Effects of Long Cycles in Cash Flows on Present Value

Bell, Peter N

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Peter N. Bell

Author note

Contact information: pnbell@shaw.ca, 250 588 6939.

Abstract

This paper explores how present value varies over time when the underlying cash flow has a deterministic period. I assume that cash flows are known with certainty and follow a cycle with a long or short period. When the cash flow has a short period, the present value is relatively stable over time because the present value calculation smooths out several cycles. However, when the cash flow has a long period the present value itself develops a long and large cycle. These results are driven by the mathematical definition of the present value and are relevant to the use of present value as a pricing tool in situations where the cash flows of an investment have a long cycle.

Keywords: Present Value, Investment, Simulation.

JEL Classification: C65, E44, G12.

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1. Introduction

This paper explores how the present value of an investment changes over time when the underlying cash flow is periodic. When the period is large, the present value changes substantially based on where you are in cycle. The present value generally moves in the same direction as the cash flow, but reaches a peak or trough slightly before underlying cash flow. This is caused by the mathematical definition of present value and has implications for using present value in situations where the underlying cash flow have a long cycle.

The paper uses a simple mathematical setup. The cash flow, F(t), are defined by Equation (1). I assume the cash flows start at one hundred dollars per year, F(0)=100. The cash flows move up or down from the initial value by a maximum of ten dollars over the full cycle, k=10. The period for a short cycle is ten years, c=10, and a long cycle is one hundred years, c=100.

$$F(t)=F(0)+k*\sin(2\pi t/c). \quad (1)$$

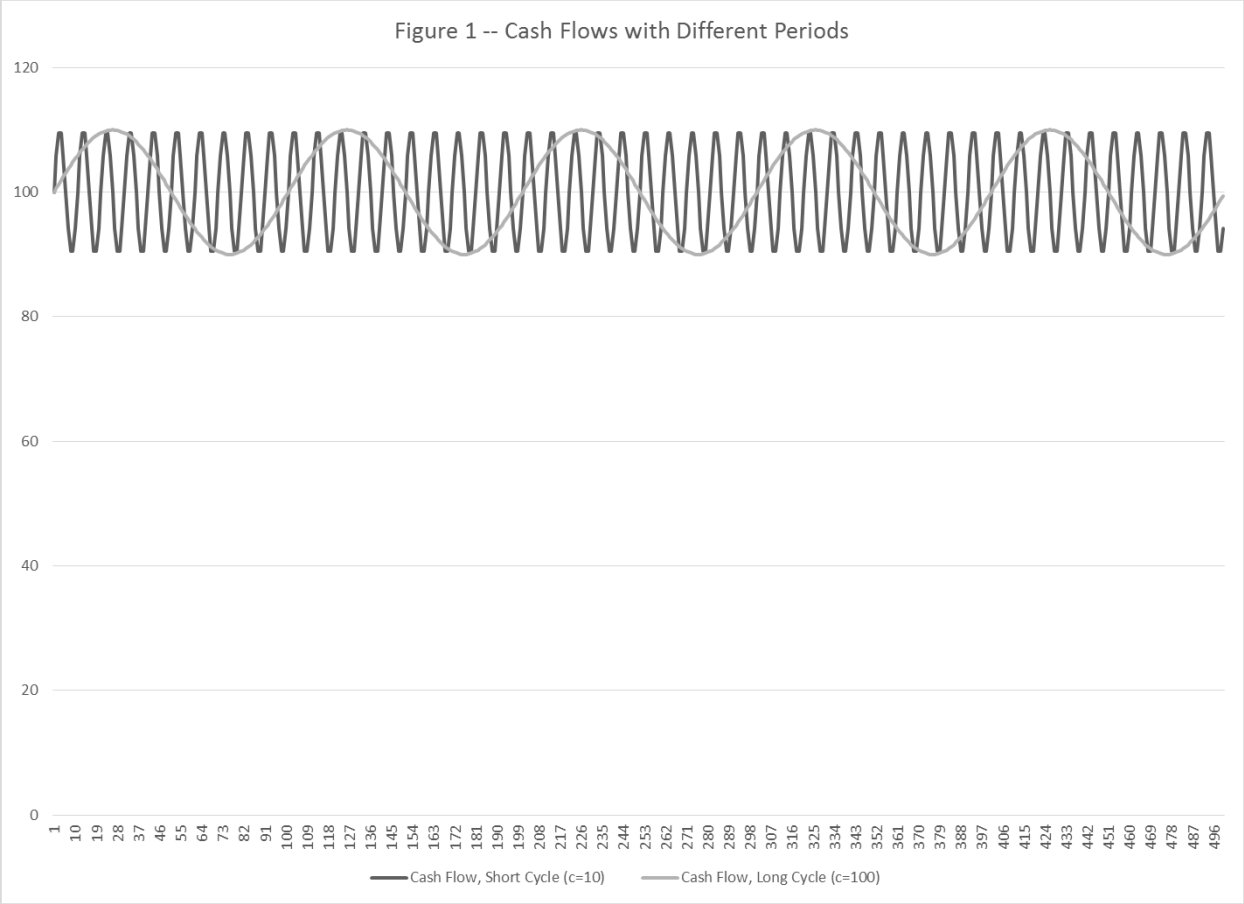
$$V(t)=\sum_{s=t}^{\infty} F(s)d^{s}. \quad (2)$$

Equation (2) describes how to calculate present value. I use a discount rate of ten percent, d=0.10, throughout. Note that the present value at a particular time, t, is defined to include all future cash flows, s=(t,∞), and excludes prior cash flows.

2. Analysis of Cases

2.1 Underlying Cash Flows

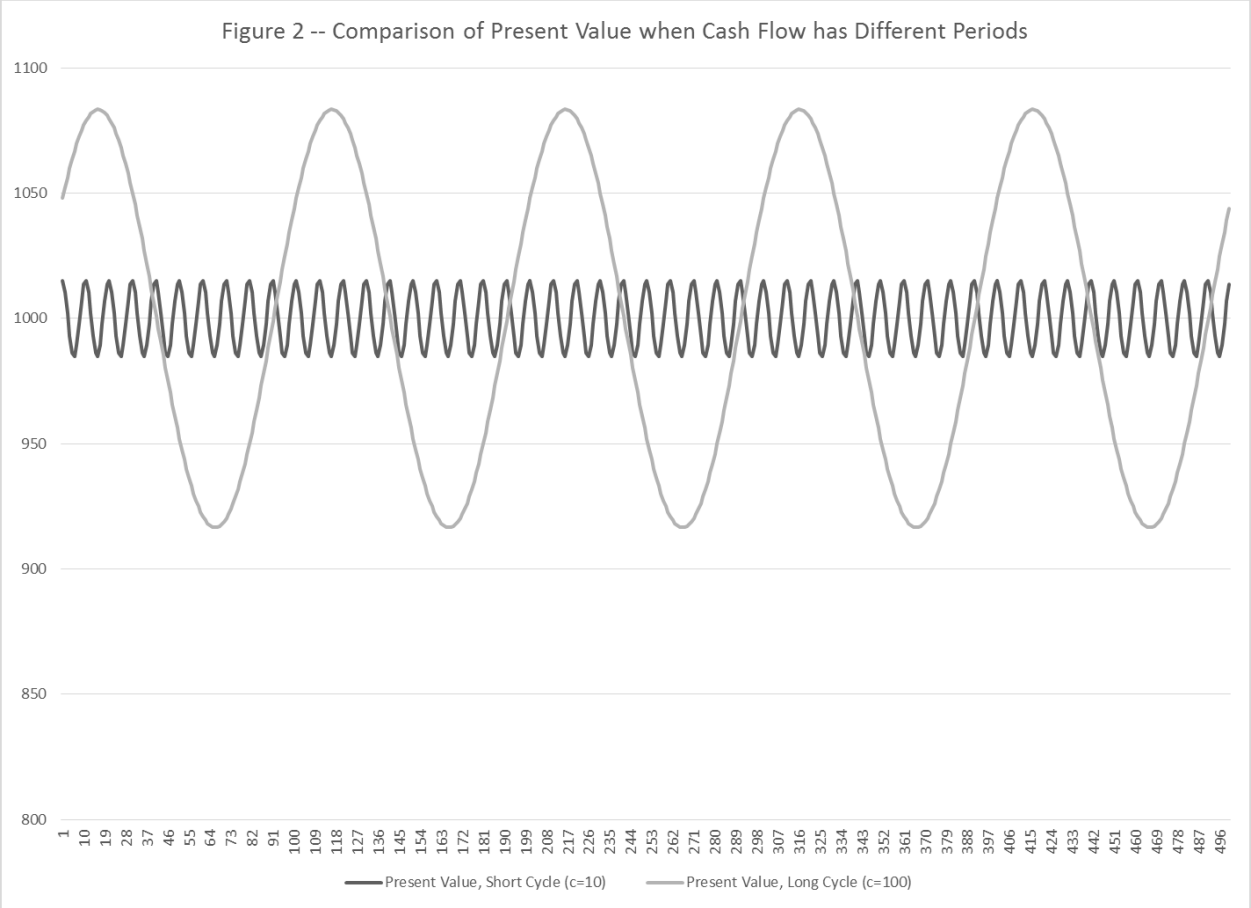
To begin, I show the underlying cash flow for both the short and long cycle.



As specified in Equation (1), the two cash flows have the same initial value and amplitude over time but different periods.

2.2 Estimates of Present Value

Figure 2 compares the present value over time, V(t), for the long and short cycles. The results show that the present value itself has a cycle caused by the cycle in the underlying cash flows. However, the shape of the cycle is different for long and short cycles.



When the cash flow has a short cycle, the present value has a short cycle with small amplitude. This occurs because the present value can effectively smooth across several generations of the cycle, which can offset each other and dampen the variation in present value. However, when cash flow have a long cycle, the present value itself has a long cycle with large amplitude. This occurs because the discount factor makes it so that future cycles have little impact on present value when cash flows have a long cycle; the present value cannot smooth over several generations of the longer cycle.

2.3 Cash Flow and Present Value with Long Cycle

Figure 3 compares the cash flow and present value for the long cycle. The cash flow and present value are generally in synch, but there is a slight difference at turning points.

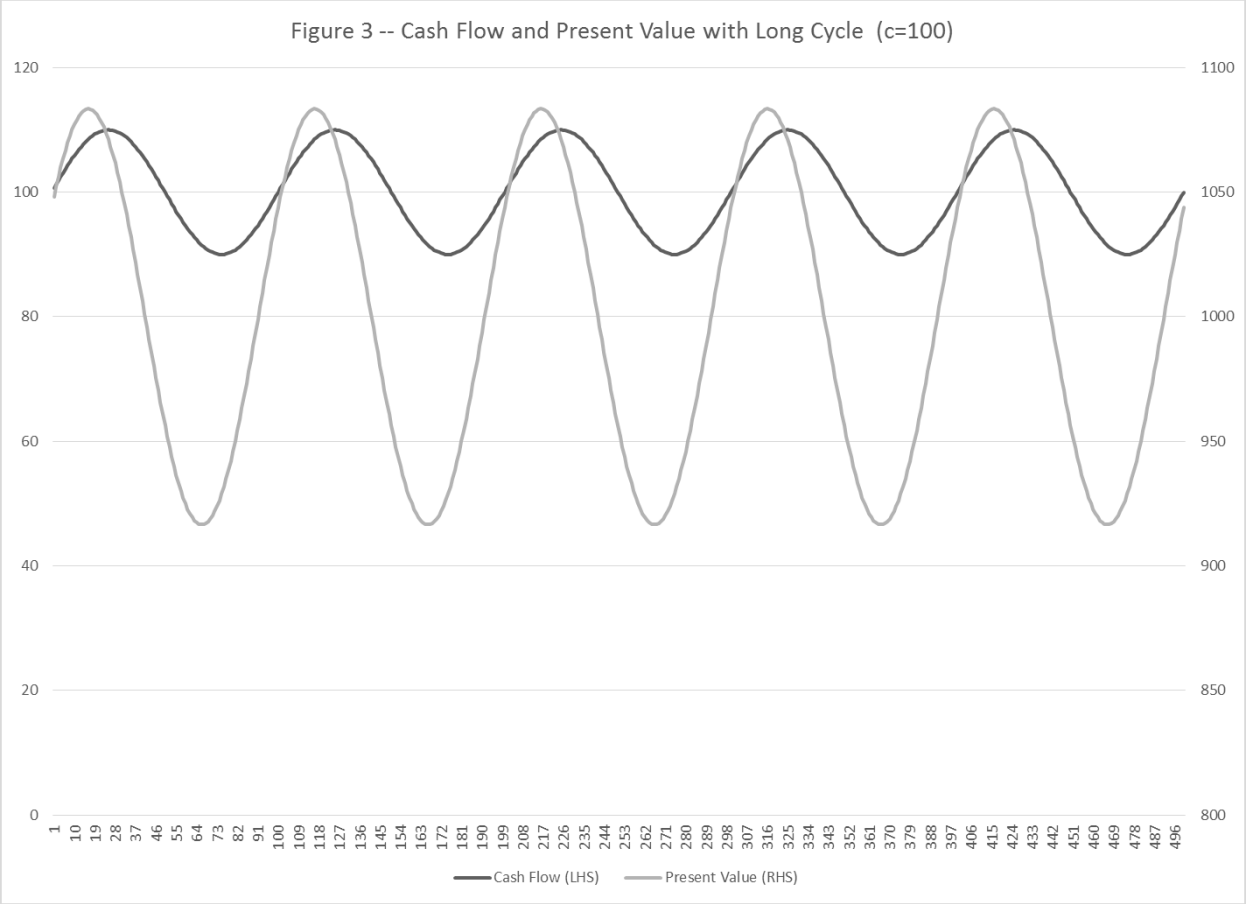


Figure 3 shows that the present value turns slightly before the cash flow. Notice how the present value reaches a peak slightly before the cash flow, this occurs because the present value includes lower cash flows as the cash flow moves past its' peak value. This phenomenon is symmetric at troughs: as the cash flows move past its' lowest values, the present value calculation begins to include larger cash flows by definition.

3. Discussion

The fact that present value can have such cyclic behaviour raises questions about the use of present value to price investments with cyclical fundamentals. The results in this paper suggest that such valuations can be unstable over time and move in long trends. Although these results are highly simplified, this idea is reflected in the folk sentiment amongst some investors that

pricing long-lived, commodity-producing assets off spot market prices can lead to overpricing at peaks and underpricing at troughs. Furthermore, the results presented in this paper raise other questions. For example, how does the valuation of a long-term mining operation vary across the lifecycle of the mine? The results in this paper suggest that such valuation can vary substantially over time, simply based on the mathematical definition of present value.