Cost Channel, Interest Rate Pass-Through and Optimal Policy under Zero Lower Bound

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Abstract

This paper analyzes optimal monetary policy under zero lower bound in the presence of cost channel. Cost channel introduces trade-off between output and inflation when economy is out of ZLB. As a result, exit time both under discretion and commitment is endogenous in the presence of cost channel. We also find that commitment outperforms discretion by promising future boom and inflation and a T-only policy closely replicates commitment both under presence and absence of cost channel. Moreover, the exit date (from ZLB) under discretion, commitment and T-only policy rises with the magnitude of demand shock given the degree of interest rate pass-through irrespective if the cost channel is present. We also show that, while exit date both under discretion and T-only policy rises with the degree of interest rate pass-through/credit market imperfection, it falls under commitment given demand shock.

JEL Classification: E63, E52, E58

Keywords: New-Keynesian Model, Cost Channel, Liquidity Trap

1 Introduction

The recent financial crisis has witnessed negative natural rate of interest due to large adverse demand shock for a number of developed countries. This forces some of the major central banks, e.g. Federal Reserve Bank, Bank of England, Bank of Japan and the European Central Bank to reduce their target interest rates to (near) zero. In this situation, monetary authority loses its ability to lower the nominal interest rate further to stimulate economy. This paper analyzes the optimal policy under ZLB without uncertainty using a New Keynesian DSGE model with cost channel.

The canonical New Keynesian DSGE model is the workhorse of modern monetary policy analysis. Optimal policy of the canonical model under ZLB has already been studied in detail by a series of paper such as, Jung, et. al. (2005), Eggertson and Woodford (2003), Adam and Billi (2006, 2007) and Nakov (2008). This paper shows that commitment outperforms discretion by promising future boom and inflation and also by postponing exit date. Chattopadhyay and Daniel (2016) show how an optimal policy can be communicated through Taylor rule. They show that a Taylor rule with properly chosen inflation target can replicate commitment. The paper also proposes a new T-only policy where exit date from ZLB is optimally chosen but monetary authority follows discretion post-exit. Chattopadhyay and Daniel (2016) shows that a T-only policy is easier to communicate and closely replicates outcome under commitment.

Even if the canonical New Keynesian model is highly popular and frequently used in policy prescription, it has certain drawbacks. One of the drawbacks of the canonical model is that, the model does not introduce any trade-off between output and inflation. As a result fluctuations of both output and inflation can be simultaneously minimized, known as "divine coincidence" in the literature. Ravenna and Walsh (2006) tackles the problem of "divine coincidence" by introducing a cost channel into the canonical New Keynesian DSGE model. They assume that firms borrow from financial intermediaries to pay for their wage bill and repays it with interest. As a result, beside output gap, interest rate also affects the marginal cost of the firm. A change in marginal cost due to interest rate affects over all inflation through Phillips curve. This is known as interest rate pass-through in the literature.

A few number of empirical studies have already been undertaken to check the importance of cost channel. However, no conclusive evidence have been found. Barth and Ramey (2001) has given evidence about the presence cost channel by estimating a VAR model using aggregate and industry level data for the period 1959-2000 for US. The paper argues that monetary policy shock should be treated as a supply side shock as the characteristics of impulse responses due to a monetary policy shock is similar to a productivity or supply shock, which is very different from the characteristics of impulse responses obtained from various other demand shocks. Christiano et. al. (2005) also gives evidence in favor of cost channel for US. Chowdhury et. al. (2006) have estimated a hybrid version of New Keynesian Phillips curve with cost channel through GMM and shows a significant presence of cost channel in majority of G-7 countries like Canada, France, Italy, UK and US. Ravenna and Walsh (2005) has also got evidence in favor of cost channel by estimating forward looking Phillips curve with cost channel using quarterly US data for the period 1960-2001. Tillman (2008) shows evidence of cost channel for US, UK and Euro area by estimating a forward looking hybrid Phillips Curve for each country using quarterly data for the time period 1960-2004. A year later, Tillman (2009) uses a rolling window GMM estimate to assess time varying nature of cost channel for US. The paper finds significant presence of cost channel in pre-Volker era and post Volker-Greenspan era.

Rabanal (2007) has on the other hand has estimated a DSGE model for US using quarterly data from 1950 to 2004 through Bayesian approach. The paper shows that transmission mechanism of monetary policy through demand side dominates the supply side effect. Later, a related study by Henzel et al. (2009) shows that though the cost channel fails to generate a price puzzle for the Euro area, its presence however, explain the initial hump in prices due to a monetary policy tightening.

Although a good number of empirical studies have been undertaken to asses the importance of cost channel, theoretical works related to the optimal policy under cost channel are very limited in number. Ravenna and Walsh (2006) and Araujo (2009) have analyzed the optimal policy both under discretion and commitment. However, these papers have not undertaken the ZLB constraint of nominal interest rate explicitly in their analysis. The model of Ravenna and Walsh (2006) is modified later by Chowdhury, et. al. (2006) by introducing financial market imperfection which affects the degree of interest rate pass-through. We have used the model of Ravenna and Walsh (2006) and Chowdhury et. al. (2006) to analyze optimal policy both under discretion and commitment under ZLB. However, for simplicity we have not introduced any uncertainty in our analysis. The demand shock in our model follows an deterministic $AR(1)$ as in Jung et. al. (2005) and Chattopadhyay and Daniel (2016).

Our analysis shows while exit time (from ZLB) in the absence of cost channel is exogenous under discretion, exit time both under discretion and commitment is endogenous due to post-exit trade-off between output gap and inflation rate introduced by cost channel. We also find that, irrespective of the of cost channel to be present or absent, commitment outperforms discretion by promising future boom and inflation and a T-only policy closely replicates commitment. This is identical with results obtained by Chattopadhyay and Daniel (2016) with no cost channel. Our paper further show that, exit date under discretion, commitment and T-only policy rises with the magnitude of demand shock given the degree of interest rate pass-through irrespective of the presence of cost channel. However, given demand shock while exit date both under discretion and T-only policy are shown to rise with the degree of interest rate pass-through/credit market imperfection, it has been shown to fall under commitment. This happens since commitment stimulates the system by promising future boom, inflation. Commitment also promise a post-exit inflation and boom too. This gives an additional stimulus to the system which rises with the degree of interest rate pass-through/credit market imperfection. The system compensates the extra stimulus by preponing the exit from liquidity trap.

Rest of the paper proceeds as follows. Section 2 describes the model. Section 3 and Section 4 analyzes the optimal policy under discretion and commitment respectively. Section 5 describes the T-only policy and Section 6 concludes.

2 The Model

Ravenna and Walsh (2006) has introduced cost channel in an otherwise canonical New Keynesian DSGE model. The paper assumes that firms borrow money to pay the wage bill of labor even before the production has taken place. The firm has to pay interest payments to financial intermediaries while paying it back. As a result, the marginal cost of firm depends both on output gap and interest rate. The presence of interest rate in marginal cost channel is the required cost channel. Araujo (2009) introduced financial market imperfection in the model of Ravenna and Wash (2006) and shows how the degree of interest rate pass-through changes with the extent of financial market imperfection. We have used the model of Ravenna and Walsh (2006) and Chowdhury et. al. (2006) for our analysis. The log-linearized model without uncertainty can be succinctly written with help of an aggregate demand or IS equation given in equation (1).

$$\begin{aligned} y_{t+1} &= y_t + \sigma\left[i_t - i - \pi_{t+1}\right] - u_t \\ &= y_t + \sigma\left[i_t - r_t^n - \pi_{t+1}\right] \end{aligned} \tag{1}$$

Equation (1) is derived by log-linearizing individual's Euler equation around zero inflationary steady state.

The supply side of the model is represented by a forward looking Phillips curve with cost channel as given in equation (2).

$$\pi_t = \beta\pi_{t+1} + \kappa\left(\sigma^{-1} + \eta\right) y_t + \kappa\delta\left(i_t - i\right) \tag{2}$$

In these equations y_t denotes the output gap; inflation (π_t) is the deviation about a long-run value of zero; i_t denotes the nominal interest rate (deposit rate), with a long-run equilibrium value of $i = r = \frac{1-\beta}{\beta}$, with r defined as the long-run real interest rate; σ represents the intertemporal elasticity of substitution with $\sigma \geq 1$, $\kappa = \frac{(1-s)(1-s\beta)}{s}$, (where $(1-s)$ is the fraction of firms choosing their price optimally in each period) represents the degree of price stickiness; $\beta \in (0,1)$ denotes the discount factor; η denotes the inverse of elasticity of Frisch labor supply and u_t represents the combination of shocks associated with preferences, technology, fiscal policy, etc.

Araujo (2009) assumes, $\iota_t^l = (1+\psi)\,\iota_t = \delta\iota_t$, where, $\iota_t = i_t - i$ is the deviation of deposit rate from long-run real interest rate and $\iota_t^l = i_t^l - i$ is deviation of lending rate from long-run real interest rate. $\psi \in (0,1)$ measures the extent of financial market imperfections governing the degree of interest rate pass through. Note, we get our canonical model without cost channel back when $\delta = 0$. Ravenna and Walsh (2006) assumes, even if rise in interest rate increases inflation rate in the presence of cost channel, the negative effect of output dominates so that, tighter monetary policy reduces inflation rate. As a result they set, $\delta \in [1, \frac{\sigma^{-1}+\eta}{\sigma^{-1}})$ when cost channel is present.[1] Note, there is no uncertainty in our model for simplicity. As a result, we assume the demand shock follows a deterministic $AR(1)$ process as given in equation (3)

$$u_t = \rho^{t-1}u_1, 0 < \rho < 1 \tag{3}$$

We define natural rate of interest as,

$$r_t^n = i - \sigma^{-1}u_t$$

Note, a large adverse demand shock puts the economy into liquidity trap with $r_t^n < 0$.

Ravenna and Walsh has also derived the loss function as the second order Taylor Series expansion of the individual's utility function around zero inflation steady state. The loss

[1] Note, unit rise in nominal interest rate reduces output gap by σ unit through IS equation, which in turn reduces inflation by $\frac{(\sigma^{-1}+\eta)}{\sigma^{-1}}$ through by Phillips curve. However, unit rises in nominal interest rate increase inflation by $\kappa\delta$ unit by Phillips curve. Therefore, negative output effect dominates when, $\delta < \frac{(\sigma^{-1}+\eta)}{\sigma^{-1}}$. Also see, section 4.1.2 for restriction on δ.

functions without uncertainty is,[2]

$$\frac{1}{2}\sum_{t=1}^{\infty}\beta^{t-1}\left(\pi_t^2+\lambda y_t^2\right),\ \lambda\in[0,\infty) \tag{4}$$

where, $\lambda=\frac{\kappa\left(\sigma^{-1}+\eta\right)}{\theta}$ is weight given to output gap relative to inflation. $\theta>1$ is own price elasticity of output.

3 Optimal Policy without Cost Channel

We use optimal policy without cost channel as a benchmark to compare the same under cost channel. As a result, this section gives a brief description of the main results of optimal policy without cost channel. Detail of the derivation and result can be found in Jung, et al. (2005) and Chattopadhyay and Daniel (2016).[3] Note, when there is no cost channel, $\delta=0$. We assume no uncertainty with deterministic demand shock as given in equation (3). We further assume that, economy in in liquidity trap till T and exit liquidity trap thereafter. This implies, once the economy exits it never falls into liquidity trap. Given this monetary authority minimizes equation (4) by choosing output, inflation and nominal interest rate given equation (1), equation (2) and the feasibility constraint $i_t\geq 0$. Monetary authority minimizes loss given future expectations under discretion and commitment endogenizes even future expectations into the process of minimization.

Some standard result of optimal policy without cost channel are, (i) exit time under discretion is exogenous and depends entirely on the time path of natural rate of interest. Economy is in liquidity trap as long as natural rate of interest rate is negative and exits as soon as natural rate of interest becomes positive. Exit time under discretion rises with magnitude and persistence of demand shock, (ii) commitment promises future boom and inflation and also postpones exit compared to discretion. These yield extra stimulus and allows commitment to outperform discretion, (iii) Taylor rule with properly chosen inflation target and its persistence can replicate both discretion and commitment and (iv) a T-only policy that only chooses exit time optimally but follows discretion after exit can closely replicate commitment. Chattopadhyay and Daniel (2016) show that the above

[2]Ravenna and Walsh assimes a fraction, $(1-\gamma_t)$ of output is consumed by government. As a result the loss function takes the form, $\frac{1}{2}E_t\sum_{t=1}^{\infty}\beta^{t-1}\left(\pi_t^2+\lambda\left(y_t-\frac{1}{\sigma^{-1}+\eta}\widehat{\gamma}_t\right)^2\right)$. We have no government expenditure. As a result, $\widehat{\gamma}_t=0$ for all t.

[3]Also see, Eggertson and Woodford (2003), Adam and Billi (2006, 2007) and Nakov (2008).

results remain unchanged even if we introduce uncertainty into the system. The optimal time path of the system under discretion, commitment and T-only policy without cost channel ($\delta = 0$) is given in Figure 2, Figure 3 and Figure 4 respectively.

4 Optimal Policy with Cost Channel

The next three subsection describes the optimal policy under ZLB when cost channel is present. Subsection 4.1 describes the optimal policy under discretion, subsection 4.2 describes optimal policy under commitment and subsection 4.3 describes the optimal policy under T-only policy. We keep on assuming no uncertainty for simplicity. As a result, demand shock is assumed to follow the deterministic time path as given in equation (3). Moreover, the economy is assumed to be in liquidity trap till period T and exits thereafter.

4.1 Optimal Policy under Discretion

Monetary authority minimizes $(\pi_t^2 + \lambda y_t^2)$ by choosing output gap, inflation rate and nominal interest rate, subject to equation (1), (2) and the feasibility constraint, $i_t \geq 0$ given future expectation of output gap and inflation rate under discretion. This implies, monetary authority optimizes each period under discretion. The Lagrangian of the problem under discretion is,

$$\mathcal{L}_D = \left\{ \begin{array}{c} -\frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right] - \phi_{1,t}\left[\sigma\left(i_t - \pi_{t+1} - r_t^n\right) - y_{t+1} + y_t\right] \\ -\phi_{2,t}\left[\pi_t - \kappa\left(\sigma^{-1} + \eta\right)y_t - \kappa\delta\left(i_t - i\right) - \beta\pi_{t+1}\right] \\ +\phi_{3,t} i_t \end{array} \right\}$$

The First Order Conditions are,

$$
\begin{aligned}
\frac{\partial L}{\partial \pi_t} &= -\pi_t - \phi_{2,t} = 0 && (5)\\
\frac{\partial L}{\partial y_t} &= -\lambda y_t - \phi_{1,t} + \kappa\left(\sigma^{-1}+\eta\right)\phi_{2,t} = 0 && (6)\\
\frac{\partial L}{\partial \phi_{1,t}} &= \sigma\left(i_t - \pi_{t+1} - r_t^n\right) - y_{t+1} + y_t = 0 && (7)\\
\frac{\partial L}{\partial \phi_{2,t}} &= \pi_t - \kappa\left(\sigma^{-1}+\eta\right)y_t - \kappa\delta\left(i_t - i\right) - \beta\pi_{t+1} = 0 && (8)\\
\frac{\partial L}{\partial i_t} &= -\sigma\phi_{1,t} + \kappa\delta\phi_{2,t} + \phi_{3,t} = 0 && (9)\\
\phi_{3,t}\frac{\partial L}{\partial \phi_{3,t}} &= \phi_{3,t} i_t = 0, \phi_{3,t} \geq 0, i_t \geq 0 \text{ with complementary slackness} && (10)
\end{aligned}
$$

Equation (5) and (6) gives,

$$\phi_{1,t} = -\left(\lambda y_t + \kappa\left(\sigma^{-1}+\eta\right)\pi_t\right) \tag{11}$$

and $\phi_{3,t} \geq 0$ implies,

$$\sigma\phi_{1,t} - \kappa\delta\phi_{2,t} \geq 0 \tag{12}$$

from equation (9) and (10).[4] To determine exit time under cost channel define, $Q_t^d = \kappa\left(\delta - \sigma\left(\sigma^{-1}+\eta\right)\right)\pi_t - \sigma\lambda y_t$. Then, equation (6), (11) and (12) gives,

$$
\begin{aligned}
i_t &= 0 \text{ till } Q_t^d > 0\\
&> 0, \text{ O.W.} && (13)
\end{aligned}
$$

Note, higher δ increases Q_t^d and delays exit. After exit, i_t is determined by equation (7). We have checked numerically that, nominal interest rate becomes positive from zero as soon as Q_t^d changes sign from positive to zero. This implies that the above exercise captures the notion of discretion. Also note that higher δ increases Q_t^d and delays exit. This happens since higher δ causes higher fluctuations in output gap and inflation rate and causes higher welfare loss. Therefore, system needs higher stimulus to compensate the welfare loss and achieves that by delaying exit.

To solve the model note, $i_t = 0$ for $t = 1, 2, ..., T^d$. Equations (1) and (2) with $i_t = 0$

[4] Note, exit date depends only on $\phi_{1,t}$ when cost channel is absent ($\delta = 0$).

gives,

$$Z_{t+1} = c + AZ_t - ar_t^n \tag{14}$$

where,

$$\begin{aligned} c &= \begin{bmatrix} -\frac{\sigma\kappa\delta i}{\beta} \\ \frac{\kappa\delta i}{\beta} \end{bmatrix}, a = \begin{bmatrix} \sigma \\ 0 \end{bmatrix}, Z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}, \\ A &= \begin{bmatrix} 1 + \frac{\sigma\kappa\left(\sigma^{-1}+\eta\right)}{\beta} & -\frac{\sigma}{\beta} \\ -\frac{\kappa\left(\sigma^{-1}+\eta\right)}{\beta} & \frac{1}{\beta} \end{bmatrix} \end{aligned}$$

A forward solution of equation (14) yields

$$Z_t = \Gamma_t^d + A^{-(T^d-t+1)} Z_{T^d+1} \tag{15}$$

where,

$$\Gamma_t^d = \sum_{k=t}^{T^d} A^{-(k-t+1)} ar_k^n - \sum_{k=t}^{T^d} A^{-(k-t+1)} c$$

Note, since one eigenvalue of the matrix A is greater than one and other less than one, higher T^d and Z_{T^d+1} gives more stimulus to the system. Now, solution of the system entails we know Z_{T^d+1} and T^d. Z_{T^d+1} is determined as follows. We know that, $i_t > 0$ and $\phi_{3,t} = 0$ for $t = T^d + 1, T^d + 2, ...$ Equation (5), (6) and (9) with $\phi_{3,t} = 0$ gives,

$$\pi_t = -\chi y_t \tag{16}$$

where,

$$\chi = \frac{\lambda}{\kappa\left[\sigma^{-1}\left(1-\delta\right)+\eta\right]}$$

We have to solve equation (7), (8) and (16) simultaneously. Equation (7), (8) gives,

$$y_{t+1} + \sigma\left(1 + \frac{\beta}{\kappa\delta}\right)\pi_{t+1} = \left[1 - \frac{\sigma^{-1}+\eta}{\sigma^{-1}\delta}\right] y_t + \frac{\sigma}{\kappa\delta}\pi_t + u_t \tag{17}$$

Note, for $\delta \in (0, \frac{\sigma^{-1}+\eta}{\sigma^{-1}})$, equation (16) is negatively sloped and equation (17) is positively sloped. Substituting equation (17) to (16) gives,

$$y_{t+1} = \mu y_t - \frac{u_t}{\sigma\chi\left(1+\frac{\beta}{\kappa\delta}\right) - 1} \tag{18}$$

where,

$$\mu = \frac{\frac{\sigma}{\kappa\delta}\left(\chi + \kappa\left(\sigma^{-1} + \eta\right)\right) - 1}{\sigma\chi\left(1 + \frac{\beta}{\kappa\delta}\right) - 1}$$

We need $\mu > 1$ to solve equation (18) forward. We see that, μ falls as δ rises and there exists a $\delta \in [1, \delta^*]$ such that, $\mu > 1$ when $\delta \in [1, \delta^*]$ and $\delta^* < \frac{\sigma^{-1}+\eta}{\sigma^{-1}}$. As a result, we restrict our analysis to $\delta \in [1, \delta^*]$ when cost channel is present.[5] Note we have, $\chi > 0$ and $\mu > 1$ for $\delta \in [1, \delta^*]$. Hence, we always get a post-exit trade-off between output gap and inflation rate in the presence of cost channel (see, Ravenna and Walsh, 2006).

Forward solution of equation (18) gives,

$$y_t = \frac{u_t}{\left[\sigma\chi\left(1 + \frac{\beta}{\kappa\delta}\right) - 1\right](\mu - \rho)}, \text{ for } t = T^d + 1, T^d + 2, ... \tag{19}$$

Substituting, equation (19) to equation (16) gives optimal post-exit inflation rate.

Figure 1 describes the post-exit optimal response of output gap and inflation rate to adverse demand shock under discretion. Note, since $\delta \in [1, \delta^*]$, equation (16) is negatively sloped and equation (17) is positively sloped. Equation (16) is denoted by AA and equation (17) is denoted by BB in Figure 1. An adverse demand shock shifts BB curve down to BB_1. This causes post-exit output gap to rise to $y_1$and inflation rate to fall to π_1.

[5]Note, output gap is a jump variable under uncertainty. As a result, we need, $\mu > 1$ for a determinate equilibrium. Ravenna and Walsh (2006), Chowdhury et. al. (2006) and Tillman (2007) have estimated $\delta \in [1, 1.4]$. We have checked that we get $\mu > 1$ as long as δ belongs to the range mentioned above.

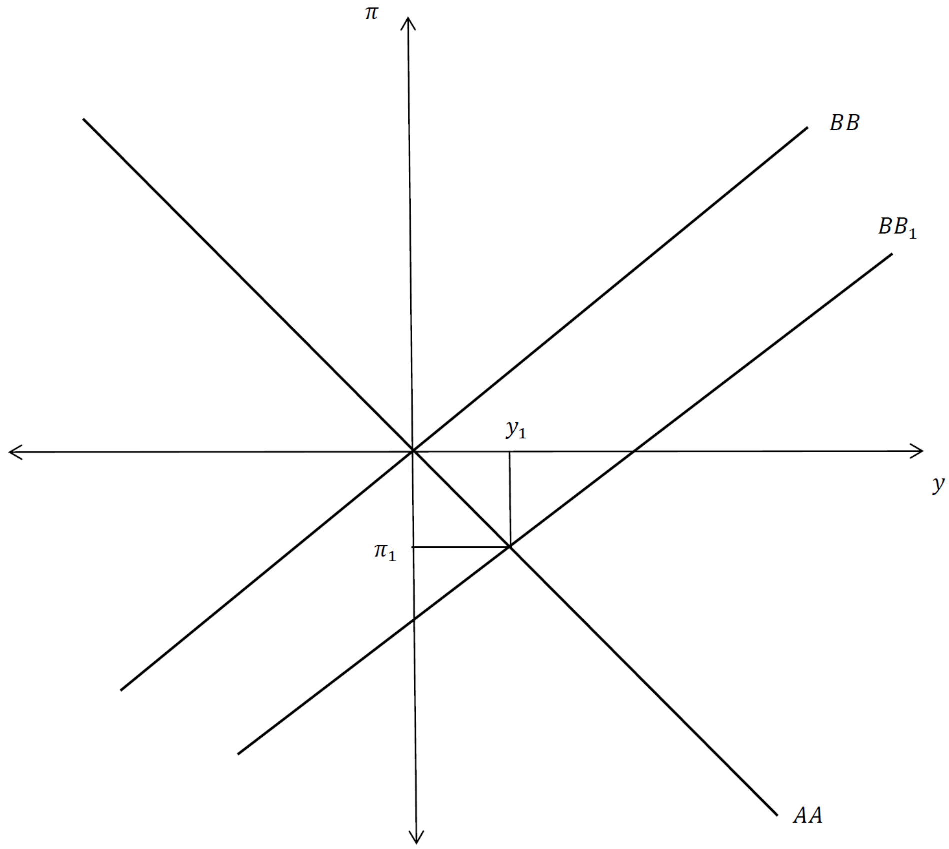

Figure 1: Post-exit Optimal Response to Adverse Demand Shock under Discretion

However, impact of δ on post-exit output gap is not monotonic. We see that, δ has threshold. When δ is below the threshold, μ falls at a faster rate and dominates the rise in $\left[\sigma\chi\left(1+\frac{\beta}{\kappa\delta}\right)-1\right]$. As result, we see higher post-exit fluctuation in output gap. However, $\left[\sigma\chi\left(1+\frac{\beta}{\kappa\delta}\right)-1\right]$ rises rapidly when when δ goes beyond the threshold, causing lower fluctuation in output gap. On the other hand, inflation falls monotonically as δ rises because χ rises monotonically with δ. In terms of Figure 1, higher δ makes the AA curve steeper and BB curve flatter.

Post-exit output gap and inflation gives,

$$Z_{T+1} = \begin{bmatrix} y_{T+1} \\ \pi_{T+1} \end{bmatrix} \tag{20}$$

Next, we calculate Z_t for $t = 1, 2, ..., T^d$ given the terminal condition Z_{T^d+1} numerically from (15) . T^d is determined as, $Q^d_{T^d} = \kappa\left(\delta - \sigma\left(\sigma^{-1} + \eta\right)\right)\pi_t - \sigma\lambda y_t > 0$ last time. Then, $T^d + 1$ is the exit time with,

$$\begin{aligned} i_t &= 0, t = 1, 2, ..., T^d \\ &> 0, t = T^{d+1}, T^d + 2, ... \end{aligned}$$

4.2 Optimal Policy under Commitment

This section analyzes the optimal policy under commitment or fully optimal policy. Here, the objective of the monetary authority is to minimize equation (4) by choosing output gap, inflation rate and nominal interest rate, subject to equation (1), (2) and the feasibility constraint, $i_t \geq 0$. Unlike discretion, monetary authority endogenizes future expectation of output gap and inflation rate in the process of loss minimization under commitment. There is no uncertainty for simplicity as before. The Lagrangian of the problem under commitment is,

$$\mathcal{L}_C = \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \begin{array}{c} -\frac{1}{2}\left[\pi_t^2 + \lambda y_t^2\right] - \phi_{1,t}\left[\sigma\left(i_t - \pi_{t+1} - r_t^n\right) - y_{t+1} + y_t\right] \\ -\phi_{2,t}\left[\pi_t - \kappa\left(\sigma^{-1} + \eta\right) y_t - \kappa\delta\left(i_t - i\right) - \beta\pi_{t+1}\right] \\ +\phi_{3,t} i_t \end{array} \right\}$$

The First Order Conditions are,

$$\begin{aligned}
\frac{\partial L}{\partial \pi_t} &= -\pi_t - \phi_{2,t} + \sigma\beta^{-1}\phi_{1,t-1} + \phi_{2,t-1} = 0 && (21)\\
\frac{\partial L}{\partial y_t} &= -\lambda y_t - \phi_{1,t} + \kappa\left(\sigma^{-1}+\eta\right)\phi_{2,t} + \beta^{-1}\phi_{1,t-1} = 0 && (22)\\
\frac{\partial L}{\partial \phi_{1,t}} &= \sigma\left(i_t - \pi_{t+1} - r_t^n\right) - y_{t+1} + y_t = 0 && (23)\\
\frac{\partial L}{\partial \phi_{2,t}} &= \pi_t - \kappa\left(\sigma^{-1}+\eta\right)y_t - \kappa\delta\left(i_t - i\right) - \beta\pi_{t+1} = 0 && (24)\\
\frac{\partial L}{\partial i_t} &= -\sigma\phi_{1,t} + \kappa\delta\phi_{2,t} + \phi_{3,t} = 0 && (25)\\
\phi_{3,t}\frac{\partial L}{\partial \phi_{3,t}} &= \phi_{3,t}i_t = 0, \phi_{3,t} \geq 0, i_t \geq 0 \text{ with complementary slackness} && (26)
\end{aligned}$$

From equation (25), $\phi_{3,t} \geq 0$ implies

$$Q_t^c = \sigma\phi_{1,t} - \kappa\delta\phi_{2,t} \geq 0 \tag{27}$$

Equation (27) gives,

$$\begin{aligned}
i_t &= 0 \text{ till } Q_t^c > 0\\
&= 0 \text{ O.W.}
\end{aligned}$$

We assume economy is in liquidity trap for $t = 1, 2, ..., T^c$ and exit liquidity trap permanently at $t = T^c + 1$. As a result, $i_t = 0$ till period T^c and $i_t > 0$ from $t = T^c + 1$ onwards. Equation (23) and (24) with $i_t = 0$ gives,

$$Z_{t+1} = c + AZ_t - ar_t^n \tag{28}$$

Solving equation (29) forward gives,

$$Z_t = \Gamma_t^c + A^{-(T^c-t+1)}Z_{T^c+1} \tag{29}$$

where,

$$\Gamma_t^c = \sum_{k=t}^{T^c} A^{-(k-t+1)}ar_k^n - \sum_{k=t}^{T^c} A^{-(k-t+1)}c$$

Note, since one eigenvalue of the matrix A is greater than one and other less than one, higher T^c and Z_{T^c+1} gives more stimulus to the system. Now, solution of the system entails we know Z_{T^c+1} and T^c. Z_{T^c+1} is determined as follows. Equation (21) and (22) gives,

$$\Phi_t = C\Phi_{t-1} - DZ_t \tag{30}$$

where,

$$\begin{aligned} \Phi_t &= \begin{bmatrix} \phi_{1,t} \\ \phi_{2,t} \end{bmatrix}, Z_t = \begin{bmatrix} y_t \\ \pi_t \end{bmatrix}, D = \begin{bmatrix} \lambda & \kappa\left(\sigma^{-1}+\eta\right) \\ 0 & 1 \end{bmatrix} \\ C &= \begin{bmatrix} \beta^{-1} + \frac{\sigma\kappa\left(\sigma^{-1}+\eta\right)}{\beta} & \kappa\left(\sigma^{-1}+\eta\right) \\ \frac{\sigma}{\beta} & 1 \end{bmatrix} \end{aligned}$$

A forward solution of equation (30) with $\Phi_1 = 0$ and using equation (29) yields,

$$\begin{aligned} \Phi_{T^c} &= -\sum_{t=1}^{T^c} C^{(T^c-t)} DZ_t \\ &= -\sum_{t=1}^{T^c} C^{(T^c-t)} D\left(\Gamma_t^c + A^{-(T^c-t+1)} Z_{T^c+1}\right) \end{aligned} \tag{31}$$

Equation (31) solves T^c such that, $Q_t^c > 0$ for the last time. It also solves Φ_{T^c}. However, to do that we need to know Z_{T^c+1}. Z_{T^c+1} is solved as follows.

Note we have $i_t > 0$ for $t = T^c+1, T^c+2, \ldots..$ We also have,

$$\phi_{2,t} = \frac{\sigma}{\kappa\delta}\phi_{1,t}, t = T^c+1, T^c+2, \ldots. \tag{32}$$

Equation (21) and (22) with equation (32) gives,

$$\pi_t = -\frac{\sigma}{\kappa\delta}\phi_{1,t} + \sigma\left(\beta^{-1} + \frac{1}{\delta\kappa}\right)\phi_{1,t-1} \tag{33}$$

and,

$$y_t = -\lambda^{-1}\left(1 - \left(\sigma^{-1}+\eta\right)\frac{\sigma}{\delta}\right)\phi_{1,t} + \lambda^{-1}\beta^{-1}\phi_{1,t-1} \tag{34}$$

Again, by eliminating i_t from equation (23) and (24) we have,

$$y_{t+1} + \sigma\left(1 + \frac{\beta}{\kappa\delta}\right)\pi_{t+1} = \left[1 - \frac{\sigma^{-1}+\eta}{\sigma^{-1}\delta}\right] y_t + \frac{\sigma}{\kappa\delta}\pi_t + u_t \tag{35}$$

Now, equation (33), (34) and (35) yields, following difference equation,

$$\phi_{1,t+1} = b_1\phi_{1,t} + b_2\phi_{1,t-1} + b_3 u_t \tag{36}$$

where

$$b_1 = \frac{\beta^{-1}\tau^2\vartheta^{-1} + \lambda^{-1}\beta^{-1} + \vartheta + \alpha^2\lambda}{\tau+\alpha}, b_2 = -\beta^{-1}, b_3 = -(\tau+\alpha)^{-1}$$

and

$$\tau = \frac{\sigma^2}{\kappa\delta}[1 + \frac{\beta}{\kappa\delta}], \alpha = [1 - \left(\sigma^{-1}+\eta\right)\frac{\sigma}{\delta}]\lambda^{-1}, \vartheta = \frac{\sigma^2}{\kappa^2\delta^2}$$

Equation (36) has two eigenvalues, $\omega_1 > 1$ and $\omega_2 \in (0,1)$. Solving equation (36) we have,

$$\begin{aligned} \phi_{1,t} &= \omega_2\phi_{1,t-1} + \frac{u_t}{(\omega_1-\rho)(\tau+\alpha)} \\ \phi_{2,t} &= \frac{\sigma}{\kappa\delta}\phi_{1,t}, t = T^c+1, T^c+2, ... \end{aligned} \tag{37}$$

Note, equation (33), (34) along with equation (37) gives,

$$Z_{T+1} = \Theta\Phi_T + Ju_{T+1} \tag{38}$$

where,

$$\begin{aligned} \Theta &= \begin{bmatrix} \lambda^{-1}[\beta^{-1} - \omega_2\{1 - (\sigma^{-1}+\eta)\frac{\sigma}{\delta}\}] & 0 \\ \beta^{-1} + \frac{1-\omega_2}{\kappa\delta} & 0 \end{bmatrix} \\ J &= \begin{bmatrix} \frac{-\lambda^{-1}[1-\left(\sigma^{-1}+\eta\right)\frac{\sigma}{\delta}]}{(\omega_1-\rho)(\tau+\alpha)} \\ \frac{\sigma}{\kappa\delta(\omega_1-\rho)(\tau+\alpha)} \end{bmatrix} \end{aligned}$$

Substituting equation (38) to equation (31) gives,

$$\Phi_{T^c} = -W^{-1}W_2 \tag{39}$$

where,

$$
\begin{aligned}
W_1 &= I + \sum_{t=1}^{T^c} C^{(T^c-t)} D A^{-(T^c-t+1)} \Theta \\
W_2 &= \sum_{t=1}^{T^c} C^{(T^c-t)} D [\Gamma_t^c + A^{-(T^c-t+1)} J u_{T+1}]
\end{aligned}
$$

Equation (39) solves T^c such that $Q^c_{T^c} > 0$ for the last time. It also solves, Φ_{T^c}. Then, equation (37) solves $\phi_{1,t}$ and $\phi_{1,t}$ for $t = T^c+1, T^c+2, ...$ This allows equation (33), (34) to solve y_t and π_t for $t = T^c+1, T^c+2, ...$ Nominal interest rate, $i_t = 0$ till period T^c and $i_t > 0$ for $t = T^c+1, T^c+2, ...$ Equation (23) solves i_t for $t = T^c+1, T^c+2, ...$ Figure 3 shows the impulse response under commitment for different δ.

4.3 The T-only Policy

Chattopadhyay and Daniel (2016) introduces a new policy known as T-only policy. Monetary authority chooses the exit time optimally under T-only policy and follows the discretionary policy post exit. This implies, pre-exit solution of output gap and inflation rate is obtained from,

$$Z_t = \Gamma_t^o + A^{-(T^o-t+1)} Z_{T^o+1}$$

and,

$$
\begin{aligned}
y_t &= \frac{u_t}{\left[\sigma\chi\left(1+\frac{\beta}{\kappa\delta}\right)-1\right](\mu-\rho)} \\
\pi_t &= -\chi y_t, t = T^o+1, T^o+2, ...
\end{aligned}
$$

where,

$$\Gamma_t^o = \sum_{k=t}^{T^o} A^{-(k-t+1)} a r_k^n - \sum_{k=t}^{T^o} A^{-(k-t+1)} c$$

and T^o is chosen optimally such that equation (4) is minimized. Figure 4 shows the impulse response under T-only policy for different δ. Note, irrespective of the presence and absence of cost channel, T-only policy replicates fully optimal policy (commitment) very closely.

5 Calibration and Impulse Response

We illustrate the base line impulse response of output gap, inflation rate, nominal interest rate and real interest rate using following parameterization.

Parameter	Description	Value	Source[6]
σ	logarithmic preference	1	AB (2006)
β	discount factor	0.99	standard
κ	measure of price stickiness	0.028	AB (2006)
η	inverse of slope of Frisch labor supply	1	RW (2006)
λ	relative weight on output gap in loss function	0.0074	AB (2006)

Table 1: Calibrated Parameters

Discount factor, $\beta = 0.99$ implies long run real interest rate, $i = \beta^{-1} - 1 = 0.0101$. $\kappa = 0.028$ implies price is highly sticky with only 16% of firm can choose their price optimally each period.[7] Moreover, above parameterization gives $\delta^* = 1.8$. Ravenna and Walsh (2006), Chowdhury et. al. (2006) and Tillman (2007) has estimated $\delta \in [1, 1.4]$. Therefore, we restrict our analysis to two extreme cases; $\delta = 1$ and 1.4. We have used no cost channel, $\delta = 0$ as our benchmark to check examine the impact of cost channel on optimal policy. Beside these, we need demand shock to be large enough to send the economy into liquidity trap and persists for a considerable period of time. Hence we set,

$$u_1 = 0.024, \rho = 0.9$$

Before, going to present the full blown analysis of impulse response let us present some important results in Table 1. These results supplements the results hidden in impulse response analysis. However, exit time as shown in Table 2 under discretion, commitment and T-only policy falls as magnitude of demand shock falls for given δ. Table also 2 shows that, while exit time under discretion and T-only policy rises but falls under commitment as δ rises. Table 2 also shows that, T-only policy closely replicates commitment for varying

[6]AB: Adam and Billi and RW: Ravenna and Walsh

[7]The slope of the Phillips curve without cost channel in Adam and Billi (2006) is $\kappa = 0.056$. Same in Ravenna and Walsh (2006) with cost channel is $\kappa\left(\sigma^{-1} + \eta\right)$. I set, $\kappa = 0.028$ such that, $\kappa\left(\sigma^{-1} + \eta\right) = 0.056$. It allows us to identify only the impact credit market imperfection or interest rate pass-through on the optimal policy.

δ.

$u = 0.021, \rho = 0.9$					
δ	T^d	T^c	T^o	$\frac{L^D}{L^c}$	$\frac{L^c}{L^{TO}}$
0	9	13	12	1.15	0.0087
1	10	13	14	17.42	0.78
1.4	11	12	15	3.30	0.99
$u = 0.018, \rho = 0.9$					
δ	T^d	T^c	T^o	$\frac{L^D}{L^c}$	$\frac{L^c}{L^{TO}}$
1	7	8	10	10.9	0.047

Table 2: Comparative Analysis

Figure 2 shows the impulse response under discretion for different degree of credit market imperfection or interest rate pass-through (δ).

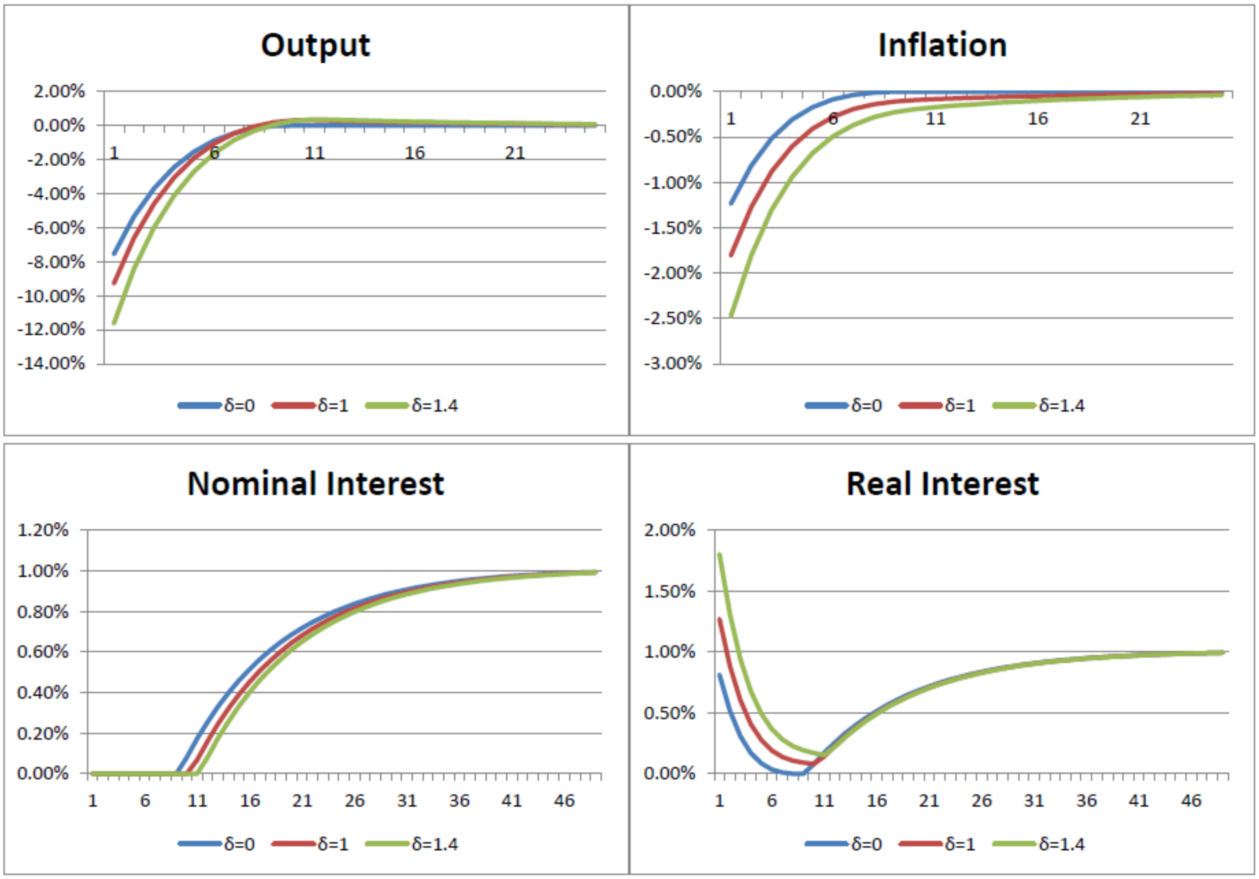

Figure 2: Impulse Response under Discretion

Figure 2 shows that large adverse demand shock puts economy into liquidity trap and causes recession and deflation. However, while post-exit output gap becomes positive and gradually converges to zero economy suffers from deflation for the entire time period. Figure 2 also shows that, fluctuations both in output gap and inflation rate rises with δ. It happens since given zero nominal interest rate during liquidity trap, higher δ causes more fluctuations in current and future inflation. This causes higher fluctuations in real interest rate and thereby output gap. As a result, exit time also rises with δ under discretion which provides extra stimulus to the system to minimize welfare loss. Figure 2 also shows that, given δ output fluctuates more than inflation rate since we have assumed that negative impact of output gap on inflation rate of a tighter monetary policy dominates the impact of interest rate on inflation. We have also checked that, while natural rate of interest

(r_t^n) becomes positive at $t = 9$, nominal interest rate becomes positive at $t = 10$. Hence, unlike without cost channel the time path of nominal interest rate under cost channel is endogenous and does not get determined by the time path of nominal interest rate only. This happens as cost channel introduces trade-off between output gap and inflation rate after exit.

Figure 3 gives the impulse response under commitment for different values of δ. Figure 3 shows that, contrary to discretion we have lower fluctuations in output gap and inflation rate and real interest rate under commitment as interest rate pass-through rises due to (i) promise of future inflation and boom and (ii) higher expected value of both inflation and output gap post-exit (Z_{T^c+1}).

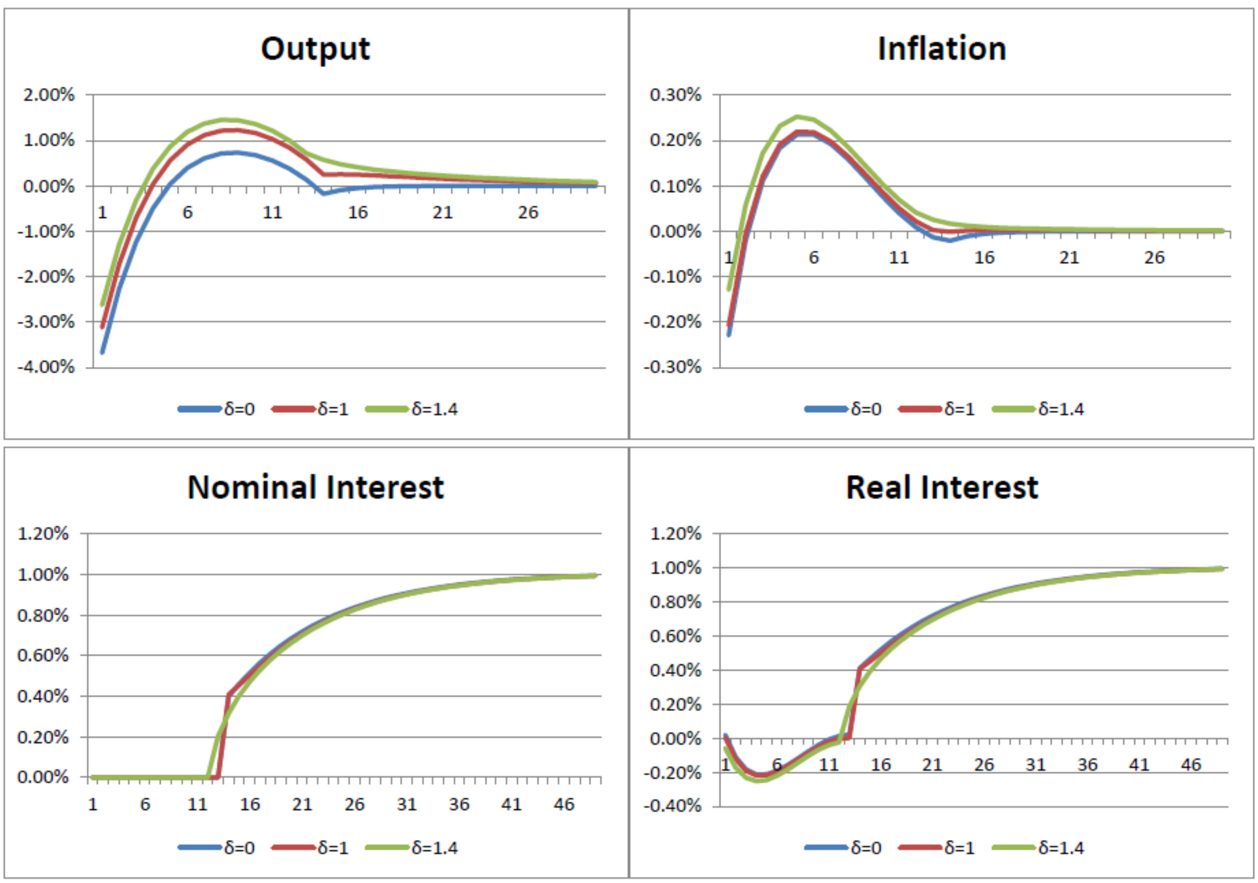

Figure 3: Impulse Response under Commitment

Promise of future boom and inflation along with higher post-exit expectation of output gap and inflation gives added stimulus to the system, causing exit date under commitment to fall as δ rises to compensate the added stimulus and minimize the welfare loss. As a result, contrary to no cost channel, exit time under commitment comes soonest in the presence of cost channel as shown in Figure 3 and furnished in Table 2.

Figure 4 portrays the impulse response under T-only policy. We know that monetary authority in a T-only policy chooses the exit date optimally and follows discretion post-exit. Chattopadhyay and Daniel (2016) show that, a T-only policy closely replicates commitment without cost channel. We see that, impulse response given in Figure 4 is very similar to Figure 3. It shows that, T-only policy closely replicate commitment even when the cost channel is present. As a result, loss under commitment is almost identical to loss under T-only policy as given in Table 2.

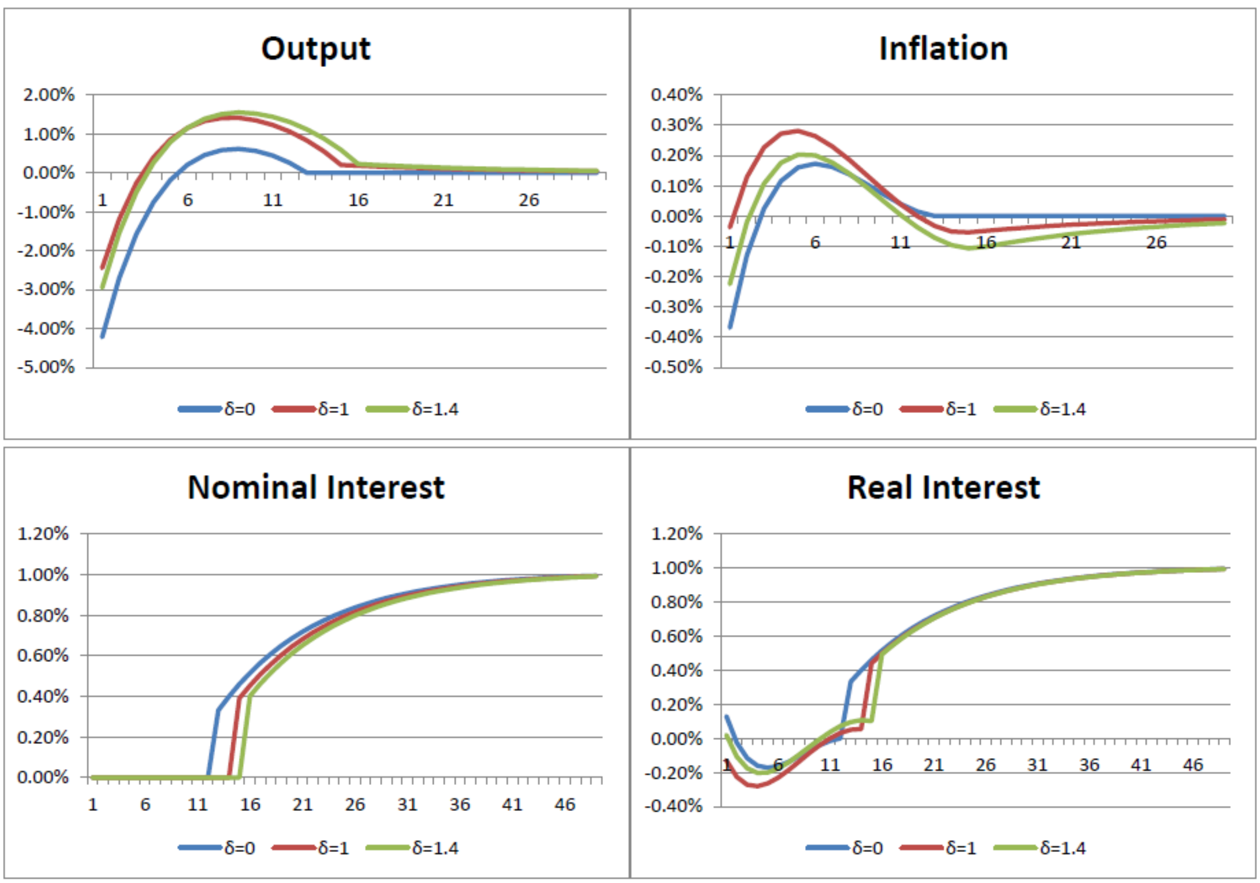

Figure 4: Impulse Response under T-only Policy

However, though impulse response under commitment and T-only policy are similar, exit date under T-only policy rises with δ but falls under commitment. To explain note, future expected output gap and inflation under T-only is lower than the same under commitment, $Z_{T^o+1} < Z_{T^c+1}$. Hence, T-only can only produce results closer to commitment if it gets a higher stimulus by postponing exit date than commitment.

6 Conclusions

The exit out of ZLB varies from one economy to other depending on its characteristics. Different degrees of interest rate pass-through (or varying strength of cost channel) originating from different extent of credit market imperfection is possibly one such characteristic leading to disparate exit dates. In absence of cost channel, exit under discretion is exogenous and the exit under commitment is endogenous. Along with this, commitment outperforms discretion by promising future boom and inflation and by delaying exit. Introduction of cost channel makes exit date endogenous under both commitment and discretion. This is due to the trade-off between inflation and output gap induced by the cost channel. Exit date rises monotonically under discretion with magnitude of demand shock and degree of interest rate pass-through. Commitment still outperforms discretion with a promise of future boom and inflation and post exit expected boom and inflation. This gives an extra stimulus to the system under commitment. System compensates the stimulus by preponing its exit compared to discretion. On the other hand, though a T-only policy promises future boom and inflation, it does not promise post exit inflation. As a result, T-only policy needs to postpone its exit to gain stimulus and producing results closer to commitment.

7 Appendix

7.1 Commitment with Cost Channel

We solve the second order difference equation (36) as follows,

$$\phi_{1,t+1} = b_1\phi_{1,t} + b_2\phi_{1,t-1} + b_3u_t$$

We assume, $\gamma_{t+1} = \phi_{1,t}$ and rewritten the second order difference equation given above as,

$$\psi_{t+2} = B\psi_{t+1} + bu_{t+1}$$

where,

$$\psi_{t+2} = \begin{bmatrix} \phi_{1,t+2} \\ \gamma_{t+2} \end{bmatrix}, B = \begin{bmatrix} b_1 & b_2 \\ 1 & 0 \end{bmatrix}, \; b = \begin{bmatrix} b_3 \\ 1 \end{bmatrix}$$

Let ω_1 and ω_2 be the two eigenvalues of B such that $\omega_1 > 1$ and $0 < \omega_2 < 1$. Now, define

$$\Omega = \begin{bmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{bmatrix}$$

and the matrix of eigenvector of B,

$$E = \begin{bmatrix} e_1 & e_2 \\ 1 & 1 \end{bmatrix}, e_i = \omega_i$$

Diagonalization gives,

$$\psi'_{t+2} = \Omega\psi'_{t+1} + E^{-1}bu_{t+1}$$

where $\psi'_{t+2} = H^{-1}\psi_{t+2}$. Now, we solve the first equation forward and second equation backward and do standard algebraic manipulation to get, equation (37) given in the text.

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