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MPRA Paper No. 72934, posted 12 Aug 2016 03:47 UTC

**FACTOR SUBSTITUTION IN DAIRY FARMING:
A COMPARISON OF ALLEN AND MORISHIMA ELASTICITIES**

by

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December 29, 1992

ABSTRACT

This paper uses a non-homothetic translog cost function to calculate Allen and Morishima elasticities of substitution (AESs and MESs) based on panel data for 11 Vermont dairy farms over a 24 year period. The results indicate that there is a substantial difference between the computed AESs and MESs. The estimates show that the AESs overstate both the substitution and complementary relationships among the inputs. Overall, the MES measures suggest that the dairy farm technology is quite flexible.

Submitted for presentation as a Selected Paper at the 1993 AAEEA-CAEA Annual Meeting, August 1-4, 1993, Orlando, Florida.

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FACTOR SUBSTITUTION IN DAIRY FARMING: A COMPARISON OF ALLEN AND MORISHIMA ELASTICITIES

Measuring the ability of firms to respond to changes in relative factor prices has been an important area of applied work in production economics. The elasticity of substitution (ES), introduced in 1932 by Hicks and refined by Allen in 1938 (AES),¹ has been widely used to quantify the sensitivity of optimal factor combinations to changes in relative factor prices. Since this early work, several different measures of factor substitution have been developed (Mundlak).

In applied analysis, the introduction of the constant elasticity of substitution production function by Arrow, Chenery, Minhas and Solow along with more recent work in flexible functional forms has provided impetus for the measurement of factor substitution. A great deal of the empirical work dealing with factor substitution has relied on the AES. Studies of this type for agriculture include the work by Ball and Chambers, Binswanger, Brown and Christensen, Ray, Sharma, Ali and Parikh, Grisley and Gitu, and Hoque and Adelaja.

Over the last decade, some researchers have questioned the usefulness of the AES as a measure of factor substitution for multi-input technologies. Chambers has argued that the AES is only a one-factor-one-price ES "... since it is a derived demand elasticity divided by a cost share" (p. 95). Hence, the AES provides no information beyond that given by the elasticity of factor demand. Chambers also argued that the Morishima Elasticity of Substitution or MES (Morishima; Blackorby and Russell, 1975), provides more economically relevant information than the AES because it is a true measure of how a two factor input ratio responds to a change in the price of one of the factors.

More recently, Blackorby and Russell (1989) have shown formally that, for a multi-input technology, the AES does not constitute a measure of the ease of substitution between a pair of inputs when their relative prices change. These authors summarize their view by bluntly stating that "...the AES is completely ... uninformative" (p. 883). In the same paper, Blackorby and Russell argue for the use of the MES.

Considering the great deal of effort that has been devoted by many researchers to develop empirical AES estimates, the findings of Blackorby and Russell invite us to take a new look at elasticity of substitution measurements. The purpose of this paper is, therefore, to examine the degree to which AESs differ from MESs in milk production. A handful of recent studies have performed similar comparisons but no one has, to our knowledge, focused on the dairy farming sector.² Furthermore, the limited number of studies that have undertaken this type of analysis is by no means sufficient to derive any empirical regularities that might exist concerning the relation between AESs and MESs.

¹The elasticity of substitution (ES) measure developed by Allen is known in the literature as the Allen ES or AES.

²Studies that have compared AESs with MESs have been published by Ball and Chambers for the US meat products industry, Taylor and Gupta for aggregate southeastern US agriculture, and McMillan and Amoaka-Tuffour for rural and urban municipalities in Australia.

The balance of the paper is divided into four sections. The next section presents a brief formulation of both the AES and the MES followed by a discussion of the empirical model and data used in the analysis. The next section contains the empirical results and the paper ends with some concluding remarks.

Allen and Morishima Elasticities of Substitution

To derive a mathematical formulation of the AES it is best to start from a dual cost function which can be expressed as

$$(1) \quad C = C(P, Y).$$

where P is a vector of variable input prices, and Y is output. Using subscripts to denote partial derivatives of the cost function in equation (1), the AES between inputs i and j can be written as

$$(2) \quad AES_{ij}(P, Y) = [C(P, Y) C_{ij}(P, Y)] / [C_i(P, Y) C_j(P, Y)].$$

Binswanger showed that the AES_{ij} can be rewritten as $AES_{ij} = E_{ij}/S_j$, where E_{ij} is the Hicksian cross price elasticity of demand, and $S_j = (X_j P_j)/C(P, Y)$ is the share of the j th input in total cost. It is important to note that the AES is symmetric, i.e., $AES_{ij} = AES_{ji}$.

According to Blackorby and Russell, the MES for the multi-input case can be computed from the cost function as $MES = \partial \ln(C_i/C_j) / \partial \ln(P_i/P_j)$, where, by Shephard's Lemma, $C_i = X_i$ and $C_j = X_j$. After some manipulation and holding P_i constant, the MES can be written as $MES_{ij} = E_{ij} - E_{ji}$, and if P_j is assumed to be constant then $MES_{ji} = E_{ji} - E_{ij}$. Koizumi has shown that

$MES_{ij} = S_j(AES_{ij} - AES_{ji})$ and $MES_{ji} = S_i(AES_{ij} - AES_{ji})$, when P_i and P_j is held constant, respectively. Thus, unlike the AES the MES is asymmetric. This asymmetry means that a one percent change in relative prices results in a different elasticity of substitution depending on whether the price of the i th or the j th factor is the one that changes (Blackorby and Russell).

Data and Empirical Model

In order to derive empirical elasticities of substitution we formulate a per cow variable cost function, which can be written in general form as

$$(3) \quad VC = f(P_i, Y, Z, T).$$

The data used to estimate this function consist of a panel of 11 Vermont dairy farms over the 24 year period from 1962 to 1985, which yields 264 observations. This data set was obtained from farmers participating in ELFAC (an electronic farm accounting system). The model incorporates a single output (Y) defined as total cwt. of milk equivalent produced per cow per year, measured as milk sales plus livestock sales divided by the price of milk. It should be noted that the farms in the sample are highly specialized and that most of the income is from milk sales.

The model includes the following five variable inputs: 1) labor (L); 2) dairy concentrates (C); 3) materials (M), including fertilizer, lime, seeds, spray, gasoline, and repairs on equipment and machinery; 4) electricity (U); and 5) veterinary and breeding costs (V). Prices are available directly from the ELFAC records or are obtained from published sources. The wage rate (P_L), the index of prices paid by farmers, which is used as a proxy for the price of veterinary and breeding costs (P_v), and the price of milk were obtained from the USDA (a). The price of dairy concentrates (P_c) is equal to expenditures divided by quantities reported by farmers. The price of materials (P_m) is a weighted average of the price indices of fertilizer, seeds, agricultural chemicals, gasoline, and machinery and equipment, which were obtained from the USDA (b). The price of electricity (P_u) is the cost per KWH and was also obtained from the USDA (b). In addition, the number of dairy cows (Z) and a time trend (T) are included to account for farm size effect and technological change, respectively. Given the panel structure of the data, a set of intercept dummies (D_k) is included in the cost function to account for firm effects.

The specific model estimated is a non-homothetic translog variable cost function which can be written as

$$\begin{aligned}
 \ln VC = \ln \beta_o &+ \sum_k A_k D_k + \sum_i \beta_i \ln P_i + \beta_z \ln Z + \beta_y \ln Y + \beta_t T \\
 &+ 1/2 \sum_i \sum_j \beta_{ij} \ln P_i \ln P_j + \sum_i \beta_{iz} \ln P_i \ln Z + \sum_i \beta_{iy} \ln P_i \ln Y \\
 &+ \sum_i \beta_{it} \ln P_i T + 1/2 \beta_{zz} (\ln Z)^2 + \beta_{zy} \ln Z \ln Y + \beta_{zt} \ln Z T \\
 &+ 1/2 \beta_{yy} (\ln Y)^2 + \beta_{yt} \ln Y T + 1/2 \beta_{tt} (T)^2 .
 \end{aligned}
 \tag{4}$$

As is normal practice (Berndt), in order to gain efficiency the above model is estimated along with the system of cost share equations which can be expressed as

$$\frac{\partial \ln VC}{\partial \ln P_i} = S_i = \beta_i + \sum_j \beta_{ij} \ln P_j + \beta_{iz} \ln Z + \beta_{iy} \ln Y + \beta_{it} T.
 \tag{5}$$

Again, according to standard practice, symmetry is imposed by setting $\beta_{ij} = \beta_{ji}$, $\beta_{iy} = \beta_{yi}$, $\beta_{it} = \beta_{ti}$, and $\beta_{iz} = \beta_{zi}$. In order to have a well behaved production technology, the cost function must satisfy the following properties: 1) monotonicity, which requires that all the estimated cost shares be positive in all prices at all data points; 2) quasiconcavity in input prices, which requires that the $n \times n$ matrix of AES_{ij} be negative semidefinite; and 3) homogeneity of degree one in input prices (Berndt). The last property is assured by setting $\sum_i \beta_i = 1$, $\sum_i \beta_{ij} = 0$, $\sum_i \beta_{iz} = 0$, $\sum_i \beta_{iy} = 0$ and $\sum_i \beta_{it} = 0$, whereas the first two properties must be checked. We also tested for homotheticity, homogeneity and Cobb-Douglas characteristics. Homotheticity requires that $\beta_{iy} = 0$, $\beta_{zy} = 0$, and $\beta_{yt} = 0$, while homogeneity also requires that $\beta_{iy} = 0$, $\beta_{zy} = 0$, $\beta_{yt} = 0$, and $\beta_{yy} = 0$. Cobb-Douglas technology (i.e., unitary elasticity of substitution, homotheticity and homogeneity) requires

$\beta_{ij}=0$, $\beta_{iy}=0$, $\beta_{zy}=0$, $\beta_{yt}=0$, $\beta_{yy}=0$, $\beta_{iz}=0$, $\beta_{it}=0$, $\beta_{zz}=0$, $\beta_{zt}=0$ and $\beta_{tt}=0$ (Christensen and Greene).

Once the translog cost function is estimated, the AES s can be computed as $AES_{ii} = (\beta_{ii} + S_i^2 - S_i)/S_i^2$, and $AES_{ij} = (\beta_{ij} + S_i \cdot S_j)/S_i \cdot S_j$, ($i \neq j$) (Ball and Chambers). By contrast, the MES s can be calculated as $MES_{ij} = (\beta_{ij}/S_i) - (\beta_{ji}/S_j) + 1$, and $MES_{ji} = (\beta_{ji}/S_j) - (\beta_{ij}/S_i) + 1$, where $\beta_{ij} = \beta_{ji}$ by symmetry.

For empirical estimation, additive disturbances, assumed to be intertemporally independent and to have a joint normal distribution with zero mean and non-zero contemporaneous covariance, are appended to the cost function and to the share equations. Since cost shares always add up to one, the sum of disturbances at each observation across equations add to zero and, hence, the error covariance matrix is singular. This implies that only $n-1$ shares are independent. Homogeneity is imposed using the price of electricity (P_u) as the numeraire, and the share equation for electricity is dropped thus avoiding the singularity of the covariance matrix. The ITSUR (iterative seemingly unrelated) procedure, which is equivalent to maximum likelihood estimation and invariant to the equation dropped, was applied (Kmenta and Gilbert; Dhrymes).

Results

The parameter estimates for the translog cost function are reported in Table 1. A total of 36 (excluding firm effects) parameters are estimated out of which 23 are significant at the 1%, three at the 5%, and two at the 10% level. Although not shown for space reasons, all of the 10 parameters (A_k) for the firm dummies (firm effects) are significant at the 5% level or better.

Table 2 shows the results of testing three hypotheses concerning the structure of production using likelihood ratio tests. The results of these tests suggest that, for these data, the non-homothetic translog cost function is the best representation of the technology. Note that homogeneity is rejected by strongly rejecting homotheticity. Consequently, both verification of regularity conditions and elasticity of substitution estimates are based on the latter specification.

Fitted values of the share equations were positive at all data points which is a necessary and sufficient condition for the cost function to be monotonic in input prices. All own price elasticities and own elasticities of substitution are negative at all observations, and the $n \times n$ matrix of AES_{ij} is negative semidefinite at the mean of the data. Hence, it can be concluded that the cost function is monotonic in prices at all data points and quasiconcave at least at the mean of the data.

The estimated average expenditure shares for the time period 1962-1985 are used to compute AES s, MES s, and price elasticities of input demand. The AES s range from high levels of substitutability - between electricity and veterinary and breeding costs - to moderate levels of complementarity - between materials and veterinary and breeding costs. Fourteen of the measures indicate inputs substitute for each other. These pairs are equally divided among high (> 1), moderate (0.51 to 0.99) and weak substitutes. Six pairs are complements - four high and two weak. By contrast, the MES s show that, at the mean of the data, all inputs are substitutes. The MES measures show less complementarity between inputs than the AES measures. Only two of the twenty elasticities (veterinary and breeding and labor, and electricity and veterinary and breeding) are greater than one, the threshold for high substitutability. Seven MES s

are between 0.51 and 0.98, indicating moderate substitutability. All other pairs have MESs below 0.5 suggesting weak substitution.

For a better understanding of the difference between AESs and MESs, we consider the effects of increases in electricity price and in wages. The AES between labor and electricity is -1.01, showing a reduction in labor use as the price of electricity rises. This is a complementary relationship. Both labor and electricity use will decrease. By symmetry of the AES, the effect on labor of a higher electricity price is exactly the same as that of a higher wage on electricity use. As argued earlier, the AES reflects only part of the impact of a change in the price of the j th factor on the quantity demanded of the i th factor. This occurs because the AES ignores own price effects. By comparison, the MES between labor and electricity indicates that these two factors are substitutes. This can be explained as follows: as the price of electricity rises the relative reduction in electricity use exceeds the relative reduction in labor use (i.e., $E_{lu} = -0.03$ and $E_{uu} = -0.09$ in Table 4). When there is an increase in wages, again the own price effect ($E_{ll} = -0.47$) is stronger than the cross price effect ($E_{ul} = -0.31$) which shows net substitution between electricity and labor.

Further inspection of Table 3 shows that those AES and MES estimates that both have positive signs vary in terms of magnitudes. In some cases, the difference between the AES and the MES is large and in other cases this difference is very small: eight of the MESs differ by more than one from their corresponding AESs. The greatest difference occurs with electricity and veterinary and breeding. The AES between electricity and veterinary and breeding is 7.66 indicating a very high substitutability. In contrast, the estimated MES between electricity and veterinary and breeding is 1.19, while the estimated MES between veterinary and breeding and electricity is 0.32, indicating much less substitutability and a clearly asymmetric pattern.

To examine the relationships between elasticity and both farm size and time, we first computed MESs for all observations. Then we calculated correlation coefficients for (a) MESs and herd sizes (average number of dairy cows per farm) and for (b) MESs and years. The results are reported in Table 5. All correlation coefficients, except the one between MES_{ue} and herd size, are highly statistically significant. The correlation coefficients reveal that four elasticities, all involving labor, are inversely related with farm size and have decreased over time, i.e., their correlation coefficients with both herd size and years are negative. By comparison, the 10 elasticities are positively related with farm size and have risen over time. The remaining six MESs - all including electricity as one of the inputs - exhibit a mixed pattern.

Concluding Comments

This paper uses a non-homothetic translog cost function to calculate Allen and Morishima elasticities of substitution based on panel data for 11 Vermont dairy farms over a 24 year period. Statistical tests confirm that this representation of the technology is consistent with the data.

The results reveal that 14 out of the 20 AESs (excluding diagonal elements) denote substitutability, while the remaining six denote complementarity. In the case of the MESs, all inputs are found to be substitutes. However, in most cases where the AES shows substitutability, the MESs show much less substitutability. These results imply that the Allen Elasticities of Substitution overstate both the substitution and complementary relationships among the inputs. This is consistent with the results reported by Ball and Chambers, McMillan and Amoaka-Tuffour, and Taylor and Gupta.

Correlation analysis between MESs and farm size, and between MESs and the time trend suggest that relationships exist between factor substitution and farm size and between factor substitution and time. Eleven out of 20 MESs have a positive relationship with farm size, and 15 MESs have increased over time. Substituting for labor, however, becomes more difficult both over time and as farm size increases. Overall, the computed MESs indicate that large farms do not have a substantial advantage in terms of ability to make substitutions between inputs and, on the other hand, dairy farm technology is becoming more flexible over time.

Finally, the results of this study reveal that there is quite a difference between empirical measures of AESs and MESs. Therefore, previous studies that have calculated AESs for dairy production should be interpreted with care. Whether the differences between AESs and MESs reported in this paper can be generalized to other industries remains to be seen.

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- USDA (b). Agricultural Prices, Washington, D.C. Various Issues.

Table 1. Parameter Estimates for the Translog Cost Function

	Intercept	Labor (L)	Concen- trates (C)	Materials (M)	Electricity (U)	Vet. Med. Breeding (V)	No. of Cows (Z)	Output (Y)	Time (T)
β_0	-0.00543 (0.02612)	0.29584 ^{a/} (0.00345)	0.46317 ^{a/} (0.00401)	0.16739 ^{a/} (0.00301)	0.03035	0.04325 ^{a/} (0.00065)	-0.16736 ^{a/} (0.02712)	0.52150 ^{a/} 0.06800	0.00480 ^{a/} (0.00139)
β_{LJ}		0.06782 ^{a/} (0.02101)	-0.6144 ^{a/} (0.01742)	-0.01182 (0.01380)	0.01882	-0.02425 ^{a/} (0.00580)	-0.08656 ^{a/} (0.00740)	-0.24268 ^{a/} 0.02746	-0.00088 (0.00058)
β_{CJ}			0.13330 ^{a/} (0.02238)	-0.0369 ^{a/} 0.01426	-0.02237	-0.01258 ^{a/} (0.00337)	0.07672 ^{a/} (0.00882)	0.12682 ^{a/} (0.03280)	-0.00094 (0.00066)
β_{MJ}				0.06773 ^{a/} (0.01582)	-0.00386	-0.01514 ^{b/} (0.00018)	0.01123 ^{c/} (0.00639)	0.08176 ^{a/} (0.02370)	0.00089 ^{c/} (0.00052)
β_{UJ}					0.00878	0.04750	-0.00445	0.00359	0.00064
β_{VJ}						0.00447 (0.01189)	0.00306 ^{b/} (0.00140)	0.0305 ^{a/} (0.00519)	0.00029 ^{b/} (0.00013)
β_{ZJ}							0.62644 ^{a/} (0.07428)	0.47378 ^{a/} (0.12298)	-0.00573 ^{b/} (0.00268)
β_{YJ}								-0.14042 (0.54775)	0.01112 (0.00719)
β_{TJ}									-0.00106 ^{a/} (0.00032)

^{a/} Significant at 1% level; ^{b/} Significant at 5% level; ^{c/} Significant at 10% level.

Table 2. Test-Statistics for Homotheticity, Homogeneity, and Cobb-Douglas Technology.

	Calc. χ^2	# of Restrictions	Critical Values	
			10%	5%
Homotheticity	95.14	6	10.64	12.59
Homogeneity	2.18	1	2.71	3.84
Cobb-Douglas Technology	289.90	11	17.28	19.68

Table 3. Allen and Morishima Elasticities of Substitution.

	Labor	Dairy Conc.	Materials	Electricity	Veterinary & Breeding
A. ALLEN ELASTICITIES					
Labor	-1.5103	0.5655	0.7641	-1.0077	2.7740
Dairy Concentrates	0.5655	-0.5566	0.4938	0.0780	-0.1242
Materials	0.7641	0.4938	-2.6002	0.2003	-1.1471
Electricity	-1.0077	0.0780	0.2003	-2.9697	7.6557
Vet. Med. Breeding	2.7740	-0.1242	-1.1471	7.6557	-19.4715
B. MORISHIMA ELASTICITIES					
Labor	0	0.5091	0.5407	0.0590	0.9758
Dairy Concentrates	0.6470	0	0.4972	0.0917	0.8487
Materials	0.7089	0.4765	0	0.0953	0.8038
Electricity	0.1566	0.2879	0.4501	0	1.1900
Vet. Med. Breeding	1.3353	0.1962	0.2335	0.3196	0

Table 4: Own and Cross Price Elasticities of Factor Demand.

	Labor	Dairy Conc.	Materials	Electricity	Vet. Med. Breeding
Labor	-0.4707	0.2566	0.1228	-0.0303	0.1217
Dairy Concentrates	0.1763	-0.2525	0.0794	0.0024	-0.0055
Materials	0.2382	0.2240	-0.4179	0.0060	-0.0503
Electricity	-0.3141	0.0354	0.0322	-0.0893	0.3358
Vet. Med. Breeding	0.8646	-0.0564	-0.1843	0.2302	-0.8542

Table 5: Correlation between Morishima Elasticities of Substitution and (a) Farm Size and (b) Time.

MES _{ij}	Avg. Cows	Time
Labor & Concentrates	-0.63	-0.63
Labor & Materials	0.41	0.73
Labor & Electricity	-0.26	0.43
Labor & Vet. Med. and Breeding	0.76	0.45
Concentrates & Materials	0.58	0.71
Concentrates & Electricity	-0.18	0.47
Concentrates & Vet. Med. Breeding	0.56	0.27
Materials & Electricity	-0.18	0.47
Materials & Vet. Med. Breeding	0.50	0.54
Electricity & Vet Med. Breeding	0.29	-0.41
Concentrates & Labor	-0.82	-0.56
Materials & Labor	-0.89	-0.41
Electricity & Labor	-0.53	0.26
Vet. Med. Breeding & Labor	-0.67	-0.40
Materials & Concentrates	0.78	0.56
Electricity & Concentrates	0.07 ^{a/}	0.56
Vet. Med. Breeding & Concentrates	0.59	0.27
Electricity & Material	0.34	0.72
Vet. Med. Breeding & Materials	0.51	0.57
Vet. Med. Breeding & Electricity	-0.26	0.43

^{a/} Highly nonsignificant.

All other coefficients are significant at 1%.