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Absolute purchasing power parity in industrial countries

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Abstract: In contrast to popular studies that focus on relative purchasing power parity, we study absolute purchasing power parity (APPP) in 21 main industrial countries. A new method in testing APPP is used. The empirical proof shows that the phenomenon that APPP holds is common, and the phenomenon that APPP does not hold is also common. In addition, most country pairs and the pooled country data indicate that the nearer the GDPs of two countries are, the more valid APPP between the two countries is.

Keywords: Absolute purchasing power parity; Real exchange rate; Coefficient restriction test

JEL Classification: F30; F31

1. Introduction

The Purchasing power parity (PPP) theory has been playing an important role in research, exchange rate policy and the foreign exchange market (Officer, 1976, Section III; MacDonald, 2007, Chapter 2), and has been one of the core theories in international finance (Krugman et al., 2010, Chapter 16). Thus, whether PPP holds or not in industrial countries has been extensively studied (Rogoff, 1996; Taylor and Taylor, 2004).

Popular papers use the unit root and cointegration tests to judge whether or not PPP holds. For example, Lothian and Taylor (1996) apply the Dickey-Fuller and the Phillips-Perron unit roots tests to the annual data spanning two centuries for dollar-sterling and franc-sterling real exchange rates (RERs). They find that PPP holds for the two RERs in the full sample periods but does not hold in the sub-period following World War II or in the floating exchange rate period. Papell (1997) applies panel unit root tests to RERs of industrial countries under the float exchange rate period. They find that the evidence against the unit root hypothesis is stronger for larger than for smaller panels, for monthly than for quarterly data, and when the German mark, rather than the United States dollar, is used as the base currency. Sollis et al. (2002) apply modified smooth transition autoregressive models to monthly series of seventeen RERs against the U.S. dollar and fourteen RERs against the deutsche mark in industrialized countries. They reveal stronger evidence against the unit root null hypothesis than does the usual Dickey-Fuller test. Koedijk et al. (2004) test the PPP hypothesis for a panel of RERs within the euro area over the period 1973–2003. They present evidence in favor of PPP for the full panel of RERs, and they show that accounting for cross-country differences within the euro area is essential. Karoglou and Morley (2012) apply the econometric method of the structure change to the bilateral US/UK RER for the sample period of January 1885 to June of 2009, and find that PPP holds in some sub-periods. Huang and Yang (2015) apply panel unit root tests to the RERs of eleven euro countries for the sample period of January 1957 to May 2013, and find that the evidence for the mean-reverting in RERs is much weaker in the post-1998 euro period than in the pre-euro period.

The PPP theory has two versions: absolute PPP and relative PPP. Absolute PPP says that a bilateral nominal exchange rate should be equal to the ratio of the (general) price levels of the two countries. Relative PPP says that the change of a bilateral nominal exchange rate should be equal to the ratio of the changes in the price levels of the two countries. Comparatively, absolute PPP is

more basic, and relative PPP only relaxes the conditions of absolute PPP. It is known that if absolute PPP holds then relative PPP must hold, but not vice versa (Taylor and Taylor, 2004, p. 137). In popular papers such as those listed above, however, the RERs are invariably constructed by consumer, producer, and wholesale price indexes rather than actual price levels. Such constructed RER is used in testing relative PPP rather than absolute PPP (Crownover et al., 1996, p. 783; Cheung et al., 2005, p. 1153). Through a search of the literature, we find that the papers focusing on relative PPP in industrial countries are countless, but the papers focusing on absolute PPP in industrial countries are very few.¹ Relative PPP has been thoroughly studied in terms of validity, convergence, and linearity or non-linearity. In contrast, absolute PPP has been scarcely studied. Concretely, the validity of APPP in industrial countries has been scarcely known since Crownover et al. (1996). Thus, it is necessary to construct RERs by actual price levels and to study the validity of absolute PPP in industrial countries.

Recently, Zhang and Zou (2014) discuss which econometric method should be used in testing absolute PPP (APPP), and analyze APPP of the 40 biggest countries using panel data. For a given country, the bilateral exchange rate between this country and its main trading partner is important. However, the panel data dimension cannot tell us whether or not APPP holds between a pair of countries in terms of the bilateral RER. That is, the validity of APPP in pairs of industrial countries is beyond their scope. Thus, in this paper, we use the time series method to discuss the validity of APPP in main industrial countries in terms of the bilateral RERs. In addition, we use different methods and data from Crownover et al. (1996) and Zhang and Zou (2014).

The contributions of this paper include the following. (1) Whether the behavior of bilateral RERs in main industrial countries (e.g., the bilateral RER between the UK and the US and that between Japan and the US) conforms to APPP is important to financial market participants and economic policy makers, but this issue is scarcely known. In this paper we proved that APPP commonly holds in the bilateral RERs in industrial countries. This conclusion should be very useful to financial market participants and economic policy makers. (2) Given that APPP holds for some RERs and not for other RERs, the condition when APPP may tend to hold is discussed. (3) Zhang and Zou (2014) have proven that the commonly used unit root and cointegration tests are invalid in testing APPP. We propose a new method in this respect. Using the method, one can analyze not only APPP in the whole periods but also the structure change of APPP in the sub-periods.

The rest of the paper is organized as follows. Section 2 gives the concept, method and data. Sections 3 and 4 investigate the validity of APPP based on various databases. Section 5 discusses whether or not the GDPP influences the validity of APPP. Section 6 concludes the paper.

2. Concept, method and data

It is useful to introduce APPP by using the term RER. In this paper, the RER is defined by Eq. (1), where P_i is the domestic price level of country i , P^* is the price level of a foreign country, PPP_i rate is P_i divided by P^* , and the nominal exchange rate NER_i is expressed as the domestic currency units per foreign currency unit. In this definition, a greater value of RER represents the local currency's appreciation against the foreign country. The RER in this definition also measures the

¹ Though some economists (e.g., Bergin, et al., 2006; Broda, 2006) construct the RER by the price level, they discuss other topics rather than the validity of absolute PPP.

relative price level between two countries in terms of a common currency. Thus, it is also called “the price level (of one country relative to the base country)” in popular databases.

$$RER_i = \frac{P_i}{NER_i \times P^*} = \frac{P_i/P^*}{NER_i} = \frac{PPP_i}{NER_i} \quad (1)$$

2.1. APPP and the Penn effect

APPP says that a bilateral nominal exchange rate should be equal to its PPP rate or two countries’ price levels should be equal when denominated in the same currency. In other words, if the RER defined in Eq. (1) is one, APPP holds; if the RER defined in Eq. (1) is not one, APPP does not hold. In practice, APPP was once used to anchor the nominal exchange rate in some countries, for example in the period between the two World Wars in the UK, Czechoslovakia, and Belgium (Officer, 1976, p. 26).

However, since Balassa (1964) and relevant studies, it is now well known that APPP often does not hold between a rich and a poor country because of the existence of the empirical regularity depicted in Fig. 1. Fig. 1 tells us that, from a global view, there is a systematic relationship between the income level and the RER: the RER tends to be positive with the income level (the RER in a low-income country is often smaller and that in a high-income country is often greater). This regularity is called the “(long-run) deviations from PPP” (Rogoff, 1996), “Balassa–Samuelson effect” (Bergin et al., 2006, Frankel, 2006), “Harrod–Balassa–Samuelson effect” (Taylor and Taylor, 2004), “Penn effect” (Samuelson, 1994, Isard, 2007), or others; The regularity and its explanations are often not differentiated. In this paper, we use the term “Penn effect.”

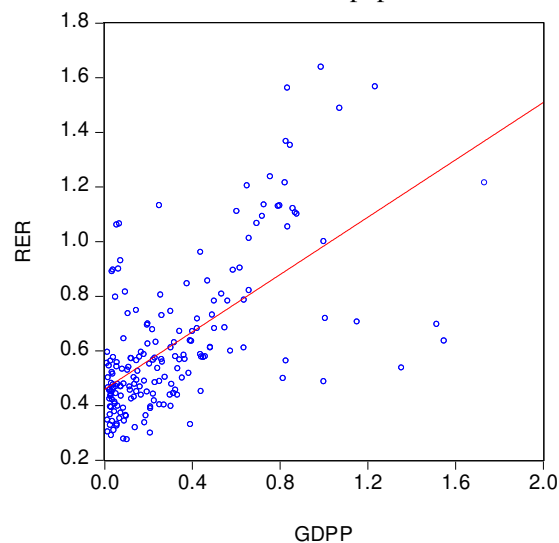


Fig. 1. Penn effect for 187 countries and areas in 2013.

Notes: Both the real exchange rate (RER, defined by Eq. (1)) and GDP per capita (GDPP, PPP (constant 2011 international \$)) are normalized, with the US = 1. A cross-section regression gives $RER = 0.460 + 0.525 GDPP$, with both constant and slope terms being significant at the 1% level.

Sources: World Development Indicators and the authors’ calculations.

Seen from the Penn effect, except for the outliers, the nearer the GDPPs of two countries are, the nearer the RERs of the two countries are. As the GDPPs in the industrial countries are relatively nearer to each other, it is expected that APPP may hold between some pairs of these countries, which is a reason for us to write this paper.

2.2. Method

It is now well known that even when the GDP of a country is very near to that of the other country (e.g., Canada and the US), APPP does not hold strictly or perfectly because of some factors such as the transportation costs, tariffs, and nontariff barriers (Rogoff, 1996, pp. 653–654). In other words, we cannot find a RER between two countries in the actual world whose value is invariably one. Further, if we test the APPP theory in accordance with whether a RER's value is invariably one, we will get the conclusion that APPP does not hold for any pair of countries. But actually, in any textbook of international finance (e.g., MacDonald, 2007, Chapter 2; Krugman et al., 2010, Chapter 16), APPP is introduced as one of the most basic and important exchange rate theories. Thus, it is wrong to test the theory in accordance with whether a RER's value is invariably one. The meaningful thing is to use some econometric method to investigate *how closely* APPP holds (how close the RER is to one) in the real world.

However, Crowover et al. (1996, p. 785) says, “Testing for absolute PPP can only be accomplished by testing for the equality between the nominal exchange rate and the ratio of price levels.” They empirically test the theory by estimating coefficients and examining coefficient restrictions. Further, Zhang and Zou (2014) discuss which econometric method should be used in testing APPP. They use empirical evidences and an example to prove that the unit root test (no matter whether using the linear or nonlinear method) is invalid in testing APPP. The two studies show that the commonly used unit root and cointegration tests in relative PPP studies are not proper (or not sufficient) in testing APPP.

In this paper, we use a test based on Eq. (2), where the RER is defined in Eq. (1), C is a constant, and no logarithmic transformation for the RER is used. Such an equation as Eq. (2) has been used to analyze the behavior of the U.S. real interest rate (e.g., Bai and Perron, 2003a; Rapach and Wohar, 2005). When Eq. (2) is used in the analysis of the interest rate, it has no particular economic meaning. However, when used in the analysis of the RER, Eq. (2) has a particular economic meaning, because the constant C in the equation measures how close the RER is to one, the equilibrium value of APPP. Concretely, we use OLS with Newey–West robust standard error to estimate Eq. (2), and then examine whether the constant, C , is equal to one. If the constant is equal to one, we think that the RER fluctuates around its equilibrium value and APPP holds. Otherwise, APPP does not hold. For the coefficient restriction test in Eq. (2), we use the Wald test. If the p -value for the Chi-squared statistic in the Wald test is greater than a usual significant level (1%, 5%, or 10%), we think that this test accepts the null hypothesis $C = 1$ and APPP holds. If the p -value is less than a usual significant level, we think that this test rejects the null hypothesis and APPP does not hold.

$$RER_t = C + u_t \quad (2)$$

For the sub-period analysis, we use the least squares with breakpoints by Bai and Perron (1998, 2003a, 2003b). The Bai and Perron method can not only identify the breakpoints but also estimate the coefficients in all sub-periods. Concretely, three tests are used: the $SupF_T(k)$, the double maximum statistics (UD_{max} and WD_{max}), and the sequential $SupF_T(l + 1/l)$. The $SupF_T(k)$ tests the null hypothesis of no structural breaks ($m = 0$) against the alternative hypothesis that there are $m = k$ breaks. The double maximum test considers the null hypothesis of no structural breaks ($m = 0$) against the alternative hypothesis of at least 1 through to M structural breaks. The double maximum test takes two forms, UD_{max} and WD_{max} . The UD_{max} statistic is the maximum value of

the $\text{Sup}F_7(k)$ statistic while the WD_{max} statistic weights the individual statistics. The sequential $\text{Sup}F_7(l + 1/l)$ procedure tests the null hypothesis of l breaks against the alternative hypothesis of $(l + 1)$ breaks. We first conduct the double maximum test to examine whether or not the breaks exist. If the double maximum test (UD_{max} and/or WD_{max}) confirms that at least one break exists, we examine the actual, fitted, and residual graphs in the three tests and choose the test whose result seems to be most reasonable. Following [Bai and Perron \(2003a, Section 6\)](#) and [Rapach and Wohar \(2005\)](#), the unit root test is not needed before applying OLS to [Eq. \(2\)](#), which reduces the econometric work.

Finally, we can see that the method based on [Eq. \(2\)](#) has some relationships and differences with the coefficient restriction and the RER misalignment distribution tests in [Zhang and Zou \(2014\)](#). (1) It also uses the Wald test to test the coefficient restriction (it is also a coefficient restriction test). In contrast, the coefficient restriction test in [Zhang and Zou \(2014\)](#) tests whether the nominal exchange rate is equal to its PPP rate, but the method based on [Eq. \(2\)](#) tests whether the RER is equal to its equilibrium value (one). However, the two kinds of methods are equivalent, because the nominal exchange rate is equal to its PPP rate if and only if the RER is equal to 1. (2) It also examines the mean of the RER as used in the RER misalignment distribution test. However, the RER misalignment distribution test examines the RER mean using a simple statistic, and the method based on [Eq. \(2\)](#) examines the RER mean in a regression analysis. (3) Compared with the coefficient restriction test in [Zhang and Zou \(2014\)](#), one does not need to do the preliminary unit root and cointegration tests before performing equation estimation when using the method based on [Eq. \(2\)](#), and the econometric steps in this method are fewer.

2.3. Data

The core data in constructing the RER defined in [Eq. \(1\)](#) is the PPP rate. Different from the price index that can be obtained from a country's statistics department, the PPP rate can only be obtained by an international price level comparison, which is often conducted by the international organizations. The two databases that supply the RER defined in [Eq. \(1\)](#) are the Penn World Table (PWT) and the World Bank's World Development Indicators (WDI). The PWT 8.0 and later versions are made by economists at the University of California, Davis and the University of Groningen; see [Feenstra et al. \(2013\)](#) for the details. The PWT 8.1 is based on the 2005 International Comparison Program, while the WDI is based on the 2011 International Comparison Program. This leads to the different values for the same variable in the two databases. In addition, the PWT 8.1 supplies data from 1950 to 2011 (a long-run period), while the WDI (the June 2015 version) supplies data from 1990 to 2013 (an updated period). Thus, the two databases are used in the paper. Using the PWT data to analyze APPP is feasible and reasonable ([Frankel, 2006, p. 260](#)).

Concretely, only the RER and GDP per capita (GDPP) for each country are needed in this paper. In the PWT 8.1, the RER is the "Price level of CGDPo (PPP/XR), price level of USA GDPo in 2005 = 1" (the variable "pl_gdpo" in the database), and the GDPP is derived from the "Output-side real GDP at chained PPPs (in mil. 2005US\$)" (the variable "rgdpo" in the database) and the "Population (in millions)" (the variable "pop" in the database). In the WDI, the RER is the "Price level ratio of PPP conversion factor (GDP) to market exchange rate" (the code "PA.NUS.PPPC.RF" in the database), and the GDPP is the "GDP per capita, PPP (constant 2011 international \$)" (the code "NY.GDP.PCAP.PP.KD" in the database). Though the WDI supplies both the PPP-converted and market exchange rate-converted GDPPs, the PWT 8.1 only supplies

the PPP-converted GDPP, thus we use the PPP-converted GDPP. Though the PWT 8.1 supplies the name “Price level of CGDPe (PPP/XR), price level of USA GDPo in 2005 = 1” (the variable “pl_gdpe”), the values for this variable are blank, thus there isn’t another choice for the RER besides the variable “pl_gdpo” in this database.

Finally, some notes about the data should be given. (1) In the following sections when the US is treated as the foreign country in Eq. (1), the RERs and GDPPs are normalized to those of the US = 1 in each year respectively. Likewise, when the UK is treated as the foreign country in Eq. (1), the RERs and GDPPs are normalized to those of the UK = 1 in each year respectively. (2) In the PWT 8.1, the whole period for Greece is 1951–2011, and those for all other countries are 1950–2011. In the WDI, the whole period is 1990–2013 for each country. (3) We chose 21 traditional, main industrial countries, the same countries as in Papell (1997) in his relative PPP studies: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, the UK, and the US.

3. Based on the PWT 8.1

In this section, we analyze the validity of APPP between each country and the US, and the validity of APPP between each country and the UK, both based on the PWT 8.1.

3.1. APPP between each country and the US: The whole period

In this section we use the US as the foreign country in Eq. (1) and analyze APPP between each country and the US. The main econometric results are given in Table 1. The coefficient estimation and test in the sub-period are the same as those in the whole period except the breakpoint analysis, thus we only give the conclusion about the breakpoint analysis for the sub-period.

Table 1. APPP between each country and the US based on the PWT 8.1.

Country	In the whole period			In the sub-period	
	C	χ^2 statistic	APPP	Breakpoint	APPP holds for
Australia	0.98 ^{***}	0.22	Holds	1973, 1983	1983–2011
Austria	0.96 ^{***}	0.31	Holds	1973, 1997	1997–2011
Belgium	1.05 ^{***}	0.90	Holds	1973	None
Canada	1.00 ^{***}	0.09	Holds	None	
Denmark	1.13 ^{***}	2.34	Holds	1973, 1987	None
Finland	1.11 ^{***}	3.55 [*]	Holds	1974, 1997	1997–2011
France	1.05 ^{***}	1.41	Holds	1973, 1987	1973–1986
Germany	1.02 ^{***}	0.07	Holds	1972, 1998	1998–2011
Greece	0.82 ^{***}	39.86 ^{***}	Doesn’t hold	1973, 2003	2003–2011
Ireland	0.91 ^{***}	3.62 [*]	Holds	1972	1972–2011
Italy	0.87 ^{***}	9.44 ^{***}	Doesn’t hold	1963, 1987	1987–2011
Japan	1.04 ^{***}	0.15	Holds	1972, 1986	1972–1985
Netherlands	0.93 ^{***}	1.01	Holds	1964, 1973	1973–2011
New Zealand	0.89 ^{***}	15.11 ^{***}	Doesn’t hold	1973, 2003	2003–2011
Norway	1.16 ^{***}	5.73 ^{**}	Holds	1970, 1997	1997–2011
Portugal	0.70 ^{***}	66.18 ^{***}	Doesn’t hold	1973, 1990	None
Spain	0.76 ^{***}	18.00 ^{***}	Doesn’t hold	1973, 1987	1987–2011

Country	In the whole period			In the sub-period	
	C	χ^2 statistic	APPP	Breakpoint	APPP holds for
Sweden	1.29 ^{***}	19.97 ^{***}	Doesn't hold	1972, 1997	1950–1971
Switzerland	1.03 ^{***}	0.07	Holds	1973, 1987	1973–1986
UK	0.93 ^{***}	2.22	Holds	1972, 1987	1972–1986

Notes: *, **, and *** indicate that the coefficient C (in Eq. (2)) or the χ^2 statistic (in the Wald test with $H_0: C = 1$) is significant at the 0.1, 0.05, and 0.01 levels, respectively. No subscript indicates that the coefficient C or the χ^2 statistic is not significant at the 0.1 level. The whole period for Greece is 1951–2011, and those for all other countries are all 1950–2011. The coefficient C in each sub-period is significant at the 0.01 level.

Sources: The PWT 8.1 and the authors' calculations.

We first analyze the whole period. Seen from Table 1, the constant C in each country is significant at the 0.01 level, spans from 0.7 to 1.3, and is not far from 1, which indicates the validity of APPP in each country to some extent. However, when we examine the Wald test to differentiate the validity, the countries are divided into two groups.

(1) For Australia, the constant (0.98) is near 1, and the Wald χ^2 statistic (0.22) is not significant at the 0.1 level, thus the null hypothesis that the constant is 1 is (strongly) accepted. That is, APPP (strongly) holds between this country and the US. Similar conclusions also appear in Austria, Belgium, Canada, Denmark, France, Germany, Japan, the Netherlands, Switzerland, and the UK. For Finland and Ireland, the null hypothesis that the constant is 1 is accepted at the 0.05 level (though rejected at the 0.1 level). For Norway, the null hypothesis that the constant is 1 is accepted at the 0.01 level (though rejected at the 0.5 level). We still think that APPP holds for these three countries: Finland, Ireland, and Norway (though less strongly than the Australia type). Thus, APPP holds for the fourteen countries.

(2) For Greece, though the constant (0.82) is not far from 1, the Wald χ^2 statistic (39.86) is significant at the 0.01 level and strongly rejects the null hypothesis that the constant is 1, thus APPP does not hold between this country and the US. Similar conclusions also appear in Italy, New Zealand, Portugal, Spain, and Sweden. That is, for all these six countries, APPP does not hold.

To illustrate, Fig. 2 gives the RERs of Canada (left part) and Portugal (right part), which can help us to understand the econometric results for the two countries. We can see that the RER of Canada basically fluctuates around the horizontal line of 1 in its whole period, which leads APPP to hold. In contrast, the RER of Portugal is mostly smaller than 1 in the whole period and only is near 1 in a very few years, which leads APPP not to hold in the whole period.

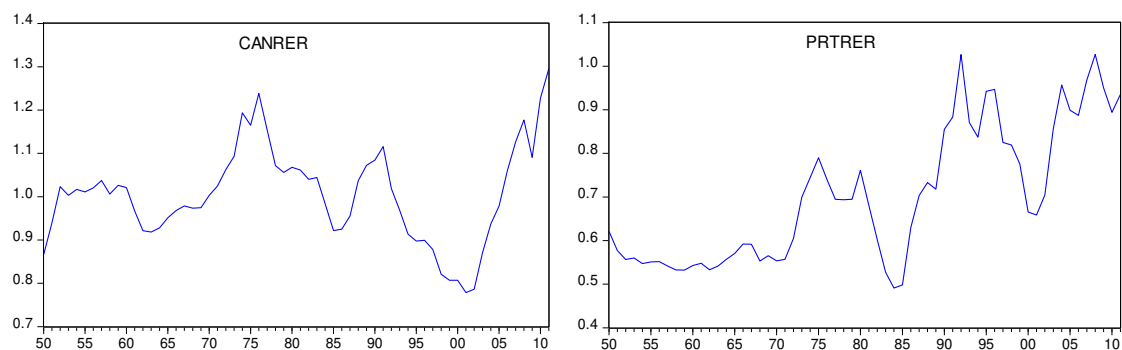


Fig. 2. The RERs of Canada (denoted CANRER) and Portugal (denoted PRTNER).

Notes: The RER of the US = 1 in each year.

Sources: The PWT 8.1 and the authors' calculations.

3.2. APPP between each country and the US: The sub-period

For the sub-period, we use the Bai and Perron method as introduced in [Section 2.2](#) to analyze the structure change. As there are only 62 observations in the whole period 1950–2011, we allow up to 2 breakpoints and use a trimming $\varepsilon = 0.15$. We use Portugal to illustrate the breakpoint analysis; the results are listed in [Table 2](#). We can see that the double maximum tests indicate that there is at least one breakpoint at the 0.05 level. In detail, they suggest the same two breakpoints: 1973 and 1990. The $\text{Sup}F_T(k)$ also suggests the same two breakpoints as the double maximum tests. However, the $\text{Sup}F_T(l + 1/l)$ tests indicate there is one breakpoint: 1990. By examining the actual, fitted, and residual graphs, we choose the breakpoints decided by the double maximum tests and the $\text{Sup}F_T(k)$ tests (1973 and 1990). They divide the whole period 1950–2011 into three regimes (sub-periods): 1950–1972, 1973–1989, and 1990–2011. In 1950–1972, the Wald test strongly rejects the null hypothesis that the constant (0.56) is equal to 1. Likewise, the null hypothesis that the constant is equal to 1 is also strongly rejected in the sub-periods 1973–1989 and 1990–2011. Thus, APPP does not hold in any sub-period for Portugal.

Table 2. The breakpoint analysis for Portugal.

Global L breaks vs none:	$\text{Sup}F_T(1)$ 43.87**	$\text{Sup}F_T(2)$ 45.67**	UD_{max} 45.67**	WD_{max} 54.27**
Sequential L+1 breaks vs. L:	$\text{Sup}F_T(1 0)$ 43.87**	$\text{Sup}F_T(2 1)$ 10.09		
Breakpoints:	1973, 1990			
Regimes:	1950–1972	1973–1989	1990–2011	
C:	0.56***	0.67***	0.87***	
χ^2 statistic:	4333.47***	91.10***	14.47***	

Notes: *, **, and *** indicate that the coefficient C (in [Eq. \(2\)](#)) or the statistic (in the Bai and Perron test and the Wald test) is significant at the 0.1, 0.05, and 0.01 levels, respectively. No subscript indicates there is no significance at the 0.1 level.

The right part of [Table 1](#) gives the conclusions about each country's sub-periods. We can see that for Canada there is no breakpoint. For the other 19 countries, there is at least one breakpoint in each whole period, and the year 1973 (1972, or 1974) is confirmed as a breakpoint in most countries, which indicates the influence of the change of the exchange rate regime on the RER. Among the 19 countries, APPP holds for at least one sub-period for 16 countries (Australia, Austria, Finland, France, Germany, Greece, Ireland, Italy, Japan, the Netherlands, New Zealand, Norway, Spain, Sweden, Switzerland, and the UK). For the remaining 3 countries (Belgium, Denmark, and Portugal), APPP holds in none of the sub-periods.

Finally the validity of PPP of the 20 countries both in the whole periods and in the sub-periods listed in [Table 1](#) can be concluded as below. In terms of the whole periods whose number is 20, APPP holds for 14 periods, and does not hold for six periods. In terms of the sub-periods whose number is 55, APPP holds for 16 sub-periods, and does not hold for 39 sub-periods. Thus, combing the validity of APPP both in the whole periods and in the sub-periods, we can conclude:

the phenomenon that APPP holds is common, and the phenomenon that APPP does not hold is also common.

3.3. APPP between each country and the UK

After knowing APPP between each country and the US, we then investigate whether the conclusion that is obtained by using the US as the base country (the foreign country in Eq. (1)) is robust when another country is used as the base country. Considering that the UK is also much influential in the world economy and the data for this country in the PWT series databases is fairly complete, we choose the UK as the new base country, and analyze APPP between each country and the UK in this section. Each country's RER against the UK can be obtained from this RER against the US divided by the UK's RER against the US.

As in Sections 3.1 and 3.2, we analyze both the whole period and the sub-period. The results are listed in Table 3. As the econometric result for the APPP between the US and the UK has been given in Table 1, we don't list the result for the same pair of countries in Table 3 anymore.

Table 3. APPP between each country and the UK based on the PWT 8.1.

Country	In the whole period			In the sub-period	
	C	χ^2 statistic	APPP	Breakpoint	APPP holds for
Australia	1.07 ^{***}	2.01	Holds	1973, 1985	1985–2011
Austria	1.02 ^{***}	0.17	Holds	1973, 1997	1997–2011
Belgium	1.13 ^{***}	13.49 ^{***}	Doesn't hold	1972, 1981	1981–2011
Canada	1.12 ^{***}	4.15 ^{**}	Holds	1979	1979–2011
Denmark	1.19 ^{***}	14.29 ^{***}	Doesn't hold	1972, 1981	1950–1971
Finland	1.19 ^{***}	29.12 ^{***}	Doesn't hold	1974, 1997	1997–2011
France	1.14 ^{***}	26.63 ^{***}	Doesn't hold	1959, 1997	1997–2011
Germany	1.09 ^{***}	4.68 ^{**}	Holds	1970, 1997	1950–1969, 1997–2011
Greece	0.89 ^{***}	10.98 ^{***}	Doesn't hold	1980	1951–1979
Ireland	0.98 ^{***}	1.45	Holds	1969, 1989	None
Italy	0.94 ^{***}	9.98 ^{***}	Doesn't hold	1968, 1978	1968–1977
Japan	1.09 ^{***}	1.73	Holds	1972, 2003	2003–2011
Netherlands	0.99 ^{***}	0.07	Holds	1969, 1980	1980–2011
New Zealand	0.97 ^{***}	0.77	Holds	1960, 1990	1960–1989
Norway	1.26 ^{***}	15.29 ^{***}	Doesn't hold	1970, 1979	None
Portugal	0.75 ^{***}	196.38 ^{***}	Doesn't hold	1979, 1992	None
Spain	0.81 ^{***}	49.21 ^{***}	Doesn't hold	1960, 1973	None
Sweden	1.40 ^{***}	43.97 ^{***}	Doesn't hold	1968, 1981	None
Switzerland	1.07 ^{***}	0.95	Holds	1973, 1997	None

Notes: *, **, and *** indicate that the coefficient C (in Eq. (2)) or the χ^2 statistic (in the Wald test with $H_0: C = 1$) is significant at the 0.1, 0.05, and 0.01 levels, respectively. No subscript indicates that the coefficient C or the χ^2 statistic is not significant at the 0.1 level. The whole period for Greece is 1951–2011, and those for all other countries are all 1950–2011. The coefficient C in each sub-period is significant at the 0.01 level.

Sources: The PWT 8.1 and the authors' calculations.

We can see that the validity of APPP against the UK is similar to that against the US. Concretely, in terms of the whole period, APPP holds for 9 countries, and does not hold for the other 10 countries. In terms of the sub-period, APPP holds in at least one sub-period for 13 countries. For

the other 6 countries, however, APPP does not hold in any sub-period. Thus we can still conclude: the phenomenon that APPP holds is common, and the phenomenon that APPP does not hold is also common.

4. Based on the WDI

After knowing APPP in the 20 countries based on the PWT 8.1, we analyze APPP based on the WDI to check for robustness in this section.

As in Section 3, both APPPs for each country against the US and the UK are investigated. As the whole period for each country is 1990–2013 (with only 24 observations), we don't analyze the sub-period. The econometric results are listed in Table 4.

Table 4. APPP between each country and the US or the UK based on the WDI.

Country	APPP against the US			APPP against the UK		
	C	χ^2 statistic	APPP	C	χ^2 statistic	APPP
Australia	1.05 ^{***}	0.40	Holds	0.98 ^{***}	0.08	Holds
Austria	1.08 ^{***}	4.70 ^{**}	Holds	1.02 ^{***}	0.30	Holds
Belgium	1.08 ^{***}	3.92 ^{**}	Holds	1.01 ^{***}	0.22	Holds
Canada	0.99 ^{***}	0.04	Holds	0.93 ^{***}	3.81 [*]	Holds
Denmark	1.35 ^{***}	58.60 ^{***}	Doesn't hold	1.27 ^{***}	63.63 ^{***}	Doesn't hold
France	1.13 ^{***}	10.19 ^{***}	Doesn't hold	1.06 ^{***}	3.07 [*]	Holds
Finland	1.20 ^{***}	18.73 ^{***}	Doesn't hold	1.13 ^{***}	11.88 ^{***}	Doesn't hold
Germany	1.10 ^{***}	7.24 ^{**}	Holds	1.04 ^{***}	0.88	Holds
Greece	0.81 ^{***}	23.79 ^{***}	Doesn't hold	0.76 ^{***}	92.75 ^{***}	Doesn't hold
Ireland	1.09 ^{***}	4.06 ^{**}	Holds	1.02 ^{***}	1.01	Holds
Italy	0.99 ^{***}	0.03	Holds	0.93 ^{***}	7.37 ^{**}	Holds
Japan	1.33 ^{***}	24.53 ^{***}	Doesn't hold	1.27 ^{***}	9.72 ^{***}	Doesn't hold
Netherlands	1.08 ^{***}	4.71 ^{**}	Holds	1.02 ^{***}	0.26	Holds
New Zealand	0.93 ^{***}	1.63	Holds	0.87 ^{***}	10.47 ^{***}	Doesn't hold
Norway	1.37 ^{***}	52.46 ^{***}	Doesn't hold	1.29 ^{***}	57.55 ^{***}	Doesn't hold
Portugal	0.80 ^{***}	70.52 ^{***}	Doesn't hold	0.75 ^{***}	243.13 ^{***}	Doesn't hold
Spain	0.90 ^{***}	7.99 ^{***}	Doesn't hold	0.85 ^{***}	34.44 ^{***}	Doesn't hold
Sweden	1.25 ^{***}	26.56 ^{***}	Doesn't hold	1.18 ^{***}	17.42 ^{***}	Doesn't hold
Switzerland	1.38 ^{***}	75.95 ^{***}	Doesn't hold	1.30 ^{***}	45.77 ^{***}	Doesn't hold
UK	1.06 ^{***}	5.95 ^{**}	Holds			

Notes: "Against the US (or the UK)" means that the US (or the UK) is the foreign country in Eq. (1). *, **, and *** indicate that the coefficient C (in Eq. (2)) or the χ^2 statistic (in the Wald test with $H_0: C = 1$) is significant at the 0.1, 0.05, and 0.01 levels, respectively. No subscript indicates that the coefficient C or the χ^2 statistic is not significant at the 0.1 level. The period for each country is 1990–2013.

Sources: The WDI database (June 2015) and the authors' calculations.

We can see that the constant C in each country (whether against the US or the UK) is significant at the 0.01 level, spans from 0.7 to 1.4, and is not far from 1, which indicates the validity of APPP in each country to some extent as in Section 3. Concretely, against the US, APPP holds for 10 countries, and it does not hold for the other 10 countries. Against the UK, APPP holds for 9

countries, and it does not hold for the other 10 countries. For 9 countries (Denmark, Finland, Greece, Japan, Norway, Portugal, Spain, Sweden, and Switzerland) where APPP does not hold whether against the US or the UK, we find APPP holds between these pairs: Denmark and Norway, Finland and Japan, Greece and Portugal, Spain and New Zealand, Sweden and Japan, and Switzerland and Norway. Thus, the conclusion obtained in [Section 3](#) remains unchanged: the phenomenon that APPP holds is common, and the phenomenon that APPP does not hold is also common.

5. Does the GDPP matter?

Now we turn to the question of why APPP holds for some countries while not for the other countries. Concretely, we will investigate whether or not the GDPP influences the validity of APPP.

5.1. APPP in Portugal

Seen from [Table 1](#) where APPP between each country and the US based on the PWT 8.1 is listed, APPP holds in the whole period and/or in at least one sub-period for 19 countries (all 20 countries except Portugal). For example for Australia, APPP holds in the whole period and in the sub-period 1983–2011. For Greece, APPP does not hold in the whole period, but holds in the sub-period 2003–2011. For Portugal, however, APPP does not hold either in the whole period or in the sub-period. Portugal is the single such country in [Table 1](#). In addition, APPP does not hold between Portugal and the UK based on the PWT 8.1 (see [Table 3](#)), and does not hold between Portugal and the US or the UK based on the other database, the WDI (see [Table 4](#)). That is, the validity of APPP between Portugal and the two base countries (the US and the UK) is the weakest among all the countries.

Then, is there a case where APPP holds between any other country and Portugal? To answer this question, we test APPP between Portugal and all the other countries except the US and the UK, based on the two databases and in the whole period. The result is listed in [Table 5](#).

Table 5. APPP between each country and Portugal in the whole period.

Country	APPP based on the PWT 8.1			APPP based on the WDI		
	<i>C</i>	χ^2 statistic	APPP	<i>C</i>	χ^2 statistic	APPP
Australia	1.43 ^{***}	43.30 ^{***}	Doesn't hold	1.32 ^{***}	13.95 ^{***}	Doesn't hold
Austria	1.36 ^{***}	30.38 ^{***}	Doesn't hold	1.36 ^{***}	195.48 ^{***}	Doesn't hold
Belgium	1.51 ^{***}	122.83 ^{***}	Doesn't hold	1.35 ^{***}	246.70 ^{***}	Doesn't hold
Canada	1.49 ^{***}	41.96 ^{***}	Doesn't hold	1.25 ^{***}	22.64 ^{***}	Doesn't hold
Denmark	1.59 ^{***}	69.10 ^{***}	Doesn't hold	1.69 ^{***}	690.02 ^{***}	Doesn't hold
France	1.52 ^{***}	208.64 ^{***}	Doesn't hold	1.42 ^{***}	151.95 ^{***}	Doesn't hold
Finland	1.60 ^{***}	113.20 ^{***}	Doesn't hold	1.51 ^{***}	93.97 ^{***}	Doesn't hold
Germany	1.46 ^{***}	56.95 ^{***}	Doesn't hold	1.39 ^{***}	88.74 ^{***}	Doesn't hold
Greece	1.19 ^{***}	24.08 ^{***}	Doesn't hold	1.02 ^{***}	0.79	Holds
Ireland	1.31 ^{***}	60.91 ^{***}	Doesn't hold	1.37 ^{***}	175.12 ^{***}	Doesn't hold
Italy	1.26 ^{***}	69.63 ^{***}	Doesn't hold	1.25 ^{***}	55.62 ^{***}	Doesn't hold
Japan	1.46 ^{***}	21.52 ^{***}	Doesn't hold	1.69 ^{***}	45.04 ^{***}	Doesn't hold

Country	APPP based on the PWT 8.1			APPP based on the WDI		
	C	χ^2 statistic	APPP	C	χ^2 statistic	APPP
Netherlands	1.32 ^{***}	21.32 ^{***}	Doesn't hold	1.35 ^{***}	249.19 ^{***}	Doesn't hold
New Zealand	1.29 ^{***}	40.32 ^{***}	Doesn't hold	1.17 ^{***}	13.08 ^{***}	Doesn't hold
Norway	1.68 ^{***}	59.45 ^{***}	Doesn't hold	1.72 ^{***}	227.39 ^{***}	Doesn't hold
Spain	1.08 ^{***}	3.55 [*]	Holds	1.13 ^{***}	18.68 ^{***}	Doesn't hold
Sweden	1.87 ^{***}	103.37 ^{***}	Doesn't hold	1.58 ^{***}	96.30 ^{***}	Doesn't hold
Switzerland	1.43 ^{***}	19.93 ^{***}	Doesn't hold	1.74 ^{***}	284.23 ^{***}	Doesn't hold

Notes: *, **, and *** indicate that the coefficient C (in Eq. (2)) or the χ^2 statistic (in the Wald test with $H_0: C = 1$) is significant at the 0.1, 0.05, and 0.01 levels, respectively. No subscript indicates that the coefficient C or the χ^2 statistic is not significant at the 0.1 level. The RER of Portugal = 1 in each year.

Sources: The PWT 8.1, the WDI, and the authors' calculations.

We can see that APPP holds between Portugal and Spain based on the PWT 8.1, and holds between Portugal and Greece based on the WDI, but does not hold between Portugal and any other country based on both databases. In addition, the constants for Spain and Greece are the closest to 1 among all the countries in each database. That is, the validity of APPP between Portugal and those two countries (Spain and Greece) is more obvious than that between Portugal and any other country. Why is that?

Based on Fig. 1 of the Penn effect, APPP tends to hold for a pair of countries whose GDPPs are near, and tends not to hold for a pair of countries whose GDPPs are far from each other. Table 6 lists the GDPPs of the 20 countries based on the PWT 8.1 and the WDI, where the two databases give the same basic information. We can see that based on the PWT 8.1, the mean GDPP of Portugal (0.35) is the lowest, and those of Greece (0.42) and Spain (0.47) are the second lowest among all 20 countries. The minimum and maximum values give similar conclusions.² The GDPP of Portugal is obviously farther from the GDPPs of the US and the UK, compared with the other countries. This may be a reason why APPP does not hold between Portugal and the US or the UK, but commonly holds between the other countries and the US or the UK. Further, the GDPPs of Greece and Spain are relatively near to that of Portugal, which may lead APPP to hold between these two countries and Portugal.

Table 6. The GDPPs of the 20 countries in the whole periods (the US = 1 in each year).

Country	Based on the PWT 8.1		Based on the WDI	
	Mean	[Min., Max.]	Mean	[Min., Max.]
Australia	0.80	[0.71, 0.88]	0.79	[0.75, 0.85]
Austria	0.63	[0.37, 0.85]	0.85	[0.82, 0.88]
Belgium	0.61	[0.48, 0.78]	0.82	[0.79, 0.86]
Canada	0.79	[0.71, 0.86]	0.81	[0.80, 0.84]
Denmark	0.73	[0.60, 0.86]	0.90	[0.83, 0.93]
Finland	0.61	[0.41, 0.80]	0.75	[0.67, 0.84]
France	0.66	[0.48, 0.77]	0.76	[0.72, 0.81]
Germany	0.62	[0.31, 0.84]	0.83	[0.76, 0.90]

² However, the correlation coefficient cannot give useful information. For example in the PWT 8.1, the correlation coefficients between the GDPPs of Australia, Canada, Greece, Portugal, and Spain and that of the US are 0.994, 0.997, 0.990, 0.990, and 0.980, respectively. They are very near and one cannot tell the difference in the GDPPs.

Country	Based on the PWT 8.1		Based on the WDI	
	Mean	[Min., Max.]	Mean	[Min., Max.]
Greece	0.42	[0.20, 0.60]	0.57	[0.47, 0.64]
Ireland	0.53	[0.32, 1.09]	0.83	[0.61, 0.98]
Italy	0.57	[0.29, 0.75]	0.78	[0.66, 0.85]
Japan	0.57	[0.18, 0.89]	0.73	[0.68, 0.83]
Netherlands	0.69	[0.50, 0.94]	0.90	[0.87, 0.94]
New Zealand	0.62	[0.55, 0.76]	0.62	[0.59, 0.65]
Norway	0.85	[0.49, 1.59]	1.26	[1.15, 1.31]
Portugal	0.35	[0.18, 0.51]	0.55	[0.51, 0.58]
Spain	0.47	[0.24, 0.69]	0.66	[0.62, 0.69]
Sweden	0.70	[0.60, 0.83]	0.82	[0.77, 0.88]
Switzerland	1.02	[0.88, 1.20]	1.10	[1.03, 1.26]
UK	0.66	[0.61, 0.76]	0.72	[0.70, 0.75]

Sources: The PWT 8.1, the WDI, and the authors' calculations.

However, even though such a rule (the nearer the GDPP is, the more valid APPP is) exists, it is not hard and fast, because there are indeed some country pairs that do not obey this rule. For example, based on the WDI, the mean GDPP of New Zealand (0.62) is nearer to that of the UK (0.72) than to that of the US (1.00), but APPP holds between New Zealand and the US and does not hold between New Zealand and the UK (see [Table 4](#)). In addition, based on the PWT 8.1, the mean GDPP of Sweden is nearer to that of the UK than to that of the US, but APPP between Sweden and the UK is not more valid than that between Sweden and the US (see [Table 3](#) and [Table 1](#)).

5.2. APPP in the pooled country data

Besides the above country pair analysis, we next analyze how the GDPP influences the validity of APPP in all countries. To do this, we pool the data of all the countries together and apply the least squares with breakpoints to the pooled time series data.

We use the PWT 8.1 and APPP against the US to illustrate our pooled method. Each observation (a country in a year) includes a pair of data: a RER and a GDPP. The observations of the US are first excluded because the country is the base country. Then we pool all the observations of the other 20 countries together and then sequence them according to the GDPPs, from low to high. Thus, we obtain two new time series, the GDPP and the RER, where the country and the year are mixed. Finally we conduct the least squares with breakpoints for the new RER, with the new GDPP as the order. The econometric conclusion is given in [Table 7](#), where we allow up to 5 breakpoints, as the observations in each situation are large enough.

Table 7. The pooled time series data analysis.

Database	Against	GDPP range in each interval					
		Does APPP hold in the corresponding interval?					
PWT 8.1	US	[0.18, 0.44]	[0.44, 0.57]	[0.57, 0.63]	[0.64, 0.70]	[0.70, 0.75]	[0.75, 1.59]
		No	No	No	No	No	No
PWT 8.1	UK	[0.28, 0.68]	[0.69, 0.92]	[0.92, 2.18]			
		No	Yes	No			

Database	Against	GDPP range in each interval					
		Does APPP hold in the corresponding interval?					
WDI	US	[0.47, 0.63]	[0.64, 0.73]	[0.73, 0.79]	[0.79, 0.83]	[0.83, 0.91]	[0.91, 1.31]
		No	Yes	No	Yes	No	No
WDI	UK	[0.66, 0.89]	[0.89, 1.03]	[1.03, 1.10]	[1.10, 1.16]	[1.16, 1.29]	[1.29, 1.85]
		No	Yes	Yes	Yes	No	No

Notes: “Against the US (or the UK)” means that the US (or the UK) is the foreign country in Eq. (1). When against the US, the RER and GDPP of the US = 1 in each year; when against the UK, the RER and GDPP of the UK = 1 in each year. The observations in the PWT 8.1 and the WDI are 1239 and 480, respectively.

Sources: The PWT 8.1, the WDI, and the authors’ calculations.

We can see that when against the US and in each database, APPP does not hold when the GDPP is smaller than 0.6. But except this common result, no clear conclusion can be obtained. When against the UK, however, a common, clear conclusion can be obtained from the two databases. That is, the nearer the GDPPs of some observations are to the UK’s GDPPs, the more valid APPP is for the RERs between these observations and the UK. Concretely, based on the PWT 8.1, APPP holds in the interval [0.69, 0.92], but does not hold in the GDPP intervals [0.28, 0.68] and [0.92, 2.18]. In other words, APPP holds when the GDPP is between 69% and 92% of the GDPP of the UK, but does not hold when the GDPP is between 28% and 68% or between 92% and 218% of the GDPP of the UK. Likewise, based on the WDI, APPP holds in three continued GDPP intervals where the GDPP is near to the GDPP of the UK ([0.89, 1.03], [1.03, 1.10], and [1.10, 1.16]), but does not hold in the other intervals where the GDPP is far from the GDPP of the UK ([0.66, 0.89], [1.16, 1.29], and [1.29, 1.85]).

6. Conclusion

In this paper we regress the RER on a constant to see whether or not it fluctuates around its equilibrium value (one) by a coefficient restriction test. If the coefficient restriction test confirms that the mean of the RER is equal to its equilibrium value, we think that APPP holds; otherwise, we think that APPP doesn’t hold. Then we apply this method to investigate the validity of APPP in 21 industrial countries.

As the values for the RERs in different databases are different, two main databases (the PWT 8.1 and the WDI) are used. In addition, both the whole period and the sub-period are analyzed. Different databases and different period dimensions both show that the phenomenon that APPP holds is common, and the phenomenon that APPP does not hold is also common. APPP holds between Portugal and the two countries with the nearest GDPP levels (Greece and Spain), but it does not hold between Portugal and any other country with a greater GDPP level. The pooled country data also indicates that APPP may tend to hold for a pair of countries whose GDPPs are near. However, as the RER in each concrete country is idiosyncratic, the validity of APPP between a pair of arbitrary two industrial countries can be further studied.

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Final notes:

Section 5.1 is revised in this version.