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QUANTITY COMPETITION IN THE PRESENCE OF STRATEGIC CONSUMERS

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Abstract. Oligopolistic retailers decide on the initial inventories of an undifferentiated limited-lifetime product offered to strategic consumers. A manufacturer sets the first-period (full) price, while the second-period (clearance) price is determined by a market clearing process. The resulting symmetric pure-strategy equilibria may lead to no sales in the first or second period (Cournot outcome versus collusion), and sales in both periods with the clearance price above or at the salvage value. The equilibria possess a comprehensive set of monotonic properties. In particular, increasing strategic behavior can benefit retailers and hurt consumers, increasing competition may harm the local economy, and high levels of strategic behavior may insure against oversupply that leads to clearance sales at the salvage value. The welfare-optimal number of retailers can lead to the above-cost clearance price.

1. Introduction

In the current global economy, it is common for major transnational manufacturers to introduce a new product in local markets. Characteristic examples include a December 23, 2013 launch of Samsung Galaxy Grand 2 in India, an introduction of Brazuca (The Official Match Ball of the 2014 FIFA World Cup) by Adidas in Brazil, and a 2013 introduction of six new Ford models in China. The rapid pace of fashion, innovation, and technological progress limits the lifetime of products, making them obsolete within a relatively short time. At the time of entry, the manufacturer may wield considerable market power and an ability to control the initial price but, eventually, competing innovation or fashion takes its course and reduces consumer willingness to pay for the product. A local market for such limited-lifetime product may have an arbitrary number of retailers that initially sell it at the manufacturer-controlled price, but eventually engage in clearance sales to liquidate the remaining inventory.

A similar process takes place in local markets for transportation services. Cargo forwarders and aggregator companies have to negotiate seasonal allotment contracts with carriers that ultimately provide the underlying capacity. The predominant fixed-commitment contracts (see Pompeo and Sapountzis (2002)) force forwarders to offer discounts for any unused capacity with the approach of the departure time. In their roles, a carrier is similar to a manufacturer, while forwarders and aggregators are similar to retailers.

A major operational decision faced by retailers and cargo forwarders is determining the quantity of the product that they are going to supply to the market. At the strategic level, this decision involves more than just procuring a certain inventory of the product; it can include choosing which retail outlets carry the product or even opening new outlets, allocating the warehouse capacity, making shipping arrangements, sizing the sales staff, and making other marketing and operational decisions. All these complicating aspects contribute to product cost and supply inflexibility. The same factors increase the importance of the product quantity decision which, in isolation, is relatively easy to formalize.

On the consumer side of the market, we see a population that is accustomed to quick changes in fashion, the emergence of new products, and their limited life cycles. Consumers are familiar with

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typical price trajectories resulting from intertemporal price discrimination by the sellers, and may engage in strategic (forward-looking) shopping behavior by delaying the purchase until the period of price reductions. In doing so, consumers realize that delaying the purchase may reduce the sense of novelty and their enjoyment of the product, but they still make this calculated trade-off. Sitting between a manufacturer with near-monopoly power and strategic consumers, retailers must make their best of the situation while aggressively competing for their market shares.

This setting brings the following research questions. First and foremost, what are the effects of strategic consumer behavior on retailer inventory decisions and profits? A common view is that strategic consumer behavior is detrimental for retailers, but is this necessarily true? Does the speed of reduction in product value play a substantial role in these effects? Better yet, do consumers themselves necessarily benefit from their strategic behavior? The answer is not obvious because consumer behavior drives competitive responses from the retailers. Finally, do the increases in strategic consumer behavior and retailer competition benefit the local economy?

In addressing these questions, we consider a stylized two-period model where a manufacturer sets the first-period list price (manufacturer suggested retail price or MSRP) for a limited-lifetime product, and identical retailers engage in quantity competition by making inflexible first-period supply decisions. The history of resale price maintenance by the manufacturer traces back to the nineteenth century and "has been one of the most controversial antitrust topics ever since" (Orbach (2008)). The phenomenon of MSRP is considered as a legal practice for branded goods and is "informally" used for non-branded products. There are a number of explanations as to why manufacturers offer MSRP and retailers accept it. In particular, collusive retailers can force a manufacturer to declare a desirable first-period price (Orbach (2008)), or retailers may be forbidden to carry the product when there is a lack of competition among manufacturers. A review by Butz (1996) concludes that "manufacturers have many, many instruments" to punish or reward retailers in order to control the retail price "and to some extent will do so whether or not the law permits it." Retailers may follow MSRP under repeated interactions even when this price is non-binding since the manufacturer uses it to communicate private information on marginal cost and consumer demand to the retailers (Buehler and Gärtner (2013)).

Since the main intention of first-period operational decisions, associated with the quantity decisions, is to increase first-period sales, we assume that the first-period demand and the resulting sales are non-decreasing in the initial order quantities. In the first period, regular consumers plan their purchases according to their expectations of the second-period price. Because of the capacity commitments of the retailers, the second-period (clearance) sales are described by the Nash-Cournot model. The lower bound on the second-period price is provided by the salvage value since there is usually a large number of bargain-hunter consumers who are ready to absorb the excess supply at a sufficiently low price.

We answer the research questions by deriving a closed-form solution for the rational expectations symmetric equilibrium (RESE) in pure strategies for the proposed generalized Nash-Cournot model. This analytical tractability is a distinguishing feature of our approach to an otherwise unwieldy problem. The equilibrium permits a complete characterization and takes one of the following forms:

(1) When the MSRP is sufficiently high relative to consumer valuations, all consumers delay their purchases until the second period effectively turning the market into a one-period Nash-Cournot.
(2) When the MSRP is relatively low, the market reduces to the first period only because retailers limit the amount of product they supply to the market. This is essentially an indirect collusive outcome facilitated by MSRP.
(3) For intermediate values of the first-period price and a sufficiently low salvage value, RESE results in sales during both periods, as well as a second-period price higher than the salvage value.

(4) In the same range of the first-period price as form 3, and with a sufficiently high salvage value, RESE still results in sales in both periods. However, the second-period sales take place at the salvage value. This "salvaging" outcome is not attractive to the retailers because they incur a large loss in the second period due to product oversupply.

Within *each type*, the equilibrium is unique. Across all types, the unique equilibrium always exists under the conditions of RESE 1 and 2, but it may not be unique in the complementary case. For the latter, we provide a sufficient condition that guarantees that RESE 3 exists and is unique. This condition requires the unit cost to be high compared to the salvage value. The equilibria exhibit *stability* in terms of consumer expectations and the strategies used by the individual retailers, which is manifested by convergence of realized expectations or retailer actions to equilibrium under a linear adjustment process.

We characterize all possible outcomes for the *entire range* of feasible values of the first-period price. These results shed light on the nature of the interaction between oligopolistic retailers and strategic consumers in various regimes allowing, e.g., to find a profit-maximizing MSRP for the manufacturer. If the manufacturer operates only in a single market, this value of the MSRP in combination with other parameters, including the number of retailers and the level of strategic behavior, will determine the type of equilibrium. We show in §5.3 that, for the manufacturer, the most beneficial markets have intermediate values of the ratio of MSRP to the highest consumer valuation and RESE 4 is possible. As a rule, however, transnational manufacturers operate in multiple markets with notably different valuations for the same product whereas, to comply with anti-dumping regulations, prices must be comparable when converted to local currencies. Consequently, comparable currency-denominated MSRP values may substantially vary across the markets when they are expressed in terms of the maximum consumer valuation, leading to different outcomes.

The equilibrium possesses intuitive monotonic properties, while it also delivers several unexpected insights. It is intuitive that, when the number of retailers increases, the total supply of the product does not decrease, the resulting second-period price falls, the total profit of retailers decreases, and the total surplus of consumers increases. However, it is not always true that the aggregate welfare (the sum of the total profit and consumer surplus) increases with the level of competition. For example, when a relative decrease in consumer valuations for the product between two periods is very small, the aggregate welfare may be increasing, decreasing, or it may even attain an internal maximum. From a regulator's point of view, the corresponding optimal market structure would involve, respectively, a monopolistic retailer, a perfect competition, or an oligopoly. For the third form of RESE, the maximum of the aggregate welfare with respect to the level of competition results in a clearance price above the unit cost.

The response of equilibrium to changes in the level of strategic consumer behavior is more complex. The quantity supplied to the market never increases with an increase in strategic behavior. This means that retailers always respond to strategic behavior by reducing supply despite competitive pressures. As a result, retailers may *capitalize* on strategic behavior since the total profit may be non-monotonic. Typically, total profit decreases as consumers become more strategic, e.g., when the relative decrease in valuations between the two periods is large or in a monopoly. However, there are two distinct cases leading to profit gains resulting from the equilibrium response of retailers to strategic consumer behavior:

- the "continuous gain" is characterized by continuously increasing profit in the level of strategic behavior; this gain may happen when the second-period sales are either profitable or at loss, but only when the relative decrease in valuations is small and the level of strategic behavior is high;
- the "discontinuous gain" occurs at various levels of strategic behavior and the relative decrease in valuations, but only when the difference between the unit cost and the salvage

value is relatively small; profit increases because retailers reduce inventories in response to increased strategic behavior, which leads to the switch from RESE 4 to RESE 3.

The most pronounced form of these phenomena is the "boundary-value gain", i.e., profit with myopic consumers is less than with fully strategic consumers. This may occur only when a strong first-period inventory competition leads to the second-period sales below cost.

Since RESE 4 is unfavorable for the retailers, they would generally prefer to avoid it whenever possible. It is then particularly noteworthy that an increase in strategic consumer behavior may prevent salvaging equilibrium from taking place. We provide a sufficient condition to rule out RESE 4 in the form of a lower bound on the level of strategic behavior.

The total consumer surplus is not generally monotonic and may attain maximum at an intermediate level of strategic behavior. Thus, the consumer population as a whole does not necessarily benefit from becoming more strategic, and may, in fact, lose by being "too strategic." Similarly, the aggregate welfare is generally non-monotonic: it may attain a maximum that tends to arise for high levels of retailer competition and a small relative decrease in valuations. Non-monotonicity of the aggregate welfare is characterized in closed form for the case of salvaging equilibrium.

We present a review of related literature in §2, describe the model in §3, and state the characterization of equilibrium as well as a sufficient condition for its existence and uniqueness in §4. We analyze equilibrium properties and some extensions of the model in §5 and the properties of consumer surplus and the aggregate welfare in §6. Finally, §7 provides a summary of monotonic properties and outlines several possibilities for extending and applying the proposed model. All mathematical proofs are presented in the online appendix.

2. Quantity decisions and strategic consumers

One of the first to recognize the importance of strategic consumer behavior is Lazear (1986) who studies a monopoly pricing problem with fixed inventory. Among a variety of two-period settings, Lazear considers a given population of strategic buyers whose valuations for a fashion good decrease by a fixed factor in the second period.

Coase (1972) has initiated a study of strategic buyer behavior in an intertemporal pricing problem faced by a durable good monopolist. The essence of his famous conjecture is that "the competitive outcome may be achieved even if there is but a single supplier". As one of the possible solutions to this problem, Coase proposes to restrict the quantity of the good supplied to the market through contractual or other arrangements.

These early studies have led to further research in consumer behavior in the context of intertemporal pricing. Shen and Su (2007) survey results involving strategic consumer models, and Aviv et al. (2009) review the research on the mitigation of strategic consumer behavior. We focus our attention on results where quantity-based decisions of sellers affect strategic consumers. For a monopolistic retailer, Cachon and Swinney (2009) consider a two-period model with uncertain demand and find that the optimal choice of the initial inventory and subsequent markdown is better than committing to a price even in the presence of strategic consumers. Moreover, an opportunity to replenish the inventory at the beginning of the second period is much more valuable for the retailer in the presence of strategic consumers than when all consumers are myopic.

Su (2007) considers a deterministic model of monopolistic pricing and rationing policy for a fixed inventory of a limited-lifetime product. The market consists of four segments characterized by one of the two fixed valuation levels (high- or low-valuation consumer types) and one of the two given values of waiting costs (patient or impatient consumers). Su shows that market heterogeneity may lead to profit gains from the increased strategic behavior of low-valuation consumers when high-valuation consumers are myopic (impatient). In this case, the retailer sells the product at a high price to the arriving high-valuation consumers, while the arriving low-valuation patient (fully strategic) consumers are waiting for clearance. When the market of low-valuation consumers becomes large enough, the monopolist drops the price, effectively exploiting a price discrimination scheme. This effect relies on the threat of stockouts for high-valuation consumers, which increases their willingness to pay, and on the proportional rationing rule used in the model.

Liu and van Ryzin (2008) concur that "capacity decisions can be even more important than price in terms of influencing strategic consumer behavior"; they study the effects of capacity decision when prices are fixed while consumers have full information and can be risk-averse. The decision is expressed in terms of consumer rationing risk. Liu and van Ryzin find that capacity rationing can mitigate strategic consumer behavior, but it is not profitable for risk-neutral consumers. Under competition, the effectiveness of capacity rationing is reduced, and there exists a critical number of firms beyond which rationing never occurs in equilibrium.

These studies suggest that retailers are most challenged by strategic consumer behavior when there is a large number of competitors, consumers are risk-neutral, and the market is homogeneous with respect to the level of strategic behavior. Moreover, when consumers do not know the total supply of the product, it is impossible to use strategic rationing to control their behavior. Our study fills the gap in the existing results for this challenging setting.

Without strategic consumers or two-period demand, quantity competition has been considered by Sherali et al. (1983) for a homogeneous product in a leader-follower framework, and by Farahat and Perakis (2011) and Kluberg and Perakis (2012) for differentiated products. Kreps and Scheinkman (1983) argued that the first-stage capacity commitment by duopolistic firms selling an undifferentiated product yields a Cournot outcome *even* if the equilibrium production and prices are determined by price competition in the second stage.

3. Model description

We consider a two-period market for a limited-lifetime product with an arbitrary number of identical retailers. All the retailers have the same per-unit cost c and offer the product to *regular* consumers at the same first-period price $p_1 > c$. This assumption is not unusual "in a competitive retail market, where retailers frequently stock identical products, sell them at the same suggested retail prices, and at nearly identical costs from manufacturers" (Liu and van Ryzin (2008)).

If there is some product remaining after the first period, retailers engage in clearance sales in the second (clearance) period. Since the product offerings are undifferentiated, the retailers lower their prices until all remaining inventory is cleared, that is, second-period price p_2 (identical for all retailers) is sufficiently low for the total clearance demand to equal the total remaining inventory. Similarly to Cachon and Swinney (2009), we assume that, in the second period, there is an infinite number of bargain-hunting consumers who can buy any remaining product at per-unit salvage value $s < c$. As a result, p_2 never goes below s. The salvage value also allows for the possibility of inventory buy-back contracts of retailers with the manufacturer, or the availability of alternative sales channels for the retailers.

Each retailer maximizes its profit by selecting the initial inventory level. The resulting game among the retailers is similar to the classical Nash-Cournot model, but with a substantially distinct two-period structure and a special role played by MSRP in the first period.

We now describe the market dynamics. Let retailers be indexed by set I of size $n = |I|$, and retailer $i \in I$ product supply and sales in the first period be y^i and q^i. Since the second-period market is cleared, each retailer's second-period supply and sales are equal to $y^i - q^i$. Denote the total first-period product supply and sales as $Y = \sum_{i \in I} y^i$ and $Q = \sum_{i \in I} q^i$, respectively. Then the total second-period supply is $Y - Q$ and the retailer i profit is

$$r^i = -cy^i + p_1 q^i + p_2(y^i - q^i). \tag{1}$$

First-period sales q^i are determined based on a consumer decision model.

3.1. **Consumer decision model.** The consumer decision model describes two aspects: demand allocation between two periods and among the retailers. We will start with the first one.

3.1.1. *Demand allocation between two periods.* We normalize the number of regular consumers to one, and let first-period valuations v be drawn from a uniform distribution on the interval $[0, 1]$. Normalization of valuations effectively expresses revenue and inventory as "unitless" quantities and MSRP as a share of maximum valuation. In order to capture a typical decrease in valuations for seasonal and limited-lifetime products, we introduce factor $\beta \in [0, 1]$: if the consumer's first-period valuation is v, the second-period valuation becomes βv. Two logical restrictions ensure non-trivial equilibrium results. First, inequality $\beta > c$ guarantees that the highest-valuation consumer is prepared to pay more than the unit cost in the second period. If this restriction does not hold, the clearance price can never be above the unit cost. We also suppose that $p_1 > \frac{s}{\beta}$ to ensure that salvage value s is less than the highest second-period valuation βp_1 of regular consumers who are forced to delay their purchases by MSRP. Similarly to $\beta > c$, this restriction supports a non-trivial second-period outcome in an equilibrium with a substantial role of regular consumers.

The availability of information about total supply of the product varies among the markets. Some markets, such as land or real estate, have nearly perfect information, an assumption used in Liu and van Ryzin (2008). In many other markets, total system-wide inventory is unobservable, which reduces the ability of retailers to use rationing as a tool for stimulating first-period demand from strategic consumers. When consumers do not observe total supply, they cannot infer exact price p_2 and probability $\alpha \in \{0, 1\}$ that the product is available in the second period.

Assumption 1. *Consumers do not know the total product supply and make their decisions based on: (a) expected probability that there are second-period sales* $\bar{\alpha} \in \{0, 1\}$; *(b) expected second-period price* $\bar{p}_2$.

Given these expectations, consumers decide whether a first or second-period purchase maximizes their intertemporal surplus, which is similar to Lazear (1986), Su (2007), and Cachon and Swinney (2009):

Assumption 2. *When the product is available, a consumer with valuation v buys in the first period if the first-period surplus $\sigma_1 \triangleq v - p_1$ is not less than the expected second-period surplus $\sigma_2 \triangleq \bar{\alpha}\rho(\beta v - \bar{p}_2)^+$, where $\rho \in [0, 1)$ is a discount factor.*

Consumers with $v < p_1$ never buy in the first period because such a purchase would result in a negative surplus. The proposition below describes the first-period demand, where we use the notation $a \vee b \triangleq \max\{a, b\}$, $a \wedge b \triangleq \min\{a, b\}$.

Lemma 1. *Given consumer expectations, surplus-maximizing behavior is to buy in the first period if $v \geq v^{\min}$, where the unique valuation threshold is given by $v^{\min} = p_1 \vee \left(\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1 - \bar{\alpha}\rho\beta} \wedge 1\right)$. The resulting total first-period demand is $D = 1 - v^{\min}$.*

Undervaluation of the surplus from delaying a purchase means that even for a product that does not depreciate much by the second period, i.e., β is near one, consumers with any valuation may myopically ignore the second period during the first-period deliberations, i.e., have $\rho = 0$. The value of ρ may depend on the market targeted by the product, e.g., for age- or culture-oriented products, and on the consumer confidence in the stability of the financial situation. As ρ increases, consumers place more emphasis on the second period in their wait-or-buy decisions. Thus, unlike β, which models an objective decrease in valuations, ρ is a subjective parameter of the consumers describing the level of their strategic behavior. The essence of the distinct roles of β and ρ has been succinctly captured by Pigou (1932): "Everybody prefers present [i.e., $\rho < 1$] pleasures or satisfaction of given magnitude to future pleasures and satisfaction of equal magnitude [i.e., $\beta = 1$], even when the latter are perfectly certain to occur." Consumer discount rates can be estimated in practice, as Busse et al. (2013) illustrate in the context of car purchases.

3.1.2. *Demand allocation among retailers.* Since consumers have no preferences among the retailers, the marketing and distribution efforts are the only differentiating aspect. It is recognized both in practice and in research (e.g., Balakrishnan et al. (2004)), that typical consumer behavior results in larger sales of a particular retailer if the product is presented to consumers at a larger number of retail outlets, in larger quantities on store shelves, and in more ads. We consider *attraction* $a^i(y^i)$ as a measure of retailer i efforts that are not decreasing in the retailer's inventory.

Assumption 3. *The function $a^i(y^i)$ is continuous, not decreasing in y^i, and $a^i(0) = 0$. Consumers do not know the functional form of $a^i(y^i)$ and observe only the resulting vector of attraction values.*

Identical retailers operate under alike conditions and use similar recipes for creating the firm's attractions, i.e., $a^i(y^i) = a(y^i)$ for all $i \in I$. Moreover, any two identical retailers with the same attraction have equal market shares, and the market share of any retailer decreases by the same amount if the attraction of *any* other retailer is increased by a particular amount. These properties, complemented by a simple assumption that zero attraction leads to zero market share, satisfy the conditions of the market share theorem of Bell et al. (1975), which claims that the functional form of the market share of retailer i, in this case, is $a(y^i)/\sum_{j\in I} a(y^j)$. Thus, effectively, consumers observe market shares.

The *first-period demand* d^i of retailer i, determined by its market share, depends not only on y^i via $a(y^i)$ but also, inversely, on $\mathbf{y}^{-i}$ – the vector of inventories of the others. Assumption 1 implies that demand d^i is homogeneous of degree zero: $d^i(ky^i, k\mathbf{y}^{-i}) = d^i(y^i, \mathbf{y}^{-i})$ for any $k > 0$, i.e., total supply cannot influence d^i when the ratios y^i/Y, $i \in I$ remain the same. Since the total first-period demand $D = \sum_{i\in I} d^i(y^i, \mathbf{y}^{-i})$ is also homogeneous of degree zero, retailer i market share $d^i(y^i, \mathbf{y}^{-i})/D$, is homogeneous of degree zero. The following lemma specifies the functional form of $a(y^i)$.

Lemma 2. *If retailer i market share is homogeneous of degree zero and has functional form $a(y^i)/\sum_{j\in I} a(y^j)$, where $a(y)$ is continuous in y, $a(y)$ has the unique functional form $a(y) = a(1)y^\gamma$.*

By choosing the scale of attraction so that $a(1) = 1$, we obtain the functional form for demand d^i :

$$d^i(y^i, \mathbf{y}^{-i}) \triangleq D\frac{\left(y^i\right)^\gamma}{\sum_{j\in I}\left(y^j\right)^\gamma}, i \in I, \tag{2}$$

where $\gamma \in [0, 1]$ is the *inventory elasticity of attraction* or inventory elasticity of demand, normalized by the market share of other retailers (Online Appendix B). Function (2) is a symmetric form of a widely used *general attraction model* (e.g., Monahan (1987), Gallego et al. (2006)). The case $\gamma = 0$ means that a retailer's attraction does not depend on y^i, and $d^i \equiv \frac{D}{n}$ for any $y^i > 0$ and $i \in I$. This case was used in §4.4 of Liu and van Ryzin (2008) to study the effect of rationing on strategic behavior of risk-averse consumers. An empirical study of Naert and Weverbergh (1981) concludes that the attraction model is "more than just a theoretically interesting specification." This model "may have a significantly better prediction power than the more classic market share specifications." This conclusion is supported by later studies (e.g., Klapper and Herwartz (2000)).

Since product is undifferentiated and the retailers are identical, consumers buy from any retailer with available product. If the combined supply of retailers is insufficient to satisfy the combined demand, one of the rationing rules can be used. For example, according to the surplus-maximizing rule (see Tirole (1988)), consumers buy in the order of their valuations. The following lemma shows that retailers have no stockouts independently of the rationing rule.

Lemma 3. *Consider any $\bar{Y} \geq 1 - v^{\min}$, symmetric inventory profile $(\frac{\bar{Y}}{n}, \ldots, \frac{\bar{Y}}{n}) \in \mathbb{R}^n_+$, and any behavior of consumers under stockouts in the first period. For any $i \in I$, let $\mathbf{y}^{-i} = (\frac{\bar{Y}}{n}, \ldots, \frac{\bar{Y}}{n}) \in \mathbb{R}^{n-1}_+$. The following claims hold: (I) any profit-maximizing response of retailer i to $\mathbf{y}^{-i}$ must satisfy $y^i \geq \breve{y}^i$, where $\breve{y}^i$ is the unique positive solution to $\breve{y}^i = d^i(\breve{y}^i, \mathbf{y}^{-i})$; (II) for any $y^i \geq \breve{y}^i$, (a)*

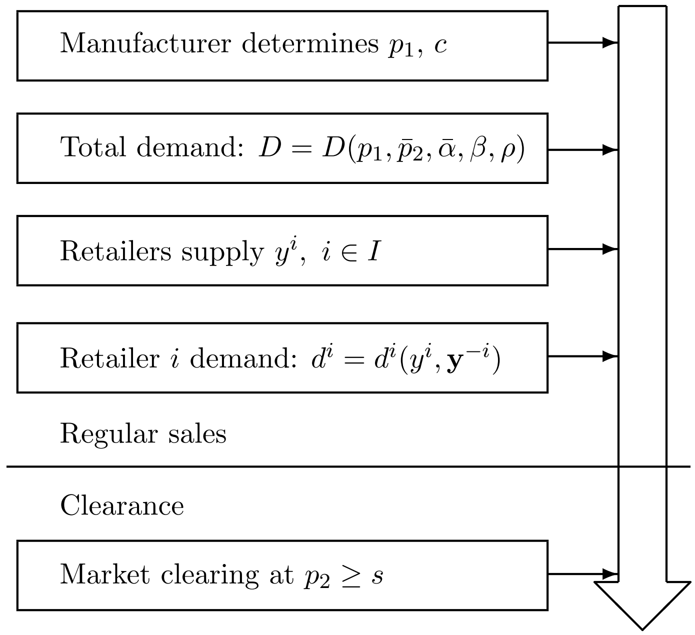

FIGURE 1. Market timeline

stockouts are impossible, (b) the total first-period sales are $Q = 1 - v^{\min}$, *the individual first-period sales are* $q^i = d^i(y^i, \mathbf{y}^{-i})$, *the resulting second-period inventories are* $y^i - q^i, i \in I$ *and (c) the second-period price is*

$$p_2 = s \vee [\beta(1 - Y)]. \tag{3}$$

The timing of main events in the game and the corresponding inputs are outlined in Figure 1.

3.2. **Rational expectations equilibrium.** Lemma 1 identifies rational consumer behavior for *given* expectations, MSRP values, and behavioral parameters ρ, β. In particular, it specifies valuation levels of consumers who purchase in the first period. However, these results are insufficient to identify how consumer expectations *form.* While it is possible to look for equilibrium behavior of retailers for given expectations, our ultimate goal is to find equilibria that can be sustained in the long run. Therefore, we need to close the loop by identifying expectations that are *rational.* That is, *the equilibrium inventory levels of the retailers must lead to precisely the same observed product availability and clearance prices as expected by the consumers.* This notion works well for equilibrium since consumers can infer the second-period price from the recognition of their own and retailers' rationality.

When a completely new technology, such as a personal computer or a cell phone, is introduced to the market, consumers may not be able to form rational expectations about the appearance of subsequent versions of the product and the resulting pricing policies. However, in mature markets, manufacturers regularly launch similar products, or new models of the same product, and consumers, getting accustomed to price-drop patterns, adjust their expectations about future pricing policies to closely match their observations. Adjustments are no longer needed if the expectations coincide with the eventual observations. On the other hand, retailers regularly conduct market research to estimate current consumer expectations.

Using this notion of rationality, the rational expectations symmetric Cournot-Nash equilibrium (RESE) in pure strategies is defined as follows:

(1) Given consumer expectations and $\mathbf{y}^{-i}$, let the best response of retailer i be $BR^i(\mathbf{y}^{-i}, \bar{p}_2, \bar{\alpha}) = \arg\max_{y^i} r^i(y^i, \mathbf{y}^{-i}, \bar{p}_2, \bar{\alpha})$.

(2) Let $\hat{y} = \hat{y}(\bar{p}_2, \bar{\alpha})$ denote a symmetric Nash-Cournot equilibrium inventory level for given expectations, i.e., $\hat{y}(\bar{p}_2, \bar{\alpha}) = BR^i\left[(\hat{y}, \ldots, \hat{y}), \bar{p}_2, \bar{\alpha}\right]$, where $(\hat{y}, \ldots, \hat{y}) \in \mathbb{R}_+^{n-1}$, and $\hat{Y}(\bar{p}_2, \bar{\alpha}) = n\hat{y}(\bar{p}_2, \bar{\alpha})$ be the corresponding total inventory.

(3) The tuple (Y^*, p_2^*, α^*) is a RESE if $Y^* = \hat{Y}(p_2^*, \alpha^*)$, $p_2^* = s \vee [\beta(1 - Y^*)]$, and either $\alpha^* = 0$, if $Y^* = 1 - v^*$, or $\alpha^* = 1$, if $Y^* > 1 - v^*$ where v^* is the equilibrium value of $v^{\min}$.

From now on, r^* denotes the equilibrium profit of a retailer. Equilibrium values may be specified for the type of RESE, e.g., $r^{*,3}$ or $Y^{*,1}$, if necessary.

3.3. **Discussion of model assumptions.** We conclude this section with the discussion of specific implications of model assumptions. Some of the assumptions are quite common and well understood. For example, consumers are modeled as homogeneous in the level of strategic behavior ρ and relative valuation decrease β. This assumption is applicable to any products targeting specific market segments. The value of ρ may also be tied to the average time value of money (rate of return), which is relatively homogeneous for all consumers. The assumption of retailer symmetry is common for qualitative models of competition, when retailers do not differ in their cost structure or brand value. For the current model, these differences are not important since the offerings of the retailers are undifferentiated, and the manufacturer supplies the product to retailers at the same wholesale price.

The information structure of the model is rather general. Indeed, it is relatively rare for the total product supply in the market to be visible to consumers while the market share effort, such as the number of outlets, does signal to consumers the relative market power of the retailers. Generally, consumer expectations about the second-period price and rationing risk may or may not be probabilistic in this context but, for pure strategy (deterministic) equilibria, deterministic expectations are consistent with retailer behavior. As we show below, there is a considerable amount of insight even from the pure-strategy case.

The assumption of a fixed first-period price (list price or MSRP) has justifications both in practice and prior literature. Indeed, retailers would rarely start the selling season by deviating from a well-publicized list price (such as \$160 for Brazuca or \$9,500 for a brand new Chevrolet Sail in China) and, as was already mentioned, they are inclined to honor a non-binding MSRP in the context of repeated interactions with the manufacturer. On the other hand, the (second-period) clearance price does vary and may be considerably lower than the first-period one. The Cournot-Nash approach to modeling equilibrium is one of the classical models for this case.

The following sections focus on the case of $\gamma = 1$ in the first-period demand (2) as most results hold for any $\gamma \in (0, 1]$, and this case is more reader-friendly than for intermediate values of γ. Some of the effects weaken when γ goes to zero and disappear for $\gamma = 0$. The robustness of the main results with respect to changes in γ, including the closed-form analysis for $\gamma = 0$, is shown in the online appendix B.

4. Characterization of RESE

The restriction of rationality for consumer expectations immediately implies the following conclusions about the equilibrium.

Lemma 4. *In any rational expectations equilibrium, (1) $p_2 < \beta p_1$ if there are sales in the second period; (2) $Y \geq 1 - p_1$, which holds as an equality only if there are no sales in the second period; (3) $\rho\beta Y < 1 - p_1$ if there are sales in both periods and $p_2 > s$; $\rho\beta Y \geq 1 - p_1$ and $p_2 \geq c$ if there are sales only in the second period; and (4) $v^{\min} = p_1$ if and only if $\bar{\alpha} = 0$ or $\rho = 0$.*

Since $v^{\min} \geq p_1$, part (2) of Lemma 4 justifies the assumption of Lemma 3 for a RESE.

4.1. **No-salvaging RESE.** We start by providing closed-form expressions for three of the possible equilibrium cases.

Theorem 5. *A unique RESE with the stated structure exists if and only if the respective conditions hold:*

- **RESE 1 (No sales in the first period):** $v^* = 1, \alpha^* = 1, p_2^* = c + \frac{\beta-c}{n+1}, Y^* = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$, *and* $r^* = \frac{(\beta-c)^2}{(n+1)^2\beta}$ *under condition* $p_1 \geq 1 - \frac{n}{n+1}\rho(\beta - c) \triangleq P_1$.
- **RESE 2 (No sales in the second period):** $v^* = p_1, \alpha^* = 0, Y^* = 1 - p_1$, *and* $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$ *under condition* $p_1 \leq \frac{nc}{n-1+\beta} \triangleq P_2$.
- **RESE 3 (Sales in both periods, $p_2^* > s$):** $v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta}$, $\alpha^* = 1$, $p_2^* = \beta(1 - Y^*)$, *where* Y^* *is the larger root of a quadratic equation, and* $r^* = \frac{1}{n}\left[(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)\right]$, *under condition* $P_2 < p_1 < P_1$ *and one of the following:*
  (a) $\frac{n-1}{n}(p_1 - s)(1 - v^*)Y^* \leq (c - s)\left(1 - \frac{s}{\beta}\right)^2$, *or (b) condition (a) does not hold,* $Y^* < 1 - \frac{s}{\beta}$, *and* $r^* \geq \tilde{r}^i \triangleq \left\{\sqrt{(p_1 - s)(1 - v^*)} - \sqrt{\frac{n-1}{n}Y^*(c - s)}\right\}^2$, *where* $\tilde{r}^i$ *is the maximum profit of a firm deviating from this RESE in such a way that* $p_2 = s$ *(the total inventory is greater than* $1 - \frac{s}{\beta}$*).*

The equilibrium characteristics Y^*, v^*, *and* r^* *are continuous on the boundaries between these forms of RESE. Moreover, in RESE 3,* $Y^* \geq \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$.

If the initial consumer expectations of the second-period price are such that $\bar{p}_2^0 < p_2^*$, *the game is repeated, and expectations follow a linear adjustment process, then the sequence of games converges to* $\bar{p}_2 = p_2^*$ *for any sufficiently small speed of adjustment.*

Equilibrium RESE 1 describes scenarios with high p_1 when there are no sales in the first period and all consumers wait for clearance sales. Inequality $p_1 \geq P_1$ implies that this outcome is possible only if consumers are strategic ($\rho > 0$), except for a degenerate case $p_1 = 1$. The area of RESE 1 inputs increases in (i) ρ because more consumers delay the purchase, (ii) n since rational strategic consumers expect a lower second-period price when competition grows, and (iii) difference $\beta - c$ because retailer profit increases in $\beta - c$ and consumer second-period valuations increase in β. The form of this RESE completely matches a one-period Nash-Cournot outcome.

RESE 2 is the opposite: p_1 is low (high-valuation market), all consumers whose valuations are higher than p_1 buy in the first period, and there are no sales in the second period. Condition $p_1 \leq P_2$ implies that the existence of this RESE does not depend on ρ because $\bar{\alpha} = 0$ — rational consumers do not expect second-period sales and, by Lemma 1, the equilibrium valuation threshold of the first-period buyers is $v^* = p_1$ regardless of ρ. Also, RESE 2 input area shrinks in β and n, disappearing for $\beta = 1$ and $n \to \infty$. The "β-effect" results from increasing attractiveness of the second-period market when retailers can gain from two-period price discrimination, and the "n-effect" results from increasing quantity competition for the market share that may force retailers to procure more inventory than just for the first period. The input area increases in c because the second-period profit approaches zero in c faster than the first-period profit, decreasing the relative attractiveness of the second-period sales. Retailers divide the profit associated with the total supply that is just enough to cover the first-period market. Since the supply is determined by an externally set MSRP, retailer competition is reduced to market sharing and we can interpret this outcome as an MSRP-facilitated collusion. In either of the first two equilibria, the intertemporal effect of competition is (locally) eliminated and, consequently, Y^* and r^* do not depend on ρ.

RESE 3 describes scenarios with intermediate p_1 leading to sales in both periods. It provides a bridge between the opposites: a competitive Cournot outcome of RESE 1 and an MSRP-enabled collusion of RESE 2. Conditions (a) and (b) correspond to different attractiveness of salvage-value sales for a potential deviator from RESE 3 that increases inventory. Condition (a) means that the deviator profit monotonically decreases, i.e., for the inputs that satisfy this condition, RESE 3 is stable with respect to small parameter deviations given that p_1 is sufficiently far from the boundary. Under condition (b), deviator profit has a local maximum with $p_2 = s$ but this maximum does not exceed the profit under RESE 3. The inputs for which (b) holds are near the boundary of RESE 3 existence where this equilibrium may be unstable with respect to parameter misestimation.

For a monopolist, RESE 3 takes a simpler form described in the following corollary. In particular, condition $P_2 < p_1 < P_1$ is necessary and sufficient.

Corollary 6. *For $n = 1$ and any $\frac{c}{\beta} < p_1 < 1 - \frac{\rho}{2}(\beta - c)$, RESE is $v^* = \frac{2p_1 - \rho c}{2 - \rho\beta}$, $\alpha^* = 1$, $p_2^* = \frac{\beta v^* + c}{2}$, $Y^* = 1 - \frac{1}{2}\left(\frac{c}{\beta} + v^*\right)$.*

Since price and quantity decisions are equivalent for a monopoly, this corollary provides a characterization of the price-skimming policy when the first-period price is externally regulated. The second-period price for a monopolist in our model always exceeds the unit cost (because $v^* \geq p_1 > \frac{c}{\beta}$ in RESE 1 and 3). On the other hand, increasing competition may drive the second-period price below cost, which we demonstrate in a market for a durable good with myopic consumers and some $n > 2$. The second-period price in this case remains above cost in a duopoly.

Corollary 7. *For $\beta = 1$, $\rho = 0$, and $c < p_1 < 1$, RESE 1 and 2 cannot be realized and, in RESE 3, the second-period price is below cost if and only if $n > 2 + \frac{p_1 - c}{1 - p_1}$.*

Increasing competition not only decreases the second-period price below cost, but undermines the very existence of RESE 3. Indeed, condition (a) in RESE 3 holds for any $n \geq 1$ only if s is sufficiently low. On the contrary, for large s, condition (a) may not hold. Moreover, one can show that the condition $r^* > \tilde{r}^i$ will then be violated for all sufficiently large n (this is the case presented below in Corollary 8). This means that growing competition provides an incentive for a retailer to deviate from this form of RESE by increasing supply beyond the point where $p_2 = s$. Despite the resulting losses in the second period, this deviation can be profitable because the first-period market share of the deviating retailer is dramatically higher. Hence, growing competition may result in the non-existence of RESE 3 even though condition $P_2 < p_1 < P_1$ holds.

Corollary 8. *If condition $P_2 < p_1 < P_1$ holds and condition (a) of RESE 3 existence is violated in the limit of $n \to \infty$, RESE 3 does not exist for all sufficiently large n.*

This result shows that we need to refine our understanding of the equilibrium and conditions for its existence. For monopoly ($n = 1$), Theorem 5 exhaustively covers all feasible parameter values. Starting from duopoly, condition $P_2 < p_1 < P_1$ may not guarantee the existence of RESE 3. The result presented below shows that, in the same p_1-range, there may exist one more form of RESE with sales in both periods and $p_2^* = s$.

4.2. **Salvaging RESE.** The best response in the retailer game depends on $Y^{-i} \triangleq Y - y^i$ — total inventory less the inventory of retailer i. If $Y^{-i} < 1 - \frac{s}{\beta}$, retailer i has control over the second-period price. Namely, $p_2 > s$ if $y^i < 1 - \frac{s}{\beta} - Y^{-i}$ (no salvaging) or $p_2 = s$ if $y^i \geq 1 - \frac{s}{\beta} - Y^{-i}$ (salvaging). If $Y^{-i} \geq 1 - \frac{s}{\beta}$, salvaging is forced on retailer i, i.e., $p_2 = s$ regardless of supply y^i. Condition $Y^{-i} < 1 - \frac{s}{\beta}$ is used in a symmetric form with $Y^{-i} = \frac{n-1}{n}Y^*$ in the following characterization of the last equilibrium form further referred to as RESE 4.

Theorem 9 ("Salvaging" RESE 4: sales in both periods, $p_2^* = s$)**.** *RESE with $\alpha^* = 1, p_2^* = s, v^* = \frac{p_1 - \rho s}{1 - \rho\beta}, Y^* = \frac{n-1}{n}\frac{p_1 - s}{c - s}(1 - v^*)$, and $r^* = \frac{p_1 - s}{n^2}(1 - v^*)$ exists if and only if one of the following mutually exclusive conditions hold:*

(a) *salvaging is forced on retailers, i.e., $\frac{n-1}{n}Y^* \geq 1 - \frac{s}{\beta}$;*

(b) *condition (a) does not hold, and $\left(\beta\left(1 - \frac{s}{\beta}\right)^2 + (p_1 - \beta)(1 - v^*)\right)\frac{n-1}{n}\frac{Y^*}{c + \beta v^* - 2s} \geq \left(1 - \frac{s}{\beta}\right)^2$;*

(c) *conditions (a) and (b) do not hold, $Y^* > 1 - \frac{s}{\beta}$, and there are no real roots of the equation*

$$2Y^3 - \left(2 - v^* - \frac{c}{\beta} + \frac{n-1}{n}Y^*\right)Y^2 + \left(1 - \frac{p_1}{\beta}\right)(1 - v^*)\frac{n-1}{n}Y^* = 0 \tag{4}$$

in the interval $(1 - v^*, 1 - \frac{s}{\beta})$, or $r^* \geq \tilde{r}^i(\tilde{Y})$, where $\tilde{r}^i(\tilde{Y})$ is the maximum profit of a firm deviating from this RESE in such a way that $p_2 > s$, and $\tilde{Y}$ is the only root of (4) in the interval $\left(1 - v^*, 1 - \frac{s}{\beta}\right)$.

If the initial consumer expectations of the second-period price are such that $\bar{p}_2^0 > s$, the game is repeated, and expectations follow a linear adjustment process, then the sequence of games converges to $\bar{p}_2 = s$ for any sufficiently small speed of adjustment.

Unlike RESE 1-3, RESE 4 cannot exist for $n = 1$. This can be seen, e.g., from the expression for Y^*. The larger n is, the easier retailers find themselves in RESE with $p_2^* = s$. Similar to RESE 3, conditions (b) and (c) correspond to different attractiveness of a higher second-period price for a potential deviator from RESE 4 that decreases inventory. Condition (b) means that the deviator profit monotonically increases in inventory, i.e., for the inputs that satisfy (b), RESE 4 is stable with respect to small parameter changes when p_1 is sufficiently far from the boundary. The first part of condition (c) — no real roots of (4) in the interval $\left(1 - v^*, 1 - \frac{s}{\beta}\right)$ — means that the deviator profit has no local maximum with $p_2 > s$, whereas inequality $r^* \geq \tilde{r}^i(\tilde{Y})$ requires that when this maximum exists at $y^i = \tilde{Y} - \frac{n-1}{n}Y^*$, it does not exceed the profit under RESE 4. The inputs where RESE 4 exists only by the second part of (c) are close to the boundary of RESE 4 existence where this equilibrium may be unstable with respect to parameter misestimation. Conditions (a)-(c) hold if $c - s$ is sufficiently small, i.e., the cost is largely compensated by salvaging any excess units, which makes this outcome attractive for the retailers. This form of equilibrium results in market overcapacity and salvage value sales of a significant portion of the total supply.

Theorem 9 implies a necessary condition $v^* < 1$, which means that there is positive demand in the first period. This condition is equivalent to the upper bound $p_1 < 1 - \rho(\beta - s) \triangleq P_4$ signifying that a relatively high MSRP precludes salvaging outcome. Alternatively, this condition represents an upper bound on the level of strategic behavior:

$$\rho < \frac{1 - p_1}{\beta - s}. \tag{5}$$

As long as the product is durable enough for $1 - p_1 < \beta - s$ to hold, highly strategic (with ρ near one) consumers guarantee that the salvaging outcome is impossible. Since $P_4 < P_1$ (the bound that separates RESE 1 and 3), P_4 separates RESE 4 and 3.

We now turn to the question of equilibrium uniqueness. By Theorem 5, RESE 1, 2, and 3 are mutually exclusive since the corresponding p_1-ranges do not intersect. The result below shows that RESE 1, 2, and 4 are also mutually exclusive. Moreover, part (b) guarantees that condition (a) of Theorem 5 holds for p_1-range of RESE 3 and, at the same time, RESE 4 cannot exist.

Proposition 10. *A unique RESE exists if any of the following conditions hold: (a) $p_1 \geq P_1$, or $p_1 \leq P_2$, or (b.1) $P_2 < p_1 < P_1$ and (b.2) $\frac{n-1}{n}(p_1 - s)(1 - p_1) \leq (c - s)\left(1 - \frac{s}{\beta}\right)$.*

Condition (b.2) trivially holds for $n = 1$. In general, it has the form of a lower bound on $c - s$, i.e., the unit cost is sufficiently high compared to the salvage value. The condition holds for any $n > 1$ and p_1, if it holds for $n \to \infty$ and $p_1 = \frac{1}{2}(1 + s)$ (p_1 maximizing the left-hand side). The resulting stronger inequality is $c - s \geq \frac{(1-s)^2}{4(1-s/\beta)}$, which holds, e.g., for $c = 0.25$ and $s = 0$. Thus, when the unit cost is relatively high, retailers avoid the unfavorable "salvaging" outcome.

The analysis of this section leaves a possibility that RESE does not exist. This is indeed the case, but the fraction of model inputs where this may occur is very small. Combining all conditions in

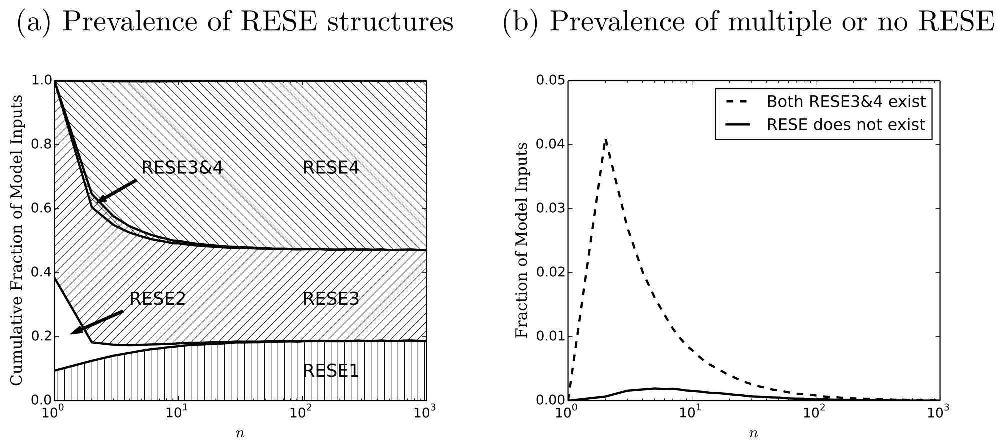

FIGURE 2. Fractions of model inputs resulting in a particular RESE structure for given n

Theorems 5 and 9, we can determine which of the four types of equilibria exist (if any) for any given set of inputs $(n, \rho, \beta, c, s, p_1)$ satisfying the feasibility conditions $0 \leq \rho < 1$, $0 \leq s < c < \beta \leq 1$, and $\frac{s}{\beta} \vee c < p_1 \leq 1$. We have performed this analysis for 1,000,000 randomly (according to uniform distribution) sampled feasible model inputs for different values of $1 \leq n \leq 1,000$. The results are presented in Figure 2. Subgraph (a) is an area plot that shows the fractions of inputs resulting in a particular equilibrium structure (RESE 1, 2, or 3 only, both RESE 3 and 4, RESE 4 only) as the heights of the respective shaded areas for each n. As n increases, RESE 2 disappears and the prevalence of RESE 1 and 4 grows with RESE 4 reaching more than 50% of model inputs. Subgraph (b) shows the fractions of inputs resulting in both RESE 3 and 4 as well as non-existence of equilibrium. The fraction of inputs where both RESE 3 and 4 exist is 4% for a duopoly and considerably less for other levels of competition. The fraction of inputs where no RESE exists is at most 0.191% (reached for $n = 5$).

5. PROPERTIES OF RESE

The results of previous sections can be used, e.g., by a manufacturer or retailer to estimate possible outcomes of entering the market. These outcomes depend on the current *levels* of competition, strategic behavior, and other parameters. For an existing market, the effects of *changes* in these parameters can be more relevant in order to anticipate possible market alterations. As to changes in consumer strategic behavior, one of their drivers is macroeconomic. When the economy is expanding, more consumers prefer to buy now than wait, and vice versa – an average consumer is more inclined to delay the purchase when the economy shrinks. For example, a study of a Fortune 500 retailer sales by Allenby et al. (1996) shows that even "fashion-forward consumers who purchase apparel early in the season are more sensitive to economic conditions and expectations than previously believed."

Various forms of Consumer Confidence Indicators report on changes in consumer behavior. For example, the Index of Consumer Confidence is defined by the Conference Board of Canada web site as "a crucial indicator of near-term sales for companies in the consumer products sector... Data is collected on each respondent's age, sex, marital status, and geographic location of residence." Using these data and other macroeconomic variables, a retailer and/or manufacturer can estimate possible changes in ρ and in market outcomes, respectively, given that the current situation is known. Lemmens et al. (2005), in an empirical study of the European markets, conclude that "the Consumer Confidence Indicators become much more homogeneous as the planning horizon is

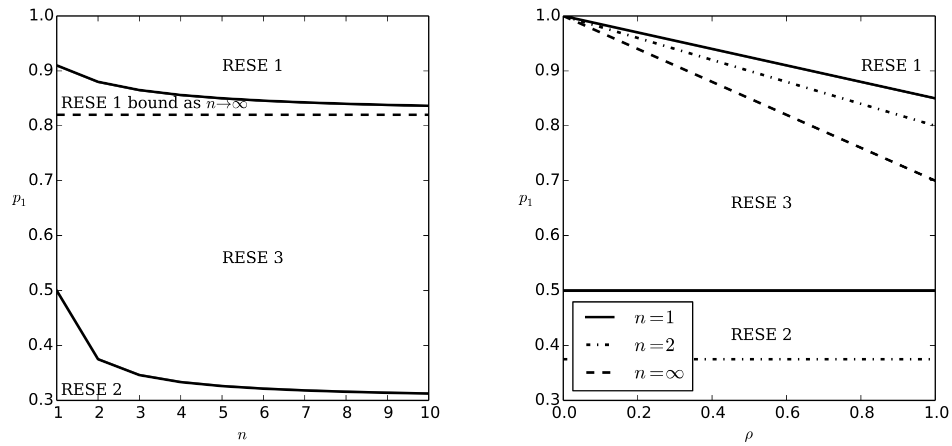

FIGURE 3. Typical boundaries between RESE 1-3 in (n, p_1) and (ρ, p_1) (example for $c = 0.3$, $s = 0.05$, $p_1 = 0.5$, $\beta = 0.6$, and, for left plot, $\rho = 0.6$)

extended." This homogeneity emerges inside of regions, and is determined by cultural, economic and geographic differences.

This section, on one hand, supports previous studies showing that equilibrium total supply increases in n and decreases in ρ. Both trends typically decrease retailers' profits. On the other hand, we specify two distinct cases when these opposing trends "compensate" each other leading to increasing profit in ρ. Increasing ρ also has different effects on consumer second-period surplus and total second-period sales depending on the consumer valuation and market situation, respectively.

5.1. **RESE 1-3 (no salvaging).** The analysis below takes into account possible switches between different forms of RESE. The requirement of a unique RESE in some statements can be guaranteed, e.g., by Proposition 10.

5.1.1. *Switches between RESE forms.* When RESE is unique, p_1-ranges indicated in Theorem 5 provide a unique mapping between input parameter values and different forms of RESE. Figure 3 illustrates how these ranges change with n and ρ:

(a) the bounds on p_1 that separate RESE 3 from RESE 1 and 2 are decreasing in n;
(b) the upper bound on p_1 in RESE 3 is decreasing, and the lower bound is constant in ρ; and
(c) the lowest possible value of p_1 that leads to RESE 1 is strictly above the highest possible value that leads to RESE 2.

These observations are summarized as follows:

Proposition 11 (Changes in RESE structure)**.** *For RESE 1-3, the following claims hold:*

(1) *(From 2 to 3 in n) If $p_1 \le \frac{c}{\beta}$, there exists $n_2 \triangleq \frac{p_1(1-\beta)}{p_1-c} \ge 1$ such that RESE is realized with sales only in the first period (RESE 2) for any $n \le n_2$, and with sales in both periods and $p_2^* > s$ (RESE 3) for any $n > n_2$.*
(2) *(From 3 to 1 in n) For any $\rho \in (0,1)$, if $1-\rho(\beta-c) < p_1 < 1-\frac{1}{2}\rho(\beta-c)$, there exists $n_1 \triangleq \frac{1-p_1}{p_1-1+\rho(\beta-c)} \ge 1$ such that RESE is realized with sales in both periods and $p_2^* > s$ (RESE 3) for any $n < n_1$, and with sales only in the second period (RESE 1) for any $n \ge n_1$.*
(3) *(From 3 to 1 in ρ) For any $n \in [1,\infty)$, if $1-\frac{n}{n+1}(\beta-c) < p_1 < 1$, there exists $\rho_1 \triangleq \frac{n+1}{n}\frac{1-p_1}{\beta-c}$ such that RESE is realized with sales in both periods and $p_2^* > s$ (RESE 3) for any $\rho < \rho_1$ and with sales only in the second period (RESE 1) for any $\rho \ge \rho_1$.*

(4) *(No switches) If* $\frac{c}{\beta} < p_1 \leq 1 - \beta + c$, *RESE is realized with sales in both periods and* $p_2^* > s$ *(RESE 3) for any* $n \geq 1$ *and* $\rho \in [0, 1)$.

The changes in equilibrium structure generally lead to shifts in sales to the second period as the levels of competition or strategic behavior increase. Next, we examine changes in the quantitative characteristics of equilibrium.

5.1.2. *Monotonicity of* Y^*, v^*, *and* nr^*. We now examine the monotonicity of v^*, Y^*, and nr^* in n and ρ within RESE 1-3 and, by continuity, between these forms of RESE.

Proposition 12. *For RESE described in Theorem 5, the following claims hold:*

(1) *The equilibrium total supply* Y^* *is non-decreasing in* n *(constant for RESE 2; increasing for RESE 1 and 3) and non-increasing in* ρ *(decreasing for RESE 3; constant for RESE 1 and 2).*
(2) v^* *is non-decreasing in* n *(constant for RESE 1, 2, and RESE 3 with* $\rho = 0$; *increasing for RESE 3 with* $\rho > 0$*) and non-decreasing in* ρ *(increasing for RESE 3; constant for RESE 1 and 2).*
(3) *The total equilibrium profit of all retailers* nr^* *is non-increasing in* n *(constant for RESE 2; decreasing for RESE 1 and 3), decreasing in* ρ *for RESE 3 with* $p_1 \geq \beta - \frac{n}{2(n+1)}(\beta - c)$ *or* $n = 1$, *and constant in* ρ *for RESE 1 and 2.*

Monotonicity of the total supply and the total profit in the level of competition agree with the theory of oligopoly and can be viewed as a sanity test for the model. On the other hand, monotonicity in the level of strategic behavior (represented by ρ) is a much finer result. The new insights of this paper are connected to the following non-trivial interaction between firms and consumers while ρ is increasing. Part (2) of Proposition 12 states that v^* is increasing in ρ when there are sales in both periods (RESE 3) and retailers effectively engage in intertemporal price discrimination. Increasing v^* means that more consumers delay their purchases, even though total supply Y^* is decreasing in ρ (by part (1)), resulting in a decreasing total number of purchases and increasing second-period price. The nature and consequences of this interaction are considered below in more detail. Part (3) of Proposition 12, for the monopoly, agrees with the existing literature that strategic consumer behavior reduces profits. We generalize this effect to the case of oligopoly when the product is not very durable (i.e., β is sufficiently low) or MSRP is relatively high.

In RESE 3, oligopolistic retailers counteract additional consumer delays in purchase resulting from increasing ρ by increasing the equilibrium second-period price $p_2^* = \beta(1 - Y^*)$. As a result, the expected surplus of waiting $\sigma_2 = \rho[\beta v - \beta(1 - Y^*)]$ may not be increasing in ρ. Indeed, its derivative in ρ is $\frac{\partial \sigma_2}{\partial \rho} = \frac{\sigma_2}{\rho} + \rho \frac{\partial Y^*}{\partial \rho}$. The first term in the RHS is the realized second-period surplus, which is non-negative for the consumers who buy in the second period. The second term reflects the equilibrium response of the oligopolistic retailers. By part (1) of Proposition 12, this term is negative for any $\rho > 0$. For RESE 3, the following corollary shows that increasing ρ has a different effect on σ_2 depending on the consumer valuation.

Corollary 13. *For RESE 1 and 3, expected surplus* σ_2 *of waiting is (1) increasing in* n *for any* $v \in [0, 1]$, *and (2) increasing in* ρ *for* $v^0 < v \leq 1$, *and decreasing in* ρ *for* $0 \leq v < v^0$, *where* $v^0 = \frac{1}{\beta}\left(p_2^* + \rho \frac{\partial p_2^*}{\partial \rho}\right) = 1 - Y^* - \rho \frac{\partial Y^*}{\partial \rho} \in \left[\frac{p_2^*}{\beta}, v^*\right)$ *is such that* $\left.\frac{\partial \sigma_2}{\partial \rho}\right|_{v=v^0} = 0$.

For consumers with high valuations, σ_2 is increasing in ρ. In particular, for the consumers with $v = v^*$, the purchase in the second period is becoming more attractive than in the first period, which means that v^* is increasing in ρ (part (2) of Proposition 12). In contrast, σ_2 is decreasing for consumers with low valuations. For example, for the second-period buyers with the lowest valuation

$v = \frac{p_2^*}{\beta}$, the second-period surplus is becoming negative, leading to a decrease in the total number of purchases. For myopic ($\rho = 0$) consumers, $v^0 = \frac{p_2^*}{\beta}$ and $\frac{\partial \sigma_2}{\partial \rho} > 0$ for all second-period buyers.

An increase in ρ leads to either an increase or decrease in the total equilibrium second-period sales $Q_2^* = Y^* - (1 - v^*)$, depending on the parameters:

Corollary 14. *For RESE 3, (1) $\frac{\partial Q_2^*}{\partial \rho} > 0$ when $n = 1$; and (2) $\frac{\partial Q_2^*}{\partial \rho} < 0$ when $n \to \infty$, $\rho = 0$, $\beta < 1$ and p_1 is near 1.*

Proposition 12 claims that $nr^{*,3}$ is decreasing in ρ for monopoly and oligopoly with a not very durable good. To show that $nr^{*,3}$ may increase in ρ, we consider a limiting case of a durable (within the time frame of the problem) product and consumers with the maximum level of strategic behavior. In this case, the consumer choice of purchase time is determined only by price.

Proposition 15. *Let $\bar{n} \triangleq \frac{1-p_1}{p_1-c}$. For $\beta = 1$ and $\rho \to 1$, RESE 2 and 4 do not exist and the equilibrium has the form*

1. *RESE 1 with $v^*|_{\rho\to1} = 1$, $Y^*|_{\rho\to1} = \frac{n}{n+1}(1-c)$, $p_2^*|_{\rho\to1} = \frac{nc+1}{n+1} < p_1$, and $nr^*|_{\rho\to1} = \frac{n(1-c)^2}{(n+1)^2}$ if $n > \bar{n}$, and*
2. *RESE 3 with $v^*|_{\rho\to1} = p_1 + n(p_1 - c)$, $Y^*|_{\rho\to1} = 1 - p_1$, $p_2^*|_{\rho\to1} = p_1$, and $nr^*|_{\rho\to1} = (p_1 - c)(1 - p_1) < \frac{n(1-c)^2}{(n+1)^2}$ if $n < \bar{n}$ and*

$$\frac{n-1}{n} \frac{(p_1 - s)(1 - p_1 - n(p_1 - c))(1 - p_1)}{(c - s)(1 - s)^2} < 1. \tag{6}$$

Moreover, when $n = \bar{n}$,

3. *the limiting cases (1) and (2) coincide, and*
4. *(boundary-value gain) for all p_1 and c such that $\bar{n} \geq 3$, we have $nr^{*,3}\big|_{\rho\to1} > nr^{*,3}\big|_{\rho=0}$.*

In both limiting scenarios, sales occur at a single price: the one-period Nash-Cournot price in part (1) and p_1 in both periods in part (2). In part (1), representing a high level of competition, no sales occur at p_1, i.e., $v^* = 1$, and price decreases to Nash-Cournot in the second period. The Nash-Cournot supply level in this case exceeds the MSRP-determined lower bound $1-p_1$ of Lemma 4. In part (2), representing a low level of competition, retailers counteract strategic behavior by reducing the total supply all the way to $1 - p_1$ which exceeds the Nash-Cournot supply level.

Thus, the extreme level of strategic behavior, in combination with MSRP, forces retailers into a collusive outcome. Part (3) shows that $n = \bar{n} = \frac{1-p_1}{p_1-c}$ plays a role of a *parameter coordination* condition ensuring that the Nash-Cournot price coincides with p_1. This condition is critical for understanding part (4) that demonstrates the total profit increase as consumer behavior changes from myopic to completely strategic.

This profit gain may appear counterintuitive because strategic consumer behavior is usually considered detrimental. However, in this case, strategic behavior prevents the second-period sales at a loss. Indeed, with completely strategic consumers, the total sales are equal to $1-p_1$ and occur at p_1. In the case of myopic consumers, the first-period sales are the same, while the second-period sales are at loss for any $n \geq 3$ according to Corollary 7. Since the increase in profit is strict, the effect presented in part (4) is quite robust. Indeed, for each $n \geq 3$, there is a continuum of model instances satisfying the parameter coordination condition. Moreover, by continuity in parameters, increased strategic behavior leads to an increase in profit in a local neighborhood of these instances. The "boundary-value" profit gain described in part (4) results in this case from the "continuous gain" since the entire range $\rho \in [0, 1)$ belongs to the same equilibrium RESE 3 (Figure 4).

Example 16. *Condition (6) is a limiting version of condition (a) of RESE 3 existence, which is less restrictive than sufficient condition (b.2) of Proposition 10. Condition (6) holds for all p_1 and $1 \leq n < \bar{n}$ if $c > \frac{1+4s}{5}$, e.g., if $s = 0$ and $c > 0.2$.*

(a) $\beta = 1$ (b) $\beta = 0.9$

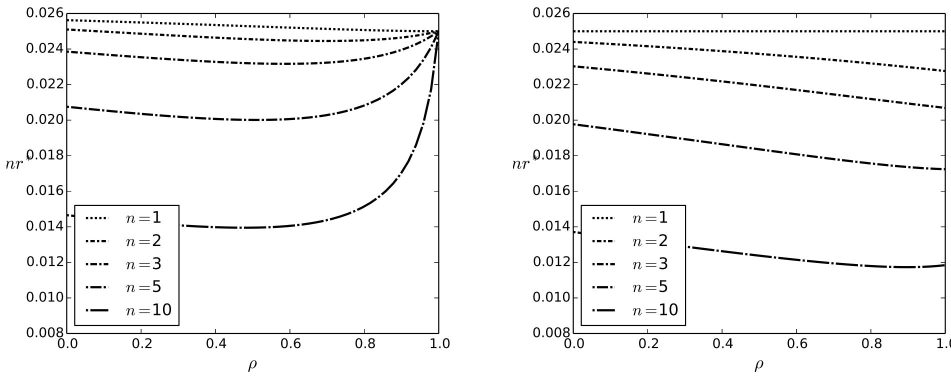

FIGURE 4. The total profit for RESE 3 with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

(a) The second-period price (b) The total profit

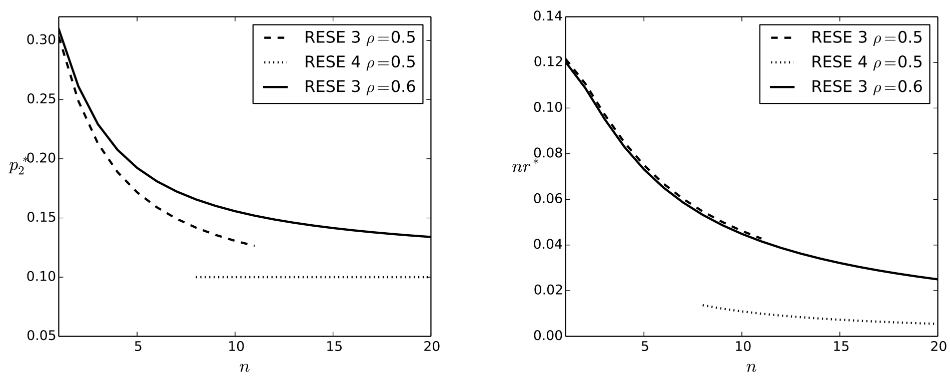

FIGURE 5. Overlap and switches between RESE 3 and 4 for $c = 0.2$, $s = 0.1$, $p_1 = 0.4$, $\beta = 0.9$

The numerical example below illustrates the behavior of the total profit in ρ. For small ρ, the total profit is decreasing in ρ (see Figure 4 (a)). On the other hand, when $\beta = 1$, the total profit is increasing for ρ near one. For all values of $n \leq \bar{n} = 10$ in this example, the total profit attains the limit $(p_1 - c)(1 - p_1)$ established in part (2) of Proposition 15. It is natural to expect that this effect of "durable-good non-monotonicity" is becoming weaker when $\beta < 1$. Indeed, Figure 4 (b) illustrates that, for $\beta = 0.9$, the total profit is decreasing in ρ for $n = 1, 2, 3$ and 5, and the increase of nr^* in ρ is small for $n = 10$. This increase can no longer compensate for the losses in nr^* resulting from increased competition.

5.2. **RESE 4 (salvaging).** The following proposition establishes the monotonic properties of Y^*, v^*, and nr^* for RESE 4. These properties partially match those of RESE 1-3.

Proposition 17. *For RESE 4, (1) v^* is constant in n and increasing in ρ; (2) Y^* is increasing in n and decreasing in ρ; and (3) nr^* is decreasing in n and decreasing in ρ.*

By part (a) of Proposition 10, RESE 4 does not exist under the conditions of RESE 1 and 2. However, RESE 3 and 4 can both exist under the same inputs. In that case, one needs to resort to focal-point arguments to predict which of the two equilibria will be realized. The example of Figure 5 illustrates this fact.

(a) $\beta = 1, n = 5, p_1 = 0.4$ (b) $\beta = 0.5, n = 10, p_1 = 0.8$

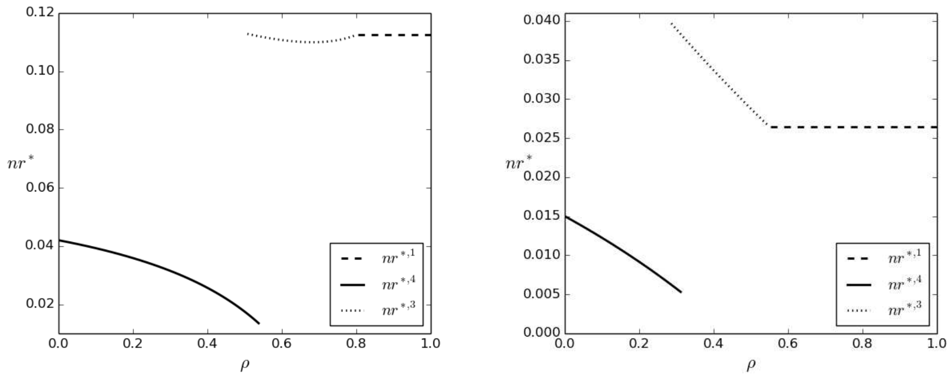

FIGURE 6. Equilibrium profit in ρ for $c = 0.1, s = 0.05$

The inputs for Figure 5 are such that inequality $P_2 < p_1 < P_1$ and condition (a) of Theorem 5 for RESE 3 existence hold for $n = 1$ but this condition does not hold for $n = \infty$. By Corollary 8, RESE 3 does not exist for sufficiently large n. At the same time, by part (b) of Proposition 10 and by rationality, RESE 4 does not exist and RESE 3 is realized uniquely for $n = 1$. It is also straightforward to check that RESE 3 can be realized for $n = 1, \ldots, 11$, while RESE 4 can be realized for $n = 8, \ldots, \infty$. For $n = 8, \ldots, 11$ either equilibrium is possible.

In line with the interpretation of rational expectations equilibrium as a structure that is self-sustaining in the long run, a possible focal point is an equilibrium with a structure that is similar to the past. Figure 5 (b) shows that RESE 4 is considerably worse for the retailers than RESE 3. However, RESE 4 may be realized because a single retailer cannot unilaterally benefit from decreasing its market share as long as others expect the RESE 4 structure and act accordingly.

Now, what is the effect of changing ρ when there is an overlap in inputs leading to RESE 3 and 4? For the data considered above, a minor increase in ρ from 0.5 to 0.6 qualitatively changes the situation because, for $\rho = 0.6$, inequality $P_2 < p_1 < P_1$ and condition (a) of RESE 3 existence hold for any $n \geq 1$ and neither of subcases (a)-(c) of Theorem 9 hold. Therefore, RESE 4 cannot exist in the scenario considered above, and this increase in ρ leads to a discontinuous profit gain and serves as an *insurance against salvaging.* Such an increase in ρ works by decreasing capacity in RESE 3 at the cost of a slight decrease in profit (compare the solid and dashed lines in Figure 5 (b)). The discontinuous profit gain can lead to the boundary-value gain even when β is small (Figure 6 b) and can be combined with the continuous gain when β is near one. As a result, equilibrium profit can have up to three local maxima in ρ (Figure 6 a).

An increase in ρ cannot always prevent retailers from realizing RESE 4, but it does reduce the fraction of inputs leading to it. Figure 7 shows that for $\rho = 0.999$ the maximum fraction of model inputs leading to RESE 4 reduces to 37.2% compared to more than 50% in Figure 2, where ρ is unrestricted. Moreover, the area of inputs leading to both RESE 3 and 4 shrinks to less than 1.25%.

5.3. **Equilibrium inventory and p_1.** As mentioned in the introduction, the paper primarily focuses on exogenous p_1, e.g., when p_1 is specified by the manufacturer-retailer agreement (Orbach (2008)). Manufacturers often operate in multiple markets with notably different valuations for the same product, but MSRP must be comparable when converted to local currencies to comply with anti-dumping regulations. In this case, the ratio of MSRP to the highest valuation on a specific market can take any value from the range $(c, 1]$ and lead to any type of RESE considered above.

However, a product may target only one specific market, or valuations on several markets might almost be the same. In this case, the manufacturer can try to negotiate p_1 to maximize its profit.

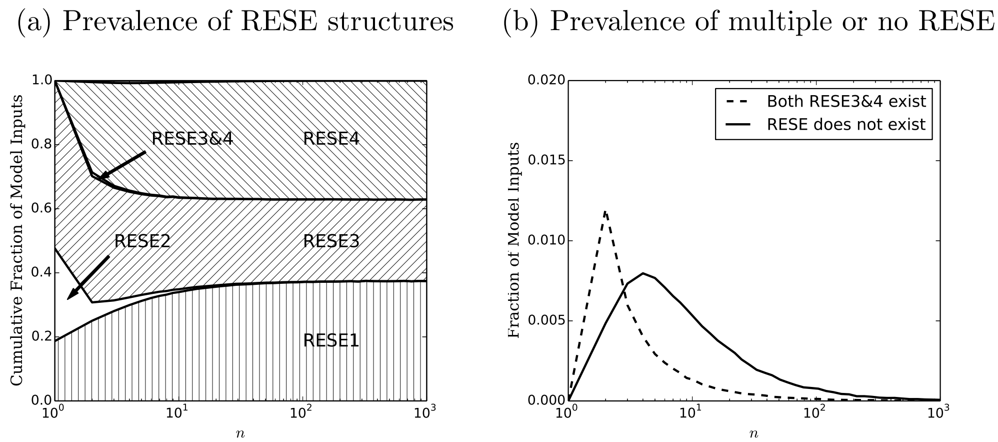

FIGURE 7. Fractions of model inputs resulting in a particular RESE structure for $\rho = 0.999$ and given n

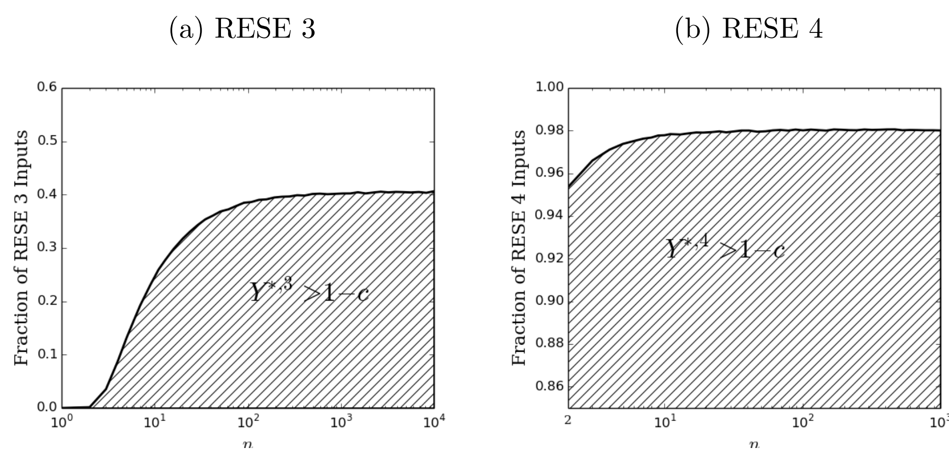

FIGURE 8. Fractions of RESE 3 and 4 inputs resulting in $Y^* > 1 - c$ for given n

Keeping all other parameters constant, such p_1 maximizes the total equilibrium inventory Y^* procured by the retailers. This section shows that RESE 4, which is the worst for the retailers, is typically the best for the manufacturer.

The simplest "benchmark" case is RESE 2 where p_1 is relatively low and $Y^* = 1 - p_1$. The supremum of the manufacturer's profit in RESE 2 is obtained as p_1 tends to c. In practice, this supremum cannot be achieved because retailer profits must be positive and consumer valuations are bounded from above. Therefore, the difference between MSRP and the unit cost, normalized by the highest valuation, is separated from zero. The following results show that, depending on the product (β) and the market situation (n, ρ, c, s), the values of p_1 leading either to RESE 3 or 4 can be more profitable for the manufacturer than $p_1 \to c$.

Proposition 18. *When the corresponding RESE exists, (1) $Y^{*,1} < 1 - c$; (2) the unique maximum of $Y^{*,4}$ in p_1 is $\bar{Y}^{*,4} = \frac{(n-1)(\bar{p}_1 - s)^2}{n(1-\rho\beta)(c-s)}$ at $p_1 = \bar{p}_1 \triangleq \frac{1}{2}(P_4 + s)$; $Y^{*,4} \geq 1 - c$ if and only if $c - s \leq \frac{n-1}{n}\frac{p_1 - s}{1-c}\frac{1-p_1-\rho(\beta - s)}{1-\rho\beta}$; (3) $Y^{*,3} < 1 - c$ for $n = 1$; for $n \to \infty$ and $p_1 \to P_2 = c, Y^{*,3} \to 1 - c$ and, if $\rho = 0$, $\left.\frac{\partial Y^{*,3}}{\partial p_1}\right|_{p_1 = P_2 + 0} > 0$.*

(a) $n = 3, \beta = 0.6$ (b) $n = 5, \beta = 0.8$

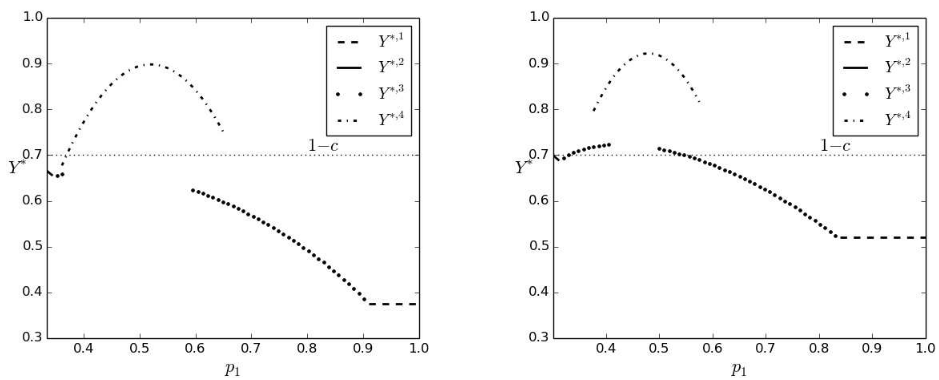

FIGURE 9. Y^* in p_1 for $\rho = 0.4, c = 0.3, s = 0.2$, and

RESE 2 is the best for the manufacturer in a market with a single retailer and $\beta < 1$ (RESE 2 must exist). For $n > 1$, consistently with Proposition 12, Figure 8 shows that the fractions of RESE 3 and 4 instances, where $Y^{*,3}$ and $Y^{*,4}$ are greater than $1 - c$, are increasing in n. For RESE 3, this fraction is zero at $n = 1$ and remains below 40% for $n > 1$, while it is at least 95% for RESE 4. Therefore, the manufacturer may prefer markets with many retailers, where the ratio of MSRP to the highest valuation takes intermediate values and RESE 4 can be realized. For Figure 8, we used the same simulation approach as for Figure 2. Typical qualitative behavior of Y^* in p_1 is illustrated in Figure 9. As a rule, the maximum values of $Y^{*,3}$ and $Y^{*,4}$ are inside the p_1-range of the corresponding RESE.

When p_1 maximizes the manufacturer profit, and the minimum first-period price is $p_1^{\min} > c$, the results of Proposition 15 are still valid with the substitution $p_1 = p_1^{\min}$. In particular, for any $p_1^{\min} \in \left(0, \frac{1+3c}{4}\right]$, we have $\bar{n} \geq 3$, and the boundary-value gain holds: $nr^{*,3}\big|_{\rho\to 1} > nr^{*,3}\big|_{\rho\to 0}$ (part 4). This profit gain disappears only with $p_1^{\min} = c$ leading to $nr^{*,3}\big|_{\rho\to 1} = nr^{*,3}\big|_{\rho\to 0} = 0$. However, the case $p_1^{\min} = c$ is infeasible and implausible in this problem.

This subsection illustrates a non-trivial nature of manufacturer-retailer interactions under oligopoly with strategic consumers. The properties of possible outcomes described in the above sections can be used to study these interactions in a two-tier supply chain framework. Such analysis includes a distinct set of research questions, e.g., the comparison of supply chain efficiency under centralized and decentralized settings with various types of contracts (see Su and Zhang (2008) for monopoly), and deserves a separate consideration.

5.4. **Retailer's discount.** Lazear (1986) (p. 25) showed that the discounted second-period profit leads to decreasing prices, which typically corresponds to increasing sales. Our setting leads to a similar result in terms of inventory. The proposition below shows that when retailers solve a non-degenerate two-period profit-maximization problem, the equilibrium inventory increases if a discount factor becomes less than one. We call a two-period problem degenerate if it reduces to one period, which happens for RESE 1, 2, and for a monopolist in RESE 3 since, for $n = 1$, the first-period demand does not depend on inventory.

Proposition 19. *If retailer i's profit is $r^i = (p_1 - c)q^i + \lambda(p_2 - c)(y^i - q^i), \lambda \in (0, 1]$, equilibrium total inventory Y^* decreases in λ for RESE 4, RESE 3 with $n > 1$ and constant for RESE 1,2, and 3 with $n = 1$. If $\lambda = (1 + \delta)^{-1}$, where δ is the interest rate between two periods, the relative increase in $Y^{*,4}$ from introducing $\lambda < 1$ is $\left(Y_\lambda^{*,4} - Y^{*,4}\right)/Y^{*,4} = \frac{p_1 - c}{p_1 - s}\delta < \delta$.*

For example, if $\lambda = 1, p_1 = 0.5, n = 10, \beta = 0.75, c = 0.1, s = 0.05$, and $\rho = 0$, then by condition (a) of Theorem 9, RESE 4 is realized with $Y^{*,4} = 4.05$. If, for the same data, retailers consider a 2% interest rate between periods, $Y_\lambda^{*,4} = 4.122$, which is around 1.8% greater than $Y^{*,4}$. For the same data, but $\rho = 0.7$, RESE 3 is realized by condition (a) of Theorem 5 with $Y^{*,3} = 0.85276$. The same 2% interest rate yields $Y_\lambda^{*,3} = 0.85346$, which is only about 0.08% greater than $Y^{*,3}$.

5.5. **RESE stability.** An equilibrium is more likely to emerge in practice if it is (a) *asymptotically locally stable*, i.e., when the initial retailers' inventories are close to an equilibrium, they converge to the equilibrium values, or (b) *globally stable*, i.e., when any initial inventories converge to an equilibrium when it is unique. In our setting, by Theorem 5 and Proposition 10, RESE is unique for any inputs except for a small fraction where both RESE 3 and 4 may exist (Figure 2). In the latter case, however, the feasible inventory ranges for RESE 3 and 4 are separated by a non-empty interval (Figure 9).

RESE 1,3, and 4, for $n \geq 2$, represent a non-degenerate game between retailers that can be reformulated as a one-period game with retailer i's payoff function $\pi^i(y^i, Y^{-i}) = y^i P(y^i, Y^{-i}) - C_i(y^i)$. Then using, e.g., Theorem 3 in al Nowaihi and Levine (1985), the following result holds.

Proposition 20. *For any inputs where RESE 1,3, or 4 exist in an open neighborhood of Y^*, a RESE is locally asymptotically stable.*

As to global stability, a seminal work of Theocharis (1960) showed that for a linear demand and constant per unit cost, the best-response discrete adjustment process $y_{t+1}^i = BR^i(Y_t^{-i}), t = 0, 1, \ldots, i \in I$, converges for $n = 2$ and any y_0^1, y_0^2. This process means that each retailer observes rivals' inventories at some time t and makes a payoff-maximizing inventory decision for $t+1$. Further studies refined this result for slower adjustment processes $y_t^i = y_{t-1}^i + k_i \left[BR^i(Y_{t-1}^{-i}) - y_{t-1}^i\right]$ or $y_t^i = y_{t-1}^i + k_i \partial\pi^i / \partial y^i$ where $k_i \in (0, 1]$ is the speed of adjustment. In particular, according to Fisher (1961), "given the number of sellers, it is always possible to find [slow enough] speeds of adjustment such that the system is stable."

6. Total consumer surplus and aggregate welfare

In this section, we examine the effects of strategic consumer behavior and retailer competition on the consumers and the local economy. In a two-period problem, the *total two-period (realized) consumer surplus* is $\Sigma^* \triangleq \Sigma_1 + \Sigma_2$, where Σ_1 is the total surplus of consumers buying in the first period and Σ_2 in the second. The second-period surplus is not discounted by ρ since ρ is a subjective behavioral parameter and such a discount would not reflect the actual surplus. In the extreme case of $\rho = 0$, such discounting would completely disregard the second-period surplus of myopic consumers. The expression for Σ^* is given by the following:

Lemma 21. *For a RESE with valuation threshold v^* and second-period price p_2^*, total consumer surplus is $\Sigma^* = (1 - v^*)\left[\frac{1+v^*}{2} - p_1\right] + \frac{(\beta v^* - p_2^*)^2}{2\beta}$, where the first term is Σ_1 and the second is Σ_2.*

Effects on the local economy can be measured in terms of the *aggregate welfare* of consumers and retailers defined as $W^* \triangleq \Sigma^* + nr^*$. The main structural result for Σ^* and W^* is

Proposition 22. *Under the conditions of Theorems 5 and 9,*

(1) *total consumer surplus Σ^* is non-decreasing in n (constant for RESE 2 and 4 and increasing for RESE 1 and 3), constant in ρ for RESE 1 and 2, and increasing in ρ for RESE 4;*
(2) *aggregate welfare W^* is*
  (2.1) *increasing in n for RESE 1, constant for RESE 2, and decreasing for RESE 4;*
  (2.2) *constant in ρ for RESE 1 and 2, and, for RESE 4, increasing in ρ for $\rho < \rho^+$ and decreasing for $\rho > \rho^+$ where $\rho^+ \triangleq \left(1 - \frac{1}{n}\frac{p_1 - s}{p_1\beta - s}\right) \Big/ \left(1 - \frac{\beta}{n}\frac{p_1 - s}{p_1\beta - s}\right)$ if $n > \frac{p_1 - s}{p_1\beta - s}$, and $\rho^+ \triangleq 0$ otherwise.*

(a) $\beta = 1$ (b) $\beta = 0.9$

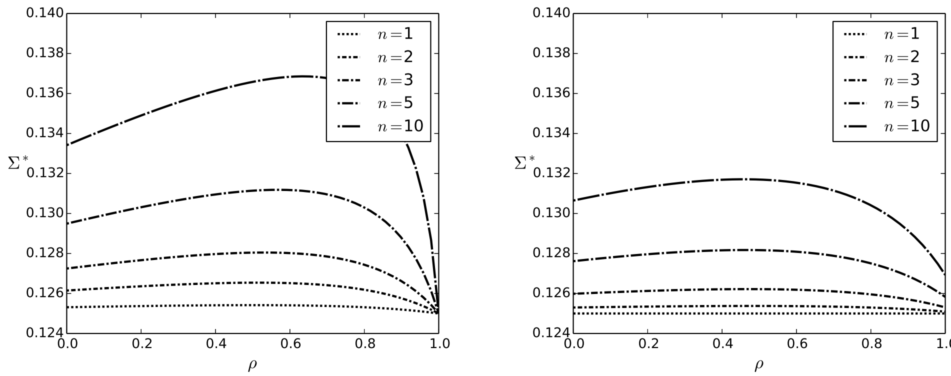

FIGURE 10. The total surplus Σ^* in ρ with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

This proposition implies that the consumer population as a whole benefits from an increase in competition. On the other hand, Σ^* may not be globally monotonic in ρ for RESE 3. The non-monotonicity is established below for the case of $\beta = 1$:

Corollary 23. *Under the conditions of RESE 3, $\beta = 1$ implies: (1) for all $n \geq 1$ and ρ sufficiently close to one, $\frac{\partial \Sigma^*}{\partial \rho} < 0$; and (2) for $n = 1$ or $n \to \infty$ and $\rho = 0$, $\frac{\partial \Sigma^*}{\partial \rho} > 0$.*

Corollary 23 and continuity of Σ^* in β imply that Σ^* has a maximum in ρ if β is sufficiently large (Figure 10 (a)). Non-monotonic behavior is less pronounced for smaller β (Figure 10 (b)).

Along with the monotonicity of Σ^*, Proposition 22 describes certain settings with monotonic aggregate welfare. The direction of monotonicity in a particular parameter varies depending on the equilibrium structure and other inputs. For example, W^* is increasing in n for RESE 1 (Nash-Cournot outcome), which matches increasing welfare results for a standard one-period Nash-Cournot equilibrium corresponding to our model. However, in other quantity competition settings, welfare may not be increasing in the level of competition. For example, Bulow et al. (1985) (§VI, Example E) claim that the welfare may decrease when a relatively inefficient retailer with high marginal costs enters a monopoly market. In our model, the aggregate welfare decreases in n for RESE 4 (salvaging outcome) because the resulting increase in product oversupply does not benefit the consumers and only decreases profits of the retailers. For RESE 4, the level of strategic behavior ρ^+ attains the internal maximum of W^* as long as $\beta < 1$ and the level of competition is sufficiently high, i.e., $n > \frac{p_1 - s}{p_1 \beta - s}$. For $n \leq \frac{p_1 - s}{p_1 \beta - s}$, W^* is decreasing for all ρ. The dependence of W^* on n was omitted in Proposition 22 for RESE 3 because this case warrants special attention:

Corollary 24 (Non-monotonicity of W^* in n)**.** *Treating n as a continuous variable and p_2^* as a function of n under the conditions of RESE 3, the following result holds:*

$$\frac{\partial W^*}{\partial n} \gtreqless 0 \text{ if and only if } p_2^* \gtreqless c \frac{(1 - \rho\beta)^2}{1 - 2\rho\beta + \rho^2\beta} + p_1 \frac{\rho\beta(1 - \beta)}{1 - 2\rho\beta + \rho^2\beta}. \tag{7}$$

The right-hand side of (7) equals c when $\rho = 0$ or $\beta = 1$ and strictly greater than c otherwise.

If there exists n^W that is strictly within the feasible interval for RESE 3 and maximizes W^*, it satisfies (7) as equality. When $\rho = 0$ or $\beta = 1$, this means that the second-period price corresponding to n^W equals the unit cost. On the other hand, when $\rho > 0$ and $\beta < 1$, the corresponding second-period price is strictly greater than the unit cost. Since, by Proposition 12, Y^* and, therefore, p_2^* are strictly monotonic for RESE 3, n^W is unique whenever it exists. From this unique value, we obtain the maximum of the aggregate welfare (the candidates for the integer-valued point of

(a) $\beta = 1$ (b) $\beta = 0.9$

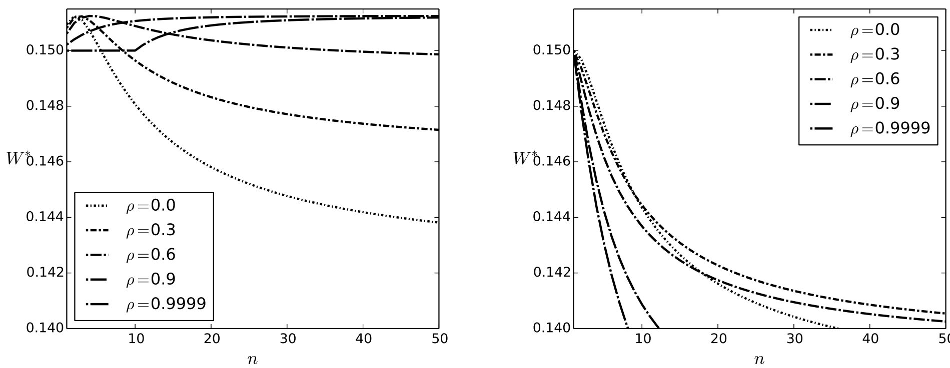

FIGURE 11. The aggregate welfare W^* in n with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

(a) $\beta = 1$ (b) $\beta = 0.9$

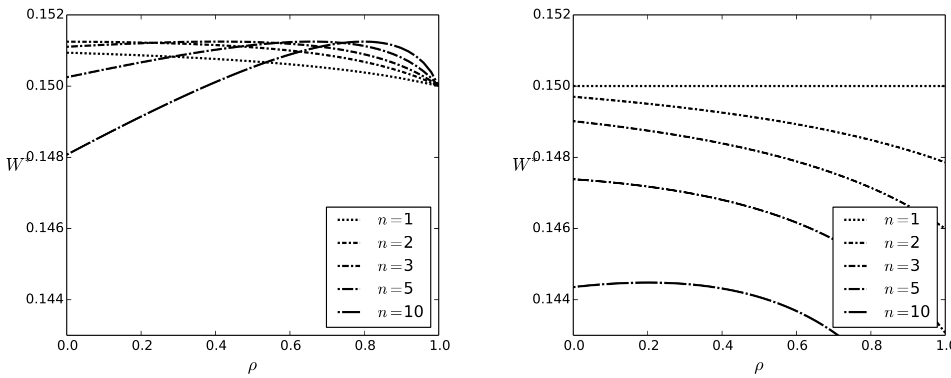

FIGURE 12. The aggregate welfare W^* in ρ with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

maximum are $\lfloor n^W \rfloor$ and $\lceil n^W \rceil$ since, generally, the solution to this equation is real-valued). We illustrate the behavior of W^* in n and ρ in Figures 11 and 12, respectively, for the same set of inputs as our earlier illustrations. For $\beta = 1$, Figure 11 (a) demonstrates that the aggregate welfare can be monotonically increasing in n (for high levels of strategic behavior), and it can also attain the maximum at intermediate values of n (for lower levels of strategic behavior). The latter illustrates Corollary 24. For $\beta = 0.9$, Figure 11 (b) shows that the aggregate welfare may remain monotonically decreasing in the whole range of ρ. In all cases presented in Figure 11 (b), the maximum value of the aggregate welfare is attained by the monopoly. These findings may provide theoretical support for a regulator introducing a policy that affects the number of independent retail chains. Figure 12 indicates that myopic consumer behavior or strategic behavior at an intermediate level may be the best for the local economy in terms of the aggregate welfare. Myopic behavior is best for low levels of competition, and the optimum level of ρ tends to increase as n increases. A smaller value of $\beta = 0.9$ leads to the optimality of myopic behavior in a wider range of n.

7. Conclusions

Even when consumers are risk-neutral and homogeneous in the level of strategic behavior, retailers can gain from increasingly strategic consumers for any level of competition. There are two distinct cases of this effect: the *continuous gain,* when the equilibrium profit increases continuously

Table 1. Summary of monotonic properties in n and ρ by equilibrium form

	Monotonicity in n				Monotonicity in ρ			
RESE	1	2	3	4	1	2	3	4
Y^*	$\nearrow$	$\equiv$	$\nearrow$	$\nearrow$	$\equiv$	$\equiv$	$\searrow$	$\searrow$
v^*	$\equiv$	$\equiv$	$\nearrow$	$\equiv$	$\equiv$	$\equiv$	$\nearrow$	$\nearrow$
nr^*	$\searrow$	$\equiv$	$\searrow$	$\searrow$	$\equiv$	$\equiv$	$\searrow$, min	$\searrow$
Σ^*	$\nearrow$	$\equiv$	$\nearrow$	$\equiv$	$\equiv$	$\equiv$	$\nearrow$, $\searrow$, max	$\nearrow$
W^*	$\nearrow$	$\equiv$	$\nearrow$, $\searrow$, max	$\searrow$	$\equiv$	$\equiv$	$\nearrow$, $\searrow$, max	$\nearrow$, $\searrow$, max

in the level of strategic behavior, and the *discontinuous gain,* when the profit increases because of the switch from the "salvaging" equilibrium to another two-period equilibrium with a higher second-period price.

The first type of gain occurs only for relatively high levels of strategic behavior and small decreases in valuations. With this gain, retailers use strategic consumer behavior to approach an outcome that is equivalent, in terms of the profit value, to a tacit collusion. The discontinuous gain occurs at various levels of strategic behavior and the relative decrease in valuations, but only when salvage sales are attractive enough, i.e., the salvage value is relatively close to the unit cost. For a manufacturer, increasing strategic behavior is always unfavorable because it decreases the total equilibrium inventory procured by the retailers. Both types of retailer profit gains are reversible. When the economy picks up, more consumers buy at the first-period price, becoming less strategic. Then, the incentive for quantity competition increases, and retailers may find themselves in the unfavorable "salvaging" outcome.

We summarize the monotonic properties of equilibrium characteristics with respect to competition level n and strategic behavior level ρ in Table 1 using $\nearrow$, $\searrow$, and $\equiv$ to indicate a monotonically increasing, decreasing, or constant property, respectively. The possibility of an internal maximum or minimum is indicated by "max" or "min", respectively. When multiple symbols are present, it means that different behaviors are possible for different inputs. The monotonicity results with respect to the level of competition are quite conclusive and are obtained in the analytic form. The direction of monotonicity can only vary for the aggregate welfare in RESE 3. The latter finding is very important, as it may affect regulatory policies with respect to the level of competition. For RESE 4, the increasing level of competition is always detrimental for the local economy.

Advantages of the presented model include its analytical tractability and natural connections to established oligopoly results. Possible extensions cover a wide range of problems in the study of competition in the presence of strategic consumers, for example: (1) analysis of policy decisions, including taxes and subsidies for the manufacturer, retailers, and/or consumers; (2) study of supply-chain coordination; (3) analysis of competition when advertisement and inventory decisions are decoupled; and (4) study of price-matching contracts as a tool to counteract strategic consumer behavior.

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Appendix A. Main text supplement

A.1. **Proof of Lemma 1 (first-period demand).** Recall that, for a first-period buyer with valuation $v \geq p_1$, the surpluses of buying in the first period and that of waiting are, respectively, $\sigma_1 = v - p_1$ and $\sigma_2 = \rho\bar{\alpha}(\beta v - \bar{p}_2)^+$. Condition $\sigma_1 \geq 0$ is equivalent to $v \geq p_1$. Condition $\sigma_1 \geq \sigma_2$ is equivalent to $v - p_1 \geq \rho\bar{\alpha}(\beta v - \bar{p}_2) \;\Leftrightarrow\; v \geq \frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta}$. Combining these inequalities, we obtain the stated expression for $v^{\min}$. Since all consumers with $v \geq v^{\min}$ would buy in the first period, the total demand is $D = 1 - v^{\min}$.

A.2. **Proof of Lemma 2 ($a(y) = y^\gamma$).** By the conditions of the lemma, equality $a(ky^i)/\sum_{j=1}^n a(ky^j) = a(y^i)/\sum_{j=1}^n a(y^j)$ holds for any $y^j > 0, j \in I$, and $k > 0$. Therefore, it holds for $y^i = y > 0$, $y^j = 1, j \neq i$, and $y^l = y, y^j = 1, l \neq i, j \neq l$. Namely,

$$\frac{a(ky)}{a(ky) + (n-1)a(k)} = \frac{a(y)}{a(y) + (n-1)a(1)} \text{ and } \frac{a(k)}{a(ky) + (n-1)a(k)} = \frac{a(1)}{a(y) + (n-1)a(1)},$$

which implies $a(ky)/a(y) = a(k)/a(1) \Leftrightarrow a(ky) = a(k)a(y)/a(1)$. Denoting $\tilde{k} \triangleq \ln k, \tilde{y} \triangleq \ln y$, and $\tilde{a}(z) \triangleq \ln(a(e^z)) - \ln a(1)$, the logarithm of the last equation is $\ln a[\exp(\tilde{k} + \tilde{y})] = \ln a[\exp(\tilde{k})] + \ln a[\exp(\tilde{y})] - \ln a(1)$ or $\tilde{a}(\tilde{k}+\tilde{y}) + \ln a(1) = \tilde{a}(\tilde{k}) + \ln a(1) + \tilde{a}(\tilde{y}) \Leftrightarrow \tilde{a}(\tilde{k}+\tilde{y}) = \tilde{a}(\tilde{k}) + \tilde{a}(\tilde{y})$. Since any continuous additive function of one variable is linear with zero intercept, we have $\tilde{a}(\tilde{y}) = \gamma\tilde{y}$ (note that by the definition of $\tilde{a}(z)$, $\tilde{a}(0)$ is, indeed, zero), which implies $a(y) = a(1)\exp[\gamma \ln y] = a(1)y^\gamma$.

A.3. **Proof of Lemma 3 (no stockouts).** *Part (I).* The existence of the unique positive solution $\breve{y}^i$ to equation $\breve{y}^i = d^i(\breve{y}^i, \mathbf{y}^{-i})$ is established in §B.3.1. Moreover, the reasoning implies that $y^i \leq d^i(y^i, \mathbf{y}^{-i})$ for any $y^i \leq \breve{y}^i$. Thus, if $y^i \leq d^i(y^i, \mathbf{y}^{-i})$, retailer i sells only in the first period and, by (1), its profit function is $r^i = (p_1 - c)y^i$, which is increasing in y^i for any $y^i \in [0, \breve{y}^i]$. Therefore, inventory y^i of a profit-maximizing retailer is never less than the first-period demand, i.e., $y^i \geq \breve{y}^i$ holds.

Part (II). Claim (a) is straightforward when $y^i \geq \breve{y}^i$ holds and when retailer i sets the inventory above the symmetric level $\bar{Y}/n$. In that case, the first-period demand of other retailers decreases compared to $\bar{Y}/n$ (constant for $\gamma = 0$), which cannot lead to stockouts.

Stockouts may potentially arise only when retailer i sets the inventory below $\bar{Y}/n$, increasing the first-period market share of other retailers above the symmetric level. In this case, we show that the first-period demand d^j of any retailer $j \neq i$ is not greater than inventory $y^j = \bar{Y}/n$. Suppose that $y^i = \breve{y}^i$, which is the minimum possible inventory of a retailer rationally responding to a symmetric profile, and that $\breve{y}^i \leq \frac{\bar{Y}}{n}$. Then $d^j = \frac{(1-v^{\min})\left(\bar{Y}/n\right)^\gamma}{(n-1)\left(\bar{Y}/n\right)^\gamma + (\breve{y}^i)^\gamma}$, and the no-stockout condition $d^j \leq \bar{Y}/n$ can be written as

$$(1 - v^{\min})\left(\bar{Y}/n\right)^{\gamma-1} \leq (n-1)\left(\bar{Y}/n\right)^\gamma + \left(\breve{y}^i\right)^\gamma.$$

Since $\breve{y}^i = d^i = \frac{(1-v^{\min})\left(\breve{y}^i\right)^\gamma}{(n-1)\left(\bar{Y}/n\right)^\gamma + (\breve{y}^i)^\gamma}$, the RHS of the last inequality equals $(1 - v^{\min})\left(\breve{y}^i\right)^{\gamma-1}$. Then $d^j \leq \bar{Y}/n$ trivially holds for $\gamma = 1$ and, for $\gamma \in [0, 1)$, is equivalent to $\breve{y}^i \leq \bar{Y}/n$ (since $\gamma < 1$), which holds by the assumption.

Part (II) (b) follows from part (II) (a).

Part (II) (c). The second-period total inventory is $Y - Q = Y - (1 - v^{\min})$. Suppose this number is positive. The number of consumers remaining in the market is $v^{\min}$, and the upper bound of their second-period valuations is $\beta v^{\min}$. Therefore, as long as $p_2 \geq s$, the market clearing condition for the second period takes the form $v^{\min}\frac{\beta v^{\min} - p_2}{\beta v^{\min}} = Y - 1 + v^{\min}$, or, equivalently, $p_2 = \beta(1 - Y)$. If $\beta(1 - Y) < s$, bargain-hunters absorb any excess supply at price s. Combining these two cases, we get the second-period price in the form $p_2 = s \vee [\beta(1 - Y)]$, which is continuous in y^i, $i \in I$.

A.4. **Profit function, its properties and inventory decisions for** $\gamma = 1$**.** By Lemma 3, the first-period total sales are $Q = 1 - v^{\min}$ and retailer i sales are $q^i = d^i(y^i, \mathbf{y}^{-i})$, which, for $\gamma = 1$, is $y^i \frac{Q}{Y}$. The second-period sales of retailer i are equal to its second-period inventory $y^i \left(1 - \frac{Q}{Y}\right)$. Then the general expression for retailer i profit, using (1) and (3), takes the form

$$r^i = -cy^i + p_1 \frac{y^i}{Y}(1 - v^{\min}) + \{s \vee [\beta(1-Y)]\} \left\{ y^i - \frac{y^i}{Y}(1 - v^{\min}) \right\}. \tag{8}$$

While this expression is continuous in all parameters and inventory y^i, it is generally not globally differentiable. Next, we consider all possible subintervals in terms of y^i. Each subinterval results in a differentiable expression for the profit function and a qualitatively distinct market outcome.

A.4.1. *No sales in the second period.* Formula (1) for profit becomes $r^i = (p_1 - c)y^i$, which yields a unique profit-maximizing inventory $y^i = \left(1 - v^{\min} - Y^{-i}\right)^+$, where $Y^{-i} = \sum_{j \neq i} y^j$, and the maximum profit $r^i = (p_1 - c)\left(1 - v^{\min} - Y^{-i}\right)^+$, leading to the following lemma:

Lemma 25. *For given model inputs and consumer expectations, retailer rationality implies that the effective domain of the inventory decision is* $y^i \geq (1 - v^{\min} - Y^{-i})^+$ *and* $(p_1 - c)(1 - v^{\min} - Y^{-i})^+$ *is the lower bound for the optimal profit.*

A.4.2. *second-period sales with* $p_2 > s$*.* If $v^{\min} > 1 - Y$ (or $y^i > 1 - v^{\min} - Y^{-i}$), there are sales in the second period. If this condition is combined with$0 < y^i < 1 - \frac{s}{\beta} - Y^{-i}$, then $p_2 > s$ and the profit is given by

$$\begin{aligned} r^i &= -cy^i + p_1 \frac{y^i}{Y}\left(1 - v^{\min}\right) + \beta(1-Y) y^i \left(1 - \frac{1 - v^{\min}}{Y}\right) \\ &= y^i \left[\beta(1-Y) - c + (p_1 - \beta(1-Y)) \frac{1 - v^{\min}}{Y}\right] && (9) \\ &= y^i \left[\beta(1-Y) - c + \beta\left(1 - v^{\min}\right) + \frac{(p_1 - \beta)(1 - v^{\min})}{Y}\right] && (10) \end{aligned}$$

with the derivative

$$\begin{aligned} \frac{\partial r^i}{\partial y^i} &= \beta(1-Y) - c + [p_1 - \beta(1-Y)] \frac{1 - v^{\min}}{Y} && (11) \\ &\quad + y^i \left[\beta\left(-1 + \frac{1 - v^{\min}}{Y}\right) - (p_1 - \beta(1-Y)) \frac{1 - v^{\min}}{Y^2}\right], \end{aligned}$$

which, using equations $Y = y^i + Y^{-i}$ and (10), can be rewritten as

$$\frac{\partial r^i}{\partial y^i} = \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta y^i + (p_1 - \beta)(1 - v^{\min}) \frac{Y^{-i}}{Y^2}. \tag{12}$$

The second derivative is

$$\frac{\partial^2 r^i}{\partial \left(y^i\right)^2} = -2\left[\beta + (p_1 - \beta)(1 - v^{\min}) \frac{Y^{-i}}{Y^3}\right]. \tag{13}$$

A.4.3. *second-period sales with* $p_2 = s$*.* This case is possible only under oligopoly ($Y^{-i} > 0$) because, for a monopolist, any price $p_2 \leq c$ is not rational. If there are sales in the second period and

$y^i \geq \left(1 - \frac{s}{\beta} - Y^{-i}\right)^+$ (or $Y \geq 1 - \frac{s}{\beta}$), then $p_2 = s$ and (8) becomes

$$\begin{aligned} r^i &= -cy^i + p_1 \frac{y^i}{Y}\left(1 - v^{\min}\right) + sy^i\left(1 - \frac{1 - v^{\min}}{Y}\right) \\ &= -(c-s)y^i + \frac{y^i}{Y}\left(p_1 - s\right)\left(1 - v^{\min}\right) \end{aligned} \tag{14}$$

with the derivative

$$\begin{aligned} \frac{\partial r^i}{\partial y^i} &= -(c-s) + \frac{Y - y^i}{Y^2}\left(p_1 - s\right)\left(1 - v^{\min}\right) \\ &= -(c-s) + \frac{Y^{-i}}{Y^2}\left(p_1 - s\right)\left(1 - v^{\min}\right), \end{aligned} \tag{15}$$

which is monotonically strictly decreasing in y^i when $v^{\min} < 1$.

A.4.4. *Properties of the profit function.* The following lemma provides the properties of retailer i profit r^i, using the continuity of r^i in y^i.

Lemma 26. *The profit function r^i is such that*

(1) *If $1 - \frac{s}{\beta} - Y^{-i} > 0$, then*

(1.1) $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} - 0} < \left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0}$;

(1.2) $r^i\left(1 - \frac{s}{\beta} - Y^{-i}\right) \leq 0$ *if and only if*

$$\frac{\left(p_1 - s\right)\left(1 - v^{\min}\right)}{\left(1 - \frac{s}{\beta}\right)(c - s)} \leq 1; \tag{16}$$

(1.3) *r^i is pseudoconcave in y^i and strictly concave if $p_1 \geq \beta v^{\min}$ on the interval $(1 - v^{\min} - Y^{-i})^+ \leq y^i \leq 1 - \frac{s}{\beta} - Y^{-i}$;*

(1.4) *r^i is strictly concave on the interval leading to $p_2 = s$, i.e. $y^i \geq 1 - \frac{s}{\beta} - Y^{-i}$; and*

(1.5) *r^i is pseudoconcave on the interval $y^i \geq (1 - v^{\min} - Y^{-i})^+$ if either*

$$\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0} \leq 0 \ \text{or} \ \left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} - 0} \geq 0.$$

(2) *If $1 - \frac{s}{\beta} - Y^{-i} \leq 0$, r^i is strictly concave on its entire domain $y^i \geq 0$.*

A.5. **Proof of Lemma 4 ($p_2 < \beta p_1$).** From Lemma 1, we have $v^{\min} = p_1$ if and only if $\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1 - \bar{\alpha}\rho\beta} \leq p_1$, which can be equivalently rewritten as $\bar{\alpha}\rho\beta p_1 \leq \bar{\alpha}\rho\bar{p}_2$. Within feasible parameter values, the later holds if and only if either $\bar{\alpha} = 0$, $\rho = 0$, or $\beta p_1 \leq \bar{p}_2$. By Lemma (25), $Y \geq 1 - v^{\min}$. Thus, either of $\rho = 0$, $\bar{\alpha} = 0$ or $\beta p_1 \leq \bar{p}_2$ implies that $Y \geq 1 - p_1$. Moreover, $Y = 1 - p_1$ means there are no sales in the second period, while $Y > 1 - p_1$ means that these sales occur at price $p_2 < \beta p_1$ according to the market clearing condition (3).

Part (1): We conclude that $\bar{p}_2 \geq \beta p_1$ would never be rational and, in any rational expectations equilibrium, we must have $p_2 < \beta p_1$.

Part (2): By the above reasoning, $\bar{\alpha} = 0$ implies $v^{\min} = p_1$ and $Y \geq 1 - p_1$. However, $Y > 1 - p_1$ in combination with $v^{\min} = p_1$ means that there *are* sales in the second period and $\bar{\alpha} = 0$ is not rational.

If $\bar{\alpha} = 1$, by part (1) and condition (3), we have $\beta(1 - Y) \leq s \vee [\beta(1 - Y)] = p_2 < \beta p_1$. Thus, $Y > 1 - p_1$ in any rational expectations equilibrium with $\bar{\alpha} = 1$.

Part (3): Since in any rational expectations equilibrium, $\bar{p}_2 = p_2$ and $\bar{\alpha} = 1$ if there are sales in the second period, Lemma 1 implies that, if there are sales in both periods, $v^{\min} < 1$, which, using (3), is equivalent to $p_1 - \rho\beta(1-Y) < 1 - \rho\beta$ or $\rho\beta Y < 1 - p_1$. If there are sales only in the second period, $p_1 - \rho\beta(1-Y) \geq 1 - \rho\beta$ or $\rho\beta Y \geq 1 - p_1$; $p_2 \geq c$ since, in this case, $r^i = (p_2 - c)y^i$, and retailers are profit-maximizing.

Part (4): Since $\bar{p}_2 \geq \beta p_1$ would never be rational, $v^{\min} = p_1$ can occur in a rational expectations equilibrium if and only if $\bar{\alpha} = 0$ or $\rho = 0$.

A.6. **Proof of Theorem 5 (RESE with $p_2^* > s$).** The theorem exhaustively considers all possible market outcomes without salvaging: no sales in the first period (RESE 1), no sales in the second period (RESE 2), and sales in both periods (RESE 3). Logically, these three outcomes are mutually exclusive but it is not obvious *a priori* that they cannot exist under the same model inputs. In the course of the proof we establish that these outcomes also do not overlap in the sense of their necessary and sufficient conditions on model parameters. Parts (1) and (2) of the RESE definition (§3.2) rely on the notion of a symmetric equilibrium for given consumer expectations. The structure of such an equilibrium is one of the major sources of necessary and sufficient conditions. Another source is the rationality of consumer expectations. We first classify the outcomes by the presence of second-period sales.

No second-period sales: RESE 2. The absence of second-period sales along with retailer rationality, by Lemma 25, means that the best response in a symmetric equilibrium occurs with $Y = 1 - v^{\min}$. Consumer rationality in this case demands that $\bar{\alpha} = 0$ and $v^{\min} = p_1$ implying that the candidate RESE is described by $v^* = p_1, Y^* = 1 - v^*$, and, therefore, $\alpha^* = 0$ and $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$. This implies that $\frac{n-1}{n}Y^* = \frac{n-1}{n}(1-p_1) < 1 - p_1 < 1 - \frac{s}{\beta}$ and condition of part (1) of Lemma 26 is satisfied.

Since, by part (1.3) of Lemma 26, r^i is pseudoconcave on the interval $(1 - v^{\min} - Y^{-i})^+ \leq y^i < 1 - \frac{s}{\beta} - Y^{-i}$, the candidate RESE exists if and only if

(i) there is a local maximum of r^i at $y^i = 1 - v^* - \frac{n-1}{n}Y^* = \frac{Y^*}{n}$ and
(ii) the profit r^i at this maximum is greater than at a potential local maximum on the interval $y^i > 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^*$.

Condition (i) is equivalent to $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - v^* - \frac{n-1}{n}Y^* + 0} \leq 0$. Since $y^i = \frac{1}{n}(1-p_1)$, the last inequality, using (11), becomes $\beta v^* - c + p_1 - \beta v^* + \frac{1}{n}(1-p_1)\left[-(p_1 - \beta v^*)\frac{1}{1-v^*}\right] \leq 0$, which, after the substitution for $v^* = p_1$ and multiplication by n, takes the form $np_1 - p_1(1-\beta) \leq nc$ or $p_1 \leq \frac{nc}{\beta+n-1} = P_2$. We showed that this condition is necessary.

Condition (ii) is satisfied if r^i is nonincreasing for $y^i > 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^*$. Since r^i is concave on this interval by part (1.4) of Lemma 26, it is nonincreasing if $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^* + 0} \leq 0$. The latter, using (15), can be written as

$$-c + s + \frac{\frac{n-1}{n}(1-p_1)}{\left(1 - \frac{s}{\beta}\right)^2}(p_1 - s)(1-p_1) \leq 0 \quad \text{or} \quad \frac{n-1}{n}\frac{(p_1 - s)(1-p_1)^2}{(c-s)\left(1 - \frac{s}{\beta}\right)^2} \leq 1. \tag{17}$$

Since $p_1 > \frac{s}{\beta}$, we have $\frac{(1-p_1)^2}{\left(1 - s/\beta\right)^2} < 1$, and (17) is implied by $(n-1)(p_1 - s) \leq n(c-s)$. The latter holds because, by (already proved as necessary) condition $p_1 \leq P_2$, $n(c-s) \geq (n-1+\beta)p_1 - ns = (n-1)(p_1 - s) + \beta p_1 - s > (n-1)(p_1 - s)$. Therefore, condition $p_1 \leq P_2$ is necessary and sufficient for the existence of RESE 2.

There are second-period sales: RESE 1 or 3. When sales in the second period do occur, a symmetric equilibrium $Y = \hat{Y}$ is provided by $\hat{Y} > 1 - v^{\min}$, which is an internal maximum of the profit function for each retailer. For $p_2 > s$, the first-order optimality condition $\frac{\partial r^i}{\partial y^i} = 0$ is provided by setting (12) (see §A.4.2) to zero with substitutions $y^i = \frac{Y}{n}$ and $Y^{-i} = \frac{n-1}{n}Y$:

$$\begin{aligned}\beta\left(1-\frac{n-1}{n}Y\right)-c+\beta(1-v^{\min})-2\beta\frac{Y}{n}+(p_1-\beta)(1-v^{\min})\frac{n-1}{n}\frac{Y}{Y^2} &= 0\\ \text{or}\quad -\beta\frac{n+1}{n}Y-c+\beta(2-v^{\min})+(p_1-\beta)(1-v^{\min})\frac{n-1}{n}\frac{1}{Y} &= 0.\end{aligned}$$

Multiplication of the last equation by $-\frac{n}{\beta(n+1)}Y$ yields

$$Y^2 - Y\frac{n}{n+1}\left(2-v^{\min}-\frac{c}{\beta}\right)-\frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)(1-v^{\min})=0. \tag{18}$$

Equation (18) along with the relation between $v^{\min}$ and Y from Lemma 1 and inequality $Y > 1-p_1$ (from part (2) of Lemma 4) provide the necessary conditions for any equilibria with sales in the second period and $p_2 = \beta(1-Y) > s$.

It is convenient to analyze RESE existence in terms of $v^{\min}$ as a function of Y. For rational expectations $\bar{\alpha} = 1$ and $\bar{p}_2 = p_2 = \beta(1-Y)$, denote the mapping from Y to $v^{\min}$ resulting from Lemma 1 as function

$$v_1^{\min}(Y) \triangleq p_1 \vee \left(\frac{p_1-\rho\beta(1-Y)}{1-\rho\beta}\wedge 1\right). \tag{19}$$

When $\rho > 0$, this function is increasing and piecewise linear with two breakpoints. It is straightforward to check that the first break-point occurs exactly at $Y = 1-p_1$ while the second at $Y = \frac{1-p_1}{\rho\beta}$. When $\rho = 0$, $v_1^{\min} \equiv p_1$.

Equation (18) yields another mapping from Y to $v^{\min}$:

$$v_2^{\min}(Y) \triangleq 1-\frac{Y^2-Y\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)}{Y\frac{n}{n+1}+\frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)}. \tag{20}$$

When $p_1 \neq \beta$ and $n > 1$, this function is a hyperbola with a vertical asymptote $Y = \frac{n-1}{n}\left(1-\frac{p_1}{\beta}\right)$ and an asymptote with a negative slope $-\frac{n+1}{n}$. When $Y = 0$ or $Y = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$, $v_2^{\min}(Y) = 1$. Implicit differentiation of (18) yields

$$2Y-\frac{n}{n+1}\left(2-v_2^{\min}-\frac{c}{\beta}\right)+Y\frac{n}{n+1}\frac{\partial v_2^{\min}}{\partial Y}+\frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)\frac{\partial v_2^{\min}}{\partial Y}=0$$

resulting in $(n-1)(p_1-\beta)\left.\frac{\partial v_2^{\min}}{\partial Y}\right|_{Y=0} = n(\beta-c)$.

When $p_1 > \beta$ and $n > 1$, the vertical asymptote is located to the left of $Y = 0$ implying that points $(0,1)$ and $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right],1\right)$ in the $(Y, v^{\min})$-plane belong to the same branch of the hyperbola. In this case, $\left.\frac{\partial v_2^{\min}}{\partial Y}\right|_{Y=0} > 0$ and it must be true that $\frac{\partial v_2^{\min}}{\partial Y} < 0$ for all $Y \geq \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$. Relevant equilibrium candidates can only be on the downward-sloping segment of $v_2^{\min}(Y)$ to the right of $Y = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$ and in the range $p_1 \leq v^{\min} \leq 1$. This case is depicted in Figure 13 (a), where a solid curve is $v_2^{\min}(Y)$, dotted lines represent its asymptotes, and the dashed lines indicate the lower and upper bounds on the relevant range of $v^{\min}$.

When $p_1 < \beta$ and $n > 1$, the vertical asymptote is located to the right of $Y = 0$ implying that points $(0,1)$ and $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right],1\right)$ belong to different branches of the hyperbola. We have $\frac{\partial v_2^{\min}}{\partial Y} < 0$ for all Y, and the entire left branch is irrelevant since the vertical asymptote is to the left of $Y = 1-p_1$. Indeed, $\frac{n-1}{n}\left(1-\frac{p_1}{\beta}\right) < 1-p_1$ is equivalent to $np_1-(n-1)\frac{p_1}{\beta} < 1$ which always

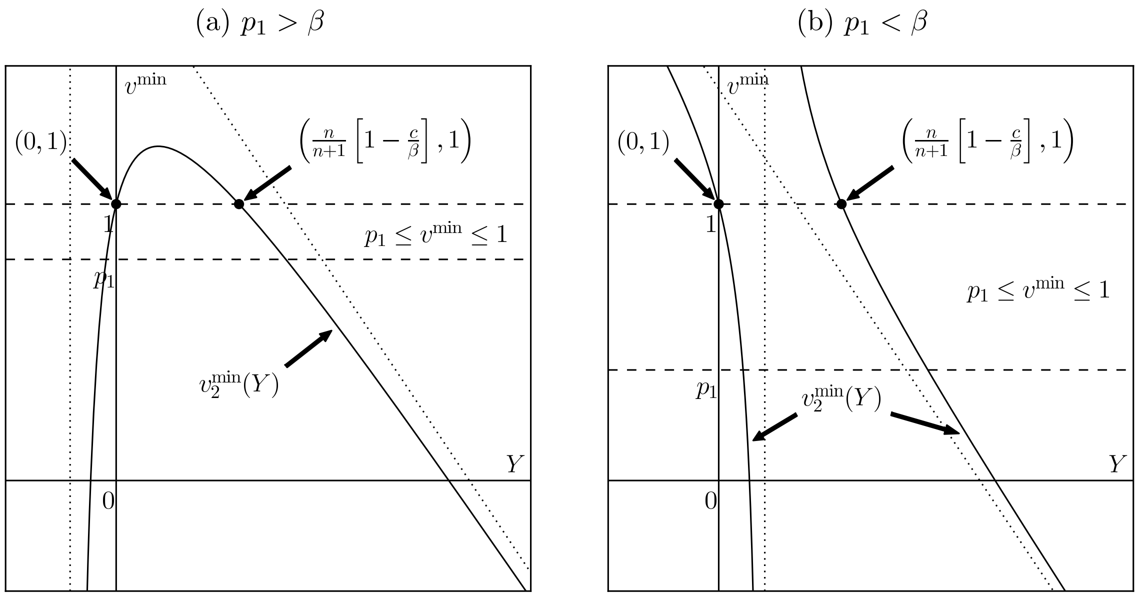


FIGURE 13. Possible appearance of $v_2^{\min}(Y)$ and the relevant range of $v^{\min}$

holds for $p_1 < \beta$. All possible equilibrium candidates are again on the downward-sloping segment of $v_2^{\min}(Y)$ to the right of $Y = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$ and in the range $p_1 \leq v^{\min} \leq 1$. This case is illustrated in Figure 13 (b).

When $p_1 = \beta$ or $n = 1$, the relevant part of $v_2^{\min}(Y)$ is decreasing linear: $v_2^{\min}(Y) = 2-\frac{c}{\beta}-\frac{n+1}{n}Y$, which also satisfies $v_2^{\min}\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right]\right) = 1$. Thus, regardless of n and the relation between p_1 and β, the geometric structure of potential equilibrium candidates is essentially the same.

RESE 1: There are no sales in the first period at a RESE if and only if $v^* = 1$. The geometric structure described above implies that such an equilibrium can be realized only if $v_1^{\min}(Y)$ intersects with $v_2^{\min}(Y)$ at a point corresponding to $Y^* = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$, i.e., $v_1^{\min}(Y^*) = 1$ or $p_1 - \rho\beta\left[1-\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right] \geq 1-\rho\beta$, which is equivalent to $p_1 \geq P_1 = 1-\frac{n}{n+1}\rho(\beta-c)$. This necessary condition is also sufficient for RESE 1. Indeed, given that $v_1^{\min}(Y^*) = 1$, the equilibrium values are in the form of RESE 1, $p_2^* = \beta\left[1-\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right] = \frac{nc+\beta}{n+1} > c > s$ and $y^i = \frac{Y^*}{n}$ indeed delivers the best response of retailer i since $Y^* = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right) < 1-\frac{c}{\beta} < 1-\frac{s}{\beta}$ and $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-\frac{n-1}{n}Y^*+0} = -c+s < 0$ implying, by part (1.5) of Lemma 26, that r^i is pseudoconcave.

The description of RESE 1 is completed by substituting p_2^*, Y^* and v^* into (10):

$$r^* = \frac{Y^*}{n}\left[\frac{\beta+nc}{n+1}-c\right] = \frac{1}{n+1}\left(1-\frac{c}{\beta}\right)\left[\frac{\beta+nc}{n+1}-c\right] = \frac{(\beta-c)}{(n+1)\beta}\frac{\beta+nc-nc-c}{n+1} = \frac{(\beta-c)^2}{(n+1)^2\beta}.$$

The p_1-ranges in RESE 1 and 2 do not overlap because the minimal lower bound for p_1 in RESE 1, which corresponds to $n \to \infty$, exceeds the maximal upper bound in RESE 2 (at $n = 1$): $1-\rho(\beta-c) > \frac{c}{\beta} \Leftrightarrow \beta(1-\rho\beta) > c(1-\rho\beta)$.

RESE 3: This case is characterized by $Y^* > 1-v^*$ (there are sales in the second period) and $p_1 \leq v^* < 1$ (there are sales in the first period) with $v^* = p_1$ only if $\rho = 0$. Translating this into the geometric structure described above, necessary conditions for RESE 3 are $v_1^{\min}\left(\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right) < 1$

and $v_2^{\min}(1-p_1) > p_1$. The first condition is equivalent to the negation of $p_1 \geq P_1$, i.e., the strict upper limit of p_1-range for RESE 3. The second condition ensures that $v_2^{\min}(Y)$ intersects $v_1^{\min}(Y)$ for $Y > 1 - p_1$ and is equivalent to

$$1 - \frac{(1-p_1)^2 - (1-p_1)\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)}{(1-p_1)\frac{n}{n+1} + \frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)} > p_1,$$

and, since $(1-p_1)\frac{n}{n+1} + \frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right) = \frac{1-p_1}{n+1} + \frac{(n-1)p_1(1-\beta)}{(n+1)\beta} > 0$, to

$$(1-p_1)\left[(1-p_1)\frac{n}{n+1} + \frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right) - (1-p_1) + \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right] > 0.$$

Collecting like terms inside $[\cdot]$ yields $(n-1+\beta)p_1 > nc$ which is the negation of the necessary and sufficient condition $p_1 \leq P_2$ of RESE 2, i.e., the strict lower limit of p_1-range for RESE 3.

Given that necessary condition $P_2 < p_1 < P_1$ holds and there are sales in both periods, the candidate point for the equilibrium, by Lemma 1, satisfies

$$v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta} \tag{21}$$

and $v^* \in [p_1, 1)$. Substitution for $v^{\min} = v^*$ into (18) results in the following equation for Y^*:

$$Y^2 - Y\frac{n}{n+1}\left(2 - \frac{p_1 - \rho\beta(1-Y)}{1-\rho\beta} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)\left(1 - \frac{p_1-\rho\beta(1-Y)}{1-\rho\beta}\right) = 0,$$

which, after collecting the terms with Y, becomes

$$\begin{aligned}&Y^2\left(1 + \frac{n}{n+1}\frac{\rho\beta}{1-\rho\beta}\right) \\ &-Y\left[\frac{n}{n+1}\left(2 - \frac{p_1-\rho\beta}{1-\rho\beta} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)\frac{\rho\beta}{1-\rho\beta}\right] - \frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)\left(1-\frac{p_1-\rho\beta}{1-\rho\beta}\right) = 0.\end{aligned} \tag{22}$$

The coefficient in front of Y^2 is

$$1 + \frac{n}{n+1}\frac{\rho\beta}{1-\rho\beta} = \frac{n+1-\rho\beta}{(n+1)(1-\rho\beta)},$$

and the coefficient in front of Y is

$$-\frac{1}{(n+1)(1-\rho\beta)}\left[n\left(2 - 2\rho\beta - p_1 + \rho\beta - \frac{c}{\beta}(1-\rho\beta)\right) - (n-1)\left(\frac{p_1}{\beta}-1\right)\rho\beta\right],$$

where the first term in the bracket $[\ldots]$ is

$$n\Big(2 - \rho\beta - p_1 - \frac{c}{\beta}(1-\rho\beta)\Big) = n(1-\rho\beta)\Big(1-\frac{c}{\beta}\Big) + n(1-p_1).$$

Then multiplication of (22) by $\frac{\beta(n+1)(1-\rho\beta)}{\beta(n+1-\rho\beta)}$ results in

$$Y^2 - \frac{(\beta-c)n(1-\rho\beta) + \beta(1-p_1)n - (p_1-\beta)\rho\beta(n-1)}{\beta(n+1-\rho\beta)}Y - \frac{(p_1-\beta)(1-p_1)(n-1)}{\beta(n+1-\rho\beta)} = 0. \tag{23}$$

By geometric structure under condition $P_2 < p_1 < P_1$, the larger root of this equation does belong to the region $Y > 1 - p_1$ and the smaller root is irrelevant.

The conditions for RESE 3 will become necessary and sufficient if (23), (21), and $P_2 < p_1 < P_1$ are complemented with the conditions guaranteeing that the larger root Y^* of (23) is such that $Y^* < 1 - \frac{s}{\beta}$ (implying $p_2^* > s$ and included as a condition of the theorem) and either

(a) the profit r^i of retailer i deviating from this RESE so that $p_2 = s$ (the total inventory is greater than $1-\frac{s}{\beta}$) has no maximum for $Y > 1 - \frac{s}{\beta}$, or

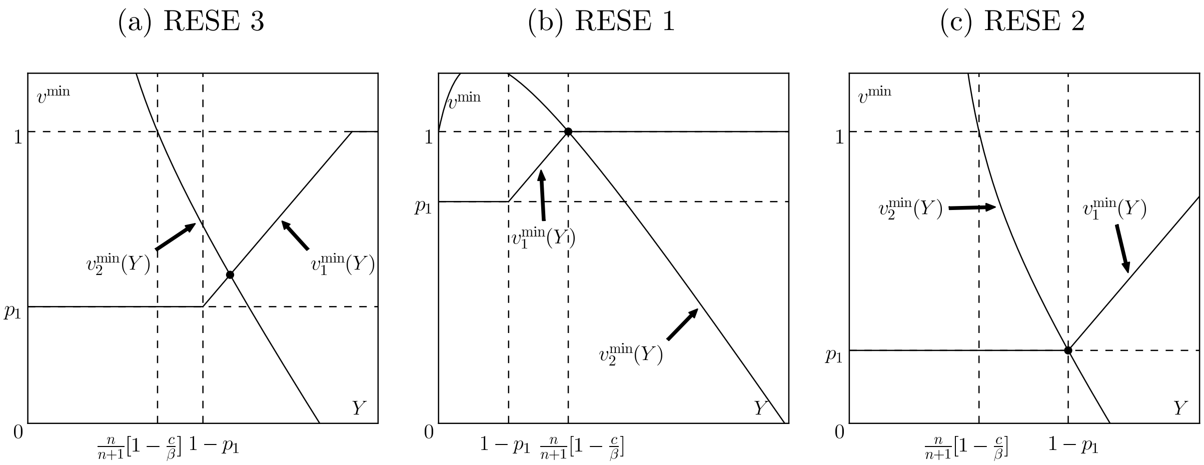


FIGURE 14. Changes in equilibrium structure from RESE 3 to RESE 1 and 2

(b) if $\tilde{r}^i = \max r^i$ exists for $Y > 1 - \frac{s}{\beta}$, then the inequality $\tilde{r}^i < r^*$ holds.

Since, by part (1.4) of Lemma 26, r^i is concave for $y^i \geq 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^*$ (or, equivalently, $Y \geq 1 - \frac{s}{\beta}$), r^i is nonincreasing for $y^i \geq 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^*$ if and only if $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^* + 0} \leq 0$. Thus, the latter condition is equivalent to (a). Using (15) with $v^{\min} = v^*$, $Y^{-i} = \frac{n-1}{n}Y^*$ and $Y = 1 - \frac{s}{\beta}$ this condition can be written as $-c + s + \frac{n-1}{n}\frac{Y^*}{\left(1-s/\beta\right)^2}(p_1 - s)(1 - v^*) \leq 0$, yielding condition (a).

If $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0} > 0$, then, since $\frac{\partial r^i}{\partial y^i}$ becomes negative for sufficiently large Y by (15), $\tilde{r}^i = \max r^i$ exists for $Y > 1 - \frac{s}{\beta}$. Therefore, the RESE exists in this case if $r^* \geq \tilde{r}^i$ (condition (b)).

In order to provide the expression for $\tilde{r}^i$, denote the maximized deviator's inventory decision by $\tilde{y}^i \triangleq \arg\max r^i > \frac{1}{n}Y^*$. As a result of this deviation, the total inventory becomes $\tilde{Y} = \tilde{y}^i + \frac{n-1}{n}Y^*$. Then, using (15) with $v^{\min} = v^*$, we obtain the following equation in $\tilde{Y}$: $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \tilde{y}^i} = 0 = -(c - s) + \frac{n-1}{n}\frac{Y^*}{\tilde{Y}^2}(p_1 - s)(1 - v^*)$, which yields $\tilde{Y} = \sqrt{\frac{n-1}{n}\frac{Y^*(p_1-s)(1-v^*)}{c-s}}$. Substitution of $Y = \tilde{Y}$ and $y^i = \tilde{Y} - \frac{n-1}{n}Y^*$ into the equation for profit (14), results in

$$\tilde{r}^i = \left\{\sqrt{\frac{n-1}{n}\frac{Y^*(p_1 - s)(1 - v^*)}{c - s}} - \frac{n-1}{n}Y^*\right\} \times \left\{-(c - s) + \frac{(p_1 - s)(1 - v^*)}{\sqrt{\frac{n-1}{n}\frac{Y^*(p_1-s)(1-v^*)}{c-s}}}\right\},$$

which, after factoring out $\frac{n-1}{n}Y^*$ from the first curly bracket and $c - s$ from the second one, becomes

$$\tilde{r}^i = \frac{n-1}{n}Y^*(c - s)\left\{\sqrt{\frac{n}{(n-1)}\frac{(p_1 - s)(1 - v^*)}{(c - s)Y^*}} - 1\right\}^2.$$

This expression can be also written as follows: $\tilde{r}^i = \left\{\sqrt{(p_1 - s)(1 - v^*)} - \sqrt{\frac{n-1}{n}Y^*(c - s)}\right\}^2$, which coincides with the expression for $\tilde{r}^i$ in the theorem statement.

Expression for r^* follows immediately from (1) and Lemma 1.

We complete the proof of the main part of the theorem by a simple observation that equilibrium characteristics are continuous on the boundaries between RESE 1 and 3 as well as RESE 2 and 3. Figure 14, in its subplot (a), depicts a typical configuration of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ when RESE 3 exists, while subplots (b) and (c) depict this configuration at the points of change to RESE 1 and 2, respectively.

RESE 3 continuously changes into RESE 1 as the intersection point of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ representing RESE 3 moves toward the point $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right],1\right)$ on $v_2^{\min}(Y)$ representing RESE 1. The latter point is to the left of all possible candidates for RESE 3 located on $v_2^{\min}(Y)$ implying that, in RESE 3, $Y^* \geq \frac{n}{n+1}\left[1-\frac{c}{\beta}\right]$. Similarly, RESE 3 continuously changes into RESE 2 as the intersection point of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ moves toward $v_1^{\min}(Y)$'s break-point $(1-p_1,p_1)$ (representing RESE 2). The continuity of r^* follows from the continuity of the expression for r^i, given by (8), in all the parameters and continuity of v^* and Y^* (using $y^i=\frac{1}{n}Y^*$).

It remains to examine the convergence under deviations from rational expectations. The geometric structure of candidates for RESE 3 and 1 implies that the areas of inputs where these RESE exist do not intersect. Suppose that (i) $\bar{\alpha}=\alpha^*=1$, i.e., one and only one of RESE 3 or 1 can be realized for given inputs; and (ii) consumer expectations of the second-period price deviate from rational ones with $\bar{p}_2^0<p_2^*$, and the game is repeated with the same inputs. As shown above, $\frac{\partial v_2^{\min}}{\partial Y}<0$, implying $\frac{\partial BR}{\partial v^{\min}}<0$, where $BR=BR[v^{\min}(\bar{p}_2)]$ is a symmetric best response, given $\bar{p}_2$. By Lemma 1, $\frac{\partial v^{\min}}{\partial \bar{p}_2}\leq 0$, which leads to $\frac{\partial BR}{\partial \bar{p}_2}\geq 0$. Then $\bar{p}_2^0<p_2^* \Rightarrow BR(\bar{p}_2^0)\leq BR(p_2^*)=Y^*<1-\frac{s}{\beta}$, implying that the realized price $p_2^0=\beta[1-BR(\bar{p}_2^0)]\geq p_2^*>s$. Moreover, for any moment of time $t\in[0,\infty)$, the realized (p_2^t) and expected ($\bar{p}_2^t$) second-period prices are such that $p_2^t\geq p_2^*>\bar{p}_2^t$ if consumer expectations follow a linear adjustment process $\bar{p}_2^{t+1}=\mu p_2^t+(1-\mu)\bar{p}_2^t$ with a sufficiently small μ. This process, using (3), can be written as $\bar{p}_2^{t+1}=\mu\beta(1-Y^t)+(1-\mu)\bar{p}_2^t$, where $Y^t=BR(\bar{p}_2^t)$. Then $\left|p_2^*-\bar{p}_2^{t+1}\right|=\left|p_2^*-\mu\left[\beta(1-BR(\bar{p}_2^t))+p_2^*-p_2^*\right]-(1-\mu)\bar{p}_2^t\right|=\left|(1-\mu)(p_2^*-\bar{p}_2^t)-\mu\beta(Y^*-BR(\bar{p}_2^t))\right|$. By the mean value theorem, there exists $\tilde{p}_2\in(p_2^*,\bar{p}_2^t)$ such that $Y^*-BR(\bar{p}_2^t)=\frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2}(p_2^*-\bar{p}_2^t)$. Then, if $\mu\beta\frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2}\leq 2(1-\mu)\Leftrightarrow \mu\leq 2/(2+\beta\frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2})<1$, we have $\left|p_2^*-\bar{p}_2^{t+1}\right|\leq(1-\mu)\left|p_2^*-\bar{p}_2^t\right|=(1-\mu)^t\left|p_2^*-\bar{p}_2^0\right|$, which goes to zero with $t\to\infty$ for any $\bar{p}_2^0<p_2^*$. Since $\frac{\partial v^{\min}}{\partial \bar{p}_2}$ is restricted for any $\rho\in[0,1)$ and, by (20), $\frac{\partial BR}{\partial v^{\min}}$ is restricted in the relevant region for Y, there exists a sufficiently small μ such that $\mu<2/(2+\beta\frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2})$ leading to the convergence of the adjustment process to p_2^*.

The adjustment process can be specified using inequalities $\frac{\partial v^{\min}}{\partial \bar{p}_2}\leq 0$ and $\bar{p}_2^0<p_2^*$, which imply $v^{\min}(\bar{p}_2^0)\geq v^*=v^{\min}(p_2^*)$. This property leads to three cases. (a) $v^*<1$ and $v^{\min}(\bar{p}_2^0)<1$, which corresponds to the adjustment process above; (b) $v^*=1$ (RESE 1 is realized at $\bar{p}_2=p_2^*$). In this case, all $\bar{p}_2^t$ are such that $v^{\min}(\bar{p}_2^t)=1$, i.e., retailers' decisions do not depend on $\bar{p}_2^t$ and the adjustment becomes $\bar{p}_2^{t+1}=\mu p_2^*+(1-\mu)\bar{p}_2^t$. Then $\left|p_2^*-\bar{p}_2^{t+1}\right|\leq|1-\mu|^t\left|p_2^*-\bar{p}_2^0\right|$, which converges to p_2^* for any $\mu\in(0,2)$ and $\bar{p}_2^0<p_2^*$. (c) $v^*<1$ and $v^{\min}(\bar{p}_2^0)=1$. In this case, the initial adjustment steps are $\bar{p}_2^{t+1}=\mu\beta(1-BR|_{v^{\min}=1})+(1-\mu)\bar{p}_2^t$. Since $\frac{\partial BR}{\partial v^{\min}}<0, \bar{p}_2^t$ in this process increases faster than for $BR|_{v^{\min}<1}$. Then, by continuity and monotonicity of $v^{\min}$ in $\bar{p}_2$ and monotonicity of $\bar{p}_2^t$ in t, there exists such $\tilde{t}$ that the adjustment process switches to case (a) and follows it for any $t\geq\tilde{t}$.

A.7. **Proof of Corollary 6 (RESE 3, monopoly).** For $n=1$, sufficient condition (a) always holds and (18) is $\left[Y-1+\frac{1}{2}\left(v^{\min}+\frac{c}{\beta}\right)\right]Y=0$, yielding $Y^*=1-\frac{1}{2}\left(v^*+\frac{c}{\beta}\right)$. The equation for v^* is $v^*=\frac{p_1-\frac{1}{2}\rho\beta\left(v^*+\frac{c}{\beta}\right)}{1-\rho\beta}$ which is equivalent to $v^*(2-\rho\beta)=2p_1-\rho c$, resulting in the equilibrium v^*. Substitution of Y^* into (3) leads to the expression for p_2^*.

A.8. **Proof of Corollary 7 (RESE 3, $\rho=0$, second-period sales at loss).** For $\beta=1$ and $\rho=0$, p_1-range in RESE 3 is $c<p_1<1$. Thus, RESE 1 and 2 cannot be realized. By the proof of Theorem 5, $Y^{*,3}\leq 1-\frac{c}{\beta}$ is equivalent to $v^*\geq v_2^{\min}\left(1-\frac{c}{\beta}\right)$ since $v_2^{\min}(Y)$, given by (20), is decreasing in the relevant range of Y. Using $Y = 1-c$ and $\beta = 1$ in (20), we get

$$v_2^{\min}(1-c) = 1 - \frac{(1-c)^2 - (1-c)^2\frac{n}{n+1}}{(1-c)\frac{n}{n+1} + \frac{n-1}{n+1}(p_1 - 1)} = 1 - \frac{(1-c)^2}{n(p_1-c)+1-p_1}.$$

Thus, under conditions of the corollary, $p_2^* \geq c$ if and only if $v^* = p_1 \geq 1 - \frac{(1-c)^2}{n(p_1-c)+1-p_1}$. Rearranging this inequality we obtain $\frac{(1-c)^2}{n(p_1-c)+1-p_1} \geq 1 - p_1$, and solving for n we get

$$n \leq \frac{1}{p_1 - c}\left(\frac{(1-c)^2}{1-p_1} - (1-p_1)\right) = \frac{2-c-p_1}{1-p_1} = 2 + \frac{p_1-c}{1-p_1}.$$

A.9. **Proof of Corollary 8 (RESE 3, perfect competition).** If $P_2 < p_1 < P_1$ for $n \to \infty$, the limits v_∞^* and Y_∞^* of, respectively, $v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta}$ and Y^* defined by (23) exist. This follows from the geometric structure of curves $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ in the limiting case (see equations (19) and (20) and their analysis in the proof of Theorem 5).

The violation of condition (a) in the limit of $n \to \infty$ means that, for some $\epsilon > 0$,

$$\frac{(p_1-s)(1-v_\infty^*)Y_\infty^*}{(c-s)\left(1-\frac{s}{\beta}\right)^2} = 1 + 2\epsilon.$$

There exists N such that condition (a) is violated for any $n > N$ by at least ϵ:

$$\frac{n-1}{n}\frac{(p_1-s)(1-v^*)Y^*}{(c-s)\left(1-\frac{s}{\beta}\right)^2} \geq 1 + \epsilon. \tag{24}$$

There are two cases: $Y_\infty^* > 1 - \frac{s}{\beta}$ and $Y_\infty^* \leq 1 - \frac{s}{\beta}$. If $Y_\infty^* > 1 - \frac{s}{\beta}$, there exists N' such that $Y^* > 1 - \frac{s}{\beta}$ for any $n > N'$ implying, by condition (b) of Theorem 5, that RESE 3 does not exist for these n, and the claim of the corollary is established. If $Y_\infty^* \leq 1 - \frac{s}{\beta}$, there exist sufficiently small $\epsilon' > 0$ and N' such that

$$\frac{1-s/\beta}{Y^*} \geq \frac{1+\epsilon'}{\sqrt{1+\epsilon}} \tag{25}$$

for all $n > N'$. Inequality (24) is equivalent to $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1-\frac{s}{\beta}-Y^{-i}+0} > 0$ (see the analysis of RESE 3 in the proof of Theorem 5) and implies that there exists $\tilde{r}^i$, which is a unique maximum of r^i for $y^i > 1 - \frac{s}{\beta} - Y^{-i}$. Using the proof of condition (b) in Theorem 5, we have

$$\tilde{r}^i = \frac{n-1}{n}Y^*(c-s)\left\{\sqrt{\frac{n}{n-1}\frac{(p_1-s)(1-v^*)}{(c-s)Y^*}} - 1\right\}^2.$$

Bounds (24) and (25) imply

$$\frac{n}{n-1}\frac{(p_1-s)(1-v^*)}{(c-s)Y^*} \geq \left(\frac{n}{n-1}\frac{1-s/\beta}{Y^*}\right)^2(1+\epsilon) \geq (1+\epsilon')^2.$$

Then, using $\frac{n-1}{n} \geq \frac{1}{2}$ and $Y^* \geq 1 - p_1$, $\tilde{r}^i$ is bounded from below as follows:

$$\tilde{r}^i \geq \frac{n-1}{n}Y^*(c-s)\left\{\sqrt{(1+\epsilon')^2} - 1\right\}^2 \geq \frac{1}{2}(1-p_1)(c-s)(\epsilon')^2$$

for all $n > \max\{N, N'\}$. That is, $\tilde{r}^i$ is separated from zero by a positive constant for all sufficiently large n. On the other hand, the following lemma immediately implies that $\lim_{n\to\infty} r^* = 0$.

Lemma 27. *The equilibrium profit in RESE 3 can be expressed as*

$$r^* \;=\; \frac{\beta}{n(1-\rho\beta)}\Big\{-(Y^*)^2+Y^*\Big[2-p_1-\frac{c}{\beta}-\rho\,(p_1-c)\Big]+\Big(\frac{p_1}{\beta}-1\Big)(1-p_1)\Big\}. \tag{26}$$

Indeed, Y^* is bounded, implying that the expression inside $\{\cdot\}$ is also bounded, while the coefficient in front of $\{\cdot\}$ tends to zero as $n\to\infty$. Then there exists $N''\geq\max\{N,N'\}$ such that $\tilde{r}^i>r^*$ for all $n>N''$ and RESE 3 does not exist.

A.10. **Proof of Theorem 9 (RESE with $p_2^*=s$).** We start by identifying candidate solutions for a symmetric equilibrium with given expectations. When $p_2=s$, the equilibrium is possible only with sales in both periods, and rationality requires that $v^{\min}<1$ and $\bar{\alpha}=1$.

By parts (1.4) and (2) of Lemma 26, the profit function is strictly concave when $y^i\geq\left(1-\frac{s}{\beta}-Y^{-i}\right)^+$. Thus, a symmetric equilibrium inventory decision is found by solving the first-order optimality condition for the best response. Since the optimum of the profit function cannot occur at $y^i=1-\frac{s}{\beta}-Y^{-i}$ by part (1.1) of Lemma 26, the candidate is found by setting the derivative of the profit (15) (§A.4.3) to zero while, by symmetry, $Y^{-i}=\frac{n-1}{n}Y$:

$$0=\frac{\partial r^i}{\partial y^i}=-(c-s)+\frac{Y^{-i}}{Y^2}\,(p_1-s)\left(1-v^{\min}\right)=-(c-s)+\frac{n-1}{nY}\,(p_1-s)\left(1-v^{\min}\right).$$

The resulting unique solution is $\hat{Y}=\frac{n-1}{n}\frac{(p_1-s)\left(1-v^{\min}\right)}{c-s}$. Combining it with rationality of expectations, we find, by Lemma 1, that $v^{\min}=v^*=\frac{p_1-\rho s}{1-\rho\beta}$ and $Y^*=\frac{n-1}{n}\frac{(p_1-s)(1-v^*)}{c-s}$. This is the only candidate point for RESE with $p_2^*=s$. The equilibrium profit of a retailer at this point is

$$r^*=\frac{1}{n}\left\{-cY^*+p_1(1-v^*)+s\left[Y^*-(1-v^*)\right]\right\}=\frac{1}{n}\left\{-(c-s)Y^*+(p_1-s)(1-v^*)\right\},$$

which yields the expression for r^* in the theorem.

We now analyze when the candidate point is indeed a RESE with $p_2^*=s$, and start by checking that it is contained within the valid ranges $p_1\leq v^*<1$ and $Y^*\geq 1-\frac{s}{\beta}$, which provide necessary conditions for RESE existence. The second condition is the domain restriction of §A.4.3. It is equivalent to $p_2^*=s$ and follows from either of the mutually exclusive cases (a), (b), and (c) in the statement of the theorem. Since the equilibrium cannot result in $Y^*=1-\frac{s}{\beta}$, by part (1.1) of Lemma 26, the second condition is strengthened to $Y^*>1-\frac{s}{\beta}$ under which cases (a), (b) and (c) become exhaustive. Since $1-\frac{s}{\beta}>0$ and Y^* is proportional to $1-v^*$, the resulting strict positivity of Y^* implies that $v^*<1$. Similarly to RESE 3, $v^*=p_1$ if $\rho=0$, and it can be shown that $v^*>p_1$ if $\rho>0$. Indeed, inequality $v^*>p_1$ is equivalent to $\frac{p_1-\rho s}{1-\rho\beta}>p_1\Leftrightarrow p_1-\rho s>p_1-p_1\rho\beta\Leftrightarrow-\rho s>-p_1\rho\beta\Leftrightarrow p_1>\frac{s}{\beta}$, which always holds in this problem.

It remains to establish that the exact conditions ensuring that $\frac{Y^*}{n}$ provides the global optimum of the profit function are indeed provided by the mutually exclusive and exhaustive (under condition $Y^*>1-\frac{s}{\beta}$) cases (a), (b), and (c).

Condition (a), i.e., $\frac{n-1}{n}Y^*\geq 1-\frac{s}{\beta}$, means, by (3), that $p_2=s$ independently of the inventory decisions of individual retailers. By part (2) of Lemma 26, the profit function is globally strictly concave in this case and $\frac{Y^*}{n}$ is indeed its unique global maximum.

In case (b) of the theorem, condition (a) does not hold, which means that $p_2=s$ may or may not hold depending on the decisions of individual retailers. Nevertheless, the maximum of the profit function is unique and occurs when $p_2=s$ as long as the profit function is strictly increasing in the interval corresponding to $p_2>s$. This is ensured by the condition $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0}\geq 0$ which, by part (1.5) of Lemma 26, implies pseudoconcavity of the profit function. Using (12), the last condition takes the following form:

$$\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0}=\beta\left(1-Y^{-i}\right)-c+\beta\left(1-v^{\min}\right)-2\beta\left(1-\frac{s}{\beta}-Y^{-i}\right)+\frac{(p_1-\beta)(1-v^{\min})Y^{-i}}{\left(1-\frac{s}{\beta}\right)^2}\geq 0,$$

which, after collecting the terms and substituting $Y^{-i}=\frac{n-1}{n}Y^*$ and $v^{\min}=v^*$, can be rewritten as

$$\left(\beta+\frac{(p_1-\beta)(1-v^*)}{\left(1-\frac{s}{\beta}\right)^2}\right)\frac{n-1}{n}Y^*\geq c+\beta v^*-2s,\quad \text{yielding condition (b).}$$

In case (c) of the theorem, condition (b) does not hold, i.e. $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0}<0$. Then, there exists a local maximum of r^i without or with the sales in the second period and $p_2>s$. In other words, there exists such an inventory decision $\tilde{y}^i$ of a deviating retailer that

$$\tilde{y}^i\triangleq\arg\max\left\{r^i(y^i)\mid y^i\in\left[\max\left\{0,1-v^*-\frac{n-1}{n}Y^*\right\},1-\frac{s}{\beta}-\frac{n-1}{n}Y^*\right)\right\}$$

or, denoting $\tilde{Y}\triangleq\tilde{y}^i+\frac{n-1}{n}Y^*$, $\tilde{Y}\in\left[\max\left\{1-v^*,\frac{n-1}{n}Y^*\right\},1-\frac{s}{\beta}\right)$. Then the equilibrium with $p_2^*=s$ exists only if

$$\tilde{r}^i\triangleq r^i(\tilde{y}^i)\leq r^*. \tag{27}$$

Consider this condition at the left boundary of the range for y^i. If $\tilde{y}^i=0$, then $\tilde{r}^i=0$ and (27) holds trivially. If $\tilde{y}^i=1-v^*-\frac{n-1}{n}Y^*=(1-v^*)\left[1-\left(\frac{n-1}{n}\right)^2\frac{p_1-s}{c-s}\right]$, then, by §A.4.1, there are no sales in the second period and $\tilde{r}^i=(1-v^*)\left[1-\left(\frac{n-1}{n}\right)^2\frac{p_1-s}{c-s}\right](p_1-c)$. After substitutions for $\tilde{r}^i$ and r^*, and multiplication of both sides by $\frac{n^2}{(1-v^*)(p_1-c)}$, condition (27) becomes $n^2-(n-1)^2\frac{p_1-s}{c-s}\leq\frac{p_1-s}{p_1-c}$, which always holds. Indeed, let $g(n)\triangleq n^2-(n-1)^2\frac{p_1-s}{c-s}$. Then $g'(n)=2n-2(n-1)\frac{p_1-s}{c-s}=2\left[-n\frac{p_1-c}{c-s}+\frac{p_1-s}{c-s}\right]$ and $g''(n)=-2\frac{p_1-c}{c-s}<0$. Therefore, the unique maximum of g, defined by the condition $g'(n)=0$, is $n_{\max}=\frac{p_1-s}{p_1-c}$ and

$$g_{\max}=g(n_{\max})=\left(\frac{p_1-s}{p_1-c}\right)^2-\left(\frac{p_1-s}{p_1-c}-1\right)^2\frac{p_1-s}{c-s}=\frac{p_1-s}{p_1-c}\left[\frac{p_1-s}{p_1-c}-\frac{(c-s)^2}{p_1-c}\frac{1}{c-s}\right]=\frac{p_1-s}{p_1-c}.$$

Finally, the RESE with $p_2^*=s$ may also exist if there exists an internal local maximum $r^i(\tilde{y}^i)\leq r^*$ with $\tilde{y}^i=\tilde{Y}-\frac{n-1}{n}Y^*$ such that $\max\left\{1-v^*,\frac{n-1}{n}Y^*\right\}<\tilde{Y}<1-\frac{s}{\beta}$ and $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\tilde{y}^i}=0$. In this case, formula (10) from §A.4.2 yields the expression for $\tilde{r}^i$ in condition (c):

$$\tilde{r}^i=\left(\tilde{Y}-\frac{n-1}{n}Y^*\right)\left[\beta\left(1-\tilde{Y}\right)-c+\beta\left(1-v^*\right)+\frac{(p_1-\beta)(1-v^*)}{\tilde{Y}}\right],$$

where $\tilde{Y}$ is a zero of the profit function derivative (12), which, in this case, is

$$0=\frac{\partial r^i}{\partial y^i}=\beta\left(1-\frac{n-1}{n}Y^*\right)-c+\beta(1-v^*)-2\beta\left(Y-\frac{n-1}{n}Y^*\right)+(p_1-\beta)(1-v^*)\frac{n-1}{n}\frac{Y^*}{Y^2}.$$

After multiplication by $-Y^2/\beta$ this equation becomes

$$2Y^3+a_2Y^2+a_0=0, \tag{28}$$

which is equation (4) if one substitutes the coefficients

$$
\begin{aligned}
a_2 &\triangleq \frac{c}{\beta} - (1-v^*) - \left(1 + \frac{n-1}{n}Y^*\right) = -(1-v^*) - \left(1 - \frac{c}{\beta}\right) - \frac{n-1}{n}Y^* < 0, \\
a_0 &\triangleq (1 - \frac{p_1}{\beta})(1-v^*)\frac{n-1}{n}Y^*.
\end{aligned}
$$

Since, by part (1.3) of Lemma 26, the profit function of the deviating retailer is pseudoconcave on the interval $(1 - v^* - \frac{n-1}{n}Y^*)^+ \leq y^i \leq 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^*$, equation (28) may have at most one root on this interval.

Any cubic equation with real coefficients has at least one and up to three real roots. If neither of the roots is relevant, it means that there is no internal maximum and the boundary maximum cannot exceed r^* as shown above. If there is a relevant root, a direct comparison between r^* and $\tilde{r}^i$ determines the existence of RESE.

Suppose that consumer expectations of the second-period price deviate from rational ones with the initial deviation $\bar{p}_2^0 > s$ and the game is repeated with the same inputs. As shown above, a symmetric best response is $BR(\bar{p}_2) = \frac{n-1}{n}\frac{(p_1-s)\left(1-v^{\min}(\bar{p}_2)\right)}{c-s}$, which is increasing in $\bar{p}_2$. For any inputs where RESE 4 exists, $Y^* = BR(s) > 1 - \frac{s}{\beta}$ and, since $\bar{p}_2^0 > s$, $BR(\bar{p}_2^0) > 1 - \frac{s}{\beta}$. Then, for any $t \geq 0$, the realized second-period price p_2^t equals s if consumer expectations follow a linear adjustment process $\bar{p}_2^{t+1} = \mu s + (1-\mu)\bar{p}_2^t$ with $\mu < 1$. Under this process, $\left|s - \bar{p}_2^{t+1}\right| = \left|(1-\mu)(s - \bar{p}_2^t)\right| = \left|(1-\mu)^t(s - \bar{p}_2^0)\right|$, which goes to zero with $t \to \infty$ for any $\mu \in (0,1)$ and $\bar{p}_2^0 \in (s, \infty)$.

A.11. **Proof of Proposition 10 (Uniqueness of RESE).** *Part (a).* We start by discussing model inputs satisfying conditions of RESE 1 and 2. By Theorem 5, these conditions rule out RESE 3 and guarantee that, *for the corresponding structure*, one and only one equilibrium exists. Thus, it remains to rule out RESE 4.

RESE 4 cannot exist under the same conditions as RESE 1 because p_1-lower bound P_1 in RESE 1 exceeds the upper bound P_4 in RESE 4: $1 - \frac{n}{n+1}\rho(\beta - c) > 1 - \rho(\beta - c) > 1 - \rho(\beta - s) \Leftrightarrow c > s$.

Moreover, RESE 4 cannot exist under the necessary and sufficient condition $p_1 \leq P_2$ for RESE 2 since the latter is incompatible with necessary condition $Y^* > 1 - \frac{s}{\beta}$ for RESE 4. Indeed, consider $Y^* = \frac{n-1}{n}\frac{p_1-s}{c-s}(1-v^*)$ for RESE 4. Condition $p_1 \leq P_2$ implies $n(c-s) \geq (n-1+\beta)p_1 - ns = (n-1)(p_1 - s) + \beta p_1 - s > (n-1)(p_1 - s)$. Since $1 - v^* \leq 1 - p_1 < 1 - \frac{s}{\beta}$, we get $Y^* < 1 - \frac{s}{\beta}$.

Part (b). It remains to show that, when conditions of RESE 1 or 2 do not hold, condition (b.2) guarantees the existence of RESE 3 and non-existence of RESE 4. Indeed, (b.2) implies that RESE 4 total equilibrium supply violates a necessary condition $Y^* > 1 - \frac{s}{\beta}$ for RESE 4 because $v^* \geq p_1$ and we have $Y^* = \frac{n-1}{n} \cdot \frac{p_1-s}{c-s}(1-v^*) \leq 1 - \frac{s}{\beta}$.

Finally, for the existence of RESE 3, we show that (b.2) implies $Y^* < 1 - \frac{s}{\beta}$ and condition (a) of Theorem 5. Indeed, as long as $Y^* < 1 - \frac{s}{\beta}$ and since $v^* \geq p_1$, the LHS of (a) is smaller than the LHS of (b.2). We show that $Y^* < 1 - \frac{s}{\beta}$ by demonstrating that $1 - \frac{s}{\beta}$ exceeds the larger root of (23) under condition (b.2). Recall, from the proof of Theorem 5, that (23) is obtained as a characterization of the intersection point (Y^*, v^*) of functions $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ in the range of $Y \geq \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$ where $v_2^{\min}(Y)$ is decreasing (see Figure 14(a)). Since the smallest possible value of $v_1^{\min}(Y)$ is p_1, $Y^* < 1 - \frac{s}{\beta}$ holds as long as $v_2^{\min}\Big(1 - \frac{s}{\beta}\Big) < p_1$, i.e.,

$$1 - \frac{\Big(1 - \frac{s}{\beta}\Big)^2 - \Big(1 - \frac{s}{\beta}\Big)\frac{n}{n+1}\Big(1 - \frac{c}{\beta}\Big)}{\frac{n}{n+1}\Big(1 - \frac{s}{\beta}\Big) + \frac{n-1}{n+1}\Big(\frac{p_1}{\beta} - 1\Big)} = 1 - \frac{\frac{1}{n+1}\Big(1 - \frac{s}{\beta}\Big)^2 + \Big(1 - \frac{s}{\beta}\Big)\frac{n}{n+1}\frac{c-s}{\beta}}{\frac{1}{n+1}\Big(1 - \frac{s}{\beta}\Big) + \frac{n-1}{n+1}\frac{p_1-s}{\beta}} < p_1, \qquad \text{or}$$

$$(n-1)(1-p_1)\frac{p_1 - s}{\beta} + (1-p_1)\Big(1 - \frac{s}{\beta}\Big) - \Big(1 - \frac{s}{\beta}\Big)^2 < n\Big(1 - \frac{s}{\beta}\Big)\frac{c-s}{\beta}.$$

The latter is implied by (b.2) since $1 - p_1 < 1 - \frac{s}{\beta}$.

A.12. **Proof of Proposition 11 (Switches between RESE).** The p_1-bounds in the claim of the proposition satisfy the following chain of inequalities for all valid model inputs: $\frac{c}{\beta} \le 1 - \beta + c < 1 - \rho(\beta - c) < 1 - \frac{n}{n+1}\rho(\beta - c) \le 1 - \frac{1}{2}\rho(\beta - c)$. The value $1 - \beta + c$ provides the exact lower bound on p_1-values corresponding to RESE 1 over all $n \ge 1$ and $\rho \in [0, 1)$, while $\frac{c}{\beta}$ provides the exact upper bound on p_1 corresponding to RESE 2. Thus, p_1 corresponding to RESE 1 for some model inputs cannot result in RESE 2 under any other inputs and *vice versa*. Consider each of the possible p_1-ranges.

Part (1): By Theorem 5, if $p_1 \le \frac{c}{\beta}$ and $n = 1$, the RESE is realized in the form 2 and not form 3. The necessary and sufficient condition for RESE 2 can be rewritten as $p_1 n - p_1(1-\beta) \le nc$ or, equivalently, $n \le n_2 = \frac{p_1(1-\beta)}{p_1 - c}$. For $n > n_2$, RESE 2 cannot exist and p_1 falls into the range of RESE 3 (and, as argued above, cannot fall into the range of RESE 1). That is, as the level of competition increases, the equilibrium with no sales in the second period (RESE 2) becomes impossible and is replaced by the equilibrium with sales in both periods (RESE 3).

Part (4): When $p_1 > \frac{c}{\beta}$, we have $n_2 < 1$, i.e. even a monopolist cannot realize RESE 2. If, in addition to this condition, $p_1 < 1 - \beta + c$, only RESE 3 is possible.

Part (2): Since the RESE 3 upper bound on p_1 is decreasing in n, RESE 3 may exist only if $p_1 < 1 - \frac{1}{2}\rho(\beta - c)$ (P_1 for $n = 1$). RESE 3 p_1-bounds imply the following bounds on $n:$

$$n_2 < n < \begin{cases} n_1 = \frac{1-p_1}{p_1 - 1 + \rho(\beta - c)} \text{ if } p_1 > 1 - \rho(\beta - c), \\ \infty \text{ otherwise.} \end{cases}$$

That is, as n increases, RESE 3 becomes impossible if $p_1 > 1 - \rho(\beta - c)$ and is replaced by RESE 1.

Part (3) of this proposition can be shown in the same way, using the boundary on p_1 between RESE 1 and 3 as a function of ρ.

A.13. **Proof of Proposition 12 (Monotonicity of Y^*, v^*, and nr^*).** *Monotonicity of v^* and Y^*.* By Theorem 5, v^* is constant in n and ρ for RESE 1 and 2; Y^* is increasing in n and constant in ρ for RESE 1 and constant in n and ρ for RESE 2. By continuity of v^* and Y^*, it remains to show the correspondent monotonicity of these values for RESE 3.

Monotonicity of v^* and Y^* in ρ. Recall that, for RESE 3, Y^* and v^* satisfy (18) for Y^*. The derivative of this equation in ρ is

$$2Y^*\frac{\partial Y^*}{\partial \rho} - \frac{\partial Y^*}{\partial \rho}\frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right) + \frac{n}{n+1}Y^*\frac{\partial v^*}{\partial \rho} + \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\frac{\partial v^*}{\partial \rho} = 0,$$

which can be written as

$$\frac{\partial Y^*}{\partial \rho}\left[2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right)\right] = -\frac{\partial v^*}{\partial \rho}\frac{1}{n+1}\left[nY^* + (n-1)\left(\frac{p_1}{\beta} - 1\right)\right]. \tag{29}$$

Since $Y^* > 1 - p_1$ (by Lemma 4), and $\frac{p_1}{\beta} \geq p_1$, the lower bound for the square bracket in the RHS is $n(1-p_1) + (n-1)(p_1 - 1) = 1 - p_1 > 0$. The square bracket in the LHS of (29) is also positive:

$$2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right) > 0 \tag{30}$$

because $Y^* > 1 - p_1 \geq 1 - v^* \geq \frac{n}{n+1}(1 - v^*)$ and $\frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$ is a lower bound for Y^* in RESE 3 (by Theorem 5).

For RESE 3, Y^* and v^* satisfy (21), which can be written as $(1-\rho\beta)v^* - \rho\beta Y^* = p_1 - \rho\beta$ with the following derivative in ρ : $(1-\rho\beta)\frac{\partial v^*}{\partial \rho} - \rho\beta\frac{\partial Y^*}{\partial \rho} = \beta(v^* + Y^* - 1)$, where the RHS is positive since $v^* \geq p_1$ and $Y^* > 1 - p_1$. The combination of the last equation with (29) results in the linear system in $\frac{\partial v^*}{\partial \rho}$ and $\frac{\partial Y^*}{\partial \rho}$ with positive a_i and b_i :

$$\begin{aligned} a_2\frac{\partial v^*}{\partial \rho} &= -a_1\frac{\partial Y^*}{\partial \rho}, \\ b_2\frac{\partial v^*}{\partial \rho} &= b_1\frac{\partial Y^*}{\partial \rho} + b_0. \end{aligned}$$

The first equation describes a straight line with zero intercept and negative slope. The second straight line goes through the points $\left(\frac{\partial Y^*}{\partial \rho}, \frac{\partial v^*}{\partial \rho}\right) = (0, b_0)$ and $\left(\frac{\partial Y^*}{\partial \rho}, \frac{\partial v^*}{\partial \rho}\right) = (-\frac{b_0}{b_1}, 0)$ with a positive slope. A unique intersection of these lines belongs to the area where $\frac{\partial v^*}{\partial \rho} > 0$ and $\frac{\partial Y^*}{\partial \rho} < 0$.

Monotonicity of v^* and Y^* in n. Denote $z \triangleq \frac{n}{n+1}$, which implies $\frac{n-1}{n+1} = 2z - 1$. Since z increases in n, monotonicity of v^* and Y^* in z is equivalent to monotonicity in n. Equation (18) for Y^* can be written as $(Y^*)^2 - Y^* z\Big(2 - v^* - \frac{c}{\beta}\Big) - (2z-1)\Big(\frac{p_1}{\beta} - 1\Big)(1 - v^*) = 0$ with the derivative in z

$$\begin{aligned} 2Y^*\frac{\partial Y^*}{\partial z} - \frac{\partial Y^*}{\partial z} z\Big(2 - v^* - \frac{c}{\beta}\Big) - Y^*\Big(2 - v^* - \frac{c}{\beta}\Big) + Y^* z\frac{\partial v^*}{\partial z} \\ - 2\Big(\frac{p_1}{\beta} - 1\Big)(1 - v^*) + (2z-1)\Big(\frac{p_1}{\beta} - 1\Big)\frac{\partial v^*}{\partial z} = 0. \end{aligned}$$

After collecting the terms with $\frac{\partial v^*}{\partial z}$ and $\frac{\partial Y^*}{\partial z}$, this equation becomes

$$\begin{aligned} \frac{\partial Y^*}{\partial z}\Big[2Y^* - z\Big(2 - v^* - \frac{c}{\beta}\Big)\Big] + \frac{\partial v^*}{\partial z}\Big[Y^* z + (2z-1)\Big(\frac{p_1}{\beta} - 1\Big)\Big] \\ = Y^*\Big(2 - v^* - \frac{c}{\beta}\Big) + 2\Big(\frac{p_1}{\beta} - 1\Big)(1 - v^*). \end{aligned} \tag{31}$$

The first square bracket in the LHS is positive by (30). The second square bracket in the LHS is also positive since it is positive for $p_1 \geq \beta$, and, for $p_1 < \beta$, it is bounded from below as follows:

$$Y^*\frac{n}{n+1} + \frac{n-1}{n+1}\Big(\frac{p_1}{\beta} - 1\Big) > \frac{n\big[Y^* - \big(1 - \frac{p_1}{\beta}\big)\big]}{n+1} > \frac{n[Y^* - (1 - p_1)]}{n+1} > 0.$$

The RHS of (31) is positive since it is linear in v^*, positive at $v^* = 1$, and positive at $v^* = p_1$:

$$\begin{aligned} & Y^*\left(2 - p_1 - \frac{c}{\beta}\right) + 2\left(\frac{p_1}{\beta} - 1\right)(1 - p_1) \\ & \geq (1 - p_1)\left[2 - p_1 - \frac{c}{\beta} + 2\frac{p_1}{\beta} - 2\right] = (1 - p_1)\left[\frac{p_1}{\beta} - p_1 + \frac{p_1 - c}{\beta}\right] > 0. \end{aligned}$$

The derivative of (21) in z is

$$\frac{\partial v^*}{\partial z} = \frac{\rho\beta}{1 - \rho\beta}\frac{\partial Y^*}{\partial z}. \tag{32}$$

If $\rho > 0$, then $\frac{\partial v^*}{\partial z}$ and $\frac{\partial Y^*}{\partial z}$ satisfy the following system with positive a_i and b_i :

$$\begin{aligned} a_1 \frac{\partial Y^*}{\partial z} &= a_2 \frac{\partial v^*}{\partial z}, \\ b_1 \frac{\partial Y^*}{\partial z} + b_2 \frac{\partial v^*}{\partial z} &= b_0. \end{aligned}$$

The first equation describes a straight line with zero intercept and positive slope. The second straight line goes through the points on the axes $\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = (\frac{b_0}{b_1}, 0)$ and $\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = (0, \frac{b_0}{b2})$ with a negative slope. A unique intersection of these lines belongs to the area where $\frac{\partial v^*}{\partial z} > 0$ and $\frac{\partial Y^*}{\partial z} > 0$.

If $\rho = 0$, (32) becomes $\frac{\partial v^*}{\partial z} = 0$ yielding the solution $\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = (\frac{b_0}{b_1}, 0)$.

Monotonicity of nr^* in n. By Theorem 5, nr^* is constant in n for RESE 2 and monotonically decreasing for RESE 1. By global continuity of nr^*, it remains to show the correspondent monotonicity of nr^* for RESE 3.

By the alternative expression (26) for RESE 3 profit (Lemma 27),

$$nr^* = \frac{\beta}{1-\rho\beta} \left\{ -\left(Y^*\right)^2 + Y^* \left[2 - p_1 - \frac{c}{\beta} - \rho\left(p_1 - c\right) \right] + \left(\frac{p_1}{\beta} - 1 \right) \left(1 - p_1\right) \right\}.$$

Denote $F \triangleq \frac{1-\rho\beta}{\beta} nr^*$. Then

$$\frac{\partial F}{\partial n} = -2Y^* \frac{\partial Y^*}{\partial n} + \frac{\partial Y^*}{\partial n} \left[2 - p_1 - \frac{c}{\beta} - \rho\left(p_1 - c\right) \right] = \frac{\partial Y^*}{\partial n} \left[2 - p_1 - \frac{c}{\beta} - \rho\left(p_1 - c\right) - 2Y^* \right]. \quad (33)$$

As shown above, Y^* is monotonically increasing in n for RESE 3. Therefore, nr^* is monotonically decreasing if and only if the square bracket in the last expression is negative. Consider two cases: $p_1 \le \frac{c}{\beta}$ and $p_1 > \frac{c}{\beta}$.

Suppose $p_1 \le \frac{c}{\beta}$. By Lemma 4, $Y^* > 1 - p_1$ in RESE 3, and, therefore, $2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^* < p_1 - \frac{c}{\beta} - \rho(p_1 - c) \le 0$.

For $p_1 > \frac{c}{\beta}$ (p_1-lower bound for RESE 3 in a monopoly), by monotonicity of Y^* in n, the RESE 3 value of Y^* for any n is bounded from below by the RESE 3 total supply in a monopoly: $Y^* \ge 1 - \frac{1}{2}\left(\frac{c}{\beta} + \frac{2p_1 - \rho c}{2 - \rho\beta}\right)$. Therefore,

$$\begin{aligned} 2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^* &\le -p_1 - \rho(p_1 - c) + \frac{2p_1 - \rho c}{2 - \rho\beta} = \frac{p_1\rho\beta - \rho(p_1 - c)(2 - \rho\beta) - \rho c}{2 - \rho\beta} \\ = \frac{\rho(p_1\beta - p_1(2 - \rho\beta) + c(1 - \rho\beta))}{2 - \rho\beta} &< \frac{\rho(p_1\beta - p_1(2 - \rho\beta) + p_1\beta(1 - \rho\beta))}{2 - \rho\beta} = \rho p_1(\beta - 1) \le 0. \end{aligned}$$

Thus, the square bracket in (33) is always negative and nr^* is monotonically decreasing in n.

Monotonicity of nr^* in ρ. By Theorem 5, nr^* does not depend on ρ for RESE 1 and 2 since there is no intertemporal effect in these cases.

By (10) with $y^i = \frac{Y^*}{n}$, total profit is $nr^* = \beta\left[Y^* - (Y^*)^2\right] - cY^* + \beta Y^*(1 - v^*) + (p_1 - \beta)(1 - v^*)$ with the derivative

$$\begin{aligned} \frac{\partial [nr^*]}{\partial \rho} &= \beta(1 - 2Y^*)\frac{\partial Y^*}{\partial \rho} - c\frac{\partial Y^*}{\partial \rho} + \beta(1 - v^*)\frac{\partial Y^*}{\partial \rho} - \beta Y^* \frac{\partial v^*}{\partial \rho} - (p_1 - \beta)\frac{\partial v^*}{\partial \rho} \\ &= -\beta\frac{\partial Y^*}{\partial \rho}\left[2Y^* - 2 + \frac{c}{\beta} + v^*\right] - \frac{\partial v^*}{\partial \rho}\left[p_1 - \beta(1 - Y^*)\right]. \end{aligned} \quad (34)$$

For $n = 1$, the first square bracket is zero (by Corollary 6), implying $\frac{\partial [nr^*]}{\partial \rho} < 0$ since $p_1 > p_2^*$ and $\frac{\partial v^*}{\partial \rho} > 0$ (by part (2) of this proposition).

For $n > 1$, let $R = \left[2Y^* - 2 + \frac{c}{\beta} + v^*\right] / \left[2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right)\right] \in [0,1)$ and consider (29) multiplied through by $\left[2Y^* - 2 + \frac{c}{\beta} + v^*\right]$, which yields

$$-\frac{\partial Y^*}{\partial \rho}\left[2Y^* - 2 + \frac{c}{\beta} + v^*\right] = \frac{\partial v^*}{\partial \rho}\frac{R}{n+1}\left[nY^* + (n-1)\left(\frac{p_1}{\beta} - 1\right)\right].$$

Since $nY^* + (n-1)\left(\frac{p_1}{\beta} - 1\right) \geq 0$ (because $Y^* \geq 1 - p_1$ by Lemma 4) and $0 \leq R < 1$, we can upper-bound the first term in (34) to obtain

$$\begin{aligned}\frac{\partial[nr^*]}{\partial \rho} &< \beta\frac{\partial v^*}{\partial \rho}\left[\frac{n}{n+1}Y^* + \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right) - Y^* - \left(\frac{p_1}{\beta} - 1\right)\right] \\ &= \frac{\beta}{n+1}\frac{\partial v^*}{\partial \rho}\left[-Y^* - 2\left(\frac{p_1}{\beta} - 1\right)\right] = \frac{2}{n+1}\frac{\partial v^*}{\partial \rho}\left[\frac{p_2^* + \beta}{2} - p_1\right].\end{aligned}$$

Since $p_2^* = \beta(1 - Y^*)$ and $Y^* \geq \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$ in RESE 3 by Theorem 5, $\frac{\partial[nr^*]}{\partial \rho} < 0$ holds when $\beta - \frac{n}{2(n+1)}(\beta - c) \leq p_1$ which yields the condition of the proposition.

A.14. **Proof of Corollary 13 (σ_2 in ρ).** The second-period surplus $\sigma_2 = \rho(\beta v - p_2^*)$ is monotonically non-decreasing in n because $p_2^* = \beta(1-Y^*)$ is non-increasing in n by part (1) or Proposition 12.

The derivative $\frac{\partial \sigma_2}{\partial \rho} = \beta v - p_2^* - \rho\frac{\partial p_2^*}{\partial \rho}$ is an increasing linear function of v that is zero at $v^0 = \frac{1}{\beta}\left(p_2^* + \rho\frac{\partial p_2^*}{\partial \rho}\right)$. For RESE 1, $v^0 < v^* = 1$ since $p_2^* < \beta p_1 < 1$, by part (1) of Lemma 4, and $\frac{\partial p_2^*}{\partial \rho} = 0$.

Minimum valuation of a consumer who purchases in the second period is $v_2^{\min} \triangleq \frac{p_2^*}{\beta} = \max\left\{\frac{s}{\beta}, 1 - Y\right\}$. For this consumer, $\left.\frac{\partial \sigma_2}{\partial \rho}\right|_{v=v_2^{\min}} = -\rho\frac{\partial p_2^*}{\partial \rho}$, which is nonpositive for RESE 3. Thus, $v^0 \geq v_2^{\min}$. Similarly, we show that $v^0 < v^*$ because, when $\rho = 0$, we have $\left.\frac{\partial \sigma_2}{\partial \rho}\right|_{v=v^*} = \beta p_1 - p_2^* > 0$, and, when $\rho > 0$, we know, by Proposition 12, that $\frac{\partial v^*}{\partial \rho} > 0$ implying $\left.\frac{\partial \sigma_2}{\partial \rho}\right|_{v=v^*} > 0$.

A.15. **Proof of Corollary 14 (Q_2 increases with ρ).** Recall that for a RESE, $Q_2 = Y^* - (1 - v^*)$, yielding $\frac{\partial Q_2}{\partial \rho} = \frac{\partial Y^*}{\partial \rho} + \frac{\partial v^*}{\partial \rho}$, which, using (29), is

$$\begin{aligned}\frac{\partial Q_2}{\partial \rho} &= \left[-\frac{\frac{n}{n+1}Y^* + \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)}{2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right)} + 1\right]\frac{\partial v^*}{\partial \rho} \quad \text{or} \\ \frac{\partial Q_2}{\partial \rho} &= \frac{\frac{n+2}{n+1}Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)}{2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right)} \cdot \frac{\partial v^*}{\partial \rho}. \qquad (35)\end{aligned}$$

Since, by Proposition 12, $\frac{\partial v^*}{\partial \rho} > 0$ and, by (30), the denominator of the fraction in the RHS of (35) is positive, the sign of $\frac{\partial Q_2}{\partial \rho}$ coincides with the sign of the numerator.

For part (1), use $n = 1$ and the corresponding $Y^* = 1 - \frac{1}{2}\left(v^* + \frac{c}{\beta}\right)$ (Corollary 6) in the numerator to get $\frac{3}{2}Y^* - \left(1 - \frac{1}{2}\left(v^* + \frac{c}{\beta}\right)\right) = \frac{1}{2}Y^* > 0$.

For part (2), $\rho = 0$ implies $v^* = p_1$, and the numerator becomes $Y^* - 1 + p_1 + \frac{c}{\beta} - \frac{p_1}{\beta}$ as $n \to \infty$. In this case, p_1-range for RESE 3 is $c < p_1 < 1$, and as $p_1 \to 1$, the total supply, given by the larger root of (23), approaches $Y^* = 1 - \frac{c}{\beta}$ implying that the numerator approaches $1 - \frac{1}{\beta} < 0$.

A.16. **Proof of Proposition 15 (boundary-value gain in RESE 3).** For $\beta = 1$, necessary condition (5) of RESE 4 becomes $\rho < \frac{1-p_1}{1-s}$ and cannot hold as $\rho \to 1$ since $p_1 > s$. Thus, RESE 4 does not exist. RESE 2 does not exist because its p_1-range is empty.

Part (1) is immediate by part (3) of Proposition 11, because, as ρ increases from 0 to 1, the switch to RESE 1 occurs at $\rho_1 = \frac{n+1}{n}\frac{1-p_1}{1-c} < 1$ if $\frac{1-p_1}{1-c} < \frac{n}{n+1}$ or, equivalently, if $n > \bar{n}$. The resulting equilibrium characteristics are obtained by substituting $\beta = 1$ in the description of RESE 1. It is immediate to check that the resulting limit of p_2^* is below p_1.

Part (2). When $n < \bar{n}$, the switch from RESE 3 to RESE 1 does not occur for any $\rho < 1$. Total supply Y^*, which is given by a larger root of (23), is continuous in ρ near $\rho = 1$. We can write (23) for $\beta = \rho = 1$ as

$$\begin{aligned} Y^2 - \left((1-p_1) + \frac{(1-p_1)(n-1)}{n}\right) Y + (1-p_1) \times \frac{(1-p_1)(n-1)}{n} \\ = \left(Y - \frac{(1-p_1)(n-1)}{n}\right)(Y - (1-p_1)) = 0, \end{aligned}$$

resulting in roots $\frac{(1-p_1)(n-1)}{n}$ and $1 - p_1$. Thus, $Y^*|_{\rho\to 1} = 1 - p_1$ and $p_2^*|_{\rho\to 1} = (1 - Y^*)|_{\rho\to 1} = p_1$. The necessary condition $Y^* < 1 - \frac{s}{\beta} = 1 - s$ of RESE 3 in Theorem 5 is satisfied for all ρ sufficiently close to 1 since $p_1 > s$.

The limit of v^* is found using

Lemma 28. *In RESE 3 with $\beta = 1$, we have $\lim_{\rho\to 1} \left.\frac{\partial Y^*}{\partial \rho}\right|_{\beta=1} = n(c - p_1)$.*

By (21) with $\beta = 1$, we have $v^* = \frac{p_1 - \rho(1-Y^*)}{1-\rho}$ for all $\rho \in [0, 1)$. Then

$$\begin{aligned} \lim_{\rho\to 1} v^* &= \frac{p_1 - p_1}{1-1} = \frac{0}{0} = \lim_{\rho\to 1} \frac{\partial\left[p_1 - \rho(1-Y^*)\right]/\partial\rho}{\partial(1-\rho)/\partial\rho} = -\lim_{\rho\to 1}\left[-(1-Y^*) + \rho\frac{\partial Y^*}{\partial\rho}\right] \\ &= \lim_{\rho\to 1}\left[1 - Y^* - \rho\frac{\partial Y^*}{\partial \rho}\right] = p_1 + n(p_1 - c) \qquad \text{(from Lemma 28 and } Y^*|_{\rho\to 1} = 1 - p_1\text{)}. \end{aligned}$$

Using the limiting values of Y^* and v^* in a strict version of condition (a) for RESE 3 existence, we obtain the sufficient existence condition of the form (6). Indeed, by continuity, condition (a) is satisfied for all ρ sufficiently close to one.

Using the expression for r^*, the limit of the total profit is

$$\begin{aligned} nr^*|_{\rho\to 1} &= \lim_{\rho\to 1}[(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)] = \lim_{\rho\to 1}[(p_1 - p_2^*)(1 - v^*) + (p_2^* - c)Y^*] \\ &= (p_1 - p_1)(1 - v^*|_{\rho\to 1}) + (p_1 - c)(1 - p_1) = (p_1 - c)(1 - p_1). \end{aligned}$$

To complete the proof of part (2), consider when $(p_1 - c)(1 - p_1) \geq \frac{n(1-c)^2}{(n+1)^2}$. With a change of variables $x = \frac{1-p_1}{1-c}$, this relation can be represented as $(1-c)^2(1-x)x \geq \frac{n(1-c)^2}{(n+1)^2}$, or, equivalently, as $(1-x)x \geq \frac{n}{(n+1)^2}$ resulting in $\frac{1}{n+1} \leq x \leq \frac{n}{n+1}$. This range does not intersect with a feasible range of x for part (2) which is given by $\frac{n}{n+1} < x < 1$ (resulting from $n < \bar{n}$). Thus, for part (2), $(p_1 - c)(1 - p_1) < \frac{n(1-c)^2}{(n+1)^2}$.

Part (3) is immediate since $n = \bar{n}$ implies that the limits in parts (1) and (2) are equal.

Part (4) follows from Corollary 7 for $n = \bar{n} = \frac{1-p_1}{p_1-c}$. In this case, the condition of the corollary becomes $\frac{1}{n} + 2 < n$, which, after solving for positive integer n, yields $n \geq 3$. Moreover, when $n = \bar{n} = \frac{1-p_1}{p_1-c}$ and $\rho = 0$, the RESE is of the form 3 because RESE 2 is impossible with $\beta = 1$ and the switch to RESE 1 occurs only in the limit of $\rho \to 1$. We also have $Y^*|_{\rho=0} > Y^*|_{\rho\to 1} = 1 - p_1$ because the total supply is (strictly) decreasing in ρ, and $p_2^*|_{\rho=0} < c$ by Corollary 7. Thus, for $\rho = 0$, the total *first period* profit is $(p_1 - c)(1 - v^*|_{\rho=0}) = (p_1 - c)(1 - p_1)$, which is exactly the same as the total profit for $\rho \to 1$, while the second-period total profit is $(p_2^* - c)(Y^* - 1 + v^*|_{\rho=0}) < 0$.

A.17. **Analysis of Example 16.** Observing that $(p_1 - s)(1 - p_1) \le \frac{1}{4}(1 - s)^2$ and using relation $1-p_1 = \bar{n}(p_1-c)$, we can strengthen (6) to $\frac{(n-1)(\bar{n}-n)}{n}\frac{p_1-c}{4(c-s)} < 1$. For RESE 3, the range of n is $[1, \bar{n})$. As a function of n, fraction $\frac{(n-1)(\bar{n}-n)}{n}$ attains its maximum of $(\sqrt{\bar{n}} - 1)^2$ in this range at $n = \sqrt{\bar{n}}$ leading to an even stronger version of the condition, i.e., $(\sqrt{\bar{n}} - 1)^2 \frac{(p_1-c)}{4(c-s)} = \frac{\left(\sqrt{1-p_1}-\sqrt{p_1-c}\right)^2}{4(c-s)} < 1$. The LHS of this inequality decreases in $p_1 > c$ and it surely holds if it holds at $p_1 = c$, i.e., if $\frac{1-c}{4(c-s)} < 1$ or $c > \frac{1+4s}{5}$. Thus, (6) holds for all p_1 and $1 \le n < \bar{n}$ if $c > \frac{1+4s}{5}$, e.g., if $s = 0$ and $c > 0.2$.

A.18. **Proof of Proposition 17 (monotonicity in RESE 4).** *Part (1).* $v^{*,4} = \frac{p_1-\rho s}{1-\rho\beta}$, which is constant in n and increasing in ρ since $\frac{\partial v^*}{\partial \rho} = \frac{-s(1-\rho\beta)+\beta(p_1-\rho s)}{(1-\rho\beta)^2} = \frac{\beta p_1 - s}{(1-\rho\beta)^2} > 0$.

Parts (2) and (3) follow directly from part (1) and the formulas for Y^* and r^*, given by Theorem 9.

A.19. **Proof of Proposition 18 ($Y^* > 1-c$).** *Part 1.* By Theorem 5, $Y^{*,1} = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$, which is maximal at $\beta = 1$, and $Y^{*,1}\big|_{\beta=1} = \frac{n}{n+1}(1 - c) < 1 - c$. Hence, $Y^{*,1} < 1 - c$ for any parameters where RESE 1 exists.

Part 2. By Theorem 9, $Y^{*,4} = \frac{n-1}{n}\frac{p_1-s}{c-s}(1 - v^*)$, which, given other parameters fixed, goes to infinity when c approaches s. The condition $Y^{*,4} \ge 1-c$, using $1-v^* = \frac{1-p_1-\rho(\beta-s)}{1-\rho\beta}$, can be written as $c - s \le \frac{n-1}{n}\frac{p_1-s}{1-c}\frac{1-p_1-\rho(\beta-s)}{1-\rho\beta}$. Since $Y^{*,4}$ is a concave quadratic function in p_1, its maximum in p_1, $\bar{Y}^{*,4}$, can be found from the condition $\frac{\partial Y^{*,4}}{\partial p_1} = 0 = \frac{n-1}{n(1-\rho\beta)(c-s)}\left[1 - p_1 - \rho(\beta - s) - (p_1 - s)\right]$, yielding $\bar{p}_1 = \frac{1}{2}[1-\rho(\beta-s)+s] = \frac{P_4+s}{2}$. Since $P_4 = 1-\rho(\beta-s) = 2\bar{p}_1-s$, we have $1 - v^*|_{p_1=\bar{p}_1} = \frac{\bar{p}_1-s}{1-\rho\beta}$ and $\bar{Y}^{*,4} = \frac{(n-1)(\bar{p}_1-s)^2}{n(1-\rho\beta)(c-s)}$. Price $\bar{p}_1$ can be in the p_1-range of RESE 4 since $\bar{p}_1$ is always below P_4 — the p_1-upper bound ($P_4 > s$), and $\bar{p}_1$ can be greater than P_2, which, by Proposition 10, is p_1-lower bound in RESE 4. Indeed, $\bar{p}_1 > P_2$ holds for any $n \ge 2$ if it holds for $n = 2$ since P_2 decreases in n. For $n = 2$, inequality $\bar{p}_1 > P_2$ can be written as $(P_4 + s)(1 + \beta) > 4c$, which holds for sufficiently small c.

Part 3. By Corollary 6, $Y^{*,3} = 1 - \frac{1}{2}\left(v^* + \frac{c}{\beta}\right)$ if $n = 1$. Then $Y^{*,3} < 1 - c$ is equivalent to $c < \frac{1}{2}\left(v^* + \frac{c}{\beta}\right)$, which holds for any $\beta \in (c, 1]$ since $v^* \ge p_1 > c$.

By Proposition 12, $Y^{*,3}$ is maximized at $n \to \infty$. By continuity of Y^* at the boundaries, $Y^{*,3} \to 1 - c$ when $p_1 \to P_2|_{n\to\infty} = c$. We will show that there are feasible inputs such that $\frac{\partial Y^{*,3}}{\partial p_1} > 0$. For example, for $n \to \infty$ and $\rho = 0$, equation (23) for Y is $Y^2 - \left[1 - \frac{c}{\beta} + 1 - p_1\right] Y - \left(\frac{p_1}{\beta} - 1\right)(1-p_1) = 0$. Derivative w.r.t. p_1 results in $2Y\frac{\partial Y}{\partial p_1} + Y - \left[1 - \frac{c}{\beta} + 1 - p_1\right]\frac{\partial Y}{\partial p_1} - \frac{1}{\beta}(1-p_1) + \left(\frac{p_1}{\beta} - 1\right) = 0$, which, for $p_1 \to c$, becomes $\frac{\partial Y}{\partial p_1}[2(1 - c) - 1 + \frac{c}{\beta} - (1 - c)] = \frac{1}{\beta}[1 - c - c(1 - \beta)]$ yielding $\frac{\partial Y}{\partial p_1} = \frac{1-c}{c(1-\beta)} - 1$. The RHS is positive since $1 - c > c(1 - \beta)$.

A.20. **Proof of Proposition 19 (discount).** The proof is similar to the corresponding parts of the proofs of Theorems 5 and 9. The expressions for Y^* result from the symmetric best responses with $r^i = (p_1 - c)q^i + \lambda(p_2 - c)(y^i - q^i), y^i = \frac{Y}{n}$, and $Y^{-i} = \frac{n-1}{n}Y$.

For RESE 1, $q^i = 0$ and, using $p_2 = \beta(1 - Y)$, $\frac{\partial r^i}{\partial y^i} = 0 = \lambda\left[\beta(1 - Y) - c - \beta\frac{Y}{n}\right]$, yielding $Y = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$ that does not depend on λ. For RESE 2, the result is obvious since $r^i = (p_1-c)y^i$ does not depend on λ. For RESE 4 with $v^* = \frac{p_1-\rho s}{1-\rho\beta}$ and $r^i = (p_1-c)(1-v^*)\frac{y^i}{Y} - \lambda(c-s)y^i\left(1-\frac{1-v^*}{Y}\right) = y^i\left\{\frac{1-v^*}{Y}[p_1-c+\lambda(c-s)] - \lambda(c-s)\right\}$, the equation for Y is

$$\begin{aligned}\frac{\partial r^i}{\partial y^i} &= 0 = \frac{1-v^*}{Y}[p_1-c+\lambda(c-s)] - \lambda(c-s) - y^i\frac{1-v^*}{Y^2}[p_1-c+\lambda(c-s)]\\ &= \frac{n-1}{n}\frac{1-v^*}{Y}[p_1-c+\lambda(c-s)] - \lambda(c-s).\end{aligned}$$

This equation yields a unique $Y_\lambda^{*,4} = \frac{n-1}{n}(1-v^*)\left[\frac{p_1-c}{\lambda(c-s)}+1\right]$, which decreases in λ and $\frac{\partial^2 r^i}{\partial (y^i)^2} = -2\frac{n-1}{n}\frac{1-v^*}{Y^2}[p_1-c+\lambda(c-s)] < 0$. If $\lambda = (1+\delta)^{-1}$, the relative change in Y^* is

$$\frac{Y_\lambda^{*,4}-Y^{*,4}}{Y^{*,4}} = \left[\frac{p_1-c}{\lambda(c-s)}+1-\frac{p_1-s}{c-s}\right]\frac{c-s}{p_1-s} = \frac{p_1-c+\lambda(c-p_1)}{\lambda(p_1-s)} = \frac{(p_1-c)(1-\lambda)}{(p_1-s)\lambda} = \frac{p_1-c}{p_1-s}\delta.$$

For RESE 3, we have

$$\begin{aligned}r^i &= (p_1-c)(1-v^*)\frac{y^i}{Y} + \lambda(p_2-c)y^i\left(1-\frac{1-v^*}{Y}\right)\\ &= y^i\left\{\lambda[\beta(1-Y)-c] + \lambda\beta(1-v^*) + \frac{1-v^*}{Y}[p_1-c(1-\lambda)-\lambda\beta]\right\},\end{aligned}$$

and Y^* results from the first-order optimality condition

$$\begin{aligned}\frac{\partial r^i}{\partial y^i} &= 0 = \lambda\beta(1-Y) - \lambda c + \lambda\beta(1-v^*)\\ &\quad + \frac{(1-v^*)}{Y}[p_1-c(1-\lambda)-\lambda\beta] + \frac{Y}{n}\left\{-\lambda\beta - \frac{(1-v^*)}{Y^2}[p_1-c(1-\lambda)-\lambda\beta]\right\}\\ &= -Y\lambda\beta\left(1+\frac{1}{n}\right) - \lambda c + \lambda\beta(2-v^*) + \frac{n-1}{n}\frac{1-v^*}{Y}[p_1-c-\lambda(\beta-c)] \qquad (36)\end{aligned}$$

as well as the equation $v^* = \frac{p_1-\rho\beta(1-Y)}{1-\rho\beta}$ that links valuation threshold to the rational second-period expectations. The remainder of the proof will formally show that Y^* is decreasing in λ. The geometric idea behind the proof is provided by a generalized version of the curve $v_2^{\min}(Y)$ in (20) that gives valuation threshold for the corresponding stationary point of the profit. Solving (36) for v^*, we obtain

$$v^* = 1 - \frac{Y^2 - Y\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)}{Y\frac{n}{n+1} + \frac{n-1}{n+1}\left(\frac{p_1-c}{\lambda\beta} - 1 + \frac{c}{\beta}\right)}. \qquad (37)$$

The generalized $v_2^{\min}(Y)$ given by the right-hand side shifts down as λ increases. Thus, the intersection point of $v_2^{\min}(Y)$ and $v_1^{\min}(Y)$ (illustrated in Figure 14(a)) shifts to the left as λ increases. Since the abscissa of the intersection point is Y^*, the claim hold in RESE 3 based on this geometric structure.

Formally, (36) multiplied by $-Y\frac{n}{\lambda\beta(n+1)}$, becomes

$$Y^2 - Y\frac{n}{n+1}\left(2-v^*-\frac{c}{\beta}\right) - \frac{n-1}{n+1}(1-v^*)\left[\frac{p_1-c}{\lambda\beta} - \left(1-\frac{c}{\beta}\right)\right] = 0,$$

implying that, for $n=1, Y$ does not depend on λ. Substitution for $1-v^* = \frac{1-p_1-\rho\beta Y}{1-\rho\beta}$ yields

$$Y^2 - Y\frac{n}{n+1}\left(\frac{1-p_1-\rho\beta Y}{1-\rho\beta} + 1 - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\frac{1-p_1-\rho\beta Y}{1-\rho\beta}\left[\frac{p_1-c}{\lambda\beta} - \left(1-\frac{c}{\beta}\right)\right] = 0.$$

The coefficient in front of Y^2 is $1+\frac{n}{n+1}\frac{\rho\beta}{1-\rho\beta}=\frac{n+1-\rho\beta}{(n+1)(1-\rho\beta)}$, and the one in front of Y is

$$-\frac{1}{(n+1)(1-\rho\beta)}\left\{n[1-p_1+\left(1-\frac{c}{\beta}\right)(1-\rho\beta)]-(n-1)\rho\beta\left[\frac{p_1-c}{\lambda\beta}-\left(1-\frac{c}{\beta}\right)\right]\right\}.$$

Multiplying the last equation by $\frac{(n+1)(1-\rho\beta)}{n+1-\rho\beta}>0$ and denoting $\tilde{\lambda}\triangleq\frac{p_1-c}{\lambda}-(\beta-c)$, we obtain

$$Y^2-\frac{(\beta-c)n(1-\rho\beta)+\beta(1-p_1)n-\rho\beta(n-1)\tilde{\lambda}}{\beta(n+1-\rho\beta)}Y-\frac{(1-p_1)(n-1)}{\beta(n+1-\rho\beta)}\tilde{\lambda}=0. \tag{38}$$

The derivative of (38) w.r.t. $\tilde{\lambda}$ is

$$2Y\frac{\partial Y}{\partial\tilde{\lambda}}+Y\frac{\rho\beta(n-1)}{\beta(n+1-\rho\beta)}-\frac{(\beta-c)n(1-\rho\beta)+\beta(1-p_1)n-\rho\beta(n-1)\tilde{\lambda}}{\beta(n+1-\rho\beta)}\frac{\partial Y}{\partial\tilde{\lambda}}-\frac{(1-p_1)(n-1)}{\beta(n+1-\rho\beta)}=0.$$

Multiplication by $\beta(n+1-\rho\beta)$ yields

$$\frac{\partial Y}{\partial\tilde{\lambda}}\left\{2Y\beta(n+1-\rho\beta)-(\beta-c)n(1-\rho\beta)-\beta(1-p_1)n+\rho\beta(n-1)\tilde{\lambda}\right\}=(n-1)(1-p_1-\rho\beta Y).$$

The RHS is zero for $n=1$ and, by part (3) of Lemma 4, positive for $n>1$. It remains to show that $\{\cdot\}>0$ for any $n\geq1$, implying $\frac{\partial Y}{\partial\lambda}\equiv0$ for $n=1$, and $\frac{\partial Y}{\partial\lambda}<0$ for $n>1$ because $\tilde{\lambda}$ decreases in λ. Since $\tilde{\lambda}$ is minimal at $\lambda=1$, it can be shown that, for any $\lambda\in(0,1]$, Y is increasing in n. The proof is identical to the corresponding part of the proof of Proposition 12 with the substitution of $\tilde{\lambda}/\beta$ for $(p_1/\beta-1)$. Then $\{\cdot\}\geq\beta\left\{2(n+1-\rho\beta)\left.Y\right|_{n=1}-(1-\frac{c}{\beta})n(1-\rho\beta)-(1-p_1)n+\rho(n-1)(p_1-\beta)\right\}=\beta\left\{n[2\left.Y\right|_{n=1}-(1-\frac{c}{\beta})(1-\rho\beta)-(1-p_1)+\rho(p_1-\beta)]+2(1-\rho\beta)\left.Y\right|_{n=1}-\rho(p_1-\beta)\right\}$, where the square bracket $[\cdot]=2\left.Y\right|_{n=1}-(1-\frac{c}{\beta})-(1-p_1)+\rho(p_1-c)$. Since $\left.Y\right|_{n=1}$ does not depend on λ, we can rewrite, by Corollary 6, the bracket $[\cdot]$ as follows: $[\cdot]=2-\frac{c}{\beta}-\frac{2p_1-\rho c}{2-\rho\beta}-1+\frac{c}{\beta}-1+p_1+\rho(p_1-c)=\frac{1}{2-\rho\beta}\left[\rho c-p_1\rho\beta+2\rho(p_1-c)-\rho^2\beta(p_1-c)\right]=\frac{\rho}{2-\rho\beta}\left[(p_1-c)(1-\rho\beta)+p_1(1-\beta)\right]\geq0$ with strict inequality for $\rho>0$. Therefore, the bracket $\{\cdot\}$ is positive if it is positive for $n=1$. $\left.\{\cdot\}\right|_{n=1}=\frac{\rho}{2-\rho\beta}\left[(p_1-c)(1-\rho\beta)+p_1(1-\beta)\right]+(1-\rho\beta)\left[2-\frac{c}{\beta}-\frac{2p_1-\rho c}{2-\rho\beta}\right]-\rho(p_1-\beta)=\frac{\rho}{2-\rho\beta}\left[p_1(2-\rho\beta)-p_1\beta\right]+(1-\rho\beta)\left[2-\frac{c}{\beta}-\frac{2p_1}{2-\rho\beta}\right]-\rho(p_1-\beta)=(1-\rho\beta)\left[2-\frac{c}{\beta}-\frac{2p_1}{2-\rho\beta}\right]-\frac{\rho p_1\beta}{2-\rho\beta}+\rho\beta$, where $\rho\beta-\frac{\rho\beta p_1}{2-\rho\beta}>\rho\beta\frac{1-\rho\beta}{2-\rho\beta}$, which leads to $\left.\{\cdot\}\right|_{n=1}>\frac{1-\rho\beta}{2-\rho\beta}\left\{\rho\beta+2\left(2-\frac{c}{\beta}-p_1\right)-2\rho\beta+\rho c\right\}=\frac{1-\rho\beta}{2-\rho\beta}\left\{2\left(2-\frac{c}{\beta}-p_1\right)-\rho(\beta-c)\right\}$. Since for $n=1,p_1<1-\frac{1}{2}\rho(\beta-c)$, the last bracket $\{\cdot\}>2\left[1-\frac{c}{\beta}+\frac{1}{2}\rho(\beta-c)\right]-\rho(\beta-c)=2\left(1-\frac{c}{\beta}\right)>0$.

A.21. **Proof of Proposition 20 (RESE stability).** RESE 1,3, and 4, for $n\geq2$, represent a non-degenerate game between retailers that can be reformulated as a one-period game with retailer i's payoff $\pi^i(y^i,Y^{-i})=y^iP(y^i,Y^{-i})-C_i(y^i)$, where $C_i(y^i)=cy^i$. For RESE 1 and 3, by (10), $P(y^i,Y^{-i})=\beta(1-Y)+\beta(1-v^*)+\frac{(p_1-\beta)(1-v^*)}{Y}$, and for RESE 4, by (14), $P(y^i,Y^{-i})=s+\frac{(p_1-s)(1-v^*)}{Y}$. By Theorem 3 in al Nowaihi and Levine (1985), a Cournot equilibrium Y^* is locally asymptotically stable if the following assumptions hold:

(A1) The equilibrium point Y^* exists and unique in an open neighborhood of Y^* and $\left(y^i\right)^*>0,i\in I$ — holds by the condition of the proposition and part (2) of Lemma 4.

(A2) P and $C_i,i\in I$ are twice continuously differentiable functions in an open neighborhood of Y^* — holds for RESE 1,3, and 4.

(A3) $\partial^2\pi^i/\partial\left(y^i\right)^2<0$ at Y^* for each $i\in I$ — holds for RESE 1,3, and 4 by Lemma 26.

(H1) $P'<C_i''$ at Y^* for each $i\in I$ — holds since $C_i''\equiv0$ and $P'<0$ for RESE 1,3, and 4.

(H2) $P' + y^i P'' \leq 0$ at Y^* for each $i \in I$ — holds strictly for RESE 1,3, and 4. Namely, for RESE 1 and 3, this inequality is $-\beta - \frac{(p_1-\beta)(1-v^*)}{Y^2} + 2y^i \frac{(p_1-\beta)(1-v^*)}{Y^3} \leq 0$, where the LHS at Y^* is $-\beta - \frac{(p_1-\beta)(1-v^*)}{(Y^*)^2} + 2\frac{Y^*}{n}\frac{(p_1-\beta)(1-v^*)}{(Y^*)^3} = -\beta - \frac{(p_1-\beta)(1-v^*)}{(Y^*)^2}\left(1-\frac{2}{n}\right) < 0$ for any $n \geq 2$. For RESE 4, the proof is similar.

A.22. **Proof of Lemma 21 (total surplus).** By the definition of $v^{\min}$, the total consumer surplus in the first period is

$$\begin{aligned}\Sigma_1 &= \int_{v^{\min}}^{1}(v-p_1)dv = \left(\frac{v^2}{2}-p_1 v\right)\Bigg|_{v^{\min}}^{1} = \frac{1}{2} - p_1 - \frac{\left(v^{\min}\right)^2}{2} + p_1 v^{\min} \\ &= \frac{1}{2}\left[1-\left(v^{\min}\right)^2\right] - p_1(1-v^{\min}) = (1-v^{\min})\left[\frac{1}{2}(1+v^{\min})-p_1\right].\end{aligned}$$

The total surplus in the second period is

$$\Sigma_2 = \int_{p_2}^{\beta v^{\min}}(\tilde{v}-p_2)\frac{d\tilde{v}}{\beta} = \frac{1}{\beta}\left(\frac{\tilde{v}^2}{2}-p_2\tilde{v}\right)\Bigg|_{p_2}^{\beta v^{\min}} = \frac{1}{\beta}\left(\beta v^{\min}-p_2\right)\left(\frac{\beta v^{\min}+p_2}{2}-p_2\right) = \frac{\left(\beta v^{\min}-p_2\right)^2}{2\beta}.$$

Hence, $\Sigma^* = \Sigma_1 + \Sigma_2 = (1-v^{\min})\left[\frac{1}{2}(1+v^{\min})-p_1\right] + \frac{\left(\beta v^{\min}-p_2\right)^2}{2\beta}$, with $v^{\min} = v^*$ for a RESE.

A.23. **Proof of Proposition 22 (Monotonicity of surplus and welfare).** *1. Monotonicity of Σ^* in n.* By Lemma 21, the derivatives $\frac{\partial\Sigma_1}{\partial n}$ and $\frac{\partial\Sigma_2}{\partial n}$ are

$$\begin{aligned}\frac{\partial\Sigma_1}{\partial n} &= \frac{1}{2}\frac{\partial v^*}{\partial n}(1-v^*) - \frac{\partial v^*}{\partial n}\left(\frac{1}{2}(1+v^*)-p_1\right) = -\frac{\partial v^*}{\partial n}(v^*-p_1); \\ \frac{\partial\Sigma_2}{\partial n} &= \frac{1}{\beta}(\beta v^*-p_2^*)\left(\beta\frac{\partial v^*}{\partial n}-\frac{\partial p_2^*}{\partial n}\right) = \frac{\partial v^*}{\partial n}(\beta v^*-p_2^*) - \frac{\partial p_2^*}{\partial n}\left(v^*-\frac{p_2^*}{\beta}\right).\end{aligned}$$

For a RESE, $\frac{\partial\Sigma_1}{\partial n} \leq 0$ since $\frac{\partial v^*}{\partial n} \geq 0$ (Proposition 12) and $v^* \geq p_1$; and $\frac{\partial\Sigma_2}{\partial n} \geq 0$ since, by Lemma 4, $\beta v^* \geq \beta p_1 > p_2^*$ and $\frac{\partial p_2^*}{\partial n} \leq 0$ since, by (3), $p_2^* = s \vee [\beta(1-Y^*)]$ and, by Propositions 12 and 17, $\frac{\partial Y^*}{\partial n} \geq 0$.

Using the expressions for $\frac{\partial\Sigma_1}{\partial n}$ and $\frac{\partial\Sigma_2}{\partial n}$, we can write

$$\frac{\partial\Sigma^*}{\partial n} = \frac{\partial v^*}{\partial n}\left[\beta v^* - p_2^* - (v^*-p_1)\right] - \frac{\partial p_2^*}{\partial n}\left(v^*-\frac{p_2^*}{\beta}\right). \tag{39}$$

By the definition of v^*, the surpluses of a consumer with valuation $v = v^*$ are equal in both periods: $\sigma_1|_{v=v^*} = v^* - p_1 = \sigma_2|_{v=v^*} = \rho(\beta v^* - p_2^*)$. Therefore, since $\rho < 1$, the square bracket in (39) is positive. Then, since $\frac{\partial v^*}{\partial n} \geq 0$ and $\frac{\partial p_2^*}{\partial n} \leq 0$, equation (39) yields $\frac{\partial\Sigma^*}{\partial n} \geq 0$ for a RESE. By Theorems 5, 9, and Proposition 12, Σ^* is constant in n for RESE 2 and 4 ($\frac{\partial v^*}{\partial n} = \frac{\partial p_2^*}{\partial n} = 0$) and monotonically increasing for RESE 1 ($\frac{\partial v^*}{\partial n} = 0, \frac{\partial p_2^*}{\partial n} < 0$) and 3 ($\frac{\partial v^*}{\partial n} > 0, \frac{\partial p_2^*}{\partial n} < 0$).

Monotonicity of Σ^* in ρ. By Theorem 5, Σ^* does not depend on ρ for RESE 1 and 2 (no intertemporal effects). In general, using the same approach as for $\frac{\partial\Sigma^*}{\partial n}$, we can write for a RESE $\frac{\partial\Sigma_1}{\partial\rho} = -\frac{\partial v^*}{\partial\rho}(v^*-p_1) \leq 0$, $\quad \frac{\partial\Sigma_2}{\partial\rho} = \frac{\partial v^*}{\partial\rho}(\beta v^*-p_2^*) - \frac{\partial p_2^*}{\partial\rho}\left(v^*-\frac{p_2^*}{\beta}\right)$. Due to the side effect of increasing ρ ($\frac{\partial p_2^*}{\partial\rho} \geq 0$), it is not obvious that $\frac{\partial\Sigma_2}{\partial\rho} \geq 0$. The derivative of total surplus is

$$\frac{\partial\Sigma^*}{\partial\rho} = \frac{\partial v^*}{\partial\rho}\left[\beta v^* - p_2^* - (v^*-p_1)\right] - \frac{\partial p_2^*}{\partial\rho}\left(v^*-\frac{p_2^*}{\beta}\right). \tag{40}$$

For RESE 4, we have $\frac{\partial p_2^*}{\partial\rho} = 0$ and, by Proposition 17, $\frac{\partial v^*}{\partial\rho} > 0$ yielding $\frac{\partial\Sigma^*}{\partial\rho} > 0$.

2. Monotonicity of W^*. Recall that *for RESE 1,* $v^* = 1$ and $p_2^* = \frac{1}{n+1}(\beta + nc)$, yielding, by Lemma 21, $\Sigma^* = \frac{1}{2\beta}\left(\beta - \frac{1}{n+1}(\beta+nc)\right)^2 = \frac{1}{2\beta}\left(\frac{n}{n+1}(\beta - c)\right)^2$ and

$$W^* = \Sigma^* + nr^* = \left(\frac{n^2}{(n+1)^2} + \frac{2n}{(n+1)^2}\right)\frac{(\beta-c)^2}{2\beta} = \frac{(n+1)^2-1}{(n+1)^2}\frac{(\beta-c)^2}{2\beta},$$

which is increasing in n and constant in ρ.

For RESE 2, $v^* = p_1, \Sigma_2 = 0$ (no second period), $\Sigma^* = \frac{1}{2}(1-p_1)^2$, and

$$W^* = \Sigma^* + nr^* = \frac{1}{2}(1-p_1)^2 + (p_1 - c)(1-p_1) = \frac{1}{2}(1-p_1)(1+p_1-2c),$$

which is constant in both n and ρ.

For RESE 4, $v^* = \frac{p_1 - \rho s}{1-\rho\beta}, p_2^* = s$, yielding Σ^* that is constant in n. Then, $W^* = \Sigma^* + nr^* = \Sigma^* + \frac{p_1 - s}{n}(1-v^*)$ is monotonically decreasing in n.

By Lemma 21, $\frac{\partial \Sigma^*}{\partial \rho}$ can be written as follows:

$$\frac{\partial \Sigma^*}{\partial \rho} = \frac{\partial \Sigma^*}{\partial v^*}\frac{\partial v^*}{\partial \rho} = \frac{\partial}{\partial v^*}\left[\frac{1}{2}(1-(v^*)^2) - p_1(1-v^*) + \frac{(\beta v^* - s)^2}{2\beta}\right]\frac{\partial v^*}{\partial \rho} = \left[-v^* + p_1 + \beta v^* - s\right]\frac{\partial v^*}{\partial \rho}.$$

Then $\frac{\partial W^*}{\partial \rho} = \frac{\partial v^*}{\partial \rho}\left[-v^* + p_1 + \beta v^* - s - \frac{p_1 - s}{n}\right]$. Since $\frac{\partial v^{*,4}}{\partial \rho} > 0$, the sign of $\frac{\partial W^*}{\partial \rho}$ coincides with the sign of $[\cdot]$, which can be written as $[\cdot] = v^*(\beta - 1) + \frac{n-1}{n}(p_1 - s)$. Hence, using $v^* = \frac{p_1 - \rho s}{1-\rho\beta}$, inequality $\frac{\partial W^*}{\partial \rho} \gtreqless 0$ is equivalent to $(p_1 - \rho s)(\beta - 1) + \frac{n-1}{n}(p_1 - s)(1-\rho\beta) \gtreqless 0$. After collecting the terms with ρ, the latter inequality becomes $\frac{n-1}{n}(p_1 - s) - p_1(1-\beta) \gtreqless \rho\left(\frac{n-1}{n}(p_1-s)\beta - (1-\beta)s\right)$ or as $p_1\beta - s - \frac{1}{n}(p_1 - s) \gtreqless \left(p_1\beta - s - \frac{\beta}{n}(p_1 - s)\right)\rho$, which, since $p_1\beta > s$, can be written as

$$1 - \frac{1}{n}\frac{p_1 - s}{p_1\beta - s} \gtreqless \left(1 - \frac{\beta}{n}\frac{p_1 - s}{p_1\beta - s}\right)\rho. \tag{41}$$

Consider two cases. If $1 > \frac{1}{n}\frac{p_1 - s}{p_1\beta - s}$, then $1 > \frac{\beta}{n}\frac{p_1-s}{p_1\beta - s}$ and (41) is equivalent to $\rho^+ \gtreqless \rho$, where ρ^+ is defined in part (2.2). If $1 \le \frac{1}{n}\frac{p_1-s}{p_1\beta-s}$, then "$>$" in (41) cannot hold for any $\rho \in (0,1)$, but "$\le$" holds for all $\rho \in [0,1)$. Thus, in the latter case, we can define ρ^+ as 0.

A.24. **Proof of Corollary 23 (RESE 3, non-monotonicity of the total surplus in ρ).** Proof is immediate from the following lemma:

Lemma 29. *If $\beta = 1$, given that RESE is unique,*

1. *for $\rho \to 1$ and $n < \bar{n} = \frac{1-p_1}{p_1 - c}$, $\frac{\partial \Sigma^*}{\partial \rho} = -n^2(p_1 - c)^2 < 0$;*
2. *for $\rho = 0$,*
   - (2.1) $\frac{\partial \Sigma^*}{\partial \rho} = \frac{1}{8}(p_1 - c) > 0$ *for $n = 1$; and*
   - (2.2) $\frac{\partial \Sigma^*}{\partial \rho} = \frac{Y^* - (1-p_1)}{Y^* + (1-p_1)}Y^*\left[Y^* - (1-c)\right] > 0$ *for $n = \infty$, where Y^* is the larger root of the equation*

$$Y^2 - Y(1 - p_1 + 1 - c) + (1-p_1)^2 = 0. \tag{42}$$

Proof: By (40) with $\beta = 1$, the derivative $\frac{\partial \Sigma^*}{\partial \rho}$ is

$$\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = \frac{\partial v^*}{\partial \rho}\left[p_1 - p_2^*\right] - \frac{\partial p_2^*}{\partial \rho}\left(v^* - p_2^*\right). \tag{43}$$

Part (1): By part (2) of Proposition 15, RESE 3 is realized if $\rho \to 1$ and $n < \bar{n}$. Then $p_2^* \to p_1$ and, if we show that $\frac{\partial v^*}{\partial \rho} < \infty$, equation (43) will become

$$\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = -\frac{\partial p_2^*}{\partial \rho}\left(v^* - p_1\right), \tag{44}$$

requiring the expressions for $\lim_{\rho\to 1} v^*|_{\beta=1}$ and $\lim_{\rho\to 1} \left.\frac{\partial p_2^*}{\partial \rho}\right|_{\beta=1}$. By (29),

$$\frac{\partial v^*}{\partial \rho} = -\frac{\partial Y^*}{\partial \rho} \cdot \frac{2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right)}{\frac{1}{n+1}\left[nY^* + (n-1)\left(\frac{p_1}{\beta} - 1\right)\right]}, \tag{45}$$

which, after substituting $\beta = 1$, $\lim_{\rho\to 1} \left.\frac{\partial Y^*}{\partial \rho}\right|_{\beta=1} = -n(p_1 - c)$ (by Lemma 28), canceling $\frac{1}{n+1}$, and considering the limit as $\rho \to 1$, becomes $\lim_{\rho\to 1} \left.\frac{\partial v^*}{\partial \rho}\right|_{\beta=1} = n(p_1 - c)\cdot \lim_{\rho\to 1} \left.\frac{2Y^*(n+1) - n(2 - v^* - c)}{nY^* - (n-1)(1-p_1)}\right|_{\beta=1}$. Using $\lim_{\rho\to 1} Y^*|_{\beta=1} = 1 - p_1$, we see that the denominator tends to $1 - p_1$. Thus, $\lim_{\rho\to 1} \left.\frac{\partial v^*}{\partial \rho}\right|_{\beta=1}$ is finite for any $n < \bar{n}$. Using (44) and $\lim_{\rho\to 1} v^* = p_1 + n(p_1 - c)$ (Proposition 15), we get

$$\begin{aligned}\lim_{\rho\to 1} \left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} &= \lim_{\rho\to 1}\left[-\frac{\partial p_2^*}{\partial \rho}\left(p_1 + n(p_1 - c) - p_1\right)\right] = \lim_{\rho\to 1}\left[-\frac{\partial p_2^*}{\partial \rho} n(p_1 - c)\right] \\ &= n(p_1 - c)\lim_{\rho\to 1}\frac{\partial Y^*}{\partial \rho} = -n^2(p_1 - c)^2.\end{aligned}$$

Part (2): Equation (43) with $\rho = 0$ (implying $v^* = p_1$) is

$$\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = (p_1 - p_2^*)\left[\frac{\partial v^*}{\partial \rho} - \frac{\partial p_2^*}{\partial \rho}\right] = \left[Y^* - (1 - p_1)\right]\left[\frac{\partial v^*}{\partial \rho} + \frac{\partial Y^*}{\partial \rho}\right]. \tag{46}$$

The derivative of (21) in ρ (with $\beta = 1$) results in

$$\frac{\partial v^*}{\partial \rho} = \frac{1}{(1-\rho)^2}\left[-\left(p_2^* + \rho\frac{\partial p_2^*}{\partial \rho}\right)(1-\rho) + (p_1 - \rho p_2^*)\right] = \frac{1}{(1-\rho)^2}\left[p_1 - p_2^* + \rho(1-\rho)\frac{\partial p_2^*}{\partial \rho}\right],$$

which, for $\rho = 0$, given $p_2^* = 1 - Y^*$, is $\left.\frac{\partial v^*}{\partial \rho}\right|_{\rho=0} = Y^* - (1 - p_1)$. By (45) with $\beta = 1$ and $\rho = 0$,

$$\frac{\partial Y^*}{\partial \rho} = -\frac{\partial v^*}{\partial \rho} \cdot \frac{nY^* - (n-1)(1-p_1)}{2Y^*(n+1) - n(1 - p_1 + 1 - c)},$$

and (46) becomes

$$\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = \left[Y^* - (1 - p_1)\right]^2\left[1 - \frac{nY^* - (n-1)(1-p_1)}{2Y^*(n+1) - n(1 - p_1 + 1 - c)}\right]. \tag{47}$$

Part (2.1): For $n = 1$, Corollary 6 with $\beta = 1$ and $\rho = 0$ yields $Y^* = 1 - \frac{1}{2}(c + p_1)$ and (47) is

$$\frac{\partial \Sigma^*}{\partial \rho} = \left[\frac{1}{2}(p_1 - c)\right]^2\left[1 - \frac{Y^*}{2Y^* - [2 - (p_1 + c)] + 2Y^*}\right] = \frac{1}{8}(p_1 - c)^2.$$

Part (2.2): After collecting the terms with n and passing to the limit as $n \to \infty$, (47) becomes

$$\lim_{n\to\infty}\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = \left[Y^* - (1 - p_1)\right]^2\left[1 - \frac{Y^* - (1 - p_1)}{2Y^* - (1 - p_1 + 1 - c)}\right] = \frac{\left[Y^* - (1 - p_1)\right]^2\left[Y^* - (1 - c)\right]}{2Y^* - (1 - p_1 + 1 - c)},$$

where (by (23) with $\beta = 1, \rho = 0$, and $n \to \infty$) Y^* is the larger root of (42), which implies that $Y^* - (1 - p_1 + 1 - c) = -\frac{(1-p_1)^2}{Y^*}$ and

$$\lim_{n\to\infty} \frac{\partial \Sigma^*}{\partial \rho}\bigg|_{\beta=1} = [Y^* - (1-p_1)]^2 \frac{Y^* - (1-c)}{Y^* - \frac{(1-p_1)^2}{Y^*}}$$

yielding the result of part (2.2). Note also, that by Corollary 7, for $n = \infty$, the second period is never profitable ($Y^* > 1 - \frac{c}{\beta} \Leftrightarrow p_2^* < c$) implying that for $\rho = 0$, $\lim_{n\to\infty} \frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} > 0$.

A.25. **Proof of Corollary 24 (Non-monotonicity of W^* in n).** By the definition of W^*, $\frac{\partial W^*}{\partial n} = \frac{\partial \Sigma^*}{\partial n} + \frac{\partial [nr^*]}{\partial n}$. Then, using (39) for $\frac{\partial \Sigma^*}{\partial n}$, (33) for $\frac{\partial [nr^*]}{\partial n}$, and equalities $\frac{\partial v^*}{\partial n} = \frac{\rho\beta}{1-\rho\beta}\frac{\partial Y^*}{\partial n}$ (by (21)) and $\frac{\partial p_2^*}{\partial n} = -\beta\frac{\partial Y^*}{\partial n}$, we get

$$\frac{\partial W^*}{\partial n} = \frac{\partial Y^*}{\partial n}\frac{\beta}{1-\rho\beta}\left\{\rho\left[\beta v^* - p_2^* - (v^* - p_1)\right] + (\beta v^* - p_2^*)\left(\frac{1}{\beta} - \rho\right) + 2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^*\right\}.$$

Since, by Proposition 12, $\frac{\partial Y^*}{\partial n} > 0$ for RESE 3, the sign of $\frac{\partial W^*}{\partial n}$ coincides with the sign of the curly bracket in the RHS, i.e., $\frac{\partial W^*}{\partial n} \gtreqless 0$ is equivalent to

$$\rho\left[\beta v^* - p_2^* - (v^* - p_1)\right] + (\beta v^* - p_2^*)\left(\frac{1}{\beta} - \rho\right) + 2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^* \gtreqless 0,$$

which, after canceling like terms and, by (3), substitution $Y^* = 1 - \frac{p_2^*}{\beta}$, becomes $\frac{p_2^* - c}{\beta} + [v^* - p_1 - \rho(v^* - c)] \gtreqless 0$. Then, using $v^* = \frac{p_1 - \rho p_2^*}{1-\rho\beta}$, the inequality for n^W (dependence of p_2^* on n^W is omitted) becomes $p_2^* \gtreqless c + \beta\left\{\frac{p_1 - \rho p_2^*}{1-\rho\beta}(\rho - 1) - \rho c + p_1\right\}$. Collecting the terms with p_2^*, we obtain

$$p_2^*\left[1 - \frac{(1-\rho)\rho\beta}{1-\rho\beta}\right] \gtreqless c(1-\rho\beta) + p_1\beta\left[1 - \frac{1-\rho}{1-\rho\beta}\right] \Leftrightarrow p_2^* \gtreqless c\frac{(1-\rho\beta)^2}{1 - 2\rho\beta + \rho^2\beta} + p_1\frac{\rho\beta(1-\beta)}{1-2\rho\beta+\rho^2\beta},$$

which yields the main claim (7). The RHS of (7) equals c if $\rho = 0$ or $\beta = 1$. For other values of ρ and β, the comparison of the RHS with c yields $c\frac{(1-\rho\beta)^2}{1-2\rho\beta+\rho^2\beta} + p_1\frac{\rho\beta(1-\beta)}{1-2\rho\beta+\rho^2\beta} > c \Leftrightarrow p_1\rho\beta(1-\beta) > c\left[1 - 2\rho\beta + \rho^2\beta - (1 - 2\rho\beta + \rho^2\beta^2)\right] \Leftrightarrow p_1\rho\beta(1-\beta) > c\rho^2\beta(1-\beta) \Leftrightarrow p_1 > \rho c$, which always holds.

A.26. **Proofs of auxiliary statements.**

A.26.1. *Proof of Lemma 26 (properties of the profit).* Part (1.1) can be shown by direct substitution of $y^i = 1 - \frac{s}{\beta} - Y^{-i}$ (which is strictly positive by the condition of part (1)) into the expressions for $\frac{\partial r^i}{\partial y^i}$ defined by (12) and (15):

$$\begin{aligned}
\frac{\partial r^i}{\partial y^i}\bigg|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} - 0} &= \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta\left(1 - \frac{s}{\beta} - Y^{-i}\right) + Y^{-i}\frac{(p_1 - \beta)(1 - v^{\min})}{\left(1 - \frac{s}{\beta}\right)^2} \\
&= -c - \beta v^{\min} + 2s + Y^{-i}\left(\beta + \frac{(p_1-\beta)(1-v^{\min})}{\left(1-\frac{s}{\beta}\right)^2}\right), \\
\frac{\partial r^i}{\partial y^i}\bigg|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0} &= -c + s + Y^{-i}\frac{(p_1 - s)\left(1 - v^{\min}\right)}{\left(1 - \frac{s}{\beta}\right)^2}.
\end{aligned}$$

These expressions imply that part (1.1) holds if and only if

$$s - \beta v^{\min} + Y^{-i}\left(\beta + \frac{(p_1-\beta)(1-v^{\min})}{\left(1-\frac{s}{\beta}\right)^2}\right) < Y^{-i}\frac{(p_1 - s)\left(1 - v^{\min}\right)}{\left(1 - \frac{s}{\beta}\right)^2},$$

which is equivalent to

$$s-\beta v^{\min} < Y^{-i}\left[\frac{(\beta-s)\left(1-v^{\min}\right)}{\left(1-\frac{s}{\beta}\right)^2}-\beta\right] = Y^{-i}\left[\frac{\beta(1-v^{\min})}{1-\frac{s}{\beta}}-\beta\right] = \frac{Y^{-i}(s-\beta v^{\min})}{1-\frac{s}{\beta}},$$

which holds because $s < \beta v^{\min}$ and, by condition of part (1), $Y^{-i} < 1 - s/\beta$.

Since r^i is continuous, i.e. $r^i\big(1-\frac{s}{\beta}-Y^{-i}-0\big) = r^i\big(1-\frac{s}{\beta}-Y^{-i}+0\big)$, and we can show part (1.2) using either (10) or (14). From (14), $r^i\Big(1-\frac{s}{\beta}-Y^{-i}\Big)$ is

$$\Big(1-\frac{s}{\beta}-Y^{-i}\Big)\left[s-c+\frac{(p_1-s)\left(1-v^{\min}\right)}{1-\frac{s}{\beta}}\right] = \Big(1-\frac{s}{\beta}-Y^{-i}\Big)(c-s)\left[\frac{(p_1-s)\left(1-v^{\min}\right)}{\left(1-\frac{s}{\beta}\right)(c-s)}-1\right],$$

which yields the result of part (1.2).

For part (1.3), rewrite the second derivative (13) of r^i as $\frac{\partial^2 r^i}{\partial (y^i)^2} = -\frac{2}{Y^3}\left[\beta Y^3 + (p_1-\beta)(1-v^{\min})Y^{-i}\right]$. Since $Y \geq 0$, the RHS of this equation is negative (r^i is strictly concave) if and only if $\beta Y^3 + (p_1 - \beta)(1-v^{\min})Y > 0$. Equality $Y = 1 - v^{\min}$ holds only at the left boundary of the domain of the profit function. For all other points in the domain $Y > 1 - v^{\min} \geq 0$ and we have

$$\begin{aligned}\beta Y^3 + (p_1-\beta)(1-v^{\min})Y &> \beta(1-v^{\min})^2 Y + (p_1-\beta)(1-v^{\min})Y \\ &= [\beta(1-v^{\min}) + p_1 - \beta](1-v^{\min})Y = [p_1 - \beta v^{\min}](1-v^{\min})Y \geq 0\end{aligned}$$

if $p_1 \geq \beta v^{\min}$ (a sufficient condition for strict concavity of r^i).

Suppose $p_1 < \beta v^{\min}$. Although r^i may be non-concave in this case, $\frac{\partial^2 r^i}{\partial (y^i)^2} = -2\beta\Big[1+(\frac{p_1}{\beta}-1)(1-v^{\min})\frac{Y^{-i}}{Y^3}\Big]$ is monotonically decreasing in y^i. Therefore, if r^i has an inflection point, this point is unique and corresponds to the total supply level $\tilde{Y}$ such that $\tilde{Y}^3 = (1-\frac{p_1}{\beta})(1-v^{\min})Y^{-i}$.

Consider an extension $\tilde{r}^i$ of r^i in the form (10) to the domain $y^i \geq (1 - v^{\min} - Y^{-i})^+$. In terms of the total supply, this domain is equivalent to $Y \geq (1-v^{\min}) \vee Y^{-i}$. We will prove that $\tilde{r}^i$ is pseudoconcave implying the claim of part (1.3) for the case of $p_1 < \beta v^{\min}$.

Equation (10), divided through by y^i, implies that $\tilde{r}^i = 0$ if and only if $y^i = 0$ or $\beta\,(1-Y) - c + \beta\left(1-v^{\min}\right) + \frac{(p_1-\beta)(1-v^{\min})}{Y} = 0$. After multiplying by $-Y/\beta$, this equation becomes

$$Y^2 - \Big(2-\frac{c}{\beta}-v^{\min}\Big)Y + \Big(1-\frac{p_1}{\beta}\Big)(1-v^{\min}) = 0. \tag{48}$$

Its properties are explored in the following lemma.

Lemma 30. *For any feasible values of $c, s, v^{\min}$, and $p_1 < \beta$, the real roots $Y_{1,2}$ of equation (48) exist and satisfy the conditions: $0 \leq Y_1 \leq 1 - v^{\min} < Y_2 \leq 2 - \left(\frac{c}{\beta} + v^{\min}\right)$ with $Y_1 = 1 - v^{\min}$ only if $v^{\min} = 1$.*

By Lemma 30, the roots $Y_{1,2}$ of (48) always exist and $0 \leq Y_1 \leq 1 - v^{\min} < Y_2$, where $Y_1 < 1 - v^{\min}$ unless $v^{\min} = 1$. Using these roots, we can express $\tilde{r}^i$ as the following function of Y: $\tilde{r}^i = -\frac{\beta}{Y}(Y - Y^{-i})(Y - Y_1)(Y - Y_2)$. Moreover, by (48), $Y_1 Y_2 = \Big(1 - \frac{p_1}{\beta}\Big)(1 - v^{\min})$, and the inflection point has the form $\tilde{Y}^3 = Y_1 Y_2 Y^{-i}$, i.e., $\tilde{Y}$ is the geometric mean of Y_1, Y_2, and Y^{-i}. Since the second derivative is decreasing, $\tilde{r}^i$ is strictly concave to the right of $\tilde{Y} - Y^{-i}$.

There are three possible locations of Y^{-i} relative to $Y_1 < Y_2$. First, if $Y^{-i} \geq Y_2$, then $1 - v^{\min} < Y^{-i}$, $\tilde{Y} < Y^{-i}$, and $\tilde{r}^i$ is nonpositive and strictly concave for all $y^i \geq (1 - v^{\min} - Y^{-i})^+$. In this case, the claim of part (1.3) holds.

Second, if $Y^{-i} \leq Y_1$, then $Y^{-i} \leq 1 - v^{\min}$, $\tilde{Y} < Y_2$, $\tilde{r}^i$ is nonnegative for $(1 - v^{\min} - Y^{-i})^+ \leq y^i \leq Y_2 - Y^{-i}$ and nonpositive for $y^i \geq Y_2 - Y^{-i}$. Since $\tilde{r}^i$ is concave for $y^i \geq \tilde{Y} - Y^{-i}$ and changes its sign from positive to negative at $Y_2 - Y^{-i} \geq \tilde{Y} - Y^{-i}$, it is also decreasing for all $y^i \geq Y_2 - Y^{-i}$. However, when $1 - v^{\min} < \tilde{Y}$, $\tilde{r}^i$ is convex in the interval $[1 - v^{\min} - Y^{-i}, \tilde{Y} - Y^{-i}]$.

Third, if $Y_1 < Y^{-i} < Y_2$, it is still true that $\tilde{Y} < Y_2$, $\tilde{r}^i$ is nonnegative for $(1 - v^{\min} - Y^{-i})^+ \leq y^i \leq Y_2 - Y^{-i}$, and nonpositive, decreasing and strictly concave for $y^i \geq Y_2 - Y^{-i}$. It is also true that, when $(1 - v^{\min}) \vee Y^{-i} < \tilde{Y}$, $\tilde{r}^i$ is convex in the interval $[(1 - v^{\min} - Y^{-i})^+, \tilde{Y} - Y^{-i}]$.

We combine the cases two and three by observing that in both of them $\tilde{r}^i$ is nonnegative for $[(1 - v^{\min} - Y^{-i})^+, Y_2 - Y^{-i}]$ and decreasing as well as concave for $y^i \geq Y_2 - Y^{-i}$. Thus, there is no local minimum for $y^i \geq Y_2 - Y^{-i}$. We complete the proof of part (1.3) using the following lemma.

Lemma 31. *If $\tilde{r}^i$ has an internal (local) minimum $(y^i)_{\min}$, then $\tilde{r}^i((y^i)_{\min}) < 0$.*

Lemma 31 implies that $\tilde{r}^i$ has no local minimum in the interval $((1 - v^{\min} - Y^{-i})^+, Y_2 - Y^{-i})$. Thus, $\tilde{r}^i$ has no internal minima in its entire domain, is strictly increasing when it is convex and, therefore, is pseudoconcave.

Parts (1.4) and (2) follow directly from (15).

Part (1.5) immediately follows from parts (1.3) and (1.4). Indeed, condition $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} + 0} \leq 0$ implies that r^i is decreasing for $y^i \geq 1 - \frac{s}{\beta} - Y^{-i}$ (by concavity on this interval). Combining this observation with pseudoconcavity for $y^i \leq 1 - \frac{s}{\beta} - Y^{-i}$, we obtain pseudoconcavity for the entire domain. Similarly, $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - \frac{s}{\beta} - Y^{-i} - 0} \geq 0$ implies that r^i is strictly increasing for $y^i \leq 1 - \frac{s}{\beta} - Y^{-i}$, again, leading to pseudoconcavity for the entire domain.

A.26.2. *Proof of Lemma 27.* The equilibrium profit, using (10) with $y^i = \frac{Y^*}{n}$ and the expression for $v^{\min} = v^*$, is $r^* = \frac{Y^*}{n}\left[\beta\,(1 - Y^*) - c + \left(\beta + \frac{p_1 - \beta}{Y^*}\right)\left(1 - \frac{p_1 - \rho\beta(1 - Y^*)}{1 - \rho\beta}\right)\right]$. After factoring out $\frac{\beta}{n(1-\rho\beta)}$ and collecting the terms with different powers of Y^*, we obtain (26).

A.26.3. *Proof of Lemma 28.* The expression for $\lim_{\rho\to 1} \frac{\partial Y^*}{\partial \rho}$ at $\beta = 1$ can be found by the implicit differentiation in (23). For brevity, we omit explicit notation indicating $\beta = 1$ throughout the proof. Denote $b_1(\rho) \triangleq \frac{(1-c)n(1-\rho) + (1-p_1)n - (p_1 - 1)\rho(n-1)}{n+1-\rho}$ and differentiate (23) with respect to ρ to obtain $2Y^* \frac{\partial Y^*}{\partial \rho} - \frac{\partial Y^*}{\partial \rho} b_1(\rho) - Y^* \frac{\partial b_1(\rho)}{\partial \rho} + \frac{\partial}{\partial \rho}\left[\frac{(1-p_1)^2(n-1)}{n+1-\rho}\right] = 0$ and

$$\frac{\partial Y^*}{\partial \rho}\left[2Y^* - b_1(\rho)\right] = Y^* \frac{\partial b_1(\rho)}{\partial \rho} - \frac{(1-p_1)^2(n-1)}{(n+1-\rho)^2}, \tag{49}$$

The limits are $\lim_{\rho\to 1} b_1(\rho) = \frac{(1-p_1)n + (1-p_1)(n-1)}{n} = 2(1-p_1) - \frac{1-p_1}{n}$, resulting in $\lim_{\rho\to 1}[2Y^* - b_1(\rho)] = \frac{1-p_1}{n}$, and for $\frac{\partial b_1(\rho)}{\partial \rho} = \frac{[-(1-c)n + (1-p_1)(n-1)](n+1-\rho) + (1-c)n(1-\rho) + (1-p_1)n + (1-p_1)\rho(n-1)}{(n+1-\rho)^2}$, it is $\lim_{\rho\to 1} \frac{\partial b_1(\rho)}{\partial \rho} = \frac{1}{n^2}\left\{[-(1-c)n + (1-p_1)(n-1)]\,n + (1-p_1)n + (1-p_1)(n-1)\right\} = \frac{1}{n^2}\left\{(1-p_1)(n-1) - (p_1 - c\right.$
The limit of the RHS of (49) is $\frac{1-p_1}{n^2}\left\{(1-p_1)(n-1) - (p_1 - c)n^2\right\} - \frac{(1-p_1)^2(n-1)}{n^2} = -(1-p_1)(p_1 - c)$. Then, from (49), we obtain the claim of the lemma.

A.26.4. *Proof of Lemma 30 (the roots of $r^i(Y) = 0$).* The discriminant of (48) is $D = \left(2 - \frac{c}{\beta} - v^{\min}\right)^2 - 4\left(1 - \frac{p_1}{\beta}\right)(1 - v^{\min}) \geq \left(2 - \frac{c}{\beta} - v^{\min}\right)^2 - 4\left(1 - \frac{c}{\beta}\right)(1 - v^{\min}) = \left(v^{\min} - \frac{c}{\beta}\right)^2 \geq 0$, where the first inequality is strict unless $v^{\min} = 1$ because $p_1 > c$, while the second inequality is strict unless $v^{\min} = \frac{c}{\beta}$. Therefore, $D > 0$, the real roots given by $Y_{1,2} = \frac{1}{2}\left(2 - \frac{c}{\beta} - v^{\min} \pm \sqrt{D}\right)$ always exist, and $Y_1 < Y_2$. Since $p_1 < \beta$, we have $4\left(1 - \frac{p_1}{\beta}\right)(1 - v^{\min}) \geq 0$ and $Y_{1,2} \in \left(0, 2 - \frac{c}{\beta} - v^{\min}\right)$.

If $v^{\min} = 1$, the roots are $Y_1 = 0$, $Y_2 = 1 - \frac{c}{\beta}$, and the claim of the lemma holds.

If $v^{\min} < 1$, then $D > \left(v^{\min} - \frac{c}{\beta}\right)^2$, and an upper bound on Y_1 is $Y_1 < 1 - \frac{1}{2}\left(\frac{c}{\beta} + v^{\min}\right) - \frac{1}{2}\left|v^{\min} - \frac{c}{\beta}\right| = 1 - \max\left\{\frac{c}{\beta}, v^{\min}\right\} \leq 1 - v^{\min}$, which, in turn, is a lower bound on Y_2 : $Y_2 > 1 - \frac{1}{2}\left(\frac{c}{\beta} + v^{\min}\right) + \frac{1}{2}\left|v^{\min} - \frac{c}{\beta}\right| = 1 - \min\left\{\frac{c}{\beta}, v^{\min}\right\} \geq 1 - v^{\min}$.

A.26.5. *Proof of Lemma 31.* Function $\tilde{r}^i$, its first and second derivatives are given, respectively, by (9), (11) and (13). If an internal local minimum of $\tilde{r}^i$ exists, it must satisfy the necessary second-order optimality conditions

$$\left.\frac{\partial \tilde{r}^i}{\partial y^i}\right|_{y^i=(y^i)_{\min}} = 0, \text{ and} \tag{50}$$

$$\left.\frac{\partial^2 \tilde{r}^i}{\partial (y^i)^2}\right|_{y^i=(y^i)_{\min}} \geq 0. \tag{51}$$

Using condition (50) and the expression for the first derivative of $\tilde{r}^i$, we obtain

$$\beta(1-Y) - c + [p_1 - \beta(1-Y)]\frac{1-v^{\min}}{Y} = -\left(y^i\right)_{\min}\beta\left[-1 + \frac{1-v^{\min}}{Y} - \left(\frac{p_1}{\beta} - (1-Y)\right)\frac{1-v^{\min}}{Y^2}\right]$$
$$= \left(y^i\right)_{\min}\beta\left[1 + \left(\frac{p_1}{\beta} - 1\right)\frac{1-v^{\min}}{Y^2}\right] \tag{52}$$

Since the LHS of (52) multiplied by y^i matches the expression for $\tilde{r}^i$, it follows that

$$\tilde{r}^i\big|_{y^i=(y^i)_{\min}} = \left(y^i\right)^2_{\min}\beta\left[1 + \left(\frac{p_1}{\beta} - 1\right)\frac{1-v^{\min}}{Y^2}\right]. \tag{53}$$

Condition (51) and the expression for the second derivative of $\tilde{r}^i$ imply that, at $y^i = \left(y^i\right)_{\min}$, $\left(\frac{p_1}{\beta} - 1\right)(1-v^{\min}) \leq -\frac{Y^3}{Y^{-i}}$. Combining this inequality with (53), we obtain $\tilde{r}^i\big|_{y^i=(y^i)_{\min}} \leq \left(y^i\right)^2_{\min}\beta\left[1 - \frac{Y}{Y^{-i}}\right] < 0$, which is strict because, here, we consider only $y^i > 0$.

Appendix B. First-period demand: general case

This section provides the derivation of the functional form of the first-period demand (2) and examines the robustness of the main results, obtained for $\gamma = 1$, with respect to variations in γ.

B.1. **Model specification.** Retailer i demand can be expressed as $d^i(y^i, \mathbf{y}^{-i}) = Dm^i(y^i, \mathbf{y}^{-i})$, where D is the total demand, $m^i(y^i, \mathbf{y}^{-i})$ — the market share of retailer i, and $\mathbf{y}^{-i}$ — the vector of inventories of the others. Since, by the assumptions of §3.1, attractions $a^i(y^i)$ are identical: $a^i(y^i) = a(y^i), i \in I$, and for a non-trivial problem some of y^i are positive, attraction vector $\mathbf{a}$ of all $a^i, i \in I$ satisfies four conditions required for the market share theorem (Bell D.E., Keeney R.L., Little J.D.C. (1975). A market share theorem. Journal of Marketing Research, 12(2), 136-141): (A1) $\mathbf{a}$ is nonnegative and nonzero: $a^j \geq 0, j \in I$, and there exists $a^i > 0$; (A2) zero attraction leads to zero market share; (A3) any two retailers with equal attraction have equal market share: $a^i(y^i) = a^j(y^j) \Rightarrow m^i(y^i, \mathbf{y}^{-i}) = m^j(y^j, \mathbf{y}^{-j})$; and (A4) the market share m^i of any retailer decreases on the same amount Δ^i if the attraction a^j of *any* other retailer j is increased by a fixed amount (Δ^i does not depend on $j \neq i$). Then, by the market share theorem applied to this symmetric case, m^i has the following functional form:

$$m^i(y^i, \mathbf{y}^{-i}) = \frac{a(y^i)}{\sum_{j \in I} a(y^j)}. \tag{54}$$

Using (54), the homogeneity of m^i (follows, by Assumption 1, from the homogeneity of d^i and D), and the continuity of $a(y)$ (Assumption 3), Lemma 2 specifies the functional form of attraction: $a(y) = a(1)(y)^\gamma$. By choosing the scale of attraction so that $a(1) = 1$, we obtain functional form (2) for demand d^i.

A feasible range for γ results from the observation that retailer i can choose y^i either not to enter the market: $y^i = 0 = a(0) = d^i(0, \mathbf{y}^{-i})$, to sell only in the first period: $y^i = \breve{y}^i \triangleq d^i(\breve{y}^i, \mathbf{y}^{-i})$, or in both: $y^i > d^i(y^i, \mathbf{y}^{-i}) \geq \breve{y}^i$ (the last inequality is strict when d^i is strictly increasing in y^i). These properties hold if d^i is concave in y^i. In extreme cases, d^i, as a function of y^i, can be a straight line with a slope less than one ($\gamma = 1$) if $y^i \geq \breve{y}^i$ or, as an opposite case, a constant if all y^i are positive and any changes in y^i are not supported by the correspondent changes in market efforts or consumers completely ignore these efforts ($\gamma = 0$).

In this model, γ is the *inventory elasticity of attraction:* $E_y(a) \triangleq \frac{\partial a}{\partial y}\frac{y}{a} = \gamma(y)^{\gamma-1}\frac{y}{(y)^\gamma} = \gamma$, or the *inventory elasticity of the first-period demand*, normalized by the market share of other retailers:

$$E_{y^i}(d^i) \triangleq \frac{\partial d^i}{\partial y^i}\frac{y^i}{d^i} = D\left[\frac{\gamma\left(y^i\right)^{\gamma-1}}{\sum_{j\in I}\left(y^j\right)^\gamma} - \frac{\left(y^i\right)^\gamma\gamma\left(y^i\right)^{\gamma-1}}{\left(\sum_{j\in I}\left(y^j\right)^\gamma\right)^2}\right]\frac{y^i}{\left(y^i\right)^\gamma D}\sum_{j\in I}\left(y^j\right)^\gamma = \gamma\left[1 - m^i\right] = \gamma\frac{\sum_{j\neq i}\left(y^j\right)^\gamma}{\sum_{j\in I}\left(y^j\right)^\gamma},$$

where $\sum_{j\neq i}\left(y^j\right)^\gamma / \sum_{j\in I}\left(y^j\right)^\gamma$ is the market share of other retailers.

The following results use some supplementary material, provided in §B.3.

B.2. **Changes in RESE structure with γ.** This section shows the effect of changing γ on the main results of this paper. For $\gamma = 1$, the structure of RESE coincides with the one described in Theorems 5 and 9. This structure continuously changes with γ by continuity of demand (2). In particular, changes in γ lead to the following effects.

I. *RESE 1 does not depend on* γ because this RESE, by the same argument as in the proof of Theorem 5, exists only when the first-period demand is zero ($v^{\min} = 1$) due to a combination of relatively high p_1, the difference $\beta - c$, the level of competition n, and strategic behavior ρ; namely, when $p_1 \geq P_1 = 1 - \frac{n}{n+1}\rho(\beta - c)$.

II. *The area of RESE 2 is decreasing in* γ, which follows from a necessary condition of existence of RESE 2 that requires the profit of a deviator from $Y^{*,2} = 1 - p_1$ be not increasing in y^i : $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \frac{1-p_1}{n}+0} \leq 0$. This inequality (§B.3.1) is equivalent to $p_1 \leq \frac{nc}{\gamma(n-1)+\beta[n(1-\gamma)+\gamma]} \triangleq P_2(\gamma)$. This bound decreases in γ from $P_2(0) = \frac{c}{\beta}$ to $P_2(1) = \frac{nc}{n-1+\beta}$ coinciding with P_2 given in Theorem 5.

III. *The area of RESE 3 is (a) increasing in* γ *along the boundary with RESE 2 and (b) decreasing along the boundary with RESE 4.* Part (a) follows from the p_1-range for RESE 3: $P_2(\gamma) < p_1 < P_1$, which results from the same geometric argument as in the proof of Theorem 5 because $Y^{*,3}(\gamma)$ is still a larger root of a quadratic equation with coefficients depending on γ (equation (61)). Part (b), for $\gamma = 0$, follows from the lack of incentive for the retailers to deviate to salvage, which is expressed in $\left.\frac{\partial r^i}{\partial y^i}\right|_{Y \geq 1-\frac{s}{\beta}} = s - c < 0$ (§B.3.1), i.e., a sufficient condition, corresponding to condition (a) in part RESE 3 of Theorem 5 always holds. The intuition is that, for $\gamma = 0$, retailers share evenly the first-period demand regardless of the inventories. Therefore, any increase in inventory does not increase the first-period market share, and possible second-period sales below cost only reduce total two-period profit. For $0 < \gamma < 1$, part (b) is checked numerically and illustrated in Figure 15 for $\gamma \in \{0, 0.4, 1\}$.

IV. *The area of RESE 4 is increasing in* γ. §B.3.1 provides a unique

$$Y^{*,4}(\gamma) = \frac{n-1}{n}\frac{p_1 - s}{c - s}\gamma(1 - v^{*,4}), \tag{55}$$

where $v^{*,4} = \frac{p_1-\rho s}{1-\rho\beta}$. This expression for $Y^{*,4}(\gamma)$ implies a sufficient condition of RESE 4 existence, namely, $\frac{n-1}{n}Y^{*,4}(\gamma) \geq 1 - \frac{s}{\beta}$ (salvaging is forced on retailers), which is

$$\gamma \geq \bar{\gamma} \triangleq \left(1 - \frac{s}{\beta}\right)\left(\frac{n}{n-1}\right)^2 \frac{c-s}{p_1-s}\frac{1-\rho\beta}{1-p_1-\rho(\beta-s)},$$

where $\bar{\gamma}$ can be sufficiently small for any feasible p_1, ρ, β, and s if c is sufficiently close to s, i.e., RESE 4 can exist for small γ but does not exist for $\gamma = 0$ (Figure 15). On the other hand, inequality $Y^{*,4} < 1 - \frac{s}{\beta}$, combined with (55), gives a sufficient condition of RESE 4 non-existence. Since $Y^{*,4}$ is increasing in n and decreasing in ρ ($v^{*,4}$ is increasing in ρ), RESE 4 does not exist for given γ and any n and ρ if $Y^{*,4} < 1 - \frac{s}{\beta}$ for $\rho = 0$ and $n \to \infty$, which is $\frac{p_1-s}{c-s}\gamma(1-p_1) < 1 - \frac{s}{\beta}$ or $\gamma < \underline{\gamma} \triangleq \frac{(1-s/\beta)(c-s)}{(p_1-s)(1-p_1)}$.

The scatterplots in Figure 15 were constructed by checking p_1-boundaries for RESE 1 and 2, and, for RESE 3 and 4, using the direct comparison of equilibrium profits with the profit of a potential deviator, according to the definition of RESE.

Since the first-period demand (2) is continuous and monotonic in γ, the case $\gamma = 0$ for RESE 3 is of a special interest as opposing to $\gamma = 1$. Although, a complete independence of market share from inventory may be an idealization for many practical settings, this case illustrates the robustness of the results of this paper and shows the direction and amplitude of the changes with respect to variations in the demand patterns. This assumption about first-period market share was used, e.g., in Liu and van Ryzin (2008), §4.4.

Proposition 32. *For $\gamma = 0$, a unique RESE 3 with $v^* = \frac{p_1+n(p_1-\rho c)}{1+n(1-\rho\beta)}, p_2^* = c + \frac{\beta p_1 - c}{1+n(1-\rho\beta)}, Y^* = \frac{1-p_1+n\left(1-\frac{c}{\beta}\right)(1-\rho\beta)}{1+n(1-\rho\beta)}$, and $r^* = \frac{1}{n}[(p_1-c)(1-v^*) + (p_2^*-c)(Y^*-1+v^*)]$ exists if and only if $\frac{c}{\beta} < p_1 < P_1$; no other equilibria exist in this area. Moreover,*

1. *$p_2^* \to c + 0$ with $n \to \infty$ for any $p_1 \in \left(\frac{c}{\beta}, P_1\right)$ or with $p_1 \to \frac{c}{\beta} + 0$ for any $n \geq 1$;*
2. *v^*, p_2^*, Y^*, and r^* are continuous at the boundaries; monotonicity of v^*, Y^* in n and ρ, and nr^* in n, stated in Proposition 12 for $\gamma = 1$ hold;*
3. *nr^* is decreasing in ρ if and only if either $n = 1$ or $p_1 \geq c + \frac{2n(\beta-c)}{(n+1)^2}$ for any $n > 1$;*
4. *nr^* attains minimum in ρ at $\rho^0 \triangleq \frac{(n+1)^2(p_1-c)-2n(\beta p_1-c)}{\beta n(n+1)(p_1-c)}$ for any $n \in (1, n^0)$, where $n^0 \triangleq \frac{\beta-p_1+\sqrt{(\beta-c)(\beta+c-2p_1)}}{p_1-c} > 2$, if and only if $p_1 < \frac{5c+4\beta}{9}$;*
5. *when $\beta = 1$, $nr^*|_{\rho\to 1} < nr^*|_{\rho=0}$ for any RESE 3 inputs; the minimum possible value of $\rho^0 = \frac{n^2+1}{n(n+1)}$ is $\bar{\rho}^0 = \rho^0\big|_{n=2} = \rho^0\big|_{n=3} = \frac{5}{6}$.*

Proposition 32 shows that, for $\gamma = 0$,

(i) RESE exists for all feasible model inputs since RESE 3 boundaries ($\frac{c}{\beta} < p_1 < P_1$) complement the boundaries of RESE 1 and 2;

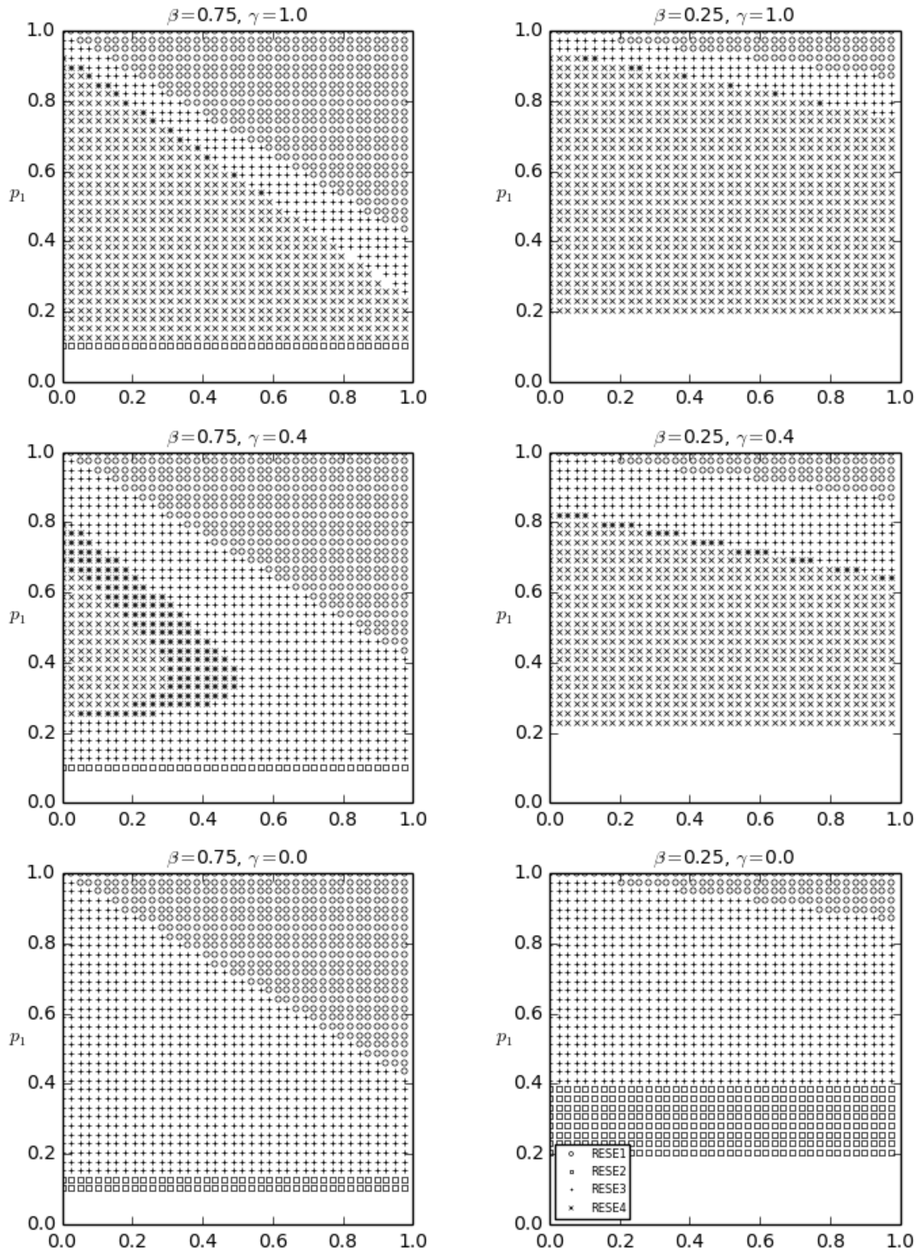
(ii) the second-period price is always above the cost for $n < \infty$;

(iii) a closed-form necessary and sufficient condition shows when nr^* is decreasing in ρ;

(iv) there exist closed-form expressions for ρ^0, the unique minimum of nr^* in ρ (part 4), and for n^0, the upper boundary of n-range where nr^* is non-monotonic in ρ;

(v) there is no effect of "boundary-value gain" (part 5); this result supports the conclusion, formulated in the discussion of Proposition 15, that under this effect, the maximum level of strategic behavior prevents the second-period sales at loss under competitive pressure ($n \geq 3$). As shown in part 1, the second-period sales are always profitable for $\gamma = 0$ because retailers have no incentive to compete for the first-period market by increasing inventories.

FIGURE 15. The (ρ, p_1)-scatterplots of the areas where a particular RESE exists for $n = 10, c = 0.1, s = 0.05$, and given γ and β



(a) $\gamma = 0.5$ (b) $\gamma = 0$

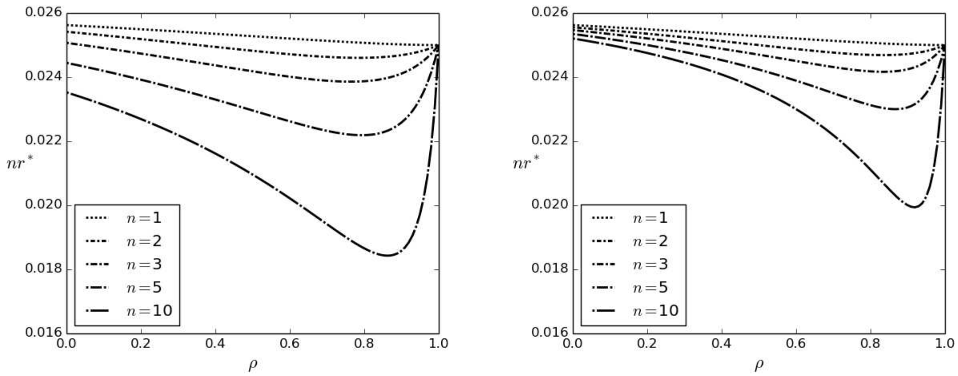


FIGURE 16. The total profit for RESE 3 with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$, $\beta = 1$

Thus, when the inventory elasticity of attraction γ decreases, the "boundary-value gain" in ρ becomes weaker (Figures 4 and 16 a) and disappears at $\gamma = 0$ (Figure 16 b); the "discontinuous gain" in ρ caused by the switch from RESE 4 to RESE 3 emerges at lower ρ (Figures 5 b and 15) and disappears at $\gamma = 0$ due to non-existence of RESE 4; the "continuous gain" in ρ (Figure 4) exists even for $\gamma = 0$. The last effect becomes less pronounced because decreasing γ weakens the first-period inventory competition and decreases the correspondent second-period losses. The point of minimum profit, ρ^0, decreases in γ (Figures 4 and 16).

B.3. **Appendix B supplement .**

B.3.1. *Profit function, its properties and inventory decisions .*
Retailer i has no sales in the second period. In this case, the general formula (1) for profit becomes $r^i = (p_1 - c)y^i$, which yields a unique profit-maximizing inventory $y^i = \breve{y}^i = d^i$. Unlike $\gamma = 1$, other retailers may have sales in the second period, implying that, in general, $\bar{\alpha} \neq 0$ and $v^{\min} \geq p_1$. Using (2) with $y^j = \frac{Y^*}{n}, j \neq i$ and $D = 1 - v^{\min}$, $\breve{y}^i$ is a root of a non-linear equation: $y^i = \frac{(1-v^{\min})\left(y^i\right)^{\gamma}}{(n-1)(Y^*/n)^{\gamma}+(y^i)^{\gamma}}$. After dividing by y^i, which eliminates the extraneous root $y^i = 0$, this equation can be written as $(n-1)\left(\frac{Y^*}{n}\right)^{\gamma} + \left(y^i\right)^{\gamma} = (1 - v^{\min})\left(y^i\right)^{\gamma-1}$ or $(n-1)\left(\frac{Y^*}{n}\right)^{\gamma} = \left(y^i\right)^{\gamma-1}(1 - v^{\min} - y^i)$, which, for $n = 1$, yields $\breve{y}^i = 1 - v^{\min}$ for any $\gamma \in [0, 1]$. When $n > 1$, this equation can be written as

$$\left(y^i\right)^{1-\gamma} = \frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}(1 - v^{\min} - y^i), \tag{56}$$

which, for $\gamma = 1$, yields $\breve{y}^i = 1 - v^{\min} - \frac{n-1}{n}Y^*$. For $\gamma < 1$, this equation has a unique positive root since the LHS is zero at $y^i = 0$ and increasing in y^i, and the RHS is a decreasing linear function in y^i, which is positive at $y^i = 0$. For $\gamma = 0$, equation (56) results in $\breve{y}^i = \frac{1-v^{\min}}{n}$, which is the maximum $\breve{y}^i$ in γ by the following lemma.

Lemma 33. *The solution of (56), $\breve{y}^i$, is decreasing in γ if $\breve{y}^i < \frac{Y^*}{n}$.*

Proof Equation (56) can be written as $\exp\left[(1-\gamma)\ln(\breve{y}^i)\right] = \exp\left[\gamma \ln\left(\frac{n}{Y^*}\right)\right]\frac{1-v^{\min}-\breve{y}^i}{n-1}$. The derivative of this equation in γ is $\left(\breve{y}^i\right)^{1-\gamma}\left[\frac{1-\gamma}{\breve{y}^i}\frac{\partial \breve{y}^i}{\partial \gamma} - \ln(\breve{y}^i)\right] = \ln\left(\frac{n}{Y^*}\right)\frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}(1 - v^{\min} - \breve{y}^i) - \frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}\frac{\partial \breve{y}^i}{\partial \gamma}$, which can be written as $\frac{\partial \breve{y}^i}{\partial \gamma}\left[\frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma} + (1-\gamma)\left(\breve{y}^i\right)^{-\gamma}\right] = \ln\left(\frac{n}{Y^*}\right)\frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}(1 - v^{\min} - \breve{y}^i) + \ln(\breve{y}^i)\left(\breve{y}^i\right)^{1-\gamma}$, where the bracket $[\cdot]$ in the LHS is positive and the RHS, by (56), becomes $\left(\breve{y}^i\right)^{1-\gamma}\left[\ln(\breve{y}^i) - \ln\left(\frac{Y^*}{n}\right)\right]$, which is negative, leading to $\frac{\partial \breve{y}^i}{\partial \gamma} < 0$■

Retailer i has sales in the second period, $p_2 > s$. Profit (1) with q^i, given by Lemma 3, becomes $r^i = p_1 d^i + p_2(y^i - d^i) - cy^i$, which, with $y^j = \frac{Y^*}{n}, j \neq i, v^{\min} = v^*$, and d^i from (2), can be written as

$$r^i = \left[p_1 - p_2(y^i)\right] \frac{(1-v^*)\left(y^i\right)^\gamma}{(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma} + \left[p_2(y^i) - c\right] y^i \tag{57}$$

where, by (3), $p_2(y^i) = \beta\left[1 - \frac{n-1}{n}Y^* - y^i\right]$. The derivative, after simplifications, is

$$\begin{aligned}\frac{\partial r^i}{\partial y^i} &= \frac{\beta(1-v^*)\left(y^i\right)^\gamma}{(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma} + [p_1 - p_2] \frac{(1-v^*)\gamma\left(y^i\right)^{\gamma-1}(n-1)\left(\frac{Y^*}{n}\right)^\gamma}{\left[(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma\right]^2} \\ &\quad -2\beta y^i + \beta\left(1 - \frac{n-1}{n}Y^*\right) - c. \end{aligned}\tag{58}$$

When $v^* = 1$, RESE takes the same form of RESE 1 as for $\gamma = 1$ (Theorem 5) because the first-period demand is zero.

When $\bar{\alpha} = 0$ and $v^* = p_1$ (no second-period sales), the necessary condition of RESE 2 existence, namely, $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \frac{1-p_1}{n}+0} \leq 0$, using formula (58) with $v^* = p_1$, becomes $\beta\frac{1-p_1}{n} + p_1(1-\beta)\gamma\frac{n-1}{n} - 2\beta\frac{1-p_1}{n} + \beta - c - \beta\frac{n-1}{n}(1-p_1) \leq 0$, which, multiplied by n, can be written as $p_1\left[(1-\beta)\gamma(n-1) + n\beta\right] \leq nc$ or $p_1 \leq \frac{nc}{\gamma(n-1)+\beta[n(1-\gamma)+\gamma]} = P_2(\gamma)$.

When $\bar{\alpha} = 1$ and $p_1 \leq v^* < 1$, a candidate for RESE 3 results from two conditions: $v^* = v^*(Y^*)$ and $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \frac{Y^*}{n}} = 0$, which, using Lemma 1 and (58), are $v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta}$ and

$$\beta\frac{1-v^*}{n} + \left[p_1 - \beta(1-Y^*)\right]\frac{(1-v^*)\gamma(n-1)}{nY^*} + \beta\left(1 - \frac{n+1}{n}Y^*\right) - c = 0. \tag{59}$$

After multiplication by $-\frac{nY^*}{\beta(n+1)}$ and collection of terms with Y^*, this equation becomes

$$(Y^*)^2 - Y^*\frac{n}{n+1}\left[\frac{1-v^*}{n}\left(1+\gamma(n-1)\right) + 1 - \frac{c}{\beta}\right] - \frac{n-1}{n+1}\gamma\left(\frac{p_1}{\beta} - 1\right)(1-v^*) = 0, \tag{60}$$

which, for $\gamma = 1$, coincides with (18). Substitution for $1 - v^* = \frac{1-p_1-\rho\beta Y^*}{1-\rho\beta}$ and collection of terms with Y^* leads to $(Y^*)^2 a_2 + Y^* a_1 + a_0 = 0$, where $a_2 = \frac{n\rho\beta(\gamma-1)+n+1-\gamma\rho\beta}{(n+1)(1-\rho\beta)} > 0$,

$$\begin{aligned} a_1 &= -\frac{[1+\gamma(n-1)](1-p_1) + n\left(1-\frac{c}{\beta}\right)(1-\rho\beta) - (n-1)\gamma\left(\frac{p_1}{\beta}-1\right)\rho\beta}{(n+1)(1-\rho\beta)}, \text{ and} \\ a_0 &= -\frac{n-1}{n+1}\gamma\left(\frac{p_1}{\beta} - 1\right)\frac{1-p_1}{1-\rho\beta}. \end{aligned}$$

After division by a_2, the last quadratic equation becomes

$$\begin{aligned}(Y^*)^2 &- \frac{(\beta-c)\,n(1-\rho\beta) + \beta[1+\gamma(n-1)](1-p_1) - \gamma\left(p_1-\beta\right)\rho\beta(n-1)}{\beta\left[n\rho\beta(\gamma-1) + n + 1 - \gamma\rho\beta\right]}Y^* \\ &- \frac{\gamma\left(p_1-\beta\right)(1-p_1)(n-1)}{\beta\left[n\rho\beta(\gamma-1) + n + 1 - \gamma\rho\beta\right]} = 0, \end{aligned}\tag{61}$$

which, for $\gamma = 1$, coincides with (23). The equilibrium inventory is the larger root of this equation since, multiplying (61) by $-a_2\frac{\beta(n+1)}{n} < 0$, we obtain the original equation (59) with substituted $v^*(Y^*)$ and multiplied by $Y^* > 0$. The LHS of this resulting equation is a quadratic function with a negative coefficient in front of $(Y^*)^2$, i.e., the LHS decreases in Y^* at the larger root, which corresponds to the maximum of profit.

Retailer i has sales in the second period, $p_2 = s$. By (57) with $p_2 = s$,

$$r^i = (p_1 - s)\frac{(1-v^*)\left(y^i\right)^\gamma}{(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma} + (s-c)y^i \quad \text{and} \tag{62}$$

$$\frac{\partial r^i}{\partial y^i} = (p_1 - s)\frac{(1-v^*)\gamma\left(y^i\right)^{\gamma-1}(n-1)\left(\frac{Y^*}{n}\right)^\gamma}{\left[(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma\right]^2} + s - c. \tag{63}$$

Profit (62) is concave in y^i since $\left(y^i\right)^\gamma$ is concave, function $\frac{Az}{B+z}$ is concave in z for any positive z, A, and B (first term of r^i) and $(s-c)y^i$ is concave.

A candidate for RESE 4 results from conditions: $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \frac{Y^*}{n}} = 0$ and $v^{*,4} = \frac{p_1 - \rho s}{1-\rho\beta}$. The latter implies the same p_1-upper bound as for $\gamma = 1$. Namely, $v^{*,4} < 1$ (there are sales in the first period) is equivalent to $p_1 < P_4 \triangleq 1-\rho(\beta - s)$. The former yields $Y^{*,4} : (p_1 - s)\frac{(1-v^*)\gamma(n-1)(Y^*/n)^{2\gamma-1}}{n^2(Y^*/n)^{2\gamma}} + s - c = 0$, which, multiplied by $\frac{Y^* n}{p_1 - s}$, gives $\frac{c-s}{p_1-s}Y^* n = (1-v^*)\gamma(n-1)$ or $Y^{*,4}(\gamma)$ in the form of (55).

B.3.2. *Proof of Proposition 32.* For $\gamma = 0$, equation (61) becomes $Y^*\left[Y^* - \frac{\beta(1-p_1)+(\beta-c)n(1-\rho\beta)}{\beta[1+n(1-\rho\beta)]}\right] = 0$ yielding a unique $Y^* > 0$. Substitution of $1 - Y^* = \frac{p_1 + n(1-\rho\beta)c/\beta}{1+n(1-\rho\beta)}$ into $p_2^* = \beta(1-Y^*)$ and $v^* = \frac{p_1 - \rho p_2^*}{1-\rho\beta}$ results in the corresponding expressions.

Condition $v^* < 1$ (there are sales in the first period) is $p_1 + n(p_1 - \rho c) < 1 + n(1-\rho\beta)$ or $p_1(n+1) < 1 + n[1-\rho(\beta - c)]$ yielding $p_1 < P_1$ — the boundary with RESE 1. Condition $v^* \geq p_1$ is $p_1 + n(p_1 - \rho c) \geq p_1 + np_1(1-\rho\beta)$ or $\rho c \leq p_1\rho\beta$, which holds for $\rho = 0$. For $\rho > 0$, it becomes $p_1 \geq \frac{c}{\beta}$.

RESE 3 exists if and only if any retailer i has no incentive to deviate neither to (i) sales in both periods with $p_2 = s$ nor to (ii) sales only in the first period. Part (i) holds since, by (63), $\left.\frac{\partial r^i}{\partial y^i}\right|_{\gamma=0} = s - c < 0$ for any y^i leading to $p_2 = s$. Part (ii) is equivalent to $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \frac{1-v^*}{n}} > 0$, which, by (58) with $\gamma = 0$, is $-2\beta\frac{1-v^*}{n} + \beta\frac{1-v^*}{n} + \beta\left(1 - \frac{n-1}{n}Y^*\right) > c$. Multiplication by $\frac{n}{\beta}$ leads to $n(1-c/\beta) - (n-1)Y^* > 1 - v^*$, and, after the substitutions of Y^* and

$$1 - v^* = \frac{1 - p_1 + n[1 - p_1 - \rho(\beta - c)]}{1 + n(1-\rho\beta)}, \tag{64}$$

the last inequality, multiplied by $1+n(1-\rho\beta) > 0$, becomes $n(1-c/\beta) + n^2(1-c/\beta)(1-\rho\beta) - (n-1)(1-p_1) - n(n-1)(1-c/\beta)(1-\rho\beta) > 1 - p_1 + n[1-p_1-\rho(\beta-c)]$ or $n(1-c/\beta) + n(1-c/\beta)(1-\rho\beta) > 2n(1-p_1) - n\rho\beta(1-c/\beta)$, which, after simplifications, yields $p_1 > \frac{c}{\beta} = P_2(0)$. This inequality implies that $v^* = p_1$ only if $\rho = 0$. Since the p_1-boundaries of RESE 3 are the negations of the boundaries of RESE 1 and RESE 2, and, for $\gamma = 0$, RESE 4 does not exist, no other equilibria exist in the area $\frac{c}{\beta} < p_1 < P_1$.

Continuity of Y^*, v^*, p_2^*, and r^* can be shown directly by substitution of the boundaries to the correspondent formulas. For example, $\left.Y^{*,3}\right|_{p_1 = c/\beta} = \frac{1-c/\beta + n(1-c/\beta)(1-\rho\beta)}{1+n(1-\rho\beta)} = 1 - c/\beta = 1 - p_1 = Y^{*,2}$.

Monotonicity of v^*, Y^*, and p_2^*. $\frac{\partial v^*}{\partial n} = \frac{(p_1-\rho c)[1+n(1-\rho\beta)] - (1-\rho\beta)[p_1 + n(p_1 - \rho c)]}{[1+n(1-\rho\beta)]^2}$ where the numerator is $\rho(p_1\beta - c) \geq 0$, so $\frac{\partial v^*}{\partial n} = 0$ only if $\rho = 0$.

$\frac{\partial v^*}{\partial \rho} = \frac{1}{[1+n(1-\rho\beta)]^2}\left\{-nc[1+n(1-\rho\beta)] + n\beta[p_1 + n(p_1 - \rho c)]\right\}$, where $\{\cdot\} = (\beta p_1 - c)(n+1) > 0$ for any RESE 3 inputs.

$\frac{\partial Y^*}{\partial n} = \frac{1}{[1+n(1-\rho\beta)]^2}\left\{(1-c/\beta)(1-\rho\beta)[1+n(1-\rho\beta)] - (1-\rho\beta)[1-p_1+n(1-c/\beta)(1-\rho\beta)]\right\}$, where $\{\cdot\} = p_1 - c/\beta > 0$.

$\frac{\partial Y^*}{\partial \rho} = \frac{1}{[1+n(1-\rho\beta)]^2} \left\{n\left(1-c/\beta\right)(-\beta)[1+n(1-\rho\beta)] + n\beta[1-p_1+n\left(1-c/\beta\right)(1-\rho\beta)]\right\}$, where $\frac{1}{n\beta}\{\cdot\} = -(p_1 - c/\beta) < 0$.

The results for Y^* imply that $p_2^* = \beta(1-Y^*)$ is decreasing in n and increasing in ρ.

Monotonicity of nr^* in n. By (57) with $\gamma = 0$,

$$nr^* = (p_1 - p_2^*)(1-v^*) + (p_2^* - c)Y^*. \tag{65}$$

Then $\frac{\partial(nr^*)}{\partial n} = -\frac{\partial p_2^*}{\partial n}(1-v^*) - \frac{\partial v^*}{\partial n}(p_1 - p_2^*) + \frac{\partial Y^*}{\partial n}(p_2^* - c) + \frac{\partial p_2^*}{\partial n}Y^*$. Substitutions for $\frac{\partial p_2^*}{\partial n} = -\beta\frac{\partial Y^*}{\partial n}$ and $\frac{\partial v^*}{\partial n} = \frac{\rho\beta}{1-\rho\beta}\frac{\partial Y^*}{\partial n}$ lead to $\frac{\partial(nr^*)}{\partial n} = \frac{\partial Y^*}{\partial n}\left\{\beta(1-v^*) - \frac{\rho\beta}{1-\rho\beta}(p_1 - p_2^*) + p_2^* - c - \beta Y^*\right\}$. Using (64) and

$$\begin{aligned} p_1 - p_2^* &= \frac{p_1(1-\beta) + n(1-\rho\beta)(p_1 - c)}{1+n(1-\rho\beta)}, &(66)\\ p_2^* - c &= \frac{\beta p_1 - c}{1+n(1-\rho\beta)}, &(67)\end{aligned}$$

the bracket $\{\cdot\}$, multiplied by $[1+n(1-\rho\beta)]$, becomes

$$\beta(1-p_1) + n\beta\left[1-p_1-\rho\beta\left(1-\frac{c}{\beta}\right)\right] - \frac{\rho\beta}{1-\rho\beta}\left[p_1(1-\beta)+n(1-\rho\beta)(p_1-c)\right]$$
$$+\beta p_1 - c - \beta(1-p_1) - \beta n\left(1-\frac{c}{\beta}\right)(1-\rho\beta)$$

$$= n\beta\left[\frac{c}{\beta} - p_1 - \rho(p_1 - c)\right] + \beta p_1 - c - \frac{\rho\beta p_1(1-\beta)}{1-\rho\beta},$$

which is decreasing in n. For $n=1$, this expression is $-\rho\beta(p_1-c) - \frac{\rho\beta p_1(1-\beta)}{1-\rho\beta} \leq 0$. Therefore, nr^* is decreasing in n for any $n \geq 1$ since $\frac{\partial(nr^*)}{\partial n} = 0$ only for $n=1$ and $\rho = 0$.

The conditions of monotonicity of nr^* in ρ. Using (65), $\frac{\partial(nr^*)}{\partial \rho} = -\frac{\partial p_2^*}{\partial \rho}(1-v^*) - \frac{\partial v^*}{\partial \rho}(p_1 - p_2^*) + \frac{\partial Y^*}{\partial \rho}(p_2^* - c) + \frac{\partial p_2^*}{\partial \rho}Y^*$, which, using $\frac{\partial p_2^*}{\partial \rho} = -\beta\frac{\partial Y^*}{\partial \rho}$, can be written as $\frac{\partial(nr^*)}{\partial \rho} = \frac{\partial p_2^*}{\partial \rho}\left[2Y^* + \frac{c}{\beta} - 2 + v^*\right] - \frac{\partial v^*}{\partial \rho}\left[p_1 - \beta(1-Y^*)\right]$. The first bracket $[\cdot]$ is zero for $n=1$ because, by (60) for $\gamma = 0$ and $n=1$, $Y^* = \frac{1}{2}\left(2 - v^* - \frac{c}{\beta}\right)$, whereas the second bracket $[\cdot] > 0$. Therefore, for $n=1$, $\frac{\partial(nr^*)}{\partial \rho} < 0$.

For $n > 1$ and $\gamma = 0$, equation (60) yields $Y^* = \frac{1-v^*}{n+1} + \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$, which can be written as

$$1 - Y^* = \frac{1}{n+1}\left(v^* + n\frac{c}{\beta}\right) \text{ or } p_2^* = \frac{\beta v^* + nc}{n+1}. \tag{68}$$

Since $\frac{\partial p_2^*}{\partial \rho} = \frac{\beta}{n+1}\frac{\partial v^*}{\partial \rho}$, $\frac{\partial(nr^*)}{\partial \rho} = \frac{\partial v^*}{\partial \rho}\frac{1}{n+1}\left\{\beta\left[2Y^* + \frac{c}{\beta} - 2 + v^*\right] - (n+1)\left[p_1 - \beta(1-Y^*)\right]\right\}$, which means that $\frac{\partial(nr^*)}{\partial \rho} \leq 0$ is equivalent to $\{\cdot\} \leq 0$ or $p_1 \geq \beta(1-Y^*) + \frac{1}{n+1}\left[c + \beta v^* - 2\beta(1-Y^*)\right] = p_2^* + \frac{c+\beta v^* - 2p_2^*}{n+1}$ or, using (68),

$$p_1 \geq c + 2\frac{\beta v^* - p_2^*}{n+1} \Leftrightarrow p_1 \geq c + \frac{2n}{n+1}(p_2^* - c). \tag{69}$$

The last inequality always holds for $n=1$ and never holds when $\rho\beta \to 1$ (leading to $p_2^* \to p_1$) and $n > 1$ since $(n+1)(p_1 - c) < 2n(p_1 - c)$ for any $n > 1$.

Condition (69) is only sufficient for monotonicity of $nr^{*,3}$ under RESE 3 because violation of this condition may take place outside the area of RESE 3 inputs, and inside this area, $nr^{*,3}$ can be monotonic. Namely, by part 3 of Proposition 11, which holds for $\gamma = 0$, RESE 3 exists only for

$\rho < \rho^1 = \frac{n+1}{n}\frac{1-p_1}{\beta-c}$, where ρ^1 can be less than one for large n. In order to take into account this bound, condition (69) can be written in terms of inputs using (67):

$$p_1 \geq c + \frac{2n}{n+1}\frac{\beta p_1 - c}{1+n(1-\rho\beta)}. \tag{70}$$

The RHS of this inequality is increasing in ρ; therefore, given other inputs fixed, $\frac{\partial(nr^*)}{\partial\rho} < 0$ for all ρ under RESE 3 if and only if (70) holds for $\rho = \rho^1$. With this ρ, condition (70) becomes

$$p_1 \geq c + \frac{2n}{n+1}\frac{(\beta p_1 - c)(\beta - c)}{(1+n)[\beta - c - \beta(1-p_1)]} \Leftrightarrow p_1 \geq c + \frac{2n}{(n+1)^2}(\beta - c). \tag{71}$$

The RHS of (71) decreases in n to c with $n \to \infty$. Therefore, there exists n^0 such that $\frac{\partial(nr^*)}{\partial\rho} < 0$ for any $n \geq n^0$. On the other hand, since $\frac{\partial(nr^*)}{\partial\rho} < 0$ for $n = 1$ and (71) is necessary and sufficient for $\frac{\partial(nr^*)}{\partial\rho} < 0$, nr^* attains minimum (since the RHS of 70 increases in ρ) for any $n \in (1, n^0)$, where $n^0 > 2$ if and only if (71) does not hold at least for $n = 2$, i.e., $p_1 < c + \frac{4}{9}(\beta - c) = \frac{5c+4\beta}{9}$.

n^0 can be found, e.g., from the negation of (71) bearing in mind that non-monotonicity holds for $n < n^0$, where n^0 is the larger root of the equation, corresponding to $(p_1 - c)(n^2 + 2n + 1) < 2n(\beta - c)$. The equation is $n^2 - 2n\frac{\beta - p_1}{p_1 - c} + 1 = 0$, where $\frac{\beta-p_1}{p_1-c} > 0$ since $p_1 \in (c, \beta)$. The discriminant $D = 4\left[\left(\frac{\beta-p_1}{p_1-c}\right)^2 - 1\right] > 0$ because $\beta - p_1 > p_1 - c$ is equivalent to $p_1 < \frac{\beta+c}{2}$, where the RHS is greater than $\frac{5c+4\beta}{9}$. Then the larger root is $n^0 = \frac{\beta-p_1+\sqrt{(\beta-p_1)^2-(p_1-c)^2}}{p_1-c} = \frac{\beta-p_1+\sqrt{(\beta-c)(\beta+c-2p_1)}}{p_1-c}$.

The expression for ρ^0 can be found from (70) when it holds as an equality: $(p_1-c)[1+n(1-\rho\beta)] = \frac{2n}{n+1}(\beta p_1 - c) \Leftrightarrow 1 - \rho\beta = \frac{2}{n+1}\frac{\beta p_1 - c}{p_1 - c} - \frac{1}{n} \Leftrightarrow \rho^0 = \frac{1}{\beta}\left[1 + \frac{1}{n} - \frac{2}{n+1}\frac{\beta p_1 - c}{p_1 - c}\right]$, yielding

$$\rho^0 = \frac{(n+1)^2(p_1 - c) - 2n(\beta p_1 - c)}{\beta n(n+1)(p_1 - c)}. \tag{72}$$

When $\beta = 1$, $nr^{*,3}$, using (65) and the expressions (66), (64), and (67), is

$$\begin{aligned}
nr^{*,3}\big|_{\beta=1} &= \frac{1}{[1+n(1-\rho)]^2}\left\{n(1-\rho)(p_1-c)(1-p_1) + n^2(1-\rho)(p_1-c)[1-p_1-\rho(1-c)]\right.\\
&\quad \left.+(1-p_1)(p_1-c) + n(1-\rho)(p_1-c)(1-c)\right\},\\
\lim_{\rho\to1} nr^{*,3}\big|_{\beta=1} &= (1-p_1)(p_1-c) < nr^{*,3}\big|_{\substack{\beta=1\\ \rho=0}} = \frac{1}{(1+n)^2}\left\{n(p_1-c)(1-p_1) + n^2(p_1-c)(1-p_1)\right.\\
&\quad \left.+(1-p_1)(p_1-c) + n(p_1-c)(1-c) \pm n(p_1-c)(1-p_1)\right\}\\
&= (1-p_1)(p_1-c) + \frac{n(p_1-c)^2}{(1+n)^2}.
\end{aligned}$$

Formula $\rho^0\big|_{\beta=1} = \frac{n^2+1}{n(n+1)}$ results from (72) with $\beta = 1$. Minimum of ρ^0 in n can be found from $\frac{\partial\rho^0}{\partial n} = 0 = \frac{2n^2(n+1)-(2n+1)(n^2+1)}{n^2(n+1)^2}$, which is equivalent to $n^2 - 2n - 1 = 0$ with the roots $n_{1,2} = 1 \pm \sqrt{2}$. The relevant root is $n_2 = 1 + \sqrt{2}$; direct calculation yields $\rho^0\big|_{n=2} = \frac{5}{6} = \frac{10}{12} = \rho^0\big|_{n=3}$.