Estimation of Unobserved Inflation Expectations in India using State-Space Model

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Abstract

Inflation expectations is an important marker for monetary policy makers. India being a new entrant to the group of countries that pursue inflation targeting as its monetary policy objective, estimating the inflation expectation is of paramount importance. This paper estimates the unobserved inflation expectations in India between 1993:Q1 to 2016:Q1 from the Fisher equation relation using the state space approach (Kalman Filter). We find that our results match well with the inflation forecasts made by the Survey of Professional Forecasters conducted by the Rerserve Bank of India and by the International Monetary Fund for the Indian economy. We apply the estimated series on inflation expectation to show that there is a long-run equilibrium relation between inflation expectations and monetary policy in India during the post liberalization period.

JEL Classification: E47

Keywords: Fisher Equation, Kalman Filter, Expected Inflation, India

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1 Introduction

Inflation expectations is an important marker for monetary policy makers. Both theoretically and empirically, it has been established that current inflation is influenced by expectations about future inflation and vice-versa. Especially for economies where inflation targeting is an explicit policy goal, knowledge of inflation expectations is required to attain inflation anchoring.

The inflation dynamics, as captured by the New Keynesian Phillips Curve (NKPC), has inflation expectations as one of its explanatory variables. In this context, various authors have used different specifications of inflation expectations, a comprehensive survey of which can be found in Mavroeidis et al (2014).[1]

In the Indian context, authors have modelled the unobserved inflation expectations following techniques like perfect foresight (Srinivasan et.al 2006, Patra and Kapoor, 2010, Sahu 2013), ARIMA (Patra and Ray 2010), moving average (Dua and Gaur 2010), single and double exponential smoothing (Dua and Gaur 2010) etc. Our work adds to the existing literature by augmenting another procedure for estimating expected inflation.

This paper estimates the unobserved inflation expectations in India based on the Fisher equation relation using the state space approach or the Kalman Filter (KF) method following Mishkin (1982), Fama and Gibbons (1982), Burmeister et al. (1986) and Hamilton (1994). The novelty of this methodology when compared to the other techniques is that expected inflation is estimated from a well established economic relationship which is the Fisher equation. Looking at it from another perspective, this estimation also verifies if the Fisher equation holds for the Indian economy. We find that between 1993:Q1 and 2016:Q1 the unobserved inflation expectations estimated from the Fisher equation match well with inflation forecasts made by the Survey of Professional Forecasters conducted by the Reserve Bank of India (RBI) and the forecasts by the International Monetary Fund (IMF) for the Indian economy.

Futher, we apply our estimated series on inflation expecations to test if there exists a long-run equilibrium relation between expected inflation and monetary policy (measured by the effective policy rate) in India during post-liberalization era.[2]We find that inflation expectation and India's monetary policy are cointegrated, thereby indicating the presence of a long-run equilibrium relation between these two variables. Using the Error Correction

[1]A more general treatment of modeling unobserved components in structural time series models can be found in Harvey (2006).

[2]We have chosen post-liberalization era for our analysis since interest before liberalization was not market determined and administered by the monetary authority.

Model (ECM), we investigate how these variables adjust in the short run when they move away from equilibrium. Additionally, we find that the monetary policy in India in the post-liberalization period indeed anchors inflation expectations.

The paper proceeds as follows. We present the methodology in Section 2, followed by the data description in Section 3. Section 4 discusses the results obtained from our analysis along with an application and Section 5 concludes this paper.

2 Methodology

Our starting point is the Fisher equation that relates the nominal interest rate and the real interest rate. It is given by the following equation:

$$rr_t^a = i_t - \pi_t^e \tag{1}$$

where, rr_t^a is the ex-ante real interest rate, i_t is the nominal interest rate captured by the policy rate and π_t^e is the expected inflation rate. Note that the ex-ante real interest rate is unobserved since the expected inflation is unobserved. The ex-post real interest rate is rr_t=$i_t - \pi_t$.

Our variable of interest here is the one period ahead expected inflation. We apply the state space approach or the Kalman Filter (KF) to the Fisher equation to estimate inflation expectation.[3] To do so, we decompose *ex-post* real interest rate rr_t, into *ex-ante* real interest rate, rr_t^a and the forecast error, $u_t = \pi_t^e - \pi_t$ as shown in equation (2). We further assume that ex-ante real interest rate follows an $AR(1)$ process as given in equation (3).[4]

$$rr_t = rr_t^a + u_t \tag{2}$$

$$rr_t^a = \rho rr_{t-1}^a + \varepsilon_t \tag{3}$$

where, $\rho \in (0,1)$ is the persistence. We assime, $u_t \sim IN(0,1)$ and $\varepsilon_t \sim IN(0,1)$ and independent of each other $\forall t$.

Note, equation (2) is our signal equation and equation (3) is our state equation. We apply KF on equation (2) and equation (3) and estimate the parameters numerically through MLE using following initial conditions, $\rho = 0.83, \pi_0^e = 8, \sigma_u^2 = 1.87$ for our

[3] See Appendix for a brief outline of Kalman Filter methodology.

[4] We have checked that ex-post real interest rate is $AR(1)$. See appendix for detail.

estimation.[5]These initial conditions yield, $rr^a_{1|0} = 4$. However, we do not have any prior information about the variance of the state variable, *ex-ante* real interest rate. Hence, we have used diffuse prior for obtaining an initial guess of the variance of state variable, $P_{1|0}$.[6] Once convergence is achieved after 7 iterations, expected inflation is calculated as, $\widehat{\pi}^e_t = i_t - \widehat{rr}^a_{t|t-1}$.

3 Data

Quarterly data on CPI for India between 1993:Q1 to 2016:Q1 has been collected from the FRED database provided by the Federal Reserve Bank of St. Louis.

For quarterly nominal interest rate for India, we calculate the effective policy rate following Patra and Kapur (2010) for the same period. As pointed out by Patra and Kapur (2010), "as regards the appropriate policy interest rate, the choice is complicated by regime shifts and consequent changes in operating procedures", thereby using the effective policy rate as a proxy for the nominal interest rate.

Later, we compare our findings with the inflation forecasts made by the International Monetary Fund available at the World Economic Outlook Database.[7] The data on Survey of Professional Forecasters (SPF) conducted by the Reserve Bank of India is available at the RBI's website starting from 2008:Q1.

4 Results

Figure 1 plots the one period ahead expected quarterly inflation for India between 1993:Q1 and 2016:Q1 generated by applying the KF to the Fisher equation along with

[5]A simple correlogram shows that, ex-post real interest rate is an process with first autocorrelation coefficient 0.89 and variance, 1.89. As a result, initial value of $\rho = 0.89$. We have decomposed the inflation rate into trend and cycle through HP filter. We set, $\pi_{1|0} = 8$, which is the inflation rate for 1992:4. Appendix shows the plot of inflation and its trend obtained from HP filter. We have achieved convergence after 7 iterations. See appendix for the correlagram and descriptive statistics of *ex-post* real interest rate.

[6]For detail see, Harvey (1989) and Koopman, Shephard and Doornik (1999).

[7]IMF publishes biannual inflation forecast for Fall and Spring from 1990 onwards. It publishes inflation forecast for one year and two year ahead. We have used only one year ahead forecast in our paper.

the actual inflation. Actual inflation and its forecast are measured in percentage.

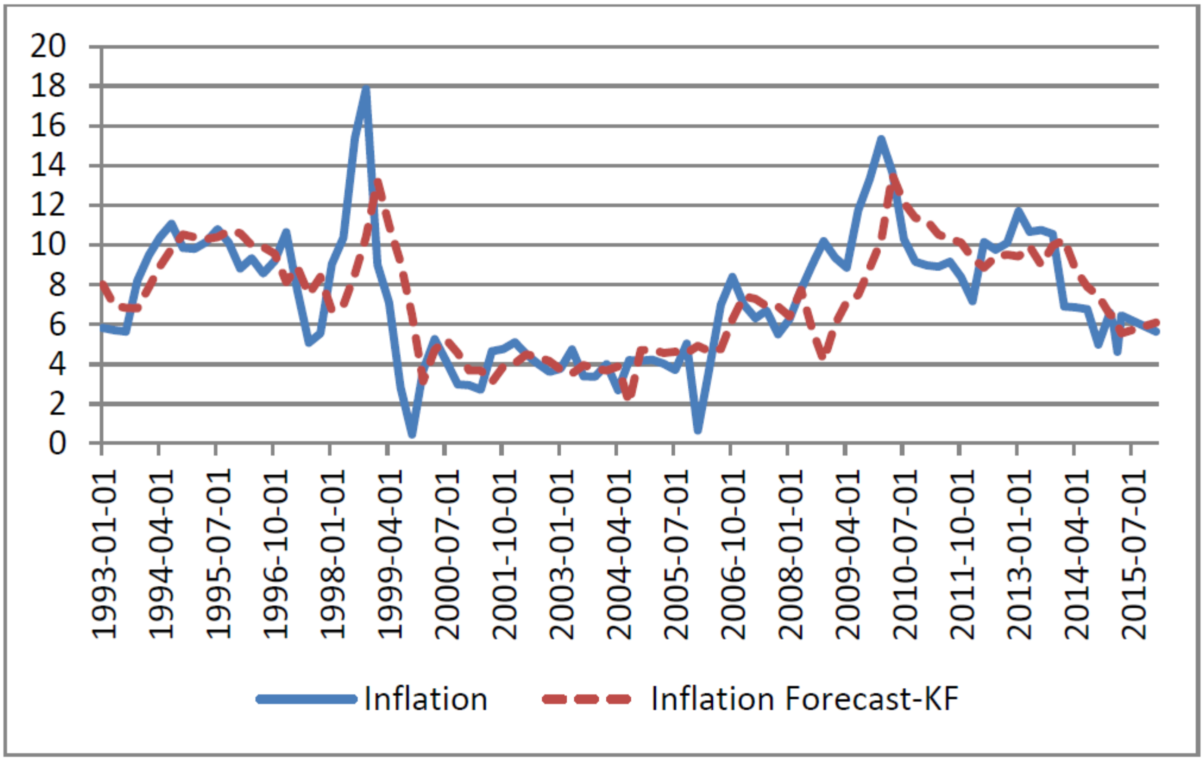

Figure 1: Inflation and Inflation Forecast

We compare the SPF inflation forecast forecast of IMF with the same obtained from KF for checking the performance of KF. Figure 2 plots the quarterly inflation forecats made by the IMF and the quarterly expected inflation obtained by our model.[8]

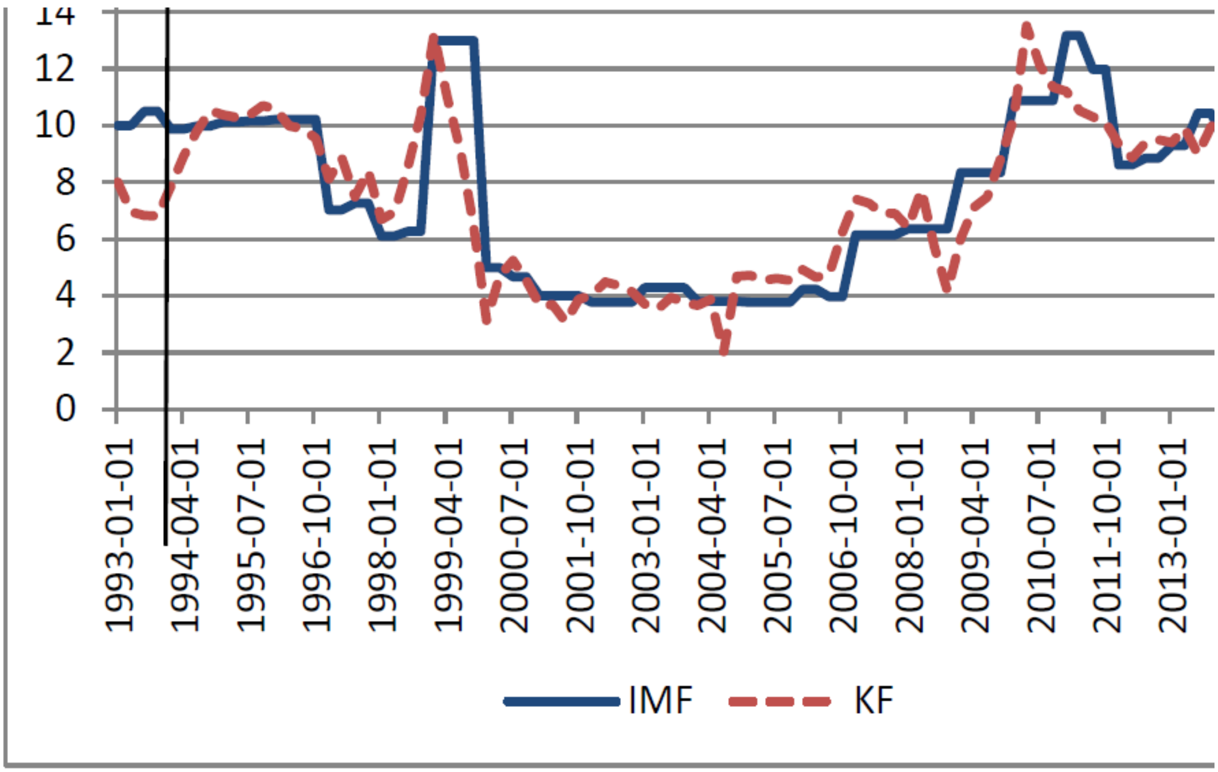

Figure 2: Inflation Forecast of IMF and KF

A t-test and F-test is undertaken for checking the equality of mean and variance of inflation forecast obtained from KF and SPF inflation forecast of IMF. The mean test gives a t-value 0.51 with degrees of freedom 178 and the test of equality of variance gives the value of F-statistic as, 1.22 with (89, 89) degrees of freedom. This shows that null hypothesis of equality of mean and variance is accepted at 5% level, which confirms that SPF inflation forecast of IMF and forecast obtained from KF are identical and one correctly represents other.[9]

Figure 3 compares the inflation forecast from KF with the same obtained from the Survey of Professional Forecasters conducted by the Reserve Bank of India. We have undertaken a t-test and F-test for checking the equality of mean and variance of inflation

[8]Note, initial forecast obtained from KF is sensitive to initial guess but it quickly converges to SPF inflation forecast of IMF as shown in Figure 2. Figure 2 shows that, forecast of KF quickly converges to IMF forecast from 1994:1 when the estimation starts from 1993:1.

[9]IMF forecast is available till 2015:2. Therefore, the test of mean and variance is done for the time period 1992:1 to 2015:2.

forecast obtained from KF and SPF inflation forecast of RBI for the time period 2008:1 to 2015:2. The mean test gives a t-value 0.15 with degrees of freedom 58 and the test of equality of variance gives the value of F-statistic as, 1.40 with (29, 29) degrees of freedom. This shows that null hypothesis of equality of mean and variance is accepted at 5% level, which confirms that SPF inflation forecast of RBI and forecast obtained from KF are identical too.

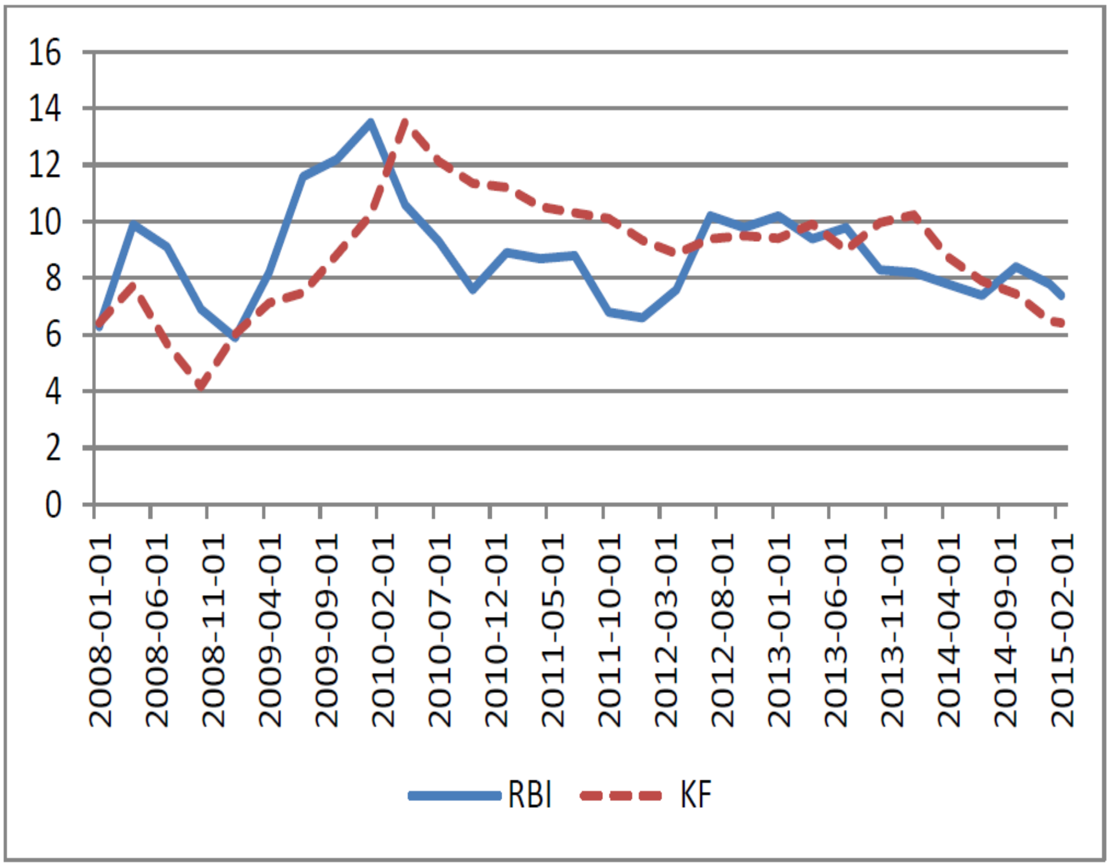

Figure 3: Inflation Forecast of RBI and KF

Overall, we find that the expected inflation estimated from the Fisher Equation using the state-space approach or the Kalman Filter, compares well with the realized inflation rate and the forecasts made by other agencies.

4.1 An Application: Monetary Policy and Inflation Forecast

We show an application of our estimated series on inflation expectation to test for the validity of an equilibrium long-run relation between monetary policy in India (captured by the effective policy rate) and expected inflation during the post-liberalization years in India. Additionally, we test if the monetary policy in India anchors inflation expectation or not.

The SPF inflation forecasts of IMF start from 1994:Q1 for India, but they provide forecasts for two alternate quarters each year, while the SPF by RBI started from 2008:Q1, thereby giving very few data points for the purpose of conducting any meaningful analysis. Thus, we use the series on expected inflation generated by our model between 1993:Q1 and 2016:Q1 that overcomes the problems of few data points and gives us a continuous quarterly series.

Table 1 presents the results of the Augmented Dickey Fuller (ADF) and the Phillips-Perron (PP) unit root test for nominal interest rate and expected inflation. It shows that both nominal interest rate and expected inflation are stationary in first difference, i.e. I(1).

Nominal Interest in Level			
Test	Model	Calculated Value	Critical Value (5% level)
ADF	No constant and trend	-1.23	-1.94
PP		-1.19	-0.94
Expected Inflation in Level			
ADF	Model	Calculated Value	Critical Value (5% level)
PP	No constant and trend	-0.84	-1.94
		-0.84	-0.94
Nominal Interest in First Difference			
ADF	No constant and trend	-8.58	-1.94
PP		-8.58	-1.94
Expected Inflation in First Difference			
ADF	No constant and trend	-8.35	-1.94
PP		-8.35	-1.94

Table 1: Unit Root Test

Subsequently we conduct the cointegration test to rule out the possibility of spurious regression. Table 2 shows the result obtained from Johansen cointegration test. While the λ_{trace} test in Table 2 shows that, we have at most one cointegrating vector, $\lambda_{\max}$ test confirms that we have exactly one cointegrating vector between nominal interest rate and interest rate. The cointegrating vector normalized with respect to nominal interest rate is, (1,-1.01). The t-value associated with expected inflation is, 11.82, which is significant even at 5% level. The presence of cointegration shows that there exists a long-run equilibrium relationship between nominal interest rate and expected inflation for India in post-reform period. The presence of cointegration with two variables guarantees the presence of error correction representation. The estimated Error Correction Model (ECM) shows that adjustment of the system in short-run if it deviates from long-run equilibrium. ECM also identifies the variables that bring the system back to equilibrium in the long-run if it deviates in the short-run.

Model: No Constant and Trend in Cointegrating Equation			
Null Hypothesis	Alternative Hypothesis		
λ_{trace} test		λ_{trace} value	Critical Value (5% level)
$r = 0$	$r > 0$	15.79	12.32
$r \leq 1$	$r > 1$	0.78	4.12
$\lambda_{\max}$ test		$\lambda_{\max}$ value	Critical Value (5% level)
$r = 0$	$r = 1$	15.00	11.22
$r = 1$	$r = 2$	0.78	4.12
Note: r is the number of cointegrating vector			

Table 2: Johansen Cointegration Test

Therefore, we estimate ECM given below with single lag to uncover the short-run and long-run dynamics. The lag length is selected on the basis of SIC as it identifies the most parsimonious system.

$$\begin{aligned} \Delta i_t &= \alpha_i \left(i_{t-1} - \beta\pi^e_{t-1}\right) + \alpha_{11}\Delta i_{t-1} + \alpha_{12}\Delta\pi^e_{t-1} + \varepsilon_{it} \\ \Delta\pi_t &= \alpha_\pi \left(i_{t-1} - \beta\pi^e_{t-1}\right) + \alpha_{21}\Delta i_{t-1} + \alpha_{22}\Delta\pi^e_{t-1} + \varepsilon_{\pi t} \end{aligned} \quad (4)$$

α_i and α_π are the speed of adjustment parameters while ε_{it} and $\varepsilon_{\pi t}$ are white noise and might be correlated. Table 3 presents the estimate of ECM given in equations (4). We find that the speed of adjustment associated with the monetary policy (or the effective policy rate) is negative and that of the expected inflation rate is positive. This implies, as expected, the two variables move in opposite direction to bring the system back in equilibrium whenever there is a short-run deviation from the equilibrium. As evident from the table, the policy rate takes the major burden to bring the system back to equlibrium since its speed of adjustment is of higher magnitude and also significant.

Coefficient	Estimated Value
$\hat{\alpha}_i$	-0.07 (-2.57)*
$\hat{\alpha}_\pi$	0.02 (0.58)
$\hat{\alpha}_{11}$	0.14 (1.26)
$\hat{\alpha}_{12}$	-0.12 (-1.72)**
$\hat{\alpha}_{21}$	-0.76 (-4.46)*
$\hat{\alpha}_{22}$	0.37 (3.33)*

Note: t-value is reported in parenthesis with * indicating significance at 5% level and ** indicating significance at 10% level.

Table 3: Error Correction Model

Further, from Table 3, we conclude that the monetary policy (or the effective policy rate) negatively Granger causes expected inflation since the coefficient is significant at the 5% level. Hence a rise in effective policy rate reduces inflation expectation. Our analysis shows that, expected inflation negatively granger cause nominal interest rate. This happens since a rise in expected inflation reduces money demand, causing an excess supply in money market. A fall in nominal interest rate removes the disequilibrium in money market by increasing the speculative demand for money. This demonstrates that the monetary policy of India has been able to anchor expected inflation during post-reform period.

5 Conclusion

This paper estimates the unobserved quarterly inflation expectations in India between 1993:Q1 and 2016:Q1 from the Fisher equation by using the state-space model. Our estimates match well with the inflation forecasts made by the Survey of Professional Forecasters conducted by the RBI and the inflation forecasts of India made by the IMF. As an application, we employ the estimated inflation expectations to establish that there is a long run equlibrium relation between the former and monetary policy (proxied by the effective policy rate). Our analysis indicates that India's monetary policy during the post-liberalization era well anchors inflation expectations.

6 Appendix

6.1 The Kalman Filter

Kalman Filter is an Unobserved Componenet Modeling (UCM) which extracts an unobserved state recursively form an observed signal equation. To explain Kalman Filter assume a signal equation,

$$y_t = A + Bs_t + u_t \tag{5}$$

Here, the observed signal is, $y_t : (N \times 1)$, unobserved state variable is $s_t : (K \times 1)$, the coefficient matrix $A : (N \times 1)$ represents intercept and $B : (N \times K)$. $u_t : (N \times 1)$ is random error. We assume, $u_t \sim N(0, \Sigma_u)$, where, $\Sigma_u : (N \times N)$.

We assume that the state equation is,

$$s_t = \Phi s_{t-1} + v_t \tag{6}$$

Here, $\Phi : (K \times K)$ and $u_t \sim N(0, \Sigma_v)$, where, $\Sigma_v : (K \times K)$. The random error term is signal $(u)$and state (v) are assumed to be independent of each other.

The objective of Kalman Filter is to estimate the unobserved state variable s_t. The unobserved state variable is estimated recirsively from equation (7) to (9) as given below.

$$\begin{aligned} s_{t|t-1} &= \Phi s_{t-1|t-1} \\ P_{t|t-1} &= \Phi P_{t-1|t-1}\Phi' + \Sigma_v \end{aligned} \tag{7}$$

$$y_{t|t-1} = A + Bs_{t|t-1}$$
$$V_{t|t-1} = BP_{t|t-1}B' + \Sigma_u \tag{8}$$

and,

$$s_{t|t} = s_{t|t-1} + P_{t|t-1}B'V_{t|t-1}^{-1}\left(y_t - y_{t|t-1}\right)$$
$$P_{t|t} = P_{t|t-1} + P_{t|t-1}B'V_{t|t-1}^{-1}BP_{t|t-1} \tag{9}$$

Here, the $y_{t|t-1} = E_{t-1}(y_t)$ is the conditional mean of y_t given the information available till $(t-1)$. The condional variance of y_t is, $V_{t|t-1}$. The best estimator of state is, $s_{t|t-1} = E_{t-1}(s_t)$ with variance, $P_{t|t-1}$. Here, the Kalman gain is,

$$K_t = P_{t|t-1}B'V_{t|t-1}^{-1}$$

The Kalman gain is determined as a factor that minimizes the variance of state update, $s_{t|t}$. The variance of state update is, $P_{t|t}$.

The parameter of the model is $\Theta = (A, B, \Phi, \Sigma_u, \Sigma_v)$. Note,

$$y_t \sim N\left(y_{t|t-1}, V_{t|t-1}\right)$$

Here, $y_{t|t-1}$ and $V_{t|t-1}$ and non-linear function of Θ. As a result, Θ is estimated numerically by Maximum Likelihood Esttimation as follows,

$$\min_{\Theta} imizel = \frac{1}{T}\sum_{t=1}^{T} l_t$$
$$l_t = -\frac{N}{2}\log(2\pi) - \frac{1}{2}\log\left(V_{t|t-1}\right) - \frac{1}{2}\left(y_t - y_{t|t-1}\right)' V_{t|t-1}^{-1}\left(y_t - y_{t|t-1}\right)$$

given the initial condition, $s_{1|0}, P_{1|0}$ and Θ_0. We can accodingly draw statistical inferences from,

$$\widehat{\Theta}_{MLE} \sim AN\left[\Theta, \left(I(\Theta)_{\Theta=\widehat{\Theta}_{MLE}}\right)^{-1}\right]$$

where, $I(\Theta)$ is the Fischer Information Matrix.

6.2 ***Ex-Post*** Real Interest Rate

Here we briefly describe the ACF and PACF of *ex-post* real interest rate. Figure 4 plots the ACF and PACF of ex-post real interest rate. Figure 4 shows that ACF decays and there is one significant spike in PACF. This shows that ex-post real interest rate is $AR(1)$ autocorrelation coefficient 0.83. The estimated autocorrelation coefficient is $\widehat{\rho}$ = 0.91 with z statistic 34.22, significant at 5% level.

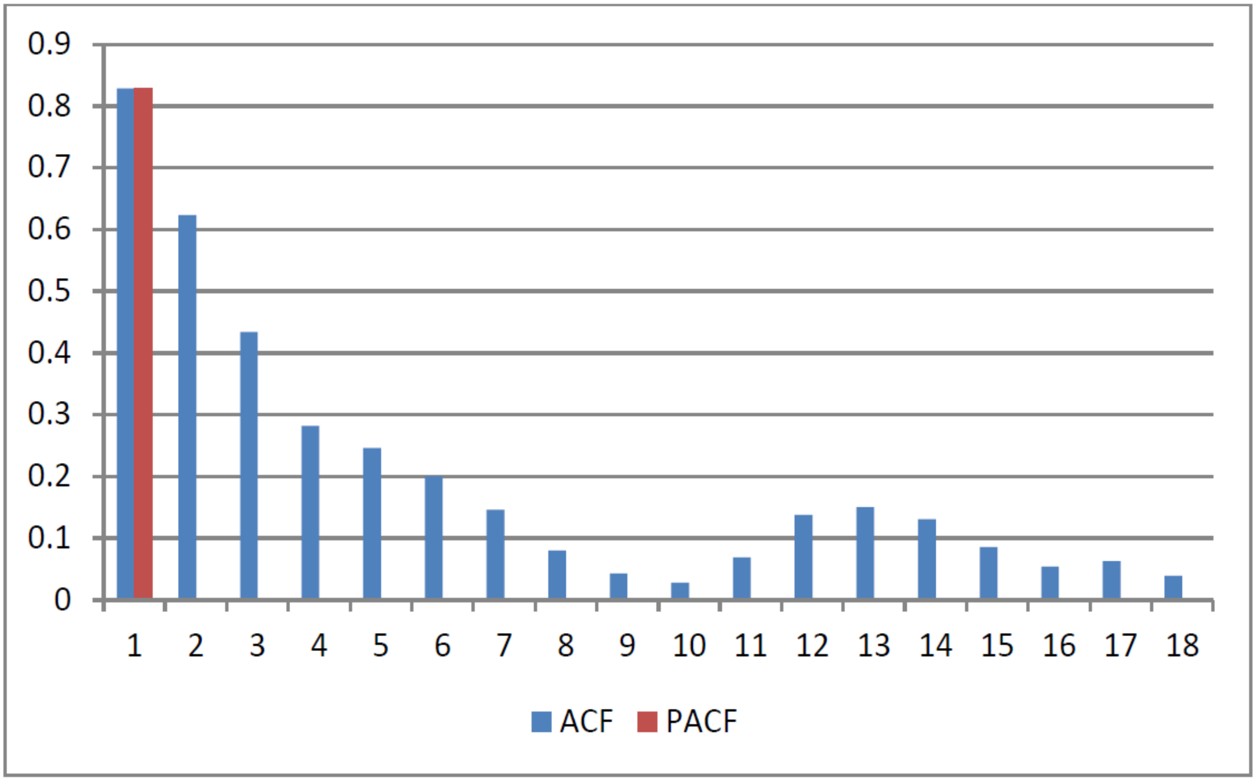

Figure 4: The Autocorrelation and Partial Autocorrelation

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