Quantity Competition in the Presence of Strategic Consumers

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Abstract: An arbitrary number of retailers compete in capacities of a homogeneous limited-lifetime product offered to strategic consumers with heterogeneous valuations and a general discount factor. The first-period price is fixed, whereas the second-period (clearance) price is determined by market clearing. We provide a closed-form characterization of symmetric pure-strategy equilibria, which may lead to no sales in the first or second period and sales in both periods with clearance price above or at salvage value. In equilibrium, increasing competition may harm local economy. Retailers reduce inventories when consumers' discount factor increases. As a result, having more strategic consumers can benefit competing retailers and insure them against sales at salvage value. Moreover, an increase in consumers' discount factor increases consumer inequality in terms of utility and may even decrease the total consumer surplus.

Keywords: oligopoly, strategic consumers, limited-lifetime product, rational expectations equilibrium

1 Introduction

In the current global economy, it is common for transnational manufacturers to introduce a new version of a product in local markets. Characteristic examples include a December 23, 2013 launch of Samsung Galaxy Grand 2 in India, an introduction of a new model of the Official Match Ball of the FIFA World Cup by Adidas every four years, called "Brazuca" in 2014, and a 2013 introduction of six new Ford models in China. The rapid pace of fashion, innovation, and technological progress limits the lifetime of these versions, making them obsolete within a relatively short time.

When the product life cycle is at the growth or maturity stages[1], demand is more predictable than at the introduction stage. Moreover, retailers are more experienced and can use focal-point pricing[2] to avoid price wars and switch to non-price forms of competition. For example, at the time of a new version entry, retailers can exploit the manufacturer suggested retail price (MSRP) as a focal point when manufacturer uses resale price maintenance (RPM). Sooner or later, competing innovation or fashion takes its course and reduces consumer willingness to pay for this version of the product. A local market for such a product may have an arbitrary number of retailers that initially sell it at MSRP, but eventually engage in clearance sales to liquidate remaining inventory.

When price competition is weak and demand is known, a major decision faced by retailers is determining the quantity of the product that they are going to supply to the market. At the strategic level, this decision involves more than just procuring a certain inventory of the product; it can include choosing which retail outlets carry the product or even opening new outlets, allocating the warehouse capacity, making shipping arrangements, sizing the sales staff, and making other marketing and operational decisions. All these aspects contribute to product cost and supply inflexibility. The same factors increase the importance of the product quantity decision which, in isolation, is relatively easy to formalize. We assume that retailers promote their firms rather than products. The latter effort usually belongs to the manufacturer implying that the changes in retailer promotions may lead only to a redistribution of market shares and do not affect the total demand of the product.[3]

On the consumer side of the market, we see a population that is accustomed to quick changes in fashion, the emergence of new models of the products with their limited lifetime. Consumers are familiar with typical price trajectories, which can result from intertemporal price discrimination by the sellers. Therefore, consumers can form relatively accurate expectations about future markdowns and, using these expectations, may engage in strategic, forward-looking, or patient, shopping behavior by delaying the purchase until the period of price reductions. In doing so, consumers realize that delaying the purchase may reduce the sense of novelty and their enjoyment of the product, but they still make this intertemporal trade-off. Sitting between a manufacturer with near-monopoly power and strategic consumers, retailers must make their best of the situation while aggressively competing for market shares.

Competition typically forces retailers to increase their supply to the market at the cost of decrease in their profits. On the other hand, the literature shows that profit-maximizing retailers shrink the inventory in response to increasing consumer's intertemporal discount factor or the "level"of strategic behavior, which we confirm in our setting. This intriguing interplay between the opposing phenomena brings the following research questions. First and foremost, what are the effects of strategic consumer behavior on retailer inventory decisions and profits? A common view is that this behavior is detrimental for retailers, but is this necessarily true under competition? Does the speed of reduction in product value play a substantial role in these effects? Better yet, do consumers themselves necessarily benefit from being strategic? The answer is not obvious because consumer behavior drives competitive responses from the retailers. Finally, do the increases in the consumer's strategic behavior and retailer competition benefit local economy?

In addressing these questions, we consider a stylized two-period model where retailers compete in quantities by making inflexible first-period supply decisions. To avoid technical complications and non-essentials, the main analysis considers identical retailers.[4] The major intention of retailer's first-period decisions, associated with the quantity decisions, is to promote the first-period sales. Therefore, we assume that the corresponding first-period demand and the resulting sales are non-decreasing in the initial order quantities. In the first period, regular consumers plan their purchases according to their expectations of the second-period price. This price is not less than the salvage value because there is usually a large number of bargain-hunter consumers who are ready to absorb the excess supply at a sufficiently low price.

Because of the capacity commitments of the retailers, we describe the second-period (clearance) sales by the Cournot-Nash model. Kreps and Scheinkman (1983) argued that the first-stage capacity commitment by duopolistic firms selling an undifferentiated product yields a Cournot outcome even if the equilibrium capacities and prices are determined by price competition in the second stage. In addition to the focal-point argument above, there are studies confirming that Cournot assumption, leading to the same price among retailers, is not implausible in cases of non-price competition, see, e.g., Schmalensee (1976); Karnani (1984), and Perakis and Sun (2014). One of the arguments is that retailers choose their promotional decisions, associated with inventories, independently, whereas price cuts are easily observable and can be matched almost instantaneously. Flath (2012) shows that the markets of music records, bicycles, and thermos bottles are appropriately described by the Cournot model. For example, the Japanese market of music records is characterized, besides plausibility of the Cournot model, by legal use of RPM system (saihan seido) and strategic consumer behavior (Nippop (2005)).

We answer the research questions by analyzing a game among retailers where the information set is determined by the manufacturer and consumer behavior characteristics. We derive a closed-form solution for the rational expectations symmetric equilibrium (RESE) in pure strategies for the proposed generalized Cournot-Nash model. This analytical tractability is a distinguishing feature of our approach to an otherwise unwieldy problem. The equilibrium permits a complete characterization and takes one of the following forms:

1. When the list price is sufficiently high relative to consumer valuations, all consumers delay their purchases until the second period, effectively turning the market into a one-period Cournot-Nash.

2. When the list price is relatively low and the relative decrease in valuations between the two periods is large (the product is not very durable), the market reduces to the first period only because retailers limit the amount of product they supply to the market. This is essentially a tacit collusive outcome facilitated by the manufacturer.

3. For intermediate values of the first-period price and a sufficiently low salvage value, RESE leads to sales in both periods as well as a second-period price higher than the salvage value.

4. In the same range of the first-period price as form 3, and with a sufficiently high salvage value, RESE still results in sales in both periods. However, the second-period sales take place at the salvage value. This "salvaging" outcome is not attractive to the retailers because they incur a large loss in the second period due to product oversupply.

Thus, possible equilibria describe situations where retailers fully follow the suggested price (RESE2), sell only part of inventory at MSRP (RESE3 or 4), and do not sell at MSRP (RESE1). Within *each type*, the equilibrium is unique. Across all types, the unique equilibrium always exists under the conditions of RESE1 and 2, but it may not be unique in the complementary case. For the latter, we provide a sufficient condition that guarantees that RESE3 exists and is unique. This condition requires the unit cost to be high compared to the salvage value.

Besides intuitive monotonic properties, the equilibria reveal several novel insights. It is intuitive that, when the number of retailers increases, the total supply of the product does not decrease, the resulting second-period price falls, the total profit of retailers decreases, and the total surplus of consumers increases. However, it is not always true that the aggregate welfare (the sum of the total profit and consumer surplus) increases with the level of competition. For example, when a relative decrease in consumer valuations of the product between two periods is small, the aggregate welfare may increase, decrease, or even attain an internal maximum. From a regulator's point of view, the corresponding optimal market structure would involve, respectively, a monopolistic retailer, a perfect competition, or an oligopoly. For the third form of RESE, the maximum of the aggregate welfare with respect to the level of competition results in a clearance price above the unit cost.

The response of equilibrium to changes in the consumer's discount factor is more complex. The total quantity supplied to the market never increases in this factor. That is, at the aggregate level, retailers always respond to increasing strategic behavior by reducing supply despite competitive pressures. As a result, retailers may *capitalize* on strategic behavior because the total profit may be non-monotonic. Typically, total profit decreases as consumers become more strategic, e.g., when the relative decrease in valuations between the two periods is large or in a monopoly. However, there are two distinct cases leading to profit gains resulting from the equilibrium response of retailers to strategic consumer behavior:

- the "continuous gain" is characterized by continuously increasing profit in the consumer's discount factor; this gain may happen when the second-period sales are either profitable or at loss, but only when the relative decrease in valuations is small and the discount factor is high;
- the "discontinuous gain" occurs at various values of the consumer's discount factor and the relative decrease in valuations, but only when the difference between the unit cost and the salvage value is relatively small; profit increases because retailers reduce inventories in response to increased strategic behavior, which leads to the switch from RESE4 to RESE3.

The most pronounced form of these phenomena is the "boundary-value gain", i.e., profit with myopic consumers is less than with fully strategic consumers. This may occur only when a strong first-period quantity competition leads to the second-period sales below cost.

As RESE4 is unfavorable for retailers, they would generally prefer to avoid it. It is then particularly noteworthy that an increase in strategic behavior may prevent salvaging equilibrium from taking place. We provide a sufficient condition to rule out RESE4 in the form of a lower bound on the consumer's discount factor. On the other hand, as shown in the appendix, the most beneficial markets for the manufacturer are those where the first-period price is not too close to the maximum consumer valuation or to the retailer's unit cost and salvaging equilibrium is possible.

The total consumer surplus is not generally monotonic and may attain maximum at an intermediate consumer's discount factor. Thus, the consumer population as a whole does not necessarily benefit from becoming more strategic, and may, in fact, lose by being "too strategic." Similarly, the aggregate welfare is generally non-monotonic: it may attain a maximum that tends to arise for high levels of retailer competition and a small relative decrease in valuations. Non-monotonicity of the aggregate welfare is characterized in closed form for the salvaging equilibrium.

We present a review of related literature in §2, describe the model in §3, and state the characterization of equilibrium as well as a sufficient condition for its existence and uniqueness in §4.

We analyze equilibrium properties in §5 and the properties of consumer surplus and the aggregate welfare in §6. Finally, §7 provides a summary of monotonic properties and outlines several possibilities for extending and applying the proposed model. All mathematical proofs are provided in the online appendix.[5]

2 Quantity decisions and strategic consumers

Coase (1972) has initiated a study of strategic buyer behavior in an intertemporal pricing problem faced by a durable good monopolist. The essence of his famous conjecture is that "the competitive outcome may be achieved even if there is but a single supplier." As one of the possible solutions to this problem, Coase proposes to restrict the quantity of the good supplied to the market through contractual or other arrangements. Further studies, e.g., Stokey (1979), formally support these conclusions. Stokey (1981) has also pointed to a tradeoff between decreasing capacity in response to strategic consumer behavior and increasing capacity by a monopolist as a deterrent against competing entrants as an area for further study. Lazear (1986) studies a monopoly pricing problem with fixed inventory and strategic consumers. Among a variety of two-period settings, he considers a given population of strategic buyers whose valuations for a fashion good decrease by a fixed factor in the second period.

These early findings have led to further research in consumer behavior in the context of intertemporal pricing. Shen and Su (2007) survey results involving strategic consumer models, and Aviv, Levin, and Nediak (2009) review the research on the mitigation of strategic consumer behavior. We focus our attention on results where quantity-based decisions of sellers affect strategic consumers. For a monopolistic retailer, Cachon and Swinney (2009) consider a two-period model with uncertain demand and find that the optimal choice of the initial inventory and subsequent markdown is better than committing to a price even in the presence of strategic consumers. Moreover, an opportunity to replenish the inventory at the beginning of the second period is much more valuable for the retailer in the presence of strategic consumers than when all consumers are myopic.

Su (2007) considers a deterministic model of monopolistic pricing and rationing policy for a fixed inventory of a limited-lifetime product. The market consists of four segments characterized by one of the two fixed valuation levels (high- or low-valuation consumer types) and one of the two given values of waiting costs (patient or impatient consumers). Su shows that market heterogeneity may lead to profit gains from the increased strategic behavior of low-valuation consumers when high-valuation consumers are myopic (impatient). In this case, the retailer sells the product at a high price to the arriving high-valuation consumers, while the arriving low-valuation patient (fully strategic) consumers are waiting for clearance. When the market of low-valuation consumers becomes large enough, the monopolist drops the price, effectively exploiting a price discrimination scheme. This effect relies on the threat of stockouts for high-valuation consumers, which increases their willingness to pay, and on the proportional rationing rule used in the model.

Liu and Ryzin (2008) concur that "capacity decisions can be even more important than price in terms of influencing strategic consumer behavior"; they study the effects of capacity decision when prices are fixed while consumers have full information and can be risk-averse. The decision is expressed in terms of consumer rationing risk. Liu and van Ryzin find that capacity rationing can mitigate strategic consumer behavior, but it is not profitable for risk-neutral consumers. Under competition, the effectiveness of capacity rationing is reduced, and there exists a critical number of firms beyond which rationing never occurs in equilibrium. Further development of this work by Huang and Liu (2015) showed that capacity rationing is also less effective under inaccurate consumer expectations about the reduced-price product availability.

These studies suggest that retailers are most challenged by strategic consumer behavior when there is a large number of competitors, consumers are risk-neutral, and the market is homogeneous with respect to the consumer's discount factor. Moreover, when consumers do not know the total supply of the product, it is impossible to use strategic rationing to control their behavior. Our study fills the gap in the existing results for this challenging setting.

3 Model description

We consider a two-period market for a limited-lifetime product with an arbitrary number of identical retailers. All the retailers have the same unit cost c and offer the product at the same first-period price $p_1 > c$. As argued in Liu and Ryzin (2008), this assumption is not unusual "in a competitive retail market, where retailers frequently stock identical products, sell them at the same suggested retail prices, and at nearly identical costs from manufacturers;" see also Huang and Liu (2015).[6] The number of *regular* consumers who arrive at the start of the first period is normalized to one and their first-period valuations v are uniformly distributed on the interval $[0, 1]$. Normalization of valuations effectively expresses revenue and inventory as "unitless" quantities and MSRP as a share of maximum valuation, i.e., $p_1 \leq 1$.

If there is some product remaining after the first period, retailers engage in clearance sales in the second (clearance) period. As the product offerings are undifferentiated, the retailers lower their prices until all remaining inventory is cleared, that is, second-period price p_2 (identical for all retailers) is sufficiently low for the total clearance demand to equal the total remaining inventory. Similarly to Cachon and Swinney (2009), we assume that, in the second period, there is an infinite number of *bargain-hunting* consumers who can buy any remaining product at per-unit salvage value $s < c$. As a result, p_2 never goes below s. The salvage value also allows for the possibility of inventory buy-back contracts of retailers with the manufacturer, or the availability of alternative sales channels for the retailers.

Each retailer maximizes its profit by selecting the initial inventory level. The resulting game among the retailers is similar to the classical Cournot-Nash model, but with a substantially distinct two-period structure.

We now describe the market dynamics. Let retailers be indexed by set I of size $n = |I|$, and retailer $i \in I$ product supply and sales in the first period be y^i and q^i. As the second-period market is cleared, each retailer's second-period supply and sales are equal to $y^i - q^i$. Denote the total first-period product supply and sales as $Y = \sum_{i \in I} y^i$ and $Q = \sum_{i \in I} q^i$ respectively. Then the total second-period supply is $Y - Q$ and the retailer i profit is

$$r^i = -cy^i + p_1 q^i + p_2(y^i - q^i). \tag{1}$$

First-period sales q^i are determined based on a consumer decision model.

3.1 Consumer decision model

The consumer decision model describes two aspects: demand allocation between two periods and among the retailers. We will start with the first one.

Demand allocation between two periods In order to capture a typical decrease in valuations for seasonal and limited-lifetime products, we introduce factor $\beta \in [0, 1]$: if the consumer's first-period valuation is v, the second-period valuation becomes βv. Two logical restrictions ensure non-trivial equilibrium results. First, inequality $\beta > c$ guarantees that the highest-valuation consumer is prepared to pay more than the unit cost in the second period. If this restriction does not hold, the clearance price can never be above the unit cost. We also suppose that $p_1 > \frac{s}{\beta}$ to ensure that salvage value s is less than the highest second-period valuation βp_1 of regular consumers who are forced to delay their purchases by MSRP. Similarly to $\beta > c$, this restriction supports a non-trivial second-period outcome in an equilibrium with a substantial role of regular consumers.

The availability of information about total supply of the product varies among the markets. Some markets, such as land or real estate, have nearly perfect information, an assumption used, e.g., in Stokey (1981) and Liu and Ryzin (2008). In many other markets, total system-wide inventory is unobservable, which reduces the ability of retailers to use rationing as a tool for stimulating first-period demand from strategic consumers. When consumers do not observe total supply, they cannot infer exact price p_2 and product availability $\alpha \in \{0, 1\}$ in the second period.

Assumption 1. *Consumers do not know the total product supply and form expectations: (a) expected availability $\bar{\alpha} \in \{0, 1\}$ of the product in the second period and (b) expected second-period price $\bar{p}_2$.*

Given these expectations, consumers decide whether a first or second-period purchase maximizes their surplus, which is similar to Lazear (1986); Su (2007), and Cachon and Swinney (2009):

Assumption 2. *In addition to their expectations, consumers know only their private valuations v, list price p_1, product durability β, and the second-period surplus discount factor $\rho \in [0, 1)$. When the product is available, a consumer with valuation v buys in the first period if the first-period surplus $\sigma_1 \triangleq v - p_1$ is not less than the expected second-period surplus $\sigma_2 \triangleq \bar{\alpha}\rho(\beta v - \bar{p}_2)^+$.*

In our setting, consumers do not consider rationing risk in the first period because there are no first-period stockouts, which is shown in Lemma 3 below. As $\sigma_2 \geq 0$, consumers with $v < p_1$ never buy in the first period because such a purchase would result in a negative surplus. The proposition below describes the first-period demand.

Lemma 1. *Given consumer expectations, surplus-maximizing behavior is to buy in the first period if $v \geq v^{\min}$, where the unique valuation threshold is given by $v^{\min} = \max\left\{p_1,\ \min\left\{\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1 - \bar{\alpha}\rho\beta},\ 1\right\}\right\}$. The resulting total first-period demand is $D = 1 - v^{\min}$.*

Undervaluation of the surplus from delaying a purchase means that even for a product that does not depreciate much by the second period, i.e., β is near one, consumers with any valuation may myopically ignore the second period during the first-period deliberations, i.e., have $\rho = 0$. The value of ρ may depend on the market targeted by the product, e.g., for age- or culture-oriented products, and on the consumer confidence in the stability of the financial situation. As ρ increases, consumers place more emphasis on the second period in their wait-or-buy decisions. Thus, unlike β, which models an objective decrease in valuations, the consumer's discount factor ρ is a subjective parameter describing the level of strategic behavior. The essence of the distinct roles of β and ρ has been succinctly captured by Pigou (1932): "Everybody prefers present [i.e., $\rho < 1$] pleasures or satisfaction of given magnitude to future pleasures and satisfaction of equal magnitude [i.e., $\beta = 1$], even when the latter are perfectly certain to occur." Frederick, Loewenstein, and O'Donoghue (2002) provide a review of empirical estimates of consumers' discount rates.

Demand allocation among retailers Because consumers have no preferences among the retailers, the marketing and distribution efforts are the only differentiating aspect. It is recognized both in practice and in research (e.g., Balakrishnan, Pangburn, and Stavrulaki (2004)) that in many markets typical consumer behavior results in larger sales of a particular retailer if the product is presented to consumers at a larger number of retail outlets, in larger quantities on store shelves, and in more ads. A review of 60 theoretical and empirical papers supporting these findings in various industries is provided in Urban (2005). Other studies consider markets where demand responds positively to product scarcity; see, e.g., a review in Yang and Zhang (2014). On the other hand, Lippman and McCardle (1997) introduce a stockout-penalty term in the retailer profit function, which may reflect, e.g., the stockout losses in the healthcare industry or a systematic profit loss due to outside options for disappointed consumers.

We consider *attraction* $a^i(y^i)$ as a measure of retailer i efforts, which, depending on the product, may include the use of the demand-promoting inventory display. To model typical retail practice, we assume that a^i is non-decreasing in the retailer's inventory. Due to a general form of a^i, this function is also called promotion or advertisement, see, e.g., Schmalensee (1976).

Assumption 3. *The function $a^i(y^i)$ is continuous, non-decreasing in y^i, and $a^i(0) = 0$. Consumers do not know the functional form of $a^i(y^i)$ and react only to the resulting vector of attraction values.*

Identical retailers operate under alike conditions and use similar recipes for creating the firm's attractions, i.e., $a^i(y^i) = a(y^i)$ for all $i \in I$. Moreover, any two identical retailers with the same attraction have equal market shares, and the market share of any retailer decreases by the same amount if the attraction of *any* other retailer is increased by a particular amount. These properties, complemented by a simple assumption that zero attraction leads to zero market share, satisfy the conditions of the market share theorem of Bell, Keeney, and Little (1975), which claims that the functional form of the market share of retailer i, in this case, is $a(y^i)/\sum_{j\in I} a(y^j)$.

The *first-period demand* d^i of retailer i, determined by its market share, depends not only on attraction $a(y^i)$ but also, inversely, on the vector of attractions (inventories) of the others $\mathbf{a}^{-i}(\mathbf{y}^{-i}) \triangleq (a(y^1), \ldots, a(y^{i-1}), a(y^{i+1}), \ldots, a(y^n))$. Since attraction is monotonic in inventory, we use the shortcut notation $d^i = d^i(y^i, \mathbf{y}^{-i})$, which implicitly presumes that d^i depends on inventories via the corresponding attractions. Recall that consumers do not know total supply and react only to relative attractiveness of retailers. The resulting market shares are proportional to the attractions regardless of the total attraction and total supply. Since retailers' promotional technologies are the same ($a^i(y^i) = a(y^i)$ for all $i \in I$), we formalize this property as the following assumption.

Assumption 4. *Retailers' market shares are homogeneous of degree zero in inventories.*

This assumption means that any changes in the total supply cannot influence market shares when the ratios y^j/Y, $j \in I$ remain the same. The following lemma specifies the functional form of $a(y^i)$.

Lemma 2. *If retailer i market share has functional form $a(y^i)/\sum_{j\in I} a(y^j)$, where $a(y)$ is continuous in y, and Assumption 4 holds, then $a(y)$ has the unique functional form $a(y) = a(1)y^\gamma$.*

A review of studies using this form for inventory-dependent monopolistic demand is in Balakrishnan, Pangburn, and Stavrulaki (2004). By choosing the scale of attraction so that $a(1) = 1$, we obtain the functional form for d^i :

$$d^i(y^i, \mathbf{y}^{-i}) \triangleq D\frac{(y^i)^\gamma}{\sum_{j\in I}(y^j)^\gamma}, i \in I, \tag{2}$$

where $\gamma \in [0, 1]$ is the *inventory elasticity of attraction* or inventory elasticity of demand, normalized by the market share of other retailers. Function (2) is a symmetric form of the *general attraction model.* This form is widely used both in theoretical and empirical studies, e.g., Schmalensee (1976), Karnani (1984), Monahan (1987), Gallego et al. (2006). Authors usually refer to Mills (1961) or Friedman (1958), who introduced this form of competitive demand or market share by assumption.[7]

An empirical study of Naert and Weverbergh (1981) concludes that the attraction model is "more than just a theoretically interesting specification." This model "may have a significantly better prediction power than the more classic market share specifications." This conclusion is supported by later studies, e.g., Klapper and Herwartz (2000). The case $\gamma = 0$ means that a retailer's attraction does not depend on y^i, and $d^i \equiv \frac{D}{n}$ for any $y^i > 0$ and $i \in I$.[8] This case was used in §4.4 of Liu and Ryzin (2008) to study the effect of rationing on strategic behavior of risk-averse consumers. Cachon (2003), in §6.5, considers a newsvendor competition model where retail demand is "divided between the n firms proportional to their stocking quantity," which matches the case of $\gamma = 1$ in our model. This case can be viewed as a fluid limit of the following simple randomized allocation model. Suppose all retailers pool their (discrete) inventory into an urn (one may think of different retailers' inventory being identified by different colors). Each customer randomly picks an item from the urn (without replacement), and the retailer to whom the item belongs is credited for the sale. In such allocation model, the case of intermediate $0 < \gamma < 1$ corresponds to pooling of attractions rather than inventories.

As product is undifferentiated and the retailers are identical, consumers buy from any retailer with available product. If the combined supply of retailers is insufficient to satisfy the combined demand, one of the rationing rules can be used. For example, according to the surplus-maximizing rule (see Tirole (1988)), consumers buy in the order of their valuations. The following lemma shows that retailers have no stockouts independently of the rationing rule.

Lemma 3. *Consider any $\bar{Y} \geq 1 - v^{\min}$, symmetric inventory profile $(\frac{\bar{Y}}{n}, \ldots, \frac{\bar{Y}}{n}) \in \mathbb{R}^n_+$, and any behavior of consumers under stockouts in the first period. For any $i \in I$, let $\mathbf{y}^{-i} = (\frac{\bar{Y}}{n}, \ldots, \frac{\bar{Y}}{n}) \in \mathbb{R}^{n-1}_+$. The following claims hold: (I) any profit-maximizing response of retailer i to $\mathbf{y}^{-i}$ must satisfy $y^i \geq \breve{y}^i$, where $\breve{y}^i$ is the unique positive solution to $\breve{y}^i = d^i(\breve{y}^i, \mathbf{y}^{-i})$; (II) for any $y^i \geq \breve{y}^i$, (a) stockouts are impossible, (b) the total first-period sales are $Q = 1 - v^{\min}$, the individual first-period sales are $q^i = d^i(y^i, \mathbf{y}^{-i})$, the resulting second-period inventories are $y^i - q^i, i \in I$, and (c) the second-period price is*

$$p_2 = \max\{s,\ \beta(1 - Y)\}. \tag{3}$$

The timing of main events in the market and the corresponding inputs are outlined in Figure 1.

3.2 Rational expectations equilibrium

Lemma 1 identifies rational consumer behavior for *given* expectations, list price p_1, and behavioral parameters ρ, β, which are the only inputs known to consumers according to Assumption 2. In particular, it specifies valuation levels of consumers who purchase in the first period. However, these results are insufficient to identify how consumer expectations *form*. Although it is possible to look for equilibrium behavior of retailers for given expectations, our ultimate goal is to find internally consistent market outcomes that can be sustained in the long run. Therefore, we close the loop by identifying expectations that are *rational*. That is, *the equilibrium inventory levels of the retailers must lead to precisely the same observed product availability and clearance prices as expected by the consumers.*

Some studies assume that all players in the game form beliefs about the actions of the other players including consumers' beliefs about retailers' inventories. For some products, however, consumers may not form such beliefs even when new versions of the product repeatedly emerge in the market. For example, a buyer of a music or video record usually does not know the number of particular records in the market and the number of consumers interested in buying this record. This buyer, however, may form beliefs about the availability of the product on sale and the clearance price because this information is observable ex post over multiple realizations of the market.

In our setting, *ex post* consumers observe only the second-period availability α and price p_2, *not* the inventory levels or market size. Given all available information, consumers *cannot even infer* the inventory levels. In such an environment, consumer expectations in terms of directly observable quantities such as the second-period availability and price are a natural model.

At the introduction stage of the product life cycle, e.g., for the very first personal computer, consumers may not be able to form rational expectations about the release of a new version of the product and the resulting pricing policies. However, at later stages, manufacturers regularly launch similar products, or new models of the same product, and consumers, getting accustomed to price-drop patterns, adjust their expectations about future pricing policies to closely match their observations. Adjustments are no longer needed if the expectations coincide with the eventual observations. On the other hand, retailers regularly conduct market research to estimate current consumer expectations. Thus, we assume that retailers operate under *complete information*.

Using this notion of rationality, the rational expectations symmetric Cournot-Nash equilibrium (RESE) in pure strategies is defined as follows:

1. Given consumer expectations and $\mathbf{y}^{-i}$, let the best response of retailer i be $BR^i(\mathbf{y}^{-i}, \bar{p}_2, \bar{\alpha}) = \arg\max_{y^i} r^i(y^i, \mathbf{y}^{-i}, \bar{p}_2, \bar{\alpha})$.

2. For given consumer expectations, let $\hat{y} = \hat{y}(\bar{p}_2, \bar{\alpha})$ denote a symmetric Cournot-Nash equilibrium inventory level in the retailer game, i.e., $\hat{y}(\bar{p}_2, \bar{\alpha}) = BR^i\left[(\hat{y}, \ldots, \hat{y}), \bar{p}_2, \bar{\alpha}\right],$ where $(\hat{y}, \ldots, \hat{y}) \in \mathbb{R}_+^{n-1}$, and $\hat{Y}(\bar{p}_2, \bar{\alpha}) = n\hat{y}(\bar{p}_2, \bar{\alpha})$ be the corresponding total inventory.

3. The tuple (Y^*, p_2^*, α^*) is a RESE if $Y^* = \hat{Y}(p_2^*, \alpha^*)$, $p_2^* = \max\{s,\ \beta(1 - Y^*)\}$, and either $\alpha^* = 0$, if $Y^* = 1 - v^*$, or $\alpha^* = 1$, if $Y^* > 1 - v^*$ where v^* is the equilibrium value of $v^{\min}$.

From now on, r^* denotes the equilibrium profit of a retailer. Equilibrium values may be specified for the type of RESE, e.g., $r^{*,3}$ or $Y^{*,1}$ if necessary.

3.3 Discussion of model assumptions

We conclude this section with the discussion of specific implications of model assumptions. Some of the assumptions are quite common and well understood. For example, consumers are modeled as homogeneous in the discount factor ρ and relative valuation decrease β. This assumption is applicable to any products targeting specific market segments. The value of ρ may also be tied to the average time value of money (rate of return), which is relatively homogeneous for all consumers. Some empirical studies, e.g., Hausman (1979), claim a dependence of the discount rate on income (which serves sometimes as a proxy for product valuation). Other studies, however, show that the discount rate does not vary significantly with income, see, e.g., Houston (1983). The assumption of retailer symmetry is common for studying the effects of the *level* of competition, when retailers do not differ in their cost structure or brand value. We relax this assumption in Supplementary Document by showing that different costs provide additional interesting insights.

The information structure of the model is rather general. Indeed, it is relatively rare for the total product supply in the market to be visible to consumers whereas the market share effort, such as the number of outlets, does signal to consumers the relative market power of the retailers. We assume that the total demand is predetermined by the manufacturer's promotional efforts. As noted in Balakrishnan, Pangburn, and Stavrulaki (2004), larger quantities of the product on store shelves may attract additional consumers. These additional consumers at a particular retailer may generally come from the populations of consumers either with or without the original intention of buying the product. By ignoring the latter part, we disregard the cases when a consumer comes to a store to buy a different product and buys, in addition, the product under consideration only because it is displayed in large quantities. If that happened, it would increase the demand of the retailer, who may potentially deviate from an equilibrium by increasing inventory. Hence, the area of the equilibrium with the first-period sales only (RESE2) would be slightly less, whereas the areas of the equilibria with the sales in both periods (RESE3 and RESE4), which provide the main insights of this study, would be slightly greater.

Generally, consumer expectations about the second-period price and rationing risk may or may not be probabilistic in this context but, for pure strategy (deterministic) equilibria, deterministic expectations are consistent with retailer behavior. As we show below, there is a considerable amount of insight even from the pure-strategy case.

We focus on the case of $\gamma = 1$ in the first-period demand (2). This case serves as a useful magnifying glass for examination of our research questions because most of the results hold for any $\gamma \in (0, 1]$, and the case of $\gamma = 1$ is more reader-friendly than the cases of intermediate γ. An empirical evidence of sales proportional to inventory levels ($\gamma = 1$) in apparel industry was first studied in Wolfe (1968). Some of the effects found in our paper weaken when γ goes to zero and disappear for $\gamma = 0$. The robustness of the results with respect to changes in γ, including the closed-form analysis for $\gamma = 0$, is provided in Supplementary Document.

Unlike the case of $\gamma = 1$, the Cournot model may lessen the effects of profit gains from increasing consumer's discount factor. These gains are more notable when retailers suffer more from competition (greater n, γ, a higher-cost retailer). Meanwhile, Davidson and Deneckere (1986) argue, using mixed-strategies equilibria and rationing rules different from the one in Kreps and Scheinkman (1983), that the Cournot model underestimates the degree of competitiveness in markets with quantity precommitments.

We also assume, for simplicity, that retailers do not discount second-period profits. A usual assumption is that retailers use a market interest rate, i.e., a 2% rate yields a discount factor 0.98, whereas we assume that it equals one. Meanwhile, empirical studies suggest that consumer discount rates can be much higher (up to 300% or $\rho = 0.25$) even for relatively expensive items, see a review in Frederick, Loewenstein, and O'Donoghue (2002). We show in Supplementary Document that retailer's discount factor insignificantly changes the structure of equilibria and does not lead to additional insights.

4 Characterization of RESE

The rationality of consumer expectations immediately implies the following result.

Lemma 4. *In any rational expectations equilibrium, (1) $p_2 < \beta p_1$ if there are sales in the second period; (2) $Y \geq 1 - p_1$, which holds as an equality only if there are no sales in the second period; (3) $\rho\beta Y < 1 - p_1$ if there are sales in both periods and $p_2 > s$; $\rho\beta Y \geq 1 - p_1$ and $p_2 \geq c$ if there are sales only in the second period; and (4) $v^{\min} = p_1$ if and only if $\bar{\alpha} = 0$ or $\rho = 0$.*

Because $v^{\min} \geq p_1$, part 2 of Lemma 4 justifies the assumption of Lemma 3 for a RESE.

4.1 No-salvaging RESE

We start by providing closed-form expressions for three of the possible equilibrium cases. The values of p_1 close to the maximum valuation (i.e., p_1 near 1) may seldom arise in practice. Nevertheless, we consider the entire range of p_1 for theoretical completeness.

Theorem 1. *A unique RESE with the stated structure exists if and only if the respective conditions hold:*

RESE1 (No sales in the first period) $v^* = 1, \alpha^* = 1, p_2^* = c + \frac{\beta - c}{n+1}, Y^* = \frac{n}{n+1}\left(1 - c/\beta\right),$ *and* $r^* = \frac{(\beta-c)^2}{(n+1)^2\beta}$ *under condition* $p_1 \geq 1 - \frac{n}{n+1}\rho(\beta - c) \triangleq P_1$.

RESE2 (No sales in the second period) $v^* = p_1, \alpha^* = 0, Y^* = 1 - p_1,$ *and* $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$ *under condition* $p_1 \leq \frac{nc}{n-1+\beta} \triangleq P_2$.

RESE3 (Sales in both periods, $p_2^* > s$) $v^* = \frac{p_1 - \rho\beta(1-Y^*)}{1-\rho\beta}$, $\alpha^* = 1$, $p_2^* = \beta(1 - Y^*)$, *where* Y^* *is the larger root of a quadratic equation, and* $r^* = \frac{1}{n}\left[(p_1 - c)(1 - v^*) + (p_2^* - c)(Y^* - 1 + v^*)\right]$, *under condition* $P_2 < p_1 < P_1$ *and one of the following:*

(a) $\frac{n-1}{n}(p_1 - s)\left(1 - v^*\right)Y^* \leq (c - s)\left(1 - s/\beta\right)^2$, *or (b) condition (a) does not hold,* $Y^* < 1 - \frac{s}{\beta}$, *and* $r^* \geq \tilde{r}^i \triangleq \left\{\sqrt{(p_1 - s)\left(1 - v^*\right)} - \sqrt{\frac{n-1}{n}Y^*\left(c - s\right)}\right\}^2$, *where* $\tilde{r}^i$ *is the maximum profit of a firm deviating from this RESE in such a way that* $p_2 = s$ *(total inventory exceeds* $1 - s/\beta$*).*

The equilibrium characteristics Y^*, v^*, and r^* are continuous on the boundaries between these forms of RESE. Moreover, in RESE3, $Y^* \geq \frac{n}{n+1}(1 - c/\beta)$.

If the initial consumer expectations of the second-period price are such that $\bar{p}_2^0 < p_2^*$, the game is repeated, and expectations follow a linear adjustment process, then the sequence of games converges to $\bar{p}_2 = p_2^*$ for any sufficiently small speed of adjustment.

Remark 1. *One can consider rational expectation equilibria in the case of asymmetric decisions y^i of otherwise symmetric firms. We show in the appendix that such equilibria are possible only when there are no second-period sales. These equilibria are analogous to RESE2 with the same total inventory $Y^* = 1 - p_1$. There are no analogous asymmetric equilibria for RESE1 and RESE3.*

In practice, market outcomes corresponding to RESE1 can be identified by very small first-period sales primarily arising, e.g., from slight heterogeneity in the consumer's discount factor. Therefore, RESE1 describes practical scenarios where overwhelming majority of consumers wait for clearance sales. This scenario is common in retail because it describes, e.g., MSRP use solely to make discounts seem more dramatic than they actually are because, effectively, p_1 is not the actual selling price. In this case, the actual time duration of the first period can be very small, which can be captured by β close to one.

Inequality $p_1 \geq P_1$ implies that RESE1 is possible only if consumers are strategic ($\rho > 0$), except for a degenerate case $p_1 = 1$. The area of RESE1 inputs increases in ρ because consumers are more prone to delay the purchase and this outcome with no first-period sales happens at a lower p_1, in n because competition drives lower the second-period price, increasing the second-period consumer surplus, in the difference $\beta - c$ because retailer second-period profit increases in $\beta - c$ and consumer second-period valuations increase in β. This form of RESE completely matches a one-period Cournot-Nash outcome.

RESE2 is the opposite to RESE1: p_1 is low (high-valuation market), all consumers whose valuations are higher than p_1 buy in the first period, and there are no sales in the second period.

Condition $p_1 \le P_2$ implies that the existence of this RESE does not depend on ρ because $\bar{\alpha} = 0$, i.e., consumers do not expect second-period sales and, by Lemma 1, the equilibrium valuation threshold of the first-period buyers is $v^* = p_1$ regardless of ρ. The input area of RESE2 shrinks in β and n, disappearing for $\beta = 1$ and $n \to \infty$. The "β-effect" stems from increasing profitability of the second-period market when retailers can gain from two-period price discrimination. The "n-effect" results from increasing quantity competition for the market share, which may force retailers to procure more inventory than just for the first period.

The input area of RESE2 increases in c because the second-period profit approaches zero in c faster than the first-period profit, decreasing the relative attractiveness of the second-period sales. Retailers divide the profit associated with the total supply that is just enough to cover the first-period market. Because the supply is determined by an externally set MSRP, retailer competition is reduced to market sharing and we can interpret this outcome as an MSRP-facilitated collusion. In either of the first two equilibria, the intertemporal effect of competition is (locally) eliminated and, consequently, Y^* and r^* do not depend on ρ.

RESE3 describes scenarios with intermediate p_1 leading to sales in both periods with price discrimination between high and low valuation consumers. It provides a bridge between the opposites: a competitive Cournot outcome of RESE1 and an MSRP-enabled collusion of RESE2. Conditions (a) and (b) correspond to different attractiveness of salvage-value sales for a potential deviator from RESE3 that increases inventory. Condition (a) means that the deviator profit monotonically decreases, i.e., for the inputs that satisfy this condition, RESE3 is stable with respect to small parameter deviations given that p_1 is sufficiently far from the boundary. Under condition (b), deviator profit has a local maximum with $p_2 = s$ but this maximum does not exceed the profit under RESE3. The inputs satisfying (b) are near the boundary of RESE3 existence where this equilibrium may be unstable with respect to parameter misestimation.

In reality, consumer expectations may deviate from actual market outcome. The last paragraph of Theorem 1 provides a simple sufficient condition of convergence for possibly misaligned second-period price expectations when similar products (with the same β) are repeatedly introduced to the market with the same parameters. This condition assumes that retailers can accurately identify consumer expectations. If consumers incorrectly assume no sales in the second period, the equilibrium outcome, by Lemma 1, is the same as with myopic consumers ($\rho = 0$). If consumers assume there are sales in the second period but make mistake about the second-period price, the structure of RESE1 remains the same because it does not depend on expectations whenever $v^* = 1$. As to RESE3, the following result implies the upper bound on change in $\hat{Y}$ (the total symmetric equilibrium inventory with $\bar{p}_2 \ne p_2$) due to the effect of expectation errors.

Corollary 1. *Under RESE3,* $\frac{\partial \hat{Y}}{\partial \bar{p}_2} \le \frac{\rho}{1-\rho\beta} \max\left\{1, \frac{p_1 - c}{\beta - c}\right\}$.

A detailed study of the effects of misaligned second-period availability expectations on retailer rationing policy is provided in Huang and Liu (2015).

For a monopolist, RESE3 takes a simpler form described in the following corollary. In particular, condition $P_2 < p_1 < P_1$, which becomes $\frac{c}{\beta} < p_1 < 1 - \frac{\rho}{2}(\beta - c)$, is necessary and sufficient.

Corollary 2. *For* $n = 1$ *and any* $\frac{c}{\beta} < p_1 < 1 - \frac{\rho}{2}(\beta - c)$*, RESE is* $v^* = \frac{2p_1 - \rho c}{2 - \rho\beta}$, $\alpha^* = 1$, $p_2^* = \frac{\beta v^* + c}{2}$, $Y^* = 1 - \frac{1}{2}\left(c/\beta + v^*\right)$.

Because price and quantity decisions are equivalent for a monopoly, this corollary provides a characterization of the price-skimming policy when the first-period price is externally regulated. Monopolistic second-period price always exceeds the unit cost in our model (because $v^* \ge p_1 > c/\beta$ in RESE1 and 3). Increasing competition may drive the second-period price below cost, which we demonstrate in a market for a durable good with myopic consumers and some $n > 2$. The second-period price in this case remains above cost in a duopoly.

Corollary 3. *For $\beta = 1$, $\rho = 0$, and $c < p_1 < 1$, RESE1 and RESE2 cannot be realized and, in RESE3, the second-period price is below cost if and only if $n > 2 + \frac{p_1 - c}{1 - p_1}$.*

Increasing competition not only decreases the second-period price below cost, but undermines the very existence of RESE3. Indeed, condition (a) in RESE3 holds for any $n \geq 1$ only if s is sufficiently low. However, if there exists a liquidation channel with a salvage value s close to unit cost c, condition (a) may not hold. Moreover, one can show that the condition $r^* > \tilde{r}^i$ will then be violated for all sufficiently large n (this case is presented below in Corollary 4). This means that growing competition provides an incentive for a retailer to deviate from this form of RESE by increasing supply beyond the point where $p_2 = s$. Despite the resulting losses in the second period, this deviation can be profitable because of the increasing first-period market share, which compensates for the second-period loss. Hence, growing competition may result in the non-existence of RESE3 even though condition $P_2 < p_1 < P_1$ holds.

Corollary 4. *If condition $P_2 < p_1 < P_1$ holds and condition (a) of RESE3 existence is violated in the limit of $n \to \infty$, RESE3 does not exist for all sufficiently large n.*

This result calls for refinement of our understanding of the equilibrium and conditions for its existence. For monopoly ($n = 1$), Theorem 1 exhaustively covers all feasible parameter values. Starting from duopoly, condition $P_2 < p_1 < P_1$ may not guarantee the existence of RESE3. The result presented below shows that, in the same p_1-range, there may exist one more form of RESE with sales in both periods and $p_2^* = s$.

4.2 Salvaging RESE

The best response in the retailer game depends on $Y^{-i} \triangleq Y - y^i$ — total inventory less the inventory of retailer i. If $Y^{-i} < 1 - s/\beta$, retailer i can influence the second-period price. Namely, $p_2 > s$ if $y^i < 1 - s/\beta - Y^{-i}$ (no salvaging) or $p_2 = s$ if $y^i \geq 1 - s/\beta - Y^{-i}$ (salvaging). If $Y^{-i} \geq 1 - s/\beta$, salvaging is forced on retailer i, i.e., $p_2 = s$ regardless of supply y^i. Condition $Y^{-i} < 1 - s/\beta$ is used in a symmetric form with $Y^{-i} = \frac{n-1}{n} Y^*$ in the following characterization of the last equilibrium form further referred to as RESE4.

Theorem 2 ("Salvaging" RESE4: sales in both periods, $p_2^* = s$)**.** *RESE with $\alpha^* = 1, p_2^* = s, v^* = \frac{p_1 - \rho s}{1 - \rho\beta}, Y^* = \frac{n-1}{n}\frac{p_1 - s}{c - s}(1 - v^*)$, and $r^* = \frac{p_1 - s}{n^2}(1 - v^*)$ exists if and only if one of the following holds:*

(a) salvaging is forced on retailers, i.e., $\frac{n-1}{n} Y^* \geq 1 - \frac{s}{\beta}$;

(b) condition (a) does not hold, and $\left(\beta\left(1 - \frac{s}{\beta}\right)^2 + (p_1 - \beta)(1 - v^*)\right)\frac{n-1}{n}\frac{Y^*}{c + \beta v^* - 2s} \geq \left(1 - \frac{s}{\beta}\right)^2$;

(c) conditions (a) and (b) do not hold, $Y^* > 1 - \frac{s}{\beta}$, and there are no real roots of the equation

$$2Y^3 - \left(2 - v^* - c/\beta + \frac{n-1}{n} Y^*\right) Y^2 + (1 - p_1/\beta)(1 - v^*)\frac{n-1}{n} Y^* = 0 \tag{4}$$

in the interval $(1 - v^*, 1 - \frac{s}{\beta})$, or there is only one real root of (4) $\tilde{Y} \in \left(1 - v^*, 1 - \frac{s}{\beta}\right)$ and $r^* \geq \tilde{r}^i(\tilde{Y})$, where $\tilde{r}^i(\tilde{Y})$ is the maximum profit of a firm deviating from this RESE in such a way that $p_2 > s$.

If the initial consumer expectations of the second-period price are such that $\bar{p}_2^0 > s$, the game is repeated, and expectations follow a linear adjustment process, then the sequence of games converges to $\bar{p}_2 = s$ for any sufficiently small speed of adjustment.

Remark 2. *As we show in the appendix, there are no equilibria with asymmetric decisions y^i for otherwise symmetric firms with sales in both periods and $p_2^* = s$.*

One practical realization of this equilibrium outcome is a retailer use of liquidation channels for excess inventory (such as `www.salvagesale.com`, `liquidations.walmart.com`, and `www.liquidationchannel.com`). RESE4 provides an example of an overinvestment (in this case, in inventory), which is a known phenomenon in real economies. Unlike RESE1-3, RESE4 cannot exist for $n = 1$ because a monopolist would not have an incentive to overinvest in this setting. This can be seen, e.g., from the expression for Y^*. The larger n is, the easier retailers find themselves in RESE with $p_2^* = s$. Similar to RESE3, conditions (b) and (c) correspond to different attractiveness of a higher second-period price for a potential deviator from RESE4 that decreases inventory. Condition (b) means that the deviator profit monotonically increases in inventory, i.e., for the inputs that satisfy (b), RESE4 is stable with respect to small parameter changes when p_1 is sufficiently far from the boundary. The first part of condition (c) — no real roots of (4) in the interval $\left(1 - v^*, 1 - \frac{s}{\beta}\right)$ — means that the deviator profit has no local maxima with $p_2 > s$, whereas inequality $r^* \geq \tilde{r}^i(\tilde{Y})$ requires that when such a maximum exists at $y^i = \tilde{Y} - \frac{n-1}{n} Y^*$, it does not exceed the profit under RESE4. The inputs where RESE4 exists only by the second part of (c) are close to the boundary of RESE4 existence where this equilibrium may be unstable with respect to parameter misestimation. Conditions (a)-(c) hold if $c - s$ is sufficiently small, i.e., the cost is largely compensated by salvaging any excess units, which makes this outcome attractive for the retailers.

The last paragraph of Theorem 2, similar to the one in Theorem 1, provides a sufficient convergence condition for misaligned second-period price expectations when a new version of the product (with the same β) is repeatedly introduced to the market with the same parameters, and retailers do not make mistakes about consumer expectations. The effect of expectation error on the total inventory is limited by the following result.

Corollary 5. *Under RESE4,* $\frac{\partial \hat{Y}}{\partial \bar{p}_2} = \frac{\rho}{1-\rho\beta} \frac{n-1}{n} \frac{p_1 - s}{c-s}$.

Theorem 2 implies a necessary condition $v^* < 1$, which means that there is positive demand in the first period. This condition is equivalent to the upper bound $p_1 < 1 - \rho(\beta - s) \triangleq P_4$ signifying that a relatively high MSRP precludes salvaging outcome. Alternatively, this condition represents an upper bound on the consumer's discount factor:

$$\rho < (1 - p_1)/(\beta - s). \tag{5}$$

As long as the product is durable enough for $1 - p_1 < \beta - s$ to hold, highly strategic (with ρ near one) consumers guarantee that the salvaging outcome is impossible. Because $P_4 < P_1$ (the bound that separates RESE1 and 3), P_4 separates RESE4 and 3.

We now turn to the question of equilibrium uniqueness. By Theorem 1, RESE1, 2, and 3 are mutually exclusive because the corresponding p_1-ranges do not intersect. The result below shows that RESE1, 2, and 4 are also mutually exclusive. Moreover, part (b) guarantees that condition (a) of Theorem 1 holds for p_1-range of RESE3 and, at the same time, RESE4 cannot exist.

Proposition 1. *A unique RESE exists and is of the form stated if any of the following conditions hold: (a) RESE1 if $p_1 \geq P_1$, or RESE2 if $p_1 \leq P_2$, or RESE3 if (b.1) $P_2 < p_1 < P_1$ and (b.2)* $\frac{n-1}{n}(p_1 - s)(1 - p_1) \leq (c - s)\,(1 - s/\beta)$.

Condition (b.2) trivially holds for $n = 1$. In general, it has the form of a lower bound on $c - s$, i.e., the unit cost is sufficiently high compared to the salvage value. The condition holds for any p_1 and $n > 1$ if it holds for $n \to \infty$ and $p_1 = \frac{1}{2}(1 + s)$ (p_1 maximizing the left-hand side). The resulting stronger inequality is $c - s \geq \frac{(1-s)^2}{4(1-s/\beta)}$, which holds, e.g., for $c = 0.25$ and $s = 0$. Thus, when the unit cost is relatively high, retailers avoid the unfavorable "salvaging" outcome.

The analysis of this section leaves a possibility that RESE does not exist. This is indeed the case, but the fraction of model inputs where this may occur is very small. Combining all conditions in Theorems 1 and 2, we can determine which of the four types of equilibria exist (if any) for any given set of inputs $(n, \rho, \beta, c, s, p_1)$ satisfying the feasibility conditions $0 \leq \rho < 1$, $0 \leq s < c < \beta \leq 1$, and $\max\{s/\beta, c\} < p_1 \leq 1$. We have performed this analysis for 1,000,000 randomly (according to uniform distribution) sampled feasible model inputs for different values of $1 \leq n \leq 1,000$. The results are presented in Figure 2. Subgraph (a) is an area plot that shows the fractions of inputs resulting in a particular equilibrium structure (RESE1, 2, or 3 only, both RESE3 and 4, RESE4 only) as the heights of the respective shaded areas for each n. As n increases, RESE2 disappears and the prevalence of RESE1 and 4 grows with RESE4 reaching more than 50% of model inputs. Subgraph (b) shows the fractions of inputs resulting in both RESE3 and 4 as well as non-existence of equilibrium. The fraction of inputs where both RESE3 and 4 exist is 4% for a duopoly and considerably less for other levels of competition. The fraction of inputs where no RESE exists is at most 0.191% (reached for $n = 5$).

5 Properties of RESE

The results of previous sections can be used, e.g., by a manufacturer or retailer to estimate possible outcomes of entering the market. These outcomes depend on the current *levels* of competition, strategic behavior, and other parameters. For an existing market, the effects of *changes* in these parameters can be more relevant in order to anticipate possible market alterations. As to changes in consumer strategic behavior, one of their drivers is macroeconomic. When the economy is expanding, more consumers prefer to buy now than wait, and vice versa – an average consumer is more inclined to delay the purchase when the economy shrinks. For example, a study of a Fortune 500 retailer sales by Allenby, Jen, and Leone (1996) shows that even "fashion-forward consumers who purchase apparel early in the season are more sensitive to economic conditions and expectations than previously believed."

Various forms of Consumer Confidence Indicators report on changes in consumer behavior. For example, the Index of Consumer Confidence is defined by the Conference Board of Canada web site as "a crucial indicator of near-term sales for companies in the consumer products sector... Data is collected on each respondent's age, sex, marital status, and geographic location of residence." Using these data and other macroeconomic variables, a retailer and/or manufacturer can estimate possible changes in ρ and, respectively, in market outcomes given that the current situation is known. Lemmens, Croux, and Dekimpe (2005), in an empirical study of the European markets, conclude that "the Consumer Confidence Indicators become much more homogeneous as the planning horizon is extended." This homogeneity emerges inside of regions, and is determined by cultural, economic and geographic differences.

A major macroeconomic driver of consumer intertemporal choice is the economy's interest rate. The "substitution effect" refers to an increase in interest rate that encourages consumers to save more and defer some of their purchases (increasing ρ). A review of Thimme (2015) shows that empirical estimates of the sensitivity of consumer intertemporal choice to the interest rate (the elasticity of intertemporal substitution) essentially varies across markets and groups of consumers.

The study implies that a forecast of possible changes in market outcomes depending on the interest rates should be market-specific.

This section partly supports previous studies showing that equilibrium total supply increases in n and decreases in ρ. Both trends typically decrease retailers' profits. On the other hand, we specify two distinct cases where these opposing trends "compensate" each other leading to increasing profit in ρ. It is also noteworthy, that increasing ρ has different effects on consumer second-period surplus and total second-period sales depending on the consumer valuation and market situation, respectively.

5.1 RESE1-3 (no salvaging)

The analysis below accounts for possible switches between different forms of RESE. The requirement of a unique RESE existence can be guaranteed, e.g., by Proposition 1.

Switches between RESE forms When RESE is unique, p_1-ranges indicated in Theorem 1 provide a unique mapping between input parameter values and different forms of RESE. Figure 3 illustrates how these ranges change with n and ρ:

(a) the bounds on p_1 that separate RESE3 from RESE1 and 2 are decreasing in n;

(b) the upper bound on p_1 in RESE3 is decreasing, and the lower bound is constant in ρ; and

(c) the lowest possible value of p_1 that leads to RESE1 is strictly above the highest possible value that leads to RESE2.

These observations are summarized as follows:

Proposition 2 (Changes in RESE structure)**.** *For RESE1-3, the following claims hold:*

1. *(From 2 to 3 in n) If $p_1 \leq \frac{c}{\beta}$, there exists $n_2 \triangleq \frac{p_1(1-\beta)}{p_1-c} \geq 1$ such that RESE can only be realized with sales only in the first period (RESE2) for $n \leq n_2$, and with sales in both periods and $p_2^* > s$ (RESE3) for $n > n_2$.*

2. *(From 3 to 1 in n) For any $\rho \in (0,1)$, if $1-\rho(\beta-c) < p_1 < 1-\frac{1}{2}\rho(\beta-c)$, there exists $n_1 \triangleq \frac{1-p_1}{p_1-1+\rho(\beta-c)} \geq 1$ such that RESE can only be realized with sales in both periods and $p_2^* > s$ (RESE3) for $n < n_1$, and with sales only in the second period (RESE1) for $n \geq n_1$.*

3. *(From 3 to 1 in ρ) For any $n \in [1,\infty)$, if $1-\frac{n}{n+1}(\beta-c) < p_1 < 1$, there exists $\rho_1 \triangleq \frac{n+1}{n}\frac{1-p_1}{\beta-c}$ such that RESE can only be realized with sales in both periods and $p_2^* > s$ (RESE3) for $\rho < \rho_1$ and with sales only in the second period (RESE1) for $\rho \geq \rho_1$.*

4. *(No switches) If $\frac{c}{\beta} < p_1 \leq 1-\beta+c$, RESE can only be realized with sales in both periods and $p_2^* > s$ (RESE3) for $n \geq 1$ and $\rho \in [0,1)$.*

The changes in equilibrium structure generally lead to shifts in sales to the second period as the levels of competition or strategic behavior increase. Next, we examine changes in the quantitative characteristics of equilibrium.

Monotonicity of Y^*, v^*, and nr^* We now examine the monotonicity of v^*, Y^*, and nr^* in n and ρ within RESE1-3 and, by continuity, between these forms of RESE.

Proposition 3. *For RESE described in Theorem 1, the following claims hold:*

1. *The equilibrium total supply Y^* is non-decreasing in n (constant for RESE2; increasing for RESE1 and 3) and non-increasing in ρ (decreasing for RESE3; constant for RESE1 and 2).*

2. *v^* is non-decreasing in n (constant for RESE1, 2, and RESE3 with $\rho = 0$; increasing for RESE3 with $\rho > 0$) and non-decreasing in ρ (increasing for RESE3; constant for RESE1 and 2).*

3. *The total equilibrium profit of all retailers nr^* is non-increasing in n (constant for RESE2; decreasing for RESE1 and 3), decreasing in ρ for RESE3 with $p_1 \geq \beta - \frac{n}{2(n+1)}(\beta - c)$ or $n = 1$, and constant in ρ for RESE1 and 2.*

Monotonicity of the total supply and the total profit in the level of competition agree with the theory of oligopoly and can be viewed as a sanity test for the model. On the other hand, monotonicity in the consumer's discount factor ρ is a much finer result. The new insights of this study are connected to the following non-trivial interaction between firms and consumers while ρ is increasing. Part 2 of Proposition 3 states that v^* is increasing in ρ when there are sales in both periods (RESE3) and retailers effectively engage in intertemporal price discrimination. Increasing v^* means that more consumers delay their purchases, even though total supply Y^* is decreasing in ρ (part 1), resulting in a decreasing total number of purchases and increasing second-period price. The nature and consequences of this interaction are considered below in more detail. Part 3 of Proposition 3 agrees with the existing literature that strategic consumer behavior reduces monopoly profits. We generalize this effect to the case of oligopoly when the product is not very durable (i.e., β is sufficiently low), the level of competition is low, or the cost and MSRP are relatively high.

In RESE3, an increase in ρ leads to additional consumer delays in purchase and a response of oligopolistic retailers by increasing the equilibrium second-period price $p_2^* = \beta\,(1 - Y^*)$. As a result, the expected surplus of waiting $\sigma_2 = \rho\,[\beta v - \beta\,(1 - Y^*)]$ may not be increasing in ρ. Indeed, its derivative in ρ is $\frac{\partial \sigma_2}{\partial \rho} = \frac{\sigma_2}{\rho} + \rho \frac{\partial Y^*}{\partial \rho}$. The first term in the RHS is the realized second-period surplus, which is non-negative for the consumers who buy in the second period. The second term reflects the equilibrium response of the oligopolistic retailers. By part 1 of Proposition 3, this term is negative for any $\rho > 0$. For RESE3, the following corollary shows that increasing ρ has a different effect on σ_2 depending on the consumer valuation.

Corollary 6. *For RESE1 and 3, expected surplus σ_2 of waiting is (1) increasing in n for any $v \in [0, 1]$, and (2) increasing in ρ for $v^0 < v \leq 1$, and decreasing in ρ for $0 \leq v < v^0$, where $v^0 = \frac{1}{\beta}\left(p_2^* + \rho \frac{\partial p_2^*}{\partial \rho}\right) = 1 - Y^* - \rho \frac{\partial Y^*}{\partial \rho} \in \left[\frac{p_2^*}{\beta}, v^*\right)$ is such that $\left.\frac{\partial \sigma_2}{\partial \rho}\right|_{v = v^0} = 0$.*

For consumers with high valuations, σ_2 is increasing in ρ. In particular, for the consumers with $v = v^*$, the purchase in the second period is becoming more attractive than in the first period, which means that v^* is increasing in ρ (part 2 of Proposition 3). In contrast, σ_2 is decreasing for consumers with low valuations. For example, the second-period surplus of the second-period buyers with the lowest valuation $v = \frac{p_2^*}{\beta}$ is becoming negative, leading to a decrease in the total number of purchases. For myopic ($\rho = 0$) consumers, $v^0 = \frac{p_2^*}{\beta}$ and $\frac{\partial \sigma_2}{\partial \rho} > 0$ for all second-period buyers. Since income is often considered as a proxy for product valuation, we can interpret this result as an increasing inequality in terms of utility facilitated by a homogeneous increase in strategic behavior. This effect cannot be captured by utilitarian welfare function such as the total consumer surplus.

An increase in ρ leads to either an increase or decrease in the total equilibrium second-period sales $Q_2^* = Y^* - (1 - v^*)$, depending on the parameters:

Corollary 7. *For RESE3, (1) $\frac{\partial Q_2^*}{\partial \rho} > 0$ when $n = 1$; and (2) $\frac{\partial Q_2^*}{\partial \rho} < 0$ when $n \to \infty$, $\rho = 0$, $\beta < 1$ and p_1 is near 1.*

Proposition 3 claims that $nr^{*,3}$ is decreasing in ρ for monopoly and oligopoly with a not very durable good. However, by the same Proposition, the equilibrium inventory $Y^{*,3}$ (the only retailer decision determining the profit) is increasing in n and decreasing in ρ. Therefore, one may expect that for some model inputs there may exist such ρ^0 that a further increase in ρ, by decreasing $Y^{*,3}$, can "compensate" not only for losses in the first-period sales but also for profit losses due to a high level of competition. By part 3 of Proposition 3, this effect can be expected only if the product is durable (β is high) and the cost is low, which leads to a relatively high second-period profit, and for high levels of competition when the loss from competition is significant.

To show that $nr^{*,3}$ may increase in ρ, we consider a limiting case of a durable (within the time frame of the problem) product and consumers with the maximum discount factor. In this case, the consumer choice of purchase time is determined only by price.

Proposition 4. *Let $\bar{n} \triangleq \frac{1-p_1}{p_1-c}$. For $\beta = 1$ and $\rho \to 1$, RESE2 and 4 do not exist and the equilibrium has the form*

(1) RESE1 with $v^*|_{\rho\to 1} = 1$, $Y^*|_{\rho\to 1} = \frac{n}{n+1}(1-c)$, $p_2^*|_{\rho\to 1} = \frac{nc+1}{n+1} < p_1$, and $nr^*|_{\rho\to 1} = \frac{n(1-c)^2}{(n+1)^2}$ if $n > \bar{n}$, and

(2) RESE3 with $v^*|_{\rho\to 1} = p_1 + n(p_1 - c)$, $Y^*|_{\rho\to 1} = 1 - p_1$, $p_2^*|_{\rho\to 1} = p_1$, and $nr^*|_{\rho\to 1} = (p_1 - c)(1 - p_1) < \frac{n(1-c)^2}{(n+1)^2}$ if $n < \bar{n}$ and

$$\frac{n-1}{n} \frac{(p_1 - s)(1 - p_1 - n(p_1 - c))(1 - p_1)}{(c - s)(1 - s)^2} < 1. \tag{6}$$

Moreover, when $n = \bar{n}$,

(3) the limiting cases (1) and (2) coincide, and

(4) (boundary-value gain) for all p_1 and c such that $\bar{n} \geq 3$, we have $nr^{*,3}\big|_{\rho\to 1} > nr^{*,3}\big|_{\rho=0}$.

In both limiting scenarios, sales occur at a single price: the one-period Cournot-Nash price in part 1 and p_1 in both periods in part 2. In part 1, representing a high level of competition, no sales occur at p_1, i.e., $v^* = 1$, and price decreases to Cournot-Nash in the second period. The Cournot-Nash supply level in this case exceeds the lower bound $1 - p_1$ of Lemma 4. In part 2, representing a low level of competition, retailers counteract strategic behavior by reducing the total supply all the way to $1 - p_1$ which exceeds in this case the Cournot-Nash supply.

Thus, the maximum level of consumer strategic behavior forces retailers into a collusion-like outcome. Part 3 shows that $n = \bar{n} = \frac{1-p_1}{p_1-c}$ plays a role of a *parameter coordination* condition ensuring that the Cournot-Nash price coincides with p_1. This condition is critical for understanding part 4 that demonstrates the total profit increase as consumer behavior changes from myopic to fully strategic.

Under Proposition 4, fully strategic behavior prevents the second-period sales at a loss. Indeed, with fully strategic consumers, the total sales are equal to $1 - p_1$ and occur at p_1. In the case of myopic consumers, the first-period sales are the same, whereas the second-period sales are at loss for any $n \geq 3$ by Corollary 3. Because the increase in profit is strict, the effect presented in part 4 is quite robust. Indeed, for each $n \geq 3$, there is a continuum of model instances satisfying the parameter coordination condition. Moreover, by continuity in parameters, increased strategic behavior leads to an increase in profit in a local neighborhood of these instances. The boundary-value profit gain described in part 4 results from the continuous gain because the entire range $\rho \in [0, 1)$ belongs to the same equilibrium RESE3 (Figure 4).

The following example shows that input area of condition (6) is not empty.

Example 1. *Condition (6) is a limiting version of condition (a) of RESE3 existence, which is less restrictive than sufficient condition (b.2) of Proposition 1. Condition (6) holds for all* p_1 *and* $1 \leq n < \bar{n}$ *if* $c > \frac{1+4s}{5}$*, e.g., if* $s = 0$ *and* $c > 0.2$*.*

The numerical example below illustrates the behavior of the total profit in ρ. For small ρ, the profit decreases in ρ (see Figure 4 (a)). On the other hand, when $\beta = 1$, the profit increases for ρ near one. For all values of $n \leq \bar{n} = 10$ in this example, the total profit attains the limit $(p_1 - c)(1 - p_1)$ established in part 2 of Proposition 4. It is natural to expect that this effect of "durable-good non-monotonicity" weakens for $\beta < 1$. Indeed, Figure 4 (b) illustrates that, for $\beta = 0.9$, the profit decreases in ρ for $n = 1, 2, 3$, and 5, and an increase of nr^* in ρ is small for $n = 10$. This increase can no longer compensate for the losses in nr^* resulting from increased competition.

5.2 RESE4 (salvaging)

The following proposition establishes the monotonic properties of Y^*, v^*, and nr^* for RESE4. These properties partially match those of RESE1-3.

Proposition 5. *For RESE4, (1)* v^* *is constant in* n *and increasing in* ρ*; (2)* Y^* *is increasing in* n *and decreasing in* ρ*; and (3)* nr^* *is decreasing in* n *and decreasing in* ρ*.*

By part (a) of Proposition 1, RESE4 does not exist under the conditions of RESE1 and 2. However, both RESE3 and 4 can exist for the same inputs. In that case, one needs to resort to focal-point arguments to predict which of the two equilibria will be realized. The example of Figure 5 illustrates this fact.

The inputs for Figure 5 are such that inequality $P_2 < p_1 < P_1$ holds and condition (a) of Theorem 1 for RESE3 existence holds for $n = 1$ but does not hold for $n = \infty$. By Corollary 4, RESE3 does not exist for sufficiently large n. At the same time, by part (b) of Proposition 1 and by rationality, RESE4 does not exist and RESE3 is realized uniquely for $n = 1$. It is also straightforward to check that RESE3 can be realized for $n = 1, \ldots, 11$, whereas RESE4 can be realized for $n = 8, \ldots, \infty$. For $n = 8, \ldots, 11$ either equilibrium is possible.

In line with the interpretation of rational expectations equilibrium as a structure that is self-sustaining in the long run, a possible focal point is an equilibrium with a structure that is similar to the past. Figure 5 (b) shows that RESE4 is considerably worse for the retailers than RESE3. However, RESE4 may be realized because a single retailer cannot unilaterally benefit from decreasing its market share as long as others expect the RESE4 structure and act accordingly.

Now, what is the effect of changing ρ when there is an overlap in inputs leading to RESE3 and 4? For the data considered above, a minor increase in ρ from 0.5 to 0.6 qualitatively changes the situation because, for $\rho = 0.6$, inequality $P_2 < p_1 < P_1$ and condition (a) of RESE3 existence hold for any $n \geq 1$ and neither of subcases (a)-(c) of Theorem 2 hold. Therefore, RESE4 cannot exist in the scenario considered above, and this increase in ρ leads to the discontinuous profit gain and serves as an *insurance against salvaging*. Such an increase in ρ works by decreasing capacity in RESE3 at the cost of a slight decrease in profit (compare the solid and dashed lines in Figure 5 (b)). The discontinuous profit gain can lead to the boundary-value gain even when β is small (Figure 6 b) and can be combined with the continuous gain when β is near one. As a result, equilibrium profit can have up to three local maxima in ρ (Figure 6 a).

An increase in ρ cannot always prevent retailers from RESE4, but it does reduce the fraction of inputs leading to it. Figure 7 shows that for $\rho = 0.999$ the maximum fraction of model inputs leading to RESE4 reduces to 37.2% compared to more than 50% in Figure 2, where ρ is unrestricted. Moreover, the area of RESE3 and 4 coexistence shrinks to less than 1.25%.

6 Total consumer surplus and aggregate welfare

In this section, we examine the effects of strategic consumer behavior and retailer competition on the consumers and the local economy. In a two-period problem, the *total two-period (realized) consumer surplus* is $\Sigma^* \triangleq \Sigma_1 + \Sigma_2$, where Σ_1 and Σ_2 are the total surpluses of consumers buying in the first and second periods. Σ_2 is not discounted by ρ because ρ is a subjective behavioral parameter and such a discount would not reflect the actual surplus. In the extreme case of $\rho = 0$, such discounting would completely disregard the second-period surplus of myopic consumers. The expression for Σ^* is given by the following:

Lemma 5. *For a RESE with valuation threshold v^* and second-period price p_2^*, total consumer surplus is $\Sigma^* = (1 - v^*)\left[\frac{1+v^*}{2} - p_1\right] + \frac{(\beta v^* - p_2^*)^2}{2\beta}$, where the first term is Σ_1 and the second is Σ_2.*

The effects on the local economy can be measured in terms of the *aggregate welfare* $W^* \triangleq \Sigma^* + nr^*$. The main structural result for Σ^* and W^* is

Proposition 6. *Under the conditions of Theorems 1 and 2,*

1. *total consumer surplus Σ^* is non-decreasing in n (constant for RESE2 and 4 and increasing for RESE1 and 3), constant in ρ for RESE1 and 2, and increasing in ρ for RESE4;*

2. *aggregate welfare W^* is*

   (2.1) increasing in n for RESE1, constant for RESE2, and decreasing for RESE4;

   (2.2) constant in ρ for RESE1 and 2, and, for RESE4, increasing in ρ for $\rho < \rho^+$ and decreasing for $\rho > \rho^+$ where $\rho^+ \triangleq \left(1 - \frac{1}{n}\frac{p_1 - s}{p_1\beta - s}\right) \Big/ \left(1 - \frac{\beta}{n}\frac{p_1 - s}{p_1\beta - s}\right)$ if $n > \frac{p_1 - s}{p_1\beta - s}$, and $\rho^+ \triangleq 0$ otherwise.

This proposition implies that the consumer population as a whole benefits from an increase in competition. On the other hand, Σ^* may not be globally monotonic in ρ for RESE3. The non-monotonicity is established below for the case of $\beta = 1$:

Corollary 8. *Under the conditions of RESE3, $\beta = 1$ implies: (1) for all $n \geq 1$ and ρ sufficiently close to one, $\frac{\partial \Sigma^*}{\partial \rho} < 0$; and (2) for $n = 1$ or $n \to \infty$ and $\rho = 0$, $\frac{\partial \Sigma^*}{\partial \rho} > 0$.*

Corollary 8 and continuity of Σ^* in β imply that Σ^* has a maximum in ρ if β is sufficiently large (Figure 8 (a)). Non-monotonic behavior is less pronounced for smaller β (Figure 8 (b)).

Along with the monotonicity of Σ^*, Proposition 6 describes certain settings with monotonic aggregate welfare. The direction of monotonicity in a particular parameter varies depending on the equilibrium structure and other inputs. For example, W^* is increasing in n for RESE1 (Cournot-Nash outcome), which matches increasing welfare results for a standard one-period Cournot-Nash equilibrium corresponding to our model. However, in other quantity competition settings, welfare may not be increasing in the level of competition. For example, Bulow, Geanakoplos, and Klemperer (1985) (§VI, Example E) claim that welfare may decrease when a retailer with high marginal costs enters a monopoly market. In our model, the aggregate welfare decreases in n for RESE4 (salvaging outcome) because the resulting increase in product oversupply does not benefit the consumers and only decreases profits of the retailers. For RESE4, the discount factor ρ^+ attains the internal maximum of W^* as long as $\beta < 1$ and the level of competition is sufficiently high, i.e., $n > \frac{p_1 - s}{p_1\beta - s}$. For $n \leq \frac{p_1 - s}{p_1\beta - s}$, W^* decreases for all ρ. The dependence of W^* on n was omitted in Proposition 6 for RESE3 because this case warrants special attention:

Corollary 9 (Non-monotonicity of W^* in n)**.** *Treating n as a continuous variable and p_2^* as a function of n under the conditions of RESE3, the following result holds:*

$$\frac{\partial W^*}{\partial n} \gtreqless 0 \text{ if and only if } p_2^* \gtreqless c\frac{(1-\rho\beta)^2}{1-2\rho\beta+\rho^2\beta} + p_1\frac{\rho\beta(1-\beta)}{1-2\rho\beta+\rho^2\beta}. \tag{7}$$

The right-hand side of (7) equals c when $\rho = 0$ or $\beta = 1$ and strictly greater than c otherwise.

If there exists n^W that is strictly within the feasible interval for RESE3 and maximizes W^*, it satisfies (7) as equality. When $\rho = 0$ or $\beta = 1$, this means that the second-period price corresponding to n^W equals the unit cost. On the other hand, when $\rho > 0$ and $\beta < 1$, the corresponding second-period price is strictly greater than the unit cost. By Proposition 3, Y^* and, therefore, p_2^* are strictly monotonic for RESE3. Then n^W is unique whenever it exists. From this unique value, we obtain the maximum of the aggregate welfare (the candidates for the integer-valued point of maximum are $\lfloor n^W \rfloor$ and $\lceil n^W \rceil$ because, generally, the solution to this equation is real-valued). We illustrate the behavior of W^* in n and ρ in Figures 9 and 10, respectively, for the same set of inputs as our earlier illustrations. For $\beta = 1$, Figure 9 (a) demonstrates that the aggregate welfare can be monotonically increasing in n (for high levels of strategic behavior), and it can also attain the maximum at intermediate values of n (for lower levels of strategic behavior). The latter illustrates Corollary 9. For $\beta = 0.9$, Figure 9 (b) shows that the aggregate welfare may remain monotonically decreasing in the whole range of ρ. In all cases presented in Figure 9 (b), the maximum value of the aggregate welfare is attained by the monopoly. These findings may provide theoretical support for a regulator introducing a policy that affects the number of independent retail chains. Figure 10 indicates that myopic consumer behavior or strategic behavior at an intermediate level may be the best for the local economy in terms of the aggregate welfare. Myopic behavior is the best for low levels of competition, and the welfare-maximizing ρ tends to increase in n. A smaller value of $\beta = 0.9$ leads to the optimality of myopic behavior in a wider range of n.

7 Conclusions

Even when consumers are risk-neutral and have the same discount factor, retailers can gain from increasingly strategic consumers for any level of competition. There are two distinct cases of this effect: the *continuous gain,* when the equilibrium profit increases continuously in the consumer's discount factor, and the *discontinuous gain,* when the profit increases because of the switch from the "salvaging" equilibrium to another two-period equilibrium with a higher second-period price.

The first type of gain occurs only for relatively high levels of strategic behavior and small decreases in valuations. With this gain, retailers use strategic consumer behavior to approach an outcome that is equivalent, in terms of the profit value, to a tacit collusion. The discontinuous gain occurs at various levels of strategic behavior and the relative decrease in valuations, but only when salvage sales are attractive enough, i.e., the salvage value is relatively close to the unit cost. For a manufacturer, increasing strategic behavior is always unfavorable because it decreases the total equilibrium inventory procured by the retailers. Both types of retailer profit gains are reversible. When consumer confidence increases, more consumers buy at the first-period price, becoming less strategic. The incentive for quantity competition increases, and retailers may find themselves in the unfavorable "salvaging" outcome.

We summarize the monotonic properties of equilibrium characteristics with respect to competition level n and strategic behavior level ρ in Table 1 using $\nearrow$, $\searrow$, and $\equiv$ to indicate a monotonically increasing, decreasing, or constant property, respectively. The possibility of an internal maximum or minimum is indicated by "max" or "min", respectively. When multiple symbols are present, it means that different behaviors are possible for different inputs. The direction of monotonicity with respect to the level of competition can only vary for the aggregate welfare in RESE3. The latter finding may be important for regulatory policies with respect to competition. For RESE4, the increasing level of competition is always detrimental for the local economy.

Advantages of the presented model include its analytical tractability and natural connections to established oligopoly results. Possible extensions cover a wide range of problems in the study of competition in the presence of strategic consumers, for example: (1) analysis of policy decisions, including taxes and subsidies for the manufacturer, retailers, and/or consumers; (2) study of supply-chain coordination; (3) analysis of competition when advertisement and inventory decisions are decoupled; and (4) study of price-matching contracts as a tool to counteract strategic consumer behavior.

Notes

[1]E.g., 3D TVs are currently at the introduction stage, blueray discs/DVR – at the growth stage, DVD – at the maturity stage, video cassette – at the decline stage (http://productlifecyclestages.com Accessed 15 February 2016).

[2]A brief review of studies, history, and empirical evidence of focal-point pricing including RPM is in Appendix.

[3]Some studies consider market share competitions of firms that produce and sell their products. E.g., Schmalensee (1976) assumes that total demand increases in total promotions as a power function; Karnani (1984) assumes, for simplicity, exogenously fixed constant total demand.

[4]This assumption is relaxed in Supplementary Document by considering retailers with different costs.

[5]Some extensions of the model are presented in Supplementary Document.

[6]A review of demand allocation models with the same fixed retailer price is in §6.5 of Cachon (2003). A relaxation of the "same cost" assumption is provided in Supplementary Document.

[7]Market share in the form "us/(us + them)" can be obtained by assuming that firm's market share is proportional to the marketing effort. This form can also be derived from random utility models of individual consumer choice. The latter approach requires an assumption of the double exponential distribution for the joint distribution of random utilities of a customer from buying a product from different retailers, see a review in Cooper and Nakanishi (1988). The formula "us/(us + them)" in the form of the MNL model results also from the assumption of customer rational inattention, see Matějka and McKay (2015). The latter approach, which assumes that information about the differences among retailers is costly, does not work in our setting because our firm attractions are easily observable.

[8]Even though attractions are not continuous at 0 in this case, we demonstrate in Supplementary Document that the analysis is still possible.

References

Allenby, G., L. Jen, and R. Leone (1996). "Economic Trends and Being Trendy: The Influence of Consumer Confidence on Retail Fashion Sales". In: *Journal of Business & Economic Statistics* 14(1), pp. 103–111.

Aviv, Y., Y. Levin, and M. Nediak (2009). "Counteracting Strategic Consumer Behavior in Dynamic Pricing Systems". In: *Consumer-Driven Demand and Operations Management Models*. Ed. by S. Netessine and C. Tang. Springer, pp. 323–352.

Balakrishnan, A., M. Pangburn, and E. Stavrulaki (2004). ""Stack Them High, Let'em Fly": Lot-Sizing Policies When Inventories Stimulate Demand". In: *Management Science* 50(5), pp. 630–644.

Bell, D., R. Keeney, and J. Little (1975). "A Market Share Theorem". In: *Journal of Marketing Research* 12(2), pp. 136–141.

Bulow, J., J. Geanakoplos, and P. Klemperer (1985). "Multimarket Oligopoly: Strategic Substitutes and Complements". In: *Journal of Political Economy* 93(3), pp. 488–511.

Cachon, G. (2003). "Supply Chain Coordination with Contracts". In: *Supply chain management: Design, coordination and operation*. Ed. by S. Graves and A. Kok. North-Holland: Amsterdam. Chap. 6.

Cachon, G. and R. Swinney (2009). "Purchasing, Pricing, and Quick Response in the Presence of Strategic Consumers". In: *Management Science* 55(3), pp. 497–511.

Coase, R. (1972). "Durability and Monopoly". In: *The Journal of Law and Economics* 15(1), pp. 143–149.

Cooper, L. and M. Nakanishi (1988). *Market-share analysis: Evaluating competitive marketing effectiveness*. Kluwer Academic Publishers Boston.

Davidson, C. and R. Deneckere (1986). "Long-run competition in capacity, short-run competition in price, and the Cournot model". In: *RAND Journal of Economics* 17(3), pp. 404–415.

Flath, D. (2012). "Are There Any Cournot Industries in Japan?" In: *Japanese Economy* 39(2), pp. 3–36.

Frederick, S., G. Loewenstein, and T. O'Donoghue (2002). "Time discounting and time preference: A critical review". In: *Journal of Economic Literature* 40(2), pp. 351–401.

Friedman, L. (1958). "Game-Theory Models in the Allocation of Advertising Expenditures". In: *Oper. Res.* 6(5), pp. 699–709.

Gallego, G. et al. (2006). "Price Competition with the Attraction Demand Model: Existence of Unique Equilibrium and Its Stability". In: *Manuf. Serv. Oper. Man.* 8(4), pp. 359–375.

Hausman, J. (1979). "Individual discount rates and the purchase and utilization of energy-using durables". In: *The Bell Journal of Economics* 10(1), pp. 33–54.

Houston, D. (1983). "Implicit discount rates and the purchase of untried, energy-saving durable goods". In: *Journal of Consumer Research* 10(2), pp. 236–246.

Huang, T. and Q. Liu (2015). "Strategic Capacity Management When Customers Have Boundedly Rational Expectations". In: *Production and Operations Management* 24(12), pp. 1852–1869.

Karnani, A. (1984). "The Value of Market Share and the Product Life Cycle – A Game-Theoretic Model". In: *Management Science* 30(6), pp. 696–712.

Klapper, D. and H. Herwartz (2000). "Forecasting market share using predicted values of competitive behavior: further empirical results". In: *International Journal of Forecasting* 16(3), pp. 399 –421.

Kreps, D. and J. Scheinkman (1983). "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes". In: *The Bell Journal of Economics* 14(2), pp. 326–337.

Lazear, E. (1986). "Retail Pricing and Clearance Sales". In: *American Economic Review* 76, pp. 14–32.

Lemmens, A., C. Croux, and M. Dekimpe (2005). *The European Consumer: United In Diversity?* Tech. rep. ERS-2005-022-MKT. ERIM Report Series Research in Management.

Lippman, S. and K. McCardle (1997). "The competitive newsboy". In: *Operations research* 45(1), pp. 54–65.

Liu, Q. and G. van Ryzin (2008). "Strategic capacity rationing to induce early purchases". In: *Management Science* 54(6), pp. 1115–1131.

Matějka, F. and A. McKay (2015). "Rational inattention to discrete choices: A new foundation for the multinomial logit model". In: *American Economic Review* 105(1), pp. 272–98.

Mills, H. (1961). "A Study in Promotional Competition". In: *Mathematical Models and Methods in Marketing*. Ed. by F. Bass and et al. Irwin Homewood, IL, pp. 245–301.

Monahan, G. (1987). "The structure of equilibria in market share attraction models". In: *Man. Sci.* 33(2), pp. 228–243.

Naert, P. and M. Weverbergh (1981). "On the Prediction Power of Market Share Attraction Models". In: *Journal of Marketing Research* 18(2), pp. 146–153.

Nippop (2005). *Saihan Seido – Japan's Resale Price Maintenance System*. Accessed on October 13, 2015. URL: http://nippop.com/features/Saihan_Seido___Japan_s_Resale_Price_Maintenance_System/.

Perakis, G. and W. Sun (2014). "Efficiency analysis of Cournot competition in service industries with congestion". In: *Management Science* 60(11), pp. 2684–2700.

Pigou, A. (1932). *The economics of welfare*. Macmillan and Co., Limited.

Schmalensee, R. (1976). "A Model of Promotional Competition in Oligopoly". In: *The Review of Economic Studies* 43(3), pp. 493–507.

Shen, Z. and X. Su (2007). "Customer Behavior Modeling in Revenue Management and Auctions: A Review and New Research Opportunities". In: *Production and Operations Management* 16(6), pp. 713–728.

Stokey, N. (1979). "Intertemporal Price Discrimination". In: *The Quarterly Journal of Economics* 93(3), pp. 355–371.

Stokey, N. (1981). "Rational expectations and durable goods pricing". In: *The Bell Journal of Economics* 12(1), pp. 112–128.

Su, X. (2007). "Inter-temporal pricing with strategic customer behavior". In: *Management Science* 53(5), pp. 726–741.

Thimme, J. (2015). "Intertemporal Substitution in Consumption: A Literature Review". In: *Available at SSRN*.

Tirole, J. (1988). *The Theory of Industrial Organization*. MIT Press.

Urban, T. (2005). "Inventory models with inventory-level-dependent demand: A comprehensive review and unifying theory". In: *European Journal of Operational Research* 162(3), pp. 792–804.

Wolfe, H. (1968). "A model for control of style merchandise". In: *Industrial Management Review* 9(2), pp. 69–82.

Yang, N. and R. Zhang (2014). "Dynamic pricing and inventory management under inventory-dependent demand". In: *Operations Research* 62(5), pp. 1077–1094.

8 Tables and figures

	Monotonicity in n				Monotonicity in ρ			
RESE	1	2	3	4	1	2	3	4
Y^*	$\nearrow$	$\equiv$	$\nearrow$	$\nearrow$	$\equiv$	$\equiv$	$\searrow$	$\searrow$
v^*	$\equiv$	$\equiv$	$\nearrow$	$\equiv$	$\equiv$	$\equiv$	$\nearrow$	$\nearrow$
nr^*	$\searrow$	$\equiv$	$\searrow$	$\searrow$	$\equiv$	$\equiv$	$\searrow$, min	$\searrow$
Σ^*	$\nearrow$	$\equiv$	$\nearrow$	$\equiv$	$\equiv$	$\equiv$	$\nearrow$, $\searrow$, max	$\nearrow$
W^*	$\nearrow$	$\equiv$	$\nearrow$, $\searrow$, max	$\searrow$	$\equiv$	$\equiv$	$\nearrow$, $\searrow$, max	$\nearrow$, $\searrow$, max

Table 1: Summary of monotonic properties in n and ρ by equilibrium form

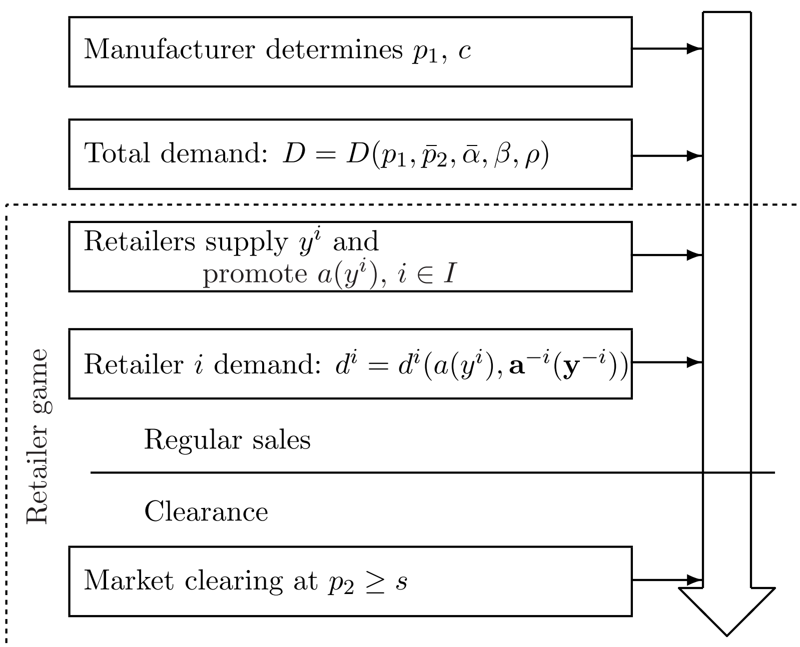


Figure 1: Market timeline

(a) Prevalence of RESE structures (b) Prevalence of multiple or no RESE

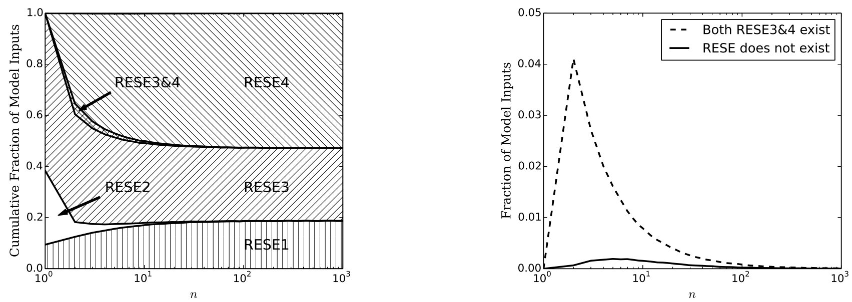


Figure 2: Fractions of model inputs resulting in a particular RESE structure for given n

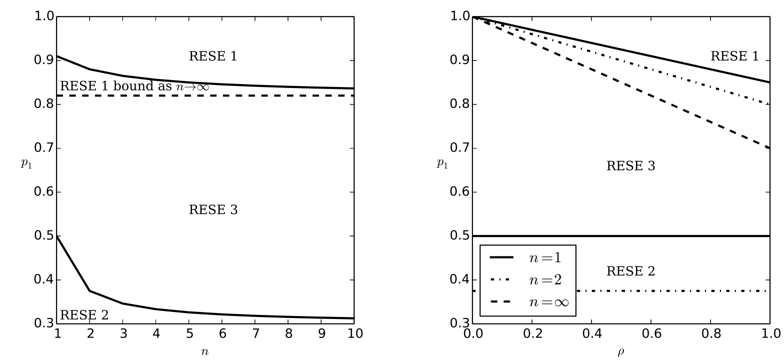


Figure 3: Typical boundaries between RESE1-3 in (n, p_1) and (ρ, p_1) (example for $c = 0.3$, $s = 0.05$, $p_1 = 0.5$, $\beta = 0.6$, and, for left plot, $\rho = 0.6$)

(a) $\beta = 1$ (b) $\beta = 0.9$

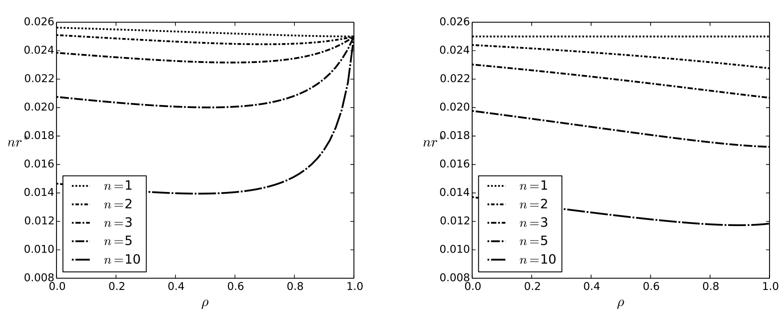


Figure 4: The total profit for RESE3 with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

(a) The second-period price (b) The total profit

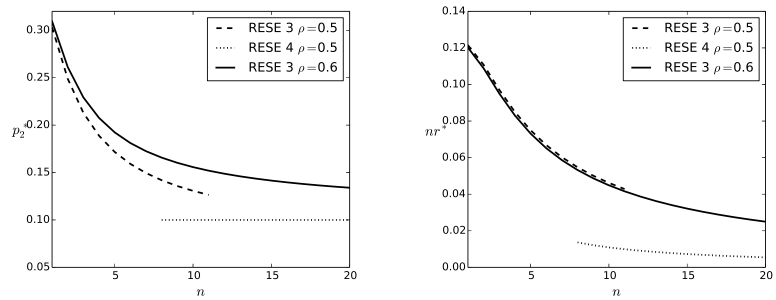


Figure 5: Overlap and switches between RESE3 and 4 for $c = 0.2$, $s = 0.1$, $p_1 = 0.4$, $\beta = 0.9$

(a) $\beta = 1, n = 5, p_1 = 0.4$ (b) $\beta = 0.5, n = 10, p_1 = 0.8$

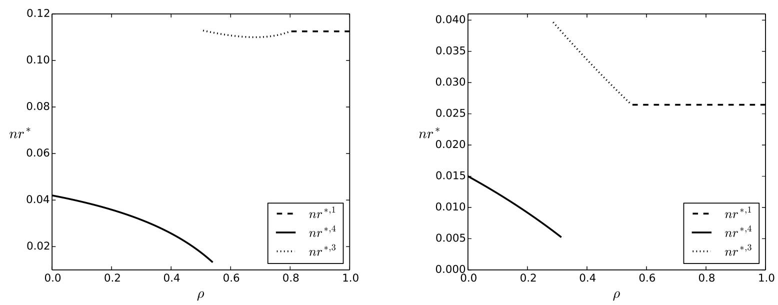


Figure 6: Equilibrium profit in ρ for $c = 0.1, s = 0.05$

(a) Prevalence of RESE structures (b) Prevalence of multiple or no RESE

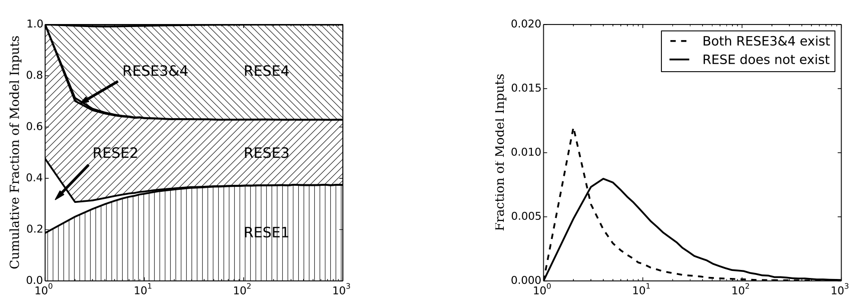


Figure 7: Fractions of model inputs resulting in a particular RESE structure for $\rho = 0.999$ and given n

(a) $\beta = 1$ (b) $\beta = 0.9$

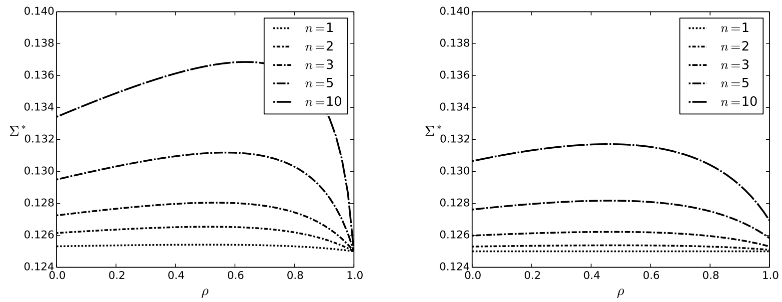


Figure 8: The total surplus Σ^* in ρ with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

(a) $\beta = 1$ (b) $\beta = 0.9$

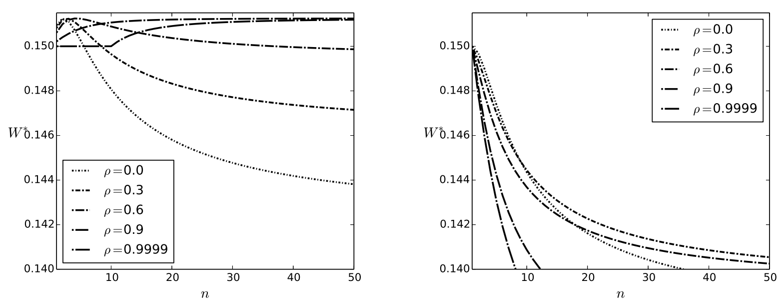


Figure 9: The aggregate welfare W^* in n with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

(a) $\beta = 1$ (b) $\beta = 0.9$

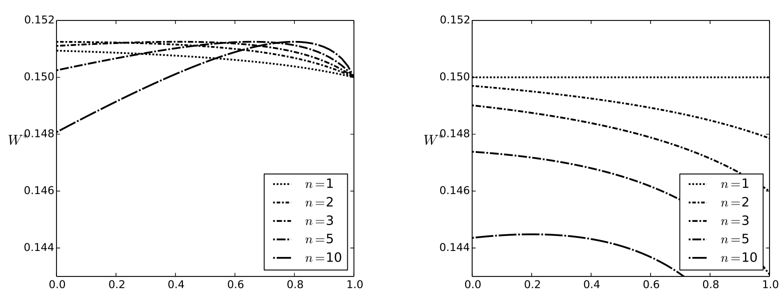


Figure 10: The aggregate welfare W^* in ρ with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$

Appendix: Proofs and supplementary results

A Main text supplement

A.1 RPM and focal point pricing

The history of RPM traces back to the nineteenth century and "has been one of the most controversial antitrust topics ever since" (Orbach (2008)). Available evidence indicates that the scale of RPM use is essentially underestimated and the effects of this phenomenon on economy require close attention. When RPM was illegal in the USA, Butz (1996) quoted antitrust authorities arguing that RPM is "ubiquitous" and "endemic", "but based upon 'winks and nods' rather than written agreements that could be used in court." Butz concludes that "manufacturers have many, many instruments" to punish or reward retailers in order to control the retail price "and to some extent will do so whether or not the law permits it."

RPM is attracting growing attention after the USA Supreme Court declared in 2007 that all manufacturer-imposed vertical price fixing should be evaluated using a rule of reason approach. According to the Court, "Absent vertical price restraints, the retail services that enhance interbrand competition might be underprovided. This is because discounting retailers can free ride on retailers who furnish services and then capture some of the increased demand those services generate" (`www.ftc.gov`, accessed 5 August, 2015). Using the data before and after the Court decision, MacKay and Smith (2014) show that, after 2007, RPM became even more common. The explicit evidence mentioned in the literature included "manufacturers and suppliers of childcare and maternity gear, light fixtures and home accessories, pet food and supplies, and rental cars. Sony has publicly used minimum RPM on electronics such as camcorders and video game consoles, and as of mid-2012, Sony and Samsung began enforcing minimum RPM on their televisions. Other retailers do not comment on whether or not they enter minimum RPM agreements, perhaps due to negative consumer sentiment associated with higher prices."

The growing importance of studying RPM-controlled markets provides a justification for assuming exogenously fixed price, which facilitates model tractability. The assumption of the "same-for-all-firms-price" is common for examining the effects of non-price competition, see, e.g., Schmalensee (1976); Karnani (1984), and Kouvelis and Zhao (2011). According to Holland, Rossouw, and Staples (2015), almost the same prices across retailers in local markets can result from focal points other than MSRP, e.g., from market norms, conventions, culture, and even government lows and regulations. Coordination mechanisms include price leadership (see a practical case in Andreoli-Versbach and Franck (2015)), cost-plus pricing, price lining, and use of round numbers. Holland, Rossouw, and Staples (2015) claim that some of these focal points and mechanisms can be "pervasive . . . not just in highly concentrated markets" as was previously believed starting from the work of Chamberlain. Proctor (2015) lists the factors that facilitate price uniformity, such as homogeneity of firms and products, transparency of price, stability of demand, high entry barriers, past cartels or a history of collusion, etc. In some cases, government can support exogenously fixed prices (e.g., Nippop (2005)) to improve social welfare. These cases are possible when price competition leads to inferior quality of the product, which is shown both theoretically, see Spiegler (2006), and empirically, see Huck, Lünser, and Tyran (2016). Practice-oriented article Maddah, Bish, and Munroe (2011) complements the above studies by pointing that "Exogenous pricing is somewhat justified for popular, fast moving, competitive items, for which pricing is a complex matter."

Given a variety of possible scenarios for the focal-point pricing, the range of feasible values for the first-period price can be rather wide. For example, collusive retailers can force a manufacturer to declare a desirable first-period price (Orbach (2008)). Retailers may also follow the suggested price under repeated interactions even when this price is non-binding because the manufacturer uses it to communicate private information on marginal cost and consumer demand to the retailers (Buehler and Gärtner (2013)). The main text of this study assumes a fixed first-period price and focuses on the effects of strategic consumer behavior on markets with non-price competition. We provide some mechanisms leading to the same price across retailers in Supplementary Document.

We consider all possible outcomes for any reasonable value of the first-period price and interaction between oligopolistic retailers and strategic consumers in the corresponding regimes allowing, e.g., to find a profit-maximizing list price for the manufacturer. If the manufacturer operates only in a single market, this value of the list price in combination with other parameters, including the number of retailers and consumer's discount factor, will determine the structure of equilibrium. As a rule, however, transnational manufacturers operate in multiple markets with notably different valuations for the same product whereas, to maintain a consistent brand image, to combat cross-border/market consumer diversion due to online sales, or, possibly, to comply with anti-dumping regulations[9], prices are often set to be comparable when converted to local currencies. As an example, Table 2 indicates prices of Apple Watch (Sport, 38mm model) in GBP across different launch regions. The average price without tax is £240.71 with a standard deviation of £4.71, which is approximately 2% of the average. Consequently, comparable currency-denominated MSRP values may substantially vary across the markets when they are expressed in terms of the maximum consumer valuation, leading to different outcomes.

Launch region	Price (£)	Tax rate	Price without tax
UK	299	20%	249.17
US (California)	257	9%	235.78
US (Montana)	236	0%	236.00
Canada (Quebec)	280	15%	243.48
Canada (Alberta)	256	5%	243.81
Australia	258	10%	234.55
France	288	20%	240.00
Japan	262	8%	242.59
China	282	17%	241.03

Table 2: Official prices of Apple Watch (Sport, 38mm). Source: Griffiths and Woollaston (2015)

A.2 Proof of Lemma 2 ($a(y) = y^\gamma$)

By the conditions of the lemma, equality $a(ky^i)/\sum_{j=1}^n a(ky^j) = a(y^i)/\sum_{j=1}^n a(y^j)$ holds for any $y^j > 0, j \in I$, and $k > 0$. Therefore, it holds for $y^i = y > 0$, $y^j = 1, j \neq i$, and $y^l = y, y^j = 1, l \neq i, j \neq l$. Namely,

$$\frac{a(ky)}{a(ky) + (n-1)a(k)} = \frac{a(y)}{a(y) + (n-1)a(1)} \text{ and } \frac{a(k)}{a(ky) + (n-1)a(k)} = \frac{a(1)}{a(y) + (n-1)a(1)},$$

which implies $a(ky)/a(y) = a(k)/a(1) \Leftrightarrow a(ky) = a(k)a(y)/a(1)$. Denoting $\tilde{k} \triangleq \ln k, \tilde{y} \triangleq \ln y$, and $\tilde{a}(z) \triangleq \ln(a(e^z)) - \ln a(1)$, the logarithm of the last equation is $\ln a[\exp(\tilde{k} + \tilde{y})] = \ln a[\exp(\tilde{k})] + \ln a[\exp(\tilde{y})] - \ln a(1)$ or $\tilde{a}(\tilde{k}+\tilde{y}) + \ln a(1) = \tilde{a}(\tilde{k}) + \ln a(1) + \tilde{a}(\tilde{y}) \Leftrightarrow \tilde{a}(\tilde{k}+\tilde{y}) = \tilde{a}(\tilde{k}) + \tilde{a}(\tilde{y})$. Because any continuous additive function of one variable is linear with zero intercept, we have $\tilde{a}(\tilde{y}) = \gamma\tilde{y}$ (note that by the definition of $\tilde{a}(z)$, $\tilde{a}(0)$ is, indeed, zero), which implies $a(y) = a(1)\exp[\gamma \ln y] = a(1)y^\gamma$.

A.3 Profit function, its properties and inventory decisions for $\gamma = 1$

By Lemma 3, the first-period total sales are $Q = 1 - v^{\min}$ and retailer i sales are $q^i = d^i(y^i, \mathbf{y}^{-i})$, which, for $\gamma = 1$, is $y^i \frac{Q}{Y}$. The second-period sales of retailer i are equal to its second-period inventory $y^i \left(1 - \frac{Q}{Y}\right)$. Then the general expression for retailer i profit, using (1) and (3), takes the form

$$r^i = -cy^i + p_1 \frac{y^i}{Y}(1 - v^{\min}) + \max\{s, \beta(1 - Y)\} \left\{ y^i - \frac{y^i}{Y}(1 - v^{\min}) \right\}. \tag{8}$$

Although this expression is continuous in all parameters and inventory y^i, it is generally not globally differentiable. Next, we consider all possible subintervals in terms of y^i. Each subinterval results in a differentiable expression for the profit function and a qualitatively distinct market outcome.

No sales in the second period Formula (1) for profit becomes $r^i = (p_1 - c)y^i$, which yields a unique profit-maximizing inventory $y^i = \left(1 - v^{\min} - Y^{-i}\right)^+$, where $Y^{-i} = \sum_{j \neq i} y^j$, and the maximum profit $r^i = (p_1 - c)\left(1 - v^{\min} - Y^{-i}\right)^+$, leading to the following lemma:

Lemma 6. *For given model inputs and consumer expectations, retailer rationality implies that the effective domain of the inventory decision is $y^i \geq (1 - v^{\min} - Y^{-i})^+$ and $(p_1 - c)(1 - v^{\min} - Y^{-i})^+$ is the lower bound for the optimal profit.*

Second-period sales with $p_2 > s$ If $v^{\min} > 1 - Y$ (or $y^i > 1 - v^{\min} - Y^{-i}$), there are sales in the second period. If this condition is combined with$0 < y^i < 1 - s/\beta - Y^{-i}$, then $p_2 > s$ and the profit is given by

$$\begin{aligned} r^i &= -cy^i + p_1 \frac{y^i}{Y}\left(1 - v^{\min}\right) + \beta(1 - Y)\, y^i \left(1 - \frac{1 - v^{\min}}{Y}\right) \\ &= y^i \left[\beta(1 - Y) - c + (p_1 - \beta(1 - Y)) \frac{1 - v^{\min}}{Y}\right] \qquad (9) \\ &= y^i \left[\beta(1 - Y) - c + \beta\left(1 - v^{\min}\right) + \frac{(p_1 - \beta)(1 - v^{\min})}{Y}\right] \qquad (10) \end{aligned}$$

with the derivative $\frac{\partial r^i}{\partial y^i} =$

$$\begin{aligned} &= y^i \left[\beta\left(-1 + \frac{1 - v^{\min}}{Y}\right) - \frac{(p_1 - \beta(1 - Y))(1 - v^{\min})}{Y^2}\right] + \beta(1 - Y) - c + \frac{[p_1 - \beta(1 - Y)](1 - v^{\min})}{Y} \\ &= \beta(1 - Y) - c + \beta(1 - v^{\min}) - \beta y^i + (p_1 - \beta)(1 - v^{\min})(Y - y^i)/Y^2, \qquad (11) \end{aligned}$$

which, using equations $Y = y^i + Y^{-i}$ and (10), can be rewritten as

$$\frac{\partial r^i}{\partial y^i} = \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta y^i + (p_1 - \beta)(1 - v^{\min}) \frac{Y^{-i}}{Y^2}. \tag{12}$$

The second derivative is

$$\frac{\partial^2 r^i}{\partial (y^i)^2} = -2\left[\beta + (p_1 - \beta)(1 - v^{\min}) \frac{Y^{-i}}{Y^3}\right]. \tag{13}$$

Second-period sales with $p_2 = s$ This case is possible only under oligopoly, i.e., $Y^{-i} > 0$ (for a monopolist, any $p_2 \le c$ is not rational) and only for $v^{\min} < 1$ (there are first-period sales, otherwise profit is negative). If there are sales in the second period and $y^i \ge \left(1 - s/\beta - Y^{-i}\right)^+$ (or $Y \ge 1 - s/\beta$), then $p_2 = s$ and (8) becomes

$$\begin{aligned} r^i &= -cy^i + p_1 y^i \left(1 - v^{\min}\right)/Y + sy^i \left[1 - (1 - v^{\min})/Y\right] \\ &= -(c-s)y^i + y^i \left(p_1 - s\right)\left(1 - v^{\min}\right)/Y \end{aligned} \tag{14}$$

with the derivative

$$\frac{\partial r^i}{\partial y^i} = -(c-s) + \frac{Y - y^i}{Y^2}\left(p_1 - s\right)\left(1 - v^{\min}\right) = -(c-s) + Y^{-i}\left(p_1 - s\right)\left(1 - v^{\min}\right)/Y^2, \tag{15}$$

which is monotonically strictly decreasing in y^i when $v^{\min} < 1$.

Properties of the profit function The following lemma provides the properties of retailer i profit r^i, using the continuity of r^i in y^i.

Lemma 7. *The profit function r^i is such that*

1. *If $1 - s/\beta - Y^{-i} > 0$, then*

   (1.1) $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} < \left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0}$;

   (1.2) $r^i(1 - s/\beta - Y^{-i} \le 0$ *if and only if*
   $$\frac{\left(p_1 - s\right)\left(1 - v^{\min}\right)}{(1 - s/\beta)(c - s)} \le 1; \tag{16}$$

   (1.3) r^i *is pseudoconcave in* y^i *and strictly concave if* $p_1 \ge \beta v^{\min}$ *on the interval* $(1 - v^{\min} - Y^{-i})^+ \le y^i \le 1 - s/\beta - Y^{-i}$*;*

   (1.4) r^i *is strictly concave on the interval leading to* $p_2 = s$*, i.e.* $y^i \ge 1 - s/\beta - Y^{-i}$*; and*

   (1.5) r^i *is pseudoconcave on the interval* $y^i \ge (1 - v^{\min} - Y^{-i})^+$ *if either*
   $$\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0} \le 0 \ \text{or} \ \left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} \ge 0.$$

2. *If $1 - s/\beta - Y^{-i} \le 0$, r^i is strictly concave on its entire domain $y^i \ge 0$.*

Possibility of asymmetric equilibria When there are no sales in the second period, profit-maximizing inventory $y^i = \left(1 - v^{\min} - Y^{-i}\right)^+$ is determined up to a redistribution of inventory among the retailers. In this case, the model allows for a continuum of combinations of profit-maximizing y^i, satisfying $\sum_{i=1}^n y^i = Y = 1 - v^{\min}$.

When there are sales in the second period ($y^i > \left(1 - v^{\min} - Y^{-i}\right)^+$), parts 1.3 and 1.4 of Lemma 7 imply that in both cases $p_2 > s$ and $p_2 = s$, profit-maximizing y^i results from $\frac{\partial r^i}{\partial y^i} = 0$.

When $p_2 > s$, using (11) for $\frac{\partial r^i}{\partial y^i}$, for any y^i and y^j ($j \ne i$) satisfying $\frac{\partial r^i}{\partial y^i} = \frac{\partial r^j}{\partial y^j} = 0$ we have $\frac{\partial r^i}{\partial y^i} - \frac{\partial r^j}{\partial y^j} = 0 = (y^j - y^i)\left[\beta + (p_1 - \beta)(1 - v^{\min})/Y^2\right]$, yielding $y^j = y^i$ because the bracket $[\cdot]$ is always positive. Indeed, $[\cdot] > 0 \Leftrightarrow p_1(1 - v^{\min}) + \beta[Y^2 - (1 - v^{\min})] > 0$. As $v^{\min} \ge p_1$, by part 2 of Lemma˜4, $Y^2 > (1 - p_1)(1 - v^{\min})$. Then $p_1(1 - v^{\min}) + \beta[Y^2 - (1 - v^{\min})] > (1 - v^{\min})[p_1 - \beta p_1] \ge 0$.

When there are sales in both periods ($v^{\min} < 1$) and $p_2 = s$, the first equation in (15) implies that any y^i and y^j ($j \ne i$), satisfying $\frac{\partial r^i}{\partial y^i} = \frac{\partial r^j}{\partial y^j} = 0$, are such that $\frac{\partial r^i}{\partial y^i} - \frac{\partial r^j}{\partial y^j} = 0 = (y^j - y^i)(p_1 - s)(1 - v^{\min})/Y^2$, i.e., $y^j = y^i$ because $(p_1 - s)(1 - v^{\min})/Y^2 > 0$.

A.4 Proof of Lemma 1 (first-period demand)

Recall that, for a first-period buyer with valuation $v \geq p_1$, the surpluses of buying in the first period and that of waiting are, respectively, $\sigma_1 = v - p_1$ and $\sigma_2 = \rho\bar{\alpha}(\beta v - \bar{p}_2)^+$. Condition $\sigma_1 \geq 0$ is equivalent to $v \geq p_1$. Condition $\sigma_1 \geq \sigma_2$ is equivalent to $v - p_1 \geq \rho\bar{\alpha}(\beta v - \bar{p}_2) \;\Leftrightarrow\; v \geq \frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta}$. Combining these inequalities, we obtain the stated expression for $v^{\min}$. Because all consumers with $v \geq v^{\min}$ would buy in the first period, the total demand is $D = 1 - v^{\min}$.

A.5 Proof of Lemma 3 (no stockouts)

Part I. The existence of the unique positive solution $\breve{y}^i$ to equation $\breve{y}^i = d^i(\breve{y}^i, \mathbf{y}^{-i})$ is established in §C.1. Moreover, the reasoning implies that $y^i \leq d^i(y^i, \mathbf{y}^{-i})$ for any $y^i \leq \breve{y}^i$. Thus, if $y^i \leq d^i(y^i, \mathbf{y}^{-i})$, retailer i sells only in the first period and, by (1), its profit function is $r^i = (p_1 - c)y^i$, which is increasing in y^i for any $y^i \in [0, \breve{y}^i]$. Therefore, inventory y^i of a profit-maximizing retailer is never less than the first-period demand, i.e., $y^i \geq \breve{y}^i$ holds.

Part II. Claim (a) is straightforward when $y^i \geq \breve{y}^i$ holds and when retailer i sets the inventory above the symmetric level $\bar{Y}/n$. In that case, the first-period demand of other retailers decreases compared to $\bar{Y}/n$ (constant for $\gamma = 0$), which cannot lead to stockouts.

Stockouts may potentially arise only when retailer i sets the inventory below $\bar{Y}/n$, increasing the first-period market share of other retailers above the symmetric level. In this case, we show that the first-period demand d^j of any retailer $j \neq i$ is not greater than inventory $y^j = \bar{Y}/n$. Suppose that $y^i = \breve{y}^i$, which is the minimum possible inventory of a retailer rationally responding to a symmetric profile, and that $\breve{y}^i \leq \frac{\bar{Y}}{n}$. Then $d^j = \frac{(1-v^{\min})\left(\bar{Y}/n\right)^\gamma}{(n-1)\left(\bar{Y}/n\right)^\gamma + (\breve{y}^i)^\gamma}$, and the no-stockout condition $d^j \leq \bar{Y}/n$ can be written as

$$(1 - v^{\min})\left(\bar{Y}/n\right)^{\gamma-1} \leq (n-1)\left(\bar{Y}/n\right)^\gamma + \left(\breve{y}^i\right)^\gamma.$$

As $\breve{y}^i = d^i = \frac{(1-v^{\min})\left(\breve{y}^i\right)^\gamma}{(n-1)\left(\bar{Y}/n\right)^\gamma + (\breve{y}^i)^\gamma}$, the RHS of the last inequality equals $(1 - v^{\min})\left(\breve{y}^i\right)^{\gamma-1}$. Then $d^j \leq \bar{Y}/n$ trivially holds for $\gamma = 1$ and, for $\gamma \in [0, 1)$, is equivalent to $\breve{y}^i \leq \bar{Y}/n$ (because $\gamma < 1$), which holds by the assumption.

Part II (b) follows from part II (a).

Part II (c). The second-period total inventory is $Y - Q = Y - (1 - v^{\min})$. Suppose this number is positive. The number of consumers remaining in the market is $v^{\min}$, and the upper bound of their second-period valuations is $\beta v^{\min}$. Therefore, as long as $p_2 \geq s$, the market clearing condition for the second period takes the form $v^{\min}\frac{\beta v^{\min} - p_2}{\beta v^{\min}} = Y - 1 + v^{\min}$, or, equivalently, $p_2 = \beta(1 - Y)$. If $\beta(1 - Y) < s$, bargain-hunters absorb any excess supply at price s. Combining these two cases, we get the second-period price in the form $p_2 = \max\{s, \beta(1 - Y)\}$, which is continuous in y^i, $i \in I$.

A.6 Proof of Lemma 4 ($p_2 < \beta p_1$)

From Lemma 1, we have $v^{\min} = p_1$ if and only if $\frac{p_1 - \bar{\alpha}\rho\bar{p}_2}{1-\bar{\alpha}\rho\beta} \leq p_1$, which can be equivalently rewritten as $\bar{\alpha}\rho\beta p_1 \leq \bar{\alpha}\rho\bar{p}_2$. Within feasible parameter values, the later holds if and only if either $\bar{\alpha} = 0$, $\rho = 0$, or $\beta p_1 \leq \bar{p}_2$. By Lemma (6), $Y \geq 1 - v^{\min}$. Thus, either of $\rho = 0$, $\bar{\alpha} = 0$ or $\beta p_1 \leq \bar{p}_2$ implies that $Y \geq 1 - p_1$. Moreover, $Y = 1 - p_1$ means there are no sales in the second period, whereas $Y > 1 - p_1$ means that these sales occur at price $p_2 < \beta p_1$ according to the market clearing condition (3).

Part 1: We conclude that $\bar{p}_2 \geq \beta p_1$ would never be rational and, in any rational expectations equilibrium, we must have $p_2 < \beta p_1$.

Part 2: By the above reasoning, $\bar{\alpha} = 0$ implies $v^{\min} = p_1$ and $Y \geq 1 - p_1$. However, $Y > 1 - p_1$ in combination with $v^{\min} = p_1$ means that there *are* sales in the second period and $\bar{\alpha} = 0$ is not rational.

If $\bar{\alpha} = 1$, by part 1 and condition (3), we have $\beta(1-Y) \leq \max\{s, \beta(1-Y)\} = p_2 < \beta p_1$. Thus, $Y > 1 - p_1$ in any rational expectations equilibrium with $\bar{\alpha} = 1$.

Part 3: Because in any rational expectations equilibrium, $\bar{p}_2 = p_2$ and $\bar{\alpha} = 1$ if there are sales in the second period, Lemma 1 implies that, if there are sales in both periods, $v^{\min} < 1$, which, using (3), is equivalent to $p_1 - \rho\beta(1-Y) < 1 - \rho\beta$ or $\rho\beta Y < 1 - p_1$. If there are sales only in the second period, $p_1 - \rho\beta(1-Y) \geq 1 - \rho\beta$ or $\rho\beta Y \geq 1 - p_1$; $p_2 \geq c$ because, in this case, $r^i = (p_2 - c)y^i$, and retailers are profit-maximizing.

Part 4: As $\bar{p}_2 \geq \beta p_1$ would never be rational, $v^{\min} = p_1$ can occur in a rational expectations equilibrium if and only if $\bar{\alpha} = 0$ or $\rho = 0$.

A.7 Proof of Theorem 1 (RESE with $p_2^* > s$)

The theorem exhaustively considers all possible market outcomes without salvaging: no sales in the first period (RESE1), no sales in the second period (RESE2), and sales in both periods (RESE3). Logically, these three outcomes are mutually exclusive but it is not obvious *a priori* that they cannot exist under the same model inputs. In the course of the proof we establish that these outcomes also do not overlap in the sense of their necessary and sufficient conditions on model parameters. Parts 1 and 2 of the RESE definition (§3.2) rely on the notion of a symmetric equilibrium for given consumer expectations. The structure of such an equilibrium is one of the major sources of necessary and sufficient conditions. Another source is the rationality of consumer expectations. We first classify the outcomes by the presence of second-period sales.

No second-period sales: RESE2 The absence of second-period sales along with retailer rationality, by Lemma 6, means that the best response in a symmetric equilibrium occurs with $Y = 1 - v^{\min}$. Consumer rationality in this case demands that $\bar{\alpha} = 0$ and $v^{\min} = p_1$ implying that the candidate RESE is described by $v^* = p_1, Y^* = 1 - v^*$, and, therefore, $\alpha^* = 0$ and $r^* = \frac{1}{n}(p_1 - c)(1 - p_1)$. This implies that $\frac{n-1}{n}Y^* = \frac{n-1}{n}(1 - p_1) < 1 - p_1 < 1 - \frac{s}{\beta}$ and condition of part 1 of Lemma 7 is satisfied.

Because, by part 1.3 of Lemma 7, r^i is pseudoconcave on the interval $(1 - v^{\min} - Y^{-i})^+ \leq y^i < 1 - s/\beta - Y^{-i}$, the candidate RESE exists if and only if

(i) there is a local maximum of r^i at $y^i = 1 - v^* - \frac{n-1}{n}Y^* = \frac{Y^*}{n}$ and

(ii) the profit r^i at this maximum is greater than at a potential local maximum on the interval $y^i > 1 - \frac{s}{\beta} - \frac{n-1}{n}Y^*$.

Condition (i) is equivalent to $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - v^* - \frac{n-1}{n}Y^* + 0} \leq 0$. As $y^i = \frac{1}{n}(1 - p_1)$, the last inequality, using (11), becomes $\beta v^* - c + p_1 - \beta v^* + \frac{1}{n}(1 - p_1)\left[-(p_1 - \beta v^*)\frac{1}{1 - v^*}\right] \leq 0$, which, after the substitution for $v^* = p_1$ and multiplication by n, takes the form $np_1 - p_1(1 - \beta) \leq nc$ or $p_1 \leq \frac{nc}{\beta + n - 1} = P_2$. We showed that this condition is necessary.

Condition (ii) is satisfied if r^i is nonincreasing for $y^i > 1 - s/\beta - \frac{n-1}{n}Y^*$. Because r^i is concave on this interval by part 1.4 of Lemma 7, it is nonincreasing if $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - \frac{n-1}{n}Y^* + 0} \leq 0$. The latter, using (15), can be written as

$$-c+s+\frac{\frac{n-1}{n}(1-p_1)}{(1-s/\beta)^2}(p_1-s)(1-p_1)\le 0 \quad \text{or} \quad \frac{n-1}{n}\frac{(p_1-s)(1-p_1)^2}{(c-s)(1-s/\beta)^2}\le 1. \tag{17}$$

As $p_1 > s/\beta$, we have $\frac{(1-p_1)^2}{(1-s/\beta)^2} < 1$, and (17) is implied by $(n-1)(p_1-s)\le n(c-s)$. The latter holds because, by (already proved as necessary) condition $p_1 \le P_2$, $n(c-s)\ge(n-1+\beta)p_1-ns = (n-1)(p_1-s)+\beta p_1 - s > (n-1)(p_1-s)$. Therefore, condition $p_1\le P_2$ is necessary and sufficient for the existence of RESE2.

There are second-period sales: RESE1 or 3 When sales in the second period do occur, a symmetric equilibrium $Y=\hat{Y}>1-v^{\min}$, by Lemma 7, is an internal maximum of the profit function for each retailer. For $p_2>s$, the first-order condition $\frac{\partial r^i}{\partial y^i}=0$ is provided by setting (12) to zero with substitutions $y^i=\frac{Y}{n}$ and $Y^{-i}=\frac{n-1}{n}Y$:

$$\begin{aligned}\beta\left(1-\frac{n-1}{n}Y\right)-c+\beta(1-v^{\min})-2\beta\frac{Y}{n}+(p_1-\beta)(1-v^{\min})\frac{n-1}{n}\frac{Y}{Y^2} &= 0\\ \text{or}\quad -\beta\frac{n+1}{n}Y-c+\beta(2-v^{\min})+(p_1-\beta)(1-v^{\min})\frac{n-1}{n}\frac{1}{Y} &= 0.\end{aligned}$$

Multiplication of the last equation by $-\frac{n}{\beta(n+1)}Y$ yields

$$Y^2-Y\frac{n}{n+1}\left(2-v^{\min}-\frac{c}{\beta}\right)-\frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)(1-v^{\min})=0. \tag{18}$$

Equation (18) along with the relation between $v^{\min}$ and Y from Lemma 1 and inequality $Y > 1-p_1$ (from part 2 of Lemma 4) provide the necessary conditions for any equilibria with sales in the second period and $p_2=\beta(1-Y)>s$.

It is convenient to analyze RESE existence in terms of $v^{\min}$ as a function of Y. For rational expectations $\bar{\alpha}=1$ and $\bar{p}_2=p_2=\beta(1-Y)$, denote the mapping from Y to $v^{\min}$ resulting from Lemma 1 as function

$$v_1^{\min}(Y)\triangleq\max\left\{p_1,\ \min\left\{\frac{p_1-\rho\beta(1-Y)}{1-\rho\beta},\ 1\right\}\right\}. \tag{19}$$

When $\rho>0$, this function is increasing and piecewise linear with two breakpoints. It is straightforward to check that the first break-point occurs exactly at $Y=1-p_1$ whereas the second at $Y=\frac{1-p_1}{\rho\beta}$. When $\rho=0$, $v_1^{\min}\equiv p_1$.

Equation (18) yields another mapping from Y to $v^{\min}$:

$$v_2^{\min}(Y)\triangleq 1-\frac{Y^2-Y\frac{n}{n+1}(1-c/\beta)}{Y\frac{n}{n+1}+\frac{n-1}{n+1}(p_1/\beta-1)}. \tag{20}$$

When $p_1\neq\beta$ and $n>1$, this function is a hyperbola with a vertical asymptote $Y=\frac{n-1}{n}(1-p_1/\beta)$ and an asymptote with a negative slope $-\frac{n+1}{n}$. When $Y=0$ or $Y=\frac{n}{n+1}(1-c/\beta)$, $v_2^{\min}(Y)=1$. Implicit differentiation of (18) yields

$$2Y-\frac{n}{n+1}\left(2-v_2^{\min}-\frac{c}{\beta}\right)+Y\frac{n}{n+1}\frac{\partial v_2^{\min}}{\partial Y}+\frac{n-1}{n+1}\left(\frac{p_1}{\beta}-1\right)\frac{\partial v_2^{\min}}{\partial Y}=0$$

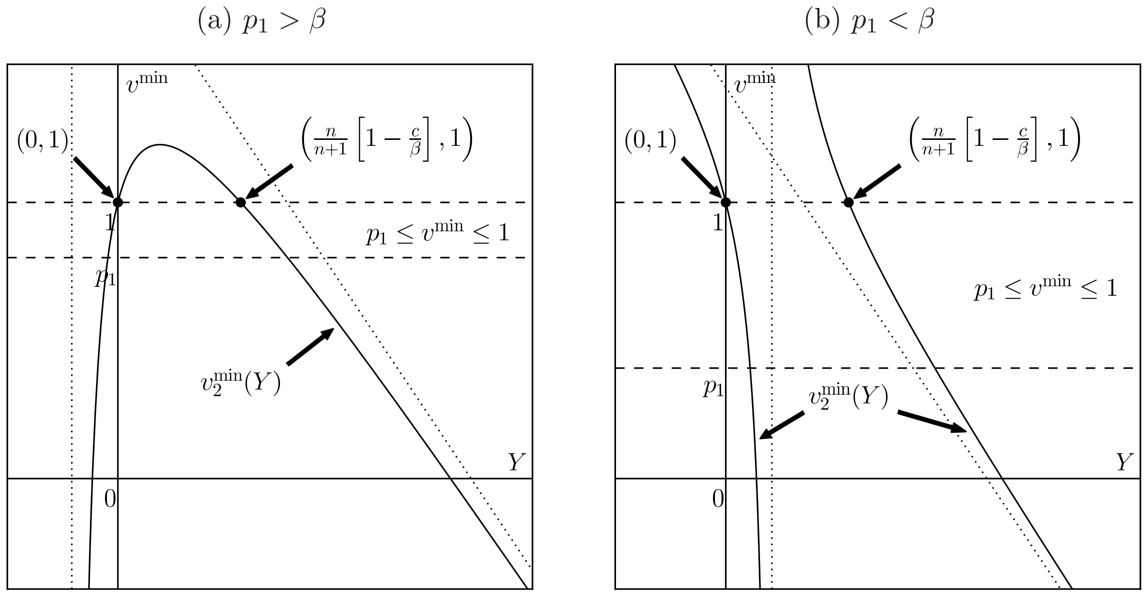


Figure 11: Possible appearance of $v_2^{\min}(Y)$ and the relevant range of $v^{\min}$

resulting in $(n-1)(p_1-\beta)\left.\frac{\partial v_2^{\min}}{\partial Y}\right|_{Y=0} = n(\beta - c)$.

When $p_1 > \beta$ and $n > 1$, the vertical asymptote is located to the left of $Y = 0$ implying that points $(0,1)$ and $\left(\frac{n}{n+1}\left[1-c/\beta\right],1\right)$ in the $(Y, v^{\min})$-plane belong to the same branch of the hyperbola. In this case, $\left.\frac{\partial v_2^{\min}}{\partial Y}\right|_{Y=0} > 0$ and it must be true that $\frac{\partial v_2^{\min}}{\partial Y} < 0$ for all $Y \ge \frac{n}{n+1}(1-c/\beta)$. Relevant equilibrium candidates can only be on the downward-sloping segment of $v_2^{\min}(Y)$ to the right of $Y = \frac{n}{n+1}(1-c/\beta)$ and in the range $p_1 \le v^{\min} \le 1$. This case is depicted in Figure 11 (a), where a solid curve is $v_2^{\min}(Y)$, dotted lines represent its asymptotes, and the dashed lines indicate the lower and upper bounds on the relevant range of $v^{\min}$.

When $p_1 < \beta$ and $n > 1$, the vertical asymptote is located to the right of $Y = 0$ implying that points $(0,1)$ and $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right],1\right)$ belong to different branches of the hyperbola. We have $\frac{\partial v_2^{\min}}{\partial Y} < 0$ for all Y, and the entire left branch is irrelevant because the vertical asymptote is to the left of $Y = 1-p_1$. Indeed, $\frac{n-1}{n}\left(1-\frac{p_1}{\beta}\right) < 1-p_1$ is equivalent to $np_1-(n-1)\frac{p_1}{\beta} < 1$ which always holds for $p_1 < \beta$. All possible equilibrium candidates are again on the downward-sloping segment of $v_2^{\min}(Y)$ to the right of $Y = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$ and in the range $p_1 \le v^{\min} \le 1$. This case is illustrated in Figure 11 (b).

When $p_1 = \beta$ or $n = 1$, the relevant part of $v_2^{\min}(Y)$ is decreasing linear: $v_2^{\min}(Y) = 2-\frac{c}{\beta}-\frac{n+1}{n}Y$, which also satisfies $v_2^{\min}\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right]\right) = 1$. Thus, regardless of n and the relation between p_1 and β, the geometric structure of potential equilibrium candidates is essentially the same.

RESE1: There are no sales in the first period at a RESE if and only if $v^* = 1$. The geometric structure described above implies that such an equilibrium can be realized only if $v_1^{\min}(Y)$ intersects with $v_2^{\min}(Y)$ at a point corresponding to $Y^* = \frac{n}{n+1}\left(1-\frac{c}{\beta}\right)$, i.e., $v_1^{\min}(Y^*) = 1$ or $p_1 - \rho\beta\left[1-\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)\right] \ge 1-\rho\beta$, which is equivalent to $p_1 \ge P_1 = 1-\frac{n}{n+1}\rho\left(\beta-c\right)$. This necessary condition is also sufficient for RESE1. Indeed, given that $v_1^{\min}(Y^*) = 1$, the equilibrium values are in the form of RESE1, $p_2^* = \beta\left[1 - \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)\right] = \frac{nc+\beta}{n+1} > c > s$ and $y^i = \frac{Y^*}{n}$ indeed delivers the best response of retailer i because $Y^* = \frac{n}{n+1}(1 - c/\beta) < 1 - c/\beta < 1 - s/\beta$ and $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - \frac{n-1}{n}Y^* + 0} = -c + s < 0$ implying, by part 1.5 of Lemma 7, that r^i is pseudoconcave.

The description of RESE1 is completed by substituting p_2^*, Y^* and v^* into (10):

$$r^* = \frac{Y^*}{n}\left[\frac{\beta + nc}{n+1} - c\right] = \frac{1}{n+1}\left(1 - \frac{c}{\beta}\right)\left[\frac{\beta + nc}{n+1} - c\right] = \frac{(\beta - c)}{(n+1)\beta}\frac{\beta + nc - nc - c}{n+1} = \frac{(\beta - c)^2}{(n+1)^2\beta}.$$

The p_1-ranges in RESE1 and 2 do not overlap because the minimal lower bound for p_1 in RESE1, which corresponds to $n \to \infty$, exceeds the maximal upper bound in RESE2 (at $n = 1$): $1 - \rho(\beta - c) > c/\beta \;\Leftrightarrow\; \beta(1 - \rho\beta) > c(1 - \rho\beta)$.

RESE3: This case is characterized by $Y^* > 1 - v^*$ (there are sales in the second period) and $p_1 \le v^* < 1$ (there are sales in the first period) with $v^* = p_1$ only if $\rho = 0$. Translating this into the geometric structure described above, necessary conditions for RESE3 are $v_1^{\min}\left(\frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)\right) < 1$ and $v_2^{\min}(1 - p_1) > p_1$. The first condition is equivalent to the negation of $p_1 \ge P_1$, i.e., the strict upper limit of p_1-range for RESE3. The second condition ensures that $v_2^{\min}(Y)$ intersects $v_1^{\min}(Y)$ for $Y > 1 - p_1$ and is equivalent to

$$1 - \frac{(1 - p_1)^2 - (1 - p_1)\frac{n}{n+1}(1 - c/\beta)}{(1 - p_1)\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta - 1)} > p_1,$$

and, because $(1 - p_1)\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta - 1) = \frac{1-p_1}{n+1} + \frac{(n-1)p_1(1-\beta)}{(n+1)\beta} > 0$, to

$$(1 - p_1)\left[(1 - p_1)\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta - 1) - (1 - p_1) + \frac{n}{n+1}(1 - c/\beta)\right] > 0.$$

Collecting like terms inside $[\cdot]$ yields $(n - 1 + \beta)p_1 > nc$ which is the negation of the necessary and sufficient condition $p_1 \le P_2$ of RESE2, i.e., the strict lower limit of p_1-range for RESE3.

Given that necessary condition $P_2 < p_1 < P_1$ holds and there are sales in both periods, the candidate point for the equilibrium, by Lemma 1, satisfies

$$v^* = \frac{p_1 - \rho\beta(1 - Y^*)}{1 - \rho\beta} \tag{21}$$

and $v^* \in [p_1, 1)$. Substitution for $v^{\min} = v^*$ into (18) results in the following equation for Y^*:

$$Y^2 - Y\frac{n}{n+1}\left(2 - \frac{p_1 - \rho\beta(1 - Y)}{1 - \rho\beta} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\left(1 - \frac{p_1 - \rho\beta(1 - Y)}{1 - \rho\beta}\right) = 0,$$

which, after collecting the terms with Y, becomes

$$\begin{aligned} &Y^2\left(1 + \frac{n}{n+1}\frac{\rho\beta}{1 - \rho\beta}\right) \\ &-Y\left[\frac{n}{n+1}\left(2 - \frac{p_1 - \rho\beta}{1 - \rho\beta} - \frac{c}{\beta}\right) - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\frac{\rho\beta}{1 - \rho\beta}\right] - \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right)\left(1 - \frac{p_1 - \rho\beta}{1 - \rho\beta}\right) = 0. \end{aligned} \tag{22}$$

The coefficient in front of Y^2 is

$$1 + \frac{n}{n+1}\frac{\rho\beta}{1 - \rho\beta} = \frac{n + 1 - \rho\beta}{(n+1)(1 - \rho\beta)},$$

and the coefficient in front of Y is

$$-\frac{1}{(n+1)(1-\rho\beta)}\left\{n\left[2-2\rho\beta-p_1+\rho\beta-(1-\rho\beta)c/\beta\right]-(n-1)\left(p_1/\beta-1\right)\rho\beta\right\},$$

where the first term in the bracket $\{\dots\}$ is

$$n[2-\rho\beta-p_1-(1-\rho\beta)c/\beta]=n(1-\rho\beta)(1-c/\beta)+n(1-p_1).$$

Then multiplication of (22) by $\frac{\beta(n+1)(1-\rho\beta)}{\beta(n+1-\rho\beta)}$ results in

$$Y^2-\frac{(\beta-c)n(1-\rho\beta)+\beta(1-p_1)n-(p_1-\beta)\rho\beta(n-1)}{\beta(n+1-\rho\beta)}Y-\frac{(p_1-\beta)(1-p_1)(n-1)}{\beta(n+1-\rho\beta)}=0. \quad (23)$$

By geometric structure under condition $P_2<p_1<P_1$, the larger root of this equation does belong to the region $Y>1-p_1$ and the smaller root is irrelevant.

The conditions for RESE3 will become necessary and sufficient if (23), (21), and $P_2<p_1<P_1$ are complemented with the conditions guaranteeing that the larger root Y^* of (23) is such that $Y^*<1-\frac{s}{\beta}$ (implying $p_2^*>s$ and included as a condition of the theorem) and either

(a) the profit r^i of retailer i deviating from this RESE so that $p_2=s$ (the total inventory is greater than $1-\frac{s}{\beta}$) has no maximum for $Y>1-\frac{s}{\beta}$, or

(b) if $\tilde{r}^i=\max r^i$ exists for $Y>1-\frac{s}{\beta}$, then the inequality $\tilde{r}^i<r^*$ holds.

Because, by part 1.4 of Lemma 7, r^i is concave for $y^i\geq 1-s/\beta-\frac{n-1}{n}Y^*$ (or, equivalently, $Y\geq 1-s/\beta$), r^i is nonincreasing for $y^i\geq 1-s/\beta-\frac{n-1}{n}Y^*$ if and only if $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-s/\beta-\frac{n-1}{n}Y^*+0}\leq 0$. Thus, the latter condition is equivalent to (a). Using (15) with $v^{\min}=v^*$, $Y^{-i}=\frac{n-1}{n}Y^*$ and $Y=1-s/\beta$ this condition can be written as $-c+s+\frac{n-1}{n}\frac{Y^*}{(1-s/\beta)^2}\left(p_1-s\right)\left(1-v^*\right)\leq 0$, yielding condition (a).

If $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-s/\beta-Y^{-i}+0}>0$, then, since $\frac{\partial r^i}{\partial y^i}$ becomes negative for sufficiently large Y by (15), $\tilde{r}^i=\max r^i$ exists for $Y>1-s/\beta$. Therefore, RESE exists in this case if $r^*\geq\tilde{r}^i$ (condition (b)).

In order to provide the expression for $\tilde{r}^i$, denote the maximized deviator's inventory decision by $\tilde{y}^i\triangleq\arg\max r^i>\frac{1}{n}Y^*$. As a result of this deviation, the total inventory becomes $\tilde{Y}=\tilde{y}^i+\frac{n-1}{n}Y^*$. Then, using (15) with $v^{\min}=v^*$, we obtain the following equation in $\tilde{Y}$: $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\tilde{y}^i}=0=-(c-s)+\frac{n-1}{n}\frac{Y^*}{\tilde{Y}^2}\left(p_1-s\right)\left(1-v^*\right)$, which yields $\tilde{Y}=\sqrt{\frac{n-1}{n}\frac{Y^*(p_1-s)(1-v^*)}{c-s}}$. Substitution of $Y=\tilde{Y}$ and $y^i=\tilde{Y}-\frac{n-1}{n}Y^*$ into the equation for profit (14), results in

$$\tilde{r}^i=\left\{\sqrt{\frac{n-1}{n}\frac{Y^*\left(p_1-s\right)\left(1-v^*\right)}{c-s}}-\frac{n-1}{n}Y^*\right\}\times\left\{-(c-s)+\frac{(p_1-s)(1-v^*)}{\sqrt{\frac{n-1}{n}\frac{Y^*(p_1-s)(1-v^*)}{c-s}}}\right\},$$

which, after factoring out $\frac{n-1}{n}Y^*$ from the first curly bracket and $c-s$ from the second one, becomes

$$\tilde{r}^i=\frac{n-1}{n}Y^*\left(c-s\right)\left\{\sqrt{\frac{n}{(n-1)}\frac{(p_1-s)(1-v^*)}{(c-s)Y^*}}-1\right\}^2.$$

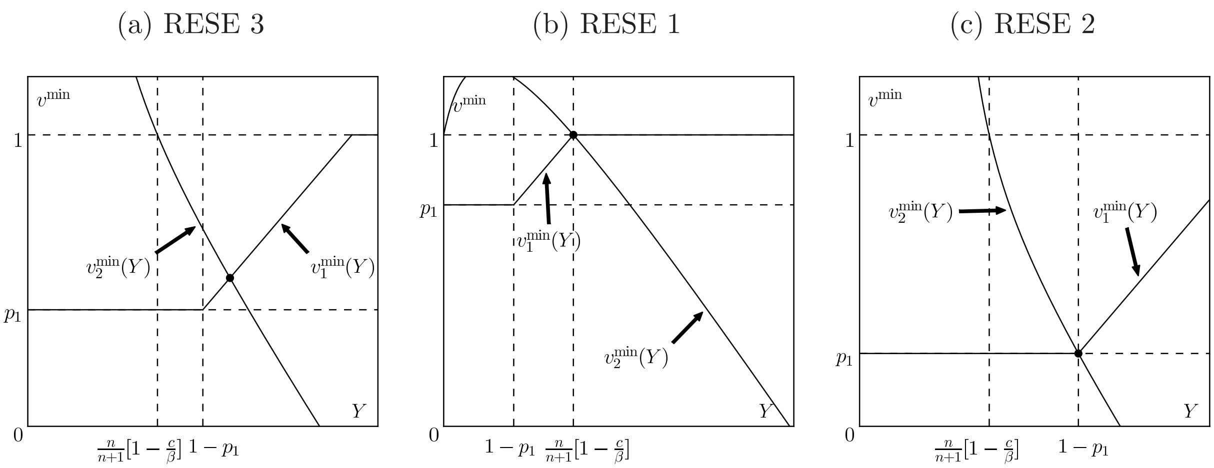


Figure 12: Changes in equilibrium structure from RESE 3 to RESE 1 and 2

This expression can be also written as follows: $\tilde{r}^i = \left\{\sqrt{(p_1-s)(1-v^*)} - \sqrt{\frac{n-1}{n}Y^*(c-s)}\right\}^2$, which coincides with the expression for $\tilde{r}^i$ in the theorem statement.

Expression for r^* follows immediately from (1) and Lemma 1.

We complete the proof of the main part of the theorem by a simple observation that equilibrium characteristics are continuous on the boundaries between RESE1 and 3 as well as RESE2 and 3. Figure 12, in its subplot (a), depicts a typical configuration of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ when RESE3 exists, whereas subplots (b) and (c) depict this configuration at the points of change to RESE1 and 2, respectively.

RESE3 continuously changes into RESE1 as the intersection point of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ representing RESE3 moves toward the point $\left(\frac{n}{n+1}\left[1-\frac{c}{\beta}\right], 1\right)$ on $v_2^{\min}(Y)$ representing RESE1. The latter point is to the left of all possible candidates for RESE3 located on $v_2^{\min}(Y)$ implying that, in RESE3, $Y^* \geq \frac{n}{n+1}\left[1-\frac{c}{\beta}\right]$. Similarly, RESE3 continuously changes into RESE2 as the intersection point of $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ moves toward $v_1^{\min}(Y)$'s break-point $(1-p_1, p_1)$ (representing RESE2). The continuity of r^* follows from the continuity of the expression for r^i, given by (8), in all the parameters and continuity of v^* and Y^* (using $y^i = \frac{1}{n}Y^*$).

It remains to examine the convergence under deviations from rational expectations. The geometric structure of candidates for RESE3 and 1 implies that the areas of inputs where these RESE exist do not intersect. Suppose that (i) $\bar{\alpha} = \alpha^* = 1$, i.e., one and only one of RESE3 or 1 can be realized for given inputs; and (ii) consumer expectations of the second-period price deviate from rational ones with $\bar{p}_2^0 < p_2^*$, and the game is repeated with the same inputs. As shown above, $\frac{\partial v_2^{\min}}{\partial Y} < 0$, implying $\frac{\partial BR}{\partial v^{\min}} < 0$, where $BR = BR[v^{\min}(\bar{p}_2)]$ is a symmetric best response, given $\bar{p}_2$. By Lemma 1, $\frac{\partial v^{\min}}{\partial \bar{p}_2} \leq 0$, which leads to $\frac{\partial BR}{\partial \bar{p}_2} \geq 0$. Then $\bar{p}_2^0 < p_2^* \Rightarrow BR(\bar{p}_2^0) \leq BR(p_2^*) = Y^* < 1 - \frac{s}{\beta}$, implying that the realized price $p_2^0 = \beta[1 - BR(\bar{p}_2^0)] \geq p_2^* > s$. Moreover, for any moment of time $t \in [0, \infty)$, the realized (p_2^t) and expected ($\bar{p}_2^t$) second-period prices are such that $p_2^t \geq p_2^* > \bar{p}_2^t$ if consumer expectations follow a linear adjustment process $\bar{p}_2^{t+1} = \mu p_2^t + (1-\mu)\bar{p}_2^t$ with a sufficiently small μ. This process, using (3), can be written as $\bar{p}_2^{t+1} = \mu\beta(1-Y^t) + (1-\mu)\bar{p}_2^t$, where $Y^t = BR(\bar{p}_2^t)$. Then $\left|p_2^* - \bar{p}_2^{t+1}\right| = \left|p_2^* - \mu\left[\beta(1 - BR(\bar{p}_2^t)) + p_2^* - p_2^*\right] - (1-\mu)\bar{p}_2^t\right| = \left|(1-\mu)(p_2^* - \bar{p}_2^t) - \mu\beta(Y^* - BR(\bar{p}_2^t))\right|$. By the mean value theorem, there exists $\tilde{p}_2 \in (p_2^*, \bar{p}_2^t)$ such that $Y^* - BR(\bar{p}_2^t) = \frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2}(p_2^* - \bar{p}_2^t)$. Then, if $\mu\beta\frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2} \leq 2(1-\mu) \Leftrightarrow \mu \leq 2/(2 + \beta\frac{\partial BR(\tilde{p}_2)}{\partial \bar{p}_2}) < 1$, we have $\left|p_2^* - \bar{p}_2^{t+1}\right| \leq (1-\mu)\left|p_2^* - \bar{p}_2^t\right| = (1-\mu)^t\left|p_2^* - \bar{p}_2^0\right|$, which goes to zero with $t \to \infty$ for any $\bar{p}_2^0 < p_2^*$. As $\frac{\partial v^{\min}}{\partial \bar{p}_2}$ is restricted for any $\rho \in [0,1)$ and, by (20), $\frac{\partial BR}{\partial v^{\min}}$ is restricted in the relevant region for Y, there exists a sufficiently small μ such that $\mu < 2/(2+\beta\frac{\partial BR(\bar{p}_2)}{\partial \bar{p}_2})$ leading to the convergence of the adjustment process to p_2^*.

The adjustment process can be specified using inequalities $\frac{\partial v^{\min}}{\partial \bar{p}_2} \leq 0$ and $\bar{p}_2^0 < p_2^*$, which imply $v^{\min}(\bar{p}_2^0) \geq v^* = v^{\min}(p_2^*)$. This property leads to three cases. (a) $v^* < 1$ and $v^{\min}(\bar{p}_2^0) < 1$, which corresponds to the adjustment process above; (b) $v^* = 1$ (RESE1 is realized at $\bar{p}_2 = p_2^*$). In this case, all $\bar{p}_2^t$ are such that $v^{\min}(\bar{p}_2^t) = 1$, i.e., retailers' decisions do not depend on $\bar{p}_2^t$ and the adjustment becomes $\bar{p}_2^{t+1} = \mu p_2^* + (1-\mu)\bar{p}_2^t$. Then $\left|p_2^* - \bar{p}_2^{t+1}\right| \leq |1-\mu|^t\left|p_2^* - \bar{p}_2^0\right|$, which converges to p_2^* for any $\mu \in (0,2)$ and $\bar{p}_2^0 < p_2^*$. (c) $v^* < 1$ and $v^{\min}(\bar{p}_2^0) = 1$. In this case, the initial adjustment steps are $\bar{p}_2^{t+1} = \mu\beta(1 - BR|_{v^{\min}=1}) + (1-\mu)\bar{p}_2^t$. As $\frac{\partial BR}{\partial v^{\min}} < 0, \bar{p}_2^t$ in this process increases faster than for $BR|_{v^{\min}<1}$. Then, by continuity and monotonicity of $v^{\min}$ in $\bar{p}_2$ and monotonicity of $\bar{p}_2^t$ in t, there exists such $\tilde{t}$ that the adjustment process switches to case (a) and follows it for any $t \geq \tilde{t}$.

A.8 Proof of Corollary 1 (irrational $\bar{p}_2$, RESE3)

The derivative $\frac{\partial \hat{Y}}{\partial \bar{p}_2}$ can be written as $\frac{\partial \hat{Y}}{\partial \bar{p}_2} = \frac{\partial \hat{Y}}{\partial v^{\min}}\frac{\partial v^{\min}}{\partial \bar{p}_2}$, which, by the implicit function theorem, is $\frac{\partial v^{\min}}{\partial \bar{p}_2}/\frac{\partial v^{\min}}{\partial \hat{Y}}$. By Lemma 1, $\frac{\partial v^{\min}}{\partial \bar{p}_2} = -\frac{\bar{\alpha}\rho}{1-\bar{\alpha}\rho\beta}$, which is $-\frac{\rho}{1-\rho\beta}$ under RESE3. By (20), $\frac{\partial v^{\min}}{\partial \hat{Y}} =$

$$
\begin{aligned}
\frac{\partial v_2^{\min}}{\partial \hat{Y}} &= \frac{\left[2\hat{Y} - \frac{n}{n+1}(1-c/\beta)\right]\left[\hat{Y}\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta - 1)\right] - \left[\hat{Y}^2 - \hat{Y}\frac{n}{n+1}(1-c/\beta)\right]\frac{n}{n+1}}{\left[\hat{Y}\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta-1)\right]^2} \\
&= \frac{\hat{Y}^2\frac{n}{n+1} + 2\hat{Y}\frac{n-1}{n+1}(p_1/\beta - 1) - \frac{n(n-1)}{(n+1)^2}(1-c/\beta)(p_1/\beta-1)}{\left[\hat{Y}\frac{n}{n+1} + \frac{n-1}{n+1}(p_1/\beta-1)\right]^2}. \qquad (24)
\end{aligned}
$$

As both $\frac{\partial v^{\min}}{\partial \bar{p}_2}$ and $\frac{\partial v_2^{\min}}{\partial \hat{Y}}$ are negative (the latter – by the geometric argument in the proof of Theorem 1 illustrated in Figure 11), the upper bound on $\frac{\partial \hat{Y}}{\partial \bar{p}_2}$ corresponds to the upper bound on $\frac{\partial v_2^{\min}}{\partial \hat{Y}}$, which, for $p_1 \geq \beta$, (see Figure 11 (a)) is provided by $\frac{\partial v_2^{\min}}{\partial \hat{Y}} \leq \frac{\partial v_2^{\min}}{\partial \hat{Y}}|_{\hat{Y}=\frac{n}{n+1}(1-c/\beta)} =$

$$
= -\frac{\frac{n}{n+1}(1-c/\beta)\left[\left(\frac{n}{n+1}\right)^2(1-c/\beta) + \frac{n-1}{n+1}(p_1/\beta-1)\right]}{\left[\left(\frac{n}{n+1}\right)^2(1-c/\beta) + \frac{n-1}{n+1}(p_1/\beta-1)\right]^2} = -\frac{1-c/\beta}{\frac{n}{n+1}(1-c/\beta) + \frac{n-1}{n}(p_1/\beta-1)},
$$

where denominator is positive, increases in n with the limit $(p_1-c)/\beta$ when $n \to \infty$. Therefore, $\frac{\partial \hat{Y}}{\partial \bar{p}_2} \leq \frac{\rho}{1-\rho\beta}\frac{p_1-c}{\beta-c}$ when $p_1 \geq \beta$.

When $p_1 < \beta$, we have (see Figure 11 (b)) $\frac{\partial v_2^{\min}}{\partial \hat{Y}} \leq \frac{\partial v_2^{\min}}{\partial \hat{Y}}|_{\hat{Y}=1-p_1} =$

$$
= \frac{(1-p_1)^2\frac{n}{n+1} + 2\frac{n-1}{n+1}(p_1/\beta-1)(1-p_1) - \frac{n(n-1)}{(n+1)^2}(1-c/\beta)(p_1/\beta-1)}{\left(\frac{n}{n+1}\right)^2\left[1-p_1+\frac{n-1}{n}(p_1/\beta-1)\right]^2}.
$$

The last term in the numerator can be written as $-\frac{n(n-1)}{(n+1)^2}\left[1-p_1/\beta+(p_1-c)/\beta\right](p_1/\beta-1) = \frac{n(n-1)}{(n+1)^2}(1-p_1/\beta)^2 + \frac{n(n-1)}{(n+1)^2}(1-p_1/\beta)(p_1-c)/\beta$. Then the numerator is

$$
\frac{n}{n+1}\left\{(1-p_1)^2 + 2\frac{n-1}{n}(p_1/\beta-1)(1-p_1) + \frac{n-1}{n+1}(1-p_1/\beta)^2 + \frac{n-1}{n+1}(1-p_1/\beta)(p_1-c)/\beta\right\}.
$$

By adding and subtracting $\left[\frac{n-1}{n}(p_1/\beta-1)\right]^2$, we have $\{\cdot\} = \left[1-p_1+\frac{n-1}{n}(p_1/\beta-1)\right]^2 + \frac{n-1}{n}(p_1/\beta-1)^2\left(\frac{n}{n+1}-\frac{n-1}{n}\right) + \frac{n-1}{n+1}(1-p_1/\beta)(p_1-c)/\beta$, where $\frac{n}{n+1}-\frac{n-1}{n} = \frac{1}{n(n+1)}$. Then

$$\frac{\partial v_2^{\min}}{\partial \hat{Y}}|_{\hat{Y}=1-p_1} = -\frac{n+1}{n}\left[1+\frac{\frac{n-1}{n^2(n+1)}(p_1/\beta-1)^2+\frac{n-1}{n+1}(1-p_1/\beta)(p_1-c)/\beta}{\left[1-p_1+\frac{n-1}{n}(p_1/\beta-1)\right]^2}\right] \le -\frac{n+1}{n} < -1,$$

i.e., $\frac{\partial \hat{Y}}{\partial \bar{p}_2} \le \frac{\rho}{1-\rho\beta}$ when $p_1 < \beta$.

A.9 Proof of Corollary 2 (RESE3, monopoly)

For $n=1$, sufficient condition (a) always holds and (18) is $\left[Y-1+\frac{1}{2}\left(v^{\min}+\frac{c}{\beta}\right)\right]Y=0$, yielding $Y^*=1-\frac{1}{2}\left(v^*+\frac{c}{\beta}\right)$. The equation for v^* is $v^*=\frac{p_1-\frac{1}{2}\rho\beta\left(v^*+\frac{c}{\beta}\right)}{1-\rho\beta}$ which is equivalent to $v^*(2-\rho\beta) = 2p_1-\rho c$, resulting in the equilibrium v^*. Substitution of Y^* into (3) leads to the expression for p_2^*.

A.10 Proof of Corollary 3 (RESE3, $\rho=0$, second-period sales at loss)

For $\beta=1$ and $\rho=0$, p_1-range in RESE3 is $c<p_1<1$. Thus, RESE1 and 2 cannot be realized. By the proof of Theorem 1, $Y^{*,3} \le 1-c/\beta$ is equivalent to $v^* \ge v_2^{\min}(1-c/\beta)$ because $v_2^{\min}(Y)$, given by (20), is decreasing in the relevant range of Y. Using $Y=1-c$ and $\beta=1$ in (20), we get

$$v_2^{\min}(1-c) = 1-\frac{(1-c)^2-(1-c)^2\frac{n}{n+1}}{(1-c)\frac{n}{n+1}+\frac{n-1}{n+1}(p_1-1)} = 1-\frac{(1-c)^2}{n(p_1-c)+1-p_1}.$$

Thus, under conditions of the corollary, $p_2^* \ge c$ if and only if $v^*=p_1 \ge 1-\frac{(1-c)^2}{n(p_1-c)+1-p_1}$. Rearranging this inequality we obtain $\frac{(1-c)^2}{n(p_1-c)+1-p_1} \ge 1-p_1$, and solving for n we get

$$n \le \frac{1}{p_1-c}\left(\frac{(1-c)^2}{1-p_1}-(1-p_1)\right) = \frac{2-c-p_1}{1-p_1} = 2+\frac{p_1-c}{1-p_1}.$$

A.11 Proof of Corollary 4 (RESE3, perfect competition)

If $P_2 < p_1 < P_1$ for $n\to\infty$, the limits v_∞^* and Y_∞^* of, respectively, $v^*=\frac{p_1-\rho\beta(1-Y^*)}{1-\rho\beta}$ and Y^* defined by (23) exist. This follows from the geometric structure of curves $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ in the limiting case (see equations (19) and (20) and their analysis in the proof of Theorem 1).

The violation of condition (a) in the limit of $n\to\infty$ means that, for some $\epsilon>0$,

$$\frac{(p_1-s)(1-v_\infty^*)Y_\infty^*}{(c-s)(1-s/\beta)^2} = 1+2\epsilon.$$

There exists N such that condition (a) is violated for any $n>N$ by at least ϵ:

$$\frac{n-1}{n}\frac{(p_1-s)(1-v^*)Y^*}{(c-s)(1-s/\beta)^2} \ge 1+\epsilon. \tag{25}$$

There are two cases: $Y_\infty^* > 1-s/\beta$ and $Y_\infty^* \le 1-s/\beta$. If $Y_\infty^* > 1-s/\beta$, there exists N' such that $Y^* > 1-s/\beta$ for any $n>N'$ implying, by condition (b) of Theorem 1, that RESE3 does not exist for these n, and the claim of the corollary is established. If $Y^*_\infty \le 1 - s/\beta$, there exist sufficiently small $\epsilon' > 0$ and N' such that

$$\frac{1 - s/\beta}{Y^*} \ge \frac{1 + \epsilon'}{\sqrt{1+\epsilon}} \tag{26}$$

for all $n > N'$. Inequality (25) is equivalent to $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0} > 0$ (see the analysis of RESE3 in the proof of Theorem 1) and implies that there exists $\tilde{r}^i$, which is a unique maximum of r^i for $y^i > 1 - s/\beta - Y^{-i}$. Using the proof of condition (b) in Theorem 1, we have

$$\tilde{r}^i = \frac{n-1}{n} Y^* (c - s) \left\{ \sqrt{\frac{n}{n-1} \frac{(p_1 - s)(1 - v^*)}{(c - s) Y^*}} - 1 \right\}^2.$$

Bounds (25) and (26) imply

$$\frac{n}{n-1} \frac{(p_1 - s)(1 - v^*)}{(c - s) Y^*} \ge \left(\frac{n}{n-1} \frac{1 - s/\beta}{Y^*} \right)^2 (1 + \epsilon) \ge (1 + \epsilon')^2.$$

Then, using $\frac{n-1}{n} \ge \frac{1}{2}$ and $Y^* \ge 1 - p_1$, $\tilde{r}^i$ is bounded from below as follows:

$$\tilde{r}^i \ge \frac{n-1}{n} Y^* (c - s) \left\{ \sqrt{(1+\epsilon')^2} - 1 \right\}^2 \ge \frac{1}{2} (1 - p_1)(c - s)(\epsilon')^2$$

for all $n > \max\{N, N'\}$. That is, $\tilde{r}^i$ is separated from zero by a positive constant for all sufficiently large n. On the other hand, the following lemma immediately implies that $\lim_{n\to\infty} r^* = 0$.

Lemma 8. *The equilibrium profit in RESE 3 can be expressed as*

$$r^* \;=\; \frac{\beta}{n(1 - \rho\beta)} \Big\{ -(Y^*)^2 + Y^* \Big[2 - p_1 - \frac{c}{\beta} - \rho\,(p_1 - c) \Big] + \Big(\frac{p_1}{\beta} - 1 \Big)(1 - p_1) \Big\}. \tag{27}$$

Indeed, Y^* is bounded, implying that the expression inside $\{\cdot\}$ is also bounded, whereas the coefficient in front of $\{\cdot\}$ tends to zero as $n \to \infty$. Then there exists $N'' \ge \max\{N, N'\}$ such that $\tilde{r}^i > r^*$ for all $n > N''$ and RESE3 does not exist.

A.12 Proof of Theorem 2 (RESE with $p_2^* = s$)

We start by identifying candidate solutions for a symmetric equilibrium with given expectations. When $p_2 = s$, the equilibrium is possible only with sales in both periods, and rationality requires that $v^{\min} < 1$ and $\bar{\alpha} = 1$.

By parts 1.4 and 2 of Lemma 7, the profit function is strictly concave when $y^i \ge \left(1 - s/\beta - Y^{-i}\right)^+$ and (by part 1.1) the optimum cannot occur at $y^i = 1 - s/\beta - Y^{-i}$. Then the candidate is found by setting the derivative of the profit to zero. Using (15) for $\frac{\partial r^i}{\partial y^i}$ and, by symmetry, $Y^{-i} = \frac{n-1}{n} Y$,

$$0 = \frac{\partial r^i}{\partial y^i} = -(c - s) + \frac{Y^{-i}}{Y^2} (p_1 - s)\left(1 - v^{\min}\right) = -(c - s) + \frac{n-1}{nY} (p_1 - s)\left(1 - v^{\min}\right).$$

The unique solution is $\hat{Y} = \frac{n-1}{n} \frac{(p_1 - s)\left(1 - v^{\min}\right)}{c - s}$. Rationality of expectations, by Lemma 1, leads to $v^{\min} = v^* = \frac{p_1 - \rho s}{1 - \rho\beta}$, which gives us $Y^* = \frac{n-1}{n} \frac{(p_1 - s)(1 - v^*)}{c - s}$ and the equilibrium profit

$$r^* = \frac{1}{n} \left\{ -cY^* + p_1(1 - v^*) + s\left[Y^* - (1 - v^*)\right] \right\} = \frac{1}{n} \left\{ -(c - s)Y^* + (p_1 - s)(1 - v^*) \right\},$$

which yields the expression for r^* in the theorem.

We now analyze when the candidate point is indeed a RESE with $p_2^* = s$, and start by checking that it is contained within the valid ranges $p_1 \le v^* < 1$ and $Y^* \ge 1 - s/\beta$, which provide necessary conditions for RESE existence. The second condition is the domain restriction of §A.3. It is equivalent to $p_2^* = s$ and follows from either of the mutually exclusive cases (a), (b), and (c) in the statement of the theorem. Because the equilibrium cannot result in $Y^* = 1 - s/\beta$, by part 1.1 of Lemma 7, the second condition is strengthened to $Y^* > 1 - s/\beta$ under which cases (a), (b) and (c) become exhaustive. As $1 - s/\beta > 0$ and Y^* is proportional to $1 - v^*$, the resulting strict positivity of Y^* implies that $v^* < 1$. Similarly to RESE3, $v^* = p_1$ if $\rho = 0$, and it can be shown that $v^* > p_1$ if $\rho > 0$. Indeed, inequality $v^* > p_1$ is equivalent to $\frac{p_1 - \rho s}{1-\rho\beta} > p_1 \Leftrightarrow p_1 - \rho s > p_1 - p_1\rho\beta \Leftrightarrow -\rho s > -p_1\rho\beta \Leftrightarrow p_1 > s/\beta$, which always holds in this problem.

It remains to establish that the exact conditions ensuring that $\frac{Y^*}{n}$ provides the global optimum of the profit function are indeed provided by the mutually exclusive and exhaustive (under condition $Y^* > 1 - s/\beta$) cases (a), (b), and (c).

Condition (a), i.e., $\frac{n-1}{n}Y^* \ge 1 - \frac{s}{\beta}$, means, by (3), that $p_2 = s$ independently of the inventory decisions of individual retailers. By part 2 of Lemma 7, the profit function is globally strictly concave in this case and $\frac{Y^*}{n}$ is indeed its unique global maximum.

In case (b) of the theorem, condition (a) does not hold, which means that $p_2 = s$ may or may not hold depending on the decisions of individual retailers. Nevertheless, the maximum of the profit function is unique and occurs when $p_2 = s$ as long as the profit function is strictly increasing in the interval corresponding to $p_2 > s$. This is ensured by the condition $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} \ge 0$ which, by part 1.5 of Lemma 7, implies pseudoconcavity of the profit function. Using (12), the last condition takes the following form:

$$\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0} = \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta\left(1 - \frac{s}{\beta} - Y^{-i}\right) + \frac{(p_1-\beta)(1-v^{\min})Y^{-i}}{(1-s/\beta)^2} \ge 0,$$

which, after collecting the terms and substituting $Y^{-i} = \frac{n-1}{n}Y^*$ and $v^{\min} = v^*$, can be rewritten as

$$\left(\beta + \frac{(p_1-\beta)(1-v^*)}{(1-s/\beta)^2}\right)\frac{n-1}{n}Y^* \ge c + \beta v^* - 2s, \quad \text{yielding condition (b).}$$

In case (c) of the theorem, condition (b) does not hold, i.e. $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-s/\beta-Y^{-i}-0} < 0$. Then, there exists a local maximum of r^i without or with the sales in the second period and $p_2 > s$. In other words, there exists such an inventory decision $\tilde{y}^i$ of a deviating retailer that

$$\tilde{y}^i \triangleq \arg\max\left\{r^i(y^i) \mid y^i \in \left[\max\left\{0, 1 - v^* - \frac{n-1}{n}Y^*\right\}, 1 - s/\beta - \frac{n-1}{n}Y^*\right)\right\}$$

or, denoting $\tilde{Y} \triangleq \tilde{y}^i + \frac{n-1}{n}Y^*$, $\tilde{Y} \in \left[\max\left\{1 - v^*, \frac{n-1}{n}Y^*\right\}, 1 - s/\beta\right)$. Then the equilibrium with $p_2^* = s$ exists only if

$$\tilde{r}^i \triangleq r^i(\tilde{y}^i) \le r^*. \tag{28}$$

Consider this condition at the left boundary of the range for y^i. If $\tilde{y}^i = 0$, then $\tilde{r}^i = 0$ and (28) holds trivially. If $\tilde{y}^i = 1 - v^* - \frac{n-1}{n}Y^* = (1-v^*)\left[1 - \left(\frac{n-1}{n}\right)^2 \frac{p_1-s}{c-s}\right]$, then, by §A.3, there are no sales in the second period and $\tilde{r}^i = (1-v^*)\left[1 - \left(\frac{n-1}{n}\right)^2 \frac{p_1-s}{c-s}\right](p_1 - c)$. After substitutions for $\tilde{r}^i$ and r^*, and multiplication of both sides by $\frac{n^2}{(1-v^*)(p_1-c)}$, condition (28) becomes $n^2 - (n-1)^2 \frac{p_1-s}{c-s} \le \frac{p_1-s}{p_1-c}$, which always holds. Indeed, let $g(n) \triangleq n^2 - (n-1)^2 \frac{p_1 - s}{c - s}$. Then $g'(n) = 2n - 2(n-1)\frac{p_1-s}{c-s} = 2\left[-n\frac{p_1-c}{c-s} + \frac{p_1-s}{c-s}\right]$ and $g''(n) = -2\frac{p_1-c}{c-s} < 0$. Therefore, the unique maximum of g, defined by the condition $g'(n) = 0$, is $n_{\max} = \frac{p_1-s}{p_1-c}$ and

$$g_{\max} = g(n_{\max}) = \left(\frac{p_1-s}{p_1-c}\right)^2 - \left(\frac{p_1-s}{p_1-c} - 1\right)^2 \frac{p_1-s}{c-s} = \frac{p_1-s}{p_1-c}\left[\frac{p_1-s}{p_1-c} - \frac{(c-s)^2}{p_1-c}\frac{1}{c-s}\right] = \frac{p_1-s}{p_1-c}.$$

Finally, the RESE with $p_2^* = s$ may also exist if there exists an internal local maximum $r^i(\tilde{y}^i) \leq r^*$ with $\tilde{y}^i = \tilde{Y} - \frac{n-1}{n}Y^*$ such that $\max\left\{1 - v^*, \frac{n-1}{n}Y^*\right\} < \tilde{Y} < 1 - \frac{s}{\beta}$ and $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = \tilde{y}^i} = 0$. In this case, formula (10) from §A.3 yields the expression for $\tilde{r}^i$ in condition (c):

$$\tilde{r}^i = \left(\tilde{Y} - \frac{n-1}{n}Y^*\right)\left[\beta\left(1 - \tilde{Y}\right) - c + \beta\left(1 - v^*\right) + \frac{(p_1 - \beta)(1 - v^*)}{\tilde{Y}}\right],$$

where $\tilde{Y}$ is a zero of the profit function derivative (12), which, in this case, is

$$0 = \frac{\partial r^i}{\partial y^i} = \beta\left(1 - \frac{n-1}{n}Y^*\right) - c + \beta(1 - v^*) - 2\beta\left(Y - \frac{n-1}{n}Y^*\right) + (p_1 - \beta)(1 - v^*)\frac{n-1}{n}\frac{Y^*}{Y^2}.$$

After multiplication by $-Y^2/\beta$ this equation becomes

$$2Y^3 + a_2 Y^2 + a_0 = 0, \tag{29}$$

which is equation (4) if one substitutes the coefficients

$$\begin{aligned} a_2 &\triangleq c/\beta - (1 - v^*) - \left(1 + \frac{n-1}{n}Y^*\right) = -(1 - v^*) - (1 - c/\beta) - \frac{n-1}{n}Y^* < 0, \\ a_0 &\triangleq (1 - p_1/\beta)(1 - v^*)\frac{n-1}{n}Y^*. \end{aligned}$$

Because, by part 1.3 of Lemma 7, the profit function of the deviating retailer is pseudoconcave on the interval $(1 - v^* - \frac{n-1}{n}Y^*)^+ \leq y^i \leq 1 - s/\beta - \frac{n-1}{n}Y^*$, equation (29) may have at most one root on this interval.

Any cubic equation with real coefficients has at least one and up to three real roots. If neither of the roots is relevant, it means that there is no internal maximum and the boundary maximum cannot exceed r^* as shown above. If there is a relevant root, a direct comparison between r^* and $\tilde{r}^i$ determines the existence of RESE.

Suppose that consumer expectations of the second-period price deviate from rational ones with the initial deviation $\bar{p}_2^0 > s$ and the game is repeated with the same inputs. As shown above, a symmetric best response is $BR(\bar{p}_2) = \frac{n-1}{n}\frac{(p_1-s)\left(1-v^{\min}(\bar{p}_2)\right)}{c-s}$, which is increasing in $\bar{p}_2$. For any inputs where RESE4 exists, $Y^* = BR(s) > 1 - s/\beta$ and, because $\bar{p}_2^0 > s, BR(\bar{p}_2^0) > 1 - s/\beta$. Then, for any $t \geq 0$, the realized second-period price p_2^t equals s if consumer expectations follow a linear adjustment process $\bar{p}_2^{t+1} = \mu s + (1 - \mu)\bar{p}_2^t$ with $\mu < 1$. Under this process, $\left|s - \bar{p}_2^{t+1}\right| = \left|(1 - \mu)(s - \bar{p}_2^t)\right| = \left|(1 - \mu)^t(s - \bar{p}_2^0)\right|$, which goes to zero with $t \to \infty$ for any $\mu \in (0, 1)$ and $\bar{p}_2^0 \in (s, \infty)$.

A.13 Proof of Corollary 5 (irrational $\bar{p}_2$, RESE4)

The result follows from $\frac{\partial \hat{Y}}{\partial \bar{p}_2} = \frac{\partial \hat{Y}}{\partial v^{\min}}\frac{\partial v^{\min}}{\partial \bar{p}_2}$, where, by Theorem 2, $\frac{\partial \hat{Y}}{\partial v^{\min}} = -\frac{n-1}{n}\frac{p_1-s}{c-s}$ and, by Lemma 1, $\frac{\partial v^{\min}}{\partial \bar{p}_2} = -\frac{\rho}{1-\rho\beta}$.

A.14 Proof of Proposition 1 (Uniqueness of RESE)

Part (a). We start by discussing model inputs satisfying conditions of RESE1 and 2. By Theorem 1, these conditions rule out RESE3 and guarantee that, *for the corresponding structure*, one and only one equilibrium exists. Thus, it remains to rule out RESE4.

RESE4 cannot exist under the same conditions as RESE1 because p_1-lower bound P_1 in RESE1 exceeds the upper bound P_4 in RESE4: $1 - \frac{n}{n+1}\rho(\beta - c) > 1 - \rho(\beta - c) > 1 - \rho(\beta - s) \Leftrightarrow c > s$.

Moreover, RESE4 cannot exist under the necessary and sufficient condition $p_1 \leq P_2$ for RESE2 because the latter is incompatible with necessary condition $Y^* > 1 - s/\beta$ for RESE4. Indeed, consider $Y^* = \frac{n-1}{n}\frac{p_1-s}{c-s}(1-v^*)$ for RESE4. Condition $p_1 \leq P_2$ implies $n(c-s) \geq (n-1+\beta)p_1 - ns = (n-1)(p_1 - s) + \beta p_1 - s > (n-1)(p_1 - s)$. As $1 - v^* \leq 1 - p_1 < 1 - s/\beta$, we get $Y^* < 1 - s/\beta$.

Part (b). It remains to show that, when conditions of RESE1 or 2 do not hold, condition (b.2) guarantees the existence of RESE3 and non-existence of RESE4. Indeed, (b.2) implies that RESE4 total equilibrium supply violates a necessary condition $Y^* > 1 - s/\beta$ for RESE4 because $v^* \geq p_1$ and we have $Y^* = \frac{n-1}{n} \cdot \frac{p_1-s}{c-s}(1 - v^*) \leq 1 - s/\beta$.

Finally, for the existence of RESE3, we show that (b.2) implies $Y^* < 1 - s/\beta$ and condition (a) of Theorem 1. Indeed, as long as $Y^* < 1 - s/\beta$ and because $v^* \geq p_1$, the LHS of (a) is smaller than the LHS of (b.2). We show that $Y^* < 1 - s/\beta$ by demonstrating that $1 - s/\beta$ exceeds the larger root of (23) under condition (b.2). Recall, from the proof of Theorem 1, that (23) is obtained as a characterization of the intersection point (Y^*, v^*) of functions $v_1^{\min}(Y)$ and $v_2^{\min}(Y)$ in the range of $Y \geq \frac{n}{n+1}(1 - c/\beta)$ where $v_2^{\min}(Y)$ is decreasing (see Figure 12(a)). Because the smallest possible value of $v_1^{\min}(Y)$ is p_1, $Y^* < 1 - s/\beta$ holds as long as $v_2^{\min}(1 - s/\beta) < p_1$, i.e.,

$$1 - \frac{(1 - s/\beta)^2 - (1 - s/\beta)\frac{n}{n+1}(1 - c/\beta)}{\frac{n}{n+1}(1 - s/\beta) + \frac{n-1}{n+1}(p_1/\beta - 1)} = 1 - \frac{\frac{1}{n+1}(1 - s/\beta)^2 + (1 - s/\beta)\frac{n}{n+1}(c - s)/\beta}{\frac{1}{n+1}(1 - s/\beta) + \frac{n-1}{n+1}(p_1 - s)/\beta} < p_1, \qquad \text{or}$$

$$(n-1)(1 - p_1)(p_1 - s)/\beta + (1 - p_1)(1 - s/\beta) - (1 - s/\beta)^2 < n(1 - s/\beta)(c - s)/\beta.$$

The latter is implied by (b.2) because $1 - p_1 < 1 - s/\beta$.

A.15 Proof of Proposition 2 (Switches between RESE)

The p_1-bounds in the claim of the proposition satisfy the following chain of inequalities for all valid model inputs: $c/\beta \leq 1 - \beta + c < 1 - \rho(\beta - c) < 1 - \frac{n}{n+1}\rho(\beta - c) \leq 1 - \frac{1}{2}\rho(\beta - c)$. The value $1 - \beta + c$ provides the exact lower bound on p_1-values corresponding to RESE1 over all $n \geq 1$ and $\rho \in [0, 1)$, whereas c/β provides the exact upper bound on p_1 corresponding to RESE2. Thus, p_1 corresponding to RESE1 for some model inputs cannot result in RESE2 under any other inputs and *vice versa*. Consider each of the possible p_1-ranges.

Part 1: By Theorem 1, if $p_1 \leq c/\beta$ and $n = 1$, the RESE is realized in the form 2 and not form 3. The necessary and sufficient condition for RESE2 can be rewritten as $p_1 n - p_1(1 - \beta) \leq nc$ or, equivalently, $n \leq n_2 = \frac{p_1(1-\beta)}{p_1 - c}$. For $n > n_2$, RESE2 cannot exist and p_1 falls into the range of RESE3 (and, as argued above, cannot fall into the range of RESE1). That is, as the level of competition increases, the equilibrium with no sales in the second period (RESE2) becomes impossible and is replaced by the equilibrium with sales in both periods (RESE3).

Part 4: When $p_1 > \frac{c}{\beta}$, we have $n_2 < 1$, i.e. even a monopolist cannot realize RESE2. If, in addition to this condition, $p_1 < 1 - \beta + c$, only RESE3 is possible.

Part 2: Because the RESE3 upper bound on p_1 is decreasing in n, RESE3 may exist only if

$p_1 < 1 - \frac{1}{2}\rho(\beta - c)$ (P_1 for $n = 1$). RESE3 p_1-bounds imply the following bounds on n :

$$n_2 < n < \begin{cases} n_1 = \frac{1-p_1}{p_1-1+\rho(\beta-c)} \text{ if } p_1 > 1 - \rho(\beta - c), \\ \infty \text{ otherwise.} \end{cases}$$

That is, as n increases, RESE3 becomes impossible if $p_1 > 1 - \rho(\beta - c)$ and is replaced by RESE1.

Part 3 of this proposition can be shown in the same way, using the boundary on p_1 between RESE1 and 3 as a function of ρ.

A.16 Proof of Proposition 3 (Monotonicity of Y^*, v^*, and nr^*)

Monotonicity of v^* and Y^*. By Theorem 1, v^* is constant in n and ρ for RESE1 and 2; Y^* is increasing in n and constant in ρ for RESE1 and constant in n and ρ for RESE2. By continuity of v^* and Y^*, it remains to show the correspondent monotonicity of these values for RESE3.

Monotonicity of v^* and Y^* in ρ. Recall that, for RESE3, Y^* and v^* satisfy (18) for Y^*. The derivative of this equation in ρ is

$$2Y^* \frac{\partial Y^*}{\partial \rho} - \frac{\partial Y^*}{\partial \rho} \frac{n}{n+1} \left(2 - v^* - \frac{c}{\beta}\right) + \frac{n}{n+1} Y^* \frac{\partial v^*}{\partial \rho} + \frac{n-1}{n+1} \left(\frac{p_1}{\beta} - 1\right) \frac{\partial v^*}{\partial \rho} = 0,$$

which can be written as

$$\frac{\partial Y^*}{\partial \rho} \left[2Y^* - \frac{n}{n+1} \left(2 - v^* - \frac{c}{\beta}\right)\right] = -\frac{\partial v^*}{\partial \rho} \frac{1}{n+1} \left[nY^* + (n-1) \left(\frac{p_1}{\beta} - 1\right)\right]. \tag{30}$$

As $Y^* > 1 - p_1$ (by Lemma 4), and $\frac{p_1}{\beta} \geq p_1$, the lower bound for the square bracket in the RHS is $n(1 - p_1) + (n - 1)(p_1 - 1) = 1 - p_1 > 0$. The square bracket in the LHS of (30) is also positive:

$$2Y^* - \frac{n}{n+1} (2 - v^* - c/\beta) > 0 \tag{31}$$

because $Y^* > 1 - p_1 \geq 1 - v^* \geq \frac{n}{n+1}(1 - v^*)$ and $\frac{n}{n+1}(1 - c/\beta)$ is a lower bound for Y^* in RESE3 (by Theorem 1).

For RESE3, Y^* and v^* satisfy (21), which can be written as $(1 - \rho\beta)v^* - \rho\beta Y^* = p_1 - \rho\beta$ with the following derivative in ρ : $(1 - \rho\beta)\frac{\partial v^*}{\partial \rho} - \rho\beta\frac{\partial Y^*}{\partial \rho} = \beta(v^* + Y^* - 1)$, where the RHS is positive because $v^* \geq p_1$ and $Y^* > 1 - p_1$. The combination of the last equation with (30) results in the linear system in $\frac{\partial v^*}{\partial \rho}$ and $\frac{\partial Y^*}{\partial \rho}$ with positive a_i and b_i :

$$\begin{aligned} a_2 \frac{\partial v^*}{\partial \rho} &= -a_1 \frac{\partial Y^*}{\partial \rho}, \\ b_2 \frac{\partial v^*}{\partial \rho} &= b_1 \frac{\partial Y^*}{\partial \rho} + b_0. \end{aligned}$$

The first equation describes a straight line with zero intercept and negative slope. The second straight line goes through the points $\left(\frac{\partial Y^*}{\partial \rho}, \frac{\partial v^*}{\partial \rho}\right) = (0, b_0)$ and $\left(\frac{\partial Y^*}{\partial \rho}, \frac{\partial v^*}{\partial \rho}\right) = (-\frac{b_0}{b_1}, 0)$ with a positive slope. A unique intersection of these lines belongs to the area where $\frac{\partial v^*}{\partial \rho} > 0$ and $\frac{\partial Y^*}{\partial \rho} < 0$.

Monotonicity of v^* and Y^* in n. Denote $z \triangleq \frac{n}{n+1}$, which implies $\frac{n-1}{n+1} = 2z - 1$. As z increases in n, monotonicity of v^* and Y^* in z is equivalent to monotonicity in n. Equation (18) for Y^* can be written as $(Y^*)^2 - Y^* z\left(2 - v^* - \frac{c}{\beta}\right) - (2z - 1)\left(\frac{p_1}{\beta} - 1\right)(1 - v^*) = 0$ with the derivative in z

$$2Y^* \frac{\partial Y^*}{\partial z} - \frac{\partial Y^*}{\partial z} z\left(2 - v^* - \frac{c}{\beta}\right) - Y^*\left(2 - v^* - \frac{c}{\beta}\right) + Y^* z \frac{\partial v^*}{\partial z}$$
$$- 2\left(\frac{p_1}{\beta} - 1\right)(1 - v^*) + (2z - 1)\left(\frac{p_1}{\beta} - 1\right) \frac{\partial v^*}{\partial z} = 0.$$

After collecting the terms with $\frac{\partial v^*}{\partial z}$ and $\frac{\partial Y^*}{\partial z}$, this equation becomes

$$\frac{\partial Y^*}{\partial z}\Big[2Y^* - z\Big(2 - v^* - \frac{c}{\beta}\Big)\Big] + \frac{\partial v^*}{\partial z}\Big[Y^* z + (2z-1)\Big(\frac{p_1}{\beta} - 1\Big)\Big]$$
$$= Y^*\Big(2 - v^* - \frac{c}{\beta}\Big) + 2\Big(\frac{p_1}{\beta} - 1\Big)(1 - v^*). \quad (32)$$

The first square bracket in the LHS is positive by (31). The second square bracket in the LHS is also positive because it is positive for $p_1 \geq \beta$, and, for $p_1 < \beta$, it is bounded from below as follows:

$$Y^* \frac{n}{n+1} + \frac{n-1}{n+1}\Big(\frac{p_1}{\beta} - 1\Big) > \frac{n\big[Y^* - \big(1 - \frac{p_1}{\beta}\big)\big]}{n+1} > \frac{n[Y^* - (1 - p_1)]}{n+1} > 0.$$

The RHS of (32) is positive because it is linear in v^*, positive at $v^* = 1$, and positive at $v^* = p_1$:

$$Y^*\left(2 - p_1 - \frac{c}{\beta}\right) + 2\left(\frac{p_1}{\beta} - 1\right)(1 - p_1)$$
$$\geq (1 - p_1)\left[2 - p_1 - \frac{c}{\beta} + 2\frac{p_1}{\beta} - 2\right] = (1 - p_1)\left[\frac{p_1}{\beta} - p_1 + \frac{p_1 - c}{\beta}\right] > 0.$$

The derivative of (21) in z is

$$\frac{\partial v^*}{\partial z} = \frac{\rho\beta}{1 - \rho\beta}\frac{\partial Y^*}{\partial z}. \quad (33)$$

If $\rho > 0$, then $\frac{\partial v^*}{\partial z}$ and $\frac{\partial Y^*}{\partial z}$ satisfy the following system with positive a_i and b_i :

$$\begin{aligned} a_1\frac{\partial Y^*}{\partial z} &= a_2\frac{\partial v^*}{\partial z}, \\ b_1\frac{\partial Y^*}{\partial z} + b_2\frac{\partial v^*}{\partial z} &= b_0. \end{aligned}$$

The first equation describes a straight line with zero intercept and positive slope. The second straight line goes through the points on the axes $\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = \left(\frac{b_0}{b_1}, 0\right)$ and $\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = \left(0, \frac{b_0}{b2}\right)$ with a negative slope. A unique intersection of these lines belongs to the area where $\frac{\partial v^*}{\partial z} > 0$ and $\frac{\partial Y^*}{\partial z} > 0$.

If $\rho = 0$, (33) becomes $\frac{\partial v^*}{\partial z} = 0$ yielding the solution $\left(\frac{\partial Y^*}{\partial z}, \frac{\partial v^*}{\partial z}\right) = \left(\frac{b_0}{b_1}, 0\right)$.

Monotonicity of nr^* in n. By Theorem 1, nr^* is constant in n for RESE2 and monotonically decreasing for RESE1. By global continuity of nr^*, it remains to show the correspondent monotonicity of nr^* for RESE3.

By the alternative expression (27) for RESE3 profit (Lemma 8),

$$nr^* = \frac{\beta}{1 - \rho\beta}\left\{-(Y^*)^2 + Y^*\left[2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c)\right] + \left(\frac{p_1}{\beta} - 1\right)(1 - p_1)\right\}.$$

Denote $F \triangleq \frac{1-\rho\beta}{\beta} nr^*$. Then

$$\frac{\partial F}{\partial n} = -2Y^*\frac{\partial Y^*}{\partial n} + \frac{\partial Y^*}{\partial n}\left[2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c)\right] = \frac{\partial Y^*}{\partial n}\left[2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^*\right]. \quad (34)$$

As shown above, Y^* is monotonically increasing in n for RESE3. Therefore, nr^* is monotonically decreasing if and only if the square bracket in the last expression is negative. Consider two cases: $p_1 \leq \frac{c}{\beta}$ and $p_1 > \frac{c}{\beta}$.

Suppose $p_1 \leq \frac{c}{\beta}$. By Lemma 4, $Y^* > 1-p_1$ in RESE3, and, therefore, $2-p_1-\frac{c}{\beta}-\rho(p_1-c)-2Y^* < p_1 - \frac{c}{\beta} - \rho(p_1-c) \leq 0$.

For $p_1 > \frac{c}{\beta}$ (p_1-lower bound for RESE3 in a monopoly), by monotonicity of Y^* in n, the RESE3 value of Y^* for any n is bounded from below by the RESE3 total supply in a monopoly: $Y^* \geq 1 - \frac{1}{2}\left(\frac{c}{\beta} + \frac{2p_1-\rho c}{2-\rho\beta}\right)$. Therefore,

$$2 - p_1 - \frac{c}{\beta} - \rho(p_1-c) - 2Y^* \leq -p_1 - \rho(p_1-c) + \frac{2p_1-\rho c}{2-\rho\beta} = \frac{p_1\rho\beta - \rho(p_1-c)(2-\rho\beta) - \rho c}{2-\rho\beta}$$
$$= \frac{\rho(p_1\beta - p_1(2-\rho\beta) + c(1-\rho\beta))}{2-\rho\beta} < \frac{\rho(p_1\beta - p_1(2-\rho\beta) + p_1\beta(1-\rho\beta))}{2-\rho\beta} = \rho p_1(\beta-1) \leq 0.$$

Thus, the square bracket in (34) is always negative and nr^* is monotonically decreasing in n.

Monotonicity of nr^* in ρ. By Theorem 1, nr^* does not depend on ρ for RESE1 and 2 because there is no intertemporal effect in these cases.

By (10) with $y^i = \frac{Y^*}{n}$, total profit is $nr^* = \beta\left[Y^* - (Y^*)^2\right] - cY^* + \beta Y^*(1-v^*) + (p_1-\beta)(1-v^*)$ with the derivative

$$\begin{aligned}\frac{\partial[nr^*]}{\partial\rho} &= \beta(1-2Y^*)\frac{\partial Y^*}{\partial\rho} - c\frac{\partial Y^*}{\partial\rho} + \beta(1-v^*)\frac{\partial Y^*}{\partial\rho} - \beta Y^*\frac{\partial v^*}{\partial\rho} - (p_1-\beta)\frac{\partial v^*}{\partial\rho} \\ &= -\beta\frac{\partial Y^*}{\partial\rho}\left[2Y^* - 2 + \frac{c}{\beta} + v^*\right] - \frac{\partial v^*}{\partial\rho}\left[p_1 - \beta(1-Y^*)\right]. \end{aligned} \tag{35}$$

For $n = 1$, the first square bracket is zero (by Corollary 2), implying $\frac{\partial[nr^*]}{\partial\rho} < 0$ because $p_1 > p_2^*$ and $\frac{\partial v^*}{\partial\rho} > 0$ (by part 2 of this proposition).

For $n > 1$, let $R = [2Y^* - 2 + c/\beta + v^*] / \left[2Y^* - \frac{n}{n+1}(2 - v^* - c/\beta)\right] \in [0,1)$. Then (30) can be written as $-\frac{\partial Y^*}{\partial\rho}[2Y^* - 2 + c/\beta + v^*] = \frac{\partial v^*}{\partial\rho}\frac{R}{n+1}[nY^* + (n-1)(p_1/\beta - 1)]$ when multiplied through by $[2Y^* - 2 + c/\beta + v^*]$. As $nY^* + (n-1)(p_1/\beta - 1) \geq 0$ (because $Y^* \geq 1 - p_1$ by Lemma 4) and $0 \leq R < 1$, we can upper-bound the first term in (35) to obtain

$$\begin{aligned}\frac{\partial[nr^*]}{\partial\rho} &< \beta\frac{\partial v^*}{\partial\rho}\left[\frac{n}{n+1}Y^* + \frac{n-1}{n+1}\left(\frac{p_1}{\beta} - 1\right) - Y^* - \left(\frac{p_1}{\beta} - 1\right)\right] \\ &= \frac{\beta}{n+1}\frac{\partial v^*}{\partial\rho}\left[-Y^* - 2\left(\frac{p_1}{\beta} - 1\right)\right] = \frac{2}{n+1}\frac{\partial v^*}{\partial\rho}\left[\frac{p_2^* + \beta}{2} - p_1\right].\end{aligned}$$

As $p_2^* = \beta(1-Y^*)$ and $Y^* \geq \frac{n}{n+1}(1 - c/\beta)$ in RESE3 by Theorem 1, $\frac{\partial[nr^*]}{\partial\rho} < 0$ holds when $\beta - \frac{n}{2(n+1)}(\beta - c) \leq p_1$ which yields the condition of the proposition.

A.17 Proof of Corollary 6 (σ_2 in ρ)

The second-period surplus $\sigma_2 = \rho(\beta v - p_2^*)$ is monotonically non-decreasing in n because $p_2^* = \beta(1-Y^*)$ is non-increasing in n by part 1 or Proposition 3.

The derivative $\frac{\partial\sigma_2}{\partial\rho} = \beta v - p_2^* - \rho\frac{\partial p_2^*}{\partial\rho}$ linearly increases in v and equals zero at $v^0 = \frac{1}{\beta}\left(p_2^* + \rho\frac{\partial p_2^*}{\partial\rho}\right)$. For RESE1, $v^0 < v^* = 1$ because $p_2^* < \beta p_1 < 1$, by part 1 of Lemma 4, and $\frac{\partial p_2^*}{\partial\rho} = 0$.

Minimum valuation of a second-period buyer is $v_2^{\min} \triangleq p_2^*/\beta = \max\{s/\beta, 1-Y\}$. For this consumer, $\left.\frac{\partial\sigma_2}{\partial\rho}\right|_{v=v_2^{\min}} = -\rho\frac{\partial p_2^*}{\partial\rho}$, which is nonpositive for RESE3. Thus, $v^0 \geq v_2^{\min}$. Similarly, we show that $v^0 < v^*$ because, when $\rho = 0$, we have $\left.\frac{\partial\sigma_2}{\partial\rho}\right|_{v=v^*} = \beta p_1 - p_2^* > 0$, and, when $\rho > 0$, we know, by Proposition 3, that $\frac{\partial v^*}{\partial\rho} > 0$ implying $\left.\frac{\partial\sigma_2}{\partial\rho}\right|_{v=v^*} > 0$.

A.18 Proof of Corollary 7 (Q_2 increases with ρ)

Recall that for a RESE, $Q_2 = Y^* - (1 - v^*)$, yielding $\frac{\partial Q_2}{\partial \rho} = \frac{\partial Y^*}{\partial \rho} + \frac{\partial v^*}{\partial \rho}$, which, using (30), is

$$\begin{aligned}
\frac{\partial Q_2}{\partial \rho} &= \left[- \frac{\frac{n}{n+1} Y^* + \frac{n-1}{n+1}(p_1/\beta - 1)}{2Y^* - \frac{n}{n+1}(2 - v^* - c/\beta)} + 1 \right] \frac{\partial v^*}{\partial \rho} \quad \text{or} \\
\frac{\partial Q_2}{\partial \rho} &= \frac{\frac{n+2}{n+1} Y^* - \frac{n}{n+1}(2 - v^* - c/\beta) - \frac{n-1}{n+1}(p_1/\beta - 1)}{2Y^* - \frac{n}{n+1}(2 - v^* - c/\beta)} \cdot \frac{\partial v^*}{\partial \rho}. \qquad (36)
\end{aligned}$$

Because, by Proposition 3, $\frac{\partial v^*}{\partial \rho} > 0$ and, by (31), the denominator of the fraction in the RHS of (36) is positive, the sign of $\frac{\partial Q_2}{\partial \rho}$ coincides with the sign of the numerator.

For part 1, use $n = 1$ and the corresponding $Y^* = 1 - \frac{1}{2}(v^* + c/\beta)$ (Corollary 2) in the numerator to get $\frac{3}{2} Y^* - \left[1 - \frac{1}{2}(v^* + c/\beta)\right] = \frac{1}{2} Y^* > 0$.

For part 2, $\rho = 0$ implies $v^* = p_1$, and the numerator becomes $Y^* - 1 + p_1 + c/\beta - p_1/\beta$ as $n \to \infty$. In this case, p_1-range for RESE3 is $c < p_1 < 1$, and as $p_1 \to 1$, the total supply, given by the larger root of (23), approaches $Y^* = 1 - c/\beta$ implying that the numerator approaches $1 - 1/\beta < 0$.

A.19 Proof of Proposition 4 (boundary-value gain in RESE3)

For $\beta = 1$, necessary condition (5) of RESE4 becomes $\rho < \frac{1-p_1}{1-s}$ and cannot hold as $\rho \to 1$ because $p_1 > s$. Thus, RESE4 does not exist. RESE2 does not exist because its p_1-range is empty.

Part 1 is immediate by part 3 of Proposition 2, because, as ρ increases from 0 to 1, the switch to RESE1 occurs at $\rho_1 = \frac{n+1}{n} \frac{1-p_1}{1-c} < 1$ if $\frac{1-p_1}{1-c} < \frac{n}{n+1}$ or, equivalently, if $n > \bar{n}$. The resulting equilibrium characteristics are obtained by substituting $\beta = 1$ in the description of RESE1. It is immediate to check that the resulting limit of p_2^* is below p_1.

Part 2. When $n < \bar{n}$, the switch from RESE3 to RESE1 does not occur for any $\rho < 1$. Total supply Y^*, which is given by a larger root of (23), is continuous in ρ near $\rho = 1$. We can write (23) for $\beta = \rho = 1$ as

$$\begin{aligned}
Y^2 - \left((1 - p_1) + \frac{(1 - p_1)(n - 1)}{n} \right) Y + (1 - p_1) \times \frac{(1 - p_1)(n - 1)}{n} \\
= \left(Y - \frac{(1 - p_1)(n - 1)}{n} \right) (Y - (1 - p_1)) = 0,
\end{aligned}$$

resulting in roots $\frac{(1-p_1)(n-1)}{n}$ and $1 - p_1$. Thus, $Y^*|_{\rho \to 1} = 1 - p_1$ and $p_2^*|_{\rho \to 1} = (1 - Y^*)|_{\rho \to 1} = p_1$. The necessary condition $Y^* < 1 - \frac{s}{\beta} = 1 - s$ of RESE3 in Theorem 1 is satisfied for all ρ sufficiently close to 1 because $p_1 > s$.

The limit of v^* is found using

Lemma 9. *In RESE 3 with $\beta = 1$, we have* $\lim_{\rho \to 1} \left. \frac{\partial Y^*}{\partial \rho} \right|_{\beta = 1} = n(c - p_1)$.

By (21) with $\beta = 1$, we have $v^* = \frac{p_1 - \rho(1 - Y^*)}{1 - \rho}$ for all $\rho \in [0, 1)$. Then

$$\begin{aligned}
\lim_{\rho \to 1} v^* &= \frac{p_1 - p_1}{1 - 1} = \frac{0}{0} = \lim_{\rho \to 1} \frac{\partial \left[p_1 - \rho(1 - Y^*) \right] / \partial \rho}{\partial (1 - \rho) / \partial \rho} = - \lim_{\rho \to 1} \left[-(1 - Y^*) + \rho \frac{\partial Y^*}{\partial \rho} \right] \\
&= \lim_{\rho \to 1} \left[1 - Y^* - \rho \frac{\partial Y^*}{\partial \rho} \right] = p_1 + n(p_1 - c) \qquad \text{(from Lemma 9 and } Y^*|_{\rho \to 1} = 1 - p_1).
\end{aligned}$$

Using the limiting values of Y^* and v^* in a strict version of condition (a) for RESE3 existence, we obtain the sufficient existence condition of the form (6). Indeed, by continuity, condition (a) is satisfied for all ρ sufficiently close to one.

Using the expression for r^*, the limit of the total profit is

$$\begin{aligned} nr^*|_{\rho\to 1} &= \lim_{\rho\to 1}[(p_1-c)(1-v^*)+(p_2^*-c)(Y^*-1+v^*)] = \lim_{\rho\to 1}[(p_1-p_2^*)(1-v^*)+(p_2^*-c)Y^*] \\ &= (p_1-p_1)(1-v^*|_{\rho\to 1})+(p_1-c)(1-p_1) = (p_1-c)(1-p_1). \end{aligned}$$

To complete the proof of part 2, consider when $(p_1-c)(1-p_1) \geq \frac{n(1-c)^2}{(n+1)^2}$. With a change of variables $x = \frac{1-p_1}{1-c}$, this relation can be represented as $(1-c)^2(1-x)x \geq \frac{n(1-c)^2}{(n+1)^2}$, or, equivalently, as $(1-x)x \geq \frac{n}{(n+1)^2}$ resulting in $\frac{1}{n+1} \leq x \leq \frac{n}{n+1}$. This range does not intersect with a feasible range of x for part 2 which is given by $\frac{n}{n+1} < x < 1$ (resulting from $n < \bar{n}$). Thus, for part 2, $(p_1-c)(1-p_1) < \frac{n(1-c)^2}{(n+1)^2}$.

Part 3 is immediate because $n = \bar{n}$ implies that the limits in parts 1 and 2 are equal.

Part 4 follows from Corollary 3 for $n = \bar{n} = \frac{1-p_1}{p_1-c}$. In this case, the condition of the corollary becomes $\frac{1}{n}+2 < n$, which, after solving for positive integer n, yields $n \geq 3$. Moreover, when $n = \bar{n} = \frac{1-p_1}{p_1-c}$ and $\rho = 0$, the RESE is of the form 3 because RESE2 is impossible with $\beta = 1$ and the switch to RESE1 occurs only in the limit of $\rho \to 1$. We also have $Y^*|_{\rho=0} > Y^*|_{\rho\to 1} = 1-p_1$ because the total supply is (strictly) decreasing in ρ, and $p_2^*|_{\rho=0} < c$ by Corollary 3. Thus, for $\rho = 0$, the total *first period* profit is $(p_1-c)(1-v^*|_{\rho=0}) = (p_1-c)(1-p_1)$, which is exactly the same as the total profit for $\rho \to 1$, whereas the second-period total profit is $(p_2^*-c)(Y^*-1+v^*|_{\rho=0}) < 0$.

A.20 Analysis of Example 1

Observing that $(p_1-s)(1-p_1) \leq \frac{1}{4}(1-s)^2$ and using relation $1-p_1 = \bar{n}(p_1-c)$, we can strengthen (6) to $\frac{(n-1)(\bar{n}-n)}{n}\frac{p_1-c}{4(c-s)} < 1$. For RESE3, the range of n is $[1,\bar{n})$. As a function of n, fraction $\frac{(n-1)(\bar{n}-n)}{n}$ attains its maximum of $(\sqrt{\bar{n}}-1)^2$ in this range at $n = \sqrt{\bar{n}}$ leading to an even stronger version of the condition, i.e., $(\sqrt{\bar{n}}-1)^2\frac{(p_1-c)}{4(c-s)} = \frac{\left(\sqrt{1-p_1}-\sqrt{p_1-c}\right)^2}{4(c-s)} < 1$. The LHS of this inequality decreases in $p_1 > c$ and it surely holds if it holds at $p_1 = c$, i.e., if $\frac{1-c}{4(c-s)} < 1$ or $c > \frac{1+4s}{5}$. Thus, (6) holds for all p_1 and $1 \leq n < \bar{n}$ if $c > \frac{1+4s}{5}$, e.g., if $s = 0$ and $c > 0.2$.

A.21 Proof of Proposition 5 (monotonicity in RESE4)

Part 1. $v^{*,4} = \frac{p_1-\rho s}{1-\rho\beta}$, which is constant in n and increasing in ρ because $\frac{\partial v^*}{\partial \rho} = \frac{-s(1-\rho\beta)+\beta(p_1-\rho s)}{(1-\rho\beta)^2} = \frac{\beta p_1 - s}{(1-\rho\beta)^2} > 0$.

Parts 2 and 3 follow directly from part 1 and the formulas for Y^* and r^*, given by Theorem 2.

A.22 Proof of Lemma 5 (total surplus)

By the definition of $v^{\min}$, the total consumer surplus in the first period is

$$\begin{aligned} \Sigma_1 &= \int_{v^{\min}}^1 (v-p_1)dv = \left(\frac{v^2}{2}-p_1 v\right)\Big|_{v^{\min}}^1 = \frac{1}{2}-p_1-\frac{\left(v^{\min}\right)^2}{2}+p_1 v^{\min} \\ &= \frac{1}{2}\left[1-\left(v^{\min}\right)^2\right]-p_1(1-v^{\min}) = (1-v^{\min})\left[\frac{1}{2}(1+v^{\min})-p_1\right]. \end{aligned}$$

The total surplus in the second period is

$$\Sigma_2 = \int_{p_2}^{\beta v^{\min}} (\tilde{v} - p_2) \frac{d\tilde{v}}{\beta} = \frac{1}{\beta}\left(\frac{\tilde{v}^2}{2} - p_2\tilde{v}\right)\Big|_{p_2}^{\beta v^{\min}} = \frac{1}{\beta}\left(\beta v^{\min} - p_2\right)\left(\frac{\beta v^{\min} + p_2}{2} - p_2\right) = \frac{\left(\beta v^{\min} - p_2\right)^2}{2\beta}.$$

Hence, $\Sigma^* = \Sigma_1 + \Sigma_2 = (1 - v^{\min})\left[\frac{1}{2}(1 + v^{\min}) - p_1\right] + \frac{\left(\beta v^{\min} - p_2\right)^2}{2\beta}$, with $v^{\min} = v^*$ for a RESE.

A.23 Proof of Proposition 6 (Monotonicity of surplus and welfare)

1. Monotonicity of Σ^* *in* n. By Lemma 5, the derivatives $\frac{\partial \Sigma_1}{\partial n}$ and $\frac{\partial \Sigma_2}{\partial n}$ are

$$\begin{aligned}
\frac{\partial \Sigma_1}{\partial n} &= \frac{1}{2}\frac{\partial v^*}{\partial n}(1 - v^*) - \frac{\partial v^*}{\partial n}\left(\frac{1}{2}(1 + v^*) - p_1\right) = -\frac{\partial v^*}{\partial n}(v^* - p_1); \\
\frac{\partial \Sigma_2}{\partial n} &= \frac{1}{\beta}(\beta v^* - p_2^*)\left(\beta\frac{\partial v^*}{\partial n} - \frac{\partial p_2^*}{\partial n}\right) = \frac{\partial v^*}{\partial n}(\beta v^* - p_2^*) - \frac{\partial p_2^*}{\partial n}\left(v^* - \frac{p_2^*}{\beta}\right).
\end{aligned}$$

For a RESE, $\frac{\partial \Sigma_1}{\partial n} \leq 0$ because $\frac{\partial v^*}{\partial n} \geq 0$ (Proposition 3) and $v^* \geq p_1$; and $\frac{\partial \Sigma_2}{\partial n} \geq 0$ because, by Lemma 4, $\beta v^* \geq \beta p_1 > p_2^*$ and $\frac{\partial p_2^*}{\partial n} \leq 0$ because, by (3), $p_2^* = \max\{s, \beta(1 - Y^*)\}$ and, by Propositions 3 and 5, $\frac{\partial Y^*}{\partial n} \geq 0$.

Using the expressions for $\frac{\partial \Sigma_1}{\partial n}$ and $\frac{\partial \Sigma_2}{\partial n}$, we can write

$$\frac{\partial \Sigma^*}{\partial n} = \frac{\partial v^*}{\partial n}\left[\beta v^* - p_2^* - (v^* - p_1)\right] - \frac{\partial p_2^*}{\partial n}\left(v^* - \frac{p_2^*}{\beta}\right). \tag{37}$$

By the definition of v^*, the surpluses of a consumer with valuation $v = v^*$ are equal in both periods: $\sigma_1|_{v=v^*} = v^* - p_1 = \sigma_2|_{v=v^*} = \rho(\beta v^* - p_2^*)$. Therefore, because $\rho < 1$, the square bracket in (37) is positive. Then, because $\frac{\partial v^*}{\partial n} \geq 0$ and $\frac{\partial p_2^*}{\partial n} \leq 0$, equation (37) yields $\frac{\partial \Sigma^*}{\partial n} \geq 0$ for a RESE. By Theorems 1, 2, and Proposition 3, Σ^* is constant in n for RESE2 and 4 ($\frac{\partial v^*}{\partial n} = \frac{\partial p_2^*}{\partial n} = 0$) and monotonically increasing for RESE1 ($\frac{\partial v^*}{\partial n} = 0, \frac{\partial p_2^*}{\partial n} < 0$) and 3 ($\frac{\partial v^*}{\partial n} > 0, \frac{\partial p_2^*}{\partial n} < 0$).

Monotonicity of Σ^* *in* ρ. By Theorem 1, Σ^* does not depend on ρ for RESE1 and 2 (no intertemporal effects). In general, using the same approach as for $\frac{\partial \Sigma^*}{\partial n}$, we can write for a RESE $\frac{\partial \Sigma_1}{\partial \rho} = -\frac{\partial v^*}{\partial \rho}(v^* - p_1) \leq 0, \quad \frac{\partial \Sigma_2}{\partial \rho} = \frac{\partial v^*}{\partial \rho}(\beta v^* - p_2^*) - \frac{\partial p_2^*}{\partial \rho}\left(v^* - \frac{p_2^*}{\beta}\right)$. Due to the side effect of increasing ρ ($\frac{\partial p_2^*}{\partial \rho} \geq 0$), it is not obvious that $\frac{\partial \Sigma_2}{\partial \rho} \geq 0$. The derivative of total surplus is

$$\frac{\partial \Sigma^*}{\partial \rho} = \frac{\partial v^*}{\partial \rho}\left[\beta v^* - p_2^* - (v^* - p_1)\right] - \frac{\partial p_2^*}{\partial \rho}\left(v^* - \frac{p_2^*}{\beta}\right). \tag{38}$$

For RESE4, we have $\frac{\partial p_2^*}{\partial \rho} = 0$ and, by Proposition 5, $\frac{\partial v^*}{\partial \rho} > 0$ yielding $\frac{\partial \Sigma^*}{\partial \rho} > 0$.

2. Monotonicity of W^*. Recall that *for RESE1*, $v^* = 1$ and $p_2^* = \frac{1}{n+1}(\beta + nc)$, yielding, by Lemma 5, $\Sigma^* = \frac{1}{2\beta}\left(\beta - \frac{1}{n+1}(\beta + nc)\right)^2 = \frac{1}{2\beta}\left(\frac{n}{n+1}(\beta - c)\right)^2$ and

$$W^* = \Sigma^* + nr^* = \left(\frac{n^2}{(n+1)^2} + \frac{2n}{(n+1)^2}\right)\frac{(\beta - c)^2}{2\beta} = \frac{(n+1)^2 - 1}{(n+1)^2}\frac{(\beta - c)^2}{2\beta},$$

which is increasing in n and constant in ρ.

For RESE2, $v^* = p_1, \Sigma_2 = 0$ (no second period), $\Sigma^* = \frac{1}{2}(1-p_1)^2$, and

$$W^* = \Sigma^* + nr^* = \frac{1}{2}(1-p_1)^2 + (p_1 - c)(1-p_1) = \frac{1}{2}(1-p_1)(1+p_1-2c),$$

which is constant in both n and ρ.

For RESE4, $v^* = \frac{p_1 - \rho s}{1-\rho\beta}, p_2^* = s$, yielding Σ^* that is constant in n. Then, $W^* = \Sigma^* + nr^* = \Sigma^* + \frac{p_1 - s}{n}(1-v^*)$ is monotonically decreasing in n.

By Lemma 5, $\frac{\partial \Sigma^*}{\partial \rho}$ can be written as follows:

$$\frac{\partial \Sigma^*}{\partial \rho} = \frac{\partial \Sigma^*}{\partial v^*}\frac{\partial v^*}{\partial \rho} = \frac{\partial}{\partial v^*}\left[\frac{1}{2}(1-(v^*)^2) - p_1(1-v^*) + \frac{(\beta v^* - s)^2}{2\beta}\right]\frac{\partial v^*}{\partial \rho} = \left[-v^* + p_1 + \beta v^* - s\right]\frac{\partial v^*}{\partial \rho}.$$

Then $\frac{\partial W^*}{\partial \rho} = \frac{\partial v^*}{\partial \rho}\left[-v^* + p_1 + \beta v^* - s - \frac{p_1 - s}{n}\right]$. As $\frac{\partial v^{*,4}}{\partial \rho} > 0$, the sign of $\frac{\partial W^*}{\partial \rho}$ coincides with the sign of $[\cdot]$, which can be written as $[\cdot] = v^*(\beta - 1) + \frac{n-1}{n}(p_1 - s)$. Hence, using $v^* = \frac{p_1 - \rho s}{1 - \rho\beta}$, inequality $\frac{\partial W^*}{\partial \rho} \gtreqless 0$ is equivalent to $(p_1 - \rho s)(\beta - 1) + \frac{n-1}{n}(p_1 - s)(1 - \rho\beta) \gtreqless 0$. After collecting the terms with ρ, the latter inequality becomes $\frac{n-1}{n}(p_1 - s) - p_1(1-\beta) \gtreqless \rho\left(\frac{n-1}{n}(p_1 - s)\beta - (1-\beta)s\right)$ or as $p_1\beta - s - \frac{1}{n}(p_1 - s) \gtreqless \left(p_1\beta - s - \frac{\beta}{n}(p_1 - s)\right)\rho$, which, because $p_1\beta > s$, can be written as

$$1 - \frac{1}{n}\frac{p_1 - s}{p_1\beta - s} \gtreqless \left(1 - \frac{\beta}{n}\frac{p_1 - s}{p_1\beta - s}\right)\rho. \tag{39}$$

Consider two cases. If $1 > \frac{1}{n}\frac{p_1 - s}{p_1\beta - s}$, then $1 > \frac{\beta}{n}\frac{p_1 - s}{p_1\beta - s}$ and (39) is equivalent to $\rho^+ \gtreqless \rho$, where ρ^+ is defined in part 2.2. If $1 \le \frac{1}{n}\frac{p_1 - s}{p_1\beta - s}$, then "$>$" in (39) cannot hold for any $\rho \in (0,1)$, but "$\le$" holds for all $\rho \in [0,1)$. Thus, in the latter case, we can define ρ^+ as 0.

A.24 Proof of Corollary 8 (RESE3, non-monotonicity of the total surplus in ρ)

Proof is immediate from the following lemma:

Lemma 10. *If $\beta = 1$, given that RESE is unique,*

1. *for $\rho \to 1$ and $n < \bar{n} = \frac{1-p_1}{p_1 - c}$, $\frac{\partial \Sigma^*}{\partial \rho} = -n^2(p_1 - c)^2 < 0$;*
2. *for $\rho = 0$,*
   - *(2.1) $\frac{\partial \Sigma^*}{\partial \rho} = \frac{1}{8}(p_1 - c) > 0$ for $n = 1$; and*
   - *(2.2) $\frac{\partial \Sigma^*}{\partial \rho} = \frac{Y^* - (1-p_1)}{Y^* + (1-p_1)}Y^*\left[Y^* - (1-c)\right] > 0$ for $n = \infty$, where Y^* is the larger root of the equation*

$$Y^2 - Y(1 - p_1 + 1 - c) + (1-p_1)^2 = 0. \tag{40}$$

Proof: By (38) with $\beta = 1$, the derivative $\frac{\partial \Sigma^*}{\partial \rho}$ is

$$\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = \frac{\partial v^*}{\partial \rho}\left[p_1 - p_2^*\right] - \frac{\partial p_2^*}{\partial \rho}\left(v^* - p_2^*\right). \tag{41}$$

Part 1: By part 2 of Proposition 4, RESE3 is realized if $\rho \to 1$ and $n < \bar{n}$. Then $p_2^* \to p_1$ and, if we show that $\frac{\partial v^*}{\partial \rho} < \infty$, equation (41) will become

$$\left.\frac{\partial \Sigma^*}{\partial \rho}\right|_{\beta=1} = -\frac{\partial p_2^*}{\partial \rho}\left(v^* - p_1\right), \tag{42}$$

requiring the expressions for $\lim_{\rho\to 1} v^*|_{\beta=1}$ and $\lim_{\rho\to 1} \frac{\partial p_2^*}{\partial\rho}\Big|_{\beta=1}$. By (30),

$$\frac{\partial v^*}{\partial \rho} = -\frac{\partial Y^*}{\partial \rho} \cdot \frac{2Y^* - \frac{n}{n+1}\left(2 - v^* - \frac{c}{\beta}\right)}{\frac{1}{n+1}\left[nY^* + (n-1)\left(\frac{p_1}{\beta} - 1\right)\right]}, \tag{43}$$

which, after substituting $\beta = 1$, $\lim_{\rho\to 1} \frac{\partial Y^*}{\partial \rho}\Big|_{\beta=1} = -n(p_1 - c)$ (by Lemma 9), canceling $\frac{1}{n+1}$, and considering the limit as $\rho \to 1$, becomes $\lim_{\rho\to 1} \frac{\partial v^*}{\partial \rho}\Big|_{\beta=1} = n(p_1-c)\cdot \lim_{\rho\to 1} \frac{2Y^*(n+1)-n(2-v^*-c)}{nY^*-(n-1)(1-p_1)}\Big|_{\beta=1}$. Using $\lim_{\rho\to 1} Y^*|_{\beta=1} = 1 - p_1$, we see that the denominator tends to $1 - p_1$. Thus, $\lim_{\rho\to 1} \frac{\partial v^*}{\partial \rho}\Big|_{\beta=1}$ is finite for any $n < \bar{n}$. Using (42) and $\lim_{\rho\to 1} v^* = p_1 + n(p_1 - c)$ (Proposition 4), we get

$$\begin{aligned}
\lim_{\rho\to 1} \frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} &= \lim_{\rho\to 1}\left[-\frac{\partial p_2^*}{\partial \rho}\left(p_1 + n(p_1 - c) - p_1\right)\right] = \lim_{\rho\to 1}\left[-\frac{\partial p_2^*}{\partial \rho} n(p_1 - c)\right] \\
&= n(p_1 - c)\lim_{\rho\to 1} \frac{\partial Y^*}{\partial \rho} = -n^2(p_1 - c)^2.
\end{aligned}$$

Part 2: Equation (41) with $\rho = 0$ (implying $v^* = p_1$) is

$$\frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} = (p_1 - p_2^*)\left[\frac{\partial v^*}{\partial \rho} - \frac{\partial p_2^*}{\partial \rho}\right] = [Y^* - (1-p_1)]\left[\frac{\partial v^*}{\partial \rho} + \frac{\partial Y^*}{\partial \rho}\right]. \tag{44}$$

The derivative of (21) in ρ (with $\beta = 1$) results in

$$\frac{\partial v^*}{\partial \rho} = \frac{1}{(1-\rho)^2}\left[-\left(p_2^* + \rho\frac{\partial p_2^*}{\partial \rho}\right)(1-\rho) + (p_1 - \rho p_2^*)\right] = \frac{1}{(1-\rho)^2}\left[p_1 - p_2^* + \rho(1-\rho)\frac{\partial p_2^*}{\partial \rho}\right],$$

which, for $\rho = 0$, given $p_2^* = 1 - Y^*$, is $\frac{\partial v^*}{\partial \rho}\Big|_{\rho=0} = Y^* - (1 - p_1)$. By (43) with $\beta = 1$ and $\rho = 0$,

$$\frac{\partial Y^*}{\partial \rho} = -\frac{\partial v^*}{\partial \rho} \cdot \frac{nY^* - (n-1)(1-p_1)}{2Y^*(n+1) - n(1 - p_1 + 1 - c)},$$

and (44) becomes

$$\frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} = [Y^* - (1-p_1)]^2\left[1 - \frac{nY^* - (n-1)(1-p_1)}{2Y^*(n+1) - n(1 - p_1 + 1 - c)}\right]. \tag{45}$$

Part 2.1: For $n = 1$, Corollary 2 with $\beta = 1$ and $\rho = 0$ yields $Y^* = 1 - \frac{1}{2}(c + p_1)$ and (45) is

$$\frac{\partial \Sigma^*}{\partial \rho} = \left[\frac{1}{2}(p_1 - c)\right]^2\left[1 - \frac{Y^*}{2Y^* - [2 - (p_1 + c)] + 2Y^*}\right] = \frac{1}{8}(p_1 - c)^2.$$

Part 2.2: After collecting the terms with n and passing to the limit as $n \to \infty$, (45) becomes

$$\lim_{n\to\infty} \frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} = [Y^* - (1-p_1)]^2\left[1 - \frac{Y^* - (1-p_1)}{2Y^* - (1 - p_1 + 1 - c)}\right] = \frac{[Y^* - (1-p_1)]^2\,[Y^* - (1-c)]}{2Y^* - (1 - p_1 + 1 - c)},$$

where (by (23) with $\beta = 1, \rho = 0$, and $n \to \infty$) Y^* is the larger root of (40), which implies that $Y^* - (1 - p_1 + 1 - c) = -\frac{(1-p_1)^2}{Y^*}$ and

$$\lim_{n\to\infty} \frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} = [Y^* - (1-p_1)]^2\,\frac{Y^* - (1-c)}{Y^* - \frac{(1-p_1)^2}{Y^*}}$$

yielding the result of part 2.2. Note also, that by Corollary 3, for $n = \infty$, the second period is never profitable ($Y^* > 1 - \frac{c}{\beta} \Leftrightarrow p_2^* < c$) implying that for $\rho = 0$, $\lim_{n\to\infty} \frac{\partial \Sigma^*}{\partial \rho}\Big|_{\beta=1} > 0$.

A.25 Proof of Corollary 9 (Non-monotonicity of W^* in n)

By the definition of W^*, $\frac{\partial W^*}{\partial n} = \frac{\partial \Sigma^*}{\partial n} + \frac{\partial [nr^*]}{\partial n}$. Then, using (37) for $\frac{\partial \Sigma^*}{\partial n}$, (34) for $\frac{\partial [nr^*]}{\partial n}$, and equalities $\frac{\partial v^*}{\partial n} = \frac{\rho\beta}{1-\rho\beta}\frac{\partial Y^*}{\partial n}$ (by (21)) and $\frac{\partial p_2^*}{\partial n} = -\beta\frac{\partial Y^*}{\partial n}$, we get

$$\frac{\partial W^*}{\partial n} = \frac{\partial Y^*}{\partial n}\frac{\beta}{1-\rho\beta}\left\{\rho\left[\beta v^* - p_2^* - (v^* - p_1)\right] + (\beta v^* - p_2^*)\left(\frac{1}{\beta} - \rho\right) + 2 - p_1 - \frac{c}{\beta} - \rho(p_1 - c) - 2Y^*\right\}.$$

Because, by Proposition 3, $\frac{\partial Y^*}{\partial n} > 0$ for RESE3, the sign of $\frac{\partial W^*}{\partial n}$ coincides with the sign of the curly bracket in the RHS, i.e., $\frac{\partial W^*}{\partial n} \gtreqless 0$ is equivalent to

$$\rho\left[\beta v^* - p_2^* - (v^* - p_1)\right] + (\beta v^* - p_2^*)\left(1/\beta - \rho\right) + 2 - p_1 - c/\beta - \rho(p_1 - c) - 2Y^* \gtreqless 0,$$

which, after substitution of $Y^* = 1 - \frac{p_2^*}{\beta}$, by (3), becomes $\frac{p_2^* - c}{\beta} + \left[v^* - p_1 - \rho(v^* - c)\right] \gtreqless 0$. Then, using $v^* = \frac{p_1 - \rho p_2^*}{1-\rho\beta}$, the inequality for n^W (dependence of p_2^* on n^W is omitted) becomes $p_2^* \gtreqless c + \beta\left\{\frac{p_1 - \rho p_2^*}{1-\rho\beta}(\rho - 1) - \rho c + p_1\right\}$. Collecting the terms with p_2^*, we obtain

$$p_2^*\left[1 - \frac{(1-\rho)\rho\beta}{1-\rho\beta}\right] \gtreqless c(1-\rho\beta) + p_1\beta\left[1 - \frac{1-\rho}{1-\rho\beta}\right] \Leftrightarrow p_2^* \gtreqless c\frac{(1-\rho\beta)^2}{1-2\rho\beta+\rho^2\beta} + p_1\frac{\rho\beta(1-\beta)}{1-2\rho\beta+\rho^2\beta},$$

which yields the main claim (7). The RHS of (7) equals c if $\rho = 0$ or $\beta = 1$. For other values of ρ and β, the comparison of the RHS with c yields $c\frac{(1-\rho\beta)^2}{1-2\rho\beta+\rho^2\beta} + p_1\frac{\rho\beta(1-\beta)}{1-2\rho\beta+\rho^2\beta} > c \Leftrightarrow p_1\rho\beta(1-\beta) > c\left[1 - 2\rho\beta + \rho^2\beta - (1 - 2\rho\beta + \rho^2\beta^2)\right] \Leftrightarrow p_1\rho\beta(1-\beta) > c\rho^2\beta(1-\beta) \Leftrightarrow p_1 > \rho c$, which always holds.

A.26 Proofs of auxiliary statements

Proof of Lemma 7 (properties of the profit) Part 1.1 can be shown by direct substitution of $y^i = 1 - s/\beta - Y^{-i}$ (which is strictly positive by the condition of part 1) into the expressions for $\frac{\partial r^i}{\partial y^i}$ defined by (12) and (15): $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} - 0} =$

$$\begin{aligned}
&= \beta\left(1 - Y^{-i}\right) - c + \beta\left(1 - v^{\min}\right) - 2\beta\left(1 - \frac{s}{\beta} - Y^{-i}\right) + \frac{Y^{-i}(p_1 - \beta)(1 - v^{\min})}{(1 - s/\beta)^2} \\
&= -c - \beta v^{\min} + 2s + Y^{-i}\left(\beta + \frac{(p_1 - \beta)(1 - v^{\min})}{(1 - s/\beta)^2}\right),
\end{aligned}$$

$$\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i = 1 - s/\beta - Y^{-i} + 0} = -c + s + Y^{-i}\frac{(p_1 - s)\left(1 - v^{\min}\right)}{(1 - s/\beta)^2}.$$

These expressions imply that part 1.1 holds if and only if

$$s - \beta v^{\min} + Y^{-i}\left(\beta + \frac{(p_1 - \beta)(1 - v^{\min})}{(1 - s/\beta)^2}\right) < Y^{-i}\frac{(p_1 - s)\left(1 - v^{\min}\right)}{(1 - s/\beta)^2},$$

which is equivalent to

$$s - \beta v^{\min} < Y^{-i}\left[\frac{(\beta - s)\left(1 - v^{\min}\right)}{(1 - s/\beta)^2} - \beta\right] = Y^{-i}\left[\frac{\beta(1 - v^{\min})}{1 - s/\beta} - \beta\right] = \frac{Y^{-i}(s - \beta v^{\min})}{1 - s/\beta},$$

which holds because $s < \beta v^{\min}$ and, by condition of part 1, $Y^{-i} < 1 - s/\beta$.

As r^i is continuous, i.e. $r^i(1-s/\beta-Y^{-i}-0)=r^i(1-s/\beta-Y^{-i}+0)$, and we can show part 1.2 using either (10) or (14). From (14), $r^i(1-s/\beta-Y^{-i})$ is

$$\Big(1-\frac{s}{\beta}-Y^{-i}\Big)\left[s-c+\frac{(p_1-s)\left(1-v^{\min}\right)}{1-s/\beta}\right]=\Big(1-\frac{s}{\beta}-Y^{-i}\Big)(c-s)\left[\frac{(p_1-s)\left(1-v^{\min}\right)}{(1-s/\beta)(c-s)}-1\right],$$

which yields the result of part 1.2.

For part 1.3, rewrite (13) as $\frac{\partial^2 r^i}{\partial (y^i)^2}=-\frac{2}{Y^3}\left[\beta Y^3+(p_1-\beta)(1-v^{\min})Y^{-i}\right]$. As $Y\geq 0$, the RHS of this equation is negative (r^i is strictly concave) if and only if $\beta Y^3+(p_1-\beta)(1-v^{\min})Y>0$. Equality $Y=1-v^{\min}$ holds only at the left boundary of the domain of the profit function. For all other points in the domain $Y>1-v^{\min}\geq 0$ and we have

$$\begin{aligned}\beta Y^3+(p_1-\beta)(1-v^{\min})Y &> \beta(1-v^{\min})^2Y+(p_1-\beta)(1-v^{\min})Y\\ &=[\beta(1-v^{\min})+p_1-\beta](1-v^{\min})Y=[p_1-\beta v^{\min}](1-v^{\min})Y\geq 0\end{aligned}$$

if $p_1\geq \beta v^{\min}$ (a sufficient condition for strict concavity of r^i).

Suppose $p_1<\beta v^{\min}$. Although r^i may be non-concave in this case, $\frac{\partial^2 r^i}{\partial (y^i)^2}=-2\beta\Big[1+(\frac{p_1}{\beta}-1)(1-v^{\min})\frac{Y^{-i}}{Y^3}\Big]$ is monotonically decreasing in y^i. Therefore, if r^i has an inflection point, this point is unique and corresponds to the total supply level $\tilde{Y}$ such that $\tilde{Y}^3=(1-\frac{p_1}{\beta})(1-v^{\min})Y^{-i}$.

Consider an extension $\tilde{r}^i$ of r^i in the form (10) to the domain $y^i\geq(1-v^{\min}-Y^{-i})^+$. In terms of the total supply, this domain is equivalent to $Y\geq\max\{(1-v^{\min}),Y^{-i}\}$. We will prove that $\tilde{r}^i$ is pseudoconcave implying the claim of part 1.3 for the case of $p_1<\beta v^{\min}$.

Equation (10), divided through by y^i, implies that $\tilde{r}^i=0$ if and only if $y^i=0$ or $\beta\,(1-Y)-c+\beta\left(1-v^{\min}\right)+\frac{(p_1-\beta)(1-v^{\min})}{Y}=0$. After multiplying by $-Y/\beta$, this equation becomes

$$Y^2-(2-c/\beta-v^{\min})Y+(1-p_1/\beta)(1-v^{\min})=0. \tag{46}$$

Its properties are explored in the following lemma.

Lemma 11. *For any feasible values of $c,s,v^{\min}$, and $p_1<\beta$, the real roots $Y_{1,2}$ of equation (46) exist and satisfy the conditions: $0\leq Y_1\leq 1-v^{\min}<Y_2\leq 2-\left(c/\beta+v^{\min}\right)$ with $Y_1=1-v^{\min}$ only if $v^{\min}=1$.*

By Lemma 11, the roots $Y_{1,2}$ of (46) always exist and $0\leq Y_1\leq 1-v^{\min}<Y_2$, where $Y_1<1-v^{\min}$ unless $v^{\min}=1$. Using these roots, we can express $\tilde{r}^i$ as the following function of Y: $\tilde{r}^i=-\frac{\beta}{Y}(Y-Y^{-i})(Y-Y_1)(Y-Y_2)$. Moreover, by (46), $Y_1Y_2=(1-p_1/\beta)(1-v^{\min})$, and the inflection point has the form $\tilde{Y}^3=Y_1Y_2Y^{-i}$, i.e., $\tilde{Y}$ is the geometric mean of Y_1, Y_2, and Y^{-i}. Because the second derivative is decreasing, $\tilde{r}^i$ is strictly concave to the right of $\tilde{Y}-Y^{-i}$.

There are three possible locations of Y^{-i} relative to $Y_1<Y_2$. First, if $Y^{-i}\geq Y_2$, then $1-v^{\min}<Y^{-i}$, $\tilde{Y}<Y^{-i}$, and $\tilde{r}^i$ is nonpositive and strictly concave for all $y^i\geq(1-v^{\min}-Y^{-i})^+$. In this case, the claim of part 1.3 holds.

Second, if $Y^{-i}\leq Y_1$, then $Y^{-i}\leq 1-v^{\min}$, $\tilde{Y}<Y_2$, $\tilde{r}^i$ is nonnegative for $(1-v^{\min}-Y^{-i})^+\leq y^i\leq Y_2-Y^{-i}$ and nonpositive for $y^i\geq Y_2-Y^{-i}$. Because $\tilde{r}^i$ is concave for $y^i\geq\tilde{Y}-Y^{-i}$ and changes its sign from positive to negative at $Y_2-Y^{-i}\geq\tilde{Y}-Y^{-i}$, it is also decreasing for all $y^i\geq Y_2-Y^{-i}$. However, when $1-v^{\min}<\tilde{Y}$, $\tilde{r}^i$ is convex in the interval $[1-v^{\min}-Y^{-i},\tilde{Y}-Y^{-i}]$.

Third, if $Y_1<Y^{-i}<Y_2$, it is still true that $\tilde{Y}<Y_2$, $\tilde{r}^i$ is nonnegative for $(1-v^{\min}-Y^{-i})^+\leq y^i\leq Y_2-Y^{-i}$, and nonpositive, decreasing and strictly concave for $y^i\geq Y_2-Y^{-i}$. It is also true that, when $\max\{(1-v^{\min}),Y^{-i}\}<\tilde{Y}$, $\tilde{r}^i$ is convex in the interval $[(1-v^{\min}-Y^{-i})^+,\tilde{Y}-Y^{-i}]$.

We combine the cases two and three by observing that in both of them $\tilde{r}^i$ is nonnegative for $[(1-v^{\min}-Y^{-i})^+, Y_2-Y^{-i}]$ and decreasing as well as concave for $y^i \geq Y_2 - Y^{-i}$. Thus, there is no local minimum for $y^i \geq Y_2 - Y^{-i}$. We complete the proof of part 1.3 using the following lemma.

Lemma 12. *If $\tilde{r}^i$ has an internal (local) minimum $(y^i)_{\min}$, then $\tilde{r}^i((y^i)_{\min}) < 0$.*

Lemma 12 implies that $\tilde{r}^i$ has no local minimum in the interval $((1-v^{\min}-Y^{-i})^+, Y_2-Y^{-i})$. Thus, $\tilde{r}^i$ has no internal minima in its entire domain, is strictly increasing when it is convex and, therefore, is pseudoconcave.

Parts 1.4 and 2 follow directly from (15).

Part 1.5 immediately follows from parts 1.3 and 1.4. Indeed, condition $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}+0} \leq 0$ implies that r^i is decreasing for $y^i \geq 1-\frac{s}{\beta}-Y^{-i}$ (by concavity on this interval). Combining this observation with pseudoconcavity for $y^i \leq 1-\frac{s}{\beta}-Y^{-i}$, we obtain pseudoconcavity for the entire domain. Similarly, $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=1-\frac{s}{\beta}-Y^{-i}-0} \geq 0$ implies that r^i is strictly increasing for $y^i \leq 1-\frac{s}{\beta}-Y^{-i}$, again, leading to pseudoconcavity for the entire domain.

Proof of Lemma 8 The equilibrium profit, using (10) with $y^i = \frac{Y^*}{n}$ and the expression for $v^{\min} = v^*$, is $r^* = \frac{Y^*}{n}\left[\beta\left(1-Y^*\right)-c+\left(\beta+\frac{p_1-\beta}{Y^*}\right)\left(1-\frac{p_1-\rho\beta(1-Y^*)}{1-\rho\beta}\right)\right]$. After factoring out $\frac{\beta}{n(1-\rho\beta)}$ and collecting the terms with different powers of Y^*, we obtain (27).

Proof of Lemma 9 The expression for $\lim_{\rho\to1}\frac{\partial Y^*}{\partial\rho}$ at $\beta=1$ can be found by the implicit differentiation in (23). For brevity, we omit explicit notation indicating $\beta=1$ throughout the proof. Denote $b_1(\rho) \triangleq \frac{(1-c)n(1-\rho)+(1-p_1)n-(p_1-1)\rho(n-1)}{n+1-\rho}$ and differentiate (23) with respect to ρ to obtain $2Y^*\frac{\partial Y^*}{\partial\rho}-\frac{\partial Y^*}{\partial\rho}b_1(\rho)-Y^*\frac{\partial b_1(\rho)}{\partial\rho}+\frac{\partial}{\partial\rho}\left[\frac{(1-p_1)^2(n-1)}{n+1-\rho}\right]=0$ and

$$\frac{\partial Y^*}{\partial\rho}\left[2Y^*-b_1(\rho)\right] = Y^*\frac{\partial b_1(\rho)}{\partial\rho}-\frac{(1-p_1)^2(n-1)}{(n+1-\rho)^2}, \tag{47}$$

The limits are $\lim_{\rho\to1}b_1(\rho) = \frac{(1-p_1)n+(1-p_1)(n-1)}{n} = 2(1-p_1)-\frac{1-p_1}{n}$, resulting in $\lim_{\rho\to1}[2Y^*-b_1(\rho)] = \frac{1-p_1}{n}$, and for $\frac{\partial b_1(\rho)}{\partial\rho} = \frac{[-(1-c)n+(1-p_1)(n-1)](n+1-\rho)+(1-c)n(1-\rho)+(1-p_1)n+(1-p_1)\rho(n-1)}{(n+1-\rho)^2}$, it is $\lim_{\rho\to1}\frac{\partial b_1(\rho)}{\partial\rho} = \frac{1}{n^2}\left\{\left[-(1-c)n+(1-p_1)(n-1)\right]n+(1-p_1)n+(1-p_1)(n-1)\right\}$, where $\{\cdot\} = (1-p_1)(n-1)-(p_1-c)n^2$. The limit of the RHS of (47) is $\frac{1-p_1}{n^2}\left\{(1-p_1)(n-1)-(p_1-c)n^2\right\}-\frac{(1-p_1)^2(n-1)}{n^2} = -(1-p_1)(p_1-c)$. Then, from (47), we obtain the claim of the lemma.

Proof of Lemma 11 (the roots of $r^i(Y)=0$) The discriminant of (46) is $D = (2-c/\beta-v^{\min})^2-4(1-p_1/\beta)(1-v^{\min}) \geq (2-c/\beta-v^{\min})^2-4(1-c/\beta)(1-v^{\min}) = (v^{\min}-c/\beta)^2 \geq 0$, where the first inequality is strict unless $v^{\min}=1$ because $p_1>c$, whereas the second inequality is strict unless $v^{\min}=c/\beta$. Therefore, $D>0$, the real roots given by $Y_{1,2}=\frac{1}{2}(2-c/\beta-v^{\min}\pm\sqrt{D})$ always exist, and $Y_1<Y_2$. As $p_1<\beta$, we have $4\left(1-p_1/\beta\right)\left(1-v^{\min}\right)\geq0$ and $Y_{1,2}\in\left(0,2-c/\beta-v^{\min}\right)$.

If $v^{\min}=1$, the roots are $Y_1=0$, $Y_2=1-c/\beta$, and the claim of the lemma holds.

If $v^{\min}<1$, then $D>\left(v^{\min}-c/\beta\right)^2$, and an upper bound on Y_1 is $Y_1<1-\frac{1}{2}(c/\beta+v^{\min})-\frac{1}{2}\left|v^{\min}-c/\beta\right| = 1-\max\left\{c/\beta,v^{\min}\right\}\leq1-v^{\min}$, which, in turn, is a lower bound on Y_2 : $Y_2 > 1-\frac{1}{2}(c/\beta+v^{\min})+\frac{1}{2}\left|v^{\min}-c/\beta\right| = 1-\min\left\{c/\beta,v^{\min}\right\}\geq1-v^{\min}$.

Proof of Lemma 12 Function $\tilde{r}^i$, its first and second derivatives are given, respectively, by (9), (11) and (13). If an internal local minimum of $\tilde{r}^i$ exists, it must satisfy the necessary second-order optimality conditions

$$\left.\frac{\partial \tilde{r}^i}{\partial y^i}\right|_{y^i=(y^i)_{\min}} = 0, \text{ and} \tag{48}$$

$$\left.\frac{\partial^2 \tilde{r}^i}{\partial \left(y^i\right)^2}\right|_{y^i=(y^i)_{\min}} \geq 0. \tag{49}$$

Using condition (48) and the expression for the first derivative of $\tilde{r}^i$, we obtain

$$\beta\left(1-Y\right)-c+\left[p_1-\beta\left(1-Y\right)\right]\frac{1-v^{\min}}{Y} = -\left(y^i\right)_{\min}\beta\left[-1+\frac{1-v^{\min}}{Y}-\left(\frac{p_1}{\beta}-\left(1-Y\right)\right)\frac{1-v^{\min}}{Y^2}\right]$$
$$= \left(y^i\right)_{\min}\beta\left[1+\left(\frac{p_1}{\beta}-1\right)\frac{1-v^{\min}}{Y^2}\right] \tag{50}$$

Because the LHS of (50) multiplied by y^i matches the expression for $\tilde{r}^i$, it follows that

$$\left.\tilde{r}^i\right|_{y^i=(y^i)_{\min}} = \left(y^i\right)^2_{\min}\beta\left[1+\left(\frac{p_1}{\beta}-1\right)\frac{1-v^{\min}}{Y^2}\right]. \tag{51}$$

Condition (49) and the expression for the second derivative of $\tilde{r}^i$ imply that, at $y^i = \left(y^i\right)_{\min}$, $\left(\frac{p_1}{\beta}-1\right)(1-v^{\min}) \leq -\frac{Y^3}{Y^{-i}}$. Combining this inequality with (51), we obtain $\left.\tilde{r}^i\right|_{y^i=(y^i)_{\min}} \leq \left(y^i\right)^2_{\min}\beta\left[1-\frac{Y}{Y^{-i}}\right] < 0$, which is strict because, here, we consider only $y^i > 0$.

Appendix References

Andreoli-Versbach, P. and J. Franck (2015). "Endogenous price commitment, sticky and leadership pricing: Evidence from the Italian petrol market". In: *International Journal of Industrial Organization* 40, pp. 32–48.

Buehler, S. and D. Gärtner (2013). "Making Sense of Nonbinding Retail-Price Recommendations". In: *American Economic Review* 103(1), pp. 335–359.

Butz, D. (1996). "Does the per se rule deter vertical price-fixing?" In: *Economic Inquiry* 34(4), pp. 770–780.

Griffiths, S. and V. Woollaston (2015). *Is Apple ripping off Britain? Watch is up to £2,500 more expensive in the UK compared to the rest of the world*. Accessed on August 22, 2015. URL: `http://www.dailymail.co.uk/sciencetech/article-3038324/Is-Apple-ripping-Britain-Watch-2-500-expensive-UK-compared-rest-world.html`.

Holland, M., J. Rossouw, and J. Staples (2015). "Focal point pricing: A challenge to the successful implementation of Section 10A (introduced by the Competition Amendment Act)". In: *South African Journal of Economic and Management Sciences* 18(3), pp. 396–409.

Huck, S., G. Lünser, and J. Tyran (2016). "Price competition and reputation in markets for experience goods: An experimental study". In: *The RAND Journal of Economics* 47(1), pp. 99–117.

Karnani, A. (1984). "The Value of Market Share and the Product Life Cycle – A Game-Theoretic Model". In: *Management Science* 30(6), pp. 696–712.

Kouvelis, P. and W. Zhao (2011). “The Newsvendor Problem and Price-Only Contract When Bankruptcy Costs Exist”. In: *Production and Operations Management* 20(6), pp. 921–936.

MacKay, A. and D. Smith (2014). “The Empirical Effects of Minimum Resale Price Maintenance”. In: *Available at SSRN: http://ssrn.com/abstract=2513533*.

Maddah, B., E. Bish, and B. Munroe (2011). “Pricing, variety, and inventory decisions for product lines of substitutable items”. In: *Planning Production and Inventories in the Extended Enterprise*. Ed. by K. Kempf, P. Keskinocak, and R. Uzsoy. Springer, pp. 367–391.

Nippop (2005). *Saihan Seido – Japan’s Resale Price Maintenance System*. Accessed on October 13, 2015. URL: `http://nippop.com/features/Saihan_Seido___Japan_s_Resale_Price_Maintenance_System/`.

Orbach, B. (2008). “Antitrust Vertical Myopia: the Allure of High Prices”. In: *Arizona Law Review* 50, pp. 261–287.

Proctor, A. (2015). “Identifying Geographic or Customer-Based Collusion”. In: *World Competition* 38(2), pp. 253–280.

Schmalensee, R. (1976). “A Model of Promotional Competition in Oligopoly”. In: *The Review of Economic Studies* 43(3), pp. 493–507.

Spiegler, R. (2006). “Competition over agents with boundedly rational expectations”. In: *Theoretical Economics* 1(2), pp. 207–231.

B Supplementary Document: Model Extensions

B.1 First-period demand: general case

This section provides the derivation of the functional form of the first-period demand (2) and examines the robustness of the main results, obtained for $\gamma = 1$, with respect to variations in γ.

Model specification Retailer i demand can be expressed as $d^i(y^i, \mathbf{y}^{-i}) = Dm^i(y^i, \mathbf{y}^{-i})$, where D is the total demand, $m^i(y^i, \mathbf{y}^{-i})$ is the market share of retailer i, and $\mathbf{y}^{-i}$ is the vector of inventories of the others. Because, by the assumptions of §3, attractions $a^i(y^i)$ are identical: $a^i(y^i) = a(y^i), i \in I$, and for a non-trivial problem some of y^i are positive, attraction vector $\mathbf{a}$ of all $a^i, i \in I$ satisfies four conditions required for the market share theorem (Bell, Keeney, and Little (1975)):

(A1) $\mathbf{a}$ is nonnegative and nonzero: $a^j \geq 0, j \in I$, and there exists $a^i > 0$;

(A2) zero attraction leads to zero market share;

(A3) any two retailers with equal attraction have equal market share: $a^i(y^i) = a^j(y^j) \Rightarrow m^i(y^i, \mathbf{y}^{-i}) = m^j(y^j, \mathbf{y}^{-j})$; and

(A4) the market share m^i of any retailer decreases on the same amount Δ^i if the attraction a^j of *any* other retailer j is increased by a fixed amount (Δ^i does not depend on $j \neq i$).

The last assumption holds only if market shares are continuous in attractions, i.e., it does not hold for Bertrand-like competition. This property reflects, e.g., that real consumers have different implicit preferences (loyalties) to different firms due, e.g., to geographical proximity, store decor styles, etc. For studying the effects of symmetric firms we must assume that these preferences are uniformly distributed among consumers. Bell, Keeney, and Little (1975) note also that (A4) do not hold "if adding an increment to a small attraction produces a different effect (on others) from adding the same amount to a large attraction" (nonlinearity) or "if changes in attraction of one seller were differentially effective on the customers of another" (asymmetry).

If (A1)-(A4) hold, then, by the market share theorem applied to this symmetric case, m^i has the following functional form:

$$m^i(y^i, \mathbf{y}^{-i}) = \frac{a(y^i)}{\sum_{j \in I} a(y^j)}. \tag{52}$$

Using (52), the homogeneity of m^i (follows, by Assumption 1, from the homogeneity of d^i and D), and the continuity of $a(y)$ (Assumption 3), Lemma 2 specifies the functional form of attraction: $a(y) = a(1)(y)^\gamma$. By choosing the scale of attraction so that $a(1) = 1$, we obtain functional form (2) for demand d^i.

A feasible range for γ results from the observation that retailer i can choose y^i either not to enter the market: $y^i = 0 = a(0) = d^i(0, \mathbf{y}^{-i})$, to sell only in the first period: $y^i = \breve{y}^i \triangleq d^i(\breve{y}^i, \mathbf{y}^{-i})$, or in both: $y^i > d^i(y^i, \mathbf{y}^{-i}) \geq \breve{y}^i$ (the last inequality is strict when d^i is strictly increasing in y^i). These properties hold if d^i is concave in y^i. In extreme cases, d^i, as a function of y^i, can be a straight line ($\gamma = 1$) with a slope less than one if $y^i \geq \breve{y}^i$ or, as an opposite case, a constant if all y^i are positive and any changes in y^i are not supported by the correspondent changes in market efforts or consumers completely ignore these efforts ($\gamma = 0$).

In this model, γ is the *inventory elasticity of attraction:* $E_y(a) \triangleq \frac{\partial a}{\partial y}\frac{y}{a} = \gamma(y)^{\gamma-1}\frac{y}{(y)^\gamma} = \gamma$, or the *inventory elasticity of the first-period demand*, normalized by the market share of other retailers:

$$E_{y^i}(d^i) \triangleq \frac{\partial d^i}{\partial y^i}\frac{y^i}{d^i} = D\left[\frac{\gamma\left(y^i\right)^{\gamma-1}}{\sum_{j\in I}\left(y^j\right)^\gamma} - \frac{\left(y^i\right)^\gamma \gamma \left(y^i\right)^{\gamma-1}}{\left(\sum_{j\in I}\left(y^j\right)^\gamma\right)^2}\right]\frac{y^i \sum_{j\in I}\left(y^j\right)^\gamma}{\left(y^i\right)^\gamma D} = \gamma\left[1 - m^i\right] = \gamma\frac{\sum_{j\neq i}\left(y^j\right)^\gamma}{\sum_{j\in I}\left(y^j\right)^\gamma},$$

where $\sum_{j\neq i}\left(y^j\right)^\gamma / \sum_{j\in I}\left(y^j\right)^\gamma$ is the market share of other retailers.

The following results use some supplementary material, provided in §C.

Changes in RESE structure with γ This section shows the effect of changing γ on the main results of this paper. For $\gamma = 1$, the structure of RESE coincides with the one described in Theorems 1 and 2. This structure continuously changes with γ by continuity of demand (2). In particular, changes in γ lead to the following effects.

I. *RESE1 does not depend on* γ because this RESE, by the same argument as in the proof of Theorem 1, exists only when the first-period demand is zero ($v^{\min} = 1$) due to a combination of relatively high p_1, the difference $\beta - c$, the level of competition n, and the consumer's discount factor ρ; namely, when $p_1 \geq P_1 = 1 - \frac{n}{n+1}\rho(\beta - c)$.

II. *The area of RESE2 is decreasing in* γ, which follows from a necessary condition of existence of RESE2 that requires the profit of a deviator from $Y^{*,2} = 1 - p_1$ be not increasing in y^i : $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\frac{1-p_1}{n}+0} \leq 0$. This inequality (§C.1) is equivalent to $p_1 \leq \frac{nc}{\gamma(n-1)+\beta[n(1-\gamma)+\gamma]} \triangleq P_2(\gamma)$. This bound decreases in γ from $P_2(0) = \frac{c}{\beta}$ to $P_2(1) = \frac{nc}{n-1+\beta}$ coinciding with P_2 given in Theorem 1.

III. *The area of RESE3 is (a) increasing in γ along the boundary with RESE2 and (b) decreasing along the boundary with RESE4.* Part (a) follows from the p_1-range for RESE3: $P_2(\gamma) < p_1 < P_1$, which results from the same geometric argument as in the proof of Theorem 1 because $Y^{*,3}(\gamma)$ is still a larger root of a quadratic equation with coefficients depending on γ (equation (59)). Part (b), for $\gamma = 0$, follows from the lack of incentive for the retailers to deviate to salvage, which is expressed in $\left.\frac{\partial r^i}{\partial y^i}\right|_{Y\geq 1-\frac{s}{\beta}} = s - c < 0$ (§C.1), i.e., a sufficient condition, corresponding to condition (a) in part RESE3 of Theorem 1 always holds. The intuition is that, for $\gamma = 0$, retailers share evenly the first-period demand regardless of the inventories. Therefore, any increase in inventory does not increase the first-period market share, and possible second-period sales below cost only reduce total two-period profit. For $0 < \gamma < 1$, part (b) is checked numerically and illustrated in Figure 13 for $\gamma \in \{0, 0.4, 1\}$.

IV. *The area of RESE4 is increasing in* γ. §C.1 provides a unique

$$Y^{*,4}(\gamma) = \frac{n-1}{n}\frac{p_1 - s}{c - s}\gamma(1 - v^{*,4}), \tag{53}$$

where $v^{*,4} = \frac{p_1-\rho s}{1-\rho\beta}$. This expression for $Y^{*,4}(\gamma)$ implies a sufficient condition of RESE4 existence, namely, $\frac{n-1}{n}Y^{*,4}(\gamma) \geq 1 - \frac{s}{\beta}$ (salvaging is forced on retailers), which is

$$\gamma \geq \bar{\gamma} \triangleq \left(1 - \frac{s}{\beta}\right)\left(\frac{n}{n-1}\right)^2 \frac{c-s}{p_1 - s}\frac{1-\rho\beta}{1 - p_1 - \rho(\beta - s)},$$

where $\bar{\gamma}$ can be sufficiently small for any feasible p_1, ρ, β, and s if c is sufficiently close to s, i.e., RESE4 can exist for small γ but does not exist for $\gamma = 0$ (Figure 13). On the other hand, inequality $Y^{*,4} < 1 - \frac{s}{\beta}$, combined with (53), gives a sufficient condition of RESE4 non-existence. As $Y^{*,4}$ is increasing in n and decreasing in ρ ($v^{*,4}$ is increasing in ρ), RESE4 does not exist for given γ and any n and ρ if $Y^{*,4} < 1 - \frac{s}{\beta}$ for $\rho = 0$ and $n \to \infty$, which is $\frac{p_1-s}{c-s}\gamma(1 - p_1) < 1 - \frac{s}{\beta}$ or $\gamma < \underline{\gamma} \triangleq \frac{(1-s/\beta)(c-s)}{(p_1-s)(1-p_1)}$.

The scatterplots in Figure 13 were constructed by checking p_1-boundaries for RESE1 and 2, and, for RESE3 and 4, using the direct comparison of equilibrium profits with the profit of a potential deviator, according to the definition of RESE.

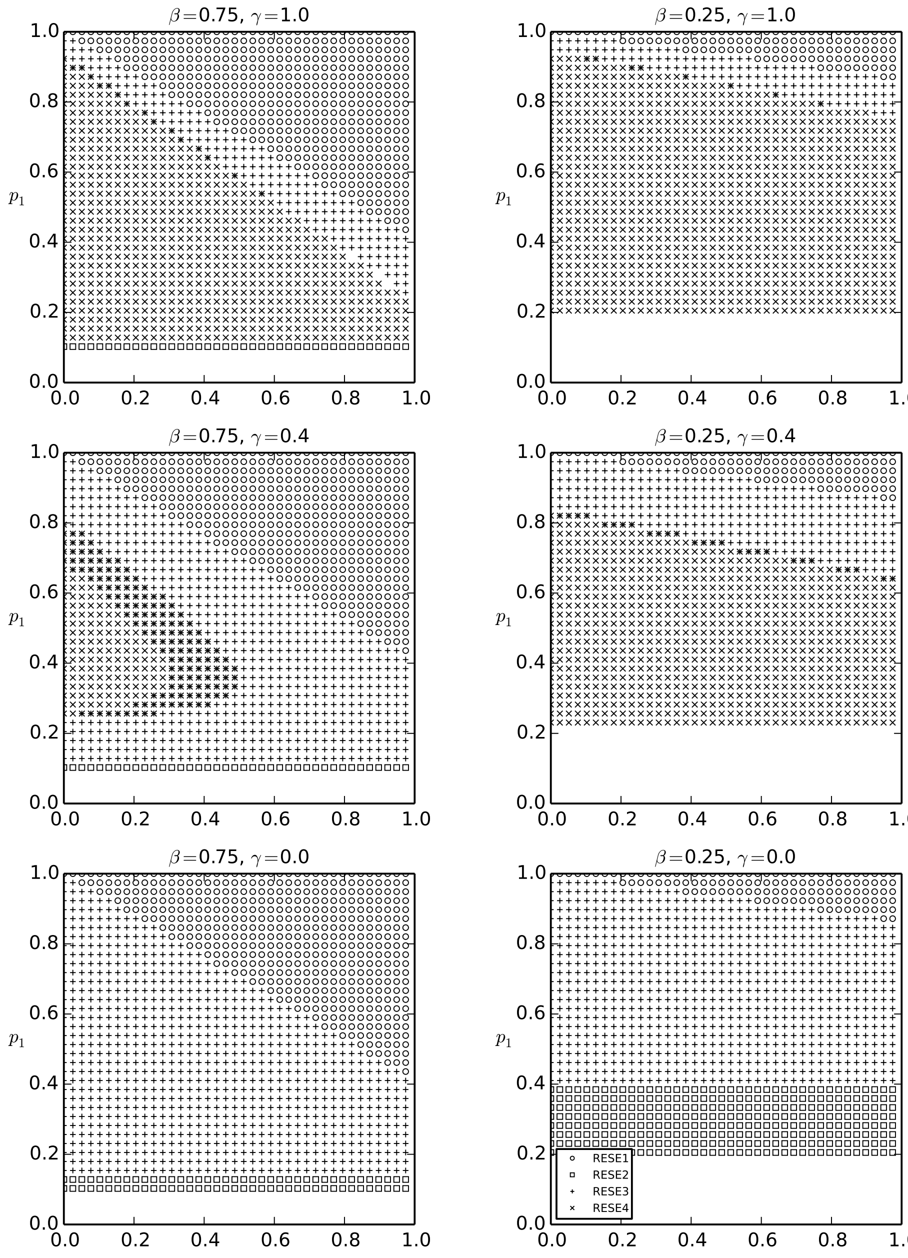


Figure 13: The (ρ, p_1)-scatterplots of the areas where a particular RESE exists for $n = 10, c = 0.1, s = 0.05$, and given γ and β

Because the first-period demand (2) is continuous and monotonic in γ, the case $\gamma = 0$ for RESE3 is of a special interest as opposing to $\gamma = 1$. Although, a complete independence of market share from inventory may be an idealization for many practical settings, this case illustrates the robustness of the results of this study and shows the direction and amplitude of the changes with respect to variations in the demand patterns. This assumption about first-period market share was used, e.g., in Liu and Ryzin (2008), §4.4.

Proposition 7. *For $\gamma = 0$, a unique RESE 3 with $v^* = \frac{p_1+n(p_1-\rho c)}{1+n(1-\rho\beta)}, p_2^* = c + \frac{\beta p_1 - c}{1+n(1-\rho\beta)}, Y^* = \frac{1-p_1+n\left(1-\frac{c}{\beta}\right)(1-\rho\beta)}{1+n(1-\rho\beta)}$, and $r^* = \frac{1}{n}[(p_1-c)(1-v^*) + (p_2^*-c)(Y^*-1+v^*)]$ exists if and only if $\frac{c}{\beta} < p_1 < P_1$; no other equilibria exist in this area. Moreover,*

1. *$p_2^* \to c+0$ with $n \to \infty$ for any $p_1 \in \left(\frac{c}{\beta}, P_1\right)$ or with $p_1 \to \frac{c}{\beta}+0$ for any $n \geq 1$;*
2. *v^*, p_2^*, Y^*, and r^* are continuous at the boundaries; monotonicity of v^*, Y^* in n and ρ, and nr^* in n, stated in Proposition 3 for $\gamma = 1$ hold;*
3. *nr^* is decreasing in ρ if and only if either $n = 1$ or $p_1 \geq c + \frac{2n(\beta-c)}{(n+1)^2}$ for any $n > 1$;*
4. *nr^* attains minimum in ρ at $\rho^0 \triangleq \frac{(n+1)^2(p_1-c)-2n(\beta p_1-c)}{\beta n(n+1)(p_1-c)}$ for any $n \in (1, n^0)$, where $n^0 \triangleq \frac{\beta-p_1+\sqrt{(\beta-c)(\beta+c-2p_1)}}{p_1-c} > 2$, if and only if $p_1 < \frac{5c+4\beta}{9}$;*
5. *when $\beta = 1$, $nr^*\big|_{\rho\to 1} < nr^*\big|_{\rho=0}$ for any RESE 3 inputs; the minimum possible value of $\rho^0 = \frac{n^2+1}{n(n+1)}$ is $\bar{\rho}^0 = \rho^0\big|_{n=2} = \rho^0\big|_{n=3} = \frac{5}{6}$.*

Proposition 7 shows that, for $\gamma = 0$,

(i) RESE exists for all feasible model inputs because RESE3 boundaries ($\frac{c}{\beta} < p_1 < P_1$) complement the boundaries of RESE1 and 2;

(ii) the second-period price is always above the cost for $n < \infty$;

(iii) a closed-form necessary and sufficient condition shows when nr^* is decreasing in ρ;

(iv) there exist closed-form expressions for ρ^0, the unique minimum of nr^* in ρ (part 4), and for n^0, the upper boundary of n-range where nr^* is non-monotonic in ρ;

(v) there is no effect of "boundary-value gain" (part 5); this result supports the conclusion, formulated in the discussion of Proposition 4, that under this effect, the maximum consumer's discount factor prevents the second-period sales at loss under competitive pressure ($n \geq 3$). As shown in part 1, the second-period sales are always profitable for $\gamma = 0$ because retailers have no incentive to compete for the first-period market by increasing inventories.

Thus, when the inventory elasticity of attraction γ decreases, the "boundary-value gain" in ρ becomes weaker (Figures 4 and 14 a) and disappears at $\gamma = 0$ (Figure 14 b); the "discontinuous gain" in ρ caused by the switch from RESE4 to RESE3 emerges at lower ρ (Figures 5 b and 13) and disappears at $\gamma = 0$ due to non-existence of RESE4; the "continuous gain" in ρ (Figure 4) exists even for $\gamma = 0$. The last effect becomes less pronounced because decreasing γ weakens the first-period inventory competition and decreases the correspondent second-period losses. The point of minimum profit, ρ^0, decreases in γ (Figures 4 and 14).

B.2 Some incentives for the same price across retailers

Market-share competing retailers may have incentives to deviate from MSRP but, according to Federal Trade Commission (`www.ftc.gov`, accessed 5 August, 2015), "[a] manufacturer ... may

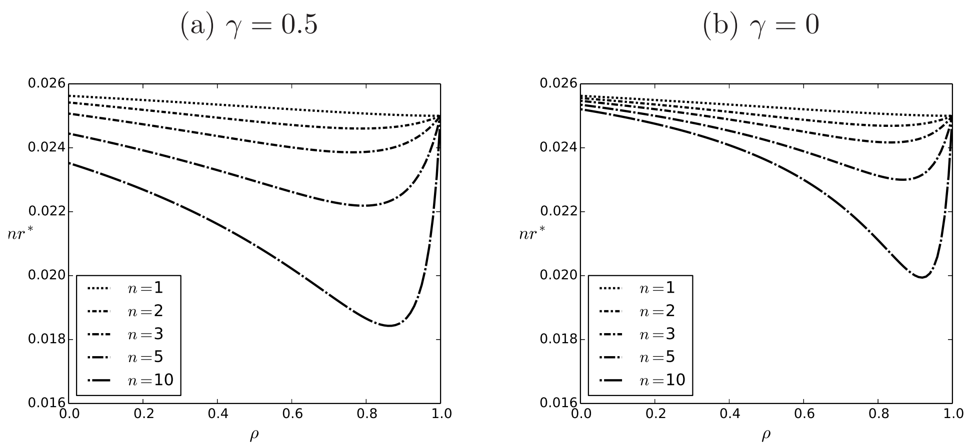


Figure 14: The total profit for RESE 3 with $c = 0.45$, $s = 0.05$, $p_1 = 0.5$, $\beta = 1$

stop dealing with a retailer that does not follow its resale price policy." In some states and countries vertical price fixing is illegal and manufacturers can indirectly influence the price by offering to the retailers who follow MSRP various cost-reducing benefits (e.g., Butz (1996), Buehler and Gärtner (2013)). Proposition below provides an upper bound on the profit of a deviating retailer assuming that this retailer obtains the entire first-period demand by setting the first-period price below MSRP. This retailer has an additional fixed cost K and its unit cost is $c^H > c$.

Proposition 8. *The profit r^i of retailer i with the first-period price below p_1 is upper-bounded by $UB^i = -K + \frac{1}{4}(1-c^H)^2$ if $\frac{\beta(1-2Y^{-i})}{2-\beta} < c^H \leq 2p_1 - 1$ or $UB^i = -K + (p_1 - c^H)(1-p_1)$ if $c^H > \max\{2p_1 - 1, \beta(p_1 - Y^{-i})\}$.*

The proof shows, in particular, that when the c^H-range in the first case is not empty, the c^H-range in the second case is $c^H > 2p_1 - 1$. The following corollary provides conditions on K and c^H guaranteeing that retailers do not deviate from MSRP under RESE1 where p_1 is the greatest among RESE and the incentive for retailers to decrease the first-period price is quite strong.

Corollary 10. *Under RESE1, the first-period price below MSRP is unprofitable for a retailer if $\frac{(3-n)\beta+2(n-1)c}{(2-\beta)(n+1)} < c^H \leq 2p_1 - 1$, and $K \geq \frac{1}{4}(1-c^H)^2 - \frac{(\beta-c)^2}{(n+1)^2\beta}$ or $c^H > \max\{2p_1 - 1, \beta p_1 - \frac{n-1}{n+1}(\beta - c)\}$ and $K \geq (p_1 - c^H)(1-p_1) - \frac{(\beta-c)^2}{(n+1)^2\beta}$.*

For example, if $n = 2, \beta = 1$, and $\frac{1+2c}{3} < c^H \leq 2p_1 - 1$, where $p_1 \geq 1 - \frac{2}{3}\rho(1-c)$ (RESE1 exists) retailers have no incentive to decrease price below p_1 because, by Corollary 10, the sufficient "no-deviation" condition becomes $K \geq \frac{1}{4}(1-c^H)^2 - \frac{(1-c)^2}{9} = \left[\frac{1-c^H}{2} - \frac{1-c}{3}\right]\left[\frac{1-c^H}{2} + \frac{1-c}{3}\right]$, which holds for any $K \geq 0$ because $\frac{1-c^H}{2} - \frac{1-c}{3} = \frac{1}{2}\left[1 - c^H - \frac{2(1-c)}{3}\right] < 0$ for $c^H > \frac{1+2c}{3}$.

B.3 Equilibrium inventory and p_1

As mentioned in the introduction, this study primarily focuses on exogenous p_1, e.g., when p_1 is specified by the manufacturer-retailer agreement (Orbach (2008)). Manufacturers often operate in multiple markets with notably different valuations for the same product, but MSRP may have to be comparable when converted to local currencies for strategic reasons (e.g., maintaining brand

(a) RESE3 (b) RESE4

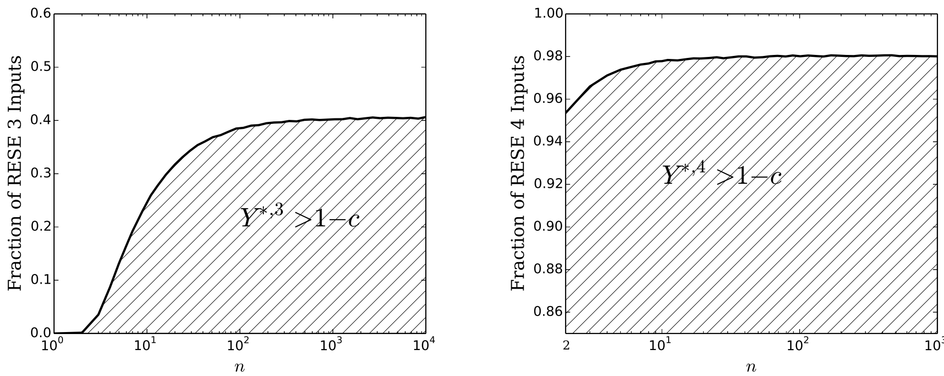


Figure 15: Fractions of RESE3 and 4 inputs resulting in $Y^* > 1 - c$ for given n

image). In this case, the ratio of MSRP to the highest valuation on a specific market can take any value from the range $(c, 1]$ and lead to any type of RESE considered above.

However, a product may target only one specific market, or valuations on several markets might almost be the same. In this case, the manufacturer can try to negotiate p_1 to improve its profit. When all other parameters are constant, the manufacturer profit in the local market is directly proportional to the total sales Y^* in this market. Thus, we consider p_1 maximizing the total equilibrium retailer inventory Y^*.

The simplest "benchmark" case is RESE2 where p_1 is relatively low and $Y^* = 1 - p_1$. The supremum of the manufacturer's sales in RESE2 is obtained as p_1 tends to c. In practice, this supremum cannot be achieved because retailer profits must be positive and consumer valuations are bounded from above. Therefore, the difference between MSRP and the unit cost, normalized by the highest valuation, is separated from zero. The following results show that, depending on the product (β) and the market situation (n, ρ, c, s), the values of p_1 leading either to RESE3 or 4 can be more profitable for the manufacturer than $p_1 \to c$ (i.e., ceteris paribus, improve the manufacturer sales beyond $1 - c$.)

Proposition 9. *When the corresponding RESE exists, (1) $Y^{*,1} < 1 - c$; (2) the unique maximum of $Y^{*,4}$ in p_1 is $\bar{Y}^{*,4} = \frac{(n-1)(\bar{p}_1 - s)^2}{n(1-\rho\beta)(c-s)}$ at $p_1 = \bar{p}_1 \triangleq \frac{1}{2}(P_4 + s)$; $Y^{*,4} \geq 1 - c$ if and only if $c - s \leq \frac{n-1}{n} \frac{p_1 - s}{1-c} \frac{1 - p_1 - \rho(\beta - s)}{1 - \rho\beta}$; (3) $Y^{*,3} < 1 - c$ for $n = 1$; for $n \to \infty$ and $p_1 \to P_2 = c, Y^{*,3} \to 1 - c$ and, if $\rho = 0$, $\left.\frac{\partial Y^{*,3}}{\partial p_1}\right|_{p_1 = P_2 + 0} > 0$.*

The proposition implies that RESE2 is the best for the manufacturer in a market with a single retailer and $\beta < 1$ (RESE2 must exist). However, using simulation we find that for most feasible combinations of ρ, β, c, s and $n > 1$ (by volume in the space of all feasible combinations of these parameters) the manufacturer, who varies p_1 while other parameters are fixed, would prefer a value that achieves RESE4, which is the worst for the retailers. Consistently with Proposition 3, Figure 15 shows that the fractions of RESE3 and 4 instances, where $Y^{*,3}$ and $Y^{*,4}$ are greater than $1 - c$, are increasing in n. For RESE3, this fraction is zero at $n = 1$ and remains below 40% for $n > 1$, whereas $Y^{*,4} > 1 - c$ for at least 95% instances of RESE4 (recall that $Y^{*,4} > 1 - s/\beta$ and $Y^{*,4}$ can be less than $1 - c$ only when $c < s/\beta$). Therefore, the manufacturer may prefer markets with many retailers, where the ratio of MSRP to the highest valuation takes intermediate values

(a) $n = 3, \beta = 0.6$ (b) $n = 5, \beta = 0.8$

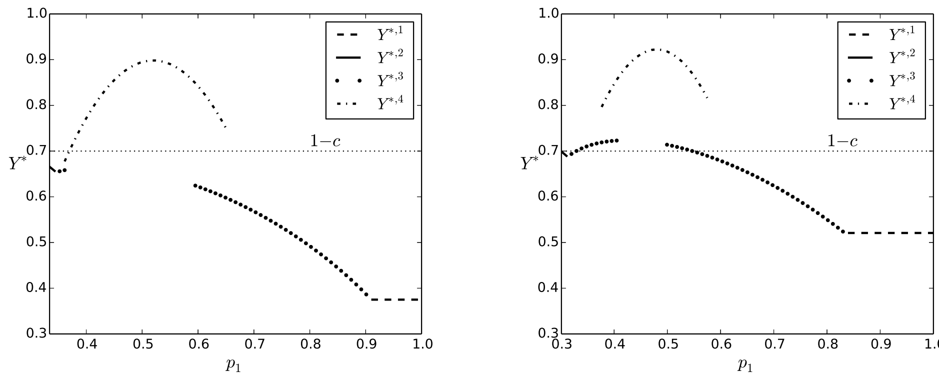


Figure 16: Y^* in p_1 for $\rho = 0.4, c = 0.3, s = 0.2$, and

and RESE4 can be realized. For Figure 15, we used the same simulation approach as for Figure 2. Typical qualitative behavior of Y^* in p_1 is illustrated in Figure 16.

Maximization of manufacturer sales with respect to p_1 does not invalidate the claim of Proposition 4 about the possibility of profit gains when consumers shift from myopic to fully strategic behavior. In practice, there is a minimum first-period price $p_1^{\min} > c$ that separates retailer margins from zero. By Proposition 4, $Y^{*,3}|_{\rho\to 1} = 1 - p_1$ so the maximum of $Y^{*,3}|_{\rho\to 1}$ is attained at $p_1^{\min}$. For any $p_1^{\min} = p_1 \in \left(c, \frac{1+3c}{4}\right]$, the value $\bar{n}$ from Proposition 4 is greater or equal to 3. Thus, part 4 still holds for $n = \bar{n}$ implying the boundary value gain $nr^{*,3}\big|_{\rho\to 1} > nr^{*,3}\big|_{\rho\to 0}$. This profit gain disappears only with $p_1^{\min} = c$ leading to $nr^{*,3}\big|_{\rho\to 1} = nr^{*,3}\big|_{\rho\to 0} = 0$. However, the case $p_1^{\min} = c$ is infeasible and implausible in this problem.

This subsection illustrates a non-trivial nature of manufacturer-retailer interactions under oligopoly with strategic consumers. The properties of possible outcomes described in the above sections can be used to study these interactions in a two-tier supply chain framework. Such analysis includes a distinct set of research questions, e.g., the comparison of supply chain efficiency under centralized and decentralized settings with various types of contracts (see Su and Zhang (2008) for monopoly), and deserves a separate consideration.

B.4 Retailer's discount

Lazear (1986) (p. 25) showed that the discounted second-period profit leads to decreasing prices, which typically corresponds to increasing sales. Our setting leads to a similar result in terms of inventory. The proposition below shows that when retailers solve a non-degenerate two-period profit-maximization problem, the equilibrium inventory increases if a discount factor becomes less than one. We call a two-period problem degenerate if it reduces to one period, which happens for RESE1, 2, and for a monopolist in RESE3 because, for $n = 1$, the first-period demand does not depend on inventory.

Proposition 10. *If retailer i's profit is $r^i = (p_1 - c)q^i + \lambda(p_2 - c)(y^i - q^i), \lambda \in (0, 1]$, equilibrium total inventory Y^* decreases in λ for RESE4, RESE3 with $n > 1$ and constant for RESE1, 2, and 3 with $n = 1$. If $\lambda = (1 + \delta)^{-1}$, where δ is the interest rate between two periods, the relative increase in $Y^{*,4}$ from introducing $\lambda < 1$ is $\left(Y_\lambda^{*,4} - Y^{*,4}\right)/Y^{*,4} = \frac{p_1 - c}{p_1 - s}\delta < \delta$.*

For example, if $\lambda = 1, p_1 = 0.5, n = 10, \beta = 0.75, c = 0.1, s = 0.05$, and $\rho = 0$, then by condition (a) of Theorem 2, RESE4 is realized with $Y^{*,4} = 4.05$. If, for the same data, retailers consider a 2% interest rate between periods, $Y_\lambda^{*,4} = 4.122$, which is around 1.8% greater than $Y^{*,4}$. For the same data, but $\rho = 0.7$, RESE3 is realized by condition (a) of Theorem 1 with $Y^{*,3} = 0.85276$. The same 2% interest rate yields $Y_\lambda^{*,3} = 0.85346$, which is only about 0.08% greater than $Y^{*,3}$.

B.5 RESE stability

An equilibrium is more likely to emerge in practice if it is (a) asymptotically locally stable, i.e., when the initial retailers' inventories are close to an equilibrium, they converge to the equilibrium values, or (b) globally stable, i.e., when any initial inventories converge to an equilibrium when it is unique. In our setting, by Theorem 1 and Proposition 1, RESE is unique for any inputs except for a small fraction where both RESE3 and 4 may exist (Figure 2). In the latter case, however, the feasible inventory ranges for RESE3 and 4 are separated by a non-empty interval (Figure 16).

RESE1, 3, and 4, for $n \geq 2$, represent a non-degenerate game between retailers that can be reformulated as a one-period game with retailer i's payoff function $\pi^i(y^i, Y^{-i}) = y^i P(y^i, Y^{-i}) - C_i(y^i)$. Then using, e.g., Theorem 3 in Nowaihi and Levine (1985), the following result holds.

Proposition 11. *For any inputs where RESE1, 3, or 4 exist in an open neighborhood of Y^* , a RESE is locally asymptotically stable.*

As to global stability, Theocharis (1960) showed that for a linear demand and constant per unit cost, the best-response discrete adjustment process $y_{t+1}^i = BR^i(Y_t^{-i}), t = 0, 1, \ldots, i \in I$, converges for $n = 2$ and any y_0^1, y_0^2. This process means that each retailer observes rivals' inventories at some time t and makes a payoff-maximizing inventory decision for $t+1$. Further studies refined this result for slower adjustment processes $y_t^i = y_{t-1}^i + k_i \left[BR^i(Y_{t-1}^{-i}) - y_{t-1}^i\right]$ or $y_t^i = y_{t-1}^i + k_i \partial\pi^i/\partial y^i$ where $k_i \in (0, 1]$ is the speed of adjustment. In particular, according to Fisher (1961), "given the number of sellers, it is always possible to find [slow enough] speeds of adjustment such that the system is stable."

B.6 Different costs

In this section, we relax the assumption of identical retailers for the case of duopoly. In particular, we examine the impact of the difference in retailer costs on the gains in retailer profits (both continuous and discontinuous), which may happen when consumers are becoming more strategic. We also provide two additional qualitative effects of strategic consumers on competing retailers. Namely, high consumer's discount factors in combination with a high first-period price can push a high-cost retailer out of the market. At the same time, equilibrium inventory of the low-cost retailer may increase in ρ.

The latter two effects can be easily illustrated in the equilibrium with second-period sales only. Similar to the symmetric case, this equilibrium (denote it as REE1) exists in a non-trivial form only when consumers are strategic, i.e., when $\rho \geq \rho_1 = \frac{3}{2}\frac{1-p_1}{\beta-\bar{c}}$, where $\bar{c} \triangleq \frac{1}{2}(c^L + c^H)$ is the average cost and indices L and H denote a low-cost and a high-cost retailers respectively (the effects of different costs on the conditions of equilibria existence are shown in §C.8). The high-cost equilibrium inventory is $y^{H,*} = \frac{1}{3}\left[1 - (2c^H - c^L)/\beta\right]^+$, which is zero whenever $c^H \geq \frac{1}{2}(c^L + \beta)$.

For the example presented in Figure 17, only REE3 exists at $\rho = 0$ with $y^{H,*} = 0.1156$ and $y^{L,*} = 0.3318$. Only REE1 exists at $\rho \geq 0.5$, *ceteris paribus*, with $y^{H,*} = 0$ and $y^{L,*} = \frac{1}{3}\left[1 - (2c^L - c^H)/\beta\right] = \frac{1}{3}$. The example shows that $y^{L,*}$ can be increasing in ρ and even "boundary-value" increasing, i.e., $y^{L,*}|_{\rho=0} < y^{L,*}|_{\rho=0.5} = y^{L,*}|_{\rho\to 1}$. This finding refines the results in the existing literature, including Proposition 3 in this paper, that the equilibrium inventory decreases in

(a) Inventories (b) Profits, surplus, welfare

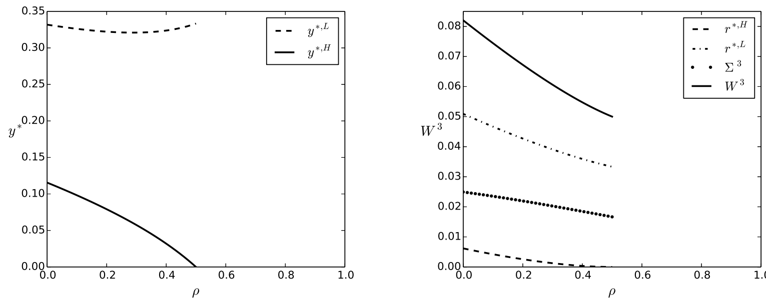


Figure 17: REE3 for high and low cost retailers at $c^L = 0.1, c^H = 0.2, p_1 = 0.95, \beta = 0.3, s = 0$

the consumer's discount factor. In this example, the total equilibrium inventory, indeed, decreases in ρ. The increase in $y^{L,*}$ is a side-effect of market monopolization: the high-cost retailer inventory goes to zero much faster than the total inventory decreases in ρ. Inequality $y^{L,*}|_{\rho=0} < y^{L,*}|_{\rho\to 1}$ can be expressed in terms of the model inputs as follows.

Proposition 12. *For any* $1 - \frac{2}{3}(\beta - \bar{c}) < p_1 < 1$, *inequality* $y^{L,*}|_{\rho=0} < y^{L,*}|_{\rho\to 1}$ *is equivalent to* $\frac{\beta(2Y^* - 2 + p_1) + c^H}{\beta(2Y^* - 2 + p_1) + \bar{c}} < \frac{2(\beta + c^H - 2c^L)}{3\beta Y^*}$, *where* $Y^* = Y^{*,3}|_{\rho=0} = \frac{1}{3}[2 - p_1 - \frac{\bar{c}}{\beta} + \sqrt{(2 - p_1 - \frac{\bar{c}}{\beta})^2 + 3(\frac{p_1}{\beta} - 1)(1 - p_1)}]$.

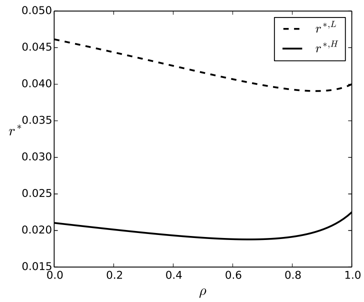
The p_1-range in this proposition includes the example above because it guarantees, similarly to part 3 of Proposition 2, that there exists such ρ_1 that if REE3 exists, it exists for $\rho < \rho_1$ whereas REE1 exists for $\rho \geq \rho_1$.

The aggregate welfare (Figure 17 (b)) decreases from $W|_{\rho=0} = 0.08$ to $W|_{\rho=0.5} = 0.05$, when the high-cost retailer is pushed out of the market. This example contrasts with the above-mentioned example in Bulow, Geanakoplos, and Klemperer (1985), where the entry of a high-cost retailer to a monopoly market decreases the aggregate welfare. This decrease happens if the (homogeneous) products of both firms (competing in quantities) with constant marginal costs are considered as strategic substitutes by a low-cost retailer, i.e., $\frac{\partial^2 r^L}{\partial y^L \partial y^H} < 0$. In our example, this derivative is also negative for both ρ. The direction of change in welfare is different in our example because the switch from duopoly to monopoly is endogenously determined by increased strategic behavior whereas in the example of Bulow, Geanakoplos, and Klemperer (1985) the high-cost retailer entry is exogenous. As shown in Figure 17 (b), W decreases in ρ because both retailer profits r^L and r^H as well as the total consumer surplus Σ^* are decreasing when ρ increases from zero to 0.5. This contrast with Bulow, Geanakoplos, and Klemperer (1985) underscores the importance of including strategic consumer behavior in the models of capacity competition.

In REE4, the low-cost retailer always considers the products of both firms as strategic complements: $\frac{\partial^2 r^{L,4}}{\partial y^L \partial y^H} = \frac{(p_1 - s)(1 - v^{*,4})(y^L - y^H)}{(y^L + y^H)^3} > 0$, whereas for the high-cost retailer the products are strategic substitutes: $\frac{\partial^2 r^{H,4}}{\partial y^H \partial y^L} = \frac{(p_1 - s)(1 - v^{*,4})(y^H - y^L)}{(y^L + y^H)^3} < 0$. In REE1, both retailers consider their products as strategic substitutes: $\frac{\partial^2 r^{L,1}}{\partial y^L \partial y^H} = \frac{\partial^2 r^{H,1}}{\partial y^H \partial y^L} = -\beta$. Because an increase in the consumer's discount factor can result in a switch from REE4 to REE3 and from REE3 to REE1 (given other parameters fixed), it obviously affects whether products are strategic complements or substitutes. Thus, the conclusions of Bulow, Geanakoplos, and Klemperer (1985) potentially depend on the consumer's discount factor.

Finally, we show how the difference in costs affects two types of profit gain due to increased

(a) $c^L = 0.45, c^H = 0.5, p_1 = 0.65, \beta = 1, s = 0$



(b) $c^L = 0.15, c^H = 0.2, p_1 = 0.5, \beta = 0.5, s = 0.1$

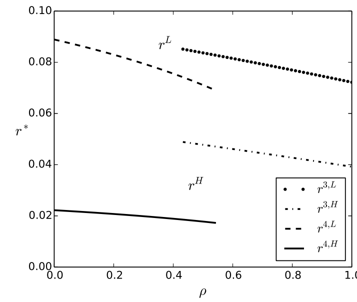


Figure 18: Profit gains of asymmetric retailers in ρ

strategic behavior, described in §5. The continuous gain (Figures 4 and 18 (a)) is possible under RESE3 and REE3 for high levels of strategic behavior and product durability. This form of gain is less pronounced for a low-cost retailer than for a high-cost one, whose increase in profit may result even in the boundary-value gain, i.e., $r^{H,*}|_{\rho=0} < r^{H,*}|_{\rho\to 1}$ (Figure 18 (a)). When the costs are the same, the continuous boundary value gain is possible only for $n \geq 3$ (Proposition 4). The discontinuous gain occurring under switches from REE4 to REE3 (Figure 18 (b)) is also more pronounced for the high-cost retailer. In the example shown, the high-cost retailer even experiences the boundary-value gain.

Hence, in case of duopoly, a refinement of the model to asymmetric costs shows that, in addition to the main insights, increasing strategic behavior may also push a high-cost retailer out of the market, lead to increasing inventory of a low-cost retailer, as well as increase retailer profits.

C Proofs and technical results of Supplementary Document

C.1 Profit function for $\gamma \in [0, 1]$, its properties and inventory decisions

Retailer i has no sales in the second period. In this case, the general formula (1) for profit becomes $r^i = (p_1 - c)y^i$, which yields a unique profit-maximizing inventory $y^i = \breve{y}^i = d^i$. Unlike $\gamma = 1$, other retailers may have sales in the second period, implying that, in general, $\bar{\alpha} \neq 0$ and $v^{\min} \geq p_1$. Using (2) with $y^j = \frac{Y^*}{n}, j \neq i$ and $D = 1 - v^{\min}$, $\breve{y}^i$ is a root of a non-linear equation: $y^i = \frac{(1-v^{\min})\left(y^i\right)^\gamma}{(n-1)(Y^*/n)^\gamma + (y^i)^\gamma}$. After dividing by y^i, which eliminates the extraneous root $y^i = 0$, this equation can be written as $(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma = (1-v^{\min})\left(y^i\right)^{\gamma-1}$ or $(n-1)\left(\frac{Y^*}{n}\right)^\gamma = \left(y^i\right)^{\gamma-1}(1-v^{\min}-y^i)$, which, for $n = 1$, yields $\breve{y}^i = 1 - v^{\min}$ for any $\gamma \in [0, 1]$. If $n > 1$, this equation can be written as

$$\left(y^i\right)^{1-\gamma} = \frac{1}{n-1}\left(\frac{n}{Y^*}\right)^\gamma (1 - v^{\min} - y^i), \tag{54}$$

which, for $\gamma = 1$, yields $\breve{y}^i = 1 - v^{\min} - \frac{n-1}{n}Y^*$. For $\gamma < 1$, this equation has a unique positive root because the LHS is zero at $y^i = 0$ and increasing in y^i, and the RHS is a decreasing linear function in y^i, which is positive at $y^i = 0$. For $\gamma = 0$, equation (54) results in $\breve{y}^i = \frac{1-v^{\min}}{n}$, which is the maximum $\breve{y}^i$ in γ by the following lemma.

Lemma 13. *The solution of (54), $\breve{y}^i$, is decreasing in γ if $\breve{y}^i < \frac{Y^*}{n}$.*

Proof Equation (54) can be written as $\exp\left[(1-\gamma)\ln(\breve{y}^i)\right] = \exp\left[\gamma\ln\left(\frac{n}{Y^*}\right)\right]\frac{1-v^{\min}-\breve{y}^i}{n-1}$. The derivative of this equation in γ is $\left(\breve{y}^i\right)^{1-\gamma}\left[\frac{1-\gamma}{\breve{y}^i}\frac{\partial\breve{y}^i}{\partial\gamma}-\ln(\breve{y}^i)\right] = \ln\left(\frac{n}{Y^*}\right)\frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}(1-v^{\min}-\breve{y}^i) - \frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}\frac{\partial\breve{y}^i}{\partial\gamma}$, which can be written as $\frac{\partial\breve{y}^i}{\partial\gamma}\left[\frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}+(1-\gamma)\left(\breve{y}^i\right)^{-\gamma}\right] = \ln\left(\frac{n}{Y^*}\right)\frac{1}{n-1}\left(\frac{n}{Y^*}\right)^{\gamma}(1-v^{\min}-\breve{y}^i)+\ln(\breve{y}^i)\left(\breve{y}^i\right)^{1-\gamma}$, where the bracket $[\cdot]$ in the LHS is positive and the RHS, by (54), becomes $\left(\breve{y}^i\right)^{1-\gamma}\left[\ln(\breve{y}^i)-\ln\left(\frac{Y^*}{n}\right)\right]$, which is negative, leading to $\frac{\partial\breve{y}^i}{\partial\gamma} < 0$■

Retailer i has sales in the second period, $p_2 > s$. Profit (1) with q^i, given by Lemma 3, becomes $r^i = p_1 d^i + p_2(y^i - d^i) - cy^i$, which, with $y^j = \frac{Y^*}{n}, j \neq i, v^{\min} = v^*$, and d^i from (2), can be written as

$$r^i = \left[p_1 - p_2(y^i)\right]\frac{(1-v^*)\left(y^i\right)^{\gamma}}{(n-1)\left(\frac{Y^*}{n}\right)^{\gamma}+\left(y^i\right)^{\gamma}} + \left[p_2(y^i)-c\right]y^i \tag{55}$$

where, by (3), $p_2(y^i) = \beta\left[1-\frac{n-1}{n}Y^*-y^i\right]$. The derivative, after simplifications, is

$$\begin{aligned}\frac{\partial r^i}{\partial y^i} &= \frac{\beta(1-v^*)\left(y^i\right)^{\gamma}}{(n-1)\left(\frac{Y^*}{n}\right)^{\gamma}+\left(y^i\right)^{\gamma}} + [p_1-p_2]\frac{(1-v^*)\gamma\left(y^i\right)^{\gamma-1}(n-1)\left(\frac{Y^*}{n}\right)^{\gamma}}{\left[(n-1)\left(\frac{Y^*}{n}\right)^{\gamma}+\left(y^i\right)^{\gamma}\right]^2} \\ &\quad -2\beta y^i + \beta\left(1-\frac{n-1}{n}Y^*\right) - c.\end{aligned} \tag{56}$$

When $v^* = 1$, RESE takes the same form of RESE1 as for $\gamma = 1$ (Theorem 1) because the first-period demand is zero.

When $\bar{\alpha} = 0$ and $v^* = p_1$ (no second-period sales), the necessary condition of RESE2 existence, namely, $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\frac{1-p_1}{n}+0} \leq 0$, using formula (56) with $v^* = p_1$, becomes $\beta\frac{1-p_1}{n} + p_1(1-\beta)\gamma\frac{n-1}{n} - 2\beta\frac{1-p_1}{n}+\beta-c-\beta\frac{n-1}{n}(1-p_1) \leq 0$, which, multiplied by n, can be written as $p_1\left[(1-\beta)\gamma(n-1)+n\beta\right] \leq nc$ or $p_1 \leq \frac{nc}{\gamma(n-1)+\beta[n(1-\gamma)+\gamma]} = P_2(\gamma)$.

When $\bar{\alpha} = 1$ and $p_1 \leq v^* < 1$, a candidate for RESE3 results from two conditions: $v^* = v^*(Y^*)$ and $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\frac{Y^*}{n}} = 0$, which, using Lemma 1 and (56), are $v^* = \frac{p_1-\rho\beta(1-Y^*)}{1-\rho\beta}$ and

$$\beta\frac{1-v^*}{n} + \left[p_1-\beta(1-Y^*)\right]\frac{(1-v^*)\gamma(n-1)}{nY^*} + \beta\left(1-\frac{n+1}{n}Y^*\right) - c = 0. \tag{57}$$

After multiplication by $-\frac{nY^*}{\beta(n+1)}$ and collection of terms with Y^*, this equation becomes

$$(Y^*)^2 - Y^*\frac{n}{n+1}\left[\frac{1-v^*}{n}\left(1+\gamma(n-1)\right)+1-\frac{c}{\beta}\right] - \frac{n-1}{n+1}\gamma\left(\frac{p_1}{\beta}-1\right)(1-v^*) = 0, \tag{58}$$

which, for $\gamma = 1$, coincides with (18). Substitution for $1-v^* = \frac{1-p_1-\rho\beta Y^*}{1-\rho\beta}$ and collection of terms with Y^* leads to $(Y^*)^2 a_2 + Y^* a_1 + a_0 = 0$, where $a_2 = \frac{n\rho\beta(\gamma-1)+n+1-\gamma\rho\beta}{(n+1)(1-\rho\beta)} > 0$,

$$\begin{aligned}a_1 &= -\frac{[1+\gamma(n-1)](1-p_1)+n\left(1-\frac{c}{\beta}\right)(1-\rho\beta)-(n-1)\gamma\left(\frac{p_1}{\beta}-1\right)\rho\beta}{(n+1)(1-\rho\beta)}, \text{ and} \\ a_0 &= -\frac{n-1}{n+1}\gamma\left(\frac{p_1}{\beta}-1\right)\frac{1-p_1}{1-\rho\beta}.\end{aligned}$$

After division by a_2, the last quadratic equation becomes

$$(Y^*)^2 - \frac{(\beta - c)\, n(1-\rho\beta) + \beta[1+\gamma(n-1)](1-p_1) - \gamma\,(p_1-\beta)\,\rho\beta(n-1)}{\beta\,[n\rho\beta(\gamma-1)+n+1-\gamma\rho\beta]} Y^*$$

$$-\frac{\gamma\,(p_1-\beta)\,(1-p_1)(n-1)}{\beta\,[n\rho\beta(\gamma-1)+n+1-\gamma\rho\beta]} = 0, \tag{59}$$

which, for $\gamma = 1$, coincides with (23). The equilibrium inventory is the larger root of this equation because, multiplying (59) by $-a_2\frac{\beta(n+1)}{n} < 0$, we obtain the original equation (57) with substituted $v^*(Y^*)$ and multiplied by $Y^* > 0$. The LHS of this resulting equation is a quadratic function with a negative coefficient in front of $(Y^*)^2$, i.e., the LHS decreases in Y^* at the larger root, which corresponds to the maximum of profit.

Retailer i has sales in the second period, $p_2 = s$. By (55) with $p_2 = s$,

$$r^i = (p_1 - s)\frac{(1-v^*)\left(y^i\right)^\gamma}{(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma} + (s-c)y^i \quad \text{and} \tag{60}$$

$$\frac{\partial r^i}{\partial y^i} = (p_1 - s)\frac{(1-v^*)\gamma\left(y^i\right)^{\gamma-1}(n-1)\left(\frac{Y^*}{n}\right)^\gamma}{\left[(n-1)\left(\frac{Y^*}{n}\right)^\gamma + \left(y^i\right)^\gamma\right]^2} + s - c. \tag{61}$$

Profit (60) is concave in y^i because $\left(y^i\right)^\gamma$ is concave, function $\frac{Az}{B+z}$ is concave in z for any positive z, A, and B (first term of r^i) and $(s-c)y^i$ is concave.

A candidate for RESE4 results from conditions: $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\frac{Y^*}{n}} = 0$ and $v^{*,4} = \frac{p_1 - \rho s}{1-\rho\beta}$. The latter implies the same p_1-upper bound as for $\gamma = 1$. Namely, $v^{*,4} < 1$ (there are sales in the first period) is equivalent to $p_1 < P_4 \triangleq 1-\rho(\beta-s)$. The former yields $Y^{*,4} : (p_1-s)\frac{(1-v^*)\gamma(n-1)(Y^*/n)^{2\gamma-1}}{n^2(Y^*/n)^{2\gamma}} + s - c = 0$, which, multiplied by $\frac{Y^*n}{p_1-s}$, gives $\frac{c-s}{p_1-s}Y^*n = (1-v^*)\gamma(n-1)$ or $Y^{*,4}(\gamma)$ in the form of (53).

C.2 Proof of Proposition 7 (equally shared demand)

For $\gamma = 0$, equation (59) becomes $Y^*\left[Y^* - \frac{\beta(1-p_1)+(\beta-c)n(1-\rho\beta)}{\beta[1+n(1-\rho\beta)]}\right] = 0$ yielding a unique $Y^* > 0$. Substitution of $1 - Y^* = \frac{p_1+n(1-\rho\beta)c/\beta}{1+n(1-\rho\beta)}$ into $p_2^* = \beta(1-Y^*)$ and $v^* = \frac{p_1-\rho p_2^*}{1-\rho\beta}$ results in the corresponding expressions.

Condition $v^* < 1$ (there are sales in the first period) is $p_1 + n(p_1 - \rho c) < 1 + n(1-\rho\beta)$ or $p_1(n+1) < 1 + n[1-\rho(\beta-c)]$ yielding $p_1 < P_1$ — the boundary with RESE1. Condition $v^* \geq p_1$ is $p_1 + n(p_1-\rho c) \geq p_1 + np_1(1-\rho\beta)$ or $\rho c \leq p_1\rho\beta$, which holds for $\rho = 0$. For $\rho > 0$, it becomes $p_1 \geq \frac{c}{\beta}$.

RESE3 exists if and only if any retailer i has no incentive to deviate neither to (i) sales in both periods with $p_2 = s$ nor to (ii) sales only in the first period. Part (i) holds because, by (61), $\left.\frac{\partial r^i}{\partial y^i}\right|_{\gamma=0} = s - c < 0$ for any y^i leading to $p_2 = s$. Part (ii) is equivalent to $\left.\frac{\partial r^i}{\partial y^i}\right|_{y^i=\frac{1-v^*}{n}} > 0$, which, by (56) with $\gamma = 0$, is $-2\beta\frac{1-v^*}{n} + \beta\frac{1-v^*}{n} + \beta\left(1 - \frac{n-1}{n}Y^*\right) > c$. Multiplication by $\frac{n}{\beta}$ leads to $n\,(1-c/\beta) - (n-1)Y^* > 1 - v^*$, and, after the substitutions of Y^* and

$$1 - v^* = \frac{1-p_1+n[1-p_1-\rho(\beta-c)]}{1+n(1-\rho\beta)}, \tag{62}$$

the last inequality, multiplied by $1+n(1-\rho\beta) > 0$, becomes $n(1-c/\beta)+n^2(1-c/\beta)(1-\rho\beta)-(n-1)(1-p_1)-n(n-1)(1-c/\beta)(1-\rho\beta) > 1-p_1+n[1-p_1-\rho(\beta-c)]$ or $n(1-c/\beta)+n(1-c/\beta)(1-\rho\beta) >$

$2n(1-p_1)-n\rho\beta(1-c/\beta)$, which, after simplifications, yields $p_1 > \frac{c}{\beta} = P_2(0)$. This inequality implies that $v^* = p_1$ only if $\rho = 0$. Because the p_1-boundaries of RESE3 are the negations of the boundaries of RESE1 and RESE2, and, for $\gamma = 0$, RESE4 does not exist, no other equilibria exist in the area $\frac{c}{\beta} < p_1 < P_1$.

Continuity of Y^*, v^*, p_2^*, and r^* can be shown directly by substitution of the boundaries to the correspondent formulas. For example, $Y^{*,3}\big|_{p_1=c/\beta} = \frac{1-c/\beta+n(1-c/\beta)(1-\rho\beta)}{1+n(1-\rho\beta)} = 1-c/\beta = 1-p_1 = Y^{*,2}$.

Monotonicity of v^*, Y^*, and p_2^*. $\frac{\partial v^*}{\partial n} = \frac{(p_1-\rho c)[1+n(1-\rho\beta)]-(1-\rho\beta)[p_1+n(p_1-\rho c)]}{[1+n(1-\rho\beta)]^2}$ where the numerator is $\rho(p_1\beta - c) \geq 0$, so $\frac{\partial v^*}{\partial n} = 0$ only if $\rho = 0$.

$\frac{\partial v^*}{\partial \rho} = \frac{1}{[1+n(1-\rho\beta)]^2}\left\{-nc[1+n(1-\rho\beta)] + n\beta[p_1 + n(p_1 - \rho c)]\right\}$, where $\{\cdot\} = (\beta p_1 - c)(n+1) > 0$ for any RESE3 inputs.

$\frac{\partial Y^*}{\partial n} = \frac{1}{[1+n(1-\rho\beta)]^2}\left\{(1-c/\beta)(1-\rho\beta)[1+n(1-\rho\beta)] - (1-\rho\beta)[1-p_1+n(1-c/\beta)(1-\rho\beta)]\right\}$, where $\{\cdot\} = p_1 - c/\beta > 0$.

$\frac{\partial Y^*}{\partial \rho} = \frac{1}{[1+n(1-\rho\beta)]^2}\left\{n(1-c/\beta)(-\beta)[1+n(1-\rho\beta)] + n\beta[1-p_1+n(1-c/\beta)(1-\rho\beta)]\right\}$, where $\frac{1}{n\beta}\{\cdot\} = -(p_1 - c/\beta) < 0$.

The results for Y^* imply that $p_2^* = \beta(1-Y^*)$ is decreasing in n and increasing in ρ.

Monotonicity of nr^* in n. By (55) with $\gamma = 0$,

$$nr^* = (p_1 - p_2^*)(1-v^*) + (p_2^* - c)Y^*. \tag{63}$$

Then $\frac{\partial(nr^*)}{\partial n} = -\frac{\partial p_2^*}{\partial n}(1-v^*) - \frac{\partial v^*}{\partial n}(p_1 - p_2^*) + \frac{\partial Y^*}{\partial n}(p_2^* - c) + \frac{\partial p_2^*}{\partial n}Y^*$. Substitutions for $\frac{\partial p_2^*}{\partial n} = -\beta\frac{\partial Y^*}{\partial n}$ and $\frac{\partial v^*}{\partial n} = \frac{\rho\beta}{1-\rho\beta}\frac{\partial Y^*}{\partial n}$ lead to $\frac{\partial(nr^*)}{\partial n} = \frac{\partial Y^*}{\partial n}\left\{\beta(1-v^*) - \frac{\rho\beta}{1-\rho\beta}(p_1 - p_2^*) + p_2^* - c - \beta Y^*\right\}$. Using (62) and

$$\begin{aligned} p_1 - p_2^* &= \frac{p_1(1-\beta) + n(1-\rho\beta)(p_1 - c)}{1+n(1-\rho\beta)}, && (64)\\ p_2^* - c &= \frac{\beta p_1 - c}{1+n(1-\rho\beta)}, && (65) \end{aligned}$$

the bracket $\{\cdot\}$, multiplied by $[1+n(1-\rho\beta)]$, becomes

$$\begin{aligned} &\beta(1-p_1) + n\beta\left[1 - p_1 - \rho\beta\left(1 - \frac{c}{\beta}\right)\right] - \frac{\rho\beta}{1-\rho\beta}\left[p_1(1-\beta) + n(1-\rho\beta)(p_1 - c)\right]\\ &+\beta p_1 - c - \beta(1-p_1) - \beta n\left(1 - \frac{c}{\beta}\right)(1-\rho\beta)\\ &\qquad = n\beta\left[\frac{c}{\beta} - p_1 - \rho(p_1 - c)\right] + \beta p_1 - c - \frac{\rho\beta p_1(1-\beta)}{1-\rho\beta}, \end{aligned}$$

which is decreasing in n. For $n = 1$, this expression is $-\rho\beta(p_1 - c) - \frac{\rho\beta p_1(1-\beta)}{1-\rho\beta} \leq 0$. Therefore, nr^* is decreasing in n for any $n \geq 1$ because $\frac{\partial(nr^*)}{\partial n} = 0$ only for $n = 1$ and $\rho = 0$.

The conditions of monotonicity of nr^* in ρ. Using (63), $\frac{\partial(nr^*)}{\partial \rho} = -\frac{\partial p_2^*}{\partial \rho}(1-v^*) - \frac{\partial v^*}{\partial \rho}(p_1 - p_2^*) + \frac{\partial Y^*}{\partial \rho}(p_2^* - c) + \frac{\partial p_2^*}{\partial \rho}Y^*$, which, using $\frac{\partial p_2^*}{\partial \rho} = -\beta\frac{\partial Y^*}{\partial \rho}$, can be written as $\frac{\partial(nr^*)}{\partial \rho} = \frac{\partial p_2^*}{\partial \rho}\left[2Y^* + \frac{c}{\beta} - 2 + v^*\right] - \frac{\partial v^*}{\partial \rho}\left[p_1 - \beta(1-Y^*)\right]$. The first bracket $[\cdot]$ is zero for $n = 1$ because, by (58) for $\gamma = 0$ and $n = 1$, $Y^* = \frac{1}{2}\left(2 - v^* - \frac{c}{\beta}\right)$, whereas the second bracket $[\cdot] > 0$. Therefore, for $n = 1$, $\frac{\partial(nr^*)}{\partial \rho} < 0$.

For $n > 1$ and $\gamma = 0$, equation (58) yields $Y^* = \frac{1-v^*}{n+1} + \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$, which can be written as

$$1 - Y^* = \frac{1}{n+1}\left(v^* + n\frac{c}{\beta}\right) \text{ or } p_2^* = \frac{\beta v^* + nc}{n+1}. \tag{66}$$

As $\frac{\partial p_2^*}{\partial \rho} = \frac{\beta}{n+1}\frac{\partial v^*}{\partial \rho}$, $\frac{\partial (nr^*)}{\partial \rho} = \frac{\partial v^*}{\partial \rho}\frac{1}{n+1}\left\{\beta\left[2Y^* + \frac{c}{\beta} - 2 + v^*\right] - (n+1)\left[p_1 - \beta(1-Y^*)\right]\right\}$, which means that $\frac{\partial (nr^*)}{\partial \rho} \leq 0$ is equivalent to $\{\cdot\} \leq 0$ or $p_1 \geq \beta(1-Y^*) + \frac{1}{n+1}\left[c + \beta v^* - 2\beta(1-Y^*)\right] = p_2^* + \frac{c+\beta v^* - 2p_2^*}{n+1}$ or, using (66),

$$p_1 \geq c + 2\frac{\beta v^* - p_2^*}{n+1} \Leftrightarrow p_1 \geq c + \frac{2n}{n+1}(p_2^* - c). \tag{67}$$

The last inequality always holds for $n = 1$ and never holds when $\rho\beta \to 1$ (leading to $p_2^* \to p_1$) and $n > 1$ because $(n+1)(p_1 - c) < 2n(p_1 - c)$ for any $n > 1$.

Condition (67) is only sufficient for monotonicity of $nr^{*,3}$ under RESE3 because violation of this condition may take place outside the area of RESE3 inputs, and inside this area, $nr^{*,3}$ can be monotonic. Namely, by part 3 of Proposition 2, which holds for $\gamma = 0$, RESE3 exists only for $\rho < \rho^1 = \frac{n+1}{n}\frac{1-p_1}{\beta - c}$, where ρ^1 can be less than one for large n. In order to take into account this bound, condition (67) can be written in terms of inputs using (65):

$$p_1 \geq c + \frac{2n}{n+1}\frac{\beta p_1 - c}{1 + n(1-\rho\beta)}. \tag{68}$$

The RHS of this inequality is increasing in ρ; therefore, given other inputs fixed, $\frac{\partial (nr^*)}{\partial \rho} < 0$ for all ρ under RESE3 if and only if (68) holds for $\rho = \rho^1$. With this ρ, condition (68) becomes

$$p_1 \geq c + \frac{2n}{n+1}\frac{(\beta p_1 - c)(\beta - c)}{(1+n)[\beta - c - \beta(1-p_1)]} \Leftrightarrow p_1 \geq c + \frac{2n}{(n+1)^2}(\beta - c). \tag{69}$$

The RHS of (69) decreases in n to c with $n \to \infty$. Therefore, there exists n^0 such that $\frac{\partial (nr^*)}{\partial \rho} < 0$ for any $n \geq n^0$. On the other hand, because $\frac{\partial (nr^*)}{\partial \rho} < 0$ for $n = 1$ and (69) is necessary and sufficient for $\frac{\partial (nr^*)}{\partial \rho} < 0$, nr^* attains minimum (because the RHS of 68 increases in ρ) for any $n \in (1, n^0)$, where $n^0 > 2$ if and only if (69) does not hold at least for $n = 2$, i.e., $p_1 < c + \frac{4}{9}(\beta - c) = \frac{5c + 4\beta}{9}$.

n^0 can be found, e.g., from the negation of (69) bearing in mind that non-monotonicity holds for $n < n^0$, where n^0 is the larger root of the equation, corresponding to $(p_1 - c)(n^2 + 2n + 1) < 2n(\beta - c)$. The equation is $n^2 - 2n\frac{\beta - p_1}{p_1 - c} + 1 = 0$, where $\frac{\beta - p_1}{p_1 - c} > 0$ because $p_1 \in (c, \beta)$. The discriminant $D = 4\left[\left(\frac{\beta - p_1}{p_1 - c}\right)^2 - 1\right] > 0$ because $\beta - p_1 > p_1 - c$ is equivalent to $p_1 < \frac{\beta + c}{2}$, where the RHS is greater than $\frac{5c + 4\beta}{9}$. Then the larger root is $n^0 = \frac{\beta - p_1 + \sqrt{(\beta - p_1)^2 - (p_1 - c)^2}}{p_1 - c} = \frac{\beta - p_1 + \sqrt{(\beta - c)(\beta + c - 2p_1)}}{p_1 - c}$.

The expression for ρ^0 can be found from (68) when it holds as an equality: $(p_1 - c)[1 + n(1 - \rho\beta)] = \frac{2n}{n+1}(\beta p_1 - c) \Leftrightarrow 1 - \rho\beta = \frac{2}{n+1}\frac{\beta p_1 - c}{p_1 - c} - \frac{1}{n} \Leftrightarrow \rho^0 = \frac{1}{\beta}\left[1 + \frac{1}{n} - \frac{2}{n+1}\frac{\beta p_1 - c}{p_1 - c}\right]$, yielding

$$\rho^0 = \frac{(n+1)^2(p_1 - c) - 2n(\beta p_1 - c)}{\beta n(n+1)(p_1 - c)}. \tag{70}$$

When $\beta = 1$, $nr^{*,3}$, using (63) and the expressions (64), (62), and (65), is

$$\begin{aligned}
nr^{*,3}\big|_{\beta=1} &= \frac{1}{[1 + n(1-\rho)]^2}\left\{n(1-\rho)(p_1 - c)(1 - p_1) + n^2(1-\rho)(p_1 - c)[1 - p_1 - \rho(1-c)]\right.\\
&\quad \left. + (1 - p_1)(p_1 - c) + n(1-\rho)(p_1 - c)(1 - c)\right\},\\
\lim_{\rho\to 1} nr^{*,3}\big|_{\beta=1} &= (1 - p_1)(p_1 - c) < nr^{*,3}\big|_{\substack{\beta=1\\ \rho=0}} = \frac{1}{(1+n)^2}\left\{n(p_1 - c)(1 - p_1) + n^2(p_1 - c)(1 - p_1)\right.\\
&\quad \left. + (1 - p_1)(p_1 - c) + n(p_1 - c)(1 - c) \pm n(p_1 - c)(1 - p_1)\right\}\\
&= (1 - p_1)(p_1 - c) + \frac{n(p_1 - c)^2}{(1+n)^2}.
\end{aligned}$$

Formula $\rho^0\big|_{\beta=1} = \frac{n^2+1}{n(n+1)}$ results from (70) with $\beta = 1$. Minimum of ρ^0 in n can be found from $\frac{\partial\rho^0}{\partial n} = 0 = \frac{2n^2(n+1)-(2n+1)(n^2+1)}{n^2(n+1)^2}$, which is equivalent to $n^2 - 2n - 1 = 0$ with the roots $n_{1,2} = 1 \pm \sqrt{2}$. The relevant root is $n_2 = 1 + \sqrt{2}$; direct calculation yields $\rho^0\big|_{n=2} = \frac{5}{6} = \frac{10}{12} = \rho^0\big|_{n=3}$.

C.3 Proof of Proposition 8 (profit of a deviator from MSRP)

Assume (too optimistically) that retailer i obtain the entire first-period demand if its first-period price is $p_1' < p_1$. This assumption overestimates retailer i's profit because in reality consumers buy at different prices due to their search costs. Moreover, the assumption treats consumers as myopic, which, by Lemma 1, maximizes the first-period demand and, as a result, the profit of a retailer who sells only in the first period. Using the general expression for profit (8) with an additional cost $K \geq 0$ and unit cost $c^H > c$, the profit upper bound, for $p_2 > s$, is $UB^i = -K - c^H y^i + p_1'(1-p_1') + \beta\,(1-Y)\,[y^i - (1-p_1')]$. Consider the problem of maximizing $UB^i(y^i, p_1')$ s.t. $1 - p_1' \leq y^i$ (by retailer rationality) and $p_1' \leq p_1$ (we consider a non-strict inequality, which also overestimates r^i). The Lagrangian of this problem is $L^i = r^i - \lambda(1 - p_1' - y^i) - \mu(p_1' - p_1)$ (which is concave because the Hessian is negative definite and the feasible region is convex) and the first-order conditions are

$$\frac{\partial L^i}{\partial p_1'} = 1 - 2p_1' + \beta\,(1-Y) + \lambda - \mu = 0, \tag{71}$$

$$\frac{\partial L^i}{\partial y^i} = -c^H + \beta(2 - Y - p_1' - y^i) + \lambda = 0, \tag{72}$$

$$\lambda, \mu \geq 0, \lambda(1 - p_1' - y^i) = 0, \mu(p_1' - p_1) = 0, \tag{73}$$

where $Y = y^i + Y^{-i}$. Profit upper bound UB^i can be written as $UB^i = -K + (1-p_1')[p_1' - \beta\,(1-Y)] + y^i\left\{\beta\,(1-Y) - c^H\right\}$, where, by (71), $[\cdot] = 1 - p_1' + \lambda - \mu$ and, by (72), $\{\cdot\} = \beta(y^i + p_1' - 1) - \lambda$. Then $UB^i = -K + (1-p_1')(1 - p_1' + \lambda - \mu) + y^i[\beta(y^i + p_1' - 1) - \lambda]$.

When $\lambda > 0$, retailer i has sales only in the first period ($y^i = 1 - p_1'$). Consider two cases: $p_1' < p_1$ ($\mu = 0$) and $p_1' = p_1$ (in the latter case system (71), (72) yields the first-period price greater than p_1 and $\mu > 0$).

When $\mu = 0$, the subtraction of doubled (71) from (72) leads to $(4-\beta)p_1' = 2 + \lambda + c^H - \beta Y^{-i}$ or $p_1' = (2 + \lambda + c^H - \beta Y^{-i})/(4-\beta)$, which, substituted into (71), yields $y^i = \frac{1}{\beta}\left[1 + \beta(1 - Y^{-i}) + \lambda - 2(2 + \lambda + c^H - \beta Y^{-i})/(4-\beta)\right] = \frac{1}{\beta(4-\beta)}[(2-\beta)\lambda + 3\beta - 2c^H - 2\beta Y^{-i} - \beta^2(1 - Y^{-i})]$. Then condition $1 - p_1' - y^i \leq 0$ is equivalent to $\beta(4-\beta)(1 - p_1' - y^i) = 4\beta - \beta^2 - [2\beta + \beta\lambda + \beta c^H - \beta^2 Y^{-i}] - [(2-\beta)\lambda + 3\beta - 2c^H - 2\beta Y^{-i} - \beta^2(1 - Y^{-i})] = -\beta - \beta c^H - 2\lambda + 2c^H + 2\beta Y^{-i} \leq 0$, which, by (73), holds either if $-\beta - \beta c^H + 2c^H + 2\beta Y^{-i} \leq 0$ leading to $\lambda = 0$ or $\lambda = \frac{1}{2}(2c^H + 2\beta Y^{-i} - \beta - \beta c^H) = \frac{1}{2}[c^H(2-\beta) + \beta(2Y^{-i} - 1)] > 0$. The latter implies $c^H > \beta(1 - 2Y^{-i})/(2-\beta)$ and holds, e.g., if $c^H > \beta/(2-\beta)$. Substitution of λ into the expressions for p_1' and $y^i = 1 - p_1'$ yields $p_1' = \frac{1}{2}(1 + c^H)$ (implying $c^H \leq 2p_1 - 1$) and $y^i = \frac{1}{2}(1 - c^H)$. Then $UB^i = -K - c^H\frac{1}{2}(1 - c^H) + \frac{1}{4}(1 - c^H)(1 + c^H) = -K + \frac{1}{4}(1 - c^H)^2$.

When $\mu > 0$, we have $p_1' = p_1$ and $y^i = 1 - p_1$. Then $UB^i = -K + (p_1 - c^H)(1 - p_1)$ and, by (72), $\lambda = c^H + \beta(Y^{-i} - p_1)$, which, substituted into (71), yields $\mu = 1 - 2p_1 + c^H$. Inequalities $\mu > 0$ and $\lambda > 0$ are equivalent to $c^H > 2p_1 - 1$ and $c^H > \beta(p_1 - Y^{-i})$ respectively.

When, for $\mu = 0$, the c^H-range $(\beta(1 - 2Y^{-i})/(2-\beta), 2p_1 - 1]$ is not empty, the c^H-range for $\mu > 0$ is $c^H > 2p_1 - 1$. Indeed, $2p_1 - 1 > \beta(1 - 2Y^{-i})/(2-\beta) \Leftrightarrow Y^{-i} > \frac{1}{2}[1 - (2-\beta)(2p_1 - 1)/\beta] = [1 - (2-\beta)p_1]/\beta$ leading to $\beta(p_1 - Y^{-i}) < \beta p_1 - [1 - (2-\beta)p_1] = 2p_1 - 1$.

C.4 Proof of Corollary 10 (no deviation from MSRP under RESE1)

By part RESE1 of Theorem 1, $Y^{-i} = \frac{n-1}{n+1}(1 - c/\beta)$ and $r^* = \frac{(\beta-c)^2}{(n+1)^2\beta}$. The deviation from MSRP is unprofitable iff $r^* \geq r^i$. By Proposition 8, this inequality holds if $\frac{(\beta-c)^2}{(n+1)^2\beta} \geq -K + \frac{1}{4}(1-c^H)^2$ if $\beta(1-2Y^{-i})/(2-\beta) < c^H \leq 2p_1 - 1$ and $\frac{(\beta-c)^2}{(n+1)^2\beta} \geq -K + (p_1 - c^H)(1-p_1)$ if $c^H > \max\{2p_1 - 1, \beta(p_1 - Y^{-i})\}$ yielding the result.

C.5 Proof of Proposition 9 ($Y^* > 1 - c$)

Part 1. By Theorem 1, $Y^{*,1} = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$, which is maximal at $\beta = 1$, and $Y^{*,1}\big|_{\beta=1} = \frac{n}{n+1}(1-c) < 1 - c$. Hence, $Y^{*,1} < 1 - c$ for any parameters where RESE1 exists.

Part 2. By Theorem 2, $Y^{*,4} = \frac{n-1}{n}\frac{p_1-s}{c-s}(1-v^*)$, which, given other parameters fixed, goes to infinity when c approaches s. The condition $Y^{*,4} \geq 1-c$, using $1 - v^* = \frac{1-p_1-\rho(\beta-s)}{1-\rho\beta}$, can be written as $c - s \leq \frac{n-1}{n}\frac{p_1-s}{1-c}\frac{1-p_1-\rho(\beta-s)}{1-\rho\beta}$. As $Y^{*,4}$ is a concave quadratic function in p_1, its maximum in p_1, $\bar{Y}^{*,4}$, can be found from the condition $\frac{\partial Y^{*,4}}{\partial p_1} = 0 = \frac{n-1}{n(1-\rho\beta)(c-s)}\left[1 - p_1 - \rho(\beta-s) - (p_1 - s)\right]$, yielding $\bar{p}_1 = \frac{1}{2}[1-\rho(\beta-s)+s] = \frac{P_4+s}{2}$. Because $P_4 = 1 - \rho(\beta-s) = 2\bar{p}_1 - s$, we have $1 - v^*|_{p_1=\bar{p}_1} = \frac{\bar{p}_1-s}{1-\rho\beta}$ and $\bar{Y}^{*,4} = \frac{(n-1)(\bar{p}_1-s)^2}{n(1-\rho\beta)(c-s)}$. Price $\bar{p}_1$ can be in the p_1-range of RESE4 because $\bar{p}_1$ is always below P_4 — the p_1-upper bound ($P_4 > s$), and $\bar{p}_1$ can be greater than P_2, which, by Proposition 1, is p_1-lower bound in RESE4. Indeed, $\bar{p}_1 > P_2$ holds for any $n \geq 2$ if it holds for $n = 2$ because P_2 decreases in n. For $n = 2$, inequality $\bar{p}_1 > P_2$ can be written as $(P_4 + s)(1+\beta) > 4c$, which holds for sufficiently small c.

Part 3. By Corollary 2, $Y^{*,3} = 1 - \frac{1}{2}\left(v^* + \frac{c}{\beta}\right)$ if $n = 1$. Then $Y^{*,3} < 1 - c$ is equivalent to $c < \frac{1}{2}(v^* + c/\beta)$, which holds for any $\beta \in (c, 1]$ because $v^* \geq p_1 > c$.

By Proposition 3, $Y^{*,3}$ is maximized at $n \to \infty$. By continuity of Y^* at the boundaries, $Y^{*,3} \to 1 - c$ when $p_1 \to P_2|_{n\to\infty} = c$. We will show that there are feasible inputs such that $\frac{\partial Y^{*,3}}{\partial p_1} > 0$. For example, for $n \to \infty$ and $\rho = 0$, equation (23) for Y is $Y^2 - \left[1 - \frac{c}{\beta} + 1 - p_1\right]Y - \left(\frac{p_1}{\beta} - 1\right)(1-p_1) = 0$. Derivative w.r.t. p_1 results in $2Y\frac{\partial Y}{\partial p_1} + Y - \left[1 - \frac{c}{\beta} + 1 - p_1\right]\frac{\partial Y}{\partial p_1} - \frac{1}{\beta}(1-p_1) + \left(\frac{p_1}{\beta} - 1\right) = 0$, which, for $p_1 \to c$, becomes $\frac{\partial Y}{\partial p_1}[2(1-c) - 1 + \frac{c}{\beta} - (1-c)] = \frac{1}{\beta}[1 - c - c(1-\beta)]$ yielding $\frac{\partial Y}{\partial p_1} = \frac{1-c}{c(1-\beta)} - 1$. The RHS is positive because $1 - c > c(1-\beta)$.

C.6 Proof of Proposition 10 (discount)

The proof is similar to the corresponding parts of the proofs of Theorems 1 and 2. The expressions for Y^* result from the symmetric best responses with $r^i = (p_1 - c)q^i + \lambda(p_2 - c)(y^i - q^i), y^i = \frac{Y}{n}$, and $Y^{-i} = \frac{n-1}{n}Y$.

For RESE1, $q^i = 0$ and, using $p_2 = \beta(1-Y)$, $\frac{\partial r^i}{\partial y^i} = 0 = \lambda\left[\beta(1-Y) - c - \beta\frac{Y}{n}\right]$, yielding $Y = \frac{n}{n+1}\left(1 - \frac{c}{\beta}\right)$ that does not depend on λ. For RESE2, the result is obvious because $r^i = (p_1 - c)y^i$ does not depend on λ. For RESE4 with $v^* = \frac{p_1-\rho s}{1-\rho\beta}$ and $r^i = (p_1 - c)(1-v^*)\frac{y^i}{Y} - \lambda(c -$

$s)y^i\left(1-\frac{1-v^*}{Y}\right)=y^i\left\{\frac{1-v^*}{Y}[p_1-c+\lambda(c-s)]-\lambda(c-s)\right\}$, the equation for Y is

$$\begin{aligned}\frac{\partial r^i}{\partial y^i} &= 0=\frac{1-v^*}{Y}[p_1-c+\lambda(c-s)]-\lambda(c-s)-y^i\frac{1-v^*}{Y^2}[p_1-c+\lambda(c-s)]\\ &= \frac{n-1}{n}\frac{1-v^*}{Y}[p_1-c+\lambda(c-s)]-\lambda(c-s).\end{aligned}$$

This equation yields a unique $Y^{*,4}_\lambda=\frac{n-1}{n}(1-v^*)\left[\frac{p_1-c}{\lambda(c-s)}+1\right]$, which decreases in λ and $\frac{\partial^2 r^i}{\partial(y^i)^2}=-2\frac{n-1}{n}\frac{1-v^*}{Y^2}[p_1-c+\lambda(c-s)]<0$. If $\lambda=(1+\delta)^{-1}$, the relative change in Y^* is

$$\frac{Y^{*,4}_\lambda-Y^{*,4}}{Y^{*,4}}=\left[\frac{p_1-c}{\lambda(c-s)}+1-\frac{p_1-s}{c-s}\right]\frac{c-s}{p_1-s}=\frac{p_1-c+\lambda(c-p_1)}{\lambda(p_1-s)}=\frac{(p_1-c)(1-\lambda)}{(p_1-s)\lambda}=\frac{p_1-c}{p_1-s}\delta.$$

For RESE3, we have

$$\begin{aligned}r^i &= (p_1-c)(1-v^*)\frac{y^i}{Y}+\lambda(p_2-c)y^i\left(1-\frac{1-v^*}{Y}\right)\\ &= y^i\left\{\lambda[\beta(1-Y)-c]+\lambda\beta(1-v^*)+\frac{1-v^*}{Y}[p_1-c(1-\lambda)-\lambda\beta]\right\},\end{aligned}$$

and Y^* results from the first-order optimality condition

$$\begin{aligned}\frac{\partial r^i}{\partial y^i} &= 0=\lambda\beta(1-Y)-\lambda c+\lambda\beta(1-v^*)\\ &\quad+\frac{(1-v^*)}{Y}[p_1-c(1-\lambda)-\lambda\beta]+\frac{Y}{n}\left\{-\lambda\beta-\frac{(1-v^*)}{Y^2}[p_1-c(1-\lambda)-\lambda\beta]\right\}\\ &= -Y\lambda\beta\left(1+\frac{1}{n}\right)-\lambda c+\lambda\beta(2-v^*)+\frac{n-1}{n}\frac{1-v^*}{Y}[p_1-c-\lambda(\beta-c)] \qquad (74)\end{aligned}$$

as well as the equation $v^*=\frac{p_1-\rho\beta(1-Y)}{1-\rho\beta}$ that links valuation threshold to the rational second-period expectations. The remainder of the proof will formally show that Y^* is decreasing in λ. The geometric idea behind the proof is provided by a generalized version of the curve $v_2^{\min}(Y)$ in (20) that gives valuation threshold for the corresponding stationary point of the profit. Solving (74) for v^*, we obtain

$$v^*=1-\frac{Y^2-Y\frac{n}{n+1}\left(1-\frac{c}{\beta}\right)}{Y\frac{n}{n+1}+\frac{n-1}{n+1}\left(\frac{p_1-c}{\lambda\beta}-1+\frac{c}{\beta}\right)}. \qquad (75)$$

The generalized $v_2^{\min}(Y)$ given by the right-hand side shifts down as λ increases. Thus, the intersection point of $v_2^{\min}(Y)$ and $v_1^{\min}(Y)$ (illustrated in Figure 12(a)) shifts to the left as λ increases. Because the abscissa of the intersection point is Y^*, the claim holds in RESE3 based on this geometric structure.

Formally, (74) multiplied by $-Y\frac{n}{\lambda\beta(n+1)}$, becomes

$$Y^2-Y\frac{n}{n+1}\left(2-v^*-\frac{c}{\beta}\right)-\frac{n-1}{n+1}(1-v^*)\left[\frac{p_1-c}{\lambda\beta}-\left(1-\frac{c}{\beta}\right)\right]=0,$$

implying that, for $n=1, Y$ does not depend on λ. Substitution for $1-v^*=\frac{1-p_1-\rho\beta Y}{1-\rho\beta}$ yields

$$Y^2-Y\frac{n}{n+1}\left(\frac{1-p_1-\rho\beta Y}{1-\rho\beta}+1-\frac{c}{\beta}\right)-\frac{n-1}{n+1}\frac{1-p_1-\rho\beta Y}{1-\rho\beta}\left[\frac{p_1-c}{\lambda\beta}-\left(1-\frac{c}{\beta}\right)\right]=0.$$

The coefficient in front of Y^2 is $1 + \frac{n}{n+1}\frac{\rho\beta}{1-\rho\beta} = \frac{n+1-\rho\beta}{(n+1)(1-\rho\beta)}$, and the one in front of Y is

$$-\frac{1}{(n+1)(1-\rho\beta)}\left\{n[1-p_1+\left(1-\frac{c}{\beta}\right)(1-\rho\beta)]-(n-1)\rho\beta\left[\frac{p_1-c}{\lambda\beta}-\left(1-\frac{c}{\beta}\right)\right]\right\}.$$

Multiplying the last equation by $\frac{(n+1)(1-\rho\beta)}{n+1-\rho\beta} > 0$ and denoting $\tilde{\lambda} \triangleq \frac{p_1-c}{\lambda} - (\beta - c)$, we obtain

$$Y^2 - \frac{(\beta-c)n(1-\rho\beta)+\beta(1-p_1)n-\rho\beta(n-1)\tilde{\lambda}}{\beta(n+1-\rho\beta)}Y - \frac{(1-p_1)(n-1)}{\beta(n+1-\rho\beta)}\tilde{\lambda} = 0. \tag{76}$$

The derivative of (76) w.r.t. $\tilde{\lambda}$ is

$$2Y\frac{\partial Y}{\partial\tilde{\lambda}}+Y\frac{\rho\beta(n-1)}{\beta(n+1-\rho\beta)}-\frac{(\beta-c)n(1-\rho\beta)+\beta(1-p_1)n-\rho\beta(n-1)\tilde{\lambda}}{\beta(n+1-\rho\beta)}\frac{\partial Y}{\partial\tilde{\lambda}}-\frac{(1-p_1)(n-1)}{\beta(n+1-\rho\beta)}=0.$$

Multiplication by $\beta(n+1-\rho\beta)$ yields

$$\frac{\partial Y}{\partial\tilde{\lambda}}\left\{2Y\beta(n+1-\rho\beta)-(\beta-c)n(1-\rho\beta)-\beta(1-p_1)n+\rho\beta(n-1)\tilde{\lambda}\right\}=(n-1)(1-p_1-\rho\beta Y).$$

The RHS is zero for $n = 1$ and, by part 3 of Lemma 4, positive for $n > 1$. It remains to show that $\{\cdot\} > 0$ for any $n \geq 1$, implying $\frac{\partial Y}{\partial\lambda} \equiv 0$ for $n = 1$, and $\frac{\partial Y}{\partial\lambda} < 0$ for $n > 1$ because $\tilde{\lambda}$ decreases in λ. Since $\tilde{\lambda}$ is minimal at $\lambda = 1$, it can be shown that, for any $\lambda \in (0, 1]$, Y increases in n. Proof is identical to the corresponding part of the proof of Proposition 3 with $\tilde{\lambda}/\beta$ substituting for $(p_1/\beta - 1)$. Then $\{\cdot\} \geq \beta\left\{2(n+1-\rho\beta)\,Y|_{n=1} - (1-\frac{c}{\beta})n(1-\rho\beta) - (1-p_1)n + \rho(n-1)(p_1-\beta)\right\}$, which is $\beta\,\{n[2\,Y|_{n=1} - (1-c/\beta)(1-\rho\beta) - (1-p_1) + \rho(p_1-\beta)] + 2(1-\rho\beta)\,Y|_{n=1} - \rho(p_1-\beta)\}$, where the square bracket $[\cdot] = 2\,Y|_{n=1} - (1-c/\beta) - (1-p_1) + \rho(p_1-c)$. As $Y|_{n=1}$ does not depend on λ, we can rewrite, by Corollary 2, the bracket $[\cdot]$ as follows: $[\cdot] = 2 - \frac{c}{\beta} - \frac{2p_1-\rho c}{2-\rho\beta} - 1 + \frac{c}{\beta} - 1 + p_1 + \rho(p_1-c) = \frac{1}{2-\rho\beta}\left[\rho c - p_1\rho\beta + 2\rho(p_1-c) - \rho^2\beta(p_1-c)\right] = \frac{\rho}{2-\rho\beta}\left[(p_1-c)(1-\rho\beta) + p_1(1-\beta)\right] \geq 0$ with strict inequality for $\rho > 0$. Therefore, the bracket $\{\cdot\}$ is positive if it is positive for $n = 1$. $\{\cdot\}|_{n=1} = \frac{\rho}{2-\rho\beta}\left[(p_1-c)(1-\rho\beta)+p_1(1-\beta)\right] + (1-\rho\beta)\left[2-\frac{c}{\beta}-\frac{2p_1-\rho c}{2-\rho\beta}\right] - \rho(p_1-\beta) = \frac{\rho}{2-\rho\beta}\left[p_1(2-\rho\beta)-p_1\beta\right]+(1-\rho\beta)\left[2-\frac{c}{\beta}-\frac{2p_1}{2-\rho\beta}\right]-\rho(p_1-\beta) = (1-\rho\beta)\left[2-\frac{c}{\beta}-\frac{2p_1}{2-\rho\beta}\right]-\frac{\rho p_1\beta}{2-\rho\beta}+\rho\beta$, where $\rho\beta - \frac{\rho\beta p_1}{2-\rho\beta} > \rho\beta\frac{1-\rho\beta}{2-\rho\beta}$, which leads to $\{\cdot\}|_{n=1} > \frac{1-\rho\beta}{2-\rho\beta}\left\{\rho\beta + 2\left(2-\frac{c}{\beta}-p_1\right) - 2\rho\beta + \rho c\right\} = \frac{1-\rho\beta}{2-\rho\beta}\left\{2\left(2-\frac{c}{\beta}-p_1\right) - \rho(\beta-c)\right\}$. Because for $n = 1, p_1 < 1 - \frac{1}{2}\rho(\beta-c)$, the last bracket $\{\cdot\} > 2\left[1-\frac{c}{\beta}+\frac{1}{2}\rho(\beta-c)\right] - \rho(\beta-c) = 2\left(1-\frac{c}{\beta}\right) > 0$.

C.7 Proof of Proposition 11 (RESE stability)

RESE1, 3, and 4, for $n \geq 2$, represent a non-degenerate game between retailers that can be reformulated as a one-period game with retailer i's payoff $\pi^i(y^i, Y^{-i}) = y^i P(y^i, Y^{-i}) - C_i(y^i)$, where $C_i(y^i) = cy^i$. For RESE1 and 3, by (10), $P(y^i, Y^{-i}) = \beta(1-Y) + \beta(1-v^*) + \frac{(p_1-\beta)(1-v^*)}{Y}$, and for RESE4, by (14), $P(y^i, Y^{-i}) = s + \frac{(p_1-s)(1-v^*)}{Y}$. By Theorem 3 in Nowaihi and Levine (1985), a Cournot equilibrium Y^* is locally asymptotically stable if the following assumptions hold:

(A1) The equilibrium point Y^* exists and unique in an open neighborhood of Y^* and $\left(y^i\right)^* > 0, i \in I$ — holds by the condition of the proposition and part 2 of Lemma 4.

(A2) P and $C_i, i \in I$ are twice continuously differentiable functions in an open neighborhood of Y^* — holds for RESE1,3, and 4.

(A3) $\partial^2\pi^i/\partial\left(y^i\right)^2 < 0$ at Y^* for each $i \in I$ — holds for RESE1,3, and 4 by Lemma 7.

(H1) $P' < C_i''$ at Y^* for each $i \in I$ — holds because $C_i'' \equiv 0$ and $P' < 0$ for RESE1,3, and 4.

(H2) $P' + y^i P'' \leq 0$ at Y^* for each $i \in I$ — holds strictly for RESE1,3, and 4. Namely, for RESE1 and 3, this inequality is $-\beta - \frac{(p_1-\beta)(1-v^*)}{Y^2} + 2y^i \frac{(p_1-\beta)(1-v^*)}{Y^3} \leq 0$, where the LHS at Y^* is $-\beta - \frac{(p_1-\beta)(1-v^*)}{(Y^*)^2} + 2\frac{Y^*}{n}\frac{(p_1-\beta)(1-v^*)}{(Y^*)^3} = -\beta - \frac{(p_1-\beta)(1-v^*)}{(Y^*)^2}\left(1-\frac{2}{n}\right) < 0$ for any $n \geq 2$. For RESE4, the proof is similar.

C.8 Formal argument for the case of different costs

The structure and conditions of REE existence We assume $\beta > c^H$, i.e., the high-cost firm may have second-period consumers with $v > c^H$.

REE1: In this case, $v^{\min} = 1$ and, using formula (8) for profit with $c = c^H$ and $y^i = y^H$, the profit of the high-cost retailer is $r^H = -c^H y^H + \beta(1-y^H-y^L)y^H$ and, similarly, $r^L = -c^L y^L + \beta(1-y^L-y^H)y^L$. The first-order optimality conditions yield $\frac{\partial r^H}{\partial y^H} = -c^H + \beta(1-y^H-y^L-y^H) = 0 \Leftrightarrow y^H = \frac{1}{2}(1-c^H/\beta - y^L)$ and, in the same way, $y^L = \frac{1}{2}(1-c^L/\beta-y^H)$. When $y^H \geq 0$, these two equations give us $y^{H,*} = \frac{1}{3}\left[1-(2c^H-c^L)/\beta\right]$ and $y^{L,*} = \frac{1}{3}\left[1-(2c^L-c^H)/\beta\right]$. Inequality $y^H \leq 0$ is equivalent to $2c^H - c^L \geq \beta$ or $c^H \geq \frac{1}{2}(c^L+\beta)$. In this case, $y^{H,*} = 0$ and retailer L is a monopolist with $y^{L,*} = \frac{1}{2}(1-c^L/\beta)$.

The expressions for $y^{L,*}$ and $y^{H,*}$ lead to $Y^{*,1} = y^{L,*} + y^{H,*} = \frac{2}{3}(1-\bar{c}/\beta)$, where $\bar{c} = \frac{1}{2}(c^L+c^H)$, and $p_2^{*,1} = \beta(1-Y^{*,1}) = \frac{1}{3}(\beta+c^H+c^L) = \bar{c} + \frac{1}{3}(\beta-\bar{c})$. Then $r^{H,1} = y^{H,*}(p_2^{*,1} - c^H) = \frac{1}{3\beta}(\beta - 2c^H + c^L)\frac{1}{3}(\beta + c^H + c^L - 3c^H) = \beta(y^{H,*})^2$ and $r^{L,1} = \beta(y^{L,*})^2$. By Lemma 5, $\Sigma^{*,1} = (\beta v^* - p_2^*)/(2\beta) = \frac{1}{2}\left[1-(\beta+c^H+c^L)/(3\beta)\right]$. The existence condition $v^* = 1$ is equivalent, by Lemma 1, to $p_1 - \rho\beta(1-Y^{*,1}) \geq 1 - \rho\beta \Leftrightarrow p_1 \geq 1 - \frac{2}{3}\rho(\beta-\bar{c}) = P_1$.

REE2: $\bar{\alpha} = \alpha^* = 0$, by Lemma 1, leads to $v^* = p_1$. Similarly to the symmetric case, retailers' profits, by their rationality, are $r^H = (p_1 - c^H)y^H$ and $r^L = (p_1 - c^L)y^L$ and, by part 1.3 of Lemma 7, r^H (r^L) is pseudoconcave on the interval $(1-p_1-y^L)^+ \leq y^H \leq 1 - \frac{s}{\beta} - y^L$ (respectively, $(1-p_1-y^H)^+ \leq y^L \leq 1 - \frac{s}{\beta} - y^H$). Then, any $y^L, y^H \geq 0$, satisfying $Y = y^L + y^H = 1 - p_1$ are the candidates for REE2, which exists iff (i) there are local maxima of r^H and r^L at $Y = 1-p_1$; (ii) this maxima are not less than possible maxima on the interval $Y > 1 - s/\beta$.

Condition (i) for r^H is equivalent to $\frac{\partial r^H}{\partial y^H}|_{y^H=1-p_1-y^L+0} \leq 0$, which, by (11) with $v^{\min} = p_1$, is $p_1 - c^H - \frac{y^H p_1(1-\beta)}{1-p_1} \leq 0 \Leftrightarrow y^H \geq \frac{(p_1-c^H)(1-p_1)}{p_1(1-\beta)} > 0$, i.e., under REE2, the inventory of the high-cost retailer is separated from zero for any $c^H < p_1$ (there are no REE2 equilibria with $y^H = 0$, i.e. high-cost retailer is pushed out of the market). Substitution for $y^H = 1 - p_1 - y^L$ yields $y^L \leq (1-p_1)\left[1 - \frac{p_1-c^H}{p_1(1-\beta)}\right]$. In the same way, $\frac{\partial r^L}{\partial y^L}|_{y^L=1-p_1-y^H+0} \leq 0 \Leftrightarrow y^L \geq \frac{(p_1-c^L)(1-p_1)}{p_1(1-\beta)}$ and, using both conditions, $y^L + y^H = 1 - p_1 \geq \frac{(2p_1-c^L-c^H)(1-p_1)}{p_1(1-\beta)} \Leftrightarrow p_1 \leq \frac{2\bar{c}}{1+\beta} = P_2$. Summarizing, $\frac{y^H}{1-p_1}$ and $\frac{y^L}{1-p_1}$ under REE2 belong to the ranges $\frac{p_1-c^H}{p_1(1-\beta)} \leq \frac{y^H}{1-p_1} \leq \frac{c^L-\beta p_1}{p_1(1-\beta)}$ and $\frac{p_1-c^L}{p_1(1-\beta)} \leq \frac{y^L}{1-p_1} \leq \frac{c^H-\beta p_1}{p_1(1-\beta)}$, and equality $y^L + y^H = 1 - p_1$ holds. The upper bound for y^H implies that duopoly REE2 exists only if $c^L > \beta p_1$.

Condition (ii) follows from $\frac{\partial r^L}{\partial y^L}|_{y^L=1-s/\beta-y^H+0} \leq 0$ because both r^H and r^L are concave for $Y \geq 1 - s/\beta$ and $\frac{\partial r^H}{\partial y^H}|_{y^H=1-s/\beta-y^L+0} \leq \frac{\partial r^L}{\partial y^L}|_{y^L=1-s/\beta-y^H+0}$, which follows from (15) with $v^{\min} = p_1$: $\frac{\partial r^H}{\partial y^H}|_{y^H=1-s/\beta-y^L+0} = -(c^H - s) + \frac{y^L(p_1-s)(1-p_1)}{(1-s/\beta)^2} \leq \frac{\partial r^L}{\partial y^L}|_{y^L=1-s/\beta-y^H+0} = -(c^L - s) + \frac{y^H(p_1-s)(1-p_1)}{(1-s/\beta)^2}$. By y^H-upper bound, $\frac{\partial r^L}{\partial y^L}|_{y^L=1-s/\beta-y^H+0} \leq -(c^L - s) + \frac{(p_1-s)(1-p_1)^2}{(1-s/\beta)^2}\frac{c^L-\beta p_1}{p_1(1-\beta)}$, where $\frac{(1-p_1)^2}{(1-s/\beta)^2} < 1$. Then $\frac{\partial r^L}{\partial y^L}|_{y^L=1-s/\beta-y^H+0} < -(c^L - s) + (p_1 - s)\frac{c^L-\beta p_1}{p_1(1-\beta)}$ where the RHS is non-positive iff

$\frac{p_1-s}{c^L-s}\frac{c^L-\beta p_1}{p_1-\beta p_1} \le 1$, which holds because $\beta p_1 > s$ and $\frac{c^L-\beta p_1}{p_1-\beta p_1} < \frac{c^L-s}{p_1-s}$ ($\frac{c^L-x}{p_1-x}$ decreases in x). Hence, REE2 exists if and only if $p_1 \le \frac{2\bar{c}}{1+\beta} = P_2$. Moreover, it can be shown that the lower p_1-bound for REE1 in the case of monopoly ($y^{H,1} = 0$ and $\bar{c} = c^L$) always exceeds P_2 with $\bar{c} = \frac{1}{2}(c^L + c^H)$.

REE3: In this case, $v^{\min} = \frac{p_1-\rho\beta(1-y^H-y^L)}{1-\rho\beta}$ and formula (9) for profit with $c = c^H$ and $y^i = y^H$ becomes $r^H = -c^H y^H + p_1\frac{y^H}{y^H+y^L}\left(1-v^{\min}\right) + \beta\left(1-y^H-y^L\right)y^H\left(1-\frac{1-v^{\min}}{y^H+y^L}\right)$. The first-order conditions, by (12), are

$$\frac{\partial r^H}{\partial y^H} = \beta\left(1-y^L\right) - c^H + \beta\left(1-v^{\min}\right) - 2\beta y^H + (p_1-\beta)(1-v^{\min})\frac{y^L}{Y^2} = 0 \text{ and} \quad (77)$$

$$\frac{\partial r^L}{\partial y^L} = \beta\left(1-y^H\right) - c^L + \beta\left(1-v^{\min}\right) - 2\beta y^L + (p_1-\beta)(1-v^{\min})\frac{y^H}{Y^2} = 0. \quad (78)$$

Summing up these equations and multiplying by $-Y/(3\beta)$, we obtain $Y^2 - Y\frac{2}{3}\left(2-v^{\min}-\bar{c}/\beta\right) - \frac{1}{3}\left(p_1/\beta-1\right)\left(1-v^{\min}\right) = 0$, which is equation (18) with $n = 2$ and $c = \bar{c}$. Using the same argument as in the symmetric case, the total equilibrium inventory Y^* is the larger root of this equation where $v^{\min} = v^{\min}(Y^*) = v^*$ (the same as in the symmetric case). Given Y^*, the expressions for y^L, y^H follow from subtraction of (78) from (77), which yields $y^L - y^H = \frac{(c^H-c^L)(Y^*)^2}{(p_1-\beta)(1-v^*)+\beta(Y^*)^2}$. As $y^L - y^H + Y^* = 2y^L$ and $y^H = Y^* - y^L$, we have $y^L = \frac{1}{2}Y^*\left[1+\frac{(c^H-c^L)Y^*}{(p_1-\beta)(1-v^*)+\beta(Y^*)^2}\right]$ and $y^H = \frac{1}{2}Y^*\left[1-\frac{(c^H-c^L)Y^*}{(p_1-\beta)(1-v^*)+\beta(Y^*)^2}\right]$. The equation for Y^* yields $(p_1-\beta)(1-v^*) = 3\beta[(Y^*)^2 - Y^*\frac{2}{3}\left(2-v^*-\bar{c}/\beta\right)]$, which leads to equilibrium inventories

$$y^H = \frac{1}{2}Y^*\left[1-\frac{c^H-c^L}{2\beta(2Y^*-2+v^*)+2\bar{c}}\right] \le y^L = \frac{1}{2}Y^*\left[1+\frac{c^H-c^L}{2\beta(2Y^*-2+v^*)+2\bar{c}}\right]. \quad (79)$$

Because the denominator of the fraction in the expression for y^H decreases in ρ (it increases in Y^* and $\frac{\partial Y^*}{\partial \rho} < 0$ similarly to the symmetric case, shown in Proposition 3), there may exist such $\rho^0 \in (0,1)$ that $y^H|_{\rho=\rho^0} = 0$ whereas $y^H|_{\rho=0} > 0$ given other inputs fixed. Condition $y^H > 0$ is equivalent to $[\cdot] > 0 \Leftrightarrow c^L > \beta(2Y^*-2+v^*) = \beta v^* - 2p_2^*$, where βv^* is the highest consumer valuation in the second period.

Considering the case of $y^H > 0$ and following the proof of Theorem 1, REE3 exists iff $\frac{2\bar{c}}{1+\beta} < p_1 < 1-\frac{2}{3}\rho(\beta-\bar{c})$ and, for both retailers, one of the conditions holds: for the high-cost retailer: (a.H) $\frac{\partial r^H}{\partial y^H}|_{y^H=1-s/\beta-y^L+0} \le 0$, which is equivalent to $y^L \le \frac{(c^H-s)(1-s/\beta)^2}{(p_1-s)(1-v^*)}$, or (b.H) condition (a.H) does not hold, $Y^* < 1-s/\beta$, and the equilibrium profit r^H is not less than the profit $\tilde{r}^H$ resulting from the deviation of this retailer from REE3 in such a way that $\tilde{Y}^H = \tilde{y}^H + y^L > 1-s/\beta$, where $\tilde{y}^H = \arg\max_{y^H > 1-s/\beta-y^L} r^H$. By (15), condition $\frac{\partial r^H}{\partial y^H}|_{y^H=\tilde{y}^H} = 0$ is $-(c^H-s) + y^L(p_1 - s)(1-v^*)/(\tilde{Y}^H)^2$ yielding $\tilde{Y}^H = \sqrt{\frac{y^L(p_1-s)(1-v^*)}{c^H-s}}$. Substitution of $\tilde{Y}^H$ and $\tilde{y}^H = \tilde{Y}^H - y^L$ into (14) leads to $\tilde{r}^H = \tilde{y}^H\left[(p_1-s)(1-v^*)/\tilde{Y}^H - (c^H-s)\right]$. Similarly, for the low-cost retailer, the conditions are (a.L) $y^H \le \frac{(c^L-s)(1-s/\beta)^2}{(p_1-s)(1-v^*)}$, or (b.L) condition (a.L) does not hold, $Y^* < 1-s/\beta$, and $r^L \ge \tilde{y}^L\left[(p_1-s)(1-v^*)/\tilde{Y}^L - (c^L-s)\right]$, where $\tilde{y}^L = \tilde{Y}^L - y^H$ and $\tilde{Y}^L = \sqrt{\frac{y^H(p_1-s)(1-v^*)}{c^L-s}}$.

Under REE3, the high-cost retailer does not supply inventory to the market ($y^H = 0$) neither resulting in $p_2 > s$ nor in $p_2 = s$ if both conditions hold: (i) $\frac{\partial r^H}{\partial y^H}|_{y^H=+0} \le 0$ and (ii) $\frac{\partial r^H}{\partial y^H}|_{y^H=1-s/\beta-y^L+0} \le 0$. By Corollary (2), $y^L = 1-\frac{1}{2}(v^{*,L}+c^L/\beta)$, where $v^{*,L} = \frac{2p_1-\rho c^L}{2-\rho\beta}$, yielding

$y^L = 1-\frac{1}{2}\frac{2c^L-\rho\beta c^L+2\beta p_1-\rho\beta c^L}{\beta(2-\rho\beta)} = 1-\frac{c^L/\beta+p_1-\rho c^L}{2-\rho\beta} = \frac{2-c^L/\beta-p_1-\rho(\beta-c^L)}{2-\rho\beta}$. Then, by (12) with $Y^{-i} = y^L$, condition (i) is $\frac{\partial r^H}{\partial y^H}|_{y^H=+0} = \beta(1-y^L)-c^H+\beta(1-v^{*,L})+(p_1-\beta)(1-v^{*,L})/y^L \leq 0 \Leftrightarrow c^H \geq \beta(2-y^L-v^{*,L})+(p_1-\beta)(1-v^{*,L})/y^L$. Condition (ii) follows from (i) because (i) implies $r^H(1-s/\beta-y^L) \leq 0$, which, by part 1.2 of Lemma 7, is equivalent to $(p_1-s)(1-v^{*,L}) \leq (1-s/\beta)(c^H-s)$. Then, using (15), $\frac{\partial r^H}{\partial y^H}|_{y^H=1-s/\beta-y^L+0} = -(c^H-s)+\frac{y^L(p_1-s)(1-v^{*,L})}{(1-s/\beta)^2} \leq (c^H-s)\left(\frac{y^L}{1-s/\beta}-1\right) < 0$.

REE4: By (15), the first-order conditions are $\frac{\partial r^H}{\partial y^H} = -(c^H-s)+y^L(p_1-s)(1-v^*)/(Y)^2 = 0$ and $\frac{\partial r^L}{\partial y^L} = -(c^L-s)+y^H(p_1-s)(1-v^*)/(Y)^2 = 0$. By summing up these equations we get $Y^* = \frac{1}{2}\frac{(p_1-s)(1-v^*)}{\bar{c}-s}$, where, by Lemma 1, $v^* = \frac{p_1-\rho s}{1-\rho\beta}$. Then $y^L = \frac{(c^H-s)}{(p_1-s)(1-v^*)}(Y^*)^2 = \frac{c^H-s}{2(\bar{c}-s)}Y^*$ and $y^H = \frac{c^L-s}{2(\bar{c}-s)}Y^*$, i.e., $y^L > y^H > 0$ whenever $c^H > c^L > s$. By (14), the equilibrium profits are $r^H = y^H[-(c^H-s)+(p_1-s)(1-v^*)/Y^*] = \frac{(c^L-s)^2}{2(\bar{c}-s)}Y^*$ and $r^L = \frac{(c^H-s)^2}{2(\bar{c}-s)}Y^*$. Similarly to the symmetric case, REE4 exists iff one of the conditions hold: (A) $y^H \geq 1-s/\beta$ (neither retailer can deviate from REE4).

(B) $y^H < 1-s/\beta, y^L \geq 1-s/\beta$ (only retailer L can potentially deviate from REE4), and one of the conditions that prevent retailer L from deviating holds: (B.1) $\frac{\partial r^L}{\partial y^L}|_{y^L=1-s/\beta-y^H-0} \geq 0 \Leftrightarrow \beta\left(1-y^H\right)-c^L+\beta\left(1-v^*\right)-2\beta\left(1-\frac{s}{\beta}-y^H\right)+\frac{(p_1-\beta)(1-v^*)y^H}{(1-s/\beta)^2} \geq 0$, which, after simplification, becomes $\left[\beta+\frac{(p_1-\beta)(1-v^*)}{(1-s/\beta)^2}\right]y^H \geq c^L+\beta v^*-2s$; or (B.2) condition (B.1) does not hold, $Y^* > 1-s/\beta$, and one of the following conditions hold: (i) internal maximizer $\tilde{y}^L$ of retailer L profit is such that $\tilde{Y}$ (which equals $\tilde{y}^L+y^H$) is not in the range $(1-v^*, 1-s/\beta)$, which is equivalent to nonexistence of real roots of the equation $2Y^3-(2-v^*-c^L/\beta+y^H)Y^2+(1-p_1/\beta)(1-v^*)y^H = 0$ in this range (in this case, the deviator profit never exceeds the equilibrium one); or (ii) $\tilde{Y} \in (1-v^*, 1-s/\beta)$ and $r^L \geq \tilde{r}^L(\tilde{Y}) = (\tilde{Y}-y^H)[\beta(1-\tilde{Y})-c^L+\beta(1-v^*)+(p_1-\beta)(1-v^*)/\tilde{Y}]$.

(C) $y^H < 1-s/\beta, y^L < 1-s/\beta$ (both retailers can potentially deviate from REE4), and, for both retailers, one of the conditions of case (B) holds. For retailer H, these conditions are: (C.1) $\left[\beta+\frac{(p_1-\beta)(1-v^*)}{(1-s/\beta)^2}\right]y^L \geq c^H+\beta v^*-2s$; or (C.2) condition (C.1) does not hold, $Y^* > 1-s/\beta$, and one of the following conditions hold: (i) there are no real roots of the equation $2Y^3-(2-v^*-c^H/\beta+y^L)Y^2+(1-p_1/\beta)(1-v^*)y^L = 0$ in the range $(1-v^*, 1-s/\beta)$; or (ii) $\tilde{Y} = \tilde{y}^H+y^L \in (1-v^*, 1-s/\beta)$ and $r^L \geq \tilde{r}^H(\tilde{Y}) = (\tilde{Y}-y^L)[\beta(1-\tilde{Y})-c^H+\beta(1-v^*)+(p_1-\beta)(1-v^*)/\tilde{Y}]$.

C.9 Proof of Proposition 12 (low-cost inventory increasing in ρ)

If $p_1 > 1-\frac{2}{3}(\beta-\bar{c})$, then, similarly to part 3 of Proposition 2, there exists such $\rho_1 = \frac{3}{2}\frac{1-p_1}{\beta-\bar{c}}$ that if REE3 exists, it exists for $\rho < \rho_1$ ($y^{L,*}|_{\rho=0} = y^{L,3}|_{\rho=0}$) whereas REE1 exists for $\rho \geq \rho_1$ ($y^{L,*}|_{\rho\to 1} = y^{L,1}$). By (79), $y^{L,3}|_{\rho=0} = \frac{1}{2}Y^*\left[1+\frac{c^H-c^L}{2\beta(2Y^*-2+p_1)+2\bar{c}}\right]$, where Y^* is the larger root of $Y^2-Y\frac{2}{3}\left(2-p_1-\bar{c}/\beta\right)-\frac{1}{3}\left(p_1/\beta-1\right)\left(1-p_1\right) = 0$, and, by part REE1 of previous subsection, $y^{L,1} = \frac{1}{3}\left[1-(2c^L-c^H)/\beta\right]$. Then $y^{L,*}|_{\rho=0} < y^{L,*}|_{\rho\to 1} \Leftrightarrow 1+\frac{c^H-c^L}{2\beta(2Y^*-2+p_1)+2\bar{c}} < \frac{2(\beta+c^H-2c^L)}{3\beta Y^*}$ yielding the result.

Supplementary References

Bell, D., R. Keeney, and J. Little (1975). “A Market Share Theorem”. In: *Journal of Marketing Research* 12(2), pp. 136–141.

Buehler, S. and D. Gärtner (2013). "Making Sense of Nonbinding Retail-Price Recommendations". In: *American Economic Review* 103(1), pp. 335–359.

Bulow, J., J. Geanakoplos, and P. Klemperer (1985). "Multimarket Oligopoly: Strategic Substitutes and Complements". In: *Journal of Political Economy* 93(3), pp. 488–511.

Butz, D. (1996). "Does the per se rule deter vertical price-fixing?" In: *Economic Inquiry* 34(4), pp. 770–780.

Fisher, F. (1961). "The stability of the Cournot oligopoly solution: The effects of speeds of adjustment and increasing marginal costs". In: *The Review of Economic Studies* 28(2), pp. 125–135.

Lazear, E. (1986). "Retail Pricing and Clearance Sales". In: *American Economic Review* 76, pp. 14–32.

Liu, Q. and G. van Ryzin (2008). "Strategic capacity rationing to induce early purchases". In: *Management Science* 54(6), pp. 1115–1131.

Nowaihi, A. al and P. Levine (1985). "The stability of the Cournot oligopoly model: A reassessment". In: *Journal of Economic Theory* 35(2), pp. 307–321.

Orbach, B. (2008). "Antitrust Vertical Myopia: the Allure of High Prices". In: *Arizona Law Review* 50, pp. 261–287.

Su, X. and F. Zhang (2008). "Strategic customer behavior, commitment, and supply chain performance". In: *Man. Sci.* 54(10), pp. 1759–1773.

Theocharis, R. (1960). "On the Stability of the Cournot Solution on the Oligopoly Problem". In: *The Review of Economic Studies* 27(2), pp. 133–134.