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Indecisiveness, Undesirability and Overload Revealed Through Rational Choice Deferral*

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Abstract

Three reasons why decision makers may defer choice are *indecisiveness* between various feasible options, *unattractiveness* of these options, and *choice overload*. This paper provides a choice-theoretic explanation for each of these phenomena by means of three deferral-permitting models of decision making that are driven by preference incompleteness, undesirability and complexity constraints, respectively. These models feature *rational* choice deferral in the sense that whenever the individual does not defer, he chooses a most preferred feasible option. As a result, choices are always consistent with the Weak Axiom of Revealed Preference. The three models suggest novel ways in which observable data can be used to recover preferences as well as their indecisiveness, desirability and complexity components or thresholds. Several examples illustrate the relevance of these models in empirical and theoretical applications.

Keywords: Choice deferral; incomplete preferences; indecisiveness; unattractiveness; choice overload; revealed preference; rational choice.

JEL Classification: D01, D03, D11

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1 Introduction

Consider an individual who must decide between keeping his existing insurance policy or replacing it with some other policy from those available to him. The status quo associated with his decision problem is his current policy, i.e. some explicit and concrete choice alternative that, in principle, may be compared with the other feasible ones. Consider a second individual who has no insurance policy and is presented with the same options that the first one is. The status quo associated with this person's decision problem is the state of being uninsured, which is qualitatively different from one that takes the form of an existing insurance policy and, in fact, may also be considered objectively inferior to *any* policy. Importantly, when the first individual maintains his status quo, he is choosing a specific option from his menu. Such a choice may reflect a genuine preference for that option over the other feasible ones or, possibly, a *status quo bias*.¹ By contrast, when the second individual maintains his status quo, he is choosing *none* of the options available to him.² A term that is often used by psychologists³ to describe this distinct type of behavior is *choice deferral*.⁴

Several models that aim to describe an individual's choices in decision problems with an explicit and concrete status quo option have been proposed.⁵ Most of them assume that whenever a decision problem does not feature such a status quo, the individual chooses one of the feasible alternatives. In so doing, these models rule out the possibility of choice deferral by construction. The purpose of this paper is to complement this body of theoretical work by focusing on the very large class of decision problems where the individual's status quo is not captured by an explicit and concrete choice option.

Experimental evidence, casual empiricism and introspection suggest that among the reasons for the occurrence of choice deferral are the following three:

1. *Unattractiveness/undesirability*: An individual may defer when faced with decision problems that feature options which are not "good enough". This source of deferral is well-documented in the consumer psychology literature.⁶ Economists have long been familiar with this type of deferral, e.g. in the context of non-binding participation constraints in principal-agent contracts. Importantly, as long as the decision maker's desirability criterion is well-defined and stable across contexts/menus, this source of deferral poses no challenge to the utility maximization paradigm. Despite this "folk wisdom", however, it appears that no choice-theoretic foundations for this kind of decision making exist.
2. *Indecisiveness/decision conflict*: Deferral may occur when the relevant menu contains multi-attribute, trade-off-generating options that are difficult to compare. Many studies in psychology and economics have demonstrated this tendency in consumer, financial, medical and managerial decision making,⁷ occasionally also rejecting alternative explanations for deferral such as rational search for more options. When this type of deferral occurs in a menu with two alternatives, for example, it suggests that the decision maker cannot compare them, contrary to what the fundamental axiom of completeness would require

¹Samuelson and Zeckhauser (1988); Knetsch (1989).

²It was precisely this difference in the nature of the status quo associated with these two types of decision problems that was utilized by Samuelson and Zeckhauser (1988) to establish a status quo bias in the health-care plan choices of Harvard University employees in the 80s. This difference was also exploited in a similar way by Knetsch (1989) when this author provided further evidence for this bias in his mug/chocolate bar exchange experiments. In both cases, a comparison was made between the choices of decision makers whose status quo did not coincide with an explicit and concrete option and those of decision makers whose status quo did take such a form.

³Tversky and Shafir (1992); Shafir et al (1993); Dhar (1999); Anderson (2003); Krijnen et al (2015).

⁴We will also use this term despite the fact that it is slightly misleading given the static nature of the decision problems that the experimental subjects in these studies were presented with, as well as the static nature of most of our analysis in this paper.

⁵Tversky and Kahneman (1991); Munro and Sugden (2003); Mandler (2004); Masatlioglu and Ok (2005, 2015); Dean (2008); Ortoleva (2010); Apesteguia and Ballester (2009, 2013); Gerasimou (2016).

⁶See, for instance, Zakay (1984), Dhar and Sherman (1996) and Mochon (2013).

⁷Greenleaf and Lehmann (1995); Redelmeier and Shafir (1995); Dhar (1997); Luce (1998); Madrian and Shea (2001); Dhar and Simonson (2003); Sawers (2005).

from his preferences. Therefore, such instances of deferral are irreconcilable with utility-maximizing behavior.

3. *Complexity/choice overload*: Decision problems that feature a large number of alternatives may cause deferral due to the increased complexity that is associated with them. Such behavior has been documented in consumer, financial as well as voting decisions.⁸ This source of deferral is also incompatible with utility maximization. For example, a certain option may be chosen over many others in a number of “simple” binary menus, while choice may be deferred in the “complex” menu that is formed by taking the union of these binary ones: All preferences revealed in the small menus are contradicted by deferral in the large menu.

In this paper we propose and axiomatically characterize three models of choice deferral, one for each of the above potential sources of this phenomenon. The common feature in all three models is that deferral is *rational* in the sense that whenever the agent decides to choose a feasible option rather than to defer, his choices are consistent with the Weak Axiom of Revealed Preference. Thus, in these three models deferral is neither due to some inconsistency in the agent’s underlying preferences nor to some context-dependent decision rule. Indeed, whenever the agent decides to choose an option, he always does so by maximizing his menu-independent preferences, similar to a standard utility maximizer. Moreover, in the tradition of revealed preference theory, all three models build on simple and easily falsifiable axioms on observable behavior, which in this case includes choice *and* deferral data. Therefore, an agent’s conformity with either model allows an outside observer to recover the agent’s preferences and other relevant entities such as his unattractiveness or complexity criteria. Moreover, as we show in Appendix 1, all three models include rational choice as a special case.

The model of undesirability-driven deferral features constrained utility maximization in the sense that the agent chooses his most preferred option at a menu if and only if this is strictly preferred to his desirability threshold, and defers otherwise. The model of indecisiveness-driven deferral portrays an agent with transitive but possibly incomplete preferences who follows the decision rule whereby an alternative is chosen from a menu if and only if it is preferred to all others in that menu. This model suggests a novel criterion for distinguishing between the concepts of indifference and indecisiveness using behavioral data. Finally, the model of overload-driven deferral is such that the agent defers at a menu if and only if its complexity exceeds the agent’s constant (i.e. menu-independent) complexity threshold, and chooses as a utility maximizer otherwise.

In section 8 we argue that the three models are informative and applicable in a variety of economic domains, such as the empirical analysis of choice datasets with deferral observations, behavioral industrial organization, robust social choice and choice under uncertainty. As far as the latter domain is concerned, in Appendix 2 we show how a two-period extension of the indecisiveness-driven deferral model can be used to provide choice-theoretic foundations for “objectively rational” or Bewley (2002) preferences which have found many applications in economics.

2 Preliminaries

The set of all possible choice alternatives is X and is assumed finite. The set \mathcal{M} denotes the collection of all subsets of X , which will be called *menus*. A choice correspondence C is a possibly multi- and empty-valued mapping from \mathcal{M} into itself that satisfies $C(A) \subseteq A$ for all $A \in \mathcal{M}$.

Our interpretation of the situation in which $C(A) = \emptyset$ for some menu $A \neq \emptyset$ is that the agent *defers choice* at A . Therefore, we will generally *not* assume the following property, which is treated as an explicit axiom instead:

⁸See, for instance, Iyengar and Lepper (2000), Iyengar et al (2004), Augenblick and Nicholson (2016) as well as the meta-analysis in Chernev et al (2015).

Nonemptiness

If $\emptyset \neq A \in \mathcal{M}$, then $C(A) \neq \emptyset$.

Surprisingly, this assumption has been relaxed (in varying degrees) in very few papers that we are aware of, such as Hurwicz (1986), Clark (1995), Gaertner and Xu (2004) and Gerasimou (2016). A discussion in favor of relaxing it, however, also appears in Kreps (2012, pp. 20-21).

Contrary to Nonemptiness, the *Weak Axiom of Revealed Preference* (WARP) which we state next is part of the axiomatic system of each of the three models that we present below.

WARP

If $x \in C(A)$, $y \in A \setminus C(A)$ and $y \in C(B)$, then $x \notin B$.

When Nonemptiness is also assumed, WARP can be written in a number of equivalent ways. Without it, however, such equivalences are no longer valid. The above statement of WARP is in the spirit of Samuelson's (1938) original version of the axiom: *If an alternative x is chosen over another alternative y in some menu, then there is no menu where y is choosable and x is feasible.* The axiom's status as a core principle of choice consistency remains intact (if not strengthened) in an environment where deferral is permissible. Intuitively, when the decision maker chooses x over y from a menu A even though she had the opportunity to defer, this may be a stronger indication that x is preferred to y relative to the case where choice from A was forced. To the extent that this is so, it becomes even more plausible to expect that a rational decision maker will not choose y from any menu where x is also feasible.

It may be useful to remark at this point that, by ruling out the possibility of deferral, most choice-theoretic models essentially allow for the terms "decision" and "choice" to be considered synonymous. In this paper where deferral is assumed to be possible we take the view that "decision" is a more general term, as it may indicate either deferral or choice of some feasible option. In light of this remark, we also emphasize from the outset that we limit the range of applicability of WARP to the individual's choices (as defined above) and not across all his decisions, which may well violate this axiom. We come back to this point in section 7 with a justification and more detailed discussion.

As far as notation is concerned, a binary relation \succsim on X denotes a (possibly incomplete) *preorder* on X , i.e. a reflexive and transitive binary relation. When this relation is assumed complete, it will be referred to as a *weak order*. The set of greatest/maximum elements of \succsim is defined and denoted by

$$B_{\succsim}(A) := \{x \in A : x \succsim y \text{ for all } y \in A\}.$$

This set is always nonempty when the preorder \succsim is complete, but generally not otherwise.

3 Indecisiveness and Maximally Dominant Choice

We open this section with an anecdote that illustrates well some key elements of the indecisiveness-driven deferral model that we develop below:

"At the bookstore, [Thomas Schelling] was presented with two attractive encyclopedias and, finding it difficult to choose between the two, ended up buying neither – this, despite the fact that had only one encyclopedia been available he would have happily bought it."

Shafir, Simonson, and Tversky (1993)

In line with this story and with the many experimental studies that have documented similar behavior, in this section we propose and analyze a model in which deferral is rooted in the agent's inability to compare some alternatives due to the incompleteness of his preferences.

Our first axiom here imposes a special upper bound on the empty-valuedness of C and plays a very important role conceptually.

Desirability

If $x \in X$, then $C(\{x\}) = \{x\}$.

This axiom is compatible with the interpretation that whenever the decision maker is faced with only one alternative, the latter is sufficiently good for him to choose it. Under this interpretation, Desirability rules out unattractiveness of the alternatives as a potential reason for deferral.⁹ Hence, it is also compatible with the interpretation that if the agent was to interpret deferral as an explicit option, he would consider it inferior to all other alternatives (see also section 7).

Contraction Consistency

If $x \in C(A)$ and $x \in B \subset A$, then $x \in C(B)$.

This standard axiom is weaker than WARP when Nonemptiness is assumed, but logically distinct from it otherwise. For example, $x \in C(A)$, $x \in B \subset A$ and $C(B) = \emptyset$ is consistent with WARP but violates Contraction Consistency. However, ruling out this kind of behavior is normatively appealing. Indeed, when choice reveals preference, x being choosable at A suggests that x is at least as good as every other alternative in A , hence in $B \subset A$ too. Therefore, to allow for the possibility that nothing is chosen from B amounts to saying that the most preferred option in the menu is not chosen, which is irrational.

Strong Expansion

If $x \in C(A)$, $y \in A$ and $y \in C(B)$, then $x \in C(A \cup B)$.

To our knowledge, this axiom is novel. It is implied by WARP when Nonemptiness is assumed and is distinct from WARP otherwise. It strengthens the well-known ‘‘Property γ ’’ (Sen, 1971) or ‘‘Expansion’’ axiom, which states that an alternative x that is choosable in both A and B is also choosable in $A \cup B$. Indeed, the proposed axiom requires that if x is chosen in the presence of y in some menu A and y is chosen in B , then x is chosen in $A \cup B$. This obviously reduces to Expansion in the special case where $x = y$. With regard to intuition, if a rational agent chooses x in the presence of y at menu A when deferral is possible, this suggests that he finds x at least as good as y and everything else in A . Likewise, the fact that y is chosen in B suggests that y is at least as good as everything else in B . Such an agent should therefore also consider x to be at least as good as everything else in $A \cup B$ and hence would choose x from this expanded menu, as required by the axiom.

Proposition 1

The following are equivalent for a choice correspondence $C : \mathcal{M} \rightarrow \mathcal{M}$:

1. C satisfies Desirability, WARP, Contraction Consistency and Strong Expansion.
2. There exists a unique preorder \succsim on X such that

$$C(A) = \begin{cases} \emptyset, & \text{iff } B_{\succsim}(A) = \emptyset \\ B_{\succsim}(A), & \text{otherwise} \end{cases} \quad (1)$$

All proofs are in Appendix 3, while the tightness of all axiomatic systems is established in Appendix 4. We will refer to (1) as the model of *maximally dominant choice* (MDC). This portrays a cautious decision maker who chooses something from a menu if and only if he can find a most preferred option in that menu, and defers otherwise. In so doing, such an agent makes fully

⁹The logical complement of this axiom, Undesirability, is introduced in the context of unattractiveness-driven deferral in the next section.

consistent choices across menus and therefore is hedged against money-pump-like manipulations that are generally possible when preferences are incomplete and the agent chooses some option that is merely undominated (Mandler, 2009; Danan, 2010).

The model's predictions are compatible in a straightforward way with experimental findings on choice from conflict-inducing multi-attribute alternatives such as those reported by Tversky and Shafir (1992) (including the anecdote in the quote that opened this section). These findings suggest that, given two alternatives x and y that dominate each other in some important attribute, many people are willing to choose x when x is the only feasible option and y when y is the only feasible option, but nothing when both x and y (and only them) are feasible. If preferences are let to coincide with the usual partial ordering on attribute space, then one would have $x \succsim x, y \succsim y$ and $x \not\succeq y, y \not\succeq x$, in which case (1) would indeed predict $C(\{x\}) = \{x\}, C(\{y\}) = \{y\}$ and $C(\{x, y\}) = \emptyset$, consistent with the findings.

In light of the discussion of the Desirability axiom and the predictions of the decision rule that is put forward in (1), an interesting feature of the MDC model is that the individual's outside option that is associated with deferral is objectively inferior to all "real" alternatives, and yet not only can it be chosen, but this fact does not by itself rule out the possibility of viewing the agent as behaving rationally when choosing among the other options. We consider this to be a virtue of the modelling approach whereby deferral is associated with an empty set of choosable options. The alternative approach that would involve modelling deferral by means of an explicit feasible option, discussed in section 7, would deprive the modeller from such a possibility. In so doing, it would also place the MDC procedure in the long list of choice models that feature WARP-inconsistent, bounded-rational behavior without being able to properly account for and highlight the fact that, unlike the other models in that class, a very important part of an MDC decision maker's decisions are actually fully consistent. As we discuss in section 8, this also has important implications for empirical work.

Although seemingly puzzling, there may be sound psychological reasons for an individual's decision to defer even if he understands that doing so is objectively worse than choosing any of the feasible options. Luce (1998), for instance, shows that some of the variation in the occurrence of deferral is due to the anticipation of negative emotions such as regret and self-blame that may follow once a decision maker who chooses in a conflict situation realizes that he has made a suboptimal choice.

Given that incomparability/indecisiveness is the only source of deferral in the MDC model, one may expect that such an individual may be able to eventually resolve his indecision by completing his preferences in the future, possibly after acquiring information about the alternatives, and then choose as a utility maximizer from any menu where he had originally deferred. In Appendix 2 we extend the model in this direction and use this extension to provide a choice-theoretic foundation for "objectively rational" Bewley (2002) preferences under uncertainty.

3.1 Revealed Preference, Indifference and Indecisiveness

MDC suggests a purely behavioral and generally applicable criterion to disentangle indifference and indecisiveness from a given set of decision observations that are consistent with it. Specifically, for an agent who has generated such data, the psychological state with respect to two alternatives x and y is

indifference *iff for every menu $A \in \mathcal{M}$ such that $x, y \in A$, it holds that $y \in C(A)$ whenever $x \in C(A)$;*

indecisiveness *iff for every menu $A \in \mathcal{M}$ such that $x, y \in A$ it holds that $x, y \notin C(A)$;*

preference (for x over y) *iff there exists a menu $A \in \mathcal{M}$ such that $x \in C(A)$ and $y \in A \setminus C(A)$.*

As in Eliaz and Ok (2006), and consistent with any model of rational choice, the agent here is revealed to be indifferent between x and y if and only if both options are always choosable whenever both are feasible and one is choosable. However, unlike the Eliaz-Ok criterion for revealed indecisiveness which necessitates one alternative to be chosen over the other in some

menu and is therefore associated with a violation of WARP, the agent in the MDC model *is revealed to be indecisive between x and y if and only if neither of these options is ever chosen in the presence of the other*. This novel criterion is the first to allow for a distinction between revealed indifference and indecisiveness that applies to WARP-consistent agents with incomplete preferences. Moreover, unlike Eliaz and Ok (2006), no a priori restrictions on the agent’s preferences are necessary for this distinction to be made (in the Eliaz-Ok model, a condition called “regularity” must be satisfied by the agent’s preferences).

Mandler (2009) proposed a distinction between the two concepts that is based on data from sequential pairwise trades of options that are not ranked by strict preference. Specifically, if such sequential trades eventually result in the decision maker owning an alternative that is either strictly better or strictly worse to the one he started off with, then Mandler’s criterion suggests that he must have been indecisive at some point along the sequence of pairs that were involved in the trades. On the other hand, in the absence of such a strict preference ranking it cannot be ruled out that the agent was indifferent throughout. Clearly, the MDC distinction between indifference and indecisiveness is compatible with Mandler’s distinction because the revealed indifference relation is transitive whereas the incomparability relation is generally not.

An interesting relationship exists between the properties of the MDC model and the revealed-preference analysis in Bernheim and Rangel (2009). These authors suggested that, in a model-free world, an alternative x may be thought of as being strictly preferred to another alternative y if there is no menu in which y is chosen and x is feasible. In the MDC model, x is revealed preferred to y if, in addition to this, there is a menu in which x is chosen *over* y . Thus, although the Bernheim-Rangel condition is necessary for x to be revealed preferred to y under the MDC model, it is not sufficient. In fact, as is evident from the preceding discussion, this condition is also necessary for the agent to be revealed indecisive between x and y in the MDC model. Moreover, it holds that *x and y are not comparable by the Bernheim-Rangel revealed preference relation if and only if they are revealed incomparable in the context of the MDC model*. Given that the Bernheim-Rangel revealed-preference criterion was proposed in a different context where an individual’s behavioral dataset was not assumed to include deferral observations (although, clearly, it need not be restricted in this way), the link established between the revealed indecisiveness relations in the two models is at least somewhat surprising.

3.2 A Dominance-Based Explanation of the Choice Overload Effect

The MDC model is also compatible with evidence suggesting the occurrence and disappearance of the so called “*choice overload*” effect. This refers to the phenomenon whereby decision makers defer choice significantly more often when faced with large/complex menus that contain many alternatives than when they are faced with smaller/simpler ones. The first evidence for this effect came from the field and was reported in Iyengar and Lepper (2000). One of the proposed explanations for choice overload is that deferral in large menus is caused by the lack of familiarity and the absence of a most preferred option.¹⁰ Intuitively, the higher the degree of incompleteness in the agent’s preferences, the more likely it is that, as the size of the menu increases, the number of incomparable pairs will become so large that no alternative is preferred to all others. This explanation is compatible with (1). The model is also compatible with the explanation of how the effect breaks down when a dominant option is added to a large menu at which choice would have otherwise been deferred (Scheibehenne et al 2010; Chernev et al 2015). Indeed, as explained above, this happens when $C(A) = \emptyset$, regardless of the size of A , and when an alternative x is added to A and is such that $x \succsim y$ for all $y \in A$, in which case $C(A \cup \{x\}) = \{x\}$. Thus, Proposition 1 offers incompleteness-driven theoretical predictions for both the occurrence and the disappearance of the choice overload effect.

¹⁰Iyengar and Lepper (2000); Scheibehenne et al 2010; Chernev et al 2015.

4 Desirability-Constrained Rationality

The next source of choice deferral that we study is inferiority of all feasible alternatives relative to an agent-specific desirability threshold. As we mentioned in the introduction, this is by no means a novel choice procedure. However, it appears that a precise choice-theoretic formulation does not exist, and therefore the revealed-preference implications associated with the behavior prescribed by it are unknown. We pin down such a set of necessary and sufficient conditions in this section and formalize some other relevant aspects in this procedure.

Our first axiom here is the logical complement of the Desirability axiom:

Undesirability

There exists $x \in X$ such that $C(\{x\}) = \emptyset$.

Indeed, if an option x is not considered to be “good enough”, then it is intuitive that the decision maker will not choose it whenever this is the only feasible option. In contract theory, for example, the Undesirability axiom manifests itself through the agent’s participation constraint: unless a contract’s expected utility exceeds the agent’s reservation utility, the agent will not choose it even if it’s the only feasible one. For some experimental evidence that are consistent with this axiom the reader is referred to Zakay (1984) and Mochon (2013).

Contractive Undesirability

If $C(A) = \emptyset$ and $B \subset A$, then $C(B) = \emptyset$.

This condition is also necessary for modelling unattractiveness-driven deferral. Intuitively, if an individual considers nothing to be sufficiently good in some menu A , then this must also be true in every submenu B of A .

We will say that a preference relation \succsim on X *quasi-rationalizes a choice correspondence* $C : \mathcal{M} \rightarrow \mathcal{M}$ if $C(A) = B_{\succsim}(A)$ whenever $C(A) \neq \emptyset$. We will also say that a weak order \succsim *quasi-rationalizes C uniquely up to an element $x^* \in X$* if for any weak order \succsim' on X where $x \succsim' y$ holds whenever $x \succsim y$ and $x, y \succ x^*$ hold, the weak order \succsim' also quasi-rationalizes C .

Proposition 2

The following are equivalent for a choice correspondence $C : \mathcal{M} \rightarrow \mathcal{M}$:

1. C satisfies WARP, Contraction Consistency, Undesirability and Contractive Undesirability.
2. There exists a weak order \succsim on X and an alternative $x^* \in X$ such that, for all $A \in \mathcal{M}$,

$$C(A) = \begin{cases} \emptyset, & \text{iff } z \in B_{\succsim}(A) \text{ implies } x^* \succsim z \\ B_{\succsim}(A), & \text{otherwise} \end{cases} \quad (2)$$

Moreover, \succsim quasi-rationalizes C uniquely up to x^* .

In this model the agent behaves as if he had complete and transitive preferences over the set X and as making decisions like a utility maximizer as long as his most preferred feasible option in the given menu is strictly preferred to some pre-specified alternative x^* . The latter sets his context-independent *undesirability threshold* in the sense that the agent defers choice from a menu if and only if the best alternative in that menu is weakly inferior to x^* . Since the behavior captured in (2) deviates from rational choice only in that some alternatives are never chosen because they are undesirable, we will refer to it as the model of *desirability-constrained rationality* (DCR).

With regard to preference revelation in the DCR model, quasi-rationalizability of the choice correspondence C by the weak order \succsim uniquely up to x^* suggests that such revelation is possible for the part of the agent’s preference order that ends on the (possibly degenerate) indifference

set where x^* belongs. This part of the agent’s preferences is completely and uniquely recoverable from his choices. The “usefulness” of his deferring behavior in this respect is limited to the identification of those alternatives in X that are undesirable in the sense that they will never be chosen. Clearly, the agent’s preference ranking of these alternatives cannot be recovered using decision data. This is the reason for the potential multitude of weak orderings that are compatible with C in the quasi-rationalizability sense that is claimed in Proposition 2. Finally, given the lack of choice data that might indicate a preference between such options, there is no way for an outside observer to know that the agent can in fact rank them in the first place. Proposition 2 merely states that such a ranking is *possible*.

It is worth emphasizing that the option x^* which sets the agent’s desirability threshold will not be feasible to him in general but influences his behavior even in decision problems where it is not. In this regard the DCR model is similar to the Tversky and Kahneman (1991) model of reference-dependent preferences with loss aversion, where an option can act as the agent’s reference point even if it is infeasible.

We conclude this section by noting that an additional benefit from identifying the behavioral axioms that characterize undesirability-driven *rational* choice deferral is that this identification can serve as the benchmark for other models where deferral is caused by *context-dependent* undesirability. Evidence for the latter has been provided by Dhar and Sherman (1996), for instance, who note that the same two alternatives can induce deferral due to unattractiveness in one context (e.g. when their unique “bad” features are emphasized) but not in another (e.g. when their unique “good” features are emphasized). Such behavior, for instance, is generally incompatible with both Contractive Undesirability and Contraction Consistency.

5 Overload-Constrained Rationality

The anecdote below illustrates nicely a special case of the behavior that we are interested in explaining in this section, as well as the mechanism through which our proposed model does so:

“As a neophyte shoe salesman, I was told never to show customers more than three pairs of shoes. If they saw more, they would not be able to decide on any of them.”¹¹

New York Times, 26 January 2004

In line with the substance of the preceding quote and related experimental findings, in our last model the agent defers in decision problems that he considers to be complex. As in Iyengar and Lepper (2000), we will refer to this third source of deferral as *choice overload*. In a previous section we argued that the MDC model of indecisiveness-driven deferral is compatible with one of the suggested explanations for this phenomenon. Here we propose a different explanation that builds on a model of utility maximization that is constrained by *complexity thresholds*.

The first additional axiom that we introduce here is standard.

Binary Choice Consistency

If $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$, then $x \in C(\{x, z\})$.

Similar to our preceding discussion of WARP, transitivity of binary choices may be considered even more appealing in deferral-permitting decision environments for individuals who aim to behave rationally in the sense that, when they do make active choices, these reflect maximization of a transitive preference relation. In some models where the revealed weak preference relation is defined by choices in binary menus this axiom ensures transitivity of that relation. In the model below it has a slightly more intricate role that will become clear shortly.

¹¹Letter to the editor of the New York Times by Milton Waxman, originally quoted in Kuksov and Villas-Boas (2010).

Deferral Monotonicity

If $B \supset A \neq \emptyset$ and $C(A) = \emptyset$, then $C(B) = \emptyset$.

To our knowledge, this axiom is novel. It suggests that if the individual finds a menu A to be complex, then he finds every menu $B \supset A$ to be complex as well, so that overload-driven deferral is monotonic with respect to set inclusion. While the intuitive appeal of this axiom is obvious at one level, we note that it can fail descriptively when insertion of a clearly superior alternative in a complex menu may override the complexity of the decision problem and nudge the individual towards that alternative. On the other hand, one may argue that the axiom is “normatively appealing” for a cognitive- or resource-constrained decision maker on the grounds that the complexity criterion that such an individual may employ incorporates a break-even cost-benefit analysis, so that once a menu is complex according to this criterion, then any menu that includes it must be at least as complex even if it contains stand-out options, possibly because their discovery may be too costly.

Proposition 3

The following are equivalent for a choice correspondence $C : \mathcal{M} \rightarrow \mathcal{M}$:

1. C satisfies Desirability, WARP, Deferral Monotonicity and Binary Choice Consistency.
2. There exist a unique preorder \succeq on X , a completion \succsim of this preorder, a function $\psi : \mathcal{M} \rightarrow \mathbb{R}$ and an integer n such that, for all $A, B \in \mathcal{M}$ and all $x, y, z \in X$

$$C(A) = \begin{cases} \emptyset, & \text{iff } \psi(A) > n \\ B_{\succsim}(A), & \text{otherwise} \end{cases} \quad (3a)$$

$$|A| = 1 \implies \psi(A) \leq n \quad (3b)$$

$$B \supset A \implies \psi(B) \geq \psi(A) \quad (3c)$$

$$\psi(\{x, y\}) \leq n \ \& \ \psi(\{y, z\}) \leq n \implies \psi(\{x, z\}) \leq n. \quad (3d)$$

The decision maker here is portrayed as a utility maximizer who is non-standard in that he employs a complexity/overload criterion that determines whether he will engage in utility maximization at some menu or whether he will *defer* at that menu instead. It will therefore be referred to as the model of *overload-constrained rationality* (OCR). Specifically, the individual’s complexity criterion here is captured by the *complexity function* ψ and *complexity threshold* n . These may correspond to the number of elements in a menu and a cut-off menu size, respectively, or to the time it takes the decision maker to make a choice from a menu and the total time he has available. Whichever the case, (3a) suggests that the agent defers when and only when the decision problem is sufficiently complex according to the pair (ψ, n) .

It follows directly from (3c) that the complexity function is increasing with respect to menu inclusion, which is a consequence of the Deferral Monotonicity axiom. It also follows from (3a) and (3b) that a decision problem consisting of a singleton menu is never complex. Finally, (3d) suggests that if the binary menus $\{x, y\}$ and $\{y, z\}$ are not complex, then the same will be true for menu $\{x, z\}$. When ψ is the cardinality function, for example, then non-complexity of one binary menu implies that all such menus must also be non-complex, consistent with (3d). Other interpretations of ψ are also compatible with this axiom. For instance, if $\psi(A)$ counts the number of seconds that the decision maker believes are required for him to go through each alternative in A and n stands for the number of seconds that he is willing to devote to this task, then (3d) is again satisfied.

Unlike the models of indecisiveness- and undesirability-driven deferral that were presented above where the agent’s preferences are fully and almost fully recoverable from decision data, respectively, in the OCR model such recovery is only partially possible. Indeed, behavior compatible with the four axioms of Proposition 3 uniquely pin down a generally incomplete preference relation \succeq which is defined by the rule $x \succeq y$ iff $x \in C(A)$ and $y \in A$ for some menu A . Given the complexity function ψ and its associated threshold n there may be alternatives x and

y for which $\psi(\{x, y\}) > n$ and $x, y \notin C(A)$ for every menu A with $\psi(A) \leq n$. For such alternatives it holds that $x \not\succeq y$ and $y \not\succeq x$. While the model is well-defined in terms of a completion of this preorder, since completions are generally not unique this fact restricts the extent to which preferences can be recovered in the OCR model.

In the special case where the complexity function ψ captures the number of alternatives in a menu, i.e. where $\psi(A) = |A|$ for all A , the model suggests a second explanation of the choice overload phenomenon that was first discussed in section 3. Specifically, in this case the model's interpretation is that the decision maker always counts the number of alternatives in a menu and if this exceeds his pre-specified menu-size threshold, he defers. This prediction is compatible with the choice-overload findings in Iyengar and Lepper (2000). Moreover, minimization of cognitive effort of the kind that is featured in the OCR model has been recognized as one of the four main drivers of choice overload in the very extensive and recent meta-analysis of this phenomenon that was carried out by Chernev et al (2015). On the other hand, unlike the explanation that is suggested by the MDC model, OCR cannot account for the observed breakdown of choice overload when a clearly dominant option is added to a menu that is considered complex.

As a corollary to Proposition 3 which may facilitate the use of the OCR model in applications we note that the stated four axioms characterize the existence of a utility function u on X , a complexity function ψ on \mathcal{M} that satisfies (3c)–(3d), and a number n , where both u and the pair (ψ, n) are unique up to strictly increasing transformations, such that, for all $A \in \mathcal{M}$,

$$C(A) = \begin{cases} \emptyset, & \text{iff } \psi(A) > n \\ \arg \max_{x \in A} u(x), & \text{otherwise} \end{cases} \quad (4)$$

Indeed, once the preference relation \succeq is completed by some weak order \succsim , the latter is representable by means of an ordinally unique utility function u , and, for any given pair (ψ, n) , (4) holds regardless of which u is chosen to represent \succsim . Similarly, for any choice of u that represents the given completion \succsim of \succeq , and any initial pair (ψ, n) , applying a strictly increasing transformation ϕ on the pair (ψ, n) results in the pair $(\phi, \phi(n))$ under which (4) still holds but for a different complexity scale.

6 Between the Three Models

In all three models the decision maker's choices satisfy WARP, Contraction Consistency and Binary Choice Consistency. In particular, since OCR predicts $C(B) \neq \emptyset$ whenever $C(A) \neq \emptyset$ and $B \subset A$, Contraction Consistency is implied by WARP. Compatibility of MDC with Binary Choice Consistency is also obvious in view of the fact that preferences in this model are transitive. To see that this axiom is also compatible with DCR, observe that $x \in C(\{x, y\})$ and $y \in C(\{y, z\})$ here suggest that both x and y are above the desirability threshold. Moreover, since preferences are transitive and the "desirable" option x must be preferred to z , we also have $x \in C(\{x, z\})$, as required.

The Desirability axiom is clearly satisfied in MDC and OCR (in the case of the latter this is due to (3b)). Its logical complement, Undesirability, is satisfied by DCR only. It is intuitive that Contractive Undesirability too is satisfied only by DCR. Indeed, moving from A where $C(A) = \emptyset$ to $B \subset A$ in MDC can render some option in B a most preferred one in that menu, and may therefore result in $C(B) \neq \emptyset$. In OCR one may have $\psi(A) > n$ but $\psi(B) \leq n$, in which case too one obtains $C(B) \neq \emptyset$.

Like MDC, the DCR model is compatible with Strong Expansion. The logic behind this is analogous to the logic that shows conformity of this model with Binary Choice Consistency. On the other hand, OCR may lead to violations of Strong Expansion. For example, if $\psi(\cdot) = |\cdot|$ and $n = 2$ we would have $x \in C(\{x, y\})$, $y \in C(\{x, z\})$ and $C(\{x, y, z\}) = \emptyset$. Finally, neither

MDC nor DCR obey Deferral Monotonicity in general. In the former case this can happen when $C(A) = \emptyset$ and the menu $B \supset A$ includes an option $x \in B \setminus A$ that is weakly preferred to all other options in B . In the latter case we have $C(A) = \emptyset$ when A consists of “undesirable” options, and $C(B) \neq \emptyset$ for every $B \supset A$ that contains at least one “desirable” alternative.

Table 1 summarizes the axiomatic similarities and differences between the three proposed models of rational choice deferral by checking conformity of each model with each of the axioms that were presented above.

Table 1: Axiom Compatibility

	Indecisiveness (MDC)	Unattractiveness (DCR)	Overload (OCR)
WARP	✓	✓	✓
Binary Choice Consistency	✓	✓	✓
Contraction Consistency	✓	✓	✓
Strong Expansion	✓	✓	×
Desirability	✓	×	✓
Undesirability	×	✓	×
Contractive Undesirability	×	✓	×
Deferral Monotonicity	×	×	✓

Although the two models are generally distinct, we note that there exist non-trivial datasets that are explainable by both MDC and OCR. For example, consider the case where $X = \{w, x, y, z\}$ and assume that choices are made from all singletons. Suppose now that $C(\{w, x\}) = \{w\}$, $C(\{y, z\}) = \{y\}$ and $C(A) = \emptyset$ for all remaining menus $A \subseteq X$. The MDC model is compatible with these choices when the preorder \succsim on X is such that $w \succ x$, $y \succ z$ and no other comparisons between distinct alternatives are possible. Yet, the OCR model is also compatible with them when: i) the complexity threshold is $n = 1$; ii) the complexity function is $\psi(\{w\}) = \psi(\{x\}) = \psi(\{y\}) = \psi(\{z\}) = \psi(\{w, x\}) = \psi(\{y, z\}) = 1$ and $\psi(A) = 2$ for all other $A \subseteq X$; iii) \succsim is any weak order on X such that $w \succ x$ and $y \succ z$. On the other hand, suppose Desirability is satisfied and $C(A) = \{w\}$ whenever $w \in A$ while $C(A) = \emptyset$ for every other non-singleton menu A . Then, the MDC model explains the dataset by means of the preorder \succsim where $w \succ x, y, z$ and no other comparisons between distinct options are possible, while the OCR model is incompatible with it.

7 Does the Approach to Modelling Deferral Matter?

We now raise the question of whether the adoption of a possibly empty-valued choice correspondence is essential for modelling rational choice deferral in the context of the three models that we analyzed above. In particular, we compare this with the approach where deferral is modelled by an explicit and always feasible option. To this end, suppose now that a decision problem comprises a menu $A \in \mathcal{M}$ together with an explicit deferral option d that is also an element of the grand choice set X . Moreover, assume that the collection of all decision problems now becomes $\mathcal{M}^* := \{A \cup \{d\} : A \in \mathcal{M}\}$. Also, let a choice correspondence here be a mapping C^* from \mathcal{M}^* into itself.

Let us first consider the effects of this alternative modelling approach on the MDC model. Modifying (1) along these lines results in the choice procedure whereby, for all $A \in \mathcal{M}$,

$$C^*(A \cup \{d\}) = \begin{cases} \{d\}, & \text{iff } B_{\succsim}(A \cup \{d\}) = \emptyset \\ B_{\succsim}(A \cup \{d\}), & \text{otherwise} \end{cases} \quad (5)$$

where \succsim is a possibly incomplete preorder on X . In order to bring (5) as close as possible to the MDC model which aims to explain deferral when the default option associated with it is

objectively inferior to all other alternatives but may still be chosen when some of these superior alternatives are incomparable, one may assume that $x \succ d$ for all $x \in X \setminus \{d\}$. Unlike the MDC model, however, (5) now predicts WARP violations whenever the agent defers at a problem $A \cup \{d\}$ in which A contains at least two options. Indeed, $\{x\} = C^*(\{x\} \cup \{d\})$ for all $x \in X$ and $\{d\} = C^*(\{x, y\} \cup \{d\})$ whenever $x \not\prec y$ and $y \not\prec x$.¹²

Modifying the OCR model to also make it compatible with this alternative approach, and assuming that \succsim is a weak order such that $x \succ d$ for all $x \neq d$, we obtain the choice procedure whereby for all $A \in \mathcal{M}$,

$$C^*(A \cup \{d\}) = \begin{cases} \{d\}, & \text{iff } \psi(A \cup \{d\}) > n \\ B_{\succsim}(A \cup \{d\}), & \text{otherwise} \end{cases} \quad (6)$$

Similar to (5) but unlike the original OCR model that was laid out in (3), this procedure necessarily leads to WARP violations whenever d is chosen in some decision problem $A \cup \{d\}$.

Finally, modifying the DCR model along the same lines we obtain

$$C^*(A \cup \{d\}) = \begin{cases} \{d\}, & \text{iff } B_{\succsim}(A \cup \{d\}) = \{d\} \\ B_{\succsim}(A \cup \{d\}), & \text{otherwise} \end{cases} \quad (7)$$

for all $A \in \mathcal{M}$. Unlike (5) and (6), and consistent with the motivation for the DCR model, the deferral option here is not necessarily inferior to everything else, while it naturally takes the role of the desirability threshold x^* in (2). For these reasons, and unlike the MDC and OCR procedures, no WARP violations are induced by (7). This fact makes DCR robust with respect to the modelling approach for deferral.

We first note that, given the dichotomy between “decision” and “choice” that we drew in the second section, and given also the importance of the modelling approach for assessing WARP consistency at the decision level in the context of the MDC and OCR models, arriving at the “right” conclusion concerning the (ir)rationality of a deferring agent’s overall behavior is to some extent a philosophical problem. One insight that the analysis of this paper does offer here, however, is that a decision maker who is non-standard in that either his preferences are incomplete or his ability to maximize them is constrained by cognitive limitations is nevertheless able to defer in ways that allow his active choices to be fully WARP-consistent, just like those of a utility maximizer. In other words, the basic analytical tools of revealed-preference theory have proved flexible enough to formally accommodate the viewpoint that certain sources of deferral can be thought of as being compatible with preference-maximizing and WARP-consistent choice. More importantly, as we also argued in section 3, fitting the above two models into a framework where deferral is captured by an explicit no-choice option, and therefore placing them in the large class of models that feature WARP-inconsistent and bounded-rational behavior, has the disadvantage of “hiding under the rug” the very sizeable fully rational component of the agent’s behavior which involves choices between non-deferral options. As we argue below, this disadvantage has negative practical implications for empirical work in such decision environments.

8 Applicability and Relevance

8.1 Empirical Analysis of Datasets with Deferral Observations

A frequently-employed criterion in empirical studies that assess the rationality of a given choice dataset is the so-called Houtman and Maks (1985) index.¹³ This counts the minimum number of changes that need to be made in a given dataset for it to be as if it was generated by

¹²We note that this choice procedure is distinct from the standard incompleteness-based explanation of status quo bias that goes back to Bewley (2002), according to which the status quo option is chosen if it is merely undominated.

¹³See, for instance, Choi et al (2007) and Choi et al (2014).

maximization of some weak order, with zero changes reflecting perfect conformity with utility maximization.¹⁴ A generalization of this rationality index that makes it particularly relevant and useful in cases where datasets include deferral observations is proposed in Costa-Gomes et al (2016). The generalized index counts the minimum number of changes that need to be made for the dataset to be as if it was generated by a possibly incomplete preorder. The MDC model provides a choice-theoretic foundation for this generalization of the Houtman-Maks index in the sense that the number of required changes in a given dataset is zero if and only if the dataset conforms perfectly with MDC. This natural generalization of the Houtman-Maks index would be impossible under the explicit-option approach to deferral that was outlined above. As far as identifying or approximating the decision process that has generated a choice & deferral dataset is concerned, Table 1 provides guidance on the kinds of axiomatic combinations one needs to look out for in the data in order to check conformity with the above three models and also to disentangle them. In principle, the distinction between DCR and MDC or OCR is easier than that between the latter two models, due to the orthogonality between the Desirability and Undesirability axioms.

8.2 Choice under Uncertainty

Bewley’s (2002) model of “objectively rational” preferences under uncertainty is based on a possibly incomplete preference relation and a set of priors over the states of the world such that one act is preferred to another if and only if its expected utility is higher under every prior in the set. In Appendix 2 we show how a two-period extension of the MDC model can be used to provide choice-theoretic foundations for the Bewley model where any first-period deferral that is a result of applying the above decision rule is resolved in the second period when the agent becomes a subjective expected utility maximizer by narrowing down his beliefs to a single prior.

8.3 Robust Social Choice

The possibility of a social welfare function that produces transitive but generally incomplete societal preferences when the choice domain contains general, risky and uncertain objects has been investigated in Fishburn (1974), Danan et al (2015) and Danan et al (2016), respectively. This approach allows for analyzing *robust* social decisions in the sense that the society’s preference ranking can be made to agree with *every* agent’s preferences, including (where relevant), their beliefs. In such contexts, if one insists that the social ranking satisfy such strong notions of Pareto efficiency, the transition from social welfare to social *choice* functions or correspondences comes about in the way dictated by the MDC model: Given a preference profile, the social planner chooses a social alternative at some menu if and only if this is weakly preferred to every alternative in the menu according to society’s preferences (hence according to every agent’s preferences), and defers otherwise.

8.4 Industrial Organization

A recent direction taken in behavioral industrial organization is motivated by the presence of potentially choice-deferring individuals in the consumer population. Specifically, in the “opt-in” version of the oligopolistic model introduced in Bachi and Spiegler (2015)¹⁵ where consumers are not endowed with some alternative, they are assumed to defer whenever they are unable to detect a preference-dominant item in the set of multi-attribute products offered by the firms. These consumers are assumed to behave exactly as described in the MDC model. In addition, Gerasimou and Papi (2015) introduce a duopolistic model where otherwise rational consumers are heterogeneously overloaded as in the special case of the OCR procedure where menu complexity is identified with cardinality.

¹⁴Apestequia and Ballester (2015) propose an alternative rationality index and also provide a detailed comparative discussion of this and other rationality criteria, including Houtman-Maks’s.

¹⁵See also Spiegler (2015).

9 Related Literature

The MDC model shares some features with the *Conflict Decision Avoidance* (CDA) (Dean, 2008) and *Extended Partial Dominance* (EPD) procedures (Gerasimou, 2016). In all three models the agent has incomplete preferences, but even when one compares their predictions in decision problems that belong to the same class, these generally differ even when preferences coincide. Specifically, in CDA the agent chooses i) a most preferred option if one exists, ii) the status quo option when the latter takes the form of a feasible option that is dominated by nothing else in the menu according to a *secondary* preference criterion; iii) an option that maximizes a *completion* of his primary preferences otherwise, irrespective of whether the status quo is modelled as an element of the menu or not. Key differences between MDC and CDA are the following: 1) MDC applies to decision problems without an explicit status quo and explains choice deferral but not status quo bias, while CDA does the opposite; 2) MDC features maximization of a *single* preference relation; 3) MDC obeys WARP. The EPD model on the other hand features WARP-inconsistent choices that are based on the joint criteria of an alternative being totally undominated *and* partially dominant according to a single acyclic preference relation. This model is compatible with both choice deferral and status quo bias, and provides a behavioral distinction between them. At the same time, the context-dependent nature of this model results in WARP violations, which are ruled out in MDC.

Frick (2015) proposed the *monotone threshold representation* (MTR) model which is somewhat similar to OCR in that both feature a menu function (called the *departure threshold* in MTR) which is weakly increasing with respect to set inclusion. The MTR model predicts increasingly inconsistent behavior as the menu size increases but, unlike OCR, does not allow for choice deferral. The two models are not nested. Indeed, unless it satisfies Nonemptiness (in which case it reduces to rational choice), OCR violates Frick's *Occasional Optimality* axiom, which weakens WARP but implies Nonemptiness. On the other hand, MTR generally violates WARP. Dean (2008) and Dean et al (2014) explain the incidence of status quo bias (but not choice deferral) in "large" menus.¹⁶ The former paper does so by means of the *information overload* model which generalizes the CDA model mentioned above and predicts that the agent's preferences are less complete in larger menus. The latter paper attributes these instances of status quo bias to the agent's limited attention, which results in a subset of the original menu ultimately being considered. In the menu-preference literature, Sarver (2008) and Ortoleva (2013) provide preference representation theorems in the domain of menus of lotteries where individuals are modelled as having a preference for smaller menus due to regret anticipation and thinking aversion, respectively. Buturak and Evren (2015) offer a choice-theoretic foundation of Sarver's model that permits choice deferral. In stochastic choice, Fudenberg and Strzalecki (2015) generalize the dynamic logit model in a way that allows the decision maker to have a preference for smaller menus.

We further note that the DCR and OCR models can be embedded in the "choice with frames" model of Salant and Rubinstein (2008) once the latter is suitably generalized to allow for choice deferral. Specifically, letting a decision problem be described by a pair (A, f) where A is a menu and f is a frame that influences decision at A , as these authors proposed, the DCR model would suggest that f is constant across all problems and coincides with the agent's desirability threshold x^* . The OCR model would also suggest that f is constant across all problems. Yet, here f would coincide with the decision maker's "complexity" pair (ψ, n) instead. It is not clear, however, if the MDC model can also be embedded in this framework.

Finally, we emphasize that there are many potential sources of choice deferral for which we have not accounted in this paper. Some of those studied by economists include consumer search, dynamically inconsistent preferences and procrastination (O'Donoghue and Rabin, 1999), procedural reasons such as choice refusal considerations (Gaertner and Xu, 2004), contextual inference and the informational content of a menu (Kamenica, 2008), bounded rationality (Manzini and Mariotti, 2014), as well as rational regret anticipation (Buturak and Evren, 2015). The reader is referred to the cited works for more details on these alternative approaches.

¹⁶The strengthening of status quo bias in large menus was first documented in Samuelson and Zeckhauser (1988).

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Appendix 1: Rational Choice as a Special Case

When preferences are complete in the MDC model of Proposition 1, the latter clearly reduces to utility maximization. From the axiomatic point of view this is equivalent to Nonemptiness being satisfied on top of the other axioms. In this case, Desirability as well as Contraction Consistency and Strong Expansion follow directly from Nonemptiness, and from Nonemptiness together with WARP, respectively. The result below provides a novel decomposition of Nonemptiness + WARP that characterize utility maximization in this context. This decomposition is in terms of the other two consistency axioms that are involved in Proposition 1. However, it also follows from this and the other statements below that the DCR model of Proposition 2 also reduces to rational choice over the entire set X when Undesirability is ruled out by Nonemptiness, and that Proposition 3 too collapses to utility maximization once this axiom is imposed.

Before stating the result we recall that a choice correspondence C satisfies *Expansion Consistency* or *Sen's β* if $x, y \in C(A)$, $B \supset A$ and $x \in C(B)$ implies $y \in C(B)$.

Proposition 4

The following are equivalent for a choice correspondence $C : \mathcal{M} \rightarrow \mathcal{M}$:

1. C satisfies Nonemptiness and WARP.
2. C satisfies Nonemptiness, Contraction Consistency and Expansion Consistency.
3. C satisfies Nonemptiness, Contraction Consistency and Strong Expansion.
4. There exists a unique weak order \succsim on X such that, for all $A \in \mathcal{M}$,

$$C(A) = B_{\succsim}(A). \quad (8)$$

Proof of Proposition 4

We will show that $2 \Leftrightarrow 3$. For the \Rightarrow direction, suppose $x \in C(A)$, $y \in A$ and $y \in C(B)$. Assume to the contrary that $x \notin C(A \cup B)$. Let $z \in C(A \cup B)$. From Contraction Consistency we have $z \in C(\{x, z\})$ and $z \in C(\{y, z\})$. Consider first the case where $y \in C(A \cup B)$. In view of the above, Contraction Consistency implies $C(\{y, z\}) = \{y, z\}$ and $C(\{x, y\}) = \{x, y\}$. Since $A \cup B \supset \{x, y\}$, it follows from $C(\{x, y\}) = \{x, y\}$, Expansion Consistency and $y \in C(A \cup B)$ that $x \in C(A \cup B)$, a contradiction. Now consider the case where $y \notin C(A \cup B)$. If $z \in A$, then $x \in C(A)$, $z \in C(A \cup B)$ and Contraction Consistency imply $C(\{x, z\}) = \{x, z\}$. Expansion Consistency now again implies $x \in C(A \cup B)$, a contradiction. If $z \in B$, then $y \in C(B)$, $z \in C(A \cup B)$ and Contraction Consistency imply $C(\{y, z\}) = \{y, z\}$. Expansion Consistency again implies $y \in C(A \cup B)$, a contradiction. This proves that Strong Expansion holds.

For the converse implication, suppose $x, y \in C(A)$, $B \supset A$ and $x \in C(B)$. We must show that $y \in C(B)$. Since $x, y \in C(A)$, Contraction Consistency implies $C(\{x, y\}) = \{x, y\}$. Since $y \in C(\{x, y\})$ and $x \in C(B)$, it follows from Strong Expansion and $y \in B$ that $y \in C(B)$. ■

The third statement in Proposition 4 is novel and clarifies that Expansion Consistency and Strong Expansion are equivalent under Nonemptiness and Contraction Consistency. It also suggests that either of these axiomatic combinations characterizes rational choice. The equivalence between the first and fourth statements is due to Arrow (1959), while that between the second and fourth is due to Sen (1971).

Appendix 2: Sequential Choice and the Resolution of Indecisiveness

Maximally Dominant Choice over Two Periods

Intuition suggests that indecisiveness between alternatives can be resolved over time, for example after reflection or suitable information acquisition. As a result, choice deferral that is driven solely by indecisiveness should vanish too. To model this, our primitive here is a *pair* of choice correspondences $C_1, C_2 : \mathcal{M} \rightarrow \mathcal{M}$. We interpret C_i as capturing the agent's behavior in period $i = 1, 2$. Given that the domain of both C_1 and C_2 is the same collection of menus, and each menu in this collection represents a decision problem, we are implicitly assuming that first-period problems neither disappear nor change in the second period.

The axioms that follow impose some structure on the pair C_1, C_2 and are mainly normative.

Eventual Nonemptiness

If $\emptyset \neq A \in \mathcal{M}$, then $C_2(A) \neq \emptyset$.

This axiom precludes indefinite deferral by requiring the decision maker to choose an alternative from every menu in the second period, regardless of whether he initially deferred at this menu or not. This restriction is relevant in decision problems where the individual must ultimately make a choice, e.g. when deciding among job offers. When the agent defers at some menu A initially but chooses from it eventually, we can think of this choice as being the result of the agent receiving additional information about the alternatives in A during the interim stage via some unmodelled process.

Sequential Choice Consistency

If $x \in C_1(A)$, then $x \in C_2(A)$.

This axiom requires the agent's second-period choice at every menu A to be consistent with the choice (if any) that would have been made by him had he faced that menu in the first period.

Proposition 5

The following are equivalent for two choice correspondences $C_1, C_2 : \mathcal{M} \rightarrow \mathcal{M}$:

1. C_1 and C_2 satisfy Desirability, WARP, Contraction Consistency, Strong Expansion, Eventual Nonemptiness and Sequential Choice Consistency.
2. There exists a unique preorder \succsim_1 and a completion \succsim_2 of this preorder such that, for all $A \in \mathcal{M}$

$$C_i(A) = \{x \in A : x \succsim_i y \text{ for all } y \in A\}, \quad i = 1, 2. \quad (9)$$

Proof of Proposition 5:

We only prove that $1 \Rightarrow 2$ (the proof of the converse implication is straightforward). Let C_1, C_2 satisfy the axioms and define the relation \succsim_1 on X by $x \succsim_1 y$ if there exists $A \in \mathcal{M}$ such that $x \in C_1(A)$ and $y \in A$. Since all singletons are included in \mathcal{M} , it follows from Desirability that \succsim_1 is reflexive. Suppose now that $x \succsim_1 y$ and $y \succsim_1 z$ for some $x, y, z \in X$. There exist $A, B \in \mathcal{M}$ such that $x \in C_1(A), y \in A$ and $y \in C_1(B), z \in B$. From Strong Expansion, $x \in C_1(A \cup B)$. Since $\{x, y, z\} \in \mathcal{M}$ by assumption and $\{x, y, z\} \subseteq A \cup B$, it follows from Contraction Consistency that $x \in C_1(\{x, y, z\})$. Hence, $x \succsim_1 z$. This shows that \succsim_1 is transitive, hence a preorder.

We must first prove that

$$C_1(A) = \{x \in A : x \succsim_1 y \text{ for all } y \in A\}. \quad (10)$$

Suppose that (10) is not satisfied. Assume that there is $A \in \mathcal{M}$ such that $x \in A$, $x \succsim_1 y$ for all $y \in A$ and $x \notin C_1(A)$. If $y \in C_1(A)$ for some $y \neq x$, then this fact and $x \succsim_1 y$ together contradict WARP. Suppose $C_1(A) = \emptyset$ instead. The postulate that $x \succsim_1 y$ for all $y \in A$ is equivalent to the postulate that for each $y \in A$ there exists $B_y \in \mathcal{M}$ such that $y \in B_y$ and $x \in C_1(B_y)$. Let $S := \bigcup \{B_y : y \in A\}$. Clearly, $A \subseteq S$. Moreover, the full domain assumption ensures that $S \in \mathcal{M}$. Repeated application of Strong Expansion gives $x \in C_1(S)$. Since $A \subseteq S$, $x \in A$ and $A \in \mathcal{M}$, it follows from Contraction Consistency that $x \in C_1(A)$. Conversely, if $x \in C_1(A)$ and $y \in A$, then, by definition of \succsim_1 , $x \succsim_1 y$. Thus, (10) holds. Uniqueness of \succsim_1 follows from the fact that all binary menus are included in \mathcal{M} .

Now define \succsim_2 on X by $x \succsim_2 y$ if there is $A \in \mathcal{M}$ such that $x \in C_2(A)$ and $y \in A$. We know from Arrow (1959) that, since Eventual Nonemptiness and WARP hold, \succsim_2 is a weak order that satisfies

$$C_2(A) = \{x \in A : x \succsim_2 y \text{ for all } y \in A\}. \quad (11)$$

Finally, from Sequential Choice Consistency we have $x \succsim_2 y$ whenever $x \succsim_1 y$. Hence, \succsim_2 is indeed a completion of \succsim_1 . \blacksquare

This can be interpreted as a two-period model in which deferral is possible in the first period but a choice is necessarily made in the second. Moreover, the choice that is made in the second period is by maximization of the agent's first-period preferences which have meanwhile been completed through some unmodelled process (e.g. via information acquisition). Following Gilboa et al (2010), one may think of \succsim_1 as capturing the "objective" but incomplete component of the agent's preferences and of \succsim_2 as capturing his "subjective" component that completes them. Being objective in the sense that the agent can convince others about the validity of the preference statements included in \succsim_1 , this part of the agent's preferences is stable and not subject to change after any additional information acquisition. This provides a theoretical justification for the preclusion of cross-period choice reversals that is featured in this model.

A Choice-Theoretic Foundation for Bewley Preferences

A specific domain in which the sequential MDC model of Proposition 5 is particularly illuminating is that of choice under uncertainty. In this domain, the finite set of outcomes X is coupled by a pair (S, Σ) , where S is a finite set of possible states of the world and Σ an algebra of events derived from S . Moreover, the objects of choice are acts that belong to the set \mathcal{F} of all Σ -measurable functions mapping S into $\Delta(X)$, the latter being the set of probability distributions on X . Let \mathcal{F} be endowed with a suitable topology and let \mathcal{D} denote the set of all convex, compact subsets of \mathcal{F} (the empty set is trivially both convex and compact). Finally, assume that the choice correspondences C_i , $i = 1, 2$, map \mathcal{D} into itself.

The benchmark model of incomplete preferences under uncertainty is Bewley (2002). In the weak-preference analogue of this model that was axiomatized in Gilboa et al (2010), the decision maker is portrayed as having "objectively rational" preferences over \mathcal{F} that are captured by a preorder \succsim (which is necessarily complete only in the subdomain of constant acts), and also as having beliefs over S that are captured by a non-singleton closed, convex set Π of probability measures on (S, Σ) . In this version of the Bewley model the agent compares two acts f and g by the following, partially applicable rule:

$$f \succsim g \iff \int_S \mathbb{E}_{f(s)} u d\pi(s) \geq \int_S \mathbb{E}_{g(s)} u d\pi(s) \quad \text{for all } \pi \in \Pi. \quad (12)$$

Here, $u : X \rightarrow \mathbb{R}$ is a von Neumann-Morgenstern utility function, which exists due to the completeness of \succsim in the subdomain of constant acts, $\Delta(X)$, while $\mathbb{E}_{f(s)} u = \sum_{x \in X} f(s)(x) u(x)$.

Being derived from a model of *preference* and not *choice* under uncertainty, the Bewley rule does not directly specify what the agent's decision is in those cases where no feasible act is preferred to all others in a given menu. However, Bewley (2002) argued that the agent *preserves the status quo* unless he can find an act that is preferred to it. Choosing from a set of insurance policies when already endowed with one of them is an example of a problem where this is possible. Yet, as we've been arguing throughout this paper, and as we also argued in Gerasimou (2016), there are also cases where the status quo does not take the form of an explicit and concrete option such as this. A suitable example would again be the problem of choosing an insurance policy when not already endowed with one. Assuming that every option is acceptable in principle, the status quo here does not actually enter the decision problem. One way forward for the decision maker here is to choose among the preference-undominated acts, as studied in Stoye (2015). Another possibility, however, is that the lack of a partially or totally dominant option in such cases can lead to deferral.

In light of the above, we can use Proposition 5 to provide an alternative choice-theoretic interpretation of the Bewley model for the case of decision problems in which the original Bewley decision rule may be inapplicable due to the different nature of the status quo and where the agent is unwilling to choose an option that is merely undominated. Specifically, given the above primitives one can model an agent faced with a menu A in \mathcal{D} as possibly deferring à la Bewley (2002) in the first period and as choosing like a subjective expected utility maximizer in the second period, as follows:

$$C_1(A) = \bigcap_{\pi \in \Pi} \arg \max_{f \in A} \int_S \mathbb{E}_{f(s)} u d\pi(s) \quad (13a)$$

$$C_2(A) = \arg \max_{f \in A} \int_S \mathbb{E}_{f(s)} u dp(s). \quad (13b)$$

Under (13) the individual chooses a most preferred act in the first period if and only if one exists; when such an act does not exist, choice is deferred to the second period, by which time he has narrowed down his beliefs to a *single* prior p from the original set Π , and chooses as a subjective expected utility maximizer according to the same, first-period utility function u that dictates his unchanged tastes.

In the same vein, one can use Proposition 5 to provide a choice-theoretic foundation of the objective vs subjective rationality model of preferences under uncertainty that was axiomatized in Gilboa et al (2010) by attaching a temporal dimension to the associated objective and subjective preference relations, as also suggested above. Here, the relation \succsim_1 is still incomplete à la Bewley while the relation \succsim_2 is a "maxmin" ambiguity-averse completion of \succsim_1 . The authors provided necessary and sufficient conditions on this pair of relations for there to exist a common set of priors Π and utility function u such that the former is represented by these two objects as in (12) and the latter according to the maxmin expected utility rule. We omit the details. Finally, we refer the reader to Pejsachowicz and Toussaert (2015) for an analysis of preferences over menus of lotteries where the primitive is also a pair of an incomplete preference relation and a completion of that relation, in which case the latter necessarily exhibits preference for flexibility.

Appendix 3: Proofs

Proof of Proposition 1:

This is included in the proof of Proposition 5 above. ■

Proof of Proposition 2:

Necessity of the axioms is straightforward to check. We prove sufficiency. To this end, we note first that the four axioms in the statement of the proposition jointly imply Strong Expansion. Suppose not. We have $x \in C(A)$, $y \in A$, $y \in C(B)$ and $C(A \cup B) = \emptyset$ (WARP is violated if

$x \notin C(A \cup B) \neq \emptyset$. It now follows from Contractive Undesirability and $C(A \cup B) = \emptyset$ that $C(A) = C(B) = \emptyset$, a contradiction.

Now let $X^* := \{x \in X : x \in C(A) \text{ for some } A \in \mathcal{M}\}$. Let $Y = X \setminus X^*$. From Undesirability, $Y \neq \emptyset$. Define the relation \triangleright on X by $x \triangleright y$ if $x \in C(A)$, $y \in A$ for some $A \in \mathcal{M}$, and the relation \succsim^* by

$$\succsim^* = \triangleright \cup \{(x, x) : x \in Y\}.$$

In view of Contraction Consistency and the definition of \succsim^* , this relation is reflexive. We will also show that the restriction of \succsim^* on X^* is complete and transitive, hence a weak order. Consider $x, y, z \in X^*$ and suppose $x \succsim^* y \succsim^* z$. Employing the same Strong-Expansion argument that was used in the proof of Proposition 5 shows that $x \succsim^* z$, which establishes transitivity. Now suppose to the contrary that $x \not\succeq^* y$ and $y \not\succeq^* x$ for some $x, y \in X^*$. It follows that $C(\{x, y\}) = \emptyset$. From Contractive Undesirability we then obtain $C(\{x\}) = C(\{y\}) = \emptyset$. Since $x, y \in X^*$, there exist $A, B \in \mathcal{M}$ such that $x \in C(A)$ and $y \in C(B)$. In light of this, $C(\{x\}) = C(\{y\}) = \emptyset$ contradicts Contraction Consistency. Therefore, \succsim^* is a reflexive relation on X and a weak order on $X^* \subset X$.

Now order the \sim^* -equivalence classes of X^* by $[x_i] \gg [x_j]$ if $x \succ^* x'$ for $x \in [x_i]$ and $x' \in [x_j]$. Since X^* is finite, there is some integer k such that $[x_1] \gg [x_2] \gg \dots \gg [x_k]$. Next, let R be an arbitrary weak order on Y with its symmetric and asymmetric parts denoted by I and P , respectively, and order the I -equivalence classes of Y by $[y_i]P[y_j]$ if yPy' for $y \in [y_i]$ and $y' \in [y_j]$. Again, there exists an integer n such that $[y_1]P[y_2]P\dots P[y_n]$. Finally, define the relation \succsim by

$$\succsim = \succsim^* \cup R \cup \bigcup_{i \leq k, j \leq n} ([x_i] \times [y_j]) \cup \bigcup_{i \leq k} ([x_i] \times [x_i]) \cup \bigcup_{j \leq n} ([y_j] \times [y_j]).$$

where

$$[x_i] \times [y_j] = \bigcup \{(x, y) : x \in [x_i], y \in [y_j]\}.$$

It is easy to check that \succsim is a weak order that extends \succsim^* from X^* to X .

Let $x^* \in Y$ be such that $x^* \succsim y$ for all $y \in Y$. To establish the first part in (2), let $C(A) = \emptyset$ for some $A \in \mathcal{M}$. Suppose to the contrary that $z \succ x^*$ for some $z \in B_{\succ}(A)$. Since, by definition, $z \succ x^*$ implies $z \succ^* x^*$, it follows from the above that $z \in X^*$ and therefore that $z \in C(D)$ for some $D \in \mathcal{M}$. In view of Contraction Consistency, this implies $C(\{z\}) = \{z\}$. Since $\{z\} \subset A$, this and $C(A) = \emptyset$ together contradict Contractive Undesirability. Thus, $C(A) = \emptyset$ implies $x^* \succsim z$ for $z \in B_{\succ}(A)$. Conversely, suppose $x^* \succsim z$ for $z \in B_{\succ}(A)$. This implies $x^* \succsim z$ for all $z \in A$. In view of the definition of x^* and Y , this further implies $A \subseteq Y$. It follows then that $z \notin C(A)$ for all $z \in A$, and therefore that $C(A) = \emptyset$. To establish the second part, let $z \succ x^*$ for $z \in B_{\succ}(A)$. From above, $C(A) \neq \emptyset$. Suppose $z \notin C(A)$ and $y \in C(A)$. It holds that $y \succ z$. But since $z \in B_{\succ}(A)$ implies $z \succ x$ for all $x \in A$, a contradiction obtains. Therefore, $C(A) \neq \emptyset$ implies $C(A) = B_{\succ}(A)$.

Finally, consider a weak order \succsim' on X such that $x \succsim y \Leftrightarrow x \succsim' y$ for all $x, y \in X^*$ such that $x, y \succ x^*$. Clearly, $B_{\succ}(A) = B_{\succ'}(A)$ for all $A \in \mathcal{M}$ such that $A \cap X^* \neq \emptyset$. Hence, $C(A) \neq \emptyset$ implies $C(A) = B_{\succ'}(A)$. Therefore, \succsim quasi-rationalizes C uniquely up to x^* . \blacksquare

Proof of Proposition 3.

For some $n \in \mathbb{N}$, define the function $\psi : \mathcal{M} \rightarrow \mathbb{R}$ by

$$\psi(A) := \begin{cases} n, & \text{if } C(A) \neq \emptyset \\ n + 1, & \text{if } C(A) = \emptyset \end{cases} \quad (14)$$

By construction, $C(A) = \emptyset$ iff $\psi(A) > n$. Moreover, Desirability, Deferral Monotonicity and Binary Choice Consistency ensure that ψ also satisfies (3b), (3c) and (3d), respectively.

We have $C(A) \neq \emptyset$ iff $\psi(A) = n$. We must show that there exists a unique preorder \supseteq and a completion \succsim of \supseteq such that $C(A) \neq \emptyset$ implies $C(A) = B_{\succ}(A)$ for all $A \in \mathcal{M}$. The proof of

this relies on a suitable adaptation of the argument in Richter (1966, p. 640). Define the relation \succeq on X by $x \succeq y$ if there is $A \in \mathcal{M}$ such that $x \in C(A)$ and $y \in A$. From Desirability, \succeq is reflexive. Now suppose $x \in C(A)$, $y \in A$ and $y \in C(B)$, $z \in B$. Since all binary menus are in \mathcal{M} by assumption, in view of WARP and (the contrapositive of) Deferral Monotonicity, these assumptions imply $x \in C\{x, y\}$ and $y \in C\{y, z\}$, respectively. It now follows from Binary Choice Consistency that $x \in C\{x, z\}$, which implies $x \succsim z$. Therefore, \succeq is transitive, hence a preorder. Since all binary menus are in \mathcal{M} , \succeq is uniquely defined.

Define the relation P on X by xPy if $x \succeq y$ and $y \not\succeq x$. Since P is the asymmetric part of the preorder \succeq , P is a strict partial order. Define the relation J on X by xJy if $x \succeq y$ and $y \succeq x$. J is the symmetric part of the preorder \succeq , hence an equivalence relation on X . Let the J -equivalence class of $x \in X$ be denoted by $[x]$. Let \mathcal{X} be the quotient set derived from X by J . Let \mathcal{P} be a relation on \mathcal{X} defined by $[x]\mathcal{P}[y]$ if xPy . The relation \mathcal{P} is a strict partial order on \mathcal{X} . From Szpilrajn's (1930) theorem, it admits an extension into a strict linear order \mathcal{R} on \mathcal{X} . Now define the relation \succsim on X by $x \succsim y$ iff xJy or $[x]\mathcal{R}[y]$. It is easy to show that \succsim is a complete preorder on X .

Suppose $C(A) \neq \emptyset$. Let $x \in C(A)$. For all $y \in A$, it holds that xPy or xJy . Hence, $x \in C(A)$ implies $x \succsim y$ for all $y \in A$. Conversely, suppose $C(A) \neq \emptyset$, $x \in A$ and $x \succsim y$ for all $y \in A$. Let $z \in C(A)$. If $x = z$, there is nothing to prove. Suppose $z \neq x$. We have zJx or $[z]\mathcal{R}[x]$. At the same time, $x \succsim z$ implies xJz or $[x]\mathcal{R}[z]$. Suppose $x\mathcal{R}y$ and $y\mathcal{R}x$. Since $x \neq z$, this violates the asymmetry of \mathcal{R} . Hence, xJz . This implies $x \in C(B)$ and $z \in B$ for some $B \in \mathcal{M}$. Suppose $x \notin C(A)$. Since $z \in C(A)$, this contradicts WARP. Therefore, $x \in C(A)$. This completes the proof that (3a) holds.

With regard to necessity of the axioms, it follows from (3a) that WARP is satisfied. It also follows from (3a) and (3c) that Deferral Monotonicity is satisfied, and from (3a) and (3b) that Desirability is satisfied. Finally, it follows from (3a) and (3d) that Binary Choice Consistency is satisfied too. ■

Appendix 4: Axiom Independence

Let $X = \{w, x, y, z\}$ and $\mathcal{M} = \{A : A \subseteq X, A \neq \emptyset\}$. Each of the examples below presents a set of choices from elements of \mathcal{M} that satisfy all but one of the axioms involved in Proposition 1.

Not WARP

$$\begin{aligned} C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\} \\ C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = \{w\}, C(\{w, z\}) = \emptyset, C(\{x, y\}) = C(\{x, z\}) = \{x\}, C(\{y, z\}) = \emptyset \\ C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = \{x\}, C(\{w, y, z\}) = \emptyset, C(\{x, y, z\}) = \{x\} \\ C(\{w, x, y, z\}) &= \{x\} \end{aligned}$$

Not Desirability

$$\begin{aligned} C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \emptyset \\ C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = \{w\}, C(\{w, z\}) = \emptyset, C(\{x, y\}) = \{x\}, C(\{x, z\}) = C(\{y, z\}) = \emptyset \\ C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset \\ C(\{w, x, y, z\}) &= \emptyset \end{aligned}$$

Not Contraction Consistency

$$C(\{w\}) = \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\}$$

$$\begin{aligned}
C(\{w, x\}) &= \emptyset, C(\{w, y\}) = C(\{w, z\}) = \{w\}, C(\{x, y\}) = C(\{x, z\}) = C(\{y, z\}) = \emptyset \\
C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = \emptyset, C(\{w, y, z\}) = \{w\}, C(\{x, y, z\}) = \emptyset \\
C(\{w, x, y, z\}) &= \{w, x\}
\end{aligned}$$

Not Strong Expansion

$$\begin{aligned}
C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\} \\
C(\{w, x\}) &= C(\{w, y\}) = C(\{w, z\}) = \{w\}, C(\{x, y\}) = C(\{x, z\}) = C(\{y, z\}) = \emptyset \\
C(\{w, x, y\}) &= \{w\}, C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset \\
C(\{w, x, y, z\}) &= \emptyset
\end{aligned}$$

Each of the examples below presents a set of choices from elements of \mathcal{M} that satisfy all but one of the axioms involved in Proposition 2.

Not WARP

$$\begin{aligned}
C(\{w\}) &= \emptyset, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{\emptyset\} \\
C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = \{w, y\}, C(\{w, z\}) = \{w\}, C(\{x, y\}) = C(\{x, z\}) = \{x\}, C(\{y, z\}) = \{y\} \\
C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = \{x\}, C(\{w, y, z\}) = \{y\}, C(\{x, y, z\}) = \{x\} \\
C(\{w, x, y, z\}) &= \{x\}
\end{aligned}$$

Not Contraction Consistency

$$\begin{aligned}
C(\{w\}) &= \{w\}, C(\{x\}) = C(\{y\}) = C(\{z\}) = \emptyset \\
C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = C(\{w, z\}) = \{w\}, C(\{x, y\}) = C(\{x, z\}) = C(\{y, z\}) = \emptyset \\
C(\{w, x, y\}) &= \{w, x\} = C(\{w, x, z\}), C(\{w, y, z\}) = \{w\}, C(\{x, y, z\}) = \emptyset \\
C(\{w, x, y, z\}) &= \{w, x\}
\end{aligned}$$

Not Undesirability

$$\begin{aligned}
C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\} \\
C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = C(\{w, z\}) = \{w\}, C(\{x, y\}) = C(\{x, z\}) = \{x\}, C(\{y, z\}) = \{y, z\} \\
C(\{w, x, y\}) &= C(\{w, x, z\}) = \{w, x\}, C(\{w, y, z\}) = \{w\}, C(\{x, y, z\}) = \{x\} \\
C(\{w, x, y, z\}) &= \{w, x\}
\end{aligned}$$

Not Contractive Undesirability

$$\begin{aligned}
C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \emptyset \\
C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = \{w\}, C(\{w, z\}) = \emptyset, C(\{x, y\}) = \{x\}, C(\{x, z\}) = C(\{y, z\}) = \emptyset \\
C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset \\
C(\{w, x, y, z\}) &= \emptyset
\end{aligned}$$

Finally, each of the examples below presents a set of choices from elements of \mathcal{M} that satisfy all but one of the axioms involved in Proposition 3.

Not WARP

$$\begin{aligned} C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\} \\ C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = \{w\}, C(\{w, z\}) = \emptyset, C(\{x, y\}) = C(\{x, z\}) = \{x\}, C(\{y, z\}) = \emptyset \\ C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = C(\{w, z\}) = \emptyset, C(\{x, y, z\}) = \{x\} \\ C(\{w, x, y, z\}) &= \emptyset \end{aligned}$$

Not Desirability

$$\begin{aligned} C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \emptyset \\ C(\{w, x\}) &= \{w, x\}, C(\{w, y\}) = \{w\}, C(\{w, z\}) = \emptyset, C(\{x, y\}) = \{x\}, C(\{x, z\}) = C(\{y, z\}) = \emptyset \\ C(\{w, x, y\}) &= \{w, x\}, C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset \\ C(\{w, x, y, z\}) &= \emptyset \end{aligned}$$

Not Binary Choice Consistency

$$\begin{aligned} C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\} \\ C(\{w, x\}) &= \emptyset, C(\{w, y\}) = \emptyset, C(\{w, z\}) = C(\{x, y\}) = C(\{x, z\}) = \emptyset, C(\{y, z\}) = \{y\} \\ C(\{w, x, y\}) &= C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset \\ C(\{w, x, y, z\}) &= \emptyset \end{aligned}$$

Not Deferral Monotonicity

$$\begin{aligned} C(\{w\}) &= \{w\}, C(\{x\}) = \{x\}, C(\{y\}) = \{y\}, C(\{z\}) = \{z\} \\ C(\{w, x\}) &= C(\{w, y\}) = C(\{w, z\}) = \{w\}, C(\{x, y\}) = C(\{x, z\}) = C(\{y, z\}) = \emptyset \\ C(\{w, x, y\}) &= \{w\}, C(\{w, x, z\}) = C(\{w, y, z\}) = C(\{x, y, z\}) = \emptyset \\ C(\{w, x, y, z\}) &= \emptyset \end{aligned}$$