al. (2004)). It is the case of Australia, New Zealand, Nigeria, the Democratic Republic of Congo, South Africa, etc. South Africa therefore belongs to this group of countries rich in natural resources where raw materials account for a significant share in their total exports. Most economists would agree - and empirical evidences show - that commodities prices are likely to be an important drivers of their economies (Cashin et al. (2004); Chen and k. Rogoff (2003)).

Largest economy in Africa until 2014, South Africa holds his wealth mainly from the export of raw materials such as gold (1^{st} world producer until 2007), platinum (1^{st} world producer), titanium, manganese (1^{st} world producer), diamond (5^{th} World Producer), coal, etc. The share of raw materials in exports is around 68% (*Source: South Africa Central Bank*). The gold has a very large share, and South Africa possess the largest reserves of the World[1] *(Source: World Gold Council)*. On the commodities market, South Africa has always been known as the world's leading of gold market, rank that has been taken by China since 2007, before the financial crisis and the general recession of 2008. Gold remains as the second source of foreign currency behind the platinum. Its production decreased by 08% between 2006 and 2007 against an increase of 12% for China over the same period. Actually South Africa is the fifth producer of gold in the world.

South Africa is also a country with good financial, economic integration and is a target of foreign capital flows. The markets are very attractive. Globalization increased the interdependence between economies. Thus, for an ideal access to the global capital market, small open economies have become highly dependent on fluctuations in global interest rates. Since the 2000s, with the rise of catch-up countries such as China, India, Brazil and especially with the trade agreements between the BRICS countries[2], the level of performance of emerging markets has increased, thus causing an increase in capital flows across different emerging markets, and this capital flows increasing have some effects on the real value of currencies. Largest producer and exporter of gold, South Africa undoubtedly belongs to countries that benefited from the financial crisis of 2007 and the period of financial stress that followed. In the real and financial areas, gold has unique qualities that enhance risk management and capital preservation for institutional and private investors. This is what economists and investors call the "*valeur refuge*". Researches have shown that a small allocation of gold makes a valuable contribution to the performance of a portfolio and protects it against downside risk without sacrificing long-term returns. These particularly qualities are considered during periods of financial stress or financial instability. The behavior of central banks towards gold has changed over these last years. From $2009-2010$, central banks became net

[1] Almost 60% of world reserves

[2] Brazil, Russia, India, China, South Africa

buyers of gold, and the demand has increased rapidly, from less than two percent (02%) in 2010 to more than nine percent (9%) in 2012 *(source: Word Gold Council)*. This change is a clear recognition of the benefits that gold can bring to a reserve portfolio. The increase of gold price is for South Africa a source of gain and thus a source of financing for their economy. However, a decline in the price represents a lack of gains. All price fluctuations affect the real value of the currency.

The modeling of the South African exchange rate is not easy as researchers may think. According to Égert (2012); it is very difficult to model the South African exchange rate, nominal, as real. The modeling of the real exchange rate used seem sensitive to periods, to the definitions of variables, to the frequency, to the data and to the estimation methods used. In this paper, our goal is to examine the nature of the relationship between the real exchange rate of the Rand, the real price of platinum, gold, and the real interest rate differential over the period going from January 2000 to September 2014. Specifically, it is to show that over the same period, through different econometric methods, our different variables may be determinants of Rand but at different horizons. In terms results, we note the absence of robustness of the results.

The rest of the paper is structured as follows: the section 2 presents the literature related to the research question; the section 3, for the definition of models that we use in our study. The section 4 analyzes and describes the data used in our study. The section 5 presents the results of different regressions and corresponding economic interpretations. Finally, the section 6 concludes our work.

2 Literature review

The literature on the exchange rate modeling is rich and varied. We focus in the first part, the session on the literature based on relationship between exchange rate and commodity prices and in the last part, the literature on the relationship between exchange rate and the real interest rate differential.

Regarding works focused on the relationship between exchange rates and commodity prices, we cannot forget to mention the work of Kenneth Rogoff on the study of relationships between countries, producers of raw materials and the real value of their specific currency. It provides the first evidence of the existence of relationship between real exchange rates and fluctuations in raw material prices for a number of developed countries rich in natural resources such as Australia, Canada and New Zealand, where the part of raw material have an important share in their total exports (Chen and k. Rogoff (2003)). First, they note that the exchange rates of those countries are driven by long-term world prices of raw materials. For those countries, world prices (in dollars) have a strong influence on their respective exchange rate. Cashin et al. (2004) have certainly done the same kind of study but they have focused their study on developing countries. They examine the existence of long-run relationship between the real exchange rates of countries that export raw materials and their respective prices. Indeed, they find that the results of Chen and k. Rogoff (2003) are consistent with theirs, but in addition they provide further evidence for a broader set of developing countries rich in raw materials. Proof; working on a sample of fifty-eight (58) countries exporting raw materials on a time interval going from 1980 to 2002, Cashin et al. (2004) show that they exist a long-run relationship between the real exchange rate and the price of raw materials, this, for $\frac{1}{3}$ of the exporting countries. The real exchange rate equilibrium of its currencies varies in function of changes in the real price of raw materials.

Apergis and Papoulakos (2013) as Chen and k. Rogoff (2003) show the importance of analyzing the relationship between the exchange rate and the price of raw materials in terms of information available to the exporters of raw materials, monetary authorities, hedge funds and international portfolio managers. Thus, they show that the real exchange rate and commodity markets are driven by *"the same set of informations"* and also that, the exchange rate of the major exporters of raw materials can be used to predict the future evolution of their exchange rate. Certainly, there is a direct relationship between the real value of a currency and the price of raw materials, but it is nevertheless clear that the exchange rate of a country is not only influenced by global commodity prices (Jeffrey (2007)). Others variables such as terms of trade and the degree of openness of an economy, the interest rate, the level of market prices influence the real value of a currency. While the literature mentioned above analyzes the effect of prices of strategic materials on the real exchange rate generally Fattouh and al. (2008), most are focusing on South Africa in analyzing the impact of fluctuations in commodity prices specifically that of gold on the real exchange rate of the Rand. They found first that, South Africa has a diversified basket of commodities that can contribute to economic progress. But, if raw materials seem to be a source of wealth, they can be a handicap for the South African economy in the long-run. Through a Markov switching error correction model (ECM-MRS), they estimate the long-term non-linear relationship between the real exchange rate of Rand and the real price of gold. To achieve its objectives, the process of Krolzig (1996) is used. Fattouh and al. (2008), found that the real equilibrium exchange rate of the South African Rand is determined by two variables: the price of gold and the interest rate differential or the uncovered interest rate parity which is based on the purchasing power parity theory. Monthly data from January 1975 to April 2007 are used.

Arezki et al. (2012) innovate on the period from 1980 to 2010, taking into account a particular event: the liberalization of capital accounts in 1995 in South Africa. They focus their analysis, firstly to the causal relationship between the real price of gold and the exchange rate before and after the impact of the liberalization of capital accounts. Thus, through a model of co-integration vector error correction *(Johansen)*, they show that the South African exchange rate caused the real price of gold before the liberalization of capital accounts and *vice versa* after. The volatility of the price of gold has become a more critical factor for the Rand volatility after liberalization than before. This is an important result because the increased volatility of the Rand may have consequences on investments and trade. The dependence of the South African economy on gold and its price, seems become more important, especially in the context of open capital markets. Jager (2012) shows that the different fundamentals of the South African economy only serve to indicate a level of possible equilibrium. Although the study provides a clear indication that the level of the exchange rate is incompatible with a set of data, the estimation in his study remains to be seen. By modeling the real effective exchange rate variable explained by real GDP per capita, economic openness, net foreign assets, the interest rate differential, the balance of the government and the real price of raw materials, in a co-integration framework over the period $1970q1$ to $2002q1$, Macdonald and Ricci (2003) show that there is a long-run relationship between all variables. But, the raise of fundamentals used are not enough to explain the evolution of the Rand especially depreciation phases. Unlike to Macdonald and Ricci (2003), the model of Jeffrey (2007) reveals no major imbalances since the early 1990*s*. However, the model does not fully explain the sharp depreciation in 2002 and seems to follow the real exchange rate observed with a lag.

The relationship between the real exchange rate and the real interest rate was at the center of macroeconomic models based on open economy. Mostly, economists find that relationship is very low. Hoffmann and MacDonald (2009), reviewing the relationship between the exchange rate and the interest rate differential. Through strong evidence-based on the procedure of Campbell and Shiller, 1987; they show that the economic link between the exchange rate and the interest rate differential is economically significant and that the real interest rate differential is a reasonable approximation of expected depreciation rate over longer horizons. Ndung'u (2000) shows that the exchange rate moves away from the level of long-run equilibrium relationship by the purchasing power parity, and such differences are driven by the interest rate differential. The deviations from the purchasing power parity are absorbed by the differential of real interest rates. The difference in relative interest rates between domestic and foreign real interest rate reflects the uncovered interest rate parity (UIP), which states that the domestic interest rate must be higher than the interest rate abroad by an amount equal to the expected depreciation of the national currency (Copeland, 1989). According to economic theory, the interest rate differential would tend to equalize across countries in the long-run, however, anecdotal evidence suggests that this is not necessarily the case. In the event that the uncovered interest rate parity is checked and that all other factors (such as a risk premium, etc.) are constant, an increase in the domestic interest rate compared to other countries would tend to attract foreign capital and cause an appreciation of the domestic currency.

Through different approaches to the South African exchange rate modeling such as "Commodity price", "Stock prices", "country risk premium", "Balassa-Samuelson effect", "real interest rate differential", (Égert (2012)) shows that the models of real exchange rate of the Rand seem sensitive to the periods, the definitions of variables, frequency data and estimation methods. Although these models do a pretty good job in the sample, their out-of-sample properties remain poor.

Our paper goes on the same lines of Égert (2012). It is an econometric investigation of some real determinants of the South African real exchange rate. The first contribution is to use different econometric methods to show the difficulty of modeling the South African real exchange rate but also the shortcomings of some methods. The second is to work in a period characterized by economic and financial instability, the period from January 2000 to September 2014, a period of economic and financial shocks which have an impact on the real determinants of the South African Rand. The last contribution consists firstly to introduce the price of platinum - a variable which is little used in economic papers, but represents the primary source of the South African currency - but also to take into account the loss of global leadership in the gold market in times of financial crisis.

3 Methodology

3.1 Co-integration and error correction models

The main principle of the theory of co-integration is the fact that economic and financial time series are non-stationary. They have at least one unit root. Indeed, applying the usual methods of econometrics poses two main problems namely the problem of "spurious regressions [3]"and especially the non-validation of some usual asymptotic properties of estimators [4]. Introduce in the economic analysis by Engle and Newbold (1974), his rigorous formalization is due to Granger, [1981, Granger (1983)], Engle and Granger (1987) and Johansen (1988), Johansen and Juselius (1992), .

If two time series $X_{1,t}$ and $X_{2,t}$ are integrated at the same order j and a linear combination of this two series is integrated zero-order $I(0)$, i.e. stationary, then we can say that $X_{1,t}$ and $X_{2,t}$ are co-integrated of order (j, j). The most studied case is when $j = 1$.

Let $X_{1,t}$ and $X_{2,t}$, two series $I(1)$[5]. Let's have the general formalization of the co-integrating relationship. The estimation of such models is done in two steps:

- The long-term equation

$$X_{1,t} = \alpha + \beta X_{2,t} + u_t \tag{1}$$

$$\Rightarrow \hat{u}_t = X_{1,t} - \hat{\alpha} - \hat{\beta} X_{2,t} \tag{2}$$

- Error correction model (*ECM*)

$$\Delta X_{1,t} = \alpha_1 + \alpha_2 \Delta X_{2,t} + \pi \hat{u}_{t-1} + \varepsilon_t \tag{3}$$

where ε_t is a *white noise* and $\hat{u}_{t-1}$ the estimated residual series of the long-term equation lagged by one period; i.e.: $\hat{u}_{t-1} = X_{1,t-1} - \hat{\alpha} - \hat{\beta} X_{2,t-1}$.

In the equation 3, α_2 represents the short-run elasticity and π the speed of adjustment. In case of existence of long-term relationship π is negative and close to -1. There are two approaches for co-integration models. We have on the one hand the univariate approach (Engle and Granger (1987)) and a second hand the multivariate approach.

3.1.1 Univariate approach: Engle & Granger

This approach focuses on the models introduced by the work of Hendry 1978; an approach that allows the modeling of adjustments which lead to a long-run equilibrium. These models introduce both the short-run dynamics and long-run equilibrium of variables. Several methods have been developed for the estimation of error correction models, as well as co-integration tests.

The most famous is the approach of Engle & Granger. The estimation is done in two steps. The first step is to estimate the long-term equation (equation 1) by the method of ordinary least squares (OLS). The second step is the estimation of the error correction model (equation 3). Engle and Granger (1987)

[3]Problems highlighted by Granger and Newbold, 1974. Perform regressions that seem statistically true but are not really at all. Therefore, estimates of regression coefficients are ineffective first, then the predictions based on the regression equations are suboptimal.

[4] In this case, usual significance tests on the estimated coefficients are not valid. For example, autoregressive models, autoregressive and moving average, autoregressive and Distributed Lags, *VAR* are valid only for stationary series

[5]The series are not stationary

have shown that if the co-integrating vector (α, β) was correctly estimated in the equation 1, then the estimators α, β, π are consistent and equivalent estimators. In addition, the standard deviations of the coefficients estimated by OLS are consistent estimators.

The co-integration test of Engle & Granger are based on the Augmented Dickey-Fuller unit root test.

$$\Delta \hat{u}_t = \phi \hat{u}_{t-1} + \sum_{i=1}^{p} \phi_i \Delta \hat{u}_{t-1} + \varepsilon_t$$

The existence of an unit root $(\phi = 0)$ in the residuals of equation 1 reflects the fact that our $X_{1,t}$ and $X_{2,t}$ are not co-integrated[6]. In contrary, the absence of unit root $(\phi < 0)$ indicating that the series in level can be co-integrated. For the decision rule, the t-statistic of Dickey-Fuller Augmented test is compared to the critical value read in the statistics table of Mackinnon or Engle & Yoo.

It is necessary to note that the approach of Engle & Granger is problematic when studies are performed simultaneously on n variables $(n > 2)$. Indeed, this approach is applicable only in the case of a single co-integrating relationship, that is to say one co-integration vector. Therefore, it does not allow to distinguish several co-integrating vectors. In addition to this, empirical studies have shown that the estimation of the long-term equation suffers from a small sample bias (Banerjee and al, 1986 ; Stock, 1987).

3.1.2 Multivariate approach

As alternative to Engle & Granger approach, there is a multivariate approach based on the maximum likelihood method for determining the number of long-term equilibrium relationship between the variables integrated of the same order whatever the standard used. This is an approach developed by Johansen and Juselius (1990). The vector error correction model (VECM) is the most used model when researchers perform a long-term relationship analysis in a multivariate framework.

We focus on the formalization of the VECM. The estimation is done in two steps. We must know that the $VECM$ is based on the vector autoregressive model (

$$VAR$$

). In contrary to the univariate approach, X_t is a vector of variables, non-stationary and all variables are integrated at the same order $(I(j))$.

Consider a vector X_t containing n variables, all $I(1)$. The procedure of vector error correction model estimation is given by:

- Long term equation

$$X_t = \phi_1 X_{t-1} + ... + \phi_p X_{t-p} + \varepsilon_t \quad (4)$$

$\varepsilon_t \sim$ White noise$(0, \Omega)$ with Ω, the variance-covariance matrix, and ϕ_i $(i = 1, ..., p)$, the matrix of size parameters $(n \times n)$, where p is the number of optimal lags.

[6]Hypothesis of co-integration test: the null hypothesis corresponds to the absence of co-integration and the alternative hypothesis, the existence of cointegration

- Error correction model

$$\Delta X_t = \phi_1 \Delta X_{t-1} + ... + \phi_{p-1} \Delta X_{t-p+1} + \pi_p X_{t-p} + \varepsilon_t \quad (5)$$

where π_i, $i = 1, ..., p$ size is $n \times n$. π_p represents the vector of speed of adjustment.

It is important to note an imbalance between the order of integration of the member of the left and right terms of the equation 5. The terms of that equation are all $I(0)$, except X_{t-p} which is $I(1)$. To restore balance, the necessary condition is that: $\pi_p X_{t-p}$ is $I(0)$. Thus the condition laid down is as follows:

$$\pi_p = -\beta \alpha^{'}$$

where $\alpha^{'}$ is a matrix of dimension (r,n) which contains the r co-integrating vectors, an β, a matrix of dimension (n,r), which contains peas associated with each vector co-integration.

Co-integration tests developed by Johansen (1991) is based on the condition that for r co-integrating relationships:

$$Rg(\pi_p) = r$$

with Rg, the range.

Two econometric tests were developed by Johansen to determine the number of vector(s) of co-integration. We have the test of *Rank* (most used) and the test of the *Maximum Eigenvalue*. Regarding co-integration tests, the null hypothesis (H_0) is that of the existence of r co-integrating relationships between the n variables, in other words in H_0 X_t is co-integrated of order or rank r.

Test of Trace

The *Trace* test is a maximum likelihood ratio of calculating the following statistic:

$$TR = -T \sum_{i=q+1}^{N} \log(1 - \hat{\lambda}_i) \quad (6)$$

The test is used to test the existence of more r co-integrating vectors. The null hypothesis is to test the existence of at most r co-integrating vectors, i.e., $Rg(\pi_p) = r$. Thus, under the null hypothesis, the TR statistic is asymptotically distributed as the law of:

$$Trace[\int_0^1 W(r) dW^{'}(r) (\int_0^1 W(r) W^{'}(r) dr)^{-1} \int_0^1 dW(r) W^{'}(r)]$$

where W is a Brownian motion of variance-covariance matrix of the identity matrix.

The null hypothesis of r relation(s) of co-integration is rejected if the TR statistic exceeds its critical value; the critical values are tabulated by Johansen and Juselius (1990) and then by Osterwald-Lenum, 1992.

Three cases are possible:

1. $Rg(\pi_p) = 0$ $(r = 0)$, meaning that the variables X_t are $I(1)$ and not co-integrated. Then, it is possible to estimate a *VAR* model on ΔX_t.

2. $R(\pi_p) = r$ with $0 < r < n$, which means that X_t is co-integrated and therefore there r co-integrating relationships. An error correction model is estimated.

3. $Rg(\pi_p) = n$, in other words, $r = n$, that is to say that π_p is full rank. This means that all variables are stationary, and a *VAR* model can be estimated on ΔX_t.

Test of the maximum eigenvalue

The test of *maximum eigenvalue* tests the presence of $r+1$ co-integrating vectors. We test the null hypothesis $Rg(\pi_p) = r$ against the alternative $Rg(\pi) = r+1$. The test statistic is given by:

$$VP_{max} = -T\log(1-\hat{\lambda}_{q+1})$$

It is important to note the absence of the intercept and deterministic trend in the error correction model. But it should be noted that during the decision making, critical test values change as soon as we introduced an intercept and/or a trend.

Indeed we can retain five options:

1. The lack of an intercept in the co-integrating error model (no linear trend) and the co-integrating relationships.

2. The presence of an intercept in the error correction model (linear trend) and in the co-integrating relationships.

3. The presence of an intercept in the co-integrating relationships but not in the error correction model (no linear trend).

4. The presence of an intercept in the error correction model (linear trend) and a trend in the co-integrating relationships.

5. The presence of an intercept in the error correction model and the co-integration relationship (quadratic trend)

Depending on the different possibilities, the econometricians estimate the model and determine the number of co-integrating vector. It may happen that two series are integrated of the same order, but there is no linear co-integrating relationship between them. Given this situation, it makes stationary series by differentiating and we focus on the analysis of the short-run dynamics through vector autoregressive processes.

3.2 Vector AutoRegressive process

Introduced in 1980 by Sims, the

VAR

process is a macroeconomic framework that has been very promising in modeling time series. This model is a generalization of autoregressive processes (AR) to multivariate case. The AR process represents the univariate case, a process which is a single equation, a univariate linear model in which the current value of a variable is explained by its own lagged values.

We call autoregressive process of order p, denoted $\text{AR}(p)$, a stationary process X_t which satisfies the following relationship:

$$X_t = \alpha_0 + \sum_{j=1}^{p} X_{t-j} + \varepsilon_t \tag{7}$$

where $\alpha_j \in R$, $(j = 1, ..., p)$, and α_0 is the intercept.

According Sims (1980), the *VAR* model compensates the shortcomings of univariate autoregressive models namely the absence of relevant tests on the causal structure; very strong restrictions on the parameters compared to that predicted by economic theory and finally deal with the inadequate treatment of expectations. A

VAR

is a n equation system of n linear models in which each variable is explained by its own lagged values, the more past values of $n-1$ other variables. This process provides a systematic way to capture the rich dynamics of several time series. The

VAR

model is based on the assumption that the evolution of the economy is well approximated by the description of the dynamic behavior of a vector of n linearly dependent variables of the past.

Consider two stationary variables and $X_{1,t}$ and $X_{2,t}$, with p the number of lags of the

VAR

process. The

VAR

model[7] describing the two variables is written as follows:

$$\begin{cases} X_{1,t} = \alpha_1 + \sum_{i=1}^{p} \beta_{1,i} X_{1,t-i} + \sum_{i=1}^{p} \eta_{1,i} X_{2,t-i} + \varepsilon_{1,t} \\ X_{2,t} = \alpha_2 + \sum_{i=1}^{p} \beta_{2,i} X_{1,t-i} + \sum_{i=1}^{p} \eta_{2,i} X_{2,t-i} + \varepsilon_{2,t} \end{cases} \tag{8}$$

where $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are two uncorrelated *white noise* process. The number of parameters to be estimated is pN^2 where p is the number of lags and n, the number of the variable in the model.
The number p of lags to be included in each equation can be determined by a number of different methods. We have the choice of the order p based on Akaike Information Criteria (AIC), Schwarz (SIC) Hannan-Quinn (HQ). We also have the choice of the minimum order p which whitens residues. The latter method allows for effective coefficients with minimal variances. But in the case where the use of information criteria is required, it is estimated a number of VARs lags for order p from 0 to h where h is the maximum number of lags.

The p chosen is that which minimizes the criteria information defined as follows:

$$AIC = \log det \hat{\sum} \varepsilon + \frac{2N^2 p}{T} \tag{9}$$

$$SIC = \log det \hat{\sum} \varepsilon + N^2 p \frac{\log T}{T} \tag{10}$$

$$HQ = \log det \hat{\sum} \varepsilon + N^2 p \frac{2 \log(\log T)}{T} \tag{11}$$

The VAR processes have proven to be powerful and reliable tools that are now rightfully in everyday use. The method used to estimate the

VAR

process is the ordinary least squares method (OLS). Thus, after estimating the

VAR

[7]The VAR model can be in recursive form or structural form. A

VAR

recursive form builds the error terms in each regression equation to be correlated with the error in the above equations. This is done by carefully including some contemporary values of explanatory variables. For a

VAR

in structural form, the macro-economists use economic theory to sort the contemporary relationships between variables (Bernanke, 1986; Blanchard and Watson, 1986; Sims, 1986). Structural

VAR

processes require "hypothesis identification" which allows correlations to be interpreted as causal.

process, it is important to perform model validation tests. We make no autocorrelation tests and test of no heteroskedasticity. We perform the test *LM* (Lagrange multiplier) and Ljung-Box for the absence of autocorrelation and White test for the absence of heteroskedasticity.

In terms of results, in the analysis of

$$VAR$$

process, the current practice is to report the impulse response functions, the causality tests and the variance decomposition of the forecast error. The objective of the variance decomposition of the forecast error is to calculate the contribution of each innovation to the variance of the error. The variance of the forecast error is written to a horizon h depending on the variance error attributed to each of the two variables. This is to calculate the ratio between each of these variances and the overall variance to obtain a relative percentage by weight. Due to the complex dynamics in the

$$VAR$$

processes, these statistics are instructive and informative than the

$$VAR$$

regression coefficients estimated by OLS, as well as statistics measuring the explanatory power of the model (R^2), which are usually more observed in the univariate models.

On the notion of Granger causality, it allows to see if the lagged values of a variable help to predict another variable. In the equation 8 for example, if the variable $X_{2,t}$ not predict $X_{1,t}$, then the coefficients delays $X_{2,t}$ $(\eta_{1,j})$ will all significantly equal to zero in the equation $X_{1,t}$ in reduced form. Causality also allows us to see which variables can be used to predict the other variable in short-run.

The latest utility of *VAR* processes concerns the analysis of the impulse responses functions. Generally, an impulse response refers to the reaction of any dynamic system in response to some external change. An impulse response describes the response of current and future values of each variable, following a one-unit increase in the present value of one of the error of the

$$VAR$$

process, assuming that error back to zero in subsequent periods and that all the other errors are zero. Impulse responses are usually calculated for recursive and structural variables. The economic analysis of response functions is only relevant if it has the degree of estimation accuracy. There are analytical methods to determine the confidence interval and for this we proceed by simulations. There are two simulation methods which are the Monte Carlo[8] and Bootstrap[9] methods.

[8] The Monte Carlo method is based on the assumption of normality of the estimators of the parameters of the average of the *VAR* model. This is verified asymptotically. The simulated trajectories is obtained by creating shocks on the estimated coefficients of the *VAR* model.

[9] In the Bootstrap method, the simulated trajectories are obtained by performing independent trials innovations of different dates, according to a uniform law or any other law. This method has the advantage of being applicable in cases of non-normality.

4 Data

In this section we describe our different variables, and perform unit root tests. In our paper the series used to compute the real series and to achieve to our goals are the price of platinum (PLA), the price of gold (GOP) (commodities prices), the nominal exchange rate (S), the long-run nominal interest rates of South Africa (i_{sa}) and United States of America (i_{us}) and finally the consumer prices index of South Africa (P) and United States of America (p_*). The main information on the series are represented in the table 1.

Series name	**Label**	**Source**	**Seasonality**	**Unit**
Nominal exchange rate	S	OECD statistics	SA	Zar per Usd
Platinum price	PLA	Global Economic Monitor	NSA	$/toz
Gold Price	GOP	Fed Economic data	NSA	US.$/per Troy Once
SA nominal interest rate	i_{sa}	Oecd statistics	SA	per cent per annum
USA nominal interest rate	i_{us}	Oecd statistics	SA	per cent per annum
SA Consumer price index	P	Oecd statistics	SA	per cent per annum
USA Consumer price index	P^*	Oecd statistics	SA	per cent per annum

$\cdot_{sa}$ *for South Africa and* $\cdot_{usa}$ *for United States of America*

OECD: Organization for Economic Cooperation and Development

SA = Seasonally adjusted and NSA = Not Seasonally adjusted

Table 1. Informations on the series

The majority of the series are seasonally adjusted, apart the prices of the two commodities. For those variables, it is important to subtract the seasonality. Most of time, we use the $X12-ARIMA$[10] process. The EViews software is used for the modeling of our data. We work on the period going from January 2000 to September 2014. The frequency of the data is monthly.

4.1 Commodities prices

South Africa is rich in natural resources. It dominates the gold market. It is also the first producer of platinum in the world, and has the world's largest reserves. The raw materials are the main wealth of the South African economy. Increases in materials prices are an added value. However, the declining of price of raw materials penalizes economic activity. For a country that has adopted a floating exchange rate regime, increases in commodity prices tend to appreciate the exchange rate and lower commodity prices tend to depreciate the exchange rate.

In this paper, we don't work with the nominal commodity prices, prices fixed on the commodity markets, but we work with the real prices. Economic decisions are mostly based on relative prices, not

[10] X12 ARIMA has the advantage of not losing data at the beginning and end of our sample

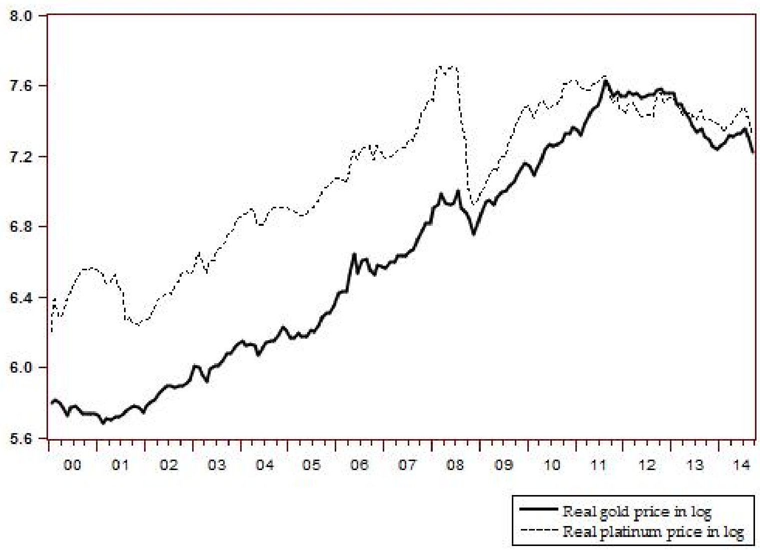


Figure 1. Real gold price *vs* Real platinum price

absolute prices.

Real Price Calculation Procedure

Real Prices are computed as follow,

$$RCP_t = P_t \times \frac{100}{CPI} \tag{12}$$

with P_t, the nominal price of raw material, CPi_t, the consumer price index for month t.

In this paper, we focus on the analysis of the real gold price and the real platinum price express in US dollar per Ounce (US dollar/Ounce). Let analyze now the evolution of those variables. On the figure 1, apart the crisis period from 2007-2008, we notice the existence of transitory shocks on the evolution of the price of gold and platinum. The raw material prices have increased over the period of the Great Moderation. The increase of price is due to the increase of demand for raw materials, increase due to the industrialization of countries like China, India, South Africa etc. The platinum price reached its peak towards (2222 US $) at the end of 2008 (Third trimester of 2008) before declining in late 2009 and begin to re-converge to its trend at the beginning of 2010.

However, on the evolution of the price of gold represented on the same figure, (figure 1), apart a slight stagnation of the price between 2000 and 2001, the value of the ounce troy of gold has been increased since 2002, reaching maximum value of 1896.5 US $per Ounce at the third quarter of 2011. Since, the value of the ounce of gold began to decline. But it is important to note that during the great recession of 2008 the value of ounce troy of gold fell to 692.50 $($4^{th}$ quarter of 2008) before rising again.

4.2 Real interest rate differential

In the calculation of the real interest rate differential, it is important to calculate the real interest rate. For this, we use the Fisher equation.

Fisher equation

The *"Fisher effect"* formula attempts to show how an expectation of inflation influences both interest rates and purchasing power. Developed as part of an overall economic theory in 1930 by the mathematical economist Irving Fisher, the formula is so widely used in the fields of economics and finance today that many consider it as stylized fact, a term used to describe observations and findings so consistent they are generally accepted as true. In finance, the Fisher equation is primarily used in Yield-To-Maturity calculations of bonds or real interest rate calculations. In economics, this equation is used to predict nominal and real interest rate behavior. The Fisher effect formula not only plays an important role in academia and business, but it also has applications that can benefit for all investors.

Formula

The Fisher effect formula breaks down into three components: the nominal interest rate (i), the real interest rate (r) and the expected inflation rate (π). The nominal interest rate is a percentage showing the price you pay for the use of money without taking inflation into account. The real interest rate is a percentage that adjusts to remove the effects of inflation and, as a result, is a measure of *"real purchasing power"*. The expected inflation rate is a percentage that varies according to current economic cycles.
The Fisher equation is represented by the following equation.

$$i_t = r_t + \pi_t^e \tag{13}$$

with π^e the expected inflation rate. All the variables in the equation 13 are expressed in logarithm terms.

$$\pi_t^e = E_t[\pi_{t+1}] \tag{14}$$

with $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}} * 100$, and P which is the inflation or the consumer price index in our case.

In our paper, we need Fisher equation to compute the real interest rate. But in our case, in the calculation of our real interest rate, we use the actual inflation rate, ie:

$$i_t = r_t + \pi_t \tag{15}$$

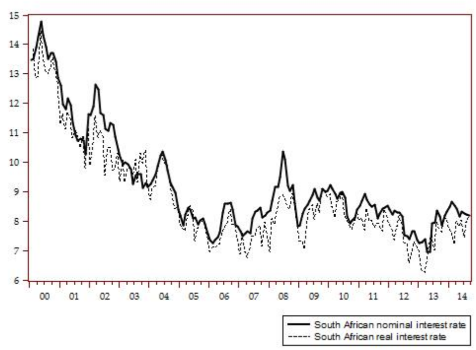
Through the equation 15, we assume that there's a perfect anticipation of interest rates. it seems more reasonable to rewrite the equation 15 as follows:

$$i_t \approx r_t + \pi_t \tag{16}$$

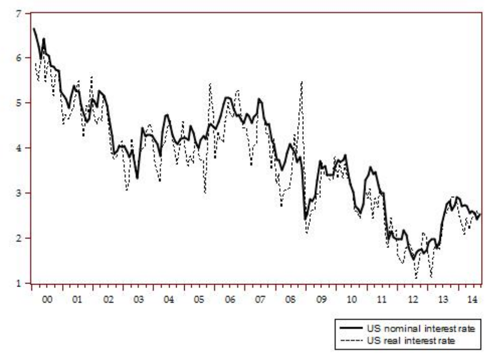
By using equation 16, we can have the expression of the real interest rate given by the following equation:

$$r_t \approx i_t - \pi_t \tag{17}$$

The real interest rate is approximately equal to difference between the nominal interest rate and the actual inflation rate. It is important to note that, on the graphic above, the real interest rate in majority is less volatile than the the nominal interest rate, especially for South Africa. Let's see now how compute the real interest rate differentials.



South Africa nominal *vs* real interest rate



US nominal *vs* real interest rate

Real interest rate differential calculation

A differential of interest rate measures the gap in interest rates between two similar assets. It is the difference in interest rates associated with two different currencies or two different economic regions. Based on the interest rate parity, deciders can create an expectation of the future exchange rate between two currencies and set the premium on the current market exchange rate futures contracts. The interest rate differential is a fundamental component of the interest rate parity theory, whereby the difference in interest rates between two countries equals the difference between the current and expected exchange rates of two currencies.

The Interest Rate Differential theory[11] claims that exchange rate movements are determined by a country's interest rate level. Countries with higher interest rates will see their currency appreciate in value and countries with lower interest rates will see their currency depreciate in value. When a country raises its interest rates, it's currency become more attractive to domestic and foreign investors so investment will flock to that country due to higher yield for that country's currency.

Formula

The real interest rate differential is calculated by using the equation 18:

$$rrid_t = r_{sa,t} - r_{us,t} \tag{18}$$

with $rrid$, the real interest rate differential, r_{sa}, the approximate long-term real interest rate of South africa and r_{us} the the approximate long-term real interest rate of United States of America.
The graphic 2 presents the evolution of the real interest rate differential. The first major finding is that

[11] This theory heavily relies on capital flows discounting a country's current account balance. Moreover, the model assumes numerous factors in check such as: political stability, inflation, economic growth, and various others.

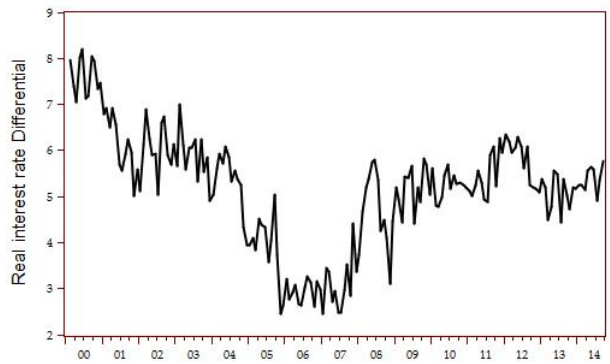


Figure 2. Real interest rate differential

during the whole period, the real interest rate in South Africa is higher than the real interest rate of the United States. This is normal, especially for a country like South Africa in full economic growth. Except the period from late 2005 to late 2007 where the gap is more stable, the real interest rate differential is more volatile. The period of stability of the gap corresponds to the period of expectations of recession with lower nominal interest rates, but also to the economic recovery period.

4.3 Real exchange rate

In the analysis of exchange rate, it is important to make the difference between nominal and real exchange rate. Two types of exchange rate are usually used.

The nominal exchange rate is the price of one currency in terms of number of units of some other currency. It is determined by demand and supply for the two currencies on the foreign exchange rate market in a floating rate regime, and is fixed in a fixed rate regime. It is "nominal" because it measures only the numerical exchange value, and does not say anything about other aspects such as the purchasing power of that currency. The real exchange rate, however describes how many of a good or service in one country can be traded for one of that good or service in another country. It incorporates in the analysis the *purchasing power and competitiveness aspect*. It is used to analyze the competitiveness.

The real exchange rate corresponds to the nominal exchange rate multiplied by the relative price index of the two countries.

$$RER_t = S_t * \frac{P_t^*}{P_t} \tag{19}$$

where RER represents the real exchange rate, S the nominal interest rate, P^* the foreign consumer price index and P the domestic consumer price index. The following equation, equation 20 is the expression in logarithm of the equation 19.

$$rer_t = s_t + p_t^* - p_t \tag{20}$$

In our work, an increase in the real exchange rate corresponds to a depreciation of the domestic currency.

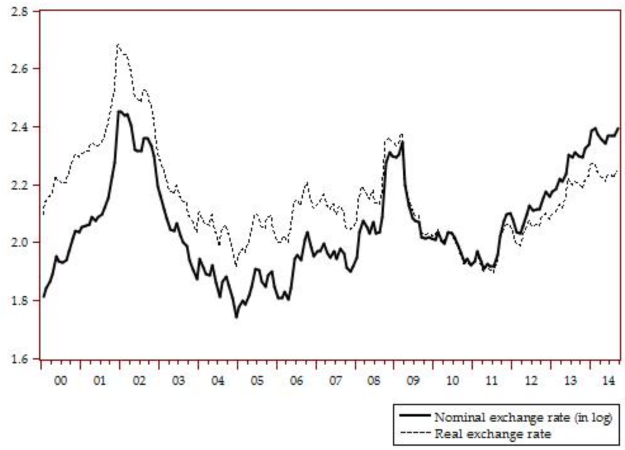


Figure 3. Nominal exchange rate *vs* Real exchange rate of South Africa

In contrast, declining real exchange rate corresponds to an appreciation of the domestic currency and thus a depreciation of the foreign currency. The figure 3 shows both the evolution of the nominal exchange rate and real South African Rand. We note in the evolution of the real value of the South African Rand, alternating phases of depreciation and appreciation. To be more specific, the South African currency has experienced a phase of depreciation reaching a record level in the last quarter of 2001 with a real value of 16.44 ZAR/US Dollar. Subsequently, there is a phase of appreciation of the South African Rand until the end of 2008, followed by a sudden depreciation in 2009 in full general recession. Then, apart a currency appreciation between 2009 and 2011, the South African Rand continues to depreciate.

4.4 Unit root test

The variables used to achieve our goals are the real exchange rate of South African Rand (rer), the real price of platinum (rpla), of gold (rgop) and the differential in real interest rates (rrid). All variables are in logarithm term. Before any concrete analysis of our data and our models, it is necessary important to see if our variables are stationary or not, i.e. to know the order of integration of the series. Two unit root tests is performed. we have the Augmented Dickey fuller test (ADF) and the Phillip Perron test (PP).

The strategy of the test is as follows. First, we perform the tests on the variables in level. If the series are stationary, then we stop there. But if not, then we differentiate the series and perform again the tests. ADF and PP tests are performed in sequential manner, i.e., starting from a model with a linear trend and a constant (equation 21), followed by a model with constant (equation 22), and finally a model without constant (equation 23). We determine the correct model taking into account the significance of the trend (model 3) Constant (model 2), and finally make the unit root test. Consider a variable X_t, with $t = 1, ..., T$,

- Model with linear trend and constant

$$\Delta X_t = \phi X_{t-1} + \sum_{i-1}^{p} \eta_i \Delta X_{t-i} + c + \beta t + \varepsilon_t \tag{21}$$

- Model with constant

$$\Delta X_t = \phi X_{t-1} + \sum_{i-1}^{p} \eta_i \Delta X_{t-i} + c + \varepsilon_t \tag{22}$$

- Model with no constant

$$\Delta X_t = \phi X_{t-1} + \sum_{i-1}^{p} \eta_i \Delta X_{t-i} + \varepsilon_t \tag{23}$$

where c represents the intercept, t, the trend and ε, a white noise.

	Level		**Difference**	
Series	ADF	PP	ADF	PP
rer	0.002786	0.095440	-10.16767	-10.11562
	(1.942655)	(1.942655)	(1.942655)	(1.942655)
	[*none*]	[*none*]	[*none*]	[*none*]
rpla	0.767882	1.020813	-6.592121	-10.67599
	(1.942655)	(1.942655)	(1.942655)	(1.942655)
	[*none*]	[*none*]	[*none*]	[*none*]
rgop	2.791916	2.434698	-12.00516	-12.13709
	(1.942655)	(1.942655)	(-2.877916)	(-2.877919)
	[*none*]	[*none*]	[*Intercept*]	[*Intercept*]
rrid	-0.896591	-1.064656	-14.00154	-20.03133
	(1.942655)	(1.942655)	(1.942655)	(1.942655)
	[*none*]	[*none*]	[*none*]	[*none*]

In (), test critical value of Mackinnon, and [] the deterministic term
The null hypothesis (H_0) corresponds to the existence of unit root.
We accept H_0 when the t-statistic are higher than the critical value.
Otherwise we reject it.

Table 2. Unit root tests

The results of unit root test is presented in the table 2.

We note that all series are not stationary in level, but in first difference, they are stationary. The real exchange rate, platinum price, price of gold, and the differential of the real interest rates are integrated of order 1, i.e. $I(1)$. It may therefore be possible to have a linear combination which is stationary, thus leading to a long-term analysis.

5 Models and empirical results

In this section we will define our models and we are interested in analyzing of the results. So we start with the examination of a potential existence of long-term relationship between the real exchange rate of Rand (*rer*), the real price of platinum (*rpla*), of gold (*rgop*) and the real interest rate differential (*rrid*).

5.1 Co-integration: Engle & Granger approach

As with any model of long-term relationship analysis, it is important to estimate the long-run equation. The long-run equation estimated is the following:

$$rer_t = \beta_0 + \beta_1 \times rpla_t + \beta_2 \times rgop_t + \beta_3 \times rrid_t + \varepsilon_t \quad (24)$$

The estimation of this model give the following result:

$$rer_t = 4.512654 - 0.495105 \times rpla_t + 0.177749 \times rgop_t - 0.006271 \times rrid_t + \varepsilon_t \quad (25)$$

With the estimation of equation 24, we recover the residual series $\hat{\varepsilon}$:

$$\hat{\varepsilon}_t = rer_t - (\hat{\beta}_0 + \hat{\beta}_1 \times rpla_t + \hat{\beta}_2 \times rgop_t + \hat{\beta}_3 \times rrid_t) \quad (26)$$

If it requires co-integration, then the test-statistic of the ADF test on the estimated residuals must be lower than the critical value read from the statistic table of Mackinnon. The ADF test on the estimated residuals give a test-statistic[12] of -3.137675. In our case the critical value of read from the statistic table of Mackinnon is -4.10. En consequence, we see that the test-statistic is higher than the critical value. We deduce the non-existence of co-integration relationship between the real exchange rate, the real price of platinum, of gold and the real interest rate differential.

In general, most of the time, without co-integration relationship (Engle & Granger approach) between variables of a model, researchers focus their analysis on the short-run relationship among variables, i.e. variable shocks analysis and its impact on others economic variables. This kind of analysis is possible through the Vector AutoRegressive processes (*VAR*).

5.2 Vector Auto Regressive process

In this subsection, we analyze the impact of shocks of each variable on other others variables. All variables are endogenous in this king of model. In the *VAR* process, all variables are stationary, it means that we work on the difference[13] of our series (ΔX) of the series.

The results of the estimated *VAR* model by OLS method are represented in the table (Appendix). For the validation of the model, we have done the test of Breush-Godfrey (LM test) and the test of Ljung-Box[14] for the absence of correlation and the test of White for the absence of Heteroskedasticity.

The null hypothesis of the Breusch-Godfrey test is the "no serial correlation" at lag order h, with $h = 1, \ldots.7$. The results of the test is presented in the table 5. We note that we accept the null hypothesis

[12] This t-statistic corresponds to the ADF model without constant: $\Delta\hat{\varepsilon}_t = \phi\hat{\varepsilon}_{t-1} + \Phi\Delta\hat{\varepsilon}_{t-1} + \varepsilon_t$

[13] $\Delta X_t \approx \log X_t - \log X_{t-1}$

[14] $Q_{LB} = T(T+2)\frac{\sum_{j=1}^{m}\rho^2}{T-j}$ where Q_{LB},T,ρ are respectively the Ljung-Box test-statistic, the number of observations, the autocorrelation

when the p-value is above the critical threshold of 5%. Thus, in light of the results presented in the table 5, we can deduce that our residuals series are not autocorrelated. For the Ljung-Box test, we hold a number of autocorrelations to order 7 too. The corresponding Ljung-Box statistic is 71.01303 and the associated p-value of 0.2556, which allows us to accept the null hypothesis of no autocorrelation. We can conclude under both tests our series are not autocorrelated.

Regarding the absence of heteroskedasticity or not, the White test (no cross terms) is performed and the results are presented in the table 3. The White test indicates that the variance of our residuals is not constant over the time.

Chi-sq	df	Prob
308.4772	240	0.0019

Table 3. *VAR* Residual Heteroskedasticity Tests

Most of the time, the mains utilities of the *VAR* processes are to analyze firsly the impulse response functions, secondly the analysis of variance decomposition of the errors, and thirdly the analysis of Granger causality test.

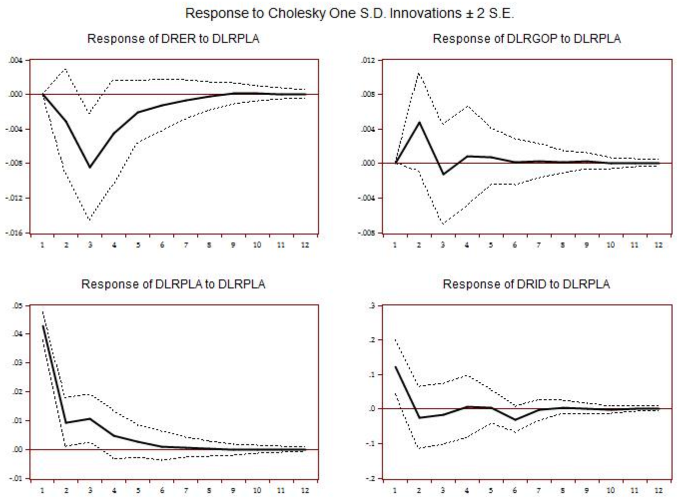
5.2.1 Impulse response functions

Once the validations tests, it seems very interesting to examine the impact of shocks for each variable. Thus, in this subsection, we analyse the impulse response functions of our model. We analyze the effect of a shock one variable on the others variable. Concretely, it is to analyze the effect of innovation on current and future variables. We consider that the magnitude of the shock is equal to one times the standard deviation and we are interested in the effects of the shock on twelve periods (12 months = one year).

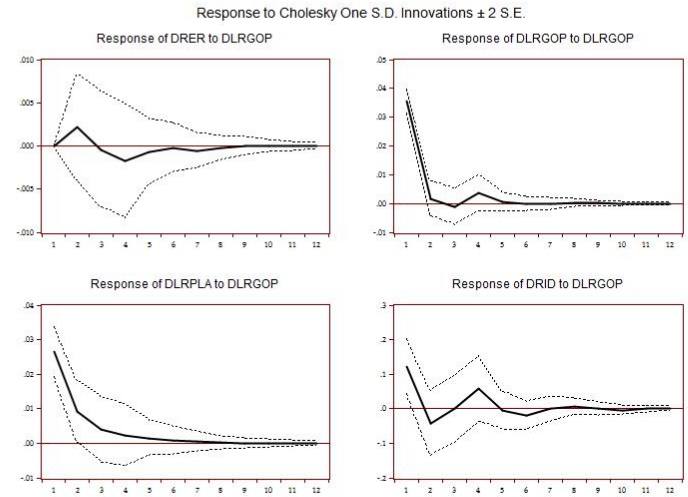
We start first by analyzing the effect of a shock of real platinum price (increase of platinum price) on the real exchange rate, the real price of gold, platinum and finally the real interest rate differential. A real platinum price shock has no immediate effect on the real exchange rate in South Africa. The impact become effective at the end of the first quarter with a real appreciation of the Rand. Beyond the third month, a rise in the real price of platinum has no effect on the real value of the Rand. A positive shock of the real price of platinum has no effect on the actual price of gold. But the real price of platinum is sensitive to its own shock until the second month. After the shock wears off. On the response of the real interest rates differential, the effect of a real shock platinum price is immediate. But beyond the first two months the increase of the price of platinum has no effect on the differential in real interest rates.

Secondly, we are interested in the effect on other variables following a positive shock of the price of gold (increase in the real price of gold). The shock of the real price of gold has no real impact on the real value of the Rand. The lack of effect is due to lower production of gold and the higher costs of production of gold in South African since early 2000. Unlike the lack of real effect on the value of the Rand, an increase in the real price of gold has an immediate and positive effect on itself and on the real platinum price and keep persistent for two months. It has also a positive effect on the real interest rate differential which is immediate at the impact. After these periods, the effect disappears.

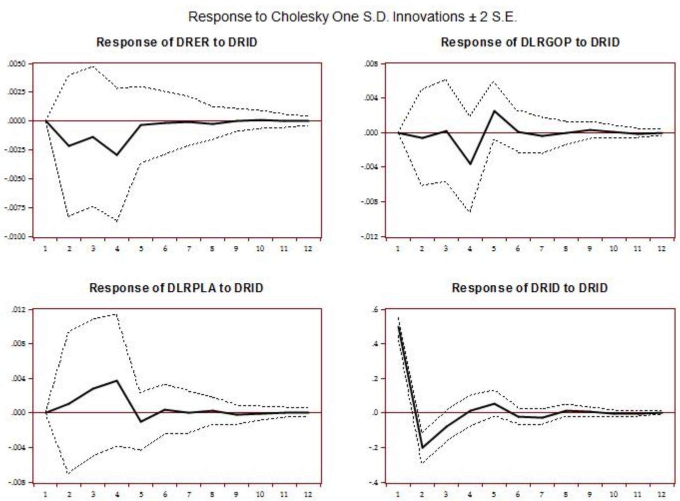
Regarding the analysis of impulse response functions, it must be said that a positive shock of the real interest rate differential has no effect on others variables, apart from itself and this during a single month. Finally, a real depreciation of the South African Rand has an immediate negative effect on the real price of gold and platinum. Then we have an improvement in the price level with a slight increase over the first two periods. An depreciation of the Rand doesn't have any effect on the real interest rate differential.



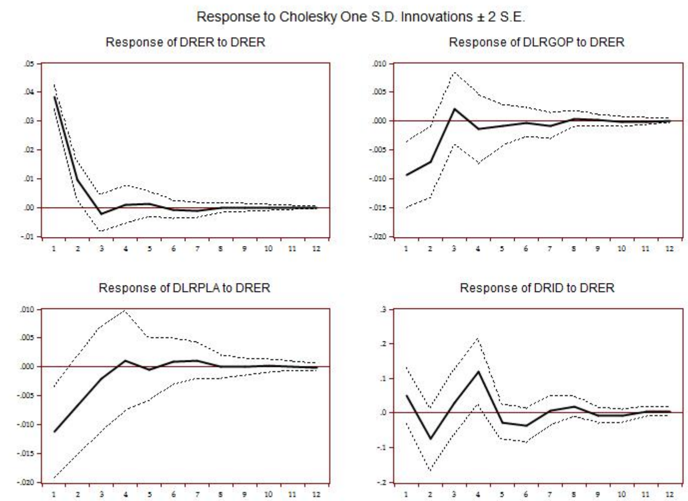
Impulse response functions *drpla*



Impulse response functions *drgop*



Impulse response functions *drrid*



Impulse response functions *drer*

5.2.2 Variance decomposition

After analyzing the impulse response functions, it is important to push the analysis and therefore to focus on the analysis of variance decomposition of the forecast error. The goal is to calculate the contribution of each of the innovations to the variance of the error. The results relating to the variance of the decomposition of the study are reported in the table **??** (Appendix). Firtly, The variance of forecast of $\Delta rgop$ is due for 9.839284% to the innovations of Δrer, for $87,074\%$ to its own innovations, for 1.756% to the innovations of $\Delta rpla$ and finally for 1.330% to the innovations of $\Delta rrid$. Secondly, the variance of forecast of $\Delta rpla$ is due for 5.833% to the innovations of Δrer, for $26,544\%$ to the innovations of $\Delta rgop$, for $66,833\%$ to its own innovations and finally for 0.789% to the innovations of $\Delta rrid$. Thirdly, the variance of forecast of drrid is 7.164% is due to the innovations of Δrer to 5.7387% of the innovations $\Delta rgop$ to 4.562476% of the innovations $\Delta rpla$ and finally for 82.53476% to its own innovations.

The results of the variance decomposition of forecast errors come support the results of the analysis of the functions of impulse responses. The real exchange rate is influenced by their own values. The price of gold is influenced by its own values, but also by the actual value of the Rand. The real price of platinum is influenced in the short-run by its own values and the real price of gold. The differential in the real interest rate is influenced only by itself. All this, is consistent with the explanations given above by analyzing the impulse response functions.

5.2.3 Granger causality

We refine our analysis by performing the no causal test of Granger, 1969. The results obtained for a number of lags $p = 3$ are shown in table 4. The null hypothesis is the hypothesis of no causal relationship.

	Δrer	$\Delta rpla$	$\Delta rgop$	$\Delta rrid$
Δrer		0.895	4.366	11.997^{**}
		$-$	$-$	$\rightarrow$
$\Delta rpla$	7.927^{**}		3.357	0.392
	$\rightarrow$		$-$	$-$
$\Delta rgop$	2.908	1.032		1.792
	$-$	$-$		$-$
$\Delta rrid$	1.551	1.337	2.427	
	$-$	$-$	$-$	

$X \rightarrow Y$ *means that* X *cause* Y

$X - Y$ *means that* X *not cause* Y

Table 4. *VAR* Granger Causality/Block Exogeneity Wald Tests

From the results shown in the table 4, we come to the conclusion that the real value of Rand influences in short-run the real interest rate differential, but also that the real price of platinum influences in short-run the real exchange rate. Economically speaking, the real price of platinum is a key determinant of Rand at the short-run.

Lags	1	2	3	4	5	6	7
LM-Stat	9.275967	14.22467	15.27051	14.38727	8.778025	22.57022	22.66844
Prob	0.9016	0.5820	0.5049	0.5699	0.9223	0.1257	0.1229

Table 5. *VAR* Residual Serial Correlation LM Tests

We find also that the real exchange rate of the Rand is a key determinant at short-run of real interest rate differential.

It is therefore logical that the real price of platinum is a key determinant of Rand in short-run. Rising platinum price leads to a real appreciation of the Rand. And a real appreciation of the Rand is followed by an increase of the real interest rates in the short-run with the aim to depreciate the currency in the long-run. We expect a depreciation by increasing interest rates in South Africa.

We are interested in short-run dynamics through a *VAR* model because the co-integration analysis through univariate approach of Engle & Granger was rejecting the hypothesis of a long-term relationship between the variables. The estimated residuals of the equation are stationary when it is a simple test of unit root ADF, but the test-statistic is quite small compared to the critical value read from the table of Mackinnon to conclude a co-integration relationship among the variables. We know that the approach of Engle & Granger is not the only possible approach to analyze a co-integration relationship. In the following section, we look at the co-integration analysis based on a multivariate approach. We speak about the vector approach, the vector error correction model: the *Johansen approach*.

5.3 Vector error correction model (VECM): Johansen approach

Consider a matrix X containing the variables of our study in level and $I(1)$.

$$X_t^{'} = (rer_t, rpla_t, rgop_t, rrid_t)$$

The estimated long-term equation based on the

$$VAR$$

modeling for a number of lags $p = 4$ is given by the following expression:

$$X_t = \Phi_0 + \Phi_1 X_{t-1} + \Phi_2 X_{t-2} + \Phi_3 X_{t-3} + \Phi_4 X_{t-4} + \varepsilon_t \tag{27}$$

with Φ_i, $i = 1, ..., 4$, vectors of parameters.

With the estimation of equation 27, we obtain the residual matrix $\hat{\varepsilon}$.

For the analysis of co-integration by Johansen's approach, it is important that the residuals are stationary. Two tests of absence of autocorrelation can be made namely Breusch-Godfrey LM test and the

Lags	1	2	3	4	5	6	7
LM-Stat	13.42	18.97	22.26	17.58	10.34	18.36	21.863
Prob	0.64	0.27	0.135	0.349	0.848	0.303	0.148

Table 6. *VAR* Residual Serial Correlation LM Tests

Ljung-Box test to ensure the stationarity of residual. In our study we perform the Breusch- Godfrey LM test to ensure the absence of correlation or not in residuals. Test results are presented in Table 6.

The LM test analysis for a number of correlations of $h = 7$, indicates the absence of correlation in the residuals.

Note: In the analysis of co-integration by Johansen approach based on the *VAR* model, it should be noted that in case of possible relationship of co-integration between the variables, a $VAR(p)$ on the variables in levels corresponds to a $VECM(p-1)$, with p the number of lags in the model.

5.3.1 Test of Trace and maximum Eigenvalue

The table 7 presents the possible various options of co-integration relationship mentioned in section 3.1.2, based on the fact that it was a trend and/or constant. The majority combination used most of the time to analyze co-integration by Johansen's approach is the third combination (presence of an intercept in the co-integrating relationships but not in the error correction model (no linear trend (see table 7)).

Data Trend	None	None	Linear	Linear	Quadratic
	No Intercept	*Intercept*	*Intercept*	*Intercept*	*Intercept*
	No Trend	*No Trend*	*No Trend*	*Trend*	*Trend*
Trace	0	1	1	0	0
Max-Eig	0	1	1	1	1

Table 7. Co-integrating Relations by Model

The test of *Trace*, tests the existence of j co-integrating relationship against $j+1$ co-integrations relations. The test of trace and maximum eigenvalues performed at the 5% level for a number of lags $p = 3$, indicates the existence of one co-integrating relationship between the real exchange rate South African, real prices of platinum, gold and the real interest rate differential. There is only one co-integration relationship between the rate of South African real exchange, the price of gold, platinum and the real interest rate differential. We can estimate our vector error correction model.

5.3.2 Results

Once sure of the number of the exiting co-integration vector, we can estimate our vector error correction model. The estimated vector model for a number of lags $p = 3$ allows us to obtain the long-term equation illustrated by the equation 28.

$$rer_{t-1} = -1.5171 \times rpla_{t-1} + 0.6812 \times rgop_{t-1} - 0.11656 \times rrid_{t-1} + 8.9783 \quad (28)$$

we can rewrite the equation 28 like:

$$\begin{array}{ccccccc} rer_t = & - & \underset{(-6.40132)}{1.5171 \times rpla_t} & + & \underset{(4.58846)}{0.6812 \times rgop_t} & - & \underset{(-3.63260)}{0.11656 \times rrid_t} & + & 8.9783 \end{array} \quad (29)$$

in (), the test-statistic.

The matrix of adjustment speed is given by the equation 30.

$$\begin{pmatrix} rer \\ \\ rpla \\ \\ rgop \\ \\ rrid \end{pmatrix} \Theta = \begin{pmatrix} \underset{(0.0913)}{0.001570} \\ \\ \underset{(-5.364)}{-0.113882} \\ \\ \underset{(-1.9879)}{-0.032293} \\ \\ \underset{(-0.487)}{-0.115295} \end{pmatrix} \quad (30)$$

5.3.3 Validation of the model

In this subsection, we turn to the step of validation of the vector error correction model. The following graph shows the evolution of the residuals of the co-integrating relationship. At first glance, the residuals

	Ljung-box		LM test	
Lags	Q-Stat	Prob	LM-Stat	Prob
1	0.430	Na*	9.083	0.9099
2	2.798	NA*	13.865	0.6087
3	6.820	NA*	17.508	0.3534
4	21.68	0.795	16.948	0.3889
5	31.220	0.9264	9.829051	0.8754
6	51.208	0.7834	20.709	0.190
7	70.320	0.661	20.516	0.198

*The test is valid only for lags larger than the *VAR* lag order

Table 8. Ljung-Box *vs* LM test

appear stationary, which is good to validate our model. We can perform a test of ADF on the residual to see if they are stationary or not, but we prefer to perform autocorrelation tests to ensure that our residuals are not autocorrelated. We perform two tests of absence of autocorrelation (LM & Ljung-Box tests).

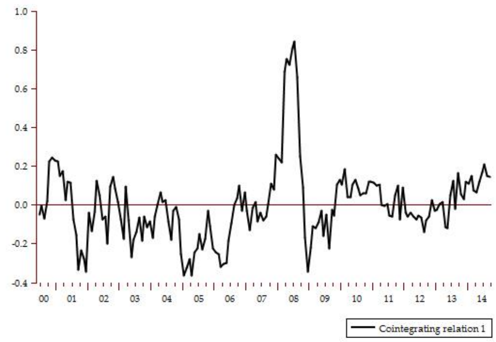


Figure 4. Evolution of co-integration residuals

5.3.4 Interpretation of results

Now, we analyze the long-run relationship and the impulse response functions.
We analyze first the co-integrating equation.

$$\begin{array}{lccccccc} rer_t = & -1.5171 \times rpla_t & + & 0.6812 \times rgop_t & - & 0.11656 \times rrid_t & + & 8.9783 \\ & (-6.40132) & & (4.58846) & & (-3.63260) & & \end{array} \quad (31)$$

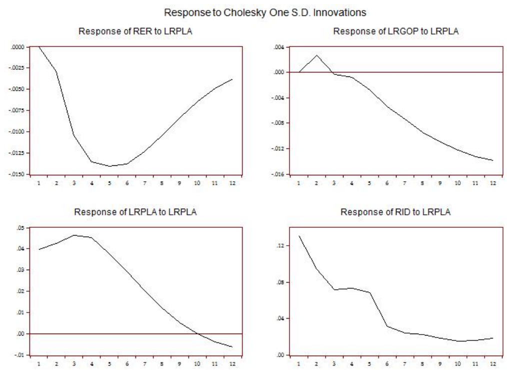
in (.), the test-statistic.

In the long-run, the rise in platinum prices *rpla* causes an appreciation of the real value of the Rand. The rise of the platinum price is for the South Africa a gain for the economy, an increase of the purchasing power. It is important to remember that South Africa is the largest platinum producer in the world.

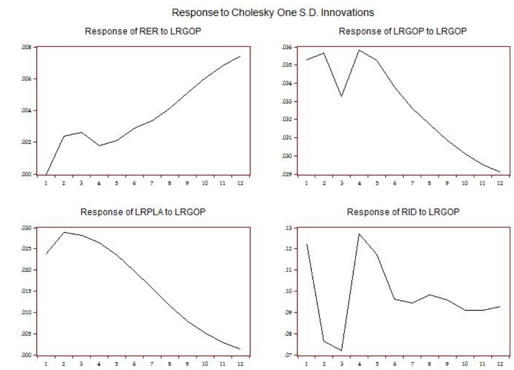
Regarding the price of gold, the rise of its real value leads at long-term to a real depreciation of the Rand. A Rise of the gold price causes an appreciation of the real value of the Rand in the short-run, an appreciation justified by a significant increase of the gain. This real appreciation of the Rand leads to an increase in purchasing power and therefore an increase in imports. This real appreciation causes a loss of competitiveness, and so an increase of imports. The increase in imports therefore results in a *Outflows of currencies*, a supply of currencies that will lead to a depreciation of the Rand in the long-run. Contrary to the depreciation of the Rand, the dollar will appreciate in the long-run. The price of gold is more volatile than the platinum price.

An increase in the real interest rate differential causes in the long-term an appreciation of the real exchange rate of Rand. Instead of increasing the differential real interest rate which results in an depreciation in the long-run through an appreciation in the short-run of the exchange rate (due to short-run capital inflows), there is a real appreciation of exchange rate.

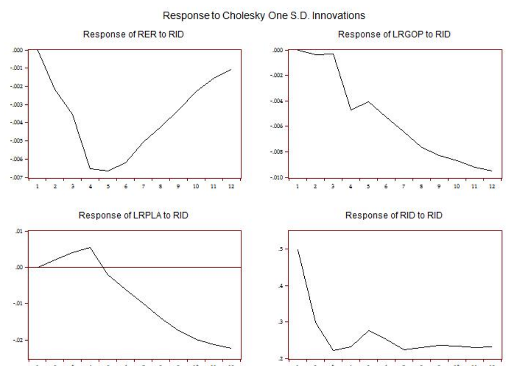
It is more interesting to confirm the analysis of the long-term equation, through the analysis of the response functions in order to see the convergence of long-term variables following the shock of certain variables. Following a rise in platinum prices in the long-run, the real exchange rate appreciates (as provided by the long-term equation), the real price of gold decreases, the gap between the real interest



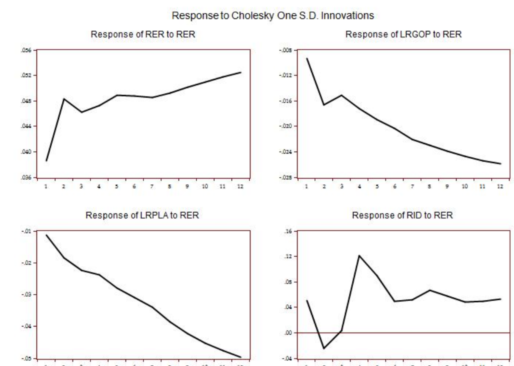
Impulse response function *rpla*



Impulse response function *rgop*



Impulse response function *rrid*



Impulse response function *rer*

Table 9. Impulse response functions

rate of south African and American decline $[0;0.04]$. The latter seems more interesting because it leads us to go along with the theory that interest rates equalize in the long-run, there is a decrease in spreads between interest rates. However an increase in the real price of gold depreciates the long-term currency. In addition to the Rand depreciate the difference between the interest rate seems more stable $[0.09;0.10]$, but the gap is greater than in the case of platinum price increase. An increase in the interest rate differential appreciate the real exchange rate, but also causes a decrease in the price of gold and platinum. A real depreciation of the Rand causes a decrease in the price of gold, platinum prices and above all a stabilization gap in interest rates between $[0.04;0.05]$.

6 conclusion

We are interested in an econometric investigation, a study on the difficulty of modeling the South African exchange rates. The aim of our study is to examine the nature of the existing relationship between the real exchange rate of the Rand, the real price of gold, platinum and the real interest rate differential over the period going from January 2000 to September 2014. This, to show that over the same period, with different econometric methods, variables can be the determinants of Rand, but at different horizons.

After analysis, we come to the conclusion that, it is difficult to analyze the South African exchange rate. The determinants of the Rand vary according to the methods used and these methods used do not allow us to have robust results. The analysis of possible long-run relationship through the co-integration approach of Engle and Granger reveals the absence of equilibrium relationship between real South African exchange rate, real platinum price, gold price and the real interest rate differential. Results which leads us to be interested in the short-run dynamic of variables. Once, the analysis performed through a vector autoregressive model, we find that the real platinum price shock has no immediate effect on the real exchange rate in South Africa. The real impact has become effective at the end of the first quarter with a real appreciation of the Rand. On the response of the real interest differential rates, the effect of a real platinum price shock is immediate. Beyond the first two months, the increase of platinum price has no effect on the real interest rate differential. The main surprise is the absence of impact of real price of gold shock on the real value of the Rand. Finally, a real depreciation of the South African Rand has an immediate and negative effect on the real price of gold and platinum.

Contrary to Engle & Granger approach, the adoption of the vector error correction model (Johansen) insinuates that there is a long-run relationship between the Rand, real price of platinum, gold and the real interest rate differential. In the long-run, the rise of platinum price causes an real appreciation of the Rand but the rise of gold price at long-term leads to depreciate the Rand. The rise of this raw materials prices is for the South Africa an add-value for his economy, an increase of the purchasing power. Finally, we have found that an increase in the real interest rate differential causes in the long-term, a real appreciation of the Rand. However, it would be very interesting to analyze the non-linearity effect of raw materials price effects on the exchange rate, i.e. analyze the effect of increases and decreases of prices on the real exchange rate South African real exchange rate.

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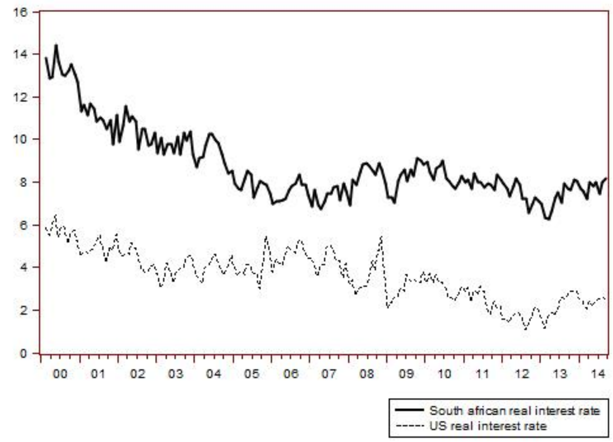
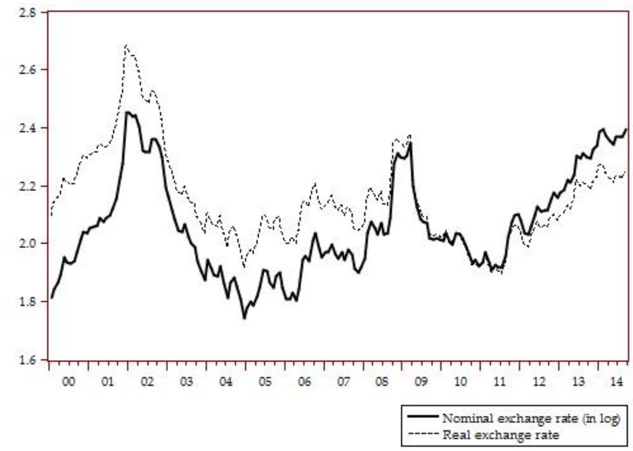
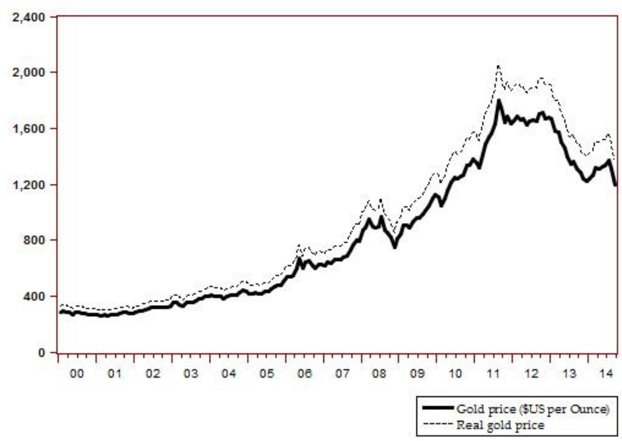
7 Appendix



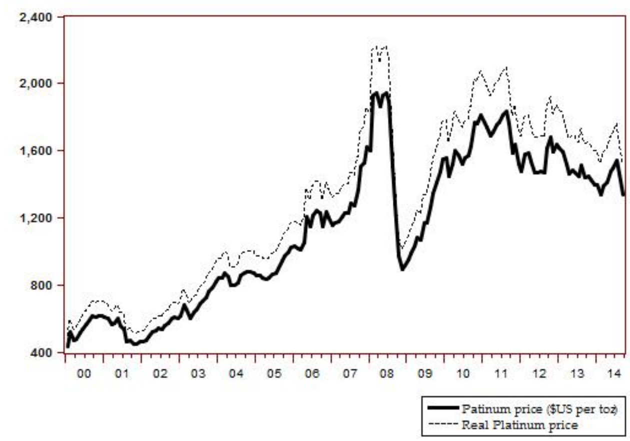
Real interest rate South africa *vs* United States



Real *vs* nominal exchange rate



Real *vs* nominal gold price



Real *vs* nominal platinum price

	RER	RGOP	RPLA	RID
Mean	2.155004	938.4588	1298.954	5.167719
Median	2.128437	763.7669	1338.657	5.281190
Maximum	2.683020	2053.306	2222.459	8.194098
Minimum	1.896708	293.0408	509.9832	2.451407
Std. Dev.	0.163633	557.0852	499.5395	1.259937
Skewness	1.059699	0.473281	-0.008689	-0.266894
Kurtosis	4.064146	1.802758	1.704814	2.978642

Table 10. Stylized fact of our series

Dependent Variable: rer				
Variable	Coefficient	Std. Error	t-Statistic	Prob
rgop	0.177749	0.041112	4.323505	0.0000
rpla	-0.495105	0.064454	-7.681474	0.0000
rrid	-0.006271	0.008438	-0.743171	0.4584
C	4.512654	0.248573	18.15424	0.0000
$R^2 = 0.468792$, obs: 176				
Sample (adjusted): 2000M02 2014M09				

Table 11. Engle & Granger: long-run equation

Lags	Q-Stat	Prob.	Adj Q-Stat	Prob.	df
1	0.449615	NA*	0.452244	NA*	NA*
2	1.910684	NA*	1.930503	NA*	NA*
3	4.940960	NA*	5.014571	NA*	NA*
4	18.57634	0.2912	18.97460	0.2700	16
5	27.07215	0.7144	27.72478	0.6829	32
6	49.26767	0.4222	50.72254	0.3667	48
7	71.01303	0.2556	73.39044	0.1974	64

Null Hypothesis: no residual autocorrelations up to lag h

Table 12. *VAR* Residual Portmanteau Tests

Joint test	-		
Chi-sq	df	Prob.	-
308.4772	240	0.0019	-

Table 13. *VAR* Residual Heteroskedasticity Tests: No Cross Terms (only levels and squares)

Variance Decomposition of Δrer*:*

Period	$S.E.$	Δrer	$\Delta rgop$	$\Delta rpla$	$\Delta rrid$
1	0.038508	100.0000	0.000000	0.000000	0.000000
2	0.039945	98.78368	0.305462	0.615787	0.295070
3	0.040894	94.51894	0.303311	4.789722	0.388025
4	0.041287	92.77821	0.474769	5.857819	0.889205
5	0.041364	92.52192	0.501820	6.082644	0.893618
6	0.041391	92.43120	0.503185	6.171020	0.894595

Variance Decomposition of $\Delta rgop$*:*

Period	$S.E.$	Δrer	$\Delta rgop$	$\Delta rpla$	$\Delta rrid$
1	0.036820	6.378043	93.62196	0.000000	0.000000
2	0.037857	9.605363	88.76676	1.602536	0.025338
3	0.037952	9.860017	88.41695	1.695328	0.027702
4	0.038322	9.807770	87.57347	1.714297	0.904459
5	0.038422	9.796141	87.14124	1.746002	1.316621
6	0.038423	9.798989	87.13653	1.747870	1.316609

Variance Decomposition of $\Delta rpla$*:*

Period	$S.E.$	Δrer	$\Delta rgop$	$\Delta rpla$	$\Delta rrid$
1	0.051676	4.761622	26.56394	68.67444	0.000000
2	0.053779	5.964247	27.52606	66.46754	0.042155
3	0.055078	5.833569	26.76498	67.09093	0.310521
4	0.055466	5.791420	26.56080	66.89549	0.752288
5	0.055555	5.778147	26.55077	66.88778	0.783311
6	0.055579	5.798045	26.54817	66.86719	0.786594

Variance Decomposition of $\Delta rrid$:

Period	$S.E.$	Δrer	$\Delta rgop$	$\Delta rpla$	$\Delta rrid$
1	0.530485	0.922028	5.421206	5.206458	88.45031
2	0.574214	2.506108	5.143692	4.627302	87.72290
3	0.580381	2.729705	5.035008	4.598715	87.63657
4	0.595500	6.591356	5.745779	4.377104	83.28576
5	0.598398	6.732620	5.697476	4.341170	83.22873
6	0.601001	7.055316	5.746357	4.569654	82.62867

Table 14. Variance Decomposition of the *VAR*