Expertise and Bias in Decision Making

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November 2004

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November 2004

This version: January 2007

Abstract

In this paper, we develop a model of a decision maker using an expert to obtain information. The expert is biased toward some favoured decision but cares also about its reputation on the market for experts. We then analyse the corresponding decision game depending on the nature of the informational linkage with the market. In the case where the expert is biased in favour of the status quo, the final decision is always biased in the same direction. Moreover, it is better to rely on experts biased against the status quo. We also show that it is optimal to publically disclose the expert report. Finally, we prove that the intuitive results that hiring an honest inside expert raises the outside expert's incentives to report truthfully holds when reports are public but not when they are secret.

Keywords: Experts, Bias, Reputation, Merger Control.

JEL Classification: D82, L40.

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*Thanks for helpful comments to Bruno Biais, Denis Gromb, Didier Laussel and Jean Tirole. We have benefited from comments by participants to the seminar at Universitat Autonoma de Barcelona and the IIOC 2005 conference in Atlanta. This work was partially written while the first author was a Marie Curie fellow at Universitat Autonoma de Barcelona. All remaining errors are ours.

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1 Introduction

In many economic environments, information plays a critical role in decision making. However, this information is not always available to the decision maker while it is known, or can be gathered, by a group of parties who are interested in the decision. For instance, the qualification of a merger as anti competitive crucially depends on the expected efficiency gains attained by the merger. To make an efficient decision, the Competition Authority can elicit this information from experts. This could be a difficult task because experts are often biased in favor of a concerned party. To mitigate this informational problem, the European Competition Authority has recently decided to create her own team of economic experts[1]. This is also the case of the agencies in charge of the evaluation of medical products. The role of those agencies is to monitor and control the quality of health products. The decisions are motivated through the advice of internal and external experts. However, external experts may have common interests with industries concerned by the decision. This creates eventual conflicts of interests which must be taken into account by the agency. Other examples of delegated expertise when some experts are biased are, among others, stock recommendations by financial analysts[2] or the evaluation of a project by scientific experts.

This paper studies how those conflicts of interests affects the efficiency of information transmission between the experts and the decision maker when the formers also care about their reputation on the market for experts. We analyze the corresponding decision game depending on the nature of the informational linkage with the market. Our results are twofold. First, relying on experts biased against the status quo enhances the efficiency of the decision. Second, there always exists an equilibrium of the game with secret reports which leads to a lower efficiency in terms of information transmission than provided by all equilibria of the game with public disclosure of the expert's report. The decision maker can therefore use these tools to mitigate the effects of conflicts of interests.

To consider this situation, we develop a framework where a Decision Authority (DA) wants to maximize the social surplus. To make an efficient decision, the DA hires at some fixed wage an outside expert, who cares about its reputation, the social surplus, but also special interests due to collusion or bribery. For instance, he may also have common interests with a concerned party, the competitors or the consumers. Experts may gather a private signal about the social surplus. We model information gathering by a private signal

[1] See Roller, Stennek and Verboven (2000) and Kuhn (2002) for more details about Europeam Competition Authority decision.

[2] Bruce (2002) analyzes what causes analysts to have such a significant bias in their recommendations. See also Morgan and Stocken (2003) for a theoretical study of this point.

which is hard information. This means that the expert cannot manipulate their signal. However, he may conceal the information he has gathered. Within this framework, the DA will always follow an expert's advice when it is informative even though there are conflicts of interests. When the expert's report is uninformative, she will base her decision on the revised expectation of the value of the surplus. The incentive of the expert to reveal his information comes for the benefits he derives from signalling to the market his ability to generate an informative signal.

In addition, we allow the DA to hire an inside expert. Inside experts' interests are perfectly aligned with the DA. However, due to time constraints or lack of expertise about this particular decision, the information they gather is less precise than the outside experts' one.

We first analyze the efficiency of information transmission when the market perfectly observes the expert's reports. When the DA cannot hire inside experts, we show that if the outside experts' bias is not too high, they will always report the truth. But they may have incentives to misreport when their bias is higher. Indeed, when misreporting moves the decision in the same direction than the expert's bias and when the gain from this is higher than the loss from reputation, the expert will conceal his information and report that he is uninformed. Moreover, the reputation gain when announcing an informative report decreases with the set of signals for which he decides to misreport. The fact that the expert misreports for some values of the signal reduces the opportunity cost of not reporting as it becomes relatively less likely that a non-reporting expert didn't receive any information. Thus misreport is to some extent self-enforcing: the more the market anticipates some misreport, the higher the incentives to misreport. This creates the possibility of multiple equilibria. We show that when the expert reports the truth more often, the DA's welfare raises and the expert's welfare diminishes. However, due to the multiplicity of equilibria, the effects of the expert's bias on the welfare depend on the equilibrium selection.

In the presence of inside experts, the outside expert reports the truth more often. This is due to the fact that the probability that an uninformed report moves the decision is reduced which makes misreporting less profitable. Moreover, hiring an inside expert raises the DA's welfare. However, when the outside experts' bias is not too high or when their expertise is superior enough, the value of hiring an inside expert decreases. In such cases, the authority may not want to bear the costs of hiring an inside expert.

We then consider the model when the expert' identity is secret. Now, there are no experts who always report the truth. This means that compared to the case of public identity, for the same value of the bias, an expert lies more when his identity cannot be known by the market. This result is not surprising because under this assumption there is no effect of reputation.

Finally, we analyze information transmission when the expert's reports are secret. Within this framework, the refinement of the market's expectations about the expert's expertise can now only be based on the final decision and not on the reports. Misreporting is therefore less costly for the expert in terms of reputation. The structure of equilibria with secret reports is similar to the case of public reports. We then compare information transmission with secret and public reports. Due to the multiplicity of equilibria, we are only able to show that for any equilibrium of the game with public report, there exists an equilibrium of the game with secret report where the expert misreports more often. In particular when considering only equilibria in which the expert lie the most, or only the ones with maximal truth telling[3], an expert misreports more often when reports are secret.

Moreover, we prove that when experts' reports are secret, hiring an inside expert reduces the outside expert's incentives to report truthfully in all pure strategy equilibria. The result is therefore reversed compared to the case of public report. Caeteris paribus, hiring an inside expert is less attractive if the reports are secret compared to the case where they are public.

This paper belongs to the literature on decision making with biased experts in favor of one cause. The nearest antecedent to our paper is Dewatripont and Tirole (1999) who analyze the advantages of competing advocacies in an incomplete contract framework. They show that, in selecting two competing agents each collecting one signal rather than one gathering two signals, the principal may improve the quality of decision-making. Our paper deviates from theirs in two respects. First, our main objective is not to discuss the role of advocacy in decision making even though we show that it is better to rely on experts biased against the status quo, but to emphasize the role of reputation concerns. Second, we analyze the efficiency of information transmission between the experts and the DA under different informational features while Dewatripont and Tirole (1999) concentrate on the case of public reports.

Several authors have analyzed the incentives of biased experts to misreport in cheap talk models. This literature has been initiated by Crawford and Sobel (1982). They show that full reporting of information is impossible when an expert is biased. Li (2003) extends this result and show that consulting two experts is better than consulting just one. The main difference with our paper is that information is hard in our model even though it can be concealed while it is non verifiable in cheap talk games. Moreover, we show that their result establishing that hiring two experts is more efficient than hiring only one is true exclusively in case reports are public. Specifically, this result cannot be generalized to the case of secret reports.

Morris (2001) introduces reputation concerns. In a repeated framework, he shows that reputation may induce the expert to misreport. However, in Morris (2001), reputation concerns are about the bias of the expert while in our model it is about the quality of expertise. This reverses the effects of reputation. Indeed, in our model reputation induces

[3]Selection of those equilibria is usual in the information transmission literature. See Shin (1998), Li (2003), or Frisell (2004) among others. For methods of refinements of cheap talk equilibria, one may refer to Farrell (1993).

the expert to report the truth more often.

We also share Ottaviani and Sorensen's (2001) result that reputation concerns does not give in general the right incentives to truthfully report information. However, we mitigate this result when the expert's identity is publicly known. We show that no matter if reports are public or secret, there are low values of the bias for which an expert always report truthfully.

The paper is organized as follows. Section 2 describes the basic model and solves it when the expert's reports are public. In Section 3, we present the model and results when the expert' identity is secret. Section 4 analyzes information transmission when the expert's reports are secret. Then, section 5 concludes. Finally, proofs are in section 6.

2 The base model

The decision authority is denoted DA and her objective is to maximize the social surplus. We assume that the social surplus is a parameter θ (say function of the efficiency gains caused by the merger). Thus, without loss of generality, we can state that the DA's objective is $y\theta$ where y is the decision that can take two values: $y = 0$ or $y = 1$ (corresponding for instance to blocking or accepting a merger). The DA is risk neutral. Let's assume that θ is continuously distributed on the interval $\left[\underline{\theta}, \overline{\theta}\right]$, with a cumulative F, a non-decreasing density function $f(.)$[4] and $\overline{\theta} > 0 > \underline{\theta}$.

To make an efficient decision, the DA hires at some fixed wage an outside expert, who cares about its reputation, the social surplus, but also special interests due to collusion or bribery. For instance, he may also be hired by a concerned party, the competitors, the consumers, or have common interests with one of them. This outside expert can gather information about θ. We normalize the model by assuming that the experts are biased in favor of the decision $y = 1$ and receive a benefit $\beta \geq 0$ when this decision is reached[5].

Prior to the consultation, experts have a particular expertise, $\alpha \in \{\underline{\alpha}, \overline{\alpha}\}$ that is unknown to all the parties. An expert with expertise $\overline{\alpha}$ (resp. $\underline{\alpha}$), gathers an informative signal with probability $\overline{p}$ (resp. $\underline{p}$). Let's denote the prior probability that an expert has expertise $\overline{\alpha}$ by (p), and the prior probability that the expert be informed by $\hat{p} = p\overline{p} + (1 - p)\underline{p}$. We assume that the signal is $s = \theta$ when observed by the expert. The expert information is thus $s \in \{\emptyset\} \times \left[\underline{\theta}, \overline{\theta}\right]$ where $\emptyset$ corresponds to no signal.

We model the experts' willingness to maintain their reputation by their future wages which depend on the refinement of the market's expectations about their expertise. In particular the expert obtains an expected premium w when being perceived of high expertise. We denote by i the market's information.

[4] To prove our results, we only need to assume that $f(\theta)$ is non-decreasing on the range $\theta < 0$.

[5] The same results emerge when experts are biased against the decision $y = 1$.

Thus, the objective of an outside expert with bias β is to maximize the following utility function:

$$U = \beta y + w \Pr(\alpha = \overline{\alpha}|i) + \gamma y E(\theta|s)$$

where the first term corresponds to the expert's bias, the second to his willingness to maintain his reputation, the third to the weight he puts on the social surplus.

Let's consider the following framework for the manipulation of information. An expert cannot report that he has gathered an informative signal if it is not the case, nor report another informative signal than the one he has gathered. In other words the informative signal is hard information and can be transmitted to the authority. However, an expert can hide that he has gathered an informative signal and report an uninformative one[6]. An interpretation for hard information is that the DA requires a formal proof in order to validate the expert's report. Any report which does not satisfy this condition is refused.

The game is then the following: the expert observe $s \in \{\emptyset\} \times [\underline{\theta}, \overline{\theta}]$, he reports $r \in \{\emptyset\} \times \{s\}$ and then the DA chooses y.

2.1 Equilibrium analysis

A strategy for an expert when he has received signal s is a function mapping the expert's private information about θ into a report. It is characterized by the set Θ of signals for which an informed expert decides to report no information. For other signals s is truthfully reported.

The strategy of the authority is a mapping from reports into probabilities of making decision $y = 1$. Let's note that as information is hard, when the expert reports an informative signal about θ, the DA follows the advice and make the corresponding decision. If the expert reports the signal $\theta < 0$, (resp. $\theta > 0$), the DA chooses $y = 0$ (resp. $y = 1$). However, when the expert reports an uninformative signal $\emptyset$, there are three cases for the DA. Let $E_{DA}(\Theta)$, the authority's expectations about θ when receiving an uninformative report:

$$E_{DA}(\Theta) = E\{\theta \mid s \in \{\emptyset\} \times \Theta\}.$$

First, when $E_{DA}(\Theta) > 0$, the DA chooses $y = 1$ as she expects that this decision may have a positive impact on the social surplus. Second, when $E_{DA}(\Theta) < 0$, she makes decision $y = 0$. Finally, when $E_{DA}(\Theta) = 0$, she is indifferent between 0 and 1 and she may therefore use a mixed strategy: $y = 1$ with probability q.

Consider now the expert. Let's first note that when he receives good news ($\theta \geq 0$), the expert will always report it to the DA. Indeed, as making an uninformative report will reduce the reputation term and will not bring any gains in this case, the expert has no incentives to misreport.

[6] This framework is also used in Laffont and Tirole (1991) and Dewatripont and Tirole (1999).

Moreover, when receiving an uninformative signal, the only thing that an expert can do is reporting it. Thus, we only have to consider the reporting strategy of an expert receiving a signal $\theta \leq 0$.

When $E_{DA}(\Theta) < 0$, the expert having signal $\theta \leq 0$ will report truthfully his signal. Indeed, the DA's decision is not affected by a negative report as without any signals she should make decision $y = 0$. Moreover, a report increases the perceived likelihood that the expert has a good expertise when

$$\Pr(\alpha = \overline{\alpha}|s = \theta) \geq \Pr(\alpha = \overline{\alpha}|\emptyset).$$

This is always satisfied as the probability that the expert has a good expertise is greater when he reports an informative signal than an uninformative one. Then, when $E_{DA}(\Theta) < 0$, the expert will always report truthfully his signal.

It means that asking for an advice from an expert biased against the status quo makes information transmission truthful. If without consulting any expert, the authority would make a decision, an expert in favor of the opposite decision reports truthfully because this rises the perceived likelihood that he has a good expertise.

Result 1 : *Relying on experts biased against the status quo enhances the efficiency of the decision.*

We immediately obtain that if $E(\theta) \leq 0$, there is a unique equilibrium in which the expert reports the signal for all values of θ. First, this strategy is an equilibrium since $E_{DA}(\emptyset) = E(\theta) \leq 0$. Second, as $E_{DA}(\Theta) < E(\theta)$ if Θ is not empty, there is no other equilibrium. in fact, as a positively biased expert only has incentives to lie when the signal he receives is negative, the DA's refined prior expectation about the efficiency gains when the expert reports an uninformative signal is always negative. Furthermore, when receiving a negative signal, the expert has no incentives to lie because in any case the DA's decision will be the same. Lying does not bring any gain but involve a loss in reputation. The expert's optimal strategy is therefore to always report truthfully.

Hence, from now on we focus on the other case:

Assumption $E(\theta) > 0$.

Notice that it is not possible that $E_{DA}(\Theta) < 0$ in this case, because this would imply that Θ is empty. Thus we have

Lemma 1 *In any equilibrium* $E_{DA}(\Theta) \geq 0$ *and* $q > 0$.

Proof. Suppose $q = 0$, then reporting $\emptyset$ induces $y = 0$, so that the expert reports θ when informed and Θ is empty, a contradiction with assumption $E(\theta) > 0$. So $q > 0$ which implies that $E_{DA}\Theta \geq 0$ ■

Let's remind that q is the probability that an authority who is indifferent between decisions $y = 0$ and $y = 1$, chooses decision $y = 1$.

When $q > 0$, the expert having signal $\theta < 0$ will report truthfully his signal when

$$w \Pr(\alpha = \overline{\alpha}|\theta) \geq q\beta + w \Pr(\alpha = \overline{\alpha}|\emptyset) + q\gamma\theta$$
$$\Leftrightarrow q(\beta + \gamma\theta) \leq w\Delta p(\Theta)$$

where we denote $\Delta p(\Theta)$ the gain of reputation when announcing an informative report when the market anticipates that signal $s \in \Theta$ induces report $\emptyset$. More formally, we have:

$$\Delta p(\Theta) = \Pr(\alpha = \overline{\alpha}|s \in [\underline{\theta}, \overline{\theta}] \setminus \Theta) - \Pr(\alpha = \overline{\alpha}|s \in \{\emptyset\} \times \Theta).$$

This defines a lower bound for the values of efficiency gains for which an expert lies. Notice that this lower bound must be negative. It means that when it is not, experts report truthfully, i.e. Θ is empty.

We are now able to characterize Θ, the set of signals for which an informed expert prefers to misreport.

Proposition 2 *Either Θ is empty or there exists $\theta^* < 0$ such that $\Theta = [\theta^*, 0]$.*

The ways in which the DA acquires information from the experts' reports are twofold. First, when reporting allows the expert to reach his favorite decision or when misreporting may cause the DA to make a too wrong decision, the expert truthfully reports the information. This enables the DA to make the right decision. Second, when reporting helps the DA to make the expert's unfavored decision, the expert withhold the information. Even though the authority cannot know when the expert misreports or has not gathered any information, she gives more importance to the states of the nature belonging to Θ when making her decision. Decision making is therefore more efficient than without any experts.

The first case is only possible if the reputation effect is strong enough. Indeed it is immediate that there is an equilibrium with full reporting ($\Theta = \emptyset$ and $q = 1$) if and only if

$$\beta \leq w\Delta p(\emptyset).$$

Indeed, this strategy is an equilibrium (given $E(\theta) > 0$) if and only if the expert announces θ even when it is very small.

When the private benefits received by an expert if his favorite decision is reached are low, there is an equilibrium in which he always report truthfully (corresponding to Θ empty). Intuitively, as in such cases the gains from moving a decision are low, it is in the expert's interest to maintain his reputation by reporting truthfully. Thus an equilibrium with full reporting only arises when the reputation effects dominate the private benefits from decision $y = 1$ being reached.

Let's now focus on the case with misreport and compute the interval Θ that induces report $\emptyset$ and the probability q of having a positive decision with no report.

The strategy of the expert is to report no signal if

$$\frac{w\Delta p([\theta^*, 0[)}{q} - \beta \leq \gamma\theta < 0.$$

Finally, when $\theta^* < 0$, we have:

$$\theta^* = \max\left\{\frac{w\Delta p([\theta^*, 0])}{\gamma q} - \frac{\beta}{\gamma}, \underline{\theta}\right\} < 0.$$

First notice that $w\Delta p([\theta^*, 0])$ increases with θ^*. The fact that the expert misreports for some values of the signal reduces the opportunity cost of not reporting as it becomes relatively less likely that a non-reporting expert didn't receive any information. Thus misreport is to some extent self-reinforcing: the more the market anticipates some misreport, the higher the incentives to misreport. This creates the possibility of multiple equilibria.

Equilibrium conditions are then

1. Either

$$\begin{aligned} E_{DA}([\theta^*, 0]) &\geq 0, \\ q &= 1, \\ \theta^* = \max\left\{\frac{w\Delta p([\theta^*, 0])}{\gamma} - \frac{\beta}{\gamma}, \underline{\theta}\right\} & \end{aligned} \tag{1}$$

2. or

$$\begin{aligned} E_{DA}([\theta^*, 0[) &= 0, \\ 0 &< q = \frac{w\Delta p([\theta^*, 0])}{\gamma\theta^* + \beta} \leq 1. \end{aligned}$$

In order to give a complete characterization of the equilibria, we use the function $\phi(.)$, defined as:

$$\phi(\theta^*) = w\Delta p([\theta^*, 0]) - \gamma\theta^*.$$

Notice that $w\Delta p(\emptyset) = \phi(0) = \lim_{\theta^* \to 0} \phi(\theta^*)$. Then we must have $\beta = \phi(\theta^*)$ for an interior pure strategy equilibrium.

Let's also remark that the authority's expectations about θ when receiving an uninformative report decreases when she anticipates that the expert will lie more often (when θ^* decreases). Together with the previous equilibrium conditions, it means that if $E_{DA}([\underline{\theta}, 0]) > 0$, a mixed strategy equilibrium cannot exist.

Proposition 3 *Suppose that* $E_{DA}([\underline{\theta}, 0]) > 0$, *then an equilibrium with misreport is characterized by* $q = 1$ *and solution* $\theta^* < 0$ *of* (1).

Thus full misreport of negative values, $\theta^* = \underline{\theta}$ is an equilibrium if and only if

$$\phi(\underline{\theta}) = w\Delta p([\underline{\theta}, 0]) - \gamma\underline{\theta} \leq \beta.$$

It means that if the authority is optimistic about the efficiency gains, an equilibrium in which an expert always lies when he observes an unfavorable signal may exist. Experts take advantage of the authority's optimism in hiding their information when it leads her to choose their favorite decision.

We are now able to state the following existence result.

Corollary 4 *When* $E_{DA}([\underline{\theta}, 0]) \geq 0$, *there exists an equilibrium.*

Proof. We have Θ empty when $\beta \leq \phi(0)$, $\Theta = [\underline{\theta}, 0]$ when $\beta \geq \phi(\underline{\theta})$ as equilibria. Suppose that $\phi(0) < \beta < \phi(\underline{\theta})$. Then $\phi(\theta) - \beta$ changes sign from positive to negative between $\theta = \underline{\theta}$ and $\theta = 0$. Thus there exists a solution θ^* to (1). When $E_{DA}([\underline{\theta}, 0]) = 0$, all that we say is valid except that there may also exists equilibria with $q < 1$. ∎

Notice that several equilibria may emerge as one does not know the variations of the function $\phi(.)$ so that the equation $\beta = \phi(\theta^*)$ may have several solutions.

Let us now turn to the case where $E_{DA}([\underline{\theta}, 0]) < 0$. Then it is not possible to have no reports for all θ, as no report would induce a decision $y = 0$ and entail an opportunity cost in terms of reputation. In fact, the authority is now less confident about the efficiency gains. When the efficiency gains are low, i.e. θ close to $\underline{\theta}$, in misreporting an expert cannot obtain his preferred decision. As we have seen above, it is therefore optimal for him to report truthfully. This also gives rise to mixed strategy equilibria as the authority may be indifferent between both decisions.

Define θ_0 as the solution of

$$E_{DA}([\theta_0, 0]) = 0.$$

The following proposition follows from the previous equilibrium conditions.

Proposition 5 *Suppose that* $E_{DA}([\underline{\theta}, 0]) \leq 0$, *then* $\Theta = [\theta^*, 0]$ *and* $q = 1$ *is an equilibrium if and only if* $\theta^* \in [\theta_0, 0]$ *and is solution of*

$$\phi(\theta^*) = \beta; \tag{2}$$

there is an equilibrium at $\theta^* = \theta_0$ *and* $q < 1$ *if and only if*

$$\phi(\theta_0) < \beta.$$

Proof. Any equilibrium must verify $\theta^* \geq \theta_0 > \underline{\theta}$ as $\theta^* < \theta_0$ implies that $q = 0$. Thus a solution with $q = 1$ must solve (2) .

There is an equilibrium at $q < 1$ if and only if $0 < q = \frac{w\Delta p([\theta^*,0[)}{\gamma\theta^*+\beta} < 1$ or

$$\begin{aligned} -\gamma\theta_0 &< \beta \\ w\Delta p([\theta_0, 0]) - \gamma\theta_0 &< \beta \end{aligned}$$

■

The equilibria that we obtain have the same form than with $E_{DA}([\underline{\theta}, 0]) > 0$. The only difference is the emergence of mixed strategy equilibria due to the fact that the authority is now less confident about the efficiency gains and may be indifferent between both decisions.

Again, equilibria exist for the same reasons than previously.

Corollary 6 *Suppose that* $E_{DA}([\underline{\theta}, 0]) \leq 0$, *then an equilibrium exists.*

Proof. The proof is the same as Corollary 5, replacing $\underline{\theta}$ by θ_0. ■

To summarize we have:

Result 2 :

i) when $\beta \leq \phi(0)$ there is an equilibrium with full transmission of the information;

ii) when $\phi(0) < \beta \leq \phi(\theta_0)$, there is an equilibrium with $\theta^* < 0$ and $q = 1$;

iii) when $\beta > \phi(\max(\underline{\theta}, \theta_0))$, there exists an equilibrium with either $(\theta^* = \underline{\theta}, q = 1)$ or $(\theta^* = \theta_0$ and $q < 1)$ depending on whether $E_{DA}([\underline{\theta}, 0])$ is positive or negative.

Obviously these possibilities are not exclusive, and several equilibria may exist. Given that $w\Delta p([\theta, 0])$ increases with θ, the comparison between $\phi(\theta_0)$ or $\phi(\underline{\theta})$ and $\phi(0)$ is ambiguous. It also appears that the structure of equilibria is governed by the shape of the function $\phi(\theta^*)$.

To provide a more complete structure of equilibria, we show in appendix that the function $\phi(.)$ is convex and that

$$w\Delta p([\theta^*, 0]) = \frac{\phi(0)}{1 + l(F(0) - F(\theta^*))}$$

with

$$l = \frac{\hat{p}}{1 - \hat{p}}$$

The term $\phi(0)$ is the value of the reputation gain when the fact that the expert receives the information or not is public. The term l is the likelihood ratio of the information.

Thanks to those properties of the function ϕ, we are able to determine when the equilibrium is unique and when there is a multiplicity of equilibria.

Proposition 7

- *If* $\gamma \geq \phi(0)lf(0) = \gamma_1$*, there exists a unique equilibrium for all* β*.*
- *Assume that* $\gamma < \gamma_1$*. Then, the equilibrium is unique for* $\beta > \phi(0)$ *(*$\theta^* < 0$*) or if* $\beta < \min_\theta \phi(\theta)$ *(*$\Theta = \emptyset$*). Otherwise there are three equilibria.*

Proof. The first part follows from $\phi'(0) \leq 0$. Indeed, as $\gamma \geq \phi(0)lf(0) = \gamma_1$, we have $\phi'(0) \leq 0$. Together with the convexity of ϕ, this gives that ϕ is decreasing. Using the characterization of equilibria, we can conclude that $\phi(0) < \phi(\max(\underline{\theta}, \theta_0))$. This implies that the three previous possibilities are exclusive, resulting in a unique equilibrium.

For the second part, let's note that $\beta < \min_\theta \phi(\theta)$ means that an expert always report truthfully. Moreover, if $\beta > \phi(0)$, as ϕ is decreasing on the range $\theta < 0$ and $\phi(\theta) > \phi(0)$, there also exists a unique equilibrium.

We consider now the case of $\min_\theta \phi(\theta) \leq \beta \leq \phi(0)$. As $\beta \leq \phi(0)$, a full reporting equilibrium always exists. We have now two different cases. In the first one, $\phi(0) < \phi(\widehat{\theta})$, it follows from convexity that the equation $\phi(\theta^*) = \beta$ has two solutions, giving rise to two equilibria. Moreover, as $\beta < \phi(\widehat{\theta})$, there is no other equilibria. In the second one, $\phi(0) > \phi(\widehat{\theta})$, convexity implies that the equation $\phi(\theta^*) = \beta$ has two solutions for $\min_\theta \phi(\theta) \leq \beta < \phi(\widehat{\theta})$ and a unique solution for $\phi(\widehat{\theta}) \leq \beta \leq \phi(0)$. Moreover, for $\phi(\widehat{\theta}) \leq \beta \leq \phi(0)$, there also exists an equilibrium with either ($\theta^* = \underline{\theta}, q = 1$) or ($\theta^* = \theta_0$ and $q < 1$) depending on whether $E_{DA}([\underline{\theta}, 0])$ is positive or negative.

We can therefore conclude that there are three equilibria when $\min_\theta \phi(\theta) \leq \beta \leq \phi(0)$.
■

Let us now describe those equilibria in more details. To simplify the notations, we denote $\widehat{\theta} = \max(\underline{\theta}, \theta_0)$. Basically, we have four cases depending on γ.

Before examining these cases let us define the values of γ for which the structure of equilibria changes.

$$\begin{aligned}
\gamma_1 &= \left\{\gamma | \gamma \geq \gamma_1 \Longleftrightarrow \phi'(0) \leq 0\right\} \\
\gamma_2 &= \left\{\gamma | \gamma \geq \gamma_2 \Longleftrightarrow \phi(0) \leq \phi(\widehat{\theta})\right\} \\
\gamma_3 &= \left\{\gamma | \gamma \geq \gamma_3 \Longleftrightarrow \phi'(\widehat{\theta}) \leq 0\right\}
\end{aligned}$$

Let's remark that we have $\gamma_3 \leq \gamma_2 \leq \gamma_1$.

1. When γ is large, $\gamma \geq \gamma_1$, we have $\phi'(0) \leq 0$ and there exists a unique equilibrium for all β. Let's remark that in this case, ϕ is decreasing on the range $[\max(\underline{\theta}, \theta_0), 0]$ and $\phi(\widehat{\theta}) \geq \phi(0)$. This is represented in Figure 1. In this case, if $\beta > \phi(0)$ the expert decides to lie when his signal belongs to the set $\left[\max\left(\widehat{\theta}, \theta^*\right), 0\right[$ and if $\beta \leq \phi(0)$ the expert always report truthfully.

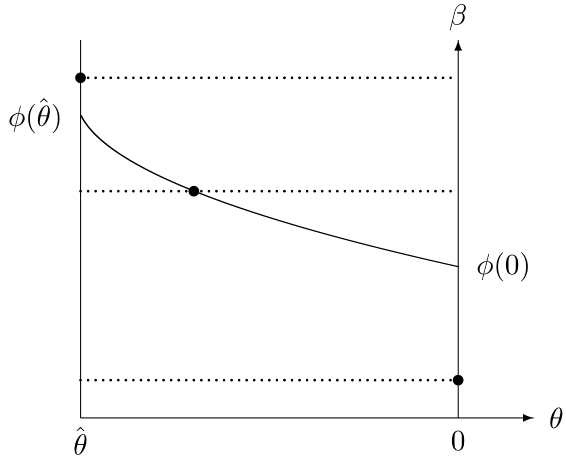

Fig. 1: *Equilibria with* $\phi'(0) \leq 0$.

2. When $\gamma_2 \leq \gamma < \gamma_1$, $\phi'(0)$ is now positive but we still have $\phi(0) \leq \phi(\widehat{\theta})$. This is represented in Figure 2. In this case, there exists a unique equilibrium, if $\beta > \phi(0)$ in which the expert decides to lie when his signal belongs to the set $\left[\max\left(\widehat{\theta}, \theta^*\right), 0\right[$, and if $\beta < \min_\theta \phi(\theta)$ in which the expert always report truthfully. However, if $\beta \in [\min_\theta \phi(\theta), \phi(0)]$, there coexists three equilibria.

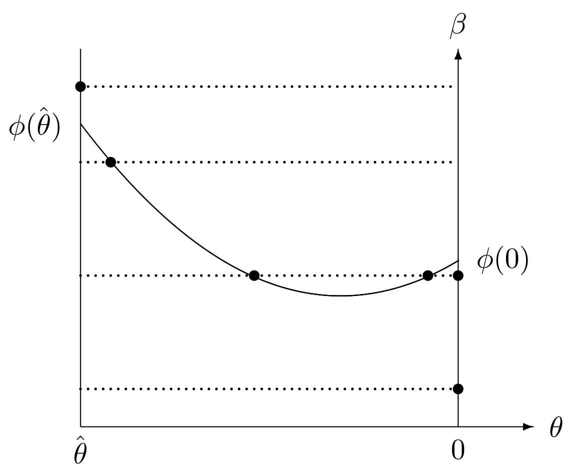

Fig. 2: *Equilibria with* $\phi'(0) \geq 0$, $\phi'(\widehat{\theta}) \leq 0$ *and* $\phi(\widehat{\theta}) \geq \phi(0)$.

3. When $\gamma_3 \leq \gamma < \gamma_2$, $\phi'(0)$ is now positive, we still have $\phi'(\widehat{\theta}) \leq 0$, but now we have $\phi(0) \geq \phi(\widehat{\theta})$. This is represented in Figure 3. Again, there exists a unique equilibrium, if $\beta > \phi(0)$ in which the expert decides to lie when his signal belongs to the set $\left[\max\left(\widehat{\theta}, \theta^*\right), 0\right[$, and if $\beta < \min_\theta \phi(\theta)$ in which the expert always report truthfully. We also have, as previously, if $\beta \in [\min_\theta \phi(\theta), \phi(0)]$, there coexists three equilibria. However, let's note that, now, when $\beta \in \left[\phi(\widehat{\theta}), \phi(0)\right]$, there exists an equilibrium where the expert lies as much as possible while it was not the case when $\gamma_2 \leq \gamma < \gamma_1$.

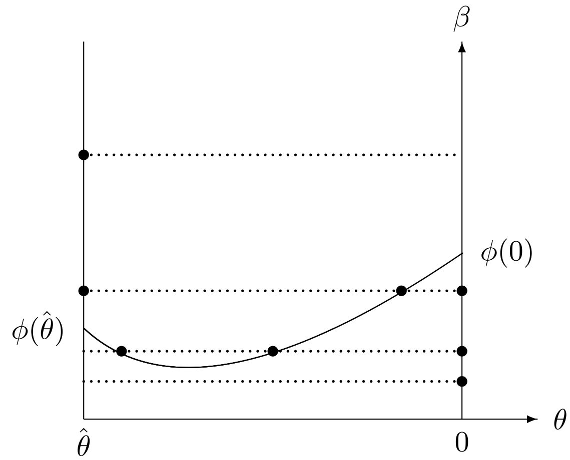

Fig. 3: *Equilibria with* $\phi'(0) \geq 0$, $\phi'(\widehat{\theta}) \leq 0$ *and* $\phi(\widehat{\theta}) \leq \phi(0)$.

4. When $\gamma > \gamma_3$, $\phi'(\widehat{\theta})$ is now positive which means that ϕ is increasing on the range $[\max(\underline{\theta}, \theta_0), 0]$. This is represented in Figure 4. Again, there exists a unique equilibrium, if $\beta > \phi(0)$ in which the expert decides to lie when his signal belongs to the set $\left[\max\left(\widehat{\theta}, \theta^*\right), 0\right[$, and if $\beta < \min_\theta \phi(\theta)$ in which the expert always report truthfully. We also have, as previously, if $\beta \in [\min_\theta \phi(\theta), \phi(0)]$, there coexists three equilibria. But now, the range of bias, β, for which there is an equilibrium where the expert lies as much as possible is raised because such an equilibrium exists for all $\beta \in [\min_\theta \phi(\theta), \phi(0)]$.

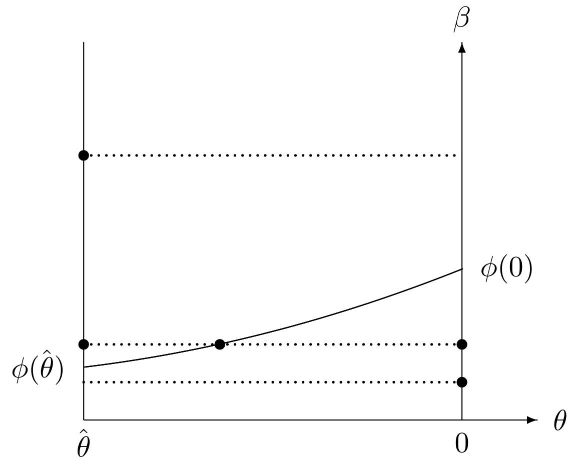

Fig. 4: *Equilibria with* $\phi'(\widehat{\theta}) \geq 0$.

2.2 Welfare analysis

In order to analyze the effects of the conflicts of interests on the different parties, we now examine how their welfare varies with the bias of the hired expert. However, due to the multiplicity of equilibria in some cases, we need to make an equilibrium selection to complete the welfare analysis. First, let's note that some of our equilibria are not stable to a little change in the beliefs. This is the case, when there are two interior equilibria, of the highest one. However, the multiplicity of equilibria survives even though we eliminate those unstable equilibria.

To cope with this multiplicity, let's apply the methodology used by Shin (1998), Li (2003), or Frisell (2004) among others. They only consider the equilibrium in which the expert always lie as much as possible and the one in which he reports the truth as often as possible. Those equilibria are the most studied in the information transmission literature in order to simplify the comparisons between equilibria.

Both equilibria have a nice and intuitive property. They are monotone in the bias. It means that when we only consider one of those equilibria, the more an expert is biased, the larger is the range in which he lies. Using our notations, along those equilibria the function $\phi(.)$ is decreasing if θ^* is interior.

To simplify matters we will focus on the case $\underline{\theta} > \theta_0$, so that $q = 1$ in all equilibria.

Let's first consider the DA's welfare. Let $\hat{p} = (1-p)\,\underline{p} + p\bar{p}$. We have

$$\begin{aligned} W_{DA} &= \hat{p}\left[1 - F\left(\theta^*\right)\right] E\left(\theta \mid \theta \geq \theta^*\right) + (1-\hat{p})\, E\left(\theta\right) \\ &= \hat{p} \int_{\theta^*}^{\overline{\theta}} \theta f\left(\theta\right) d\theta + (1-\hat{p})\, E\left(\theta\right). \end{aligned}$$

We can now study how the DA's welfare varies with θ^*. This will allow us to state what happens when the range on which the expert lies rises.

$$\frac{\partial W_{DA}}{\partial \theta^*} = -\hat{p}\theta^* f(\theta^*) > 0.$$

Not surprisingly, when the expert reports the truth more often, the DA's welfare raises. As we only consider equilibria in which the range in which the expert lies increases with the bias, this implies that the DA's welfare is non increasing in the bias of the expert.

Let's consider now the expert's welfare. Notice that in expected terms the expert's reputation gain is constant, equal to pw. Thus we have

$$\begin{aligned} W_E(\beta, \theta^*) &= pw + \hat{p}[1 - F(\theta^*)]\{\beta + \gamma E(\theta \mid \theta \geq \theta^*)\} + (1 - \hat{p})\{\beta + \gamma E(\theta)\} \\ &= pw + \hat{p}[1 - F(\theta^*)]\beta + \hat{p}\gamma \int_{\theta^*}^{\overline{\theta}} \theta f(\theta)\, d\theta + (1 - \hat{p})\{\beta + \gamma E(\theta)\}. \end{aligned}$$

Again, we study the variations of the expert's welfare. First,

$$\frac{\partial W_E}{\partial \theta^*} = -\hat{p} f(\theta^*)(\beta + \gamma\theta^*) < 0$$

Between two equilibria the expert always prefers the equilibrium with the smallest θ^*. This is because in equilibrium a decision $y = 1$ is only adopted when it benefits the expert. This implies that for any equilibrium selection where θ^* is non-decreasing with β, the welfare W_E increases with β.

Total Welfare

Now consider the following situation: the expert is the only available and has a cost C to provide the expertise. The experts knows that if he refuses to participate the decision will be $y = 1$. He then requires a wage $C + wp + \beta + \gamma E(\theta) - W_E(\beta, \theta^*)$. The total welfare gain for the pair DA-expert created by the expertise gross of the cost C is

$$\begin{aligned} WT &= W_{DA} + W_E - (wp + \beta + \gamma E(\theta)) - C \\ &= \hat{p}\left(-(1 + \gamma)\int_0^{\theta^*} \theta f(\theta)\, d\theta - F(\theta^*)\beta\right) - C \end{aligned}$$

For a fixed θ^*, notice that

$$\begin{aligned} \frac{\partial WT}{\partial \theta^*} &= -\hat{p} f(\theta^*)(\beta + (1 + \gamma)\theta^*), \\ \frac{\partial WT}{\partial \beta} &= -\hat{p}F(\theta^*) < 0. \end{aligned}$$

The higher the bias the more reluctant will be the expert to participate, as his participation raises the chance of a decision $y = 0$. Thus for a fixed θ^* it would be better to have an unbiased expert.

However, for an interior θ^*, we have $\phi(\theta^*) = \beta$, and thus

$$\begin{aligned}\frac{dWT}{d\beta} &= \frac{\partial WT}{\partial \theta^*}\frac{1}{\phi'(\theta^*)} + \frac{\partial WT}{\partial \beta} \\ &= -\hat{p}\left(\frac{f(\theta^*)(\phi(\theta^*) + (1+\gamma)\theta^*) + F(\theta^*)\phi'(\theta^*)}{\phi'(\theta^*)}\right).\end{aligned}$$

This is positive if

$$\begin{aligned}\frac{f(\theta^*)}{F(\theta^*)} &> -\frac{\phi'(\theta^*)}{\phi(\theta^*) + (1+\gamma)\theta^*} \\ and\ \phi(\theta^*) + (1+\gamma)\theta^* &= w\Delta p([\theta^*, 0]) + \theta^* > 0.\end{aligned}$$

which reduces to

$$\gamma < \frac{f(\theta^*)}{F(\theta^*)}\left(\theta^* + \frac{(1 + lF(0))\phi(0)}{(1 + l(F(0) - F(\theta^*)))^2}\right).$$

We see that it may not be the case that a benevolent principal prefers a less biased expert when the equilibrium is interior. Indeed, in some cases the total welfare gains increase with the bias of the expert. This may arise when the expert is not too affected by a loss in social surplus, i.e. γ low. An increase in the bias raises the expert's benefits from misreporting because this expert only has low social surplus concerns. When those benefits compensate the reduction of the total welfare due to a stronger bias, the benevolent principal may prefer a more biased expert.

2.3 Consultation of Inside experts

We then study if the presence of inside experts will enhance the social surplus. Inside experts' interests are perfectly aligned with the DA. Their objective is to maximize the social surplus. However, due to time constraints or lack of expertise about this particular decision, the information they gather is less precise than the outside experts' one. Let p_I be the probability that an inside expert will gather an informative signal.

Let's remark that an inside expert has no incentives to lie and thus always report truthfully. However, due to his lack of expertise, he gathers less often informative signals.

Let's now analyze precisely what happens when an inside expert may be hired.

When the outside expert receives ϕ, he is forced to report ϕ. When the outside expert observes θ, there are 2 cases: either the outside expert reports θ and the CA does not involve the inside expert, or the outside expert reports ϕ and the CA does involves the inside expert. In the latter case, when the inside expert reports ϕ, nothing is changed compared to the case without inside experts. However, the inside expert may report θ in which case the decision is reversed and the DA chooses $y = 0$. The outside expert reporting strategy may therefore change.

Thus, when $q > 0$, the outside expert having signal $\theta < 0$ will report truthfully his signal when

$$\begin{aligned} w \Pr(\alpha &= \overline{\alpha}|\theta) \geq \Pr[\text{I reports } \phi] (q\beta + w \Pr(\alpha = \overline{\alpha}|\phi) + q\gamma\theta) \\ &\quad + \Pr[\text{I reports } \theta] (w \Pr(\alpha = \overline{\alpha}|\phi)) \\ &\Leftrightarrow \quad q(\beta + \gamma\theta) \leq \frac{w\Delta p(\Theta_I)}{1 - p_I}, \end{aligned}$$

where Θ_I is the set of signals for which the informed outside expert decides to report no information when an inside expert may be hired.

This defines a new value for θ_I^*, higher than θ^*, as $(1 - p_I) < 1$.

Proposition 8 *When experts' reports are public, hiring an inside expert raises the outside expert's incentives to report truthfully.*

Let's note that hiring an inside expert has two effects on the reporting strategy. The direct effect comes from the fact that if the outside expert lies, the inside expert's report may contradict him. The indirect effect is due to the set of signals for which the informed outside expert decides to report no information which is now smaller.

We can now compute the value of hiring an inside expert for the authority which we define as the difference between the DA's welfare when she hires an inside expert and when she does not, $V_I = W_{DA}(\theta_I^*, I) - W_{DA}(\theta^*)$. We can write that as

$$\begin{aligned} V_I &= V_I^1 + V_I^2, \\ \textit{where } V_I^1 &= W_{DA}(\theta_I^*, I) - W_{DA}(\theta^*, I) \\ V_I^2 &= W_{DA}(\theta^*, I) - W_{DA}(\theta^*) \end{aligned}$$

The first term V_I^1 captures the indirect effect discussed above. The second one V_I^2 represents the value of the additional information brought by the inside expert for fixed incentives. As the DA's welfare is increasing in θ^*, and as $\theta^* \leq \theta_I^*$, V_I^1 is positive. Let's state V_I^2

$$\begin{aligned} V_I^2 &= \hat{p} \Pr(\Theta) [-p_I E(\theta \mid \theta \in \Theta)] \\ &\quad + (1 - \hat{p}) \Pr(\theta \leq 0) [-p_I E(\theta \mid \theta \leq 0)] \end{aligned}$$

As $E(\theta \mid \theta \in \Theta)$ and $E(\theta \mid \theta \leq 0)$ are non negative, V_I^2 is also non negative.

The main conclusion is first that $V_I \geq 0$, and second that $V_I^1 > 0$. Both effects enhance the social surplus. If hiring an inside expert is not too costly for the DA, this is therefore profitable to do it. Moreover, let's note that, even though relying on an inside expert would not modify the incentives of outside experts to report truthfully, this would have a positive effect on the social welfare because of the additional information they provide.

3 The model when the expert's identity is secret

This is also interesting to consider the model when the expert' identity is secret. Under this assumption, there is no effect of reputation. The expert utility is thus $U = \beta y + wp + \gamma y E(\theta|s)$.

3.1 Equilibrium analysis

When he receives good news ($\theta \geq 0$), the expert always reports it to the DA. Indeed, as making an uninformative report reduces the reputation term and do not bring any gains in this case, the expert has no incentive to misreport.

Moreover, when receiving an uninformative signal, the only thing that an expert can do is reporting it. Thus, we only have to consider the reporting strategy when an expert receives a signal $\theta \leq 0$.

When $E_{DA}(\Theta) < 0$, the expert having signal $\theta \leq 0$ report truthfully his signal because he is indifferent between lying and reporting truthfully.

When $E_{DA}(\Theta) \geq 0$, and $q > 0$, the expert having signal $\theta \leq 0$ will report truthfully his signal when

$$\theta \leq -\frac{\beta}{\gamma}$$

Let's denote as before $\widehat{\theta} = \max\{\underline{\theta}, \theta_0\}$, and $\phi_0(\theta^*) = -\gamma\theta^*$.

We have that:

i) there is no equilibrium with full transmission of the information;

ii) when $\beta \leq \phi_0(\widehat{\theta})$, there is an equilibrium with $\theta^*_{si} = -\frac{\beta}{\gamma}$ and $q = 1$;

iii) when $\beta > \phi_0(\widehat{\theta})$, there exists an equilibrium with either ($\theta^*_{si} = \underline{\theta}, q = 1$) or ($\theta^*_{si} = \theta_0$ and $q = 0$) depending on whether $E_{DA}([\underline{\theta}, 0])$ is positive or negative.

Moreover, we are able to state the following uniqueness result:

Proposition 9 *As ϕ is decreasing on $[\widehat{\theta}, 0]$, the equilibrium is unique, and $\theta^*_{si} \leq \theta^*$.*

The structure of the equilibrium is represented in Figure 5. It is the same kind of equilibrium as when γ is high enough and the expert's identity is public. Except that now, there is no expert who always report the truth. This means that compared to the case of public identity, for the same value of the bias, an expert lies more when his identity cannot be known by the market. This is quite intuitive because in the latter case, the expert does not care about his reputation.

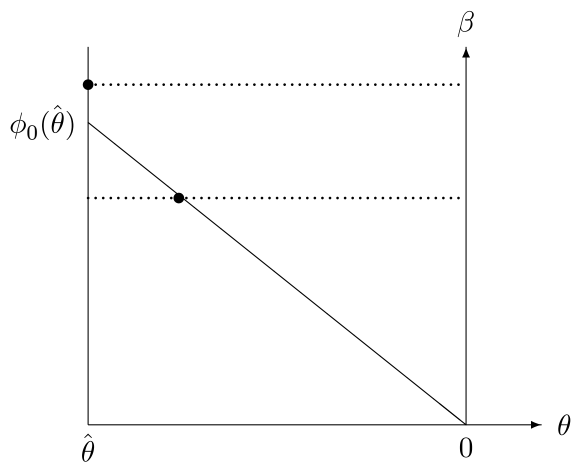

Fig. 5: *Equilibrium when identity is secret.*

4 The model when the expert's report is secret

The model is the same than before except that now, the expert's identity is public and his reports are secret. This means that the refinement of the market's expectations about the expert's expertise can only be based on the final decision and not on the reports.

Let's note that within this framework, the market has not the same abilities to determine the expert's expertise. Lying is therefore less costly for the expert in terms of reputation. Thus, we expect that the expert will lie more often when reports are secret.

4.1 Equilibrium analysis

Again, a strategy for an expert when he has received signal s is a function mapping the expert's private information about θ into a report. It is characterized by the set Θ_{sr} of signals for which the informed expert decides to report no information. For other signals, s is truthfully reported. For the same reasons than previously, we focus on the case $E(\theta) > 0$.

The strategy of the authority is a mapping from reports into probabilities q. If the expert reports the signal $\theta < 0$, (resp. $\theta > 0$), the DA chooses $y = 0$ (resp. $y = 1$). However, when the expert reports an uninformative signal $\emptyset$, there are three cases for the DA, depending on the authority's expectations $E_{DA}(\Theta_{sr})$ about θ when receiving an uninformative report .First, when $E_{DA}(\Theta_{sr}) > 0$, she chooses $y = 1$. Second, when $E_{DA}(\Theta_{sr}) < 0$, she chooses

$y = 0$. Finally, when $E_{DA}(\Theta_{sr}) = 0$, she is indifferent between 0 and 1 and she may therefore use a mixed strategy: $y = 1$ with probability q.

Let's now consider the expert's strategy. First, when receiving an uninformative signal, the only thing that an expert can do is reporting it. Thus, we only have to consider the reporting strategy when an expert receives an informative signal. As for the DA, there are three cases, depending on $E_{DA}(\Theta_{sr})$. Let's denote $\Delta s(\Theta_{sr}, q)$ the gain of reputation when the DA's decision is $y = 0$ when the market anticipates that signal $s \in \Theta_{sr}$ induces report $\emptyset$ and when the probability of $y = 1$ if the experts reports an uninformative signal $\emptyset$ is q.

$$\Delta s(\Theta_{sr}, q) = \Pr(\alpha = \overline{\alpha}|y = 0) - \Pr(\alpha = \overline{\alpha}|y = 1).$$

Then an expert with information $\theta > 0$ would report the true value if

$$\begin{gathered} q\left[\beta + w\Pr(\alpha = \overline{\alpha}|y = 1) + \gamma\theta\right] + (1-q)w\Pr(\alpha = \overline{\alpha}|y = 0) \\ > \beta + w\Pr(\alpha = \overline{\alpha}|y = 1) + \gamma\theta \end{gathered}$$

which reduces to

$$(1-q)\left(\beta + \gamma\theta - w\Delta s(\Theta_{sr}, q)\right) > 0 \tag{3}$$

The expert having signal $\theta \leq 0$ will report θ when

$$\begin{aligned} q\left[\beta + w\Pr(\alpha = \overline{\alpha}|y = 1) + \gamma\theta\right] + (1-q)w\Pr(\alpha &= \overline{\alpha}|y = 0) \\ &< w\Pr(\alpha = \overline{\alpha}|y = 0) \end{aligned}$$

or when

$$q\left(\beta + \gamma\theta - w\Delta s(\Theta_{sr}, q)\right) < 0 \tag{4}$$

Following the previous analysis of public reports, we define on $[\underline{\theta}, 0]$ the function

$$\psi(\theta^*_{sr}) = w\Delta s(\left[\theta^*_{sr}, 0\right], 1) - \gamma\theta^*_{sr}.$$

As before, the function ψ is convex if $E(\theta) > 0$, when the density f increases for negative values of θ.

Equilibria with $q = 0$.

When $E_{DA}(\Theta_{sr}) < 0$, the probability that the DA chooses $y = 1$ when the experts reports an uninformative signal is $q = 0$. Notice that $\Delta s(\Theta_{sr}, 0) \leq 0$ because a favorable decision from the DA is good news for the expert's reputation (it can only occur if the expert is informed). Thus, there is a loss of reputation when the DA's decision is $y = 0$. When he receives good news ($\theta \geq 0$), the expert will also always report it to the DA as $\beta + \gamma\theta - \Delta s(\Theta_{sr}, 0) > 0$. Now, the expert having signal $\theta \leq 0$ may or may not report truthfully his signal since her utility in lying is the same than in reporting truthfully.

Such an equilibrium would always exist if $E(\theta)$ were negative, in which case the expert would always report truthfully his signal and the DA would choose $y = 0$ if there is no report. But we have assumed that under the veil of ignorance, the DA would choose $y = 1$.

Still there exists equilibria with $q = 0$ if $E_{DA}([\underline{\theta}, 0]) \leq 0$. In particular there are equilibria where experts hide the information when θ lies in Θ_{sr} for any set such $E_{DA}(\Theta_{sr}) \leq 0.$, This holds in particular for sets $\Theta_{sr} = [\theta^*_{sr}, 0]$ where $\theta^*_{sr} \leq \theta_0$.

To select among these equilibria we suppose that there is an ε probability that the report be observed by the outside parties. We say that an equilibrium is "robust to a small perturbation in the information structure" if the game with $\varepsilon > 0$ small has an equilibrium that is close to the initial one (in terms of q and Θ_{sr}).

In the perturbed game the expert would report a negative θ when $q = 0$. Thus the above equilibrium disappears. But some close equilibrium may remain. The expert reports $\theta < 0$ when

$$q\left[\beta + \gamma\theta\right] - (1-\varepsilon)qw\Delta s(\Theta_{sr}, q) - \varepsilon w \Delta p(\Theta_{sr}) \leq 0$$

Indeed with probability ε, the report is observed so that the market update its beliefs as the in the previous section and the reputation gain for an informative report is $w\Delta p(\Theta_{sr})$. Thus there is a threshold θ^*_{sr} such that $\Theta_{sr} = [\theta^*_{sr}, 0]$. For $1 > q > 0$, we must have $\theta^*_{sr} = \theta_0$. Now we can find an equilibrium with $q < 1$ if

$$-\varepsilon w \Delta p([\theta_0, 0]) < 0 < [\beta + \gamma\theta_0] - (1-\varepsilon) w \Delta s([\theta_0, 0], 1) - \varepsilon w \Delta p([\theta_0, 0]).$$

We then obtain

Lemma 10 *There exists an equilibrium with $q = 0$ robust to a small perturbation in the information structure if $\beta > \psi(\theta_0)$. This equilibrium is characterized by $\Theta_{sr} = [\theta_0, 0]$.*

Proof. From above we must have $\Theta_{sr} = [\theta_0, 0]$. Moreover for $\varepsilon > 0$, we have

$$\begin{aligned} & [\beta + \gamma\theta_0] - (1-\varepsilon) w \Delta s([\theta_0, 0], 1) - \varepsilon w \Delta p([\theta_0, 0]) \\ > \quad & [\beta + \gamma\theta_0] - w \Delta s([\theta_0, 0], 1) = \psi(\theta_0) \geq 0. \end{aligned}$$

so the probability q is defined as

$$q_\varepsilon = \frac{\varepsilon w \Delta p([\theta_0, 0])}{[\beta + \gamma\theta_0] - (1-\varepsilon) w \Delta s([\theta_0, q_\varepsilon], 1)}$$

and converges to zero as ε goes to zero. ■

Equilibria with $q > 0$.

When $E_{DA}(\Theta_{sr}) > 0$, the probability that the DA chooses $y = 1$ when the experts reports an uninformative signal is $q = 1$ and we have $\Delta s(\Theta_{sr}, 1) > 0$. The expert having signal $\theta \geq 0$ will report truthfully his signal since her utility in lying is the same than in reporting truthfully.[7] When he receives bad news ($\theta \leq 0$), the expert will lie when

$$\beta + \gamma\theta \geq w\Delta s(\Theta_{sr}, 1)$$

Finally, when $E_{DA}(\Theta_{sr}) = 0$, the probability that the DA chooses $y = 1$ when the experts reports an uninformative signal is $q \in (0, 1)$.The expert having signal $\theta \geq 0$ will lie when

$$\theta \leq \frac{w\Delta s(\Theta_{sr}, q) - \beta}{\gamma}$$

The expert having signal $\theta \leq 0$ will lie when

$$\theta \geq \frac{w\Delta s(\Theta_{sr}, q) - \beta}{\gamma}.$$

Let's remark that an expert cannot lie in both situations as $\frac{w\Delta s(\Theta_{sr},q)-\beta}{\gamma}$ is given for an expert. When $w\Delta s(\Theta_{sr}, q) - \beta \geq 0$, the expert will report truthfully for all $\theta \leq 0$. This implies that $E_{DA}(\Theta_{sr}) > 0$. However, this is in contradiction with $E_{DA}(\Theta_{sr}) = 0$. Thus, we have $w\Delta s(\Theta_{sr}, q) - \beta \leq 0$. The expert will therefore report truthfully for all $\theta \geq 0$ and will lie when $\frac{w\Delta s(\Theta_{sr},q)-\beta}{\gamma} \leq \theta \leq 0$.

The equilibrium strategies are in the same lines than in the case of public reports. They are stated in the following proposition:

Proposition 11

- *Either* Θ_{sr} *is empty or there exists* $\theta^*_{sr} < 0$ *such that* $\Theta_{sr} = [\theta^*_{sr}, 0]$.
- *There is an equilibrium with full reporting* ($\Theta_{sr} = \emptyset$ *and* $q = 1$) *if and only if* $\beta \leq \psi(0)$.

Proof. The proof is the same as in the case with public reports. ■

Again, full reporting (Θ_{sr} empty) is only possible if the reputation effect is strong enough.

Let's now focus on situations with misreport and compute the interval Θ_{sr} that induces report $\emptyset$ and the probability q of having a positive decision with no report. Remind that θ_0 is defined as the solution of $E_{DA}([\theta_0, 0]) = 0$. From above, the equilibrium have $0 \geq \theta^*_{sr} \geq \theta_0$, $1 \geq q$, and $q = 1$ if $\theta^*_{sr} > \theta_0$.

Thus we have when $\theta_0 < \theta^*_{sr} < 0$,

$$\theta^*_{sr} = \max\left\{\theta^*_{sr} + \frac{\psi(\theta^*_{sr}) - \beta}{\gamma}, \underline{\theta}\right\},\ q = 1;$$

[7] Formally if $\varepsilon = 0$, there is always a trivial equilibria where the expert reports $\emptyset$ for a large set of negative values θ and $q = 1$, but it would not be robust to a small perturbation in the information structure.

Otherwise the equilibrium verifies:

$$\theta^*_{sr} = \theta_0;$$
$$0 < q < 1.$$

and

$$\beta + \gamma\theta_0 = w\Delta s(\Theta_{sr}, q)$$

Notice that the analysis is similar to the case of public reports for pure strategy equilibria except for the value of reputation gains: the gain from reporting θ^*_{sr} when $q = 1$ is now $\psi(\theta^*_{sr}) - \beta$. It follows that the analysis of pure strategy equilibria is the same as before, replacing the function $\phi(\theta^*_{sr})$ by the function $\psi(\theta^*_{sr})$.

In particular all the previous analysis carries over with the new ψ function if $E_{DA}([\underline{\theta}, 0]) > 0$.

Proposition 12 *Suppose that $E_{DA}([\underline{\theta}, 0]) > 0$, then any equilibrium is characterized by $q = 1$. Moreover there exists an equilibrium.*

Proof. The proof is the same as with public reports. ■

Thus

- no reporting of negative values ($\theta^*_{sr} = \underline{\theta}$) is an equilibrium if $\psi(\underline{\theta}) \leq \beta$,

- full reporting is an equilibrium if $\psi(0) \geq \beta$.

- There exists one equilibrium with partial reporting of negative values if $\max(\psi(\underline{\theta}), \psi(0)) > \beta > \min(\psi(\underline{\theta}), \psi(0))$.

- There exists two equilibria with partial reporting of negative values if $\min(\psi(\underline{\theta}), \psi(0)) > \beta > \min_{(\underline{\theta},0)} \psi(\theta)$.

In the case where $E_{DA}([\underline{\theta}, 0]) < 0$, the analysis extends only for equilibria with $q = 1$. But the analysis of mixed strategy differs from the case where the report is public. With private reports $q(w\Delta s(\Theta_{sr}, q) - \beta - \gamma\theta)$ is the net gain of reporting a negative value. For $q > 0$, the sign is the same as the sign of $w\Delta s(\Theta_{sr}, q) - \beta - \gamma\theta$.

We show in appendix that $\Delta s(\Theta_{sr}, q)$ is increasing in θ^*_{sr} and q. The reputation gain obtained by reporting $\theta < 0$ is now increasing with the probability q that the DA chooses $y = 1$ with no report. It is negative for $q = 0$ and positive for $q = 1$. This contrasts with the case of a public report.

The key difference with before is the following. When the report is observed, the reputation gain is independent of q. The probability q then affects the benefits of no report in terms of induced decision. Thus increasing q reduces the incentive to report. When the report is secret however there is no direct effect of q on the incentive to report a negative value (for $q > 0$) as the reputation gain is only obtained if the decision is

$y = 0$. But the reputation gain depends on q. Increasing q reduces the likelihood that a decision $y = 0$ follows no report, and thus increases the incentive to report θ.

We then obtain

Proposition 13 *Suppose that* $E_{DA}([\underline{\theta}, 0]) \leq 0$, *then an equilibrium exists with* $q = 1$ *if* $\max(\psi(\theta_0), \psi(0)) \geq \beta$. *An equilibrium with* $0 < q < 1$ *exists if* $w\Delta s([\theta_0, 0], 0) - \gamma\theta_0 < \beta < \psi(\theta_0)$.

Notice that $w\Delta s([\theta_0, 0], 0) < 0$, so that the first condition is trivially verified if $\theta_0 \geq -\frac{\beta}{\gamma}$. But it is possible that a decision $y = 0$ be perceived as a bad signal by the market despite a bias of the expert in favor of $y = 1$.

Corollary 14 *There exists an equilibrium with* $q < 1$ *and* $w\Delta s([\theta_0, 0], q) < 0$, *when* $\theta_0 < -\frac{\beta}{\gamma}$.

The issue of existence arises if $\beta > \max(\psi(\theta_0), \psi(0))$. Then for any $q > 0$, the expert would not announce a negative θ slightly below θ_0. But we have seen that in the case $\beta > \psi(\theta_0)$, there is an equilibrium with $\theta^*_{sr} = \theta_0$ and $q = 0$.

Notice that the function ψ is smaller than the function ϕ so that:

Proposition 15 *For any equilibrium* θ^* *of the game with public report, there exists an equilibrium of the game with secret report with* $\theta^*_{sr} \leq \theta^*$.

Proof. We have

$$\begin{aligned} \psi(\theta^*) &= w\Delta s([\theta^*, 0], 1) - \gamma\theta^* \\ &= \frac{p(1-p)\left(\bar{p} - \underline{p}\right)}{\hat{p}\left(1 - \hat{p}F(\theta^*)\right)} - \gamma\theta^*. \end{aligned}$$

And

$$\begin{aligned} \phi(\theta^*) &= w\Delta p([\theta^*, 0]) - \gamma\theta^* \\ &= \frac{(1-p)\, p\left(\bar{p} - \underline{p} + 2\overline{p}\underline{p}\right)}{\hat{p}\left(1 - \hat{p}F(\theta^*) - \hat{p}\left(1 - F(0)\right)\right)} - \gamma\theta^*. \end{aligned}$$

This is straightforward that

$$\psi(\theta^*) < \phi(\theta^*).$$

There are three cases:

1. Either $\beta \geq \phi(\max(\underline{\theta}, \theta_0))$, so that misreport of negative values greater than $\theta^* = \max(\underline{\theta}, \theta_0)$ is an equilibrium of the game with public report. In this case, we also have $\beta \geq \psi(\max(\underline{\theta}, \theta_0))$ and misreport of negative values greater than $\theta^*_{sr} = \max(\underline{\theta}, \theta_0)$ is an equilibrium of the game with secret report.

2. Either $\beta < \min_\theta \phi(\theta)$, so that full reporting of negative values is the unique equilibrium of the game with public report.

3. Either $\min_\theta \phi(\theta) \leq \beta < \phi(\max(\underline{\theta}, \theta_0))$, so that $\phi(\theta^*)$ is non increasing for all $\theta^* \in [\max(\underline{\theta}, \theta_0), Arg \min_\theta \phi(\theta)]$. In this case, as $\psi(\theta^*) < \phi(\theta^*)$, there always exists θ^*_{sr} such that $\theta^*_{sr} \leq \theta^*$, $\beta = \phi(\theta^*)$ and $\beta = \psi(\theta^*_{sr})$.

■

This result is quite interesting because it states that when one moves from public disclosure of reports to secret reports, equilibria in which the experts lie more arise. One might thus conjecture that public disclosure of the expert's report is better in terms of information transmission. But, this need not be the case for all equilibria.

However, if we only consider the equilibrium of the game with public report in which the expert always lie as much as possible and the one in which he reports the truth as often as possible, the function $\phi(.)$ is decreasing. As noted above, those equilibria are the most studied in order to cope with multiplicity in the information transmission literature. As the function ψ is smaller than the function ϕ, this implies that in all equilibria, the expert will report the truth more often when reports are public than when reports are secret.

We can thus reasonably state that public disclosure of reports is optimal in order to attain an efficient decision making.

However, there exist other equilibria in which this is not the case. This is stated in the following corollary.

Corollary 16 *There exist equilibria of the games with public and secret reports such that* $\theta^*_{sr} > \theta^*$.

Proof. The proof is straightforward and is thus omitted. ■

4.2 Consultation of Inside experts

Let us now consider as before the role of and inside expert. In the case of secret report, when $q > 0$, the outside expert having signal $\theta < 0$ will report truthfully his signal when

$$w \Pr(\alpha = \overline{\alpha}|y = 0) \geq$$
$$(1 - p_I)\, q\, [\beta + w \Pr(\alpha = \overline{\alpha}|y = 1) + \gamma\theta] + (1 - (1 - p_I)\, q)\, w \Pr(\alpha = \overline{\alpha}|y = 0)$$

or

$$(1 - p_I)\, q\, [\beta + \gamma\theta + w\, (\Pr(\alpha = \overline{\alpha}|y = 1) - \Pr(\alpha = \overline{\alpha}|y = 0))] \leq 0$$
$$\Leftrightarrow \beta + \gamma\theta \leq w\, (\Pr(\alpha = \overline{\alpha}|y = 0) - \Pr(\alpha = \overline{\alpha}|y = 1))$$

We see that the situation is different than with the case of a public contract. Because the reputation gains are directly attached to the decision and not the report, the incentive to report truthfully are unchanged at fixed market's reactions. What changes now is the market information. Consider the case $q = 1$, while a decision $y = 0$ can only occur with an informed outside expert when there is no inside expert, this is not the case when there is an inside expert. Moreover, when the DA consults an inside expert, she makes less often the decision $y = 1$. Indeed, when the outside expert misreports ($\theta \in \Theta_{sr}$) in order to reach decision $y = 0$, there is a positive probability that the inside expert reports the truth and moves the decision to $y = 1$. Thus, in the case of secret reports, reputation effects are smaller when an inside expert is consulted:

$$w\left[\Pr(\alpha = \overline{\alpha}|y = 0) - \Pr(\alpha = \overline{\alpha}|y = 1)\right] < w\Delta s(\Theta_{sr}, 1).$$

The formal derivation is made in the appendix.

The meaning of this result is that, in all pure strategy equilibria, hiring an inside expert raises the outside expert's incentives to withhold information when it is unfavorable.

We thus obtain that

Proposition 17 *When experts' reports are secret and $q = 1$, hiring an inside expert reduces the outside expert's incentives to report truthfully.*

We see that the result is reversed compared to the case of public report.

Moreover, if we use the same notations than in the section with public reports, one can remark that the term capturing the changes in the set Θ_{sr}, V_I^1, is negative. Indeed, the set of signals for which the informed outside expert decides to report no information is now larger when the DA consults inside experts. Specifically, we have $V_I < V_I^2$. Hiring an inside expert is therefore less attractive if the reports are secret compared to the case where they are public. Intuitively, in the case of public reports an inside expert may not only change the decision but may also reduce the outside expert's reputation. When experts' reports are secret, the inside expert increases the reputation of the outside expert in making a report which changes the decision. This allows an outside expert to misreport for lower values of efficiency gains since his reputation is not affected even though the inside expert's report may contradict him.

5 Conclusion

This paper examines the efficiency of information transmission between some experts an a decision maker under different informational features. We first show that hiring experts biased against the status quo always improves the efficiency of the decision. Indeed, if the decision maker were to choose a decision without any advices, an expert biased against this decision always report the truth as this increases his reputation. The second main result is that public disclosure of the expert's report enhances information transmission. Intuitively, this makes misreport more costly in terms of the expert's reputation. We also analyze the impact of an inside, unbiased expert on an outside, biased expert reporting strategy. When the experts' reports are public, we prove that the presence of the inside expert forces the outside one to report more truthfully. However, this result is reversed in the case of secret reports. The intuitive result that hiring an inside expert constrain other experts to reveal the truth is therefore only true when there is public disclosure of reports.

In this paper, the private benefits received by the experts when their favorite decision is reached are common knowledge. However it is also interesting to see what happens if those benefits are private information for the experts. It should be more difficult for the authority to anticipate misreport from a biased expert. Moreover, a natural question is whether transparency about the experts' bias is desirable for the authority.

Another extension is to build a dynamic version of this model. As experts care about their reputation, it should be interesting to analyze repeated interactions between them and the decision maker. This may induce a biased expert to report truthfully in order to enhance his reputation for having a strong expertise and in this way being consulted in future important decisions.

6 Appendix

Gain of reputation when report is public. We will first compute the gain of reputation when the report is public and when the expert announces an informative report, $\Delta p(\Theta)$:

$$\begin{aligned}
\Pr(\emptyset|\overline{\alpha}) &= 1-\overline{p}+\overline{p}\Pr(\Theta) \\
\Pr(\emptyset|\underline{\alpha}) &= 1-\underline{p}+\underline{p}\Pr(\Theta) \\
\Pr(\overline{\alpha}|\emptyset) &= \frac{p\left[1-\overline{p}\right]+p\overline{p}\Pr(\Theta)}{p\left[1-\overline{p}\right]+(1-p)\left[1-\underline{p}\right]+\left(p\overline{p}+(1-p)\underline{p}\right)\Pr(\Theta)}
\end{aligned}$$

$$\begin{aligned}
\Pr(\text{info}|\overline{\alpha}) &= \overline{p}\left[1-\Pr(\Theta)\right] \\
\Pr(\text{info}|\underline{\alpha}) &= \underline{p}\left[1-\Pr(\Theta)\right] \\
\Pr(\bar{\alpha}|\text{info}) &= \frac{p\overline{p}}{p\overline{p}+(1-p)\underline{p}}
\end{aligned}$$

This gives

$$\begin{aligned}
\Delta p(\Theta) &= \frac{p\overline{p}}{p\overline{p}+(1-p)\underline{p}}-\frac{p\left[1-\overline{p}\right]+p\overline{p}\Pr(\Theta)}{p\left[1-\overline{p}\right]+(1-p)\left[1-\underline{p}\right]+\left(p\overline{p}+(1-p)\underline{p}\right)\Pr(\Theta)} \\
&= (1-p)\,p\frac{\bar{p}-\underline{p}+2\overline{p}\underline{p}}{\hat{p}\left(1-\hat{p}\left(1-\Pr(\Theta)\right)\right)}
\end{aligned}$$

where $\hat{p}=(1-p)\underline{p}+p\overline{p}$ is the ex-ante probability of the expert being informed ■

Gain of reputation when report is secret. The general formulas for the gain of reputation when the report is secret and when the DA's decision is $y = 0$, $\Delta s(\Theta_{sr}, q)$:

$$\Delta s(\Theta_{sr}, 1) = \frac{p(1-p)\left(\bar{p} - \underline{p}\right)}{\hat{p}\left(1 - \hat{p}F(\theta^*_{sr})\right)}.$$

Notice that $\Delta s(\Theta_{sr}, 1) < \Delta p(\Theta_{sr})$.

Define

$$H(\theta^*_{sr}, q) = qF\left(\theta^*_{sr}\right) - (1-q)\left\{1 - F(0)\right\}$$

We have

$$\begin{aligned}
\Delta s(\Theta_{sr}, q) &= \frac{p(1-p)\left(\Pr\left(0|\bar{\alpha}\right) - \Pr\left(0|\underline{\alpha}\right)\right)}{\Pr\left(0\right)\Pr\left(1\right)} \\
\Pr\left(0|\bar{\alpha}\right) &= \bar{p}F\left(\theta^*_{sr}\right) + (1-q)\left\{1 - \bar{p} + \bar{p}\left[F(0) - F\left(\theta^*_{sr}\right)\right]\right\} \\
\Pr\left(0|\bar{\alpha}\right) - \Pr\left(0|\underline{\alpha}\right) &= \left(\bar{p} - \underline{p}\right)\left(qF\left(\theta^*_{sr}\right) - (1-q)\left\{1 - F(0)\right\}\right) \\
&= \left(\bar{p} - \underline{p}\right) H \\
\Pr\left(0\right) &= \hat{p}F(\theta^*_{sr}) + (1-q)\left[1 - \hat{p} + \hat{p}\left(F(0) - F(\theta^*_{sr})\right)\right] \\
&= 1 - q + \hat{p}H \\
\Pr\left(1\right) &= \hat{p}\left(1 - F(0)\right) + q\left[1 - \hat{p} + \hat{p}\left(F(0) - F(\theta^*_{sr})\right)\right] \\
&= q - \hat{p}H
\end{aligned}$$

Thus

$$\Delta s(\Theta_{sr}, q) = \frac{p(1-p)\left(\bar{p} - \underline{p}\right) H}{\left(1 - q + \hat{p}H\right)\left(q - \hat{p}H\right)}$$

And

$$\begin{aligned}
\Delta s(\Theta_{sr}, 1) &= \frac{p(1-p)\left(\bar{p} - \underline{p}\right)}{\hat{p}\left(1 - \hat{p}F(\theta^*_{sr})\right)} > 0 \\
\Delta s(\Theta_{sr}, 0) &= wp(1-p)\left(\bar{p} - \underline{p}\right)\frac{-\left(1 - F(0)\right)}{\left(1 - \hat{p}\left(1 - F(0)\right)\right)\left(\hat{p}\left(1 - F(0)\right)\right)} < 0
\end{aligned}$$

This allows us to compute the following derivatives

$$\begin{aligned}
\frac{\frac{\partial(\Delta s(\Theta_{sr}, q))}{\partial \theta^*_{sr}}}{\Delta s(\Theta_{sr}, q)} &= qf(\theta^*_{sr})\left(\frac{1}{H} - \frac{\hat{p}}{1 - q + \hat{p}H} + \frac{\hat{p}}{q - \hat{p}H}\right) \\
&= qf(\theta^*_{sr})\left(\frac{q(1 - \hat{p}H) + (\hat{p}H)^2}{H\left(1 - q + \hat{p}H\right)\left(q - \hat{p}H\right)}\right) > 0
\end{aligned}$$

$$
\begin{aligned}
\frac{\frac{\partial(\Delta s(\Theta_{sr},q))}{\partial q}}{\Delta s(\Theta_{sr},q)} &= \left(\frac{H_q}{H}+\frac{1-\hat{p}H_q}{1-q+\hat{p}H}-\frac{1-\hat{p}H_q}{q-\hat{p}H}\right) \\
&= H_q\left(\frac{q(1-\hat{p}H)+(\hat{p}H)^2}{H\left(1-q+\hat{p}H\right)\left(q-\hat{p}H\right)}\right)+\frac{1}{1-q+\hat{p}H}-\frac{1}{q-\hat{p}H} \\
&= H_q\left(\frac{q(1-\hat{p}H)+(\hat{p}H)^2}{H\left(1-q+\hat{p}H\right)\left(q-\hat{p}H\right)}\right)+\frac{2(q-\hat{p}H)-1}{\left(1-q+\hat{p}H\right)\left(q-\hat{p}H\right)} \\
&= \frac{H_q\left(q(1-\hat{p}H)+(\hat{p}H)^2\right)+H\left(2(q-\hat{p}H)-1\right)}{H\left(1-q+\hat{p}H\right)\left(q-\hat{p}H\right)} \\
&= \frac{H_q\left(q-q\hat{p}H+(\hat{p}H)^2\right)+H\left(2q-2\hat{p}H-1\right)}{H\left(1-q+\hat{p}H\right)\left(q-\hat{p}H\right)} \\
&= \frac{(1-F(0))+\left(-\hat{p}H_q+2\right)H\left(q-\hat{p}H\right)}{H\left(1-q+\hat{p}H\right)\left(q-\hat{p}H\right)}>0
\end{aligned}
$$

■

Convexity of the functions $\phi(\theta)$ and $\psi(\theta)$. We now show that those functions are convex. Let's remind that $f(\theta)$ is non-decreasing on $\theta<0$

$$
\begin{aligned}
\psi(\theta) &= w\frac{p(1-p)\left(\bar{p}-\underline{p}\right)}{\hat{p}\left(1-\hat{p}F(\theta)\right)}-\gamma\theta \\
\psi'(\theta) &= w\frac{p(1-p)\left(\bar{p}-\underline{p}\right)}{\left(1-\hat{p}F(\theta)\right)^2}f(\theta)-\gamma \\
\psi''(\theta) &= w\frac{p(1-p)\left(\bar{p}-\underline{p}\right)}{\left(1-\hat{p}F(\theta)\right)^2}\left(f'(\theta)+\frac{2\hat{p}f(\theta)^2}{1-\hat{p}F(\theta)}\right)
\end{aligned}
$$

$$
\begin{aligned}
\phi(\theta) &= \frac{\phi(0)}{\left(1+l\left(F(0)-F\left(\theta\right)\right)\right)}-\gamma\theta \\
\phi'(\theta) &= \frac{\phi(0)}{\left(1+l\left(F(0)-F\left(\theta\right)\right)\right)^2}lf(\theta)-\gamma \\
\phi''(\theta) &= \frac{\phi(0)}{\left(1+l\left(F(0)-F\left(\theta\right)\right)\right)^2}l\left(f'(\theta)+\frac{2}{1+l\left(F(0)-F\left(\theta\right)\right)}lf(\theta)^2\right)
\end{aligned}
$$

■

Proof of Proposition 17. Assume $q=1$. Let's compute the probabilities that the market anticipates that an outside expert is a good one given the decision.

Without inside experts, this gives for decision $y=0$:

$$
\begin{aligned}
\Pr\left(0|\bar{\alpha}\right) &= \bar{p}F\left(\theta\right) \\
\Pr\left(0\right) &= \hat{p}F(\theta) \\
\Pr\left(\bar{\alpha}|0\right) &= \frac{p\bar{p}}{\hat{p}}.
\end{aligned}
$$

And, for decision $y = 1$:

$$\begin{aligned}\Pr(1|\bar{\alpha}) &= \bar{p}\left[1 - F(\theta)\right] + (1 - \bar{p}) \\ \Pr(1) &= 1 - \hat{p}F(\theta) \\ \Pr(\bar{\alpha}|1) &= \frac{p\left(1 - \bar{p}F(\theta)\right)}{1 - \hat{p}F(\theta)}.\end{aligned}$$

When an inside expert is consulted, we have for decision $y = 0$:

$$\begin{aligned}\Pr_I(0|\bar{\alpha}) &= \bar{p}F(\theta) + (1 - \bar{p})\, p_I\left[F(0) - F(\theta)\right] \\ \Pr_I(0) &= \hat{p}F(\theta) + (1 - \hat{p})\, p_I\left[F(0) - F(\theta)\right] \\ \Pr_I(\bar{\alpha}|0) &= p\left[\frac{\bar{p}F(\theta) + (1 - \bar{p})\, p_I\left[F(0) - F(\theta)\right]}{\hat{p}F(\theta) + (1 - \hat{p})\, p_I\left[F(0) - F(\theta)\right]}\right].\end{aligned}$$

Finally, for decision $y = 1$:

$$\begin{aligned}\Pr_I(1|\bar{\alpha}) &= 1 - \bar{p}F(\theta) - p_I\left[F(0) - \bar{p}F(\theta)\right] \\ \Pr_I(1) &= 1 - \hat{p}F(\theta) - p_I\left[F(0) - \hat{p}F(\theta)\right] \\ \Pr_I(\bar{\alpha}|1) &= p\left[\frac{1 - \bar{p}F(\theta) - p_I\left[F(0) - \bar{p}F(\theta)\right]}{1 - \hat{p}F(\theta) - p_I\left[F(0) - \hat{p}F(\theta)\right]}\right].\end{aligned}$$

As $\bar{p} > \hat{p}$, one can easily show that $\Pr_I(\bar{\alpha}|0) < \Pr(\bar{\alpha}|0)$ and that $\Pr_I(\bar{\alpha}|1) > \Pr(\bar{\alpha}|1)$. This proves that the reputation effects are lower when an inside expert is consulted:

$$w\left[\Pr(\alpha = \overline{\alpha}|y = 0) - \Pr(\alpha = \overline{\alpha}|y = 1)\right] < w\Delta s(\Theta_{sr}, 1).$$

Finally one can state the result, when experts' reports are secret and $q = 1$, hiring an inside expert reduces the outside expert's incentives to report truthfully. ■

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