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**Abstract**

We analyse the effect of taxation in the online sport betting market. This market is characterized by its negligible marginal costs. Taxation can be on volume (General Betting Duty) or on gross profit (Gross Profit Tax). We model the two most popular online sport betting bets: fixed-odds and spread, as compared with another traditional sport betting: parimutuel.

**Keywords:** taxation, online betting market, sport betting, bookmaker.

**JEL classification:** C72, D42, L83

## 1 Introduction

Over the last years, many European countries have been regulating their online betting and gaming sector. However, this regulation has not been

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done in a uniform way throughout the different countries.

In general, the basic taxation scheme is based on two types of taxes: the General Betting Duty (GBD) is levied as a proportion of betting stakes; whereas the Gross Profits Tax (GPT) is levied as a proportion of the net revenue of the operators.

Some examples: the United Kingdom applied a 6.75% tax on GBD until October 2001, when it was replaced by a 15% tax in GPT (National Audit Office, 2005). Italy applies a 2%-5% tax on GBD (Ficom Leisure, 2011) for general sport betting and a 20% tax on GPT for spread bets (PwC, 2011). France applies a 8.5% tax on GBD (Global Betting and Gaming Consultancy, 2011) since 2010. In Germany, tax rates largely depend on the respective federal state, and they vary between 20% and 80% on GPT plus a 5% federal tax on GBD (Hofmann and Spitz, 2015). In 2011, Spanish authorities approved a law[1] that applies a 25% tax on GPT for some types of bets and a 15-22% tax on GBD for others, plus a 0.1% tax on GBD. In the next table we summarize the data:

| | GBD | GPT |
|---|---|---|
| UK | 6.75% (until 2001) | 15% (since 2001) |
| Italy | 2-5% (general) | 20% (spread) |
| France | 8.5% | - |
| Germany | 5% | 20-80% |
| Spain | 25% (parimutuel)+0.1% (all) | 15-22% (general) |

In the Spanish case, GBD has been the taxation scheme in the most traditional offline sport betting (*la quiniela*), which takes a parimutuel structure.

In a parimutuel market, a winning bet pays off a proportional share of the total stake on all outcomes. However, the most popular online sport operators are specialized in another two markets: Fixed-odds and spread. In a fixed-odd market, the operator sets the odds for each possible outcome of

[1]Ley 13/2011, de 27 de mayo, de regulación del juego (in Spanish). Boletín Oficial del Estado 127(I), 52976-53022. Available at `http://www.boe.es/boe/dias/2011/05/28/pdfs/BOE-A/2011/9280.pdf`

the match, and the bettors decide whether they accept or not these odds. In a spread market, the operator acts as an intermediator among the users, whom bargain the odds.

For sport matches, a bet of 1 monetary unit on a particular team yields a return of $\frac{1}{\pi}$ monetary units in case the team wins the match, and 0 otherwise. In this context, an odd $\pi \in (0,1)$ is defined as the probability assigned by the market. Notice that any risk-neutral bettor would find it profitable to bet at odd $\pi$ when her private probability estimation is higher than $\pi$.

In parimutuel and spread bets, the operator's profit comes from a commission on either the amount at risk or the winning amount (typically around 5% in online spread operators). In fixed-odds bets, the operator's profit comes from the odds, which should sum up more than 100%[2] for all the possible outcomes of the sport match[3].

In this paper, we model the three types of market in a general setting. The regulator decides on the general taxing scheme (either GBD or GPT) and the operators decide on their commission (parimutuel and spread operators) or odds (fixed-odds operators). We assume that the spread betting commission is applied to the winning bets (as it is typical in online spread operators), whereas commission in parimutuel betting applies to the total amount (as in the Spanish regulation).

We show that, from the online bettor's point of view, it is preferable a GPT scheme, in the following sense: In equilibrium, the odds are not affected by the taxation under GPT; whereas a GBD scheme would reduce the odds and hence the bettors' utility. As opposed, in the parimutuel market GPT and GBD provide the same effect.

These results agree with the ones presented by Smith (2000) and Paton et al. (2002, 2001), whom analyse the effect of the different taxation schemes

[2]In case the odds summed up less than 100%, it would be possible, by betting an appropriate amount of money on each possible match outcome, to win a positive amount irrespectively of the final match outcome.

[3]The sum of the odds, called *overround*, provides a way to measure the operator advantage.

in Australian, UK and USA betting markets. These results, however, are more focused on offline betting operators and government revenue. Moreover, they take into account the marginal cost of each bet. As opposed, we assume that these marginal costs are negligible.

There are other works that focus on parimutuel markets: Ottaviani and Sørensen (2009) provide a model that explains the empirical evidence of underdogs overbet. These authors argue that this bias may be due to privately informed bettors. As opposed, we prove (Corollary 3.1) that the spread bet operator would get a higher profit if the underdog wins the game.

Other works concentrate on fixed-odd markets. For example, Bag and Saha (2011, 2016) study the externalities due to bribery in sports; and Levitt (2004) argues that the operators may achieve higher profits by an accurate prediction of the match outcome.

As far as we know, no similar research has been addressed for spread markets.

The rest of the paper is organized as follows. In Section 2 we provide the model. In Section 3, we study the equilibrium payoffs in each of the three markets and provide the main results. In Section 4, we briefly compare the payoffs in the different markets. In Section 5, we present some concluding remarks.

## 2 The model

Two teams (*home* and *away*) play a competitive sport match; the match being drawn is not a possibility.

There are three types of agents in the model: A continuum set $\mathcal{B}$ of *bettors* are interested in betting, but only if the odds are attractive; a *bookmaker* that offers bets; and a *regulator* (Government) that decides on taxes.

We assume that bettors are risk neutral and try to maximize their expected profit. Each bettor $i \in \mathcal{B}$ is characterized by her individual belief (i.e. the probability) $x_i$ that the home team wins ($1 - x_i$ is the probability that the away team wins); $x_i$ is distributed following an absolutely continuous cdf with probability density function $f$ and full support over $(0, 1)$.

The bookmaker, on the other side, also wants to maximize her own profit but she does not have any belief on the true probability for the home team to win. Hence, it is not possible to estimate an expected profit for her. Instead, we assume that the bookmaker tries to maximize her monetary profit under the worst possible outcome of the match.

There are three possible types of bookmakers: Fixed-odd bookmakers, spread bookmakers and parimutuel bookmakers. Fixed-odd bookmakers decide odds $\pi_H, \pi_A \in [0, 1]$ such that any bettor that bets on the home (away) team receives $\frac{1}{\pi_H} - 1$ ($\frac{1}{\pi_A} - 1$) in case of home (away) win, and $-1$ in case of away (home) win. Spread bookmakers decide a commission $c$ on the profit of any winning bettor. Parimutuel bookmakers decide a commission $c$ on the stake of any bettor.

# 3 The non-cooperative game

The study the effect of taxation in the online market. Assume the regulator announces a tax, that could be a percentage $\upsilon$ on volume (GBD), a percentage $\rho$ on gross profit (GPT), or both. The non-cooperative game has two steps:

Step 1 The bookmaker observes $\upsilon$ and $\rho$ and announces her odds (fixed-odds) or commission (spread/parimutuel). Let $s_k$ denote this choice.

Step 2 Each bettor $i \in \mathcal{B}$ observes $s_k$, and choose to participate or not. Let $s_i(s_k)$ denote this choice.

Following Neyman (2002), we assume that, for any bettors' strategy profile, the set $\mathcal{C}$ of bettors that give any particular signal is always Borel-medible[4], and we denote its volume as $\|\mathcal{C}\|$.

[4]This is done in order to avoid meaningless strategies such as, for example, to bet iff $x_i$ is a rational number.

Given an admissible strategy profile $s = (s_k, (s_i(\cdot))_{i\in\mathcal{B}})$, we denote as $u(s) \in \mathbb{R}^{\{k\}\cup\mathcal{B}}$, or simply $u$, the final payoff allocation of the noncooperative game.

We will work with the standard concept of subgame perfect equilibrium and a natural extension of it, named strong subgame perfect equilibrium. Notice that the only proper subgames arise in Step 2..

**Definition 3.1** *A strategy profile $s = (s_k, (s_i(\cdot))_{i\in\mathcal{B}})$ is a* subgame perfect equilibrium *if two conditions hold:*

1. *For each $i \in \mathcal{B}$, each bookmaker's strategy $\widetilde{s}_k$ and all bettor i's strategy $\widetilde{s}_i(\cdot)$,*

$$u_i\left(\widetilde{s}_k, \left(s_j\left(\widetilde{s}_k\right)\right)_{j\in\mathcal{B}\setminus\{i\}}, \widetilde{s}_i\left(\widetilde{s}_k\right)\right) \leq u_i\left(\widetilde{s}_k, \left(s_j\left(\widetilde{s}_k\right)\right)_{j\in\mathcal{B}}\right).$$

2. *For all bookmaker's strategy $\widetilde{s}_k$,*

$$u_k\left(\widetilde{s}_k, \left(s_i\left(\widetilde{s}_k\right)\right)_{i\in\mathcal{B}}\right) \leq u_k\left(s_k, \left(s_i\left(s_k\right)\right)_{i\in\mathcal{B}}\right).$$

**Definition 3.2** *A strategy profile $s = (s_k, (s_i(\cdot))_{i\in\mathcal{B}})$ is a* strong subgame perfect equilibrium *if two conditions hold:*

1. *For each $\mathcal{C} \subset \mathcal{B}$, all bookmaker's strategy $\widetilde{s}_k$ and all strategy profile $(\widetilde{s}_i(\cdot))_{i\in\mathcal{C}}$,*

$$u_i\left(\widetilde{s}_k, \left(s_j\left(\widetilde{s}_k\right)\right)_{j\in\mathcal{B}\setminus\mathcal{C}}, \left(s_j\left(\widetilde{s}_k\right)\right)_{j\in\mathcal{C}}\right) \leq u_i\left(\widetilde{s}_k, \left(s_j\left(\widetilde{s}_k\right)\right)_{j\in\mathcal{B}}\right)$$

   *for all $i \in \mathcal{C}$.*

2. *For all bookmaker's strategy $\widetilde{s}_k$,*

$$u_k\left(\widetilde{s}_k, \left(s_i\left(\widetilde{s}_k\right)\right)_{i\in\mathcal{B}}\right) \leq u_k\left(s_k, \left(s_i\left(s_k\right)\right)_{i\in\mathcal{B}}\right).$$

## 3.1 Fixed-odds bookmakers

In the fixed-odd case, the bookmaker chooses odds $\pi_H$ and $\pi_A$, i.e. $s_k = (\pi_H, \pi_A) \in [0,1] \times [0,1]$. Each bettor $i \in \mathcal{B}$ observes the odds and chooses $s_i(\pi_H, \pi_A) \in \{D, H, A\}$ with the following interpretation:

- If $s_i(\pi_H, \pi_A) = D$, bettor $i$ declines to bet (abstains) and her final payoff is zero.
- If $s_i(\pi_H, \pi_A) = H$, bettor $i$ bets for the home team at odd $\pi_H$ and her final payoff is
$$u_i = \left(\frac{1}{\pi_H} - 1\right) x_i + (-1)(1 - x_i).$$
- If $s_i(\pi_H, \pi_A) = A$, bettor $i$ bets for the away team at odd $\pi_A$ and her final payoff is
$$u_i = (-1) x_i + \left(\frac{1}{\pi_A} - 1\right)(1 - x_i).$$

Let $\mathcal{B}_H = \{i \in \mathcal{B} : s_i(\pi_H, \pi_A) = H\}$ and $\mathcal{B}_A = \{i \in \mathcal{B} : s_i(\pi_H, \pi_A) = A\}$, and let $h = ||\mathcal{B}_H||$ and $a = ||\mathcal{B}_A||$ be their respective volumes. Then, the bookmaker's final payoff is

$$\begin{aligned} u_k &= (1-\rho) \min \left\{ (1-\upsilon)(h+a) - \frac{1}{\pi_H} h, (1-\upsilon)(h+a) - \frac{1}{\pi_A} a \right\} \\ &= (1-\rho) \left( (1-\upsilon)(h+a) - \max \left\{ \frac{1}{\pi_H} h, \frac{1}{\pi_A} a \right\} \right). \end{aligned}$$

The next result characterizes the (strong) subgame perfect equilibrium in the fixed-odds case:

**Theorem 3.1** *Given $\upsilon$ and $\rho$, there exists a (strong) subgame perfect equilibrium in the fixed-odds noncooperative game. Moreover, the odds in equilibrium are characterized by the maximization problem*

$$\max (1-\rho) \left\{ (1-\upsilon) \left[ \int_{\pi_H}^{1} f(t)\, dt + \int_{0}^{1-\pi_A} f(t)\, dt \right] - \frac{1}{\pi_H} \int_{\pi_H}^{1} f(t)\, dt \right\}$$

*subject to*

$$\frac{1}{\pi_H}\int_{\pi_H}^{1} f(t)\,dt = \frac{1}{\pi_A}\int_{0}^{1-\pi_A} f(t)\,dt \tag{1}$$

$$\pi_H, \pi_A \in [0,1], \pi_H + \pi_A \geq 1.$$

**Proof.** Assume w.l.o.g. $\|\mathcal{B}\| = 1$. In Step 2 of the game, it is optimal for any bettor $i \in \mathcal{B}$ with $x_i > \pi_H$ to announce $s_i(\pi_H, \pi_A) = H$. Analogously, it is optimal for any bettor $i \in \mathcal{B}_k$ with $1 - x_i > \pi_A$ to announce $s_i(\pi_H, \pi_A) = A$. Hence, $h = \int_{\pi_H}^{1} f(t)\,dt$ and $a = \int_{0}^{1-\pi_A} f(t)\,dt$. Equality (1) comes from the fact that the bookmaker wants to minimize $\max\left\{\frac{1}{\pi_H}h, \frac{1}{\pi_A}a\right\}$, that is the maximum bettors' winnings when either the home team ($\frac{1}{\pi_H}h$) or the away team ($\frac{1}{\pi_A}a$) wins. Hence, in equilibrium both amounts should be equal. For a fixed $\pi_A$, there exists a unique $\pi_H$ that satisfies (1). To see why, notice that $\phi(\pi) = \frac{1}{\pi}\int_{\pi}^{1} f(t)\,dt$ is a strictly decreasing function on $\pi \in (0,1)$ with $\phi(0^+) = +\infty$ and $\phi(1^-) = 0^+$, whereas $\psi(\pi) = \frac{1}{1-\pi}\int_{0}^{\pi} f(t)\,dt$ is a strictly increasing function on $\pi \in (0,1)$ with $\psi(0^+) = 0^+$ and $\psi(1^-) = +\infty$. Hence, for each $\pi_H$, there exists a unique $\pi_A$ with $\phi(\pi_H) = \psi(1 - \pi_A)$. Moreover, the largest $\pi_H$ is, the largest $\pi_A$ is. Let $\Pi_A : (0,1) \to (0,1)$ be the function that assigns to each $\pi_H$ its corresponding $\pi_A$. This function is well-defined, strictly increasing, and it satisfies $\Pi_A(0^+) = 0^+$ and $\Pi_A(1^-) = 1^-$. Notice that the bet volume is given by $h + a$. Since tax $\upsilon$ applies on volume, and $\rho$ applies on profit, the bookmaker would maximize

$$(1-\rho)\left\{(1-\upsilon)(h+a) - \frac{1}{\pi_H}h\right\}$$

which is the desired maximizing function. Furthermore, it is straightforward to check that there exists at least one maximizing $\pi_H \in (0,1)$. ■

From the previous result, we see that the bookmaker looks to balance the positive effect of a large volume (given by $\int_{\pi_H}^{1} f(t)\,dt + \int_{0}^{1-\pi_A} f(t)\,dt$) against the negative effect of a big prize (given by $\frac{1}{\pi_H}\int_{\pi_H}^{1} f(t)\,dt = \frac{1}{\pi_A}\int_{0}^{1-\pi_A} f(t)\,dt$). A large volume is obtained by setting low $\pi_H$ (and hence low $\pi_A$). A low prize is obtained by setting high $\pi_H$ (and hence high $\pi_A$).

The effect of $\rho$ (tax on profit) is irrelevant for the maximization problem. Hence, the optimal $\pi_H$ and $\pi_A$ are independent of the chosen $\rho$. A different issue happens with $\upsilon$, which gives less weight to the positive effect of a large volume. This suggests that the bookmaker would set a high $\pi_H$ (and high $\pi_A$), which means that the utility of the bettors is reduced.

## 3.2 Spread bookmakers

In the spread case, the bookmaker chooses commission $c \in (0, 1)$, i.e. $s_k = c \in (0, 1)$. Each bettor $i \in \mathcal{B}$ observes $c$ and chooses $s_i(c) \in \{D\} \cup \{H, A\} \times (0, 1)$ with the following interpretation:

- If $s_i(c) = D$, bettor $i$ declines to bet and her final utility is zero.
- If $s_i(c) = (H, \pi_H)$, bettor $i$ declares that she wants to bet for the home team at odd at most $\pi_H$.
- If $s_i(c) = (A, \pi_A)$, bettor $i$ declares that she wants to bet for the away team at odd at most $\pi_A$.

The bookmaker matches $(H, \pi_H)$-bettors with $(A, \pi_A)$-bettors that satisfy $\pi_H \geq 1-\pi_A$ with odds $\pi$, $1-\pi$ such that: $\pi \leq \pi_H$ and $1-\pi \leq \pi_A$. The matching is done in such a way that each $\pi$ volume of $(H, \pi_H)$-bettors is matched with a $1 - \pi$ volume of $(A, \pi_A)$-bettors. The reason is that, in case home team wins, a $1 - \pi$ volume of money is transferred from $(A, \pi_A)$-bettors to $(H, \pi_H)$-bettors, so that each $(H, \pi_H)$-bettor receives a gross winning (profit + bet):

$$\frac{1-\pi}{\pi} + 1 = \frac{1}{\pi} \geq \frac{1}{\pi_H}$$

hence granting their request to bet for the home team at odd at least $\pi_H$.

Analogously, in case away team wins, a $\pi$ volume of money is transferred from $(H, \pi_H)$-bettors to $(A, \pi_A)$-bettors, so that each $(A, \pi_A)$-bettor receives a gross winning (profit + bet):

$$\frac{\pi}{1-\pi} + 1 = \frac{1}{1-\pi} \geq \frac{1}{\pi_A}$$

hence granting their request to bet for the away team at odd at least $\pi_A$.

Hence, $\pi$ is chosen so that

$$\begin{aligned}(1-\pi)\left\|\mathcal{B}_H \cup \mathcal{B}'_H\right\| &\geq \pi\left\|\mathcal{B}_A\right\| \\ \pi\left\|\mathcal{B}_A \cup \mathcal{B}'_A\right\| &\geq (1-\pi)\left\|\mathcal{B}_H\right\|\end{aligned}$$

where

$$\begin{aligned}\mathcal{B}_H &= \{i \in \mathcal{B} : s_i = (H, \pi_H), \pi_H > \pi\} \\ \mathcal{B}'_H &= \{i \in \mathcal{B} : s_i = (H, \pi)\} \\ \mathcal{B}_A &= \{i \in \mathcal{B} : s_i = (A, \pi_A), \pi_A > 1-\pi\} \\ \mathcal{B}'_A &= \{i \in \mathcal{B} : s_i = (A, \pi)\}.\end{aligned}$$

If $s_i = (H, \pi_H)$ with $\pi_H > \pi$, bettor $i$ bets for the home team at odd $\pi$ and her final payoff is

$$u_i = (1-c)\left(\frac{1}{\pi} - 1\right) x_i + (-1)(1-x_i). \tag{2}$$

If $s_i = (A, \pi_A)$ with $\pi_A > 1-\pi$, bettor $i$ bets for the away team at odd $1-\pi$ and her final payoff is

$$u_i = (-1)x_i + (1-c)\left(\frac{1}{1-\pi} - 1\right)(1-x_i). \tag{3}$$

If $s_i = D$, or $s_i = (H, \pi_H)$ with $\pi_H < \pi$, or $s_i = (A, \pi_A)$ with $\pi_A < 1-\pi$, bettor $i$ does not bet and her final payoff is zero.

When $s_i = (H, \pi)$ or $s_i = (A, 1-\pi)$, we have two cases:

**Case 1**: $\frac{\left\|\mathcal{B}_H \cup \mathcal{B}'_H\right\|}{\pi} \leq \frac{\left\|\mathcal{B}_A \cup \mathcal{B}'_A\right\|}{1-\pi}$. If $s_i = (H, \pi)$, then bettor $i$ bets for the home team and her final payoff is (2). If $s_i = (A, 1-\pi)$, then bettor $i$ bets for the away team with probability $p_A = \frac{1-\pi}{\pi}\frac{\left\|\mathcal{B}_H \cup \mathcal{B}'_H\right\|}{\left\|\mathcal{B}'_A\right\|} - \frac{\left\|\mathcal{B}_A\right\|}{\left\|\mathcal{B}'_A\right\|}$ and her final payoff is

$$u_i = \left[(-1)x_i + (1-c)\left(\frac{1}{1-\pi} - 1\right)(1-x_i)\right] p_A.$$

**Case 2**: $\frac{\left\|\mathcal{B}_H \cup \mathcal{B}'_H\right\|}{\pi} \geq \frac{\left\|\mathcal{B}_A \cup \mathcal{B}'_A\right\|}{1-\pi}$. If $s_i = (A, 1-\pi)$, then bettor $i$ bets for the away team and her final payoff is (3). If $s_i = (H, \pi)$, then bettor $i$ bets for the home team with probability $p_H = \frac{\pi}{1-\pi}\frac{\|\mathcal{B}_A \cup \mathcal{B}'_A\|}{\|\mathcal{B}'_H\|} - \frac{\|\mathcal{B}_H\|}{\|\mathcal{B}'_H\|}$ and her final payoff is

$$u_i = \left[(1-c)\left(\frac{1}{\pi}-1\right)x_i + (-1)(1-x_i)\right]p_H.$$

We describe this protocol in the following example:

**Example 3.1** *Assume* $\|\mathcal{B}\| = 1$ *and the bets are* $D$, $(H, 0.4)$, $(H, 0.6)$, $(H, 0.8)$, $(A, 0.2)$, $(A, 0.4)$, *and* $(A, 0.6)$ *with volumes* 0.2, 0.1, 0.3, 0.1, 0.1, 0.1, *and* 0.1, *respectively, as shown in the first two columns of the following table:*

| *Bet* | *Volume* | *Matched* |
|---|---|---|
| $D$ | 0.2 | *No (abstain)* |
| $(H, 0.4)$ | 0.1 | *No* |
| $(H, 0.6)$ | 0.3 | 67% |
| $(H, 0.8)$ | 0.1 | 100% |
| $(A, 0.2)$ | 0.1 | *No* |
| $(A, 0.4)$ | 0.1 | 100% |
| $(A, 0.6)$ | 0.1 | 100% |
| *Total* | 1 | 50% |

*Under these bets,* $\pi = 0.6$ *clears the market, so that the ratio of* $H$*-bettors to* $A$*-bettors should be* $\frac{0.6}{1-0.6} = \frac{3}{2}$. *Moreover,* $\|\mathcal{B}_H\| = 0.1$, $\|\mathcal{B}'_H\| = 0.3$, $\|\mathcal{B}_A\| = 0.1$, *and* $\|\mathcal{B}'_A\| = 0.1$. *Since* $\frac{\|\mathcal{B}_H \cup \mathcal{B}'_H\|}{\pi} = \frac{0.4}{0.6} > \frac{0.2}{0.4} = \frac{\|\mathcal{B}_A \cup \mathcal{B}'_A\|}{1-\pi}$, *we are in Case 2 and there exists an excess of* $H$*-bettors that will not be matched. In particular, the whole* 0.1 *volume of* $(H, 0.8)$*-bettors matches a* $\frac{0.2}{3}$ *volume of* $(A, 0.6)$*-bettors; a* 0.05 *volume of* $(H, 0.6)$*-bettors matches the remaining* $\frac{0.1}{3}$ *volume of* $(A, 0.6)$*-bettors; finally, a* 0.15 *volume of* $(H, 0.6)$*-bettors matches the remaining* 0.1 *volume of* $(A, 0.4)$*-bettors. The remaining* 0.1 *volume of* $(H, 0.6)$*-bettors, the* 0.1 *volume of* $(A, 0.2)$*-bettors, and the* 0.1 *volume of* $(H, 0.4)$*-bettors remain unmatched.*

We now compute the bookmaker's payoff. Analogously to the previous subsection, we denote $h = \|\mathcal{B}_H\|$, $h' = \|\mathcal{B}'_H\|$, $a = \|\mathcal{B}_A\|$, and $a' = \|\mathcal{B}'_A\|$.

Now, in case the home team wins, the monetary transfer from $A$-bettors to $H$-bettors is

$$\begin{aligned}\alpha &= \left(\frac{1}{\pi}-1\right)(h+\min\{1,p_H\}\,h') \\ &= \frac{1-\pi}{\pi}\min\{h+h',h+p_Hh'\} \\ &= \min\left\{\frac{1-\pi}{\pi}(h+h'),a+a'\right\}.\end{aligned}$$

Analogously, in case the away team wins, the monetary transfer from $H$-bettors to $A$-bettors is

$$\beta = \min\left\{\frac{\pi}{1-\pi}(a+a'),h+h'\right\}.$$

Then, the total volume is $\alpha+\beta$ and the final payoff for the bookmaker is

$$u_k = (1-\rho)\,(c\min\{\alpha,\beta\}-(\alpha+\beta)\upsilon)\,.$$

The next result characterizes the (strong) subgame perfect equilibria in the spread bets case:

**Theorem 3.2** *Given $\upsilon$ and $\rho$, there exists a (strong) subgame perfect equilibrium with undominated strategies*[5] *in the spread bets noncooperative game. Moreover, the commission and odds in equilibrium are characterized by the maximization problem*

$$\max_{c\in[0,1]}(1-\rho)\,(c\min\{1-\pi,\pi\}-\upsilon)\,\gamma$$

*subject to*

$$\gamma = \frac{1}{\pi}\int_{\frac{\pi}{1-(1-\pi)c}}^{1} f(t)\,dt = \frac{1}{1-\pi}\int_0^{\frac{(1-c)\pi}{1-\pi c}} f(t)\,dt \tag{4}$$

$$\pi, c \in [0,1]\,.$$

[5]Undominated strategies are required in order to avoid meaningless equilibria of the form "everybody chooses $D$".

**Proof.** Assume w.l.o.g. $\|\mathcal{B}\| = 1$. In Step 2 of the game, it is optimal for any bettor $i \in \mathcal{B}$ to bet for the home team at odds $\pi$ when $\pi$ is such that

$$(1-c)\left(\frac{1}{\pi}-1\right)x_i + (-1)(1-x_i) > 0$$

which is equivalent to:

$$x_i > \frac{\pi}{1-(1-\pi)c}.$$

Analogously, it is optimal for any bettor $i \in \mathcal{B}$ to bet for the away team at odd $1-\pi$ when $\pi$ is such that

$$(-1)x_i + (1-c)\left(\frac{1}{1-\pi}-1\right)(1-x_i) > 0$$

which is equivalent to:

$$x_i < \frac{(1-c)\pi}{1-\pi c}.$$

Then, the unique odd $\pi^c$ that clears the market is characterized as in (4) by

$$\gamma^c = \frac{1}{\pi^c}\int_{\frac{\pi^c}{1-(1-\pi^c)c}}^{1} f(t)\,dt = \frac{1}{1-\pi^c}\int_0^{\frac{(1-c)\pi^c}{1-c\pi^c}} f(t)\,dt. \tag{5}$$

To see that $\pi^c$ exists and it is unique, let $\phi, \psi : (0,1) \longrightarrow \mathbb{R}$ be two functions defined as $\phi(\pi) = \frac{1}{\pi}\int_{\frac{\pi}{1-(1-\pi)c}}^{1} f(t)\,dt$ and $\psi(\pi) = \frac{1}{1-\pi}\int_0^{\frac{(1-c)\pi}{1-c\pi}} f(t)\,dt$ for all $\pi \in (0,1)$, respectively. It is clear that $\phi$ is strictly decreasing with $\phi(0^+) = \infty$ and $\phi(1^-) = 0^+$, and that $\psi$ is strictly increasing with $\psi(0^+) = 0^+$ and $\psi(1^-) = \infty$. Hence, there exists a unique $\pi^c$ satisfying $\phi(\pi^c) = \psi(\pi^c)$. Moreover, a subgame perfect equilibrium strategy with undominated strategies is characterized as follows:

- Each bettor $i \in \mathcal{B}$ with $x_i > \frac{\pi^c}{1-(1-\pi^c)c}$ chooses $s_i(c) = (H, \pi^c)$.
- Each bettor $i \in \mathcal{B}$ with $x_i < \frac{\pi^c(1-c)}{1-c\pi^c}$ chooses $s_i(c) = (A, 1-\pi^c)$.
- Any other bettor $i \in \mathcal{B}$ chooses $s_i(c) = D$.

This strategy profile induces $\pi = \pi^c$, and it is a strong equilibrium because no set of bettors can modify $\pi$ in its own benefit, and any other equilibria will also satisfy $\pi = \pi^c$. In general,

$$\mathcal{B}_H \cup \mathcal{B}'_H = \mathcal{B}'_A = \left\{ i \in \mathcal{B} : x_i \geq \frac{\pi^c}{1 - c\,(1 - \pi^c)} \right\} \tag{6}$$

$$\mathcal{B}_A \cup \mathcal{B}'_A = \mathcal{B}'_A = \left\{ i \in \mathcal{B} : x_i \leq \frac{\pi^c\,(1 - c)}{1 - c\pi^c} \right\} \tag{7}$$

$$\frac{h + h'}{a + a'} = \frac{h'}{a'} = \frac{\pi^c}{1 - \pi^c}. \tag{8}$$

The (strong) subgame perfect equilibrium is then completely characterized in Step 1 by the maximization of the bookmaker's payoff:

$$\max_{c \in [0,1]} (1 - \rho)\,(c \min\{\alpha, \beta\} - (\alpha + \beta)\upsilon)$$

where

$$\begin{aligned} \alpha &= \min\left\{ \frac{1 - \pi^c}{\pi^c}\,(h + h')\,, a + a' \right\} \\ &\overset{(8)}{=} \frac{1 - \pi^c}{\pi^c}\,(h + h') \overset{(6)}{=} \frac{1 - \pi^c}{\pi^c} \int_{\frac{\pi^c}{1-(1-\pi^c)c}}^{1} f(t)dt \overset{(5)}{=} (1 - \pi^c)\gamma^c. \end{aligned}$$

Analogously,

$$\beta \overset{(8)(7)}{=} \frac{\pi^c}{1 - \pi^c} \int_0^{\frac{\pi^c(1-c)}{1-c\pi^c}} f(t)dt \overset{(5)}{=} \pi^c\gamma^c$$

from where the maximization problem becomes

$$\max_{c \in [0,1]} (1 - \rho)\,(c \min\{1 - \pi^c, \pi^c\} - \upsilon)\,\gamma^c.$$

■

As $c$ increases, the percentage of winners profits increase too, but this winner profit decreases because less bettors participate. Hence, the bookmaker looks to balance the positive effect of a big commission (hence big percentage of winnings) against the negative effect on the winnings (which decreases with $c$).

Like fixed-odds bookmakers, the effect of $\rho$ (tax on profit) is irrelevant for the maximization problem. Hence, the optimal $c$ is independent of the chosen $\rho$. Again, a different issue happens with $\upsilon$, which penalizes the effect of a large volume. Hence, like fixed-odds, the bookmaker would set a higher $c$, which means that the utility of the bettors is reduced.

As opposed to fixed-odds, the spread bookmaker is not indifferent to which team will win the match. In fact, the bookmaker would always prefer the underdog (non-favorite) to win the match, as next result shows:

**Corollary 3.1** *Let $\pi$, $1-\pi$ be the odds in equilibrium. If $\pi > \frac{1}{2}$, then the bookmaker's payoff is bigger when the away team wins. If $\pi < \frac{1}{2}$, then the bookmaker's payoff is bigger when the home team wins. If $\pi = \frac{1}{2}$, then the spread bookmaker's payoff is independent of which team wins.*

**Proof.** From Theorem 3.2, $h = \int_{\frac{\pi}{1-c(1-\pi)}}^{1} f(t)\,dt$ is the volume of bettors that bet for the home team, whereas $a = \int_{0}^{\frac{\pi(1-c)}{1-c\pi}} f(t)\,dt$ is the volume of bettors that bet for the away team. When the home team wins, the gross profit of the bookmaker is $\left(\frac{1}{\pi}-1\right)ch$. When the away team wins, the gross profit of the bookmaker is $\left(\frac{1}{1-\pi}-1\right)ca$. Moreover, we have $\frac{1}{\pi}h = \frac{1}{1-\pi}a$. Hence, the bookmakers prefers the home (away) team to win when $h < a$ ($h > a$). Equivalently, the bookmaker prefers the home (away) team to win when $\frac{h}{a} < 1$ ($\frac{h}{a} > 1$). Since $\frac{h}{a} = \frac{\pi}{1-\pi}$, it only happens when $\frac{\pi}{1-\pi} < 1$ ($\frac{\pi}{1-\pi} > 1$), i.e. $\pi < \frac{1}{2}$ ($\pi > \frac{1}{2}$). The result for $\pi = \frac{1}{2}$ is straightforward. ■

## 3.3 Parimutuel bookmakers

In the parimutuel case, the bookmaker chooses commission $c \in (0,1)$, i.e. $s_k = c \in (0,1)$. Each bettor $i \in \mathcal{B}$ observes $c$ and (simultaneously) chooses $s_i(c) \in \{D, H, A\}$ with the following interpretation. Let $\mathcal{B}_H = \{i \in \mathcal{B} : s_i = H\}$ and $\mathcal{B}_A = \{i \in \mathcal{B} : s_i = A\}$:

- If $s_i(c) = D$, bettor $i$ declines to bet and her final payoff is zero.

- If $s_i(c) = H$, bettor $i$ declares that she wants to bet for the home team. Her final payoff is:

$$u_i = \frac{\|\mathcal{B}_H \cup \mathcal{B}_A\|}{\|\mathcal{B}_H\|} (1-c)\, x_i - 1.$$

- If $s_i(c) = A$, bettor $i$ declares that she wants to bet for the away team. Her final payoff is:

$$u_i = \frac{\|\mathcal{B}_H \cup \mathcal{B}_A\|}{\|\mathcal{B}_A\|} (1-c)\,(1-x_i) - 1.$$

The bookmaker's final payoff is

$$u_k = (1-\rho)\,(1-\upsilon)\, \|\mathcal{B}_H \cup \mathcal{B}_A\|\, c.$$

The next result characterizes the (strong) subgame perfect equilibria in the parimutuel case:

**Theorem 3.3** *Given $\upsilon$ and $\rho$, there exists a (strong) subgame perfect equilibrium, where each bettor $i$ bets for the home team if $x_i > \pi_H$ and for the away team if $x_i < \pi_A$ for some thresholds $\pi_H, \pi_A \in [0,1]$. Moreover, the commission and thresholds in equilibrium are characterized by the maximization problem*

$$\max_{c \in \left[0,\frac{1}{2}\right]} (1-\rho)\,(1-\upsilon) \left[\int_{\pi_H}^{1} f(t)\, dt + \int_{0}^{1-\pi_A} f(t)\, dt\right] c$$

*subject to*

$$\begin{aligned} \frac{1}{\pi_H}\int_{\pi_H}^{1} f(t)\, dt &= \frac{1}{\pi_A}\int_{0}^{1-\pi_A} f(t)\, dt \\ \pi_A + \pi_{\mathcal{B}} &= \frac{1}{1-c} \\ \pi_A, \pi_{\mathcal{B}} &\in [0,1]. \end{aligned} \tag{9}$$

**Proof.** In Step 2, an equilibrium profile $s$ is characterized by $u_i \geq 0$ for all $i \in \mathcal{B}$. That is:

- $s_i(c) = H$ iff $\frac{\|\mathcal{B}_H \cup \mathcal{B}_A\|}{\|\mathcal{B}_H\|}(1-c)\,x_i \geq 1$. Analogously, $i \in \mathcal{B}_H$ iff $x_i \geq \frac{1}{(1-c)}\frac{\|\mathcal{B}_H\|}{\|\mathcal{B}_H \cup \mathcal{B}_A\|}$, which implies that $\pi_H = \frac{1}{(1-c)}\frac{\|\mathcal{B}_H\|}{\|\mathcal{B}_H \cup \mathcal{B}_A\|}$ satisfies $\mathcal{B}_H = \{i \in \mathcal{B} : x_i \in [\pi_H, 1]\}$.

- $s_i(c) = A$ iff $\frac{\|\mathcal{B}_H \cup \mathcal{B}_A\|}{\|\mathcal{B}_A\|}(1-c)(1-x_i) \geq 1$. Analogously, $i \in \mathcal{B}_A$ iff $x_i \leq 1 - \frac{1}{(1-c)}\frac{\|\mathcal{B}_A\|}{\|\mathcal{B}_H \cup \mathcal{B}_A\|}$, which implies that $\pi_A = \frac{1}{(1-c)}\frac{\|\mathcal{B}_A\|}{\|\mathcal{B}_H \cup \mathcal{B}_A\|}$ satisfies $\mathcal{B}_A = \{i \in \mathcal{B} : x_i \in [0, 1-\pi_A]\}$.

Moreover, $\pi_H + \pi_A = \frac{1}{1-c}$. Hence, these $\pi_H$ and $\pi_A$ are characterized by:

$$\pi_H \int_0^{1-\pi_A} f(t)dt = \pi_A \int_{\pi_H}^1 f(t)dt$$
$$\pi_H + \pi_A = \frac{1}{1-c}$$

which implies

$$\frac{1}{\pi_A}\int_0^{1-\pi_A} f(t)\,dt = \frac{1}{\pi_H}\int_{\pi_H}^1 f(t)\,dt.$$

These conditions also characterize the strong equilibrium, because no subset of bettors can get advantage by changing their bets. In order to prove existence of $\pi_H$ and $\pi_A$, note first that these conditions are not possible when $c > \frac{1}{2}$. In that case, the only equilibrium is achieved with $s_i = D$ for all $i \in \mathcal{B}$, which gives the bookmaker a payoff of zero. In case $c = \frac{1}{2}$, the unique solution is $\pi_H = \pi_A = 1$, which again gives the bookmaker a payoff of zero. Assume then $c < \frac{1}{2}$, and let $d = \frac{1}{1-c} \in (1,2)$. We define $\phi : (d-1, 1) \longrightarrow \mathbb{R}$ as $\phi(\pi) = \frac{1}{\pi}\int_0^{1-\pi} f(t)dt - \frac{1}{d-\pi}\int_{d-\pi}^1 f(t)dt$. It is straightforward to check that $\phi$ is continuous, strictly decreasing, and satisfies $\phi(1-d^-) > 0$, and $\phi(1^-) < 0$. Hence, there exists a unique $\pi = \pi_A$ such that $\phi(\pi) = 0$ and, moreover, the bookmaker gets a positive payoff. ■

A straightforward corollary is the following:

**Corollary 3.2** *Assume there exists a unique (strong) subgame perfect equilibrium. Then, taxes on volume ($\upsilon$) and on profit ($\rho$) are equivalent; both decrease bookmaker's utility, but maintains the commission and the volume. Bettors utility remains unchanged.*

# 4 Type comparison in the symmetric case

Given the results in the previous section, a natural question is whether a monopolistic bookmaker would prefer to offer fixed-odds, spread bets, or parimutuel. We address this question in the symmetric case, i.e. when $f$ is symmetric:

$$f(x) = f(1-x)$$

for all $x \in (0,1)$.

This case covers situations where there is no favourite team in the sport match, or when there exists a favourite but it has a handicap that makes the match even. Such handicap bets are quite common in online betting, and allow the bookmakers to assure that the volume of bets between home and away teams are balanced. In our model, this is particularly relevant for the spread bets bookmaker, since it makes her indifferent of who is the winning team (Corollary 3.1).

The main result of this section characterizes the equilibrium payoffs and states that fixed odds and spread bets are equivalent in the symmetric case:

**Theorem 4.1** *Assume $f(x) = f(1-x)$ for all $x \in (0,1)$. Then:*

*a) Fixed-odds and spread bets yield the same payoff allocation in equilibrium. The odds and the bookmaker's payoff are given by*

$$\max_{\pi \in \left[\frac{1}{2},1\right]} (1-\rho)\left(2(1-\upsilon) - \frac{1}{\pi}\right)\int_{\pi}^{1} f(t)dt.$$

*b) The odds and the bookmaker's payoff in the parimutuel case are given by*

$$\max_{\pi \in \left[\frac{1}{2},1\right]} (1-\rho)\,(1-\upsilon)\left(2 - \frac{1}{\pi}\right)\int_{\pi}^{1} f\,(t)\,dt.$$

**Proof.** a) We focus first on the fixed-odds case characterized in Theorem 3.1. Under symmetry, (1) becomes

$$\frac{1}{\pi_H}\int_{\pi_H}^{1} f\,(t)\,dt = \frac{1}{\pi_A}\int_{\pi_A}^{1} f\,(t)\,dt.$$

This equality holds when $\pi_A = \pi_B$. Since $F(x) = \frac{1}{x}\int_x^1 f(t)\,dt$ is a strictly decreasing function, we deduce that, for each $\pi_A$, there exists a unique $\pi_H$ that satisfies $F(\pi_H) = F(\pi_A)$. Hence, (1) is equivalent to $\pi_A = \pi_B$. The other restrictions are $\pi_A, \pi_B \in [0,1]$ and $\pi_A + \pi_B \geq 1$, which become $\pi_H = \pi_A \in \left[\frac{1}{2}, 1\right]$. The maximization problem given in Theorem 3.1 becomes

$$\max_{\pi\in\left[\frac{1}{2},1\right]} (1-\rho)\left\{(1-\upsilon)\left[\int_\pi^1 f(t)dt + \int_\pi^1 f(t)dt\right] - \frac{1}{\pi}\int_\pi^1 f(t)dt\right\}$$
$$= \max_{\pi\in\left[\frac{1}{2},1\right]} (1-\rho)\left(2(1-\upsilon) - \frac{1}{\pi}\right)\int_\pi^1 f(t)dt.$$

We now focus on the spread bets characterized in Theorem 3.2. Under symmetry, (4) becomes

$$\frac{1}{\pi}\int_{\frac{\pi}{1-(1-\pi)c}}^1 f(t)\,dt = \frac{1}{1-\pi}\int_{1-\frac{(1-c)\pi}{1-\pi c}}^1 f(t)\,dt$$

equivalently,

$$\frac{1}{\pi}\int_{\frac{\pi}{1-(1-\pi)c}}^1 f(t)\,dt = \frac{1}{1-\pi}\int_{\frac{1-\pi}{1-\pi c}}^1 f(t)\,dt$$

or

$$G(\pi) = G(1-\pi)$$

where $G(x) = \frac{1}{x}\int_{\frac{x}{1-(1-x)c}}^1 f(t)\,dt$. This equality holds when $\pi = \frac{1}{2}$. It is straightforward to check that $G$ is a strictly decreasing function on $[0,1]$, and so $\pi = \frac{1}{2}$ is the only solution to $G(\pi) = G(1-\pi)$. Hence, (4) is equivalent to $\pi = \frac{1}{2}$ and $\gamma = 2\int_{\frac{1}{2-c}}^1 f(t)dt$. The maximization problem given in Theorem 3.2 becomes

$$\max_{c\in[0,1]} (1-\rho)\left(c\min\left\{1-\frac{1}{2}, \frac{1}{2}\right\} - \upsilon\right)\gamma$$
$$= \max_{c\in[0,1]} (1-\rho)\left(\frac{c}{2} - \upsilon\right)\gamma$$
$$= \max_{c\in[0,1]} (1-\rho)\left(\frac{c}{2} - \upsilon\right) 2\int_{\frac{1}{2-c}}^1 f(t)dt.$$

We now proceed by a change of variable: $\pi = \frac{1}{2-c}$, so that $c \in [0,1]$ is equivalent to $\pi \in \left[\frac{1}{2}, 1\right]$ and the maximization problem becomes

$$\max_{\pi \in \left[\frac{1}{2},1\right]} (1-\rho)\left(\frac{2-\frac{1}{\pi}}{2} - \upsilon\right) 2 \int_{\pi}^{1} f(t)dt$$
$$= \max_{\pi \in \left[\frac{1}{2},1\right]} (1-\rho)\left(2(1-\upsilon) - \frac{1}{\pi}\right) \int_{\pi}^{1} f(t)dt.$$

b) In the parimutuel case, we apply Theorem 3.3. Since (1) is equivalent to (9), we follow the same reasoning as before to deduce that $\pi_H = \pi_A$. The other restriction is $\pi_H + \pi_A = \frac{1}{1-c}$. Hence, $\pi_H = \pi_A = \frac{1}{2(1-c)}$ and the maximization problem becomes

$$\max_{c \in \left[0,\frac{1}{2}\right]} (1-\rho)(1-\upsilon)\, 2c \int_{\frac{1}{2(1-c)}}^{1} f(t)\, dt.$$

We now proceed by a change of variable: $\pi = \frac{1}{2(1-c)}$, so that $c \in \left[0, \frac{1}{2}\right]$ is equivalent to $\pi \in \left[\frac{1}{2}, 1\right]$ and the maximization problem becomes

$$\max_{\pi \in \left[\frac{1}{2},1\right]} (1-\rho)(1-\upsilon)\left(2 - \frac{1}{\pi}\right) \int_{\pi}^{1} f(t)\, dt.$$

■

We can now answer the question of which type of bets a monopolistic bookmaker would prefer to offer. If there is no tax on volume, she would be indifferent. When there is a tax on volume, she would prefer parimutuel:

**Corollary 4.1** *Assume $f(x) = f(1-x)$ for all $x \in (0,1)$. Then:*

*a) When there is no tax on volume ($\upsilon = 0$), fixed-odds, spread bets and parimutuel yield the same payoff allocation in equilibrium. The odds and the bookmaker's payoff are given by*

$$\max_{\pi \in \left[\frac{1}{2},1\right]} (1-\rho)\left(2 - \frac{1}{\pi}\right) \int_{\pi}^{1} f(t)dt.$$

*b) When there is a tax on volume ($\upsilon > 0$), parimutuel gives in equilibrium a higher payoff to the bookmaker than fixed-odds and spread bets.*

**Proof.** a) It follows from setting $\upsilon = 0$ in Theorem 4.1.

b) It follows from Theorem 4.1 and the fact that $\upsilon > 0$ implies $(1-\upsilon)\left(2-\frac{1}{\pi}\right) > 2(1-\upsilon)-\frac{1}{\pi}$ for all $\pi \in \left[\frac{1}{2}, 1\right]$. ■

# 5 Concluding remarks

In this paper, we model three different online betting markets: those given by fixed-odds, spread bets, and parimutuel, respectively. This allows us to analyse the effect of two different tax schemes: On volume (GBD) and on profit (GPT). In both fixed-odds and spread bets, respectively odds and commission are unaffected by GPT but they are by GBD. Hence, it should be expected that odds and commission to depend on the particular regulation. For example, Paddy Power Betfair, which includes one of the largest Internet spread betting companies, charges a different commission for spread bets on each country. This commission is 5% in the United Kingdom, Ireland, Italy, Gibraltar and Malta; 7% in Albany, Armenia, Croatia, Monaco, Serbia, Montenegro and Slovakia; and 6.5% in the rest of the countries, including Spain. Moreover, the company is restricted in Belgium, Greece, Germany[6], Turkey, Israel, France and Portugal, among other countries.

As opposed to other approaches in the literature, we do not assume the existence of an actual probability for the home (or away) team to win the match. Instead, the bettors are characterized by their subjective beliefs on this probability. An alternative interpretation is that each bettor is characterized by the the odd at which she is willing to bet, which includes the individual surplus of the act of betting itself. In this sense, a natural extension of the model, which does not change the results, is to assume that there are two subsets of bettors: one of them willing to bet for the home team, another willing to bet for the away team, and both characterized by the minimum odd they will bet.

[6]Betfair is only restricted in Germany for spread bets.

# References

Bag, P. and Saha, B. (2011). Match-fixing under competitive odds. *Games and Economic Behavior*, 73(2):318–344.

Bag, P. K. and Saha, B. (2016). Match-fixing in a monopoly betting market. *Journal of Economics and Management Strategy*, Forthcoming.

Ficom Leisure (2011). Update on Italy's online betting and gaming market. Available at `http://sectordeljuego.org/img_fckeditor/informeitalia.pdf`.

Global Betting and Gaming Consultancy (2011). It's all about the tax. Available at `http://www.gbgc.com/its-all-about-the-tax/`.

Hofmann, J. and Spitz, M. (2015). *Gambling 2016: A practical cross-border insight into gambling law*, chapter 11, pages 54–58. Global Legal Group.

Levitt, S. (2004). Why are gambling markets organized so differently from financial markets? *Economic Journal*, 114:223–246.

National Audit Office (2005). Gambling duties. HM customs and excise. Report by the controller and Auditor General. HC 138 Session 2004-2005. Available at `http://www.nao.org.uk/publications/0405/gambling_duties.aspx`.

Neyman, A. (2002). Values of games with infinitely many players. In Aumann, R. and Hart, S., editors, *Handbook of game theory with economic applications*, volume III, chapter 56, pages 2121–2167. Elsevier Science, Amsterdam.

Ottaviani, M. and Sørensen, P. N. (2009). Surprised by the parimutuel odds? *American Economic Review*, 99(5):2129–2134.

Paton, D., Siegel, D. S., and Williams, L. V. (2001). Gambling taxation: A comment. *Australian Economic Review*, 34(4):437–440.

Paton, D., Siegel, D. S., and Williams, L. V. (2002). A policy response to the e-commerce revolution: the case of betting taxation in the UK. *Economic Journal*, 112(480):296–314.

PwC (2011). Taxation and online sports betting in Germany: considering the relative merits of a tax on gross gaming revenue and a tax on stakes for the potential regulation of online sports betting stakes. Available at `https://gamblingcompliance.com/files/PwC%20Report%20German%20betting%20tax%202011.pdf`.

Smith, J. (2000). Gambling taxation: Public equity in the gambling business. *Australian Economic Review*, 33(2):120–144.