Central bank and asymmetric preferences: An application of sieve estimators to the U.S. and Brazil

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Central Bank and asymmetric preferences: an application of sieve estimators to the U.S. and Brazil*

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Summary

Whether central banks place the same weights on positive and negative deviations of inflation and of the output gap from their respective targets is an interesting question regarding monetary policy. The literature has been trying respond to this question using a specific asymmetric function, the linex. However, been asymmetric, is the linex the real specification? Trying answering this question, we applied a fully nonparametric estimation, the sieve estimator method, were the result could be any make sense function. This way, our results could corroborate with the quadratic or linex loss functions that have been used in the literature, or suggest an entirely new function. We applied the sieve estimator method for the United States and Brazil, a developed and an important emergent country which has consistently followed an inflation target system. The economy was modeled with forward-looking agents and central bank commitment. Our results indicate that the FED was more concerned with inflation rates below its target, but no asymmetry was found regarding inflation in the Volcker-Greenspan period neither regarding output gaps. As to Brazil, we found asymmetries regarding output gaps 2004 onwards, where the Brazilian Central Bank was more concerned with positive output gaps. Regarding inflation, we did not find any statistically significant asymmetries.

Key words: Monetary Policy. Central Bank's preference. Asymmetric preferences. Sieve estimators.

JEL: C14, E52

Highlights

1.We model asymmetries in the preferences of the FED and Brazilian Central Bank.

2. We use a nonparametric estimator for the monetary authoritys preference.

3. FED shows greater losses for inflations below the target in 1960-2011 period.

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4. Brazilian Central Bank was more concerned with positive output gaps.
5. No other asymmetries were found for the U.S. or Brazil.

1 Introduction

The literature pays special attention to how central banks conduct the monetary policy. They act to control inflation and, owing to frictions in the economy, they also affect the real side in the short run. As monetary authorities have the power to affect the output temporarily, they likely include output and employment stability among their goals, in addition to price stabilization. Nonetheless, central banks do not always explicitly disclose their goals. Actually, their reaction to changes in economic variables through the so-called reaction functions is perceived more easily. In the seminal work by Taylor [1993], the monetary authority adjusts the nominal interest rate in response to deviations of inflation from its target and from the output gap.

This monetary authoritys reaction function can be construed as the result of a model that treats the central bank as an usual economic agent, seeking to minimize a loss that depends on deviations of the inflation and the output as the consumer seeks to maximize her utility. Moreover, as consumer's decisions are limited by her budget constrain, the actions of the central bank are constrained the structure of economy, materialized usually in the investment-saving (IS) and aggregate supply (AS) curves. Therefore, observing the reaction function alone, to what extent the interest rate responds to inflation and output deviations, for example, does not allow distinguishing between how much of this effect comes from the monetary authoritys preferences and how much is imposed by the structure of the economy. Several studies try to make a distinction between these effects by estimating the central banks preferences and not only the reaction function. The traditional approach to this problem consists in modeling the economy (IS and AS curves) as linear equations, either backward-looking or forward-looking, and modeling the monetary authority as an agent that minimizes a loss that is a quadratic function of the difference between inflation and its target and of the output gap. A great advantage of this approach is its tractability, which can be solved analytically[1]. Another advantage of the quadratic loss function is that it can be seen as a second-order Taylor approximation to the expected utility function from a representative agent in a general equilibrium model with rational expectations and price frictions [Woodford, 2003, chapter 6], that is, it would represent the social preference over inflation and output.

However, the linear-quadratic approach has some drawbacks. It implies that the same weights are given in the loss function to positive and negative deviations from its target variables. For

[1] This approach consists in a dynamic programming problem with quadratic objective function and linear transition functions, which is solved by Ricatti equation. See, for instance,Ljungqvist and Sargent [2004, chapter 5].

instance, the loss the monetary authority would have with inflation one percentage point above its target would be the same as the loss resulting from inflation one percentage point below the target, which is not necessarily true: to increase its credibility, the central bank might show greater aversion to inflation levels that exceed its target, as a precautionary measure. On the other hand, when there is a risk of deflation, the monetary authority might have a greater aversion to inflation below its target and, mainly, to decreases in the price index (deflation). Second, this approach produces linear reaction functions with a constant marginal effect of the inflation over the interest rate, both for inflation rates close to or far away from their targets, and the same would occur with the output gap.

The first alternative to obtain reaction functions that are nonlinear is to assume nonlinearities in the structural functions of the economy. Theoretically, Nobay and Peel [2000] demonstrate that a discretionary policy under a nonlinear Phillips curve produces an inflationary bias if the output corresponds to the potential output. Schaling [2004], based on a theoretical model in which the Phillips curve is convex (positive deviations of the variables are more inflationary than negative deviations are disinflationary), shows that the larger the economic uncertainty, the more aggressive the monetary policy conduct should be. Using U.S. data to calibrate their model, Chiarella et al. [2003] model the economy as a dynamic system with several equations, including a Phillips curve with downwardly rigid wages and prices, and find that the equilibrium of such system is unstable. Dolado et al. [2005], based on a nonlinear Phillips curve (quadratic in terms of output gap), obtain a reaction function in which one of the terms is the interaction (multiplication) between inflation expectation and the output gap. Using monthly data, the authors found evidence of asymmetry in the monetary policy conduct for Germany, France, Spain, and for the Eurozone, but they could not reject the hypothesis of linearity for the U.S. case. Finally, Komlan [2013] found asymmetries in the preferences of the Canadian monetary authority, with a greater loss for inflation rates above its target, using a threshold model, in which the central bank has two different quadratic loss according to a threshold variable.

The second alternative to obtain nonlinear reaction functions is by modeling the monetary authoritys loss function as a possibly asymmetric function, unlike the usual quadratic function described in the literature. In this vein, Orphanides and Wieland [2000] developed a model with nonlinearities both in the structure of the economy and in the monetary authoritys preferences. They consider that the monetary authoritys loss function contains an interval on which it is linear, representing the target zone for inflation. In addition, they also allow the Phillips curve to have a linear segment. As a result, they show that there is some incentive for the monetary authority to shift away from a linear policy. Cukierman and Muscatelli [2002] develop a model in which the specification of the loss function is generic and, from a first-order condition of the monetary policy, they carry out a comparative static exercise. Kim et al. [2005] estimated the central banks loss function non-parametrically using the method proposed by Hamilton (2001)[2] and found asymmetry in the function for the pre-Volcker period, but linearity for the whole sample and for the Volcker-Greenspan period.

In the same line, Surico [2007], based on Nobay and Peel [2003], estimated the asymmetry of the U.S. central banks loss function, both for inflation and for the output gap, using a parametric function which allows nonlinearities and has the quadratic form as a special case[3]. By modeling the behavior of the monetary authority as discretionary, the author found evidence of asymmetry in the loss associated with the output gap for the pre-Volcker period, in which the central bank was far more concerned with negative than with positive output deviations. The author, however, did not reject the hypothesis of symmetry of the loss function for inflation. The method proposed by Surico [2007] was applied to Brazil by Aragón and Portugal [2010], who found asymmetry in the Brazilian central banks loss associated with inflation for the 2000-2007 period, with inflation rate below the target, causing a welfare loss higher than the above-target inflation. For the 2004-2007 subperiod, or for the output gap in the whole sample or in this subperiod, they did not find evidence of asymmetry.Naraidoo and Raputsoane [2011] also used the linex specification, combining it with target zones for inflation, and applied the model to South Africa. They found that there exist target zones for both inflation and output and that outside these zones the preferences are symmetric for inflation and asymmetric for the output gap.

The literature on the preferences regarding the monetary authoritys reaction function shows that the hypotheses of the model have been relaxed, including the hypothesis that posits that the preferences are symmetric. The linex specification used by Nobay and Peel [2003], Surico [2007] and Aragón and Portugal [2010] generalizes the loss function, but it is still a parametric assumption about the central banks behavior. The natural path for the improvement of this literature is to further reduce restrictions to the functional form of the monetary authoritys preferences, with a nonparametric estimation. Its advantage is do not impose any functional form on the loss function; this way, the estimated function could corroborate another one currently used in the literature or suggest another functional form for the loss function. Thus, the aim of this paper is to estimate the central banks loss function using a nonparametric estimation method. For that purpose, an

[2]Hamilton [2001] estimated a model of the form $y = \mu(x) + \epsilon$, where $\mu(\bullet)$ is an unknown function, considering μ, a random variable itself.

[3]This function, known as the linex specification, is given by $L(\pi_t, x_t) = \alpha^{-2}\left[e^{\alpha(\pi_t - \pi^*)} - \alpha(\pi_t - \pi^*) - 1\right] + \lambda\gamma^{-2}\left(e^{\gamma x_t} - \gamma x_t - 1\right)$, in which parameters α and γ capture the asymmetry of the function. With $\alpha = \gamma = 0$, this specification collapses in the quadratic loss function. In the original model, the function also depends on an quadratic parameter of smoothing of interest rate.

optimization model with monetary authoritys commitment is developed and, based on the resulting restrictions on the conditional moments of the variables, a sieve estimator is used to estimate the function that represents the central banks preferences. The method is applied to a developed country, the U.S., and to an emerging country, Brazil. Comparing a developed with an emerging country is important because the preferences of their monetary authorities could be rather different. Moreover, between the emerging countries, Brazil was chosen because it is the largest emerging economy that uses explicitly and consistently a system of inflation targeting. This way, it will be possible to determine whether the preferences of these two authorities are asymmetric in relation to inflation and (or) to the output gap and which the loss functions are.

However, the great liberty in the nonparametric estimation do not come at no costs. When we carry out a parametric estimation, for example a quadratic central bank loss function, $L = a\pi^2 + bx^2$, we look for two numbers in the real line, a and b, such that they minimize some criteria function which depends on these parameters and on the data observed. On the other hand, in a nonparametric estimation we look for a function, not a parameter, which lies in a functional space. It is important to realize that this search is lot harder, since there are "a lot more"[4] functions than there are pair of number in $\mathbb{R}^2$. For example, this functional space contains all quadratic functions $L = a\pi^2 + bx^2$ for all possible choices of a and b. Also, it also contains all possible linex functions, and so on. In order to address such issues, we use the sieve method proposed by Grenander [1981], which consists in minimizing the criterion function in a sequence of simpler spaces, generally of finite dimension, whose dimension increases with sample size. In other words, with this method we return, in a certain way, to a finite dimensional space, a bigger and more complex space, but still finite.

Besides this introduction, this paper is organized as follows. In the second section the economy's structure is descripted. The method used for estimation is presented in the third section. In the fourth section are presented the results, for both the U.S. and for Brazil. A brief conclusion follows.

2 The economy

The economy is described as in Svensson and Woodford [2005], been characterized by two structural equations, an IS and an AS, both increased by expectations and forward-looking, that is, they include expectations about future values of inflation and output gap. These two equations represent the log-linearization of the equilibrium of a microfounded model, as in Clarida et al. [1999], differing only as to the incorporation of an unpredictable random error unaffected by the monetary policy,

[4]Precisely, the carnality of the functional space considered here is greater than the cardinality of R^n, for all n, even $n = \infty$.

which makes changes in the interest rate by the central bank do not affect the inflation in the same period, as usual observed in the data. In order to achieve this, in both AS (equation (1)) and in IS (equation (2)) the expectation is obtained one period earlier than in the conventional literature, making the prices predefined in this economy. The model follows the hypotheses below:

Hypothesis 1. *The economy is represented by an AS and a IS curve of the form*

$$\pi_{t+1} = \beta E_t \pi_{t+2} + \kappa E_t x_{t+1} + u_{t+1} \tag{1}$$

$$x_{t+1} = E_t x_{t+2} - \phi \left(E_t i_{t+1} - E_t \pi_{t+2} - r_{t+1} \right), \tag{2}$$

where π_{t+1} *is the inflation between periods* t *and* $t+1$, x_{t+1} *is the output gap,* i_t *is the short-term nominal interest rate (deviation from its steady-state value) and* u_{t+1} *and* r_{t+1} *are error terms. In addition,* $0 \leq \beta < 1$ *is the intertemporal discount factor and* $\kappa > 0$ *and* $\phi > 0$ *are constants.* E_t *is the expectation operator conditional on information at* t.

For the following analysis, it is also necessary to make an assumption about the evolution of the error terms.

Hypothesis 2. *The error terms* u_{t+1} *and* r_{t+1} *follow the transition equations*

$$u_{t+1} = \varrho u_t + \varepsilon_{t+1} \tag{3}$$

$$r_{t+1} = \overline{r} + \omega \left(r_t - \overline{r} \right) + \eta_{t+1}, \tag{4}$$

where ε_{t+1} *and* η_{t+1} *are independent and identically distributed random variables,* $0 \leq \varrho < 1$ *and* $0 \leq \omega < 1$ *are constants and* $\overline{r}$ *is the mean natural real interest rate.*

Although this linear forward looking economy is a simplification, it is common in this literature and allows a simpler solution of the central bank's optimization problem and a fewer number of parameters to be estimated, which is an important issue due the small size of macroeconomicss time series data. Even so, this forward looking setting is an advance over some others papers on central bank's asymmetric preferences, which use a backward looking and, because of that, a discretionary equilibrium.

2.1 Optimal monetary policy

The central bank is modeled as a rational economic agent who seeks to minimize its loss, which is assumed to be a function of the deviation of inflation from its target, of the output gap and of the deviation of the interest rate from its target, given the equations that describe the economy. In the absence of these restrictions, it would set $\pi_t = \pi^*$ and $x_t = 0$ at all times; in addition, it would like to maintain the interest rate at a constant level, which should be consistent with the natural output level. The first two goals are usual in the literature; the latter, which represents the preference over smaller interest rate variability, is justified by the monetary authoritys concern with transactions frictions, as shown in Woodford [2003, 6.4.1, p. 420][5]. Like the agent's utility in the Consumer Theory, this loss can be represented by a function with certain characteristics.

Hypothesis 3. *The monetary authority can be described by a separable loss function*

$$L\left(\pi_t - \pi_t^*,\ x_t,\ i_t\right) = \tilde{L}\left(\pi_t - \pi_t^*,\ x_t\right) + \frac{\gamma}{2}\left(i_t - i_t^*\right)^2, \tag{5}$$

where π_t^* and i_t^* are the inflation and interest rate targets [6], respectively, for period t and $\gamma > 0$ is a parameter that measures the weight given by the monetary authority to interest rate smoothing.

Hypothesis 4. *The function $\tilde{L}(\bullet)$ that measures the loss associated with inflation and with the output gap,*

(i) has as minimum point $(0,\ 0)$;

(ii) crosses $\tilde{L}(0,0) = 0$;

(iii) is limited;

(iv) is supermodular, i.e.,

$$\frac{\partial^2 \tilde{L}(\bullet)}{\partial \tilde{\pi}_t^2} \geq 0,\ \ \frac{\partial^2 \tilde{L}(\bullet)}{\partial x_t^2} \geq 0,\ \ \frac{\partial^2 \tilde{L}(\bullet)}{\partial \tilde{\pi}_t \partial x_t} \geq 0, \tag{6}$$

where $\tilde{\pi}_t = \pi_t - \pi_t^*$ is the deviation of inflation from its target;

(v) belongs to the square-integrable function space, $\tilde{L}(\bullet) \in \mathcal{L}^2(\chi)$, defined in a compact set χ.

Supposing that the minimum point of the monetary authoritys loss function is on $(0,\ 0)$ is important because, otherwise, the function would imply a target (aimed to be achieved by the central bank) that differs from that specified earlier, which is currently used in the literature. The

[5] For empirical analysis of interest rate smoothing, see Clarida et al. [1998], Woodford [1999], and Sack and Wieland [2000]. On the other hand, Consolo and Favero [2009] argue that the evidences of interest rate inertia might occur due a misspecified model.

[6] Note that the monetary authoritys target for the output gap is assumed to be equal to zero, as usually described in the literature. As interests are expressed in deviations from their steady-state value, the interest target is also considered to be equal to zero.

assumptions that the function is supermodular and that it is limited warrants the uniqueness of the reaction function in relation to optimal monetary policy. Supermodularity implies that the marginal loss of inflation increases in the output gap and vice versa, a fact that also occurs in the usual quadratic loss function. Intuitively, a supermodular loss function implies that their arguments are complementary. For example, the marginal loss of a certain level of inflation is higher as further is the production from its target. In addition, supposing that the loss is square-integrable enables us to calculate the expectation of the monetary authority's future losses, assumption which is explicitly or implicitly made in all multi-period models.

Hypothesis (3) is the major contribution of this paper, as it does not assume any specific loss function. The fact that it is separable in the interest rate is a simplification of the present model, but it is recurrent in other studies in this area when a quadratic loss function is assumed (see footnote (5)).

Given the restrictions imposed by the structure of the economy, the monetary authority minimizes the expected current value of its future losses. Here it is assumed that the monetary authority does that with commitment, i.e., it takes into account the effect of its actions on the expectations of the private sector and the effects of these expectations on possible future actions. The central bank uses the short-term nominal interest rate as instrument, but this instrument also affects the expectations about inflation and about the output gap of the private sector initiative. Thus, its optimization problem is to minimize the current value of its loss (5) subject to (1) and (2), represented by the following Lagrangian:

$$\begin{aligned}\min_{\{\pi_{t+1},x_{t+1},i_{t+1}\}} \mathfrak{L} &= E_{t_0}\sum_{t=t_0}^{\infty} E_t\left[\beta^{t+1-t_0} L\left(\tilde{\pi}_{t+1},\ x_{t+1},\ \tilde{i}_{t+1}\right)\right. \\ &+ \lambda_{1,t+1}\left(\pi_{t+1}-\beta E_t\pi_{t+2}-\kappa E_t x_{t+1}-u_{t+1}\right) \\ &+ \left.\lambda_{2,t+1}\left(x_{t+1}-E_t x_{t+2}+\phi\left(E_t i_{t+1}-E_t\pi_{t+2}-r_{t+1}\right)\right)\right],\end{aligned} \tag{7}$$

where $\tilde{i}_t = i_t - i_t^*$ is the deviation of the interest rate from its target.

The first-order conditions of the problem are

$$\frac{\partial \mathfrak{L}}{\partial \pi_{t+1}} = E_t\left[\beta^{t+1-t_0}\tilde{L}_{\pi,t+1}+\lambda_{1,t+1}-\beta\lambda_{1,t}-\lambda_{2,t}\right]=0 \tag{8}$$

$$\frac{\partial \mathfrak{L}}{\partial x_{t+1}} = E_t\left[\beta^{t+1-t_0}\tilde{L}_{x,t+1}-\kappa\lambda_{1,t+1}+\lambda_{2,t+1}-\lambda_{2,t}\right]=0 \tag{9}$$

$$\frac{\partial \mathfrak{L}}{\partial i_{t+1}} = E_t\left[\beta^{t+1-t_0}\gamma\tilde{i}_{t+1}+\phi\lambda_{2,t+1}\right]=0, \tag{10}$$

$\forall t = t_0 + 1, \ldots$, where, for simplicity of notation, $\tilde{L}_{\pi,t+1} = \partial \tilde{L}(\tilde{\pi}_{t+1}, x_{t+1}) / \partial \pi_{t+1}$, analogous to $\tilde{L}_{x,t+1}$. By Isolating $\lambda_{2,t+1}$ in (10) and substituting it into (9), it is possible to solve this equation for $\lambda_{1,t+1}$, such that

$$\lambda_{1,t+1} = \kappa^{-1} \left(\beta^{t+1-t_0} \tilde{L}_{x,t+1} - \beta^{t+1-t_0} \frac{\gamma}{\phi} \tilde{i}_{t+1} + \beta^{t-t_0} \frac{\gamma}{\phi} \tilde{i}_t \right). \tag{11}$$

Substituting the multipliers in (8), one obtains Euler equation of the monetary authority under commitment for the economy described in this section,

$$E_t \left[L_{\pi,t+1} + \frac{(L_{x,t+1} - L_{x,t})}{\kappa} - \frac{\gamma}{\kappa\phi} \tilde{i}_{t+1} + \left(\frac{\gamma}{\kappa\beta\phi} + \frac{\gamma}{\kappa\phi} + \frac{\gamma}{\beta} \right) \tilde{i}_t - \frac{\gamma}{\kappa\beta\phi} \tilde{i}_{t-1} \right] = 0. \tag{12}$$

As equation (12) must be satisfied throughout the period $t = t_0 + 1, \ldots$ so that the monetary authority optimizes its preferences, it is one candidate to optimal policy [Woodford, 2003, section 7.5.1].

2.2 Reaction function

The model is based on the same assumption as Woodford [2003] concerning the central bank's behavior. The central bank must choose its instrument, i_{t+1}, using a reaction function that guarantees the validity of the optimal policy rule. That is, the interest rate i_{t+1} must guarantee that the projections of inflation and of the output gap, $E_t \pi_{t+1}$ and $E_t x_{t+1}$ are consistent with (12). First, one must postulate how private agents form their expectation: it will be assumed that they form it rationally[7], with the projections of these two variables given by the expectation in t defined in equations (1) and (2),

$$E_t x_{t+1} = E_t \left(x_{t+2} - \phi \left(i_{t+1} - \pi_{t+2} - r_{t+1} \right) \right) \tag{13}$$

$$E_t \pi_{t+1} = E_t \left(\left(\beta + \kappa\phi \right) \pi_{t+2} + \kappa x_{t+2} - \kappa\phi \left(i_{t+1} - r_{t+1} \right) + u_{t+1} \right). \tag{14}$$

Therefore, the monetary authority must, at each time t, choose i_{t+1} that satisfies (12) with x_{t+1} and π_{t+1} replaced by (13) and (14), respectively. Given the nonparametric nature of the problem, there is no explicit solution for i_{t+1} and, besides, it is not possible to guarantee a priori that this choice represents an implicit reaction function in the optimal policy rule, because the interest rate i_{t+1} consistent with the optimal policy (12) may not be unique. However, given the assumptions

[7] Although rationality is a strong assumption, it is a current one in models like this one. Relaxing it, assuming that some agents might not be fully rational, would imply, in general, in a strongest serial correlated pattern in the economic variables.

we have made in this paper, we can show that such reaction function do exist, as stated in the following proposition.

Proposition 1. *Given the assumptions* (1) *to* (4)*, there is an unique reaction function* $i_{t+1} = f(\bullet)$ *implied by the optimal policy rule* (12)*.*

Proof. Let $\varphi(i_{t+1}|\bullet)$ be the left-hand side of equation (12). There exists an unique reaction function if and only if the i_{t+1} which solves $\varphi(i_{t+1}|\bullet) = 0$ exists and is unique. In order for i_{t+1} to exist it suffices that the function $\varphi(i_{t+1}|\bullet)$ has a sign change and that the sign of $\partial\varphi(i_{t+1}|\bullet)/\partial i_{t+1}$ is equal to the sign of $\lim_{i_{t+1}\to\infty}\varphi(\infty|\bullet)$. As $\lim_{i_{t+1}\to-\infty}\varphi(-\infty|\bullet) = \infty > 0$ and $\lim_{i_{t+1}\to\infty}\varphi(\infty|\bullet) = -\infty < 0$, the function has a solution. $\partial\varphi(i_{t+1}|\bullet)/\partial i_{t+1} = -\kappa\phi\tilde{L}_{\pi\pi} - 2\phi\tilde{L}_{\pi x} - \phi/\kappa\tilde{L}_{xx} - \gamma/\kappa\phi < 0$ since all parameters are positive (hypotheses (1) and (3)) and the loss function is supermodular (hypothesis (4)), guaranteeing the uniqueness of the solution (where $\tilde{L}_{\pi\pi} = \partial^2\tilde{L}(\bullet)/\partial\pi^2$, being analogous to the other partial derivatives of the equation). □

Once confirmed that the reaction function exists, it provides the optimal short-term nominal interest rate that the central bank must choose to satisfy its monetary policy rule. In the present model, the optimal interest rate at t is a function $i_{t+1} = f(\pi_t,\ x_t,\ i_t,\ i_{t-1},\ E_t\pi_{t+2},\ E_tx_{t+2},\ E_tu_{t+1},\ E_tr_{t+1})$ of the current economic status (inflation and output gap at t and interest rate at t and $t-1$) and of the expectation at t as to the future state of the economy (inflation, $E_t\pi_{t+2}$, output gap, E_tx_{t+2}, and economic shocks, E_tu_{t+1} and E_tr_{t+1}).

3 *Sieve minimum distance estimate*

As mentioned in the previous sections, the aim of this paper is to estimate the central banks loss function. Estimating a function means estimating a parameter of an infinite-dimensional space, which could be difficult to implement, in addition to having undesirable asymptotic properties, such as inconsistency and a low convergence rate [Chen, 2007, p. 1].

Grenander [1981] proposed to estimate the parameters over a sequence of less complex spaces, usually of finite dimension, and let the dimension of these spaces grow according to the sample size, but slower so that the estimator is consistent. Roughly speaking, we replace the unknown function we want to estimate for another one which is a finite combination of some polynomials (or other basis function that can represent well the unknown function), for example $f = ax + bx^2 + cx^3 + dx^4$. Then, we return to a parametric problem. However, there is statistical issues that make the sieve method needed, and not just replacing the unknown function for a polynomial.

To guarantee the estimators desired properties, it is necessary, in addition to the growth rate of the space's complexity used in the approximation, that it approximates well the true space where the population parameter is inserted. In order for it to be a good approximator, it is necessary that the approximate space be dense in the true space. The fact that a set $\mathcal{M}$ is dense in another set, $\mathcal{N}$, implies that, for any point $n \in \mathcal{N}$, there is an point $m \in \mathcal{M}$ sufficiently close, or, equivalently, all the open balls of $\mathcal{N}$ includes a point of $\mathcal{M}$. Thus, if the approximate space is asymptotically dense in the original space, one can find an estimator for the population parameter sufficiently close when the sample size increases.

As the loss function belongs to $\mathcal{L}_2(\chi)$, $\chi \subset \mathbb{R}^2$, compact, the space to be used as approximation should be dense in this set. A space generated by a basis containing orthogonal polynomials that go from χ to $\mathbb{R}$ is dense in $\mathcal{L}_2(\chi)$.

The approximation of a function by a basis of its space has a global characteristic, in which one finds the function that is closest to the original one, with the concept of proximity given by a norm of the space that, in general, takes into account all points of the domain. This differs from what occurs with the Taylor approximation, which has a local characteristic, i.e., the level and slope of the original function are equaled in a given point of the domain, but this does not guarantee that this approximation is good in points outside from this neighborhood.

Also, for this problem, the sieve estimator has an advantage over others nonparametric estimators, such as local polynomial regression, because this last one would allow us estimate the conditional mean of the ρ function, but we are interested in the loss functions L, which is an argument of ρ. In short, the semiparametric nature of the sieve estimator, combining the unknown loss function with the parametric structure of the economy, allow us to identify L.

3.1 Loss function approximation

The basis chosen to approximate the loss function was the sequence of Chebyshev polynomials. The use of this basis, to the detriment of others that are commonly employed in the literature, such as splines and neural networks, is primarily due to the fact that polynomials have an explicit and simple form for their derivatives, which is how the monetary authoritys loss function to be estimated enters the criterion function. A second aspect is that, according to the assumptions of the economic model to be estimated, the central banks loss function is strictly concave in order to meet the banks optimization requirements. Furthermore, the function is smooth, which eliminates the one of the advantages of splines.

Because it is reasonable that the pair deviation of inflation from its target and output gap is contained in $[-1,\ 1]^2$, the domain of the polynomials did not have to be altered. Let U_n be the

Chebyshev polynomials of level n. The monetary authoritys loss function is approximated by

$$L\left(\tilde{\pi}_t, x_t, \tilde{i}_t\right) = \tilde{L}\left(\tilde{\pi}_t, x_t\right) + \frac{\gamma}{2}\left(\tilde{i}_t\right)^2, \tag{15}$$

$$\tilde{L}\left(\tilde{\pi}_t, x_t\right) = \sum_{i=1}^{K_\pi} \psi_{i,0} U_i\left(\tilde{\pi}_t\right) + \sum_{i=1}^{K_x} \psi_{0,i} U_i\left(x_t\right) + \sum_{i=1}^{k_\pi}\sum_{j=1}^{k_x} \psi_{i,j} U_i\left(\tilde{\pi}_t\right) U_j\left(x_t\right) + \psi_{0,0}. \tag{16}$$

If the complete tensor[8] had been used, one would have $K_\pi = K_x = k_\pi = k_x$. For the sake of efficiency, $K_\pi = K_x = 5$ and $k_\pi = k_x = 1$ were used here. This approximation level allows for sufficient asymmetry in the loss function, in addition to providing an interaction term between inflation and the output gap.

In order for the estimated loss function to comply with hypothesis (4), the optimization must occur with some restrictions. Guaranteeing that the minimum of the function occurs in point (0, 0) allows reducing two parameters that need be estimated. Therefore, solving the equations that imply that (0, 0) is the minimum, $\partial \tilde{L}(0,0)/\partial \tilde{\pi}_t = 0$ and $\partial \tilde{L}(0,0)/\partial x_t = 0$, it is possible to let $\psi_{1,0}$ and $\psi_{0,1}$ be a function of other parameters, $\psi_{1,0} = -\left(U_1(0)\right)^{-1}\left(\sum_{i=2}^{K_\pi} \psi_{i,0} U_1'(0) + \sum_{i=1}^{k_\pi}\sum_{j=1}^{k_x} \psi_{i,j} U_i'(0) U_j(0)\right)$ and $\psi_{0,1} = -\left(U_1(0)\right)^{-1}\left(\sum_{i=2}^{K_x} \psi_{0,i} U_1'(0) + \sum_{i=1}^{k_\pi}\sum_{j=1}^{k_x} \psi_{i,j} U_i(0) U_j'(0)\right)$, eliminating them.

As optimization problems are indifferent to affine transformations[9] in the objective function, for identifiability reasons, one must normalize the loss function to be estimated. In this paper, we normalized the function by allowing $\psi_{2,0} = 1$, which is equivalent to dividing it by $||\psi_{2,0}||$.[10] Then, we have three parameters that we do not need to estimate.

To guarantee the supermodularity of the loss function, the estimated parameters must satisfy equation (6) $\forall \tilde{\pi}, x \in (-1, 1)$[11]. This condition is checked in the estimation, but not used to eliminate other parameter because it does not allow to rewrite one parameter in function of the others, as the other conditions do.

[8] A basis for a space of functions defined in a two-dimensional set $\chi = \chi_1 \times \chi_2 \subset \mathbb{R}^2$ is a tensor of the bases of the univariate function spaces defined in χ_1 and χ_2. Let B_1 be a basis of space $\mathcal{L}_2(\chi_1)$ and B_2 a basis of space $\mathcal{L}_2(\chi_2)$. Then a basis for $\chi = \chi_1 \times \chi_2$ is the tensor (set) $B_1 \otimes B_2 = \{f(\pi, x) = b_1(\pi) b_2(x) \mid b_1 \in B_1,\ b_2 \in B_2\}$. However, the amount of elements in a tensor increases exponentially with their dimension, leading Judd [1998] to argue that it is more efficient to use only the components of the tensor $f(\pi, x) = b_1(\pi) b_2(x)$ that have a lower or the same level as a k chosen in advance (this is known as complete polynomials; a heuristic justification given by Judd [1998] is the multivariate Taylor approximation, in which the term of order k of the expansion is composed of polynomials $x_1^{i_1} \ldots x_n^{i_n}$ such that $\sum_{l=1}^{n} i_l = k$) . In this paper, this method was also used in an attempt to obtain a higher computational efficiency.

[9] Affine transformation is a linear operator of the form $z \mapsto az + b$, $a > 0$.

[10] Based on several previous estimates, it is known that $\psi_{2,0} > 0$.

[11] In practice, as the values for inflation and output gap are around zero and far from -1 and 1, supermodularity was demanded only in $(-0.9, 0.9)$. This prevents the rejection of candidate functions that do not satisfy this condition only in the limits of their support.

3.2 Conditional moments and instruments

The major parameter of interest in this paper is the central banks loss function. A natural choice for its estimation is Euler function (12). Nevertheless, this function does not contemplate all the information available in the monetary authoritys optimization problem. To include this piece of information, it is necessary to jointly estimate the structural equations of the economy, AS (1) and IS (2) equations.

Euler equation is already in the conditional moment format required by the method. On the other hand, the other two equations have to be transformed. By applying the conditional expectation to the information set at time t in (1), one obtains $E_t\left[\pi_{t+1} - \beta\pi_{t+2} - \kappa x_{t+1}\right] = E_t\left[u_{t+1}\right]$. However, as $E_t\left[u_{t+1}\right] = \varrho u_t \neq 0$, one still does not have a conditional moment whose expectation is equal to zero. It is necessary to subtract the amount $E_t\left[u_{t+1}\right] = E_t\left[\varrho u_t\right] = E_t\left[\varrho\left(\pi_t - \beta\pi_{t+1} - \kappa x_t\right)\right]$ from both sides of the equation, in which hypothesis (2) on error term structure was used. Thus, we have the conditional moment restriction

$$E_t\left[-\beta\pi_{t+2} + \left(1 + \varrho\beta\right)\pi_{t+1} - \varrho\pi_t - \kappa x_{t+1} + \varrho\kappa x_t)\right] = 0. \tag{17}$$

Analogously, from (2), it is possible to obtain the third moment restriction to be estimated,

$$E_t\left[x_{t+2} + \left(1 + \omega x_{t+1}\right) - \omega x_t + \phi\left(i_{t+1} - \omega i_t\right) - \phi\left(\pi_{t+2} - \omega\pi_{t+1}\right) - \phi\left(1 + \omega\right)\overline{r}\right] = 0 \tag{18}$$

This way, the vector with restrictions on conditional moments that characterize the economy $E\left[\boldsymbol{\rho}\left(\boldsymbol{Z},\ \boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)\right)|\boldsymbol{X}\right] = \boldsymbol{0}$ is given by equations (12), (17) and (18).

In the present model, all the variables used, $\{\pi_t\ , x_t\ , i_t,\ t = t_0, \ldots, N\}$, are endogenous, i.e., all of them are determined by the equations that describe it.

As outlined in the previous section, the first step is to estimate the expectation of conditional moments using the linear sieve estimator, as a function of the variables present in the information set. At time t, the information set is given by all the history of the endogenous variables, $I_t = \{\pi_\tau,\ x_\tau,\ i_\tau,\ \tau = t_0, \ldots, t\}$. However, in practice, it is not necessary to use all variables present in the information set. In this paper, we used only the variables from time t, in addition to those from the previous period. In this way, one has a basis with 14 elements for the approximation of $\boldsymbol{m}\left(\boldsymbol{X},\ \boldsymbol{\alpha}\right)$,

$$\boldsymbol{p} = \left(1, \pi_t, x_t, i_t, \pi_t^2, x_t^2, i_t^2, \pi_t x_t, \pi_t i_t, x_t i_t, \pi_t x_t i_t, \pi_{t-1}, x_{t-1}, i_{t-1}\right). \tag{19}$$

Once the monetary authoritys loss function has been estimated, one can verify whether or not it is quadratic. If so, its third derivative assumes value zero in all points of its domain and, therefore, the expectation of the third derivative is equal to zero.[12] Then, it is possible to test whether the loss function is not quadratic by testing whether the expectation of its third derivative is different from zero. As the expectation is a smooth functional and the loss function is estimated by the sieve method, it is possible to use the result obtained by Chen and Shen [1998], who show that these functionals converge to a normal distribution at the rate $N^{1/2}$14.[13] So, the test statistics is $\mu_q = E\left(\partial^3 L/\partial q^3\right)$, $q = \{\pi,\ x\}$.

4 Results

Table 1: Estimated parameters

	USA				Brazil			
	1960Q1-2011Q4		1982Q4-2011Q4		1999M4-2011M12		2004M01-2011M12	
Parameter	Value	SD	Value	SD	Value	SD	Value	SD
$\psi_{1,0}$	-0.1806	R	0.0401	R	-0.2446	R	-0.1036	R
$\psi_{2,0}$	1.0000	N	1.0000	N	1.0000	N	1.0000	N
$\psi_{3,0}$	-0.1486	0.1142	0.0575	0.0537	-0.2190	0.1467	-0.1003	0.0607
$\psi_{4,0}$	0.4197	0.0051	0.4139	0.0107	0.4203	0.0021	0.4155	0.0026
$\psi_{5,0}$	-0.0200	0.0344	0.0242	0.0165	-0.0415	0.0417	-0.0236	0.0174
$\psi_{0,1}$	0.0025	R	-0.0143	R	1.3557	R	1.4246	R
$\psi_{0,2}$	0.0063	0.0931	0.0492	0.1166	2.7048	0.4124	2.6995	0.6988
$\psi_{0,3}$	0.0025	0.0348	-0.0174	0.0368	1.4269	0.2196	1.4578	0.3705
$\psi_{0,4}$	0.0019	0.0276	0.0160	0.0314	1.1428	0.1714	1.1224	0.2928
$\psi_{0,5}$	0.0006	0.0093	-0.0062	0.0109	0.4019	0.0631	0.3886	0.1041
$\psi_{1,1}$	0.0036	0.0149	0.0147	0.0210	0.0052	0.0417	0.0214	0.0294
γ	0.0012	0.0262	0.0650	0.0332	0.0051	0.0110	0.1287	0.0225
β	0.9781	0.0018	0.9764	0.0028	0.9823	0.0060	0.9778	0.0040
κ	0.1184	0.3192	0.3226	0.3811	0.2883	0.2455	0.4477	0.1995
δ	0.0467	0.1705	0.6286	0.2085	0.1844	0.0787	2.8583	0.1129
ϕ	0.0257	R	0.1034	R	0.0276	R	0.0450	R
$\bar{r}$	0.0459	0.0040	0.0459	0.0058	0.0460	0.0077	0.0469	0.0084
ϱ	0.8145	0.0271	0.8120	0.0614	0.8919	0.0643	0.8888	0.0608
ω	0.7880	0.0597	0.7721	0.1204	0.9300	0.0511	0.8304	0.0817

Note: Value is the estimated value and SD is the estimated standard deviation. The standard deviation refers to bootstrapping with 500 replicates. Restricted parameters (R) are not estimated directly; i.e., a function is given by the model of the estimated parameters. N is the normalized parameter in the estimation.

[12] Note that the opposite does not hold.

[13] The result is shown in theorem 2 proposed by Chen and Shen [1998].

4.1 United States

Data from the first quarter of 1960 to the last quarter of 2011, totaling 208 observations, were used to model the preferences of the Federal Reserve System (Fed). The variables used followed Surico [2007]. The variation of the Personal Consumption Expenditure (PCE) deflator, available from the Federal Reserve Bank of St. Louis, was used as measure of inflation. The interest rate consisted of the Effective Federal Funds Rate and output was measured by the Industrial Production Index, both available at the Fed website. All the variables used were calculated as the value accumulated over the last four quarters and are revised data. The inflation target was assumed to be constant and equal to 2%. The output gap and the deviation of the interest rate from its steady-state values were calculated using the Hodrick-Prescott filter. We also used Baxter and King filter [Baxter and King, 1999] for robustness, and the results are in the Appendix (A.2).

Over the 5 decades included in the analysis, the economic policy conduct by the Fed has evidently gone through changes. Clarida et al. [2000] found evidence that the central banks reaction function changed after Paul Volcker took over in 1979. According to the authors, during the Volcker-Greenspan period, the U.S. monetary authority began to respond more to inflation expectation, whereas in the pre-Volcker era the policy was less active. Based on this fact, as in Surico [2007], the Feds loss function was estimated for the subsample corresponding to the Volcker-Greenspan period, more specifically to the period between the fourth quarter of 1982 and the last quarter of 2011. The author chose 1982 instead of 1979 because, according to him, the U.S. central banks policy instrument between 1979 and 1982 was no longer the short-term interest rate, being replaced with compulsory deposits. Table (2) contains the descriptive statistics of the U.S. series and it shows that the average inflation in the Volcker-Greenspan period was smaller than in the whole sample.

Table (1) shows the estimated parameters for the whole sample and for the subsample. As the parameter that shows the interaction between inflation and output gap was low, $\psi_{1,1} = 0.0036$, the analysis can be made, without loss of generality, by considering separately the losses with each of the variables. This result supports the current approaches available in the literature, in which the monetary authoritys loss function is separable in inflation and in the output gap. The loss function estimated for the whole sample resulting from this parameters is illustrated in Figure (1).

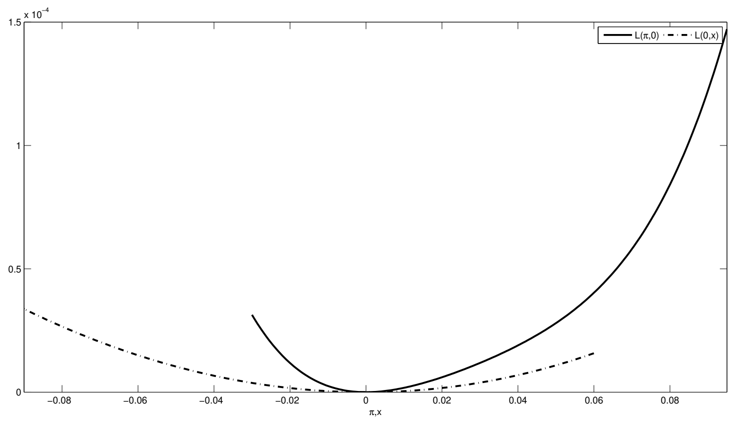
Table 2: Data statistics - USA

	1960Q1-2011Q4					1982Q4-2011Q4				
	π	x (hp)	x (bp)	i (hp)	i (bp)	π	x (hp)	x (bp)	i (hp)	i (bp)
mean	0.0355	-0.0019	-0.0022	-0.0001	0.0001	0.0255	-0.0032	-0.0034	-0.0012	-0.0010
st deviation	0.0248	0.0290	0.0282	0.0139	0.0132	0.0113	0.0271	0.0260	0.0116	0.0107
minimum	-0.0093	-0.0861	-0.0857	-0.0250	-0.0235	-0.0093	-0.0854	-0.0857	-0.0219	-0.0207
1st quartile	0.0182	-0.0171	-0.0166	-0.0110	-0.0103	0.0187	-0.0170	-0.0153	-0.0114	-0.0111
median	0.0270	-0.0004	-0.0017	-0.0011	-0.0009	0.0246	-0.0059	-0.0049	-0.0016	-0.0018
3rd quartile	0.0444	0.0182	0.0176	0.0088	0.0082	0.0348	0.0152	0.0138	0.0064	0.0069
maximum	0.1151	0.0577	0.0620	0.0463	0.0402	0.0499	0.0551	0.0532	0.0226	0.0212

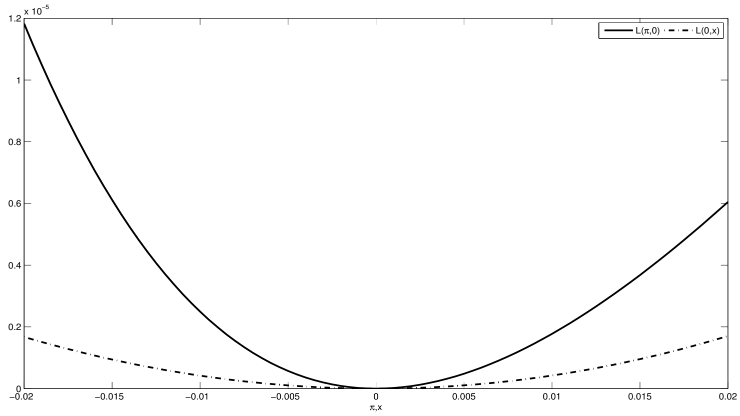
(hp): Hodrick-Prescott filter; (bk) Baxter and King filter.

Figure 1: Loss function - USA- 1960Q1 - 2011Q4

(a) all observed domain



(b) domain $(-0.02,\ 0.02)$



Note: $L(\tilde{\pi}, 0)$ shows the central banks loss function value for several inflation values, with output gap equal to zero. Analogous to $L(0, x)$. The values displayed on the upper graph represent the range of inflation and output gap. The lower graph shows only the interval $(-0.02, 0.02)$.

Table 3: Quadratic loss function test - USA

(a) 1960Q1 - 2011Q4

Statistic	Estimated value	SD	*t-value*
$\mu_\pi = E\left({}^{\partial^3 L}/_{\partial \pi^3}\right)$	-0.957	0.541	-1.768^*
$\mu_x = E\left({}^{\partial^3 L}/_{\partial x^3}\right)$	0.011	0.175	0.064

(b) 1982Q4 - 2011Q4

Statistic	Estimated value	SD	*t-value*
$\mu_\pi = E\left({}^{\partial^3 L}/_{\partial \pi^3}\right)$	0.726	0.564	1.286
$\mu_x = E\left({}^{\partial^3 L}/_{\partial x^3}\right)$	0.062	0.163	0.384

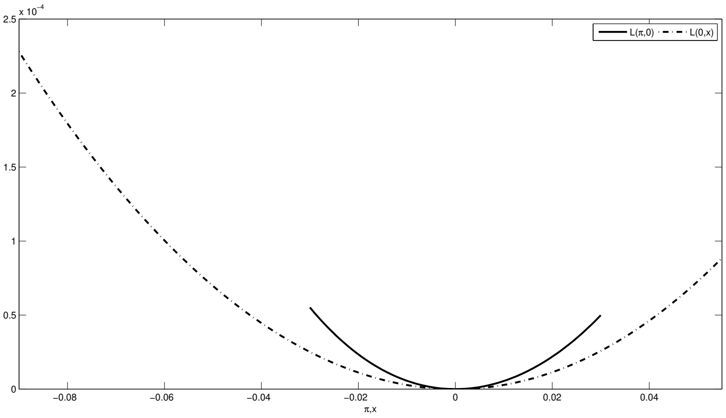
Note: The t-value corresponding to 5% is 1.65. * : significant at 5%.
The standard deviation refers to bootstrapping with 500 replicates.

According to the results, the U.S central bank showed a bigger concern with inflation than with the output gap between 1960 and 2011, as had already been pointed out in other studies: the curve of the losses associated with the inflation lies entirely above to the losses associated with the output gap. Although, as the range of the output gap was larger than the range of inflation, the lowest output gap brought a loss similar to the loss of the lowest inflation rate. Also note that the losses associated with the inflation were asymmetric to the left, with a higher loss for the central bank when the inflation was below its targets: the loss with a deviation of inflation of one percentage point below its target was 1.41 times greater than the loss with a deviation of one percentage point above its target. These results allowed to reject, at a 5% significance level, that the true loss function was quadratic in inflation, as outlined by the test in Table (3a), and, therefore, we could say it is asymmetric, with a greater concern with deviations of the inflation below its target. On the other hand, the losses associated with the output gap were rather symmetrical and it was not possible to reject, at the same level of significance, the hypothesis that the true loss function was quadratic in the output gap.

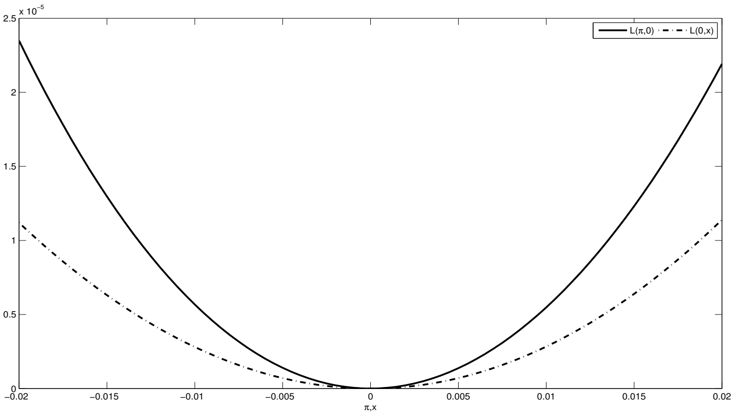
Figure (2) shows the estimated functions for the Volcker-Greenspan period. As happened for the whole period, the loss function related with the inflation lies entirely above the function related with the output gap. However, given the greater range of the output gap, a significant part of its losses were greater than the losses with the inflation. It was not possible to reject, at a 5% significance level, that both loss functions were quadratic and, therefore, we accepted them as symmetrical, although the function associated with the inflation was slight asymmetric to the right, as the positive test statistic of Table (3b) shows.

Figure 2: Loss function - USA- 1982Q4 - 2011Q4

(a) all observed domain

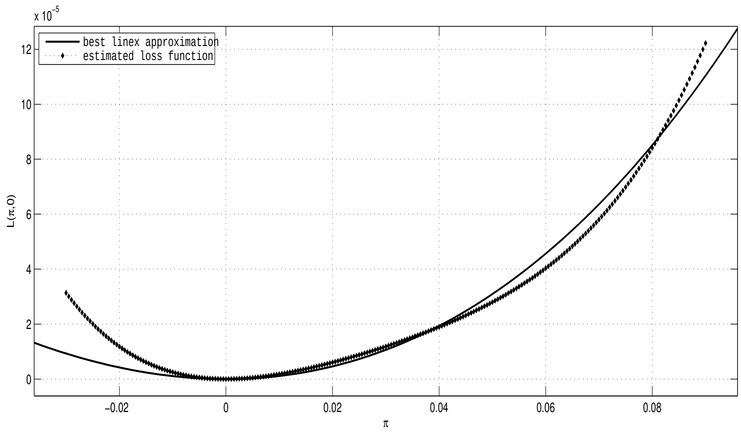


(b) domain $(-0.02,\ 0.02)$



Note: $L(\tilde{\pi}, 0)$ shows the central banks loss function value for several inflation values, with output gap equal to zero. Analogous to $L(0, x)$. The values displayed on the upper graph represent the range of inflation and output gap. The lower graph shows only the interval $(-0.02, 0.02)$.

Figure 3: Estimated $L(\tilde{\pi}, 0)$ and its best linex approximation - USA - 1960Q1 - 2011Q4



In short, we found an asymmetry in the loss function related to inflation in the whole sample, unlike from Surico [2007] who has not found asymmetries neither in the whole sample nor in the Volcker-Greenspan period. Our estimated loss function was asymmetric to the left in the whole period and, although not statistically significant, it was asymmetric to the right in the Volcker-Greenspan period, which shows a central bank more concerned about high inflations in this last period.

The fact that the sieve estimator has been able to capture an asymmetry that others papers have not shoud be due a combination of two main factors. First, here we assume a model with commitment by the central bank, and, principally, the sieve estimator is more general than the linex function assumed by others papers. Figure (3) shows this fact, with the estimated loss function related to inflation for the whole sample and its best approximation by a linex funcion. The linex function overestimates the loss with the inflation for a large range of values above the inflation target and underestimates the loss for values below its target.

4.2 Brazil

Given that the Brazilian inflation targeting regime is more recent, having started in April 1999, monthly data were used to estimate the central banks loss function. The sample contains 153 observations, from April 1999 to December 2011. To remove the effect of U.S. dollar appreciation and of the rise in inflation between 2002 and 2003 from the estimation, we also estimated the model for the period between January 2004 and December 2011.

The broad consumer price index (IPCA), which indicates the central banks actions, was used to measure inflation. Unlike the U.S. case, in Brazil, the inflation target is explicit and changes over time. As the target is for the annual inflation, it was interpolated as Aragón and Portugal [2010] did.[14] The industrial production index calculated by the Brazilian Institute of Geography and Statistics (IBGE) was used as the measure of economic output, whereas for interest rates, it was used the over-Selic rate. As with the previous estimation, all variables were used with their accumulated value over the past 12 months and both the output gap and the deviation of interests from their targets were calculated using the Hodrick-Prescott filter. For robustness, the results using the Baxter and King [1999] filter are in the Appendix (A.2).

Table (4) shows the descriptive statistics for Brazil. For both the whole period and for the subperiod the inflation was above its target. Table (1) presents the value of the parameters estimated for the Brazilian economy.

The estimation of the monetary authoritys loss function during the targeting regime is shown in Figure (4). In this period, none of these functions was statistically asymmetric, as Table (5a) shows. However both of them showed asymmetries: the loss with inflation is asymmetric to the left and the loss in output gap is asymmetric to the right. Thus, in this period the central bank seems to have been moved, especially by a reaction to the product in the growth of the economy and situations by a reaction to inflation in times of lower output.

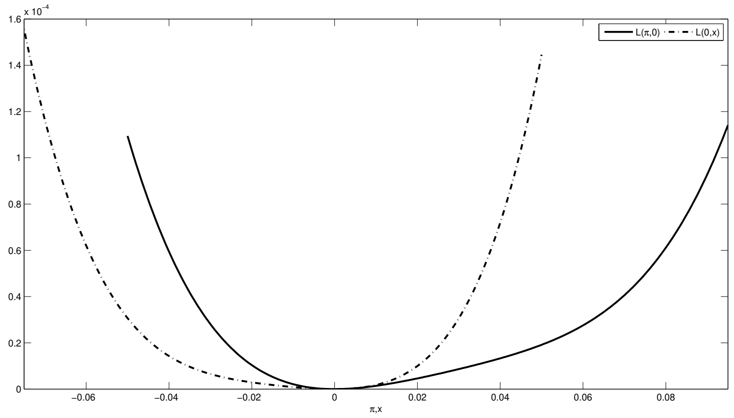
The estimates for the structural economic parameters for the 2004-2011 subperiod were similar to those of the whole sample, as shown in Table (1). The estimated loss function, shown in Figure (5), has the loss with output above the loss with inflation, showing a central bank that gives more importance to the output gap than with deviations of inflation. Again, the loss function with respect to the inflation was symmetric, but the loss with respect to the output gap was asymmetric to the right, been statistically different from a quadratic function at a 5% level of significance, as Table (5b) shows.

The estimates for the structural economic parameters for the 2004-2011 subperiod were similar to those of the whole sample, as shown in Table (1). The estimated loss function, shown in Figure (5), has the loss with output above the loss with inflation, showing a central bank that gives more

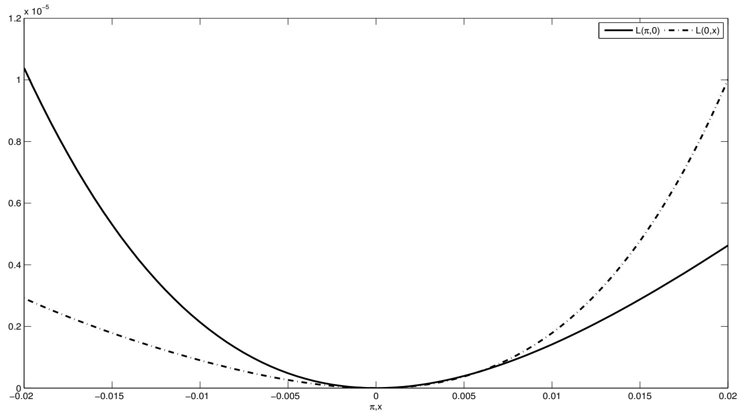
[14] Aragón and Portugal [2010] interpolate the inflation target according to equation $\pi^*_{m,t} = ({}^{12-m}/_{12})\,\pi^*_{t-1} + ({}^{j}/_{12})\,\pi^*_t$, where $\pi^*_{m,t}$ is the inflation target interpolated for month m of year t and π^*_t is the annual inflation target disclosed for year t. For 2003 and 2004, the adjusted targets of 8.5% and 5.5%, respectively, established by the 2003 Statement (*Carta Aberta*), were used.

Figure 4: Loss function - Brazil - 1999M04 - 2011M12

(a) all observed domain



(b) domain $(-0.02,\ 0.02)$



Note: $L(\tilde{\pi}, 0)$ shows the central banks loss function value for several inflation values, with output gap equal to zero. Analogous to $L(0, x)$.The values displayed on the upper graph represent the range of inflation and output gap. The lower graph shows only the interval $(-0.02, 0.02)$.

Table 4: Data statistics - Brazil

(a) 1999M4-2011M12

	π	$\bar{\pi}$	$\pi - \bar{\pi}$	x (hp)	x (bp)	i (hp)	i (bp)
mean	0.0666	0.0508	0.0158	-0.0010	-0.0013	0.0015	-0.0013
st deviation	0.0292	0.0143	0.0247	0.0223	0.0254	0.0221	0.0253
minimum	0.0296	0.0325	-0.0486	-0.0741	-0.0646	-0.0290	-0.0518
1st quartile	0.0480	0.0450	0.0006	-0.0122	-0.0181	-0.0115	-0.0165
median	0.0627	0.0450	0.0134	-0.0005	-0.0006	-0.0033	-0.0039
3rd quartile	0.0741	0.0550	0.0252	0.0142	0.0166	0.0123	0.0134
maximum	0.1724	0.0850	0.0928	0.0457	0.0519	0.0767	0.0624

(b) 2004M1-2011M12

	π	$\bar{\pi}$	$\pi - \bar{\pi}$	x (hp)	x (bp)	i (hp)	i (bp)
mean	0.0544	0.0463	0.0082	0.0007	0.0041	-0.0009	-0.0010
st deviation	0.0132	0.0033	0.0125	0.0248	0.0270	0.0128	0.0140
minimum	0.0296	0.0450	-0.0154	-0.0741	-0.0646	-0.0275	-0.0235
1st quartile	0.0443	0.0450	-0.0016	-0.0098	-0.0023	-0.0106	-0.0136
median	0.0555	0.0450	0.0092	0.0048	0.0052	-0.0021	-0.0025
3rd quartile	0.0643	0.0450	0.0174	0.0179	0.0220	0.0106	0.0082
maximum	0.0807	0.0550	0.0357	0.0457	0.0519	0.0281	0.0372

(hp): Hodrick-Prescott filter; (bk) Baxter and King filter.

importance to the output gap than with deviations of inflation. Again, the loss with respect to the inflation was symmetric, but the loss with respect to the output gap was asymmetric to the right, been statistically different from a quadratic function at a 5% level of significance.

In short, the Brazilian central bank showed in this period a different behavior from the commonly reported in other articles, giving more weight to the output gap than to the inflation. In addition, it presented a loss function with respect to output asymmetric to the right in the period 2004-2011, while the loss for inflation was not statistically different from a quadratic loss function. These results differ from Aragón and Portugal [2010] who observed a asymmetry in the loss related to inflation in the whole period. Again, it might be possible due our less restriction than the linex loss, as shown in Figure (6). There, one can see that the linex loss function overestimates the losses related to inflation above its target.

Table 5: Quadratic loss function test - Brazil

(a) 1999M04 - 2011M12

Statistic	Estimated Value	SD	*t-value*
$\mu_\pi = E\left(\partial^3 L / \partial \pi^3\right)$	-0.515	0.548	-0.940
$\mu_x = E\left(\partial^3 L / \partial x^3\right)$	1.604	2.560	0.626

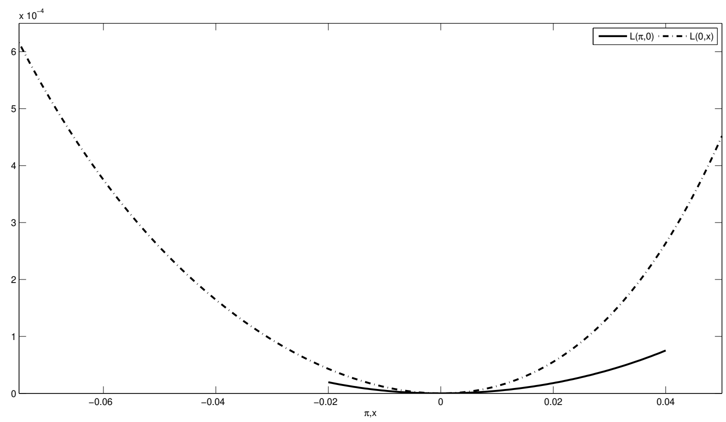
(b) 2004M01 - 2011M12

Statistic	Estimated Value	SD	*t-value*
$\mu_\pi = E\left(\partial^3 L / \partial \pi^3\right)$	0.038	0.414	0.091
$\mu_x = E\left(\partial^3 L / \partial x^3\right)$	4.500	2.332	1.930*

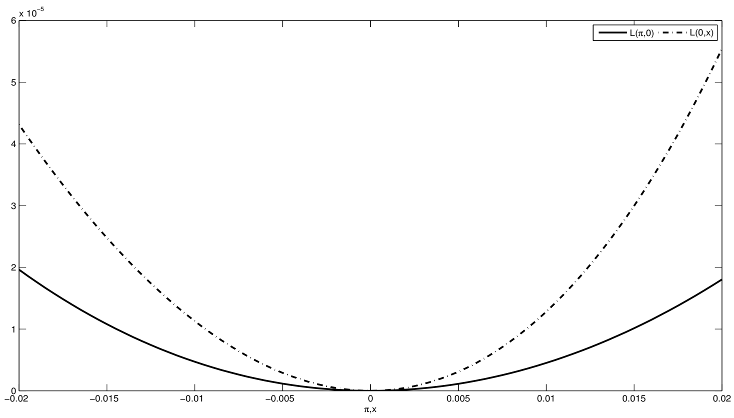
Note: The t-value corresponding to 5% is 1.65. * : significant at 5%.
The standard deviation refers to bootstrapping with 500 replicates.

Figure 5: Loss function - Brazil - 2004M1 - 2011M12

(a) all observed domain

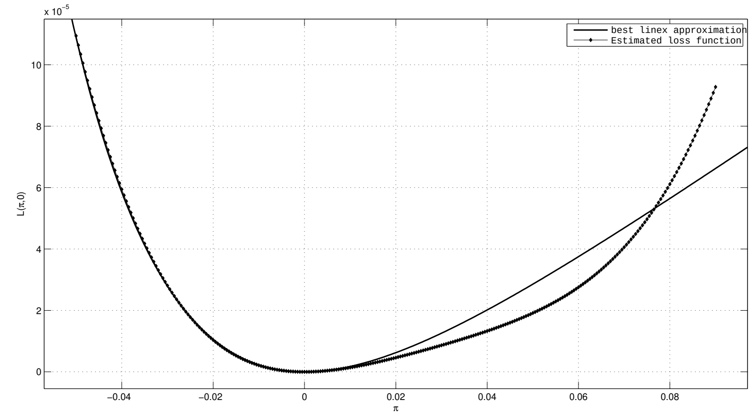


(b) domain (−0.02, 0.02)



Note: $L(\tilde{\pi}, 0)$ shows the central banks loss function value for several inflation values, with output gap equal to zero. Analogous to $L(0, x)$. The values displayed on the upper graph represent the range of inflation and output gap. The lower graph shows only the interval $(-0.02, 0.02)$.

Figure 6: Estimated $L(\tilde{\pi}, 0)$ and its best linex approximation - Brazil - 1999M4 - 2011M12



5 Conclusion

In this paper, we applied the nonparametric sieve estimator to determine the preferences of U.S. and Brazilian monetary policies. This allowed checking whether the central banks loss function was asymmetric during the analyzed period. Therefore, it was possible to test whether these monetary authorities have a larger loss associated with positive or negative deviations of inflation and of the output gap.

The economy was modeled using forward-looking agents and the maximization of the monetary authority was conducted under commitment, minimizing a generic loss function. From the first-order conditions of the problem, an optimal policy rule was obtained, been a set of restrictions over the conditional moments of the economic variables. These restrictions include the finite-dimensional parameters of the structural equations of the economy and the central banks loss function to be estimated. This way, it was possible to use the sieve method for their estimation, which informally consists in estimating the function within a less complex space than that in which it is inserted. The approximate space used here was the one generated by Chebyshev polynomials.

The novelty of this paper was the nonparametric method, which has an advantage over others nonparametric methods because it leads to a closed form solution, which is a combination of simpler functions. This way, it is easier to share the results than in a local polynomial regression, for example, where there is one value for the function at each point on its domain, but not a closed representation. Other important points are the method used here allows to test the results against a quadratic function, which is the standard of this literature, and it has as special case the linex specification, another important function. Given this, even in a possible absence of any asymmetries, the result is important because it would corroborate the other works, either with quadratic or linex functions; without an analysis as presented here, we could not know that the others related papers were meaningful.

We found evidences of asymmetries in the FEDs preferences over inflation for the whole sample (1960-2011), but not for the Volcker-Greenspan period (1982-2011), neither for preferences regarding the output gap. Even though not statistically significant, in the Volcker-Greenspan period FED showed a greater concern with inflation rate above its target. This may show an increase in FEDs concern about inflation. On the other hand, as inflation target, assumed equal to 2%, is close to zero, an asymmetry to the left as found could show a greater aversion to deflation.

On the other hand, the Brazilian central bank showed an asymmetric behavior towards output gap in the 2004-2011 period. The loss was greater with the above-target output gap than with the below-target output gap. The losses related to inflation were not statistically asymmetric both in the whole sample as in the 2004-2011 subsample. Also, the losses with the output gap were bigger than the losses with the inflation, which eventually could leading to an inflationary bias in the Brazilian economy, as the reaction function would not react as many to the inflation as it should.

This result encourages research into the reasons for this behavior of the Brazilian monetary authority. A hypothesis is that the independence of the Brazilian monetary authority is smaller than expected, causing the governments react more to the economy level of activity than to the inflation. Another hypothesis is that the method has problems with the estimation of the monetary authoritys loss function due to the two crises (the pre-election crisis of 2002 and the international financial crisis in the late 2000s). In periods of crisis, the monetary authority uses other instruments than the interest rate to fight inflation, which is not captured directly by the structure of the economy adopted in this paper (where the central banks instrument is nothing but the short-term interest rate), producing a loss function related to inflation that lies below to the one related to the output gap. A third hypothesis could be the short sampling period and, especially, the inflation rate above rather than below its target.

This paper raises some questions that could be addressed by other studies. As to the modeling of the economy, new elements could be added, such as the exchange rate, in an attempt to explain the monetary authoritys action by other means than the interest rate, as discussed for the Brazilian case. In addition, it is necessary to investigate the trajectories of the variables and the inflationary bias generated by this nonparametric approach. With regard to the sieve estimator, it would be interesting to include a Monte Carlo simulation to test its performance in finite samples. At a more advanced stage, one could try to develop a test for the functional form of the loss function whose null hypothesis includes the linex function, used in other empirical studies.

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A Appendix

A.1 Sieve minimum distance estimator

This appendix shows the generic version of the estimator, as described in Ai and Chen [2003], which describe the estimator for independent and identically distributed data. Chen and Ludvigson [2009] extend the result to time series data.

In rational expectations models, based on the maximizing behavior of the agents, one usually obtains restrictions on the conditional moments that must be satisfied by the variables.

Let$(\boldsymbol{Y}, \boldsymbol{X})$ be the variables of the economy, where $\boldsymbol{Y}$ is the vector with the endogenous data and $\boldsymbol{X}$ the vector with exogenous or predetermined data. Then the conditional moments can be written as

$$E\left[\boldsymbol{\rho}\left(\boldsymbol{Z},\ \boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)\right)|\boldsymbol{X}\right] = \boldsymbol{0}, \tag{20}$$

where $\boldsymbol{Z} = \left(\boldsymbol{Y}^{'},\ \boldsymbol{X}_z^{'}\right)$, $\boldsymbol{X}_z \subseteq \boldsymbol{X}$, $\boldsymbol{\rho}\left(\bullet\right)$ is a vector of unknown functions that place restrictions on the moments (residuals, as the expectation is equal to zero). $E\left[\boldsymbol{\rho}\left(\boldsymbol{Z},\ \boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)\right)|\boldsymbol{X}\right]$ is the expectation conditional on $\boldsymbol{X}$ of $\boldsymbol{\rho}\left(\boldsymbol{Z},\ \boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)|\boldsymbol{X}\right)$. $\boldsymbol{\theta_0}$ is a vector with parameters within a finite-dimensional space and $\boldsymbol{h_0}\left(\bullet\right) = \left(h_{01}\left(\bullet\right), \ldots, h_{0q}\left(\bullet\right)\right)$ is a vector of unknown functions. The intention is to estimate $\boldsymbol{\alpha}_0 = \left(\boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)\right)$, which makes the problem semiparametric, as one has parameters of interest both in the finite and infinite dimensions. This property perfectly suits the problem dealt with in the present paper, in which, besides the monetary authoritys preference function, there are also structural economic parameters to be estimated.

There is a sample $\{(\boldsymbol{Y}_t,\ \boldsymbol{X}_t),\ t = 1, \ldots, N\}$ for the estimation of (20), where $(\boldsymbol{Y}_t,\ \boldsymbol{X}_t)$ is a stationary β-mixing process[15]. The restrictions on conditional moments, $\boldsymbol{\rho} : \mathcal{Z} \times \mathcal{A} \to \mathbb{R}^{d_\rho}$, are known up to an unknown vector of parameters, $\boldsymbol{\alpha}_0 = \left(\boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)\right) \in \mathcal{A} = \Theta \times \mathcal{H}$, where Θ is the space of finite-dimensional parameters and $\mathcal{H} = \mathcal{H}_1 \times \ldots \times \mathcal{H}_q$ is the functional space where $\boldsymbol{h_0}$ is inserted. If the distribution function of $\boldsymbol{Y}$ given $\boldsymbol{X}$, $F_{\boldsymbol{Y}|\boldsymbol{X}}$, were known, it would be possible to calculate each component of (20) as

[15]Intuitively, a process that is β-mixing is asymptotically independent.

$$E\left[\rho_i\left(\boldsymbol{Z},\ \boldsymbol{\theta_0},\ \boldsymbol{h_0}\left(\bullet\right)\right)|\boldsymbol{x}\right] \equiv m_i\left(\boldsymbol{x},\ \boldsymbol{\alpha}\right) = \int \rho_i\left(\boldsymbol{y},\ \boldsymbol{x}_z,\ \boldsymbol{\theta},\ \boldsymbol{h}\left(\bullet\right)\right) dF_{\boldsymbol{Y}|\boldsymbol{X}=\boldsymbol{x}}\left(\boldsymbol{y}\right), \tag{21}$$

$i = 1, \ldots, d_\rho$. Let $\boldsymbol{m}\left(\boldsymbol{x},\ \boldsymbol{\alpha}\right) = \left(m_1\left(\boldsymbol{x},\ \boldsymbol{\alpha}\right), \ldots, m_{d_\rho}\left(\boldsymbol{x},\ \boldsymbol{\alpha}\right)\right)'$. Then the true parameter $\boldsymbol{\alpha}_0$ minimizes

$$\inf_{\alpha=(\theta,h)\in\Theta\times\mathcal{H}} E\left[\boldsymbol{m}\left(\boldsymbol{X},\ \boldsymbol{\alpha}\right)' \left[\boldsymbol{\Sigma}\left(\boldsymbol{X}\right)\right]^{-1} \boldsymbol{m}\left(\boldsymbol{X},\ \boldsymbol{\alpha}\right)\right], \tag{22}$$

where $\boldsymbol{\Sigma}\left(\boldsymbol{X}\right)$ is a positive definite matrix.

A natural estimator for $\boldsymbol{\alpha}_0$ would be the argument that minimizes the sample version of (22). However, to allow for this minimization, two issues have to be addressed. First, as the true functional form of $F_{\boldsymbol{Y}|\boldsymbol{X}}$, and consequently of $\boldsymbol{m}\left(\boldsymbol{X},\ \boldsymbol{\alpha}\right)$, is not known, this latter function must be estimated by any nonparametric method. Ai and Chen [2003] suggest estimating it also by the sieve estimator, more precisely by the linear sieve estimator, discussed in what follows. A second aspect is the fact that the minimization over the infinite-dimensional set $\mathcal{H}$ can be complex and result in undesirable properties in the estimators. As discussed earlier, an alternative is the sieve method, replacing $\mathcal{H}$ with a sequence of simpler sets, which are dense in the original space when the size of sample N increases. Thus, the optimization problem occurs over the set $\mathcal{H}_n = \mathcal{H}_n^1 \times \ldots \times \mathcal{H}_n^q$, simpler than the original one, and usually of finite dimension. With the treatment of these two issues, we have the sieve minimum distance (SMD) estimator,

$$\begin{aligned}\hat{\boldsymbol{\alpha}}_n &= \left(\hat{\boldsymbol{\theta}}_n,\ \hat{\boldsymbol{h}}_n\right) \\ &= \arg\min_{\alpha=(\theta,h)\in\Theta\times\mathcal{H}_n} n^{-1}\sum_{t=1}^{N} \hat{\boldsymbol{m}}\left(\boldsymbol{X}_t,\ \boldsymbol{\alpha}\right)' \left[\hat{\boldsymbol{\Sigma}}\left(\boldsymbol{X}\right)\right]^{-1} \hat{\boldsymbol{m}}\left(\boldsymbol{X}_t,\ \boldsymbol{\alpha}\right), \end{aligned} \tag{23}$$

where $\hat{\boldsymbol{\Sigma}}\left(\boldsymbol{X}\right)$ is a consistent estimator for $\boldsymbol{\Sigma}\left(\boldsymbol{X}\right)$, used to deal with some possible heteroskedasticity, and $\hat{\boldsymbol{m}}\left(\boldsymbol{X},\ \boldsymbol{\alpha}\right)$ is the linear sieve estimator for $\boldsymbol{m}\left(\boldsymbol{X},\ \boldsymbol{\alpha}\right)$.

Using the identity matrix as weighting matrix, $\hat{\boldsymbol{\Sigma}}\left(\boldsymbol{X}\right) = I$, the SMD estimator is identical with the generalized method of moments (GMM) estimator. This result is shown by Ai and Chen [2003, p. 1799].

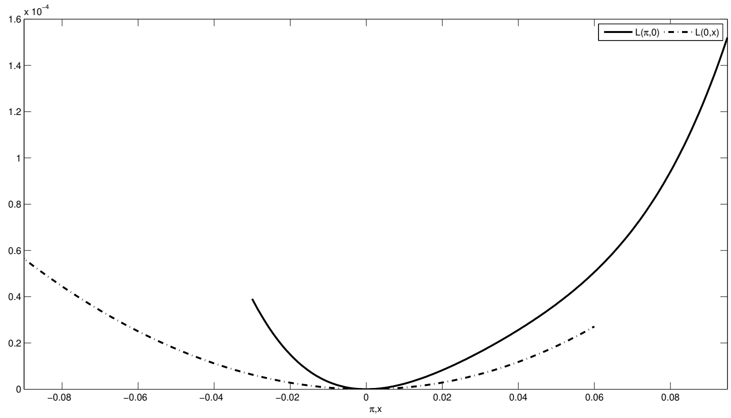
A.2 Results using the band-pass filter

Table 6: Estimated parameters using the band-pass filter

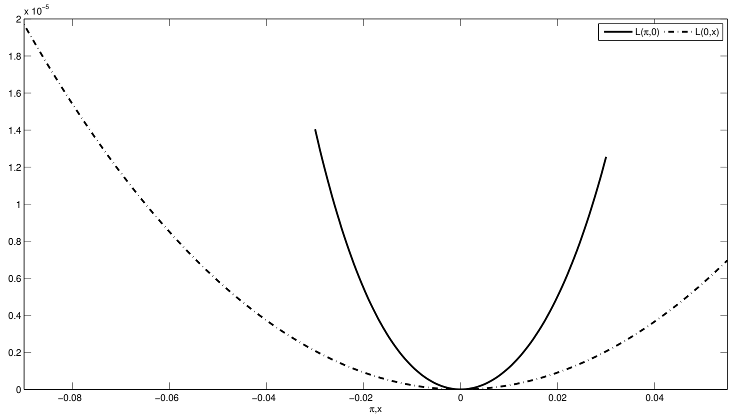
	USA		Brazil	
Parameter	1960Q1-2011Q4	1982Q4-2011Q4	1999M4-2011M12	2004M01-2011M12
$\psi_{1,0}$	-0.2207	-0.1333	-0.2434	-0.2144
$\psi_{2,0}$	1.0000	1.0000	1.0000	1.0000
$\psi_{3,0}$	-0.1840	-0.1425	-0.2090	-0.2147
$\psi_{4,0}$	0.4186	0.4213	0.4182	0.4175
$\psi_{5,0}$	-0.0264	-0.0413	-0.0340	-0.0547
$\psi_{0,1}$	-0.0027	0.0018	1.2668	1.3601
$\psi_{0,2}$	0.0120	0.0051	2.6841	2.6877
$\psi_{0,3}$	-0.0036	0.0021	1.3641	1.4287
$\psi_{0,4}$	0.0039	0.0018	1.1318	1.1347
$\psi_{0,5}$	-0.0014	0.0007	0.4004	0.4008
$\psi_{1,1}$	0.0045	0.0019	0.0146	0.0070
γ	0.0029	0.0018	0.0175	0.0179
β	0.9772	0.9658	0.9786	0.9750
κ	0.1535	0.1476	0.3077	0.3255
δ	0.0868	0.1088	0.6521	0.2378
ϕ	0.0331	0.0167	0.0269	0.0751
$\bar{r}$	0.0422	0.0394	0.0455	0.0480
ϱ	0.8366	0.5097	0.8944	0.9086
ω	0.8151	0.8605	0.8626	0.9172

Figure 7: Loss function - USA - band-pass filter

(a) 1960Q1 - 2011Q4



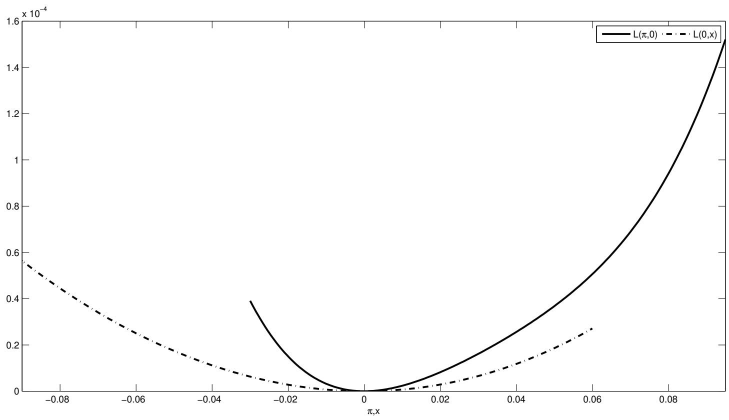
(b) 1982Q4 - 2011Q4



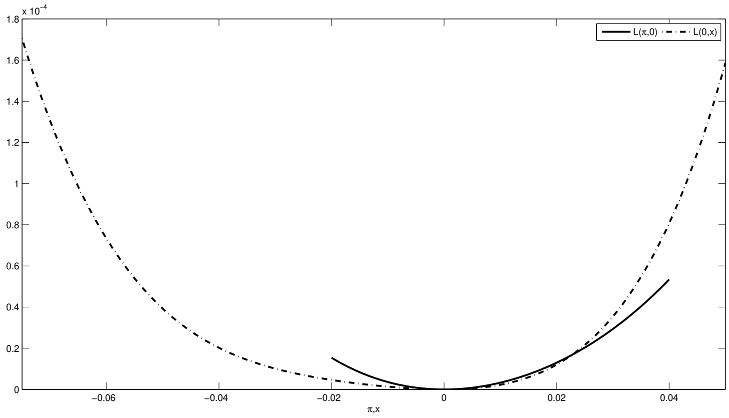
Note: $L(\tilde{\pi}, 0)$ shows the central banks loss function value for several inflation values, with output gap equal to zero. Analogous to $L(0, x)$. The values displayed on the upper graph represent the range of inflation and output gap.

Figure 8: Loss function - Brazil - band-pass filter

(a) 1999M4 - 2011M12



(b) 2004M1 - 2011M12



Note: $L(\tilde{\pi}, 0)$ shows the central banks loss function value for several inflation values, with output gap equal to zero. Analogous to $L(0, x)$. The values displayed on the upper graph represent the range of inflation and output gap.