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On the Welfare Costs of Monetary Policy*

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Abstract

This paper analyses the implications of monetary policy changes on the welfare in the U.S economy over the pre-1984 and post-1984 periods. We use a New-Keynesian model with trend inflation based on Ascari, Phaneuf and Sims (2015). First, our results show that the welfare costs respond symmetrically to a rise and a decline in trend inflation, trend growth and the level of volatility of output, output growth and inflation over the sample periods. Second, we find that changes in monetary policy and in trend inflation across the two subsamples play an important role in the shift of macroeconomic variables volatilities unconditionally and conditionally to neutral technology, marginal efficiency of investment and monetary shocks.

JEL classification: E31, E32.

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1 Introduction

The period from the late 1960s through the early 1980s in the U.S economy\(^1\) and the subsequent decline in macroeconomic variables volatilities in the Great Moderation era, have received a lot of attention in the literature.

In an early contribution, Clarida, Gali, and Gertler (2000) explored the role of monetary policy in macroeconomic stability. They estimate a Taylor-type monetary policy rule and combine it with a calibrated sticky-price framework with zero steady-state inflation. They show that there is a significant difference in the way monetary policy was conducted; interest rate policy in the Volcker period seems to have been much more responsive to changes in expected inflation than in the pre-Volcker period. They find that this difference could be an important source of the shift in macroeconomic variables volatilities. Lubik and Schorfheide (2004), Boivin and Giannoni (2006) and Zandweghe, Hirose and Kurozumi (2015) also reach the same conclusion.

However, the recent paper of Coibion and Gorodnichenko (2011) challenges this view. Based on an estimated Taylor rule and a calibrated staggered-price model with non-zero trend inflation, they attribute the shift to changes in the Feds response to macroeconomic variables and the decline in trend inflation during the Volcker disinflation. This finding is confirmed by Arias, Ascari, Branzoli and Castelnuovo (2015).

More recently, Christiano (2015) argues that the high inflation of the 1970s may have had a cost equivalent to a loss of 10 percent of GDP or more in each year that the inflation was high. However, the aforementioned literature has been silent about the implications of the welfare.

In this paper, we explore the link between non-zero trend inflation and macroeconomic variables volatility in the pre- and post-1984 periods. Then we analyze the implications of policy changes for welfare in both sample periods. The main contribution of the paper is to examine the role of non-zero steady-state inflation in the U.S economy’s shift from the pre-1984 to the post-1984 era as well as the implications for the welfare.

\(^1\)The U.S economy experienced high and volatile inflation along with several severe recessions (Clarida, Gali, and Gertler, 2000).
To address these issues, we use a medium-scale DSGE model inspired by Ascari, Phaneuf and Sims (2015). However, our approach is quite different. First, consistent with evidence in Gali and Gambetti (2009) we split the sample into 1960:I - 1983:IV and 1984:I - 2007:III subsample periods whereas they describe a full sample results. Second, in line with Clarida, Gali, and Gertler (2000), we first calibrate the Fed’s reaction function in the pre- and post-1984 periods based on estimates in the literature. We then combine it with our calibrated medium-scale New Keynesian model.

Based on evidence in the literature, we assume that structural parameters do not change over the subsample periods except for shocks parameters, trend inflation and real per capita output growth. It is worth noting that real per capita output growth originates from two main sources: trend growth in investment-specific technology (IST) and in neutral technology.

The trend growth rate of the investment shock is chosen to match the average growth rate of the relative price of investment in the data, and then the trend growth rate of the neutral shock is picked to match the observed average growth rate of output per capita, given the growth rate of the investment shock. Trend inflation is set at its observed value.

As in Ascari, Phaneuf and Sims (2015), the magnitudes of neutral technology, Marginal Efficiency of Investment (MEI), and monetary shocks in each subsample period are selected to (i) match the observed volatility of output growth in the data at respective rate of trend inflation and to (ii) hit a variance share of output growth of 35-50-15 percent for neutral, MEI, and monetary shocks, respectively.

We find the following results. First, welfare costs respond symmetrically to a rise and a decline in trend inflation, trend growth and the level of volatility of output, output growth and inflation over the pre- and post-1984 era respectively. An increase in the variance of shocks to the trend inflation process increases means welfare costs by increasing the volatilities of output and inflation in the pre-1984 period. The welfare costs in the post-1984 period are modest.

More specifically, the results show that based on means, the welfare cost of going from 0 to 4.75 percent inflation, in the pre-1984 period, is about 8.38 in the baseline case and 4 percent of consumption in the case without trend growth; in terms of steady states, it is 7.56 and 2.26 percent of consumption respectively. Consumption equivalent losses do depend on the overall level of volatility, trend growth and trend inflation.
The welfare cost of 2.29 percent inflation, in the Great Moderation period, in terms of mean is about 2.34 in the baseline case and 0.14 percent of consumption in the case without trend growth; in terms of steady states, it is around 2.26 and 0.12 percent of consumption respectively. The means change by more than the steady states and exceed the steady state losses.

Second, we find that the way monetary policy was conducted over both subsamples along with changes in trend inflation play an important role in the shift of macroeconomic variables volatilities unconditionally and conditionally to neutral technology, marginal efficiency of investment and monetary shocks. We concur with the original conclusion of Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004) and Zandweghe, Hirose and Kurozumi (2015). However, our results are in line with those of Coibion and Gorodnichenko (2011) and Arias, Ascari, Branzoli and Castelnuovo (2015).

This paper is in line with a set of papers that examines the effects of non-zero trend inflation on macroeconomic dynamics in New Keynesian models. Ascari and Ropele (2007) study the effects of non-zero trend inflation for optimal monetary policy, while Amano, Moran, Murchison, and Rennison (2009) examine the implications for the optimal rate of inflation. Amano, Ambler, and Rebei (2007) explore the implications for the time-series properties of macro variables, Ascari and Ropele (2009), and Coibion and Gorodnichenko (2011) study the effect of trend inflation on the determinacy of the model. This paper differs from theirs as it studies the effects of monetary policy changes for the welfare costs over the pre- and post-1984 periods.

The paper closest to ours, in line with the "effects of non-zero trend inflation" literature, is the one by Ascari, Phaneuf and Sims (2015). It addresses the welfare and cyclical implications of moderate trend inflation whereas we explore the monetary policy changes over the pre- and post-1984 subsample periods and its welfare implications.

Our paper is also closely related to Clarida, Gali, and Gertler (2000). However, we use sticky price and sticky wage model extended to non-zero trend inflation, trend growth and roundabout production structure whereas they use a sticky price model based on zero trend inflation. They find that changes in the Fed’s policy constitute the main source of the shift in macroeconomic variables volatilities whereas we find that both monetary policy changes and the shift in trend inflation over the subsample periods are the main causes.
The rest of the paper is organized as follows. In Section 2, we outline our baseline model specification. In Section 3, we discuss the calibration of the structural parameters. We present results in Section 4, and the last section concludes.

2 The Model

In this section, model specification is outlined. More details on full set of stationarized equilibrium conditions can be found in ?.

2.1 Firms and Price setting

2.1.1 Final Goods Producers

The final good producer uses $X_t(j)$ units of intermediate goods to produce $X_t$ units of final good. There is a continuum of intermediate goods firms indexed by $j \in (0, 1)$, producing differentiated goods. The final good is a constant elasticity of substitution aggregate of intermediate goods with $\theta > 1$, using the production technology given by:

$$X_t = \left( \int_0^1 X_t(j)^{(\frac{\theta - 1}{\theta})} dj \right)^{\frac{\theta}{\theta - 1}}. \quad (1)$$

The final goods producer maximizes profit, given a final good price, $P_t$ and taking intermediate good prices, $P_t(j)$, as given. The first-order condition gives the conditional demand for intermediate good $j$:

$$X_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\theta} X_t, \quad \forall j. \quad (2)$$

Inserting the demand function for input $j$ back into the CES aggregator gives the aggregate price index:

$$P_t^{1-\theta} = \int_0^1 P_t(j)^{1-\theta} dj. \quad (3)$$
2.1.2 Intermediate Producers

Each intermediate-good firm, indexed by $j$, uses $\tilde{K}_t(j)^2$ units of capital services, $L_t(j)$ units of labour, and intermediate inputs, $\Gamma_t(j)$, to produce $X_t(j)$ units of the intermediate good $j$. Its production function is given by:

$$X_t(j) = \max \left\{ A_t \Gamma_t(j)^\phi \left( \tilde{K}_t(j)^\alpha L_t(j)^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F, 0 \right\},$$  
where $\phi \in (0, 1)$ is the intermediate input share while $\alpha \in (0, 1)$ and $(1 - \alpha)$ are value-added share with respect to capital services and labor inputs, $F$ is a fixed cost, that is identical across firms. We choose $F$ to keep profits to be zero along a balanced growth path, given a growth factor $\Upsilon_t^3$, so that firm entries and exits are ruled out.

The neutral technology $A_t$ follows a process with both a trending and stationary component:

$$A_t = A_t^T \tilde{A}_t,$$  
where the deterministic trend component $A_t^T$ grows at the gross rate $g_A \geq 1$ in each period$^4$ such that:

$$A_t^T = g_A A_{t-1}^T.$$  

The stochastic process driving the detrended level of technology $\tilde{A}_t$ is given by

$$\tilde{A}_t = \left( \tilde{A}_{t-1} \right)^{\rho_A} \exp \left( s_A u_t^A \right),$$  
which, taking its natural logarithm, yields

$$\ln \tilde{A}_t = \rho_A \ln \tilde{A}_{t-1} + s_A u_t^A, \quad u_t^A \sim iid \,(0, 1).$$

$^2$It is the product of utilization and physical capital

$^3$For more details on growth factor, see Ascarì, Phaneuf, and Sims (2015) on page 11.

$^4$With the implicit normalization that it begins at 1 in period 0 i.e $A_0^T = 1$. 
The autoregressive parameter $\rho_A$ governs the persistence of the process and satisfies $0 \leq \rho_A < 1$. The shock is scaled by the known standard deviation equal to $s_A$ and $u_t^A$ is the innovation, drawn from a mean zero normal distribution.

**Cost Minimization**

The producer of differentiated goods $j$ is assumed to set its price, $P_t(j)$, according to Calvo pricing (Calvo, 1983) and decide in every period its quantities of intermediates, capital services, and labor input. The cost of intermediate is just the aggregate price level, $P_t$. The user cost of capital and labor are $R^k_t$ and $W_t$ (in nominal terms), respectively.

The cost-minimization problem of a typical firm choosing its inputs is given by:

$$\min \quad P_t \Gamma_t(j) + R^k_t \hat{K}_t + W_t L_t(j)$$

subject to

$$A_t \Gamma_t(j)^\phi \left( \hat{K}_t(j)^\alpha L_t(j)^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F \geq \left( \frac{P_t(j)}{P_t} \right)^{-\theta} X_t$$

The first order conditions yield the following marginal cost and conditional demand functions for the inputs used in the production of $X_t(j)$:

$$\Gamma_t(j) = \phi mc_t (X_t(j) + \Upsilon_t F), \quad (10)$$

$$\hat{K}_t(j) = \alpha (1-\phi) mc_t w_t \left( \frac{X_t(j)}{w_t} \right)$$

$$L_t(j) = (1-\alpha)(1-\phi) mc_t (X_t(j) + \Upsilon_t F). \quad (12)$$
Profit Maximization and Price Setting

Each intermediate producing firm\(^5\) chooses its price \(P_t(j)\) that maximizes the expected present discount value of its future profit. The firm problem is given by:

\[
\max_{P_t(j)} \mathbb{E}_t \sum_{h=0}^{\infty} (\xi_p)^h D_{t,t+h} (P_t(j)X_{t+h}(j) - V(X_{t+h}(j)))
\]

subject to

\[
X_{t+h}(j) = \left( \frac{P_t(j)}{P_{t+h}} \right)^{-\theta} X_{t+h}
\]

where \(D_{t,t+h}\) is the discount rate for future profits and \(V(X_t(j))\) is the total cost of producing good \(X_t(j)\). Note that \(D_{t,t+h} = \frac{\beta h \lambda_{t+h}}{\lambda_t}\). Written in real terms, it is \(\frac{P_{t+h} D_{t,t+h}}{P_t}\). Hence, the real discount factor is \(\frac{\beta h P_t \lambda_{t+h}}{P_t \lambda_t}\), which we can write as: \(\frac{\beta h \lambda'_{t+h}}{\lambda_t}\), where \(\lambda'_t = P_t \lambda_t\). The first-order condition for \(p^*_t(j)\) is:

\[
p^*_t(j) = \frac{\theta}{\theta - 1} \sum_{h=0}^{\infty} (\xi_p \beta)^h \lambda'_{t+h} \nu_{t+h}(j) \pi^{(t+1),t+h}_{t+h} X_{t+h}
\]

where \(p^*_t(j) = \frac{P_t(j)}{P_{t+h}}\) is the real optimal price and \(\nu_t\) the real marginal cost, which is equal to \(\nu'(X_{t+h}(j))\).

Since all updating firms will choose the same reset price, the optimal reset price relative to the aggregate price index becomes \(p^*_t \equiv \frac{P^*_t}{P_t}\). Then the optimal pricing condition (15) becomes:

\[
p^*_t = \frac{\theta}{\theta - 1} \frac{x_{1,t}}{x_{2,t}}
\]

\(^5\) A fraction \((1 - \xi_p)\) of these firms can optimally adjust its price (Calvo, 1983).
where $x_{1,t}$ and $x_{2,t}$ are auxiliary variables and can be written recursively as

$$x_{1,t} = \lambda_i^t \nu_t X_t + \beta \xi_p E_t(\pi_{t+1})^\theta x_{1,t+1},$$

and

$$x_{2,t} = \lambda_i^t X_t + \beta \xi_p E_t(\pi_{t+1})^{\theta-1} x_{1,t+1}.$$  

The term $\lambda_i^t$ in these equations is the marginal utility of an additional unit of real income received by household and $X_t$ is the aggregate gross output.

2.2 Households and wage setting

2.2.1 Labor aggregators

Households$^6$ supply $L_t(i)$ units of differentiated labor to labour aggregators. These firms assemble composite labor from differentiated, individual-specific labour according to the following aggregation function:

$$L_t = \left( \int_0^1 L_t(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

where $\sigma$ denotes the constant elasticity of substitution (CES) between labor types, with $\sigma > 1$. Labor aggregators are prisetakers in both their output and input markets. They sell composite labor to intermediate producers at the aggregate wage, $W_t$ while each unit of differentiated labor costs, $W_t(i)$. Thus, input demand for labor of type-$i$ is given by

$$L_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\sigma} L_t.$$

Inserting this demand function for input $i$ back into the CES aggregator yields the aggregate wage index, i.e

$$W_t^{1-\sigma} = \int_0^1 W_t(i)^{1-\sigma} di.$$  

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$^6$There is a continuum of households, indexed by $i \in (0, 1)$. 

8
2.2.2 Households

In this economy, households are monopoly suppliers of differentiated labor services. The representative household has the following expected lifetime utility\textsuperscript{7}:

\[
\max_{C_t, L_t(i), K_{t+1}, B_{t+1}, I_t, Z_t} E_0 \sum_{t=0}^{\infty} \beta^t \left( \ln \left( C_t - bC_{t-1} \right) - \eta \frac{L_t(i)1+\chi}{1+\chi} \right),
\]

subject to

\[
P_t \left( C_t + I_t + \frac{a(Z_t)K_t}{\varepsilon_{t}^{\eta}} \right) + \frac{B_{t+1}}{1+i_t} \leq W_t(i)L_t(i) + R_t^kZ_tK_t + \Pi_t + B_t + T_t,
\]

and

\[
K_{t+1} = \theta_t \varepsilon_t\left(1 - S \left( \frac{I_t}{I_{t-1}} \right) \right) I_t + (1 - \delta)K_t,
\]

with

\[
a(Z_t) = \gamma_1(Z_t - 1) + \frac{\gamma_2}{2}(Z_t - 1)^2,
\]

and

\[
S \left( \frac{I_t}{I_{t-1}} \right) = \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1}} - gI \right)^2.
\]

where \(0 < \beta < 1\) is a discount factor, \(0 < \delta < 1\) a depreciation rate, and \(0 \leq b < 1\) measures internal habit formation. \(\chi\) is the inverse of the Frisch-labor-supply elasticity. \(\gamma_1\) and \(\gamma_2\) are parameters to be calibrated. \(\kappa\) is an investment adjustment cost parameter that is strictly positive. \(B_{t+1}\) is a stock of nominal governmental bonds in \(t+1\). \(\Pi_t\) is distributed dividends from firms, and \(T_t\) is lump-sum transfer from the government net of taxes. \(Z_t\) is the level of capital utilization and \(a(Z_t)\) is the utilization adjustment cost function, with \(a(1) = 0, a'(1) = 0\), and \(a''(1) > 0\). \(S \left( \frac{I_t}{I_{t-1}} \right)\) is an investment adjustment cost, satisfying \(S(gI) = 0, S'(gI) = 0\), and \(S''(gI) > 0\), where \(gI \geq 1\) is the steady state (gross) growth rate of investment.

\textsuperscript{7}Utility is separable and we assume that households are identical with respect to non-labor choices; hence we drop the \(i\) subscripts. For details, see Erceg, Henderson and Levin (2000).
The investment-specific term follows the deterministic trend:

\[ \varepsilon_{t}^{I,\tau} = g_{t} \varepsilon_{t-1}^{I,\tau} \]  

(23)

where \( g_{t} \) is the gross growth rate and grows at the gross rate \( g_{t} \geq 1 \) in each period\(^8\).

The exogenous variable \( \vartheta_{t} \) captures the stochastic marginal efficiency of investment shock:

\[ \vartheta_{t} = (\vartheta_{t-1})^{\rho_{I}} \exp \left( s_{t} u_{t}^{r} \right) \text{ with } u_{t}^{r} \sim iid \left(0, 1 \right). \]  

(24)

The autoregressive parameter \( \rho_{I} \) governs the persistence of the process and satisfies \( 0 \leq \rho_{I} < 1 \). The shock is scaled by the known standard deviation equal to \( s_{t} \) and \( u_{t}^{r} \) is the innovation drawn from a mean zero normal distribution.

The first-order conditions for consumption, capital utilization, investment, capital and bonds are respectively:

\[ \lambda_{t}^{r} = \frac{1}{C_{t} - bC_{t-1}} - E_{t} \frac{\beta b}{C_{t+1} - bC_{t}}, \]  

(25)

where \( \lambda_{t}^{r} = P_{t} \lambda_{t} \), which is the marginal utility of an extra good;

\[ r_{t}^{k} = \frac{a'(Z_{t})}{\varepsilon_{t}^{I,\tau}}; \]  

(26)

\[ \lambda_{t}^{r} = \mu_{t} \varepsilon_{t}^{I,\tau} \vartheta_{t} \left[ 1 - S \left( \frac{I_{t}}{I_{t-1}} \right) - S' \left( \frac{I_{t}}{I_{t-1}} \right) \right] + \beta E_{t} \mu_{t+1} \varepsilon_{t+1}^{I,\tau} \vartheta_{t+1} S' \left( \frac{I_{t+1}}{I_{t}} \right) \left[ \frac{I_{t+1}}{I_{t}} \right]^{2}; \]  

(27)

\[ \mu_{t} = \beta E_{t} \lambda_{t+1}^{r} \left[ r_{t+1}^{k} Z_{t+1} - a(Z_{t+1}) \right] + \beta (1 - \delta) E_{t} \mu_{t+1}; \]  

(28)

\[ \lambda_{t}^{r} = \beta E_{t} \lambda_{t+1}^{r} (1 + i_{t}) \pi_{t+1}^{-1}. \]  

(29)

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\( ^{8} \)With the implicit normalization that it begins at 1 in period \( 0 \) i.e \( \varepsilon_{t}^{I,\tau} = 1 \)
2.2.3 Wage setting

Households get to update their wages each period with the probability \((1 - \xi_w)\). The optimal wage \(W_t(i)\) is obtained by maximizing:

\[
E_t \sum_{h=0}^{\infty} (\beta \xi_w)^h \left(-\frac{\eta}{1 + \chi} (L_{t+h}(i))^{-\sigma(1+\chi)} + \lambda_{t+h} W_t(i) L_{t+h}(i) \right).
\]

(30)

subject to

\[L_{t+h}(i) = \left(\frac{W_t(i)}{W_{t+h}}\right)^{-\sigma} L_{t+h}\]

The first order condition implies that

\[\hat{w}_t^* = \frac{\sigma}{\sigma - 1} \frac{f_{1,t}}{f_{2,t}}. \quad (31)\]

Recursively the terms \(f_{1,t}\) and \(f_{2,t}\) evolve as follows

\[f_{1,t} = \eta \left(\frac{w_t}{w_t^*}\right)^{\sigma(1+\chi)} L_t^{1+\chi} + \beta \xi_w E_t(\pi_{t+1})^{\sigma(1+\chi)} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\sigma(1+\chi)} f_{1,t+1}, \quad (32)\]

and

\[f_{2,t} = \lambda_t^f \left(\frac{w_t}{w_t^*}\right)^{\sigma} L_t + \beta \xi_w E_t(\pi_{t+1})^{\sigma-1} \left(\frac{w_{t+1}^*}{w_t^*}\right)^{\sigma} f_{2,t+1}. \quad (33)\]
2.3 Monetary Policy

Monetary policy consists of a Taylor-type rule. It responds to deviations of inflation from an exogenous steady state target, $\pi$, and to deviations of output growth from its trend level, $g_Y$, up to a stochastic shock is of the form:

\[
1 + i_t = \left(1 + i_{t-1}\right)^\rho_i \left[\left(\frac{\pi_t}{\pi}\right)^{\alpha_\pi} \left(\frac{Y_t}{Y_{t-1}}\right)^{\alpha_y}\right]^{1-\rho_i} \varepsilon_t^i. \tag{34}
\]

with $i_t$ and $i$ being respectively the nominal and steady-state value of the nominal interest rate, $\frac{\pi_t}{\pi}$ is the inflation gap and $\frac{Y_t}{Y_t}$ the output gap. The interest rate smoothing parameter is given by $\rho_i$, $\alpha_\pi$ and $\alpha_y$ are the control parameters, and $\varepsilon_t^i$ is an exogenous shock to the policy rule, where $\varepsilon_t^i \sim iid \left(0, \sigma_{\varepsilon_t}^2\right)$. To ensure determinacy, we assume that $0 \leq \rho_i < 1$, $\alpha_\pi > 1$ and $\alpha_y \geq 0$.

2.4 Aggregation

The aggregate price level and wage evolve according to:

\[
1 = \xi_p (\pi_t)^{\theta-1} + (1 - \xi_p) (p_t^*)^{1-\theta}, \tag{35}
\]

\[
w_t^{1-\sigma} = \xi_w \left(\frac{w_t-1}{\pi_t}\right)^{1-\sigma} + (1 - \xi_w) (w_t^*)^{1-\sigma}. \tag{36}
\]

With real GDP being the aggregate production of the goods, $X_t$, minus the aggregate production of intermediate inputs, $\Gamma_t$, where $\Gamma_t = \int_0^1 \Gamma_t(j) dj = \frac{\bar{V}_t}{P_t} \left(s_t \frac{X_t}{A_t} + \frac{Y_t}{A_t} F\right)$, where $s_t = \int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\theta} dj$, is a measure of price dispersion. Hence, the real GDP or aggregate net output, $Y_t$ is given by:

\[
Y_t = X_t - \Gamma_t \tag{37}
\]
Market-clearing requires that \( \int_0^1 \tilde{K}_t(j) dj = \tilde{K}_t \) and \( \int_0^1 L_t(j) dj = L_t \) respectively for capital services and labor inputs. Hence, aggregate gross output can be written as

\[
s_tX_t = A_t\Gamma_t^\phi \left( \tilde{K}_t^\alpha L_t^{1-\alpha} \right)^{1-\phi} - \Upsilon_t F, \tag{38}\]

We know that \( \int_0^1 X_t(j) dj = s_tX_t \), hence the aggregate input demands can be written as

\[
\Gamma_t(j) = \phi mc_t \left( s_tX_t(j) + \Upsilon_tF \right), \tag{39}\]

\[
\tilde{K}_t(j) = \alpha (1-\phi) \frac{mc_t}{r_t} \left( s_tX_t(j) + \Upsilon_tF \right), \tag{40}\]

\[
L_t(j) = (1-\alpha)(1-\phi) \frac{mc_t}{w_t} \left( s_tX_t(j) + \Upsilon_tF \right). \tag{41}\]

The aggregate resource constraint is therefore given by :

\[
Y_t = C_t + I_t + \frac{a(Z_t)}{\varepsilon_t^{\alpha}} K_t \tag{42}\]

### 2.5 Balanced Growth

Trend growth from the deterministic trends in neutral and investment-specific productivity, implies that a balanced-growth path exists where output, consumption, investment, intermediate inputs, and the real wage will all grow at the same rate : \( g_Y = g_I = g_r = g_w = g_\Upsilon = \frac{\Upsilon_t}{\Upsilon_{t-1}} \). In order to induce stationarity in these variables, they are scaled by the deterministic growth rate \( \Upsilon_t \). The capital stock will grow faster due to growth in investment-specific productivity, with \( \tilde{K}_t = \frac{K_t}{\Upsilon_t^{\alpha+1}} \) being stationary.

Labor hours, capital utilization and real marginal cost will be stationary, as will inflation rate and the relative reset price.
2.6 Measuring Welfare

The value function of the $i^{th}$ household is:

$$V_t(i) = \ln(C_t - bC_{t-1}) - \eta \frac{L_t(i)^{1+\chi}}{1 + \chi} + \beta E_t V_{t+1}(i)$$  \hspace{1cm} (43)

Measuring aggregate welfare is not obvious because of the heterogeneity in employment outcomes. Defining aggregate welfare as the un-weighted sum of individual welfare, then

$$V_t = \int_0^1 \left( \ln(C_t - bC_{t-1}) - \eta \frac{L_t(i)^{1+\chi}}{1 + \chi} + \beta E_t V_{t+1}(i) \right) di,$$  \hspace{1cm} (44)

since households only differ though their labor supply, this can be written as

$$V_t = \ln(C_t - bC_{t-1}) - \eta \int_0^1 N_t(i)^{1+\chi} \frac{W_t(i)}{W_t} \sigma^{(1+\chi)} di + \beta E_t V_{t+1}.$$  \hspace{1cm} (45)

Now, using the demand curve for each variety of labor, we can write this as:

$$V_t = \ln(C_t - bC_{t-1}) - \eta \int_0^1 N_t(i)^{1+\chi} \frac{W_t(i)}{W_t} \sigma^{(1+\chi)} di + \beta E_t V_{t+1}.$$  \hspace{1cm} (46)

The value function is therefore

$$V_t = \ln(C_t - bC_{t-1}) - \eta \frac{L_t^{1+\chi}}{1 + \chi} + \beta E_t v_{t+1}.$$  \hspace{1cm} (47)

The aggregate welfare in terms of aggregate variables is given by

$$W_t = \ln(C_t - bC_{t-1}) - \eta v_{t}^{w} \frac{L_t^{1+\chi}}{1 + \chi} + \beta E_t W_{t+1}.$$  \hspace{1cm} (48)
where \( v_t^w \) is a wage dispersion variable that can be written recursively as

\[
v_t^w = (1 - \xi_w) \left( \frac{w_t^*}{w_t} \right)^{\sigma(1+\chi)} + \xi_w \left( \frac{w_t\pi_t}{w_{t-1}} \right)^{\sigma(1+\chi)} v_{t-1}^w.
\] (49)

The right side term summarizes the different factors that may affect wage dispersion, and therefore the welfare costs of long-term inflation.

As in Ascari, Phaneuf, and Sims (2015), we define the consumption equivalent measure, \( \lambda \), as the constant fraction of consumption that households have to give up (or have to be given) each period in the case of zero percent inflation rate to have the same value for \( V_t \) that would be obtained alternatively in the case of 4.75 percent inflation rate in the pre-1984 period or in that of 2.29 percent inflation rate in the post-1984 period.

We consider two different consumption equivalents, one based on steady states \( \lambda_{ss} \) and the other on stochastic means \( \lambda_m \):

\[
\lambda_{ss} = 1 - \exp \left[ (1 - \beta)(V_{ss} - V_{ssB}) \right]
\] (50)

\[
\lambda_m = 1 - \exp \left[ (1 - \beta)(E(V) - E(V_B)) \right]
\] (51)

where \( B \) stands for benchmark case and the absence of a subscript denotes the alternative case. A SS subscript stands for the non-stochastic steady state, and \( E(.) \) the unconditionnal expectations operator.

### 3 Calibration

In order to generate quantitative results, the calibration of the model parameters need to be set. Tables 1 to 4 summarize our baseline model parameter values into non-shock and shock parameters over both subsamples under consideration.
3.1 Non-shock Parameters

We set our non-shock parameters (tables 1 and 2) to standard values found in the business cycle literature: The households discount factor is set to \( \beta = 0.99 \). The capital depreciation rate \( \delta = 0.025 \), that corresponds to an annual capital depreciation of 10 percent. The capital services share is set to \( 1/3 \). \( \eta = 6 \) is a scaling parameter on disutility from labor and the inverse Frisch elasticity of labor supply to \( \chi = 1 \). We set the consumption habit formation parameter \( b \) to 0.7 following Fuhrer (2000). The investment adjustment cost is set to \( \kappa = 3 \) in line with the value used in Christiano, Eichenbaum and Evans (2005). We choose the utilization cost \( \gamma_2 \) equals to 0.05 to match a capital utilization elasticity equal to 1.5 (Basu and Kimball, 1997; Dotsey and King, 2006).

Table 1: Non-Shock Parameters, Pre-1984

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \delta )</th>
<th>( \alpha )</th>
<th>( \eta )</th>
<th>( \chi )</th>
<th>( b )</th>
<th>( \kappa )</th>
<th>( \gamma_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.025</td>
<td>1/3</td>
<td>6</td>
<td>1</td>
<td>0.7</td>
<td>3</td>
<td>0.05</td>
</tr>
<tr>
<td>( \theta )</td>
<td>( \sigma )</td>
<td>( \xi_p )</td>
<td>( \xi_w )</td>
<td>( \phi )</td>
<td>( \rho_i )</td>
<td>( \alpha_\pi )</td>
<td>( \alpha_y )</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>0.66</td>
<td>0.66</td>
<td>0.61</td>
<td>0.81</td>
<td>1.65</td>
<td>0.2</td>
</tr>
</tbody>
</table>

The elasticity parameters for goods and labor are set to a uniform value \( \sigma = \theta = 6 \), implying a steady-state price and wage markups of 20 percent (Liu and Phaneuf, 2007). With \( \theta = 6 \), this implies an intermediate inputs share \( \phi \) of 0.61.

The Calvo price and wage parameters are set to \( \xi_p = 0.66 \) and \( \xi_w = 0.66 \) respectively. The Calvo price is consistent with the evidence reported in Bils and Klenow (2004) and the value assigned to the Calvo probability of wage with the evidence reported in Christiano, Eichenbaum and Evans (2005).

Based on the estimates produced by Smets and Wouters (2007), we assign the following values to parameters describing the monetary policy rule: \( \rho_i = 0.81 \), \( \alpha_\pi = 1.65 \), and \( \alpha_y = 0.2 \) (pre-1984) and \( \rho_i = 0.84 \), \( \alpha_\pi = 1.77 \), and \( \alpha_y = 0.16 \) (post-1984).
Table 2: Non-Shock Parameters, Post-1984

<table>
<thead>
<tr>
<th>β</th>
<th>δ</th>
<th>α</th>
<th>η</th>
<th>χ</th>
<th>b</th>
<th>κ</th>
<th>γ₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.99</td>
<td>0.025</td>
<td>1/3</td>
<td>6</td>
<td>1</td>
<td>0.7</td>
<td>3</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>θ</th>
<th>σ</th>
<th>ξ_p</th>
<th>ξ_w</th>
<th>φ</th>
<th>ρ_i</th>
<th>α_π</th>
<th>α_y</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>6</td>
<td>0.66</td>
<td>0.66</td>
<td>0.61</td>
<td>0.84</td>
<td>1.77</td>
<td>0.16</td>
</tr>
</tbody>
</table>

3.2 Trend inflation, Trend Growth and Shock Parameters

As in Ascari, Phaneuf and Sims (2015), we use series from the Bureau of Economic Analysis (BEA) to compute trend inflation and trend growth observed in data. However, our approach is based on a split sample. We use quarterly data covering the subsamples periods: 1960:I - 1983:IV and 1984:I - 2007:III. Following Gali and Gambetti (2009), the split before and after 1984 is a date generally viewed as the starting point of the period of enhanced stability in the US economy.

We define:

1. Investment as expenditures on new durables plus private fixed investment,

2. Consumption as consumption of non-durables and services.

We generate real series to get the corresponding deflators. We first, compute real series of the individual components by dividing by their own deflators from the National Income and Product Account (NIPA) tables. We then, compute the growth rates of the real series by using one period lagged nominal share weights.

To compute the real growth rate of non-durable and services consumption we first take the share-weighted growth rates of the real component series. We then compute price indices for consumption and investment as the ratios of the nominal to the real series. The relative price of investment is the ratio of the implied price index for investment goods to the price index for consumption goods.

The average growth rate of the relative price from each time period is -0.0029 and -0.0065. This implies a calibration of $g_I = 1.0029$ and $g_I = 1.0065$ for pre- and post-1984 respectively.
To obtain the price deflator we take the ratio between the nominal and real series. The average growth rate of the price index over the periods 1960:1-1983:IV is 0.011679 and 0.005671 for 1984:I-2007:III. This implies $\pi^* = 1.011679$ (or 1.0475 at an annual frequency) and $\pi^* = 1.005671$ (or 1.0229 at an annual frequency) for each time period respectively.

To compute real per capita GDP, we subtract the log civilian non-institutionalized population from the real GDP log-level. The average growth rates of this series over the subsample periods are 0.00534 and 0.006088 respectively. The standard deviation of output growth over each time period is 0.00932377 pre-1984 and 0.00562996 post-1984.

From the calculations above we get $g_Y = 1.00534$ and $g_Y = 1.006088$ and $g_I = 1.0029$ and $g_I = 1.0065$. To generate the corresponding output volatility, we then take $g_A^{1-\phi}$.

To calculate the shocks, we proceed as follows. Given these growth rates and $\pi^*$, we refer to the baseline model and take $s_I$, $s_A$, and $s_r$ to match output growth volatility over each subsamples. This requires taking a stand on the percentage contribution of each type of shocks to output growth volatility.

Based on the evidence produced by Justiniano and Primiceri (2008, 2010), an investment-specific shock contributes to 50 percent of this output volatility. This compares with 35 percent for the neutral technology, and for 15 percent for the monetary policy shock. If we set the AR(1) coefficients $\rho_I$ to 0.81 and $\rho_A$ to 0.95, then the resulting variances for the shocks are (see table 3): $s_I = 0.0164$, $s_A = 0.0033$ and $s_r = 0.002$ (pre-1984) and $s_I = 0.0089$, $s_A = 0.0018$ and $s_r = 0.001$ (post-1984).

<table>
<thead>
<tr>
<th>Table 3: Shock Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
</tr>
<tr>
<td>$g_A$ $g_I$ $\rho_r$ $s_r$ $\rho_I$ $s_I$ $\rho_A$ $s_A$</td>
</tr>
<tr>
<td>1.00258$^{1-\phi}$ 1.0029 0 0.002 0.81 0.0164 0.95 0.0033</td>
</tr>
<tr>
<td>Post-1984</td>
</tr>
<tr>
<td>$g_A$ $g_I$ $\rho_r$ $s_r$ $\rho_I$ $s_I$ $\rho_A$ $s_A$</td>
</tr>
<tr>
<td>1.0019$^{1-\phi}$ 1.0065 0 0.001 0.81 0.0089 0.95 0.0018</td>
</tr>
</tbody>
</table>
For the subsample post-1984 period, we have $g_A = 1.00188^{(1-\phi)}$ and $g_I = 1.00652$ with output quarterly growth rate of 1.00608. The contribution of technology progress is 1.00283 and that of marginal efficiency of investment progress is 1.00325.

4 Results

4.1 The Welfare Costs of inflation

In this section we focus on the implications of the welfare costs of inflation due to monetary policy changes and shifting trend inflation across the two subperiods. To measure these welfare costs, we consider two different sets of consumption equivalent metrics: one for means and one for steady states. For each set we compute welfare costs for going from $\pi^* = 1$ to $\pi^* = 1.0475$ (all annualized) and from $\pi^* = 1$ to $\pi^* = 1.0229$ (all annualized) respectively for pre- and post-1984 subsample periods. We produce two different sets of welfare results: one with trend growth (baseline case) and one without trend growth (alternative case).

Table 4 reports the mean consumption equivalent in the baseline case. It shows that going from 0 to 4.75 percent rate of trend inflation causes mean welfare losses increases to around 8.38 percent of consumption in the pre-1984 period. However, a decline in trend inflation at 2.29 percent annual inflation in the post-1984, causes mean welfare losses decreases to about 2.34 percent of consumption.

Table 4: Consumption Equivalents, Mean

<table>
<thead>
<tr>
<th>$\pi^*$</th>
<th>1.00 $\rightarrow$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>1.0475</td>
<td>0.0838</td>
</tr>
<tr>
<td>Post-1984</td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>1.0229</td>
<td>0.0234</td>
</tr>
</tbody>
</table>

*For more details on the two metrics see Ascari, Phaneuf and Sims (2015).*
Table 5 describes the welfare cost in terms of steady states in the baseline case. The cost of going from 0 to 4.75 percent in trend inflation is about 7.56 percent of consumption in the pre-1984 period; it’s around 2.26 percent of consumption as trend inflation rises from 0 to 2.29 percent in the post-1984 subperiod.

Table 5: Consumption Equivalents, Steady State

<table>
<thead>
<tr>
<th>$\pi^*$</th>
<th>1.00→</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>1.0475</td>
<td>0.0756</td>
</tr>
<tr>
<td>Post-1984</td>
<td></td>
</tr>
<tr>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>1.0229</td>
<td>0.0226</td>
</tr>
</tbody>
</table>

From tables 4 and 5, the welfare costs respond symmetrically to a rise and a decline in trend inflation, trend growth and the level of volatility across both subperiods. It is worth noting that the size of the magnitude is not proportional and that the welfare costs are larger in the pre-1984 period than in the post-1984 period. These results are in line with the evidence produced by Christiano (2015) in so that the high trend inflation of the 1970s, in the Great Inflation period, was more costly.

We also notice that the welfare means change by more than the steady states and exceed the steady state losses across both subsample periods. This can be illustrated by the consumption equivalent welfare changes, where the consumption equivalents based on means are about 8.38 percent at 4.75 percent in trend inflation and about 2.34 percent at 2.29 percent in trend inflation larger than on steady states.

To isolate the implications of trend inflation on welfare losses, we run our model without trend GDP growth i.e we set trend growth to 1 in the calibration. The results are reported in tables 6 and 7. Table 6 indicates that based on means the welfare cost of going from 0 to 4.75 percent inflation is about 4 percent of consumption and it is around 1.37 percent of consumption at 2.29 percent of trend inflation.
Table 6: Consumption Equivalents, Mean

<table>
<thead>
<tr>
<th>π*</th>
<th>1.00</th>
<th>→</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.0475</td>
<td>0.040</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.0229</td>
<td>0.00137</td>
</tr>
</tbody>
</table>

In terms of steady states, table 7 reveals that going from 0 to 4.75 percent of trend inflation in the pre-1984 period, causes the welfare losses to amount to around 3.57 percent of consumption and it’s about 0.12 percent of consumption at 2.29 percent of trend inflation in the post-1984 subsample period.

Table 7: Consumption Equivalents, Steady State

<table>
<thead>
<tr>
<th>π*</th>
<th>1.00</th>
<th>→</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.0475</td>
<td>0.0357</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>1.0229</td>
<td>0.00123</td>
</tr>
</tbody>
</table>

The welfare costs of inflation are larger in the baseline case (tables 4 and 5) than in the case without trend growth (tables 6 and 7). It is worth noting that the effects of trend growth on welfare come in through wage dispersion (see notes next to equations 48 and 49 for more details). The interactions between trend growth and wage dispersion in the baseline case make inflation so much more costly in the pre-1984 subperiod.

In the case without trend growth, the first order wage dispersion effects disappear and inflation costs values are quite small i.e mean and steady state welfare are much lower. However even in this
case, the welfare costs in the pre-1984 are larger than in the post-1984 subperiod, with the mean consumption equivalent welfare losses higher.

Using the alternative calibration (see table 12 in the appendix), we find the following results. First, an increase in the variance of shocks to the trend inflation process increases mean welfare costs by increasing the volatilities of output and inflation in the pre-1984 subsample period and their subsequent decline in the post-1984 period as the variance of shocks relative to trend inflation process goes down (see tables 13 and 14 in the appendix). The steady state welfare level remains stable relative to the baseline calibration as the variance of shocks increases. Second, from tables 15 and 16, welfare costs decrease not so much as in the baseline case. The basic insights obtained from this high- and low-volatility periods is that the consumption equivalent welfare losses are higher the more volatility there is.

4.2 Macroeconomic dynamics

4.2.1 Unconditional volatilities

Tables 8 reports the magnitude of volatility changes in the pre-1984 and post-1984 sample periods by showing the standard deviation for output, output growth, and inflation. As indicated in the table for the three variables aforecited, the standard deviation for the post-1984 is less than half that in the pre-1984 period.

<table>
<thead>
<tr>
<th></th>
<th>(\pi^*)</th>
<th>(\sigma(Y))</th>
<th>(\sigma(\Delta Y))</th>
<th>(\sigma(\pi))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>1.0475</td>
<td>0.1084</td>
<td>0.0120</td>
<td>0.0032</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0229</td>
<td>0.0449</td>
<td>0.0064</td>
<td>0.0015</td>
</tr>
</tbody>
</table>

The last row in table 8 gives the variations in percentage between the two sub-periods. We can see that all the aforecited variables have experienced a large decline in their volatility in the post-1984 sample period. However, the magnitude of that reduction is not proportional: -0.59 for output, -0.47 for output growth and -0.54 for inflation. Thus, the decline in the volatility of output growth is not as large as those experienced by inflation and output. In line with the above, we note that
trend inflation tends to increase output volatility in the pre-1984 period and to reduce it in the post-1984 by more than inflation and output growth.

The results so far do not allow to determine the sources of those changes in volatilities. To address this issue, we turn to the analysis of the conditional moments.

### 4.2.2 Conditional volatilities

In what follows, we analyse the sources of the changes in the standard deviations of output, output growth and inflation conditional on neutral productivity, MEI and monetary shocks.

**Table 9: Output volatility**

<table>
<thead>
<tr>
<th></th>
<th>$\pi^*$</th>
<th>$s_{\text{uinvs}}$</th>
<th>$s_{\text{ua}}$</th>
<th>$s_{ur}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>1.0475</td>
<td>0.0023</td>
<td>0.0040</td>
<td>0.0006</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0229</td>
<td>0.0017</td>
<td>0.0017</td>
<td>0.0006</td>
</tr>
<tr>
<td>Variation</td>
<td>-</td>
<td>-0.2781</td>
<td>-0.5625</td>
<td>-0.04</td>
</tr>
</tbody>
</table>

Conditional on shocks, we see that neutral productivity shocks is the main driving force behind the decline in output (see tables 9). On the other hand, we see that the volatility of the level of output is increasing in trend inflation in the pre-1984 period and decreasing in the post-1984 conditional on neutral productivity shock by far more than conditional on MEI or monetary shocks. Monetary shock has a relatively small and stable contribution to the volatility of output, output growth and inflation across the sample periods (see tables 9, 10 and 11). However, in their baseline specification, Gali and Gambetti (2009) find that nontechnology shocks appear to be the main source of the decline in the volatility of output. The main explanation can be found in the nature of their empirical approach\textsuperscript{10}, the uncertainty associated with their estimated coefficients is large.

**Table 10: Output growth volatility**

<table>
<thead>
<tr>
<th></th>
<th>$\pi^*$</th>
<th>$s_{\text{uinvs}}$</th>
<th>$s_{\text{ua}}$</th>
<th>$s_{ur}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>1.0475</td>
<td>0.0019</td>
<td>0.0017</td>
<td>0.0004</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0229</td>
<td>0.0011</td>
<td>0.0008</td>
<td>0.0003</td>
</tr>
<tr>
<td>Variation</td>
<td>-</td>
<td>-0.4232</td>
<td>-0.5084</td>
<td>-0.1235</td>
</tr>
</tbody>
</table>

\textsuperscript{10}They use a time-varying VAR approach whereas we use estimates from Smets and Wouters (2007) based on Bayesian approach to calibrate the Fed’s reaction function.
Table 10 reports the output growth volatility conditional on the three types of shocks. It shows that both neutral productivity and MEI shocks play an important role with the former as the main explanation for the decline in output growth volatility. This result concur with the findings in Justiniano and Primiceri (2006) who find that it’s the investment-specific technology shocks and in Gali and Gambetti (2009) who point to an important contribution of nontechnology shocks. However, in their alternative specification, this role of nontechnology shocks in the volatility decline is shared with technology shocks (Gali and Gambetti, 2009).

Table 11: Inflation volatility

<table>
<thead>
<tr>
<th></th>
<th>$\pi^*$</th>
<th>$s_{u\ln vs}$</th>
<th>$s_{ua}$</th>
<th>$s_{ur}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-1984</td>
<td>1.0475</td>
<td>0.0002</td>
<td>0.0005</td>
<td>0.0001</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0229</td>
<td>0.0001</td>
<td>0.0002</td>
<td>0.0001</td>
</tr>
<tr>
<td>Variation</td>
<td>-</td>
<td>-0.5797</td>
<td>-0.5411</td>
<td>-0.1363</td>
</tr>
</tbody>
</table>

Table 11 describes the inflation volatility relative to different types of shocks. Its shows that fluctuations in the inflation volatility across the two subsamples are largely accounted for by both neutral productivity and MEI shocks with monetary shocks playing a smaller role. The interaction between trend inflation and MEI shocks accounts for the larger contribution of the latter relative to that of the neutral productivity shocks.

The analysis of tables 8 through 11 points to the following findings. First, we note that neutral productivity shock play an important role as a source of the increase in output and output growth volatility in the pre-1984 and the subsequent decline in the post-1984. Second, the MEI shocks is responsible for inflation volatility by more than the neutral productivity shocks. Third, the monetary shocks has a relatively smaller contribution to the volatility of output, output growth and inflation in each sample period. Finally, the interactions between trend inflation and different types of shocks, as trend inflation goes up and down across the subsamples, account for much of the results.

In the results so far, we have assumed that investment persistence parameter $\rho_I$ is set to 0.81 and the uncertainty associated with our monetary policy estimated coefficients is small in line with Smets and Wouters (2007). In what follows, we assume that $\rho_I$ is set to 0.9 and the variation associated with our monetary policy estimated coefficients between the two subperiods is large in the spirit of Clarida, Gali, and Gertler (2000) and Gali and Gambetti (2009).
Thus, we conduct a theoretical exercise and assign the following values to parameters describing the monetary policy rule (see table 12): $\rho_i = 0.75$, $\alpha_\pi = 1.15$, and $\alpha_y = 0.125$ (in the pre-1984 period) and $\rho_i = 0.75$, $\alpha_\pi = 2.3$, and $\alpha_y = 0.125$ (in the post-1984 period).

We find the following results (see tables 17 though 20 in the appendix). First, we see from table 17 that the decline in the volatility of inflation is larger than those experienced by output and output growth. Also we note that as trend inflation goes up and down, it tends to increase inflation in the pre-1984 period and to reduce it in the post-1984 by more than output and output growth. Second, from table 18 fluctuations in output volatility is accounted for by the neutral productivity shocks and appears to display a larger downward trend than MEI and monetary shocks. The contribution of the latter is much smaller. Third, from table 20 the interaction between trend inflation and MEI shocks is responsible for inflation volatility by more than the monetary and neutral productivity shocks (Ascari, Phaneuf and Sims, 2015). Finally, from table 19 we note that the monetary shocks has a relatively larger contribution to the volatility of output growth and its subsequent decline. This resultat is consistent with Gali and Gambetti (2009).

The basic insight obtained from these two set of results is that the findings based on theoritical exercise (i.e alternative specification) are more plausible and consistent with the results in the literature than those on the baseline specification.

### 4.2.3 Impulse responses analysis

In this section, we analyse impulse responses functions relative to neutral productivity, MEI and monetary shocks. Figures display side by side the impulse responses over the pre-1984 (solid lines) and post-1984 (dashed lines) periods.

We restrict our analysis to the aforementioned variables and produce three sets of figures. In the first set, IRFs are based on the baseline calibration (see figures 1 though 3). Next, we use the alternative calibration from table 12 to generate IRFs in figures 4 though 6. Finally, to further single out the contribution of monetary policy and that of trend inflation, we conduct an experiment that produce side by side IRFs with three curves (see figures 7 though 9): solid lines are for the pre-1984 period, dashed lines for the post-1984 period and dashed-red lines relative to our experiment.
Figures 1 and 4 display the dynamic responses of variables to a neutral productivity shock. In both cases, it is noticeable that the impulse responses of output over the pre-1984 and post-1984 periods differ substantially, with the decline in amplitude in the post-1984 period being significantly large by more than in the case of MEI and monetary shocks (see also tables 9 and 18). In this perspective, neutral productivity shocks appear to be the main source of contribution to the volatility of output.

Over the sample periods the response of output to a monetary shock shows a characteristic hump shape and displays substantial persistence (see figure 3). Instead, the magnitude of the change between the pre- and post-1984 periods is smaller as reported in table 9 relative to output volatility conditional on a monetary shock. This is due to our Taylor rule estimates in the baseline calibration which are consistent with Smets and Wouters (2007). However, when taking into account the alternative calibration, the magnitude of the change varies by more than in the baseline case (see figure 6 and table 18). Thus, in both cases we see that monetary shocks have a relatively small and stable contribution to the volatility of output.

From tables 11 and 20 we have seen that fluctuations in the inflation volatility are largely accounted for by MEI shocks by more than the neutral productivity shocks. This feature of the conditional second moments is reflected by parallel changes in the inflation impulse responses in figures 2 and 5 as trend inflation goes up and down over the sample periods.

From table 19 we have noted that the monetary shocks play an important role as the main explanation for the volatility of output growth and its subsequent decline over the pre-1984 and post-1984 periods. Accordingly, the changes experienced over both periods in the output growth conditional second moments must be reflecting parallel changes in its impulse responses. Thus, the larger share of the contribution to the output growth volatility is associated with monetary shocks.

Figures 7 though 9 display IRFs from our experiment. We produce side by side IRFs with three curves. The third curve is a modification of the pre-1984 period wherein we replace the corresponding trend inflation and taylor rule estimates by those of the post-1984 period. We restrict our analysis to figure 9 which has the same features as in figure 6. The aim of our theoretical exercise is to figure out the contribution of monetary policy over both periods and the implications of trend inflation. We find that changes in the Fed’s response to macroeconomic variables (or policy response to the shocks) along with the decline in trend inflation are the main sources of the shift in macroeconomic variables volatilities (see the dashed-red lines in figure 9). We concur with the original conclusion of Clarida, Gali, and Gertler (2000), Lubik and Schorfheide (2004) and
Zandweghe, Hirose and Kurozumi (2015). However, our results are in line with those of Coibion and Gorodnichenko (2011) and Arias, Ascari, Branzoli and Castelnovo (2015).

5 Conclusion

In this paper we have provided the implications of monetary policy changes on the welfare in the U.S economy over the sample periods. We use a New-Keynesian model inspired by Ascari, Phaneuf and Sims (2015). However, our approach is quite different. We first calibrate the Fed’s reaction function in both periods based on estimates in the literature. We then combine it with our calibrated medium-scale New Keynesian model.

Our findings are consistent with the results in the literature relative to the welfare costs of inflation over both sample periods. The results show that welfare losses are larger in the pre-1984 period than in the post-1984 (Christiano, 2015). We also notice that the welfare mean change by more than the steady state and exceed the latter across both subsample periods. The basic insight obtained from this high- and low-volatility environments is that welfare losses are higher the more volatility there is (Lester, Pries and Sims, 2014).

On the other hand, we find the following results relative to macroeconomic variables volatilities. First, we see that the decline in the volatility of inflation is larger than those experienced by output and output growth. As trend inflation goes up and down, it tends to increase inflation in the pre-1984 period and to reduce it in the post-1984 by more than output and output growth. Second, fluctuations in output volatility is accounted for by the neutral productivity shocks and appears to display a larger downward trend than MEI and monetary shocks. Third, the interaction between trend inflation and MEI shocks is responsible for inflation volatility by more than the monetary and neutral productivity shocks (Ascari, Phaneuf and Sims, 2015). Finally, we note that the monetary shocks has a relatively larger contribution to the volatility of output growth and its subsequent decline (Gali and Gambetti, 2009).
References


Pancrazi, R. (2014), ‘How beneficial was the great moderation after all?’, *Journal of Economic Dynamics and Control* 46, 73–90.
A  Output and trend growth rates

Taking $g_A$ and $g_I$ to denote the deterministic trend growth rates of TFP and MEI, the growth factor $\Upsilon_t$ is then given by:

$$g_\Upsilon = g_A^{rac{1}{1-\phi(1-\alpha)}} g_I^\alpha$$

The benchmark deterministic growth over pre-1984 subsample period is given by $g_A = 1.0026^{(1-\phi)}$ and $g_I = 1.0029$ with output grows at quarterly rate: 1.00533. The contribution of technology progress is 1.003869 and that of MEI progress is 1.001461.

B  Alternative calibration

Table 12: Alternative calibration

<table>
<thead>
<tr>
<th></th>
<th>$\rho_i$</th>
<th>$\alpha_\pi$</th>
<th>$\alpha_y$</th>
<th>$s_{ua}$</th>
<th>$s_{uines}$</th>
<th>$s_{ur}$</th>
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<tbody>
<tr>
<td>Before 1984</td>
<td>0.75</td>
<td>1.15</td>
<td>0.125</td>
<td>0.0028</td>
<td>0.0117</td>
<td>0.002</td>
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<td>After 1984</td>
<td>0.75</td>
<td>2.3</td>
<td>0.125</td>
<td>0.0016</td>
<td>0.0106</td>
<td>0.0019</td>
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C The welfare costs with alternative calibration

Table 13: Consumption Equivalents, Mean

<table>
<thead>
<tr>
<th>$\pi^*$</th>
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<tbody>
<tr>
<td>Pre-1984</td>
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<tr>
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<tr>
<td>1.0475</td>
<td>0.1356</td>
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<tr>
<td>Post-1984</td>
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<td>1.0000</td>
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<tr>
<td>1.0229</td>
<td>0.0237</td>
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Table 14: Consumption Equivalents, Steady State

<table>
<thead>
<tr>
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</tr>
<tr>
<td>1.0475</td>
<td>0.0756</td>
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<td>Post-1984</td>
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<tr>
<td>1.0229</td>
<td>0.0226</td>
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D The welfare costs without trend growth (alt. calibration)

Table 15: Consumption Equivalents, Mean

<table>
<thead>
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<tbody>
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<td>Pre-1984</td>
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<tr>
<td>1.0475</td>
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<td>Post-1984</td>
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<td>1.0229</td>
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Table 16: Consumption Equivalents, Steady State

<table>
<thead>
<tr>
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<tbody>
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<td>Pre-1984</td>
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E Volatilities with alternative calibration

Table 17: Unconditional volatilities

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<th>$\sigma(Y)$</th>
<th>$\sigma(\Delta Y)$</th>
<th>$\sigma(\pi)$</th>
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Table 18: Output volatility

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<th>$s_{\text{ua}}$</th>
<th>$s_{\text{ur}}$</th>
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</thead>
<tbody>
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<td>0.0015</td>
<td>0.0031</td>
<td>0.0011</td>
</tr>
<tr>
<td>Post-1984</td>
<td>1.0229</td>
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<td>0.0013</td>
<td>0.0009</td>
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<tr>
<td>Variation</td>
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</table>

Table 19: Output growth volatility

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<th>$s_{\text{ua}}$</th>
<th>$s_{\text{ur}}$</th>
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</thead>
<tbody>
<tr>
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<td>0.0011</td>
<td>0.0008</td>
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<tr>
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<td>0.0009</td>
<td>0.0007</td>
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Table 20: Inflation volatility

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<th>$s_{\text{ua}}$</th>
<th>$s_{\text{ur}}$</th>
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</thead>
<tbody>
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<tr>
<td>Post-1984</td>
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<td>Variation</td>
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<td>-0.8311</td>
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<td>-0.5013</td>
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F IRFs with the baseline calibration

Figure 1: Neutral Shock
Figure 2: MEI Shock
Figure 3: Monetary Shock
G IRFs with alternative calibration

Figure 4: Neutral Shock
Figure 5: MEI Shock
Figure 6: Monetary Shock
H  Experiment with alternative calibration

Figure 7: Neutral Shock
Figure 8: MEI Shock
Figure 9: Monetary Shock