Secret ballots and costly information gathering: the jury size problem revisited

Guha, Brishti

Jawaharlal Nehru University

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Brishti Guha[1]

Abstract

Suppose paying attention during jury trials is costly, but that jurors do not pool information (as in contemporary Brazil, or ancient Athens). If inattentive jurors are as likely to be wrong as right, I find that small jury panels work better as long as identical jurors behave symmetrically. If not paying attention makes error more likely than not, jurors may co-ordinate on two different symmetric outcomes: a "high-attention" one or a "low attention" one. If social norms stigmatize shirking, jurors co-ordinate on the high-attention equilibrium, and a smaller jury yields better outcomes. However, increasing the jury up to a finite bound works better if norms are tolerant of shirking, in which case co-ordination on the low-attention outcome results. If the cost of attention is high, a bare majority of jurors pay attention, and efficiency increases in jury size up to a bound. The model also applies to elections and referendums.

Keywords: Jury size, pivotal voters, secret ballots, multiple equilibria, costly information.

JEL Classification: D82, K40, D72.

1. Introduction

Condorcet's jury theorem showed that if each juror were more likely to be right than wrong, the probability of arriving at a correct judgment increases in the number of jurors.[2] This theorem, however, relied on two key assumptions – independence and sincerity. It assumed that jurors make their decisions independently, and that each juror acts as if he is the only juror on the panel (that is, he acts as if his vote is pivotal). Much of the economics literature on Condorcet's theorem centers on violations of one or both of these assumptions.

[1] Centre for International Trade and Development, School of International Studies, Jawaharlal Nehru University, New Delhi 110067. Email: brishtiguha@gmail.com.

[2] McLean and Hewitt (1994).

One strand of the literature looks at how the incentives of individual jurors[3] change when they have the option of incurring an informational or effort cost to arrive at a better decision. This literature – which also encompasses voters and committees in general – emphasizes violations of Condorcet's independence assumption, pointing out that jurors may free ride on other jurors' information flows, rather than incur this cost. They can do this either if decisions are made through group deliberations, so that they can vote after listening to the opinions of more informed jurors, or if the costly information collected by the other jurors is hard, and is visible to the uninformed.

However, what if individual members on a committee *do* need to incur an effort cost to receive the right information, but *do not* have the option to free ride on the information of others? This is the primary question that this paper addresses. I also look at the effect of biases resulting from inattention, contrasting a model where inattentive jurors are more likely than not to make a mistake, with a benchmark model where inattentive jurors are equally likely to be correct and incorrect.

Free riding on informational flows is not possible if individual votes are secret, and deliberations do not occur. This is the informational environment I consider. This is, for instance, the case in Brazil, where jurors are instructed to vote privately without deliberations, and votes are aggregated using a simple majority rule[4] (Lieb 2007). It was also the practice in jury trials in classical Athens (Hansen 1991, Guha 2011), where each juror was given a hollow disc and a solid one, and cast their votes by choosing which of these discs to insert into a bronze urn, in such a manner that no one could observe which disc they had inserted (the other disc was discarded in a wooden urn so that other jurors could not infer one's vote by examining the remaining disc). The judgment was made through counting the number of solid versus hollow discs in the bronze urn after all votes had been cast, and using a simple majority rule (in later jury trials, the number of jurors was odd to avoid a hung jury[5]). The trial was settled in the course of one working day (nine and a half hours) during which jurors did not communicate or leave the courtroom. Thus, there was no opportunity to share information. Secret ballot is, of

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[3]Though this is applicable to members of any group which makes a decision through votes of individual members, I will use the term "jurors".

[4]A panel consists of seven jurors.

[5]This number was also often very large, 201 or 501.

course, also the practice in most modern elections and referendums, and one section of my paper applies my results to elections.

While proponents of secret ballot mostly cite the argument that secrecy ensures freedom from intimidation and undue influence, I focus on another advantage of secret voting without group deliberations – freedom from the worry that others would free ride on one's costly information. Thus, the independence assumption is satisfied in the environment that I consider. At the same time, however, I take into account another strand in the economics literature related to Condorcet's jury theorem, one which shows that the sincerity assumption often does not hold. Thus, voters do not automatically assume that they are pivotal. In particular, if information acquisition is costly, voters (jurors) will take their probability of being pivotal into account. In my model, jurors compute a probability of being pivotal while deciding whether to incur costly effort (and if so, with what probability). Unlike models where jurors' opinions are shared, a juror cannot know for sure if he will be pivotal or not: he cannot observe the votes of others.

I find that if uninformed jurors are as likely to be correct as incorrect, then if the cost of information or the total number of jurors on the panel are not too large, an asymmetric pure strategy equilibrium exists where a bare majority[6] of jurors acquires costly information, and the jury arrives at the correct judgment. The efficiency of this equilibrium increases in the number of jurors on the panel, up to a finite upper bound. However, a symmetric mixed strategy equilibrium overlaps with this – and indeed obtains over a larger parameter range. This mixed strategy equilibrium, where all jurors randomize between paying attention and sleeping, yields outcomes that are decreasing in jury size. Thus, if parameters support the mixed strategy equilibrium but not the pure strategy one, or if jurors – being identical in my model - are more likely to behave symmetrically than asymmetrically, small panels work best.

I then find that if inattentive jurors are more likely than not to make mistakes, the repercussions are nuanced. In particular, multiple mixed strategy equilibria coexist – a "high attention" one in which all jurors have a high probability of paying attention, and a "low attention" one. Moreover, while the outcomes in the "low attention" equilibrium improve with increasing jury size, exactly the opposite is true of the "high attention" equilibrium. Social norms

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[6] I use the term "bare majority" to denote the case where exactly (n+1)/2 out of a total of n jurors (with n odd) acquire information.

that dictate attitudes towards slacking on the job (or, in the case of elections, the general level of apathy or interest in politics) can then, by determining which of these symmetric equilibria jurors co-ordinate on, affect the implications for jury size. Thus, larger jury panels work better if norms are tolerant of slacking, while smaller jury panels would yield better outcomes if lack of effort is associated with social stigma. I also find that if informational cost is high, a "bare majority" pure strategy equilibrium obtains over a greater parameter zone than the mixed equilibria, which provides another reason for encouraging large jury sizes in this zone (as the efficiency of the pure strategy equilibrium goes up with jury size, though, again, jury size is endogenously bounded above).

I now turn to discussing the related literature. Mukhopadhyaya (2003) considers a model where identical jurors have to exert costly effort to pay attention, and can free ride on the efforts of other jurors through the deliberation process. Thus, in his model, it is sufficient for just one juror to pay attention (with a perfectly informative signal) to ensure the correct outcome, and the pure strategy equilibrium in his model is independent of jury size. He also finds a symmetric mixed strategy equilibrium where outcomes improve when the jury size is small. He argues that though there might be benefits from multiple jurors if each juror receives an imperfect signal, and jurors can pool information, smaller panels work best on average. In contrast, in my model, deliberations or free riding are not possible, but jurors take their probability of being pivotal into account. I find that the pure strategy equilibrium requires a bare majority of jurors to pay attention, rather than just one. I show that even if we eliminate one argument for increasing jury size – that multiple jurors receiving imperfect signals can pool information – by assuming that paying attention guarantees a perfect signal, there may still be arguments for increasing jury size, as when inattentive jurors are more prone to mistakes and norms are tolerant of low attention, or when a pure strategy equilibrium is likely to result (as outcomes in a pure strategy equilibrium improve with jury size).

Other papers that consider costly participation include Martinelli (2006), Koriyama and Szentes (2009), Cai (2009), and Triossi (2013). In Martinelli (2006), homogeneous voters simultaneously invest in information of potentially differing quality. He shows that as the number of voters becomes infinitely large, the probability of a correct decision converges to 1, with each voter investing only a small amount. Koriyama and Szentes (2009) show that when committee members decide whether to invest in an imperfect signal, the only equilibrium in small committees is a pure strategy one where everyone invests with probability one. In larger committees, mixed strategy outcomes prevail with some members randomizing between investing and not investing, though the number of committee members randomizing can differ. They show that though large committees may be inefficient, the welfare losses associated with them are small. In their paper, uninformed members can free ride on information acquired by others. Cai (2009) considers committee members with heterogeneous preferences who have to collect information at a cost and pass it on to a principal, and shows that optimal committee size increases in the heterogeneity of preferences. Triossi (2013) considers voters with heterogeneous skill levels whose cost of information is skill-dependent, and finds a justification, in terms of electoral outcomes, in restricting suffrage to more skilled voters. Section 4 of my paper, which applies my results to voting, contains a comparison with Triossi's results.

Another strand of the literature deals with the fact that voters may not vote sincerely. Austen-Smith and Banks (1996) showed that voters will generally not automatically act as if they are pivotal, and that sincere voting may not in fact constitute an equilibrium. Fedderson and Pesendorfer (1998) showed that, given that jurors behave differently when they are pivotal, the unanimity rule is worse than a simple majority and other non-unanimous rules, in that it can simultaneously increase the probability of convicting an innocent defendant, as well as acquitting a guilty one. Persico (2004) shows that supermajority rules are only optimal when the signals that the individual jurors receive are very accurate.

McCannon (2011) applies the Condorcet jury theorem to ancient Athenian trials, though, unlike me, he does not focus on the secret ballot aspect, or on the fact that jurors may have to exert a cost to pay attention. Instead, he focuses on deriving optimal jury size when it is costly to assemble jurors, and explains the sizes of the jury panels used in different types of classical Athenian trials.

The rest of the paper is organized as follows. Section 2 sets up and solves our benchmark model, where inattentive jurors are as likely to vote correctly as not. Section 3 solves the model under the assumption that inattentive jurors are more likely to make a mistake than not. Section 4 discusses some implications and applies our results to elections. Section 5 concludes.

2. **A Benchmark Case**

In this section, I consider a simple model without biases. A given defendant is equally likely to be either guilty or innocent. There are n jury members, who may choose either to pay attention or to sleep, or to randomize between the two options. If a juror chooses to sleep, he gets no information about the defendant's guilt or innocence, and is equally likely to vote either "guilty" or "innocent." If he pays attention, he incurs a cost c, which may be interpreted as an effort cost, and gets to know the defendant's true state with perfect accuracy. However, there is no consultation between different jurors; each casts his vote secretly, and at the end of the procedure votes are aggregated and the simple majority rule is used to decide the outcome. To rule out problems related to tie-breaking rules we will assume that n is odd. (This was actually the practice in classical Athenian jury trials, where n was odd, large – often 201 or 501 – and jurors voted by secret ballot and a simple majority rule determined the outcome).[7]

We assume that each juror's utility function is given by

$$U = p - \sigma c \tag{1}$$

Here p is the probability that a correct overall decision is reached (we normalize the juror's benefit conditional on a correct judgment to 1), σ is the probability that the juror pays attention and incurs the cost of doing so, and c is assumed to lie between 0 and 1.

Since the simple majority rule is used, each juror is going to incur the effort cost only if he thinks his vote will be pivotal. Thus, the nth juror pays attention only if his probability of being pivotal – the probability that $(n-1)/2$ of the other jurors have voted correctly, while the remaining $(n-1)/2$ have voted incorrectly, is sufficiently high.[8] We first focus on a symmetric mixed strategy equilibrium.

[7] Odd-sized jury panels are used contemporarily in several countries. Examples include Scotland (15), Brazil (7), Spain (9), and Denmark (9) (Lieb 2007). Odd-sized judge panels (with 3 judges) are used in death penalty trials in Nebraska.
[8] Since individual votes are secret, jurors do not actually observe other jurors' votes when casting their own.

2.1 Symmetric Mixed Strategy Equilibrium (SMSE)

Consider a candidate SMSE where each juror pays attention with probability σ, 0<σ<1. Then the probability of an individual juror voting correctly is σ+(1-σ)/2 =(1+σ)/2. He votes correctly if he pays attention, which happens with probability σ, while he also votes correctly half the time when he sleeps, as he chooses either state with probability half. He votes incorrectly half the time when he sleeps, or with probability (1-σ)/2.

Therefore, the probability that an individual juror is pivotal is

$$\frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!}\left(\frac{1+\sigma}{2}\right)^{(n-1)/2}\left(\frac{1-\sigma}{2}\right)^{(n-1)/2} = P(n,\sigma(n)) \tag{2}$$

The expression above captures the probability that the other jurors will be tied in their votes.

The benefit that a pivotal juror gets from paying attention is ½ ; by paying attention and voting correctly, he can ensure that the correct judgment is made, obtaining 1; while if he does not pay attention, his vote – which will break the other jurors' tie - will be correct only with probability ½, so that the correct judgment will be made only with probability ½. Thus, an individual juror's expected benefit from paying attention must equal ½ times the probability that he is pivotal. In a mixed strategy equilibrium, he is indifferent between paying attention and sleeping, so this expected benefit from paying attention must be exactly equal to his effort cost, *c:*

$$\frac{P(n,\sigma(n))}{2} = c \tag{3}$$

From (2) and (3), we obtain

$$\sigma = \sqrt{1-4\kappa^{2/(n-1)}} \tag{4}$$

where

$$\kappa = \frac{2c}{\frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!}} \tag{5}$$

Proposition 1: If the total number of jurors on an odd-sized panel is no larger than the largest odd integer n* such that $c < \frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!2^n} = c'(n)$, and there are no biases, a unique symmetric mixed strategy equilibrium exists.

Proof: From (4), the probability of an individual juror's paying attention, σ, is well-defined provided $\kappa^{2/(n-1)} < \frac{1}{4}$. From (5), this is equivalent to the restriction that $c < \frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!2^n} = c'(n)$.[9] It can be checked that $c'(n)$ is a decreasing function of *n* (as shown in Fig 1, which plots c'as a function of *n*). Thus, the smaller *n*, the likelier the restriction is to hold for a given *c*, and therefore σ is well-defined provided the number of jurors is no larger than the largest odd integer, n*, for which the restriction holds. Moreover, from (4), it is clear that σ is unique given *n*. ***QED***

Remark 1: Note that $c'(3) = 1/4$. Thus, as long as *c*<0.25, n* is at least 3 and a SMSE exists with an odd number of jurors.

If *c*>0.25, an odd numbered jury panel with just a single juror can still exist, but since this single juror is always pivotal, he always pays attention, resulting in a correct decision. Therefore, this is not relevant to mixed strategies and we focus attention on larger panels.

Example 1: Consider *c* = 0.1. Then, *n**=15. Also, $c'(201)$ =.028 and $c'(501)$=.017.

The probability of reaching a correct judgment is the probability that at least (n+1)/2 jurors vote correctly. Thus, we have

$$\sum_{j=(n+1)/2}^{n} \frac{n!}{j!(n-j)!} \left(\frac{1+\sigma}{2}\right)^j \left(\frac{1-\sigma}{2}\right)^{n-j} = p(n) \quad (6)$$

Observation 1: Given *c*, both σ(*n*) and *p*(*n*) are decreasing in *n*. So is the utility per juror, as given in equation (1).

Proof: The behavior of σ and *p* is illustrated in Figs 2 and 3 respectively for *c* = 0.1. Numerical values are provided in Table 1.

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[9]Note that the symbol prime does not denote a derivative here.

Table 1: σ,p and U for c = 0.1, odd numbered jury panels

n	σ (probability of paying attention)	p (probability of the overall judgment being correct)	U (each juror's net utility)
3	.7746	.9648	.88734
5	.5194	.9065	.85456
7	.3718	.8549	.81772
9	.2742	.8058	.77838
11	.2	.7535	.7335
13	.14	.6975	.6835
15	.08	.6238	.6158

Example 2: Consider much larger panels, where SMSE can exist for smaller values of c. For c = .02 and n=201, σ=.06 and p = .803. For c = .01 and n = 501, σ = .049 and p = .8639.

Intuitively, though jurors cannot free ride on other jurors' information flows, as ballots are secret, they take their probability of being pivotal into account while deciding whether to incur costly effort and pay attention. If the panel size increases, and each juror maintained the same probability of paying attention (fixing σ), the probability of being pivotal would shrink, so that the expected benefit of paying attention would fall below its cost. To sustain a mixed strategy equilibrium with a larger panel, therefore, it is necessary for each juror to pay attention with lower probability than with a smaller panel. This would raise an individual juror's probability of being pivotal to a point where the expected benefit of paying attention rose to balance its cost. Since jurors in larger panels are less attentive, the probability of reaching a correct judgment decreases. Moreover, the fourth column of Table 1 shows that larger panels are also less efficient in the sense that each juror is less well off, on average, in larger panels. This is not obvious, because though larger panels make less accurate decisions, they also entail lower effort cost per juror.

2.2 Asymmetric Pure Strategy Equilibrium

Since there is no information sharing, it is not enough for just one juror to pay attention in a pure strategy equilibrium, unlike in Mukhopadhyaya (2003). In a pure strategy equilibrium, a juror either pays attention or sleeps. It is immediate that no equilibrium in which every juror pays attention will exist; if *n-1* jurors are already paying attention, the *n*th juror knows for sure that he will not be pivotal, and hence does not pay attention.

Proposition 2: If the total number of jurors on an odd-sized panel is no larger than the largest odd integer n̲ such that $c < \frac{1}{2^{(n+1)/2}} = c''(n)$, and there are no biases, a pure strategy equilibrium exists in which exactly (*n+1*)/2 jurors pay attention and a correct judgment is made overall.[10]

Proof: Given that (n-1)/2 jurors are sleeping, and another (n-1)/2 jurors are paying attention, the *n*th juror's utility from paying attention is *U(a)=1-c*; if he pays attention, he incurs the cost of doing so, but also ensures that the correct decision is made overall by creating a simple majority of correct votes. If he sleeps, he saves on the cost of attention. Given that (n-1)/2 other jurors are paying attention, the correct decision will be made provided at least one of the sleeping jurors guesses correctly, which is one minus the probability that all (n+1)/2 sleeping jurors (including himself) guess wrong. Thus the correct decision is made with probability $1-\frac{1}{2^{(n+1)/2}}$, and therefore the *n*th juror's utility from sleeping is $U(s)= 1-\frac{1}{2^{(n+1)/2}}$. Provided $c < \frac{1}{2^{(n+1)/2}} = c''(n)$, *U(a)>U(s)* and therefore it is optimal for the *n*th juror to pay attention. Given that exactly (n+1)/2 jurors are paying attention, it is optimal for the other jurors to sleep since they cannot be pivotal. ***QED***

Remark 2: For the same jury size, the APSE of Proposition 2 obtains over a smaller range of *c* than the SMSE of Proposition 1; equivalently, for the same *c*, the SMSE could be supported over a larger jury panel than could the APSE. This is evident from Figure 4, which shows that while $c''(n)$ and $c'(n)$ have the same value for *n*=3, for all larger values of *n,* the former lies below the latter. For example, n̲=5 for c=0.1.

Remark 3: The overall utility of all jurors combined, in this APSE, is *n-c(n+1)/2,* so that the average utility per juror is 1-(n+1)c/2n. This is larger than the average utility per juror in SMSE for similar values of c and n: for *c=0.1,* the average utility per juror in an APSE with *n=3* is .934, and with *n =5,* is .94. These are larger than the corresponding figures in Table 1.

The above remarks highlight an interesting point. Within the class of APSE, larger panels can

[10] Note that the symbol double prime does not indicate a double derivative here.

lead to efficiency gains, measured in terms of average utility per juror. However, APSE cannot be supported for very large jury panels. If, on the other hand, jurors randomize between paying attention and sleeping, smaller panels are best as long as there are no biases.

2.3 Symmetric pure strategy equilibrium

We have already seen that no pure strategy equilibrium exists in which every juror pays attention.

Proposition 3: If $c>0.5P(n,0)=c'(n)$, a pure strategy equilibrium exists in which all jurors sleep.

Proof: The candidate equilibrium exists provided it is optimal for the *n*th juror to sleep, given that the other *n-1* jurors are sleeping. Given that they are, the probability of half the other *n-1* sleeping jurors voting correctly (on a lucky guess) and the other half voting incorrectly, is $P(n,0)=\frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!2^{n-1}}$. If the *n*th juror pays attention, he can vote correctly and increase the probability of a correct judgment from 0.5 to 1; thus, conditional on being pivotal, he gains 0.5 from paying attention, so that his expected benefit from paying attention is 0.5 times the probability that he is pivotal, yielding $0.5P(n,0)$. However, if $c>0.5P(n,0)$, his utility from sleeping is greater than his utility from paying attention. Note, however, that $0.5P(n,0)$ is exactly equal to $c'(n)$ from Proposition 1. Thus, the parameter zone supporting this equilibrium is disjoint from that supporting the SMSE of Proposition 1. ***QED***

3. Inattention, mistakes and multiple mixed equilibria

We now change one feature of the benchmark model presented above; we assume that when a juror sleeps, or, more accurately, does not pay attention (we maintain the shorthand “sleeping” for convenience), he is likely to make a mistake. Instead of guessing randomly, he now is more likely than not to make the wrong judgment. This could be conceptualized either as a bias in priors or in terms of an inattentive juror getting misleading information and not realizing that it is misleading.

Specifically, when a juror does not pay attention, we assume that he makes the wrong decision with probability $q>0.5$. Thus in a candidate SMSE, the probability that a juror votes correctly is now the sum of (i) the probability of his paying attention, σ, and (ii) the probability of his being inattentive but not making a mistake, (1-q)(1-σ). This sum simplifies to 1-q(1-σ). Similarly, the probability that a juror votes incorrectly is the probability of his being inattentive and making a mistake, q(1-σ).

Next, define a variable $\lambda = \kappa/2q$.

Proposition 4: Suppose the number of jurors in an odd-sized panel is no larger than the largest odd integer, n^{**}, such that $c < \frac{q(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!2^{n-1}} = \overline{c(n)}$ and in addition, suppose that n is large enough so that $\lambda^{2/(n-1)} > q(1-q)$. Then, if sleeping jurors are more likely than not to make mistakes, two mixed strategy equilibria exist simultaneously for a given jury size panel; a "high-attention" equilibrium, and a "low-attention" one. For n so small that $\lambda^{2/(n-1)} < q(1-q)$, only the "high-attention" equilibrium exists.

Proof: An individual juror's probability of being pivotal is now

$$\frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!}\left(q(1-\sigma)\right)^{(n-1)/2}\left(1-q(1-\sigma)\right)^{(n-1)/2} = P(n, \sigma(n)) \tag{7}$$

In the event of being pivotal, the juror could receive a benefit of 1 by paying attention, thus ensuring the correct outcome, while if he sleeps, his vote would result in the correct outcome only if he has not made a mistake, that is, with probability 1-q. Thus his benefit from paying attention conditional on being pivotal is 1-(1-q)= q. As the expected benefit from paying attention must equal its cost in equilibrium, the parallel to (3) is:

$$qP(n, \sigma(n), q) = c \tag{8}$$

Together, (7) and (8) yield two solutions for σ, the probability of paying attention:

$$\sigma_1 = \frac{2q-1+\sqrt{1-4\lambda^{2/(n-1)}}}{2q}, \sigma_2 = \frac{2q-1-\sqrt{1-4\lambda^{2/(n-1)}}}{2q} \tag{9}$$

Now, by the same logic as in Proposition 1, the expression in the square root sign is positive, and hence the two roots are well-defined, given that n is no larger than n^{**}. In addition, observe that the expression in the square root sign can be rewritten as

$$(2q-1)^2 - 4\left(\lambda^{\frac{2}{n-1}} - q(1-q)\right)$$

This is smaller than $(2q-1)^2$, hence guaranteeing that the smaller of the two roots is positive, if and only if $\lambda^{2/(n-1)}$ >*q(1-q)* (the larger root is necessarily always positive). Now the LHS of this inequality is increasing in *n* in the relevant domain where $\lambda^{2/(n-1)} < 1/4$.[11] Therefore, a "high attention" and a "low attention" MSE exist simultaneously for intermediate values of *n.* When *n* is very small, the smaller root is negative and hence irrelevant, and there is a unique "high-attention" mixed strategy equilibrium. ***QED***

Example 3: Let *c=0.1* and *q=0.8.* Then, multiple mixed strategy equilibria exist for odd-sized jury panels with no smaller than 7 and no larger than 41 jurors. With n = 3 and 5, only the high-attention equilibrium exists.

Note that the probability of reaching a correct judgment overall is now given by

$$\sum_{j=(n+1)/2}^{n} \frac{n!}{j!(n-j)!}(1-q(1-\sigma))^j (q(1-\sigma))^{n-j} = p(n) \qquad (10)$$

Observation 2: Given *c* and *q*, the probability of paying attention and of reaching an overall correct judgment decrease with jury size if jurors co-ordinate on the "high attention" equilibrium, but *increase* with jury size if jurors co-ordinate on the "low attention" equilibrium. Utility per juror decreases with jury size in the "high-attention" equilibrium (U_1) but increases with jury size in the "low-attention" equilibrium (U_2).

Proof: The behavior of σ_1 and σ_2 as a function of jury size is illustrated in Fig 5, while that of $p_1$and p_2 , the correct decision probabilities in the high and low attention equilibria as a function of jury size, is illustrated in Fig 6, for *c* = 0.1 and *q=0.8.* Numerical values are provided in Table 2. The last column of the table shows the probability of reaching an overall correct judgment when no one pays attention. Due to the presence of bias, this probability is strictly less than 0.5 and is decreasing in the number of inattentive jurors.

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[11] Graphically, this is also evident from Fig. 2 above; as (4) shows, σ decreasing in n is equivalent to $\kappa^{2/(n-1)}$ increasing in n. From the definition of λ, this would also guarantee that $\lambda^{2/(n-1)}$ is increasing in n.

Table 2: Probabilities of paying attention, an overall correct decision, and average juror utilities in the high and low attention equilibria, c=0.1, q=0.8

n	σ_1	σ_2	p_1	p_2	U_1	U_2	Prob of correct decision when no one pays attention
3	.9162		.987		.8954		.104
5	.7813		.959		.88087		.0579
7	.6956	.0544	.9353	.0647	.86574	.05926	.0333
9	.6385	.1115	.9139	.0861	.85005	.07495	.0196
11	.5975	.1525	.8942	.1058	.83445	.09055	.0116
13	.5663	.1837	.8755	.1245	.81887	.10613	.007
15	.5417	.2083	.8577	.1423	.80353	.12147	.0042

From Table 2, it is clear that while the "high-attention" equilibrium dominates the "low-attention" one for a given jury size, co-ordination may occur on either equilibrium. Thus, if social norms are tolerant of slacking off on government jobs, it is likely that jurors may co-ordinate on the "low attention" equilibrium, expecting other jurors to do so. In this case, a larger jury panel would create efficiency gains. However, if norms are such as to encourage paying attention, a smaller jury panel would be beneficial, as co-ordination on the high-attention equilibrium is more likely. In any case, the low-attention equilibrium always dominates the outcome where every one is inattentive.

Intuitively, with a larger jury size panel, the probability of being pivotal would decrease keeping each juror's probability of paying attention constant. Thus, to keep jurors indifferent between paying attention and not doing so, the probability of the other jurors' paying attention must change so as to raise the probability of being pivotal back to its old level. Now, if jurors were in the high-attention equilibrium, this would be accomplished by each juror being less attentive than before. However, if jurors were in the low-attention equilibrium, the probability of being pivotal would be raised by each juror being *more* attentive.[12]

Pure strategy equilibria

Proposition 5: An APSE in which exactly $(n+1)/2$ jurors pay attention and a correct judgment is made overall exists in the model with inattention biases if the odd-sized jury panel is no larger than the largest odd integer n' such that $c < q^{(n+1)/2} = \tilde{c}(n)$.

Proof: The proof is similar to that for Proposition 2 with the modification that now, the *n*th juror's utility from sleeping when exactly $(n-1)/2$ other jurors are paying attention and the other $(n-1)/2$ are sleeping, is $U(s)=1-q^{(n+1)/2}$, which is one minus the probability that all the sleeping jurors (including himself) make mistakes. As $U(a)$ is 1-c, as before, the result follows. ***QED***

Proposition 6: A SPSE in which all jurors sleep exists provided $c > qP(n,0,q) = \frac{q(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!}[q(1-q)]^{(n-1)/2}$.

Proof: If the other *n-1* jurors are sleeping, σ=0 and an individual juror's probability of making a mistake is q, while his probability of being correct is 1-q. Therefore, the probability of the *n*th juror being pivotal is $P(n,0,q) = \frac{(n-1)!}{\left(\frac{n-1}{2}\right)!\left(\frac{n-1}{2}\right)!}[q(1-q)]^{(n-1)/2}$, that is, he is pivotal if an equal number of the other jurors are correct and incorrect. His benefit in the event of being pivotal and paying attention is q, since he can change the majority outcome to the correct outcome with probability 1 by doing so, while if he does not, and sleeps, the correct outcome is only chosen with probability 1-q, which is the probability of the tie-breaking juror making a correct decision despite sleeping. Thus, given the condition in the Proposition, his expected benefit from paying attention is smaller than his cost of doing so, and he therefore chooses to sleep. ***QED***

Fig 7 shows the ranges of *c* and *n* over which mixed strategy equilibria, APSE, and SPSE obtain. If the cost of attention is relatively high, a larger number of jurors can be supported in an APSE than in the SMSEs. However, this pattern is reversed when the cost of attention is lower. For

[12]This last point can be clarified further. If jurors are in the low-attention equilibrium, each juror considers that a majority of the other jurors may be likely to vote incorrectly, and that therefore he has a low chance of being pivotal. Therefore, the probability of being pivotal is raised when the other jurors increase their probability of paying attention, thus raising the likelihood of enough correct votes to reach a tie.

instance, if c=0.1, the APSE can support at most 19 jurors on an odd-sized panel, while 41 could be supported under SMSE. The SPSE where all jurors sleep exists if c lies above the lowest curve. Thus, we see that multiple mixed strategy equilibria may coexist with an APSE as well as a SPSE, for example, in the region below the first two curves but above the third curve, all three types of equilibria coexist. As in Remark 3, the average utility per juror is higher in the APSE than in the mixed strategy equilibria, and is increasing in the size of the jury panel. As evident from the last column of Table 2, the average utility per juror in the SPSE where all jurors sleep (which is also equal to the probability of reaching a correct decision) is lower than in any mixed strategy equilibria, and is decreasing in the size of the jury panel.

4. Discussion and an Application to voting

In the models of the previous sections, jurors do not get the opportunity to free ride on other jurors' information flows, because voting is secret and no deliberations occur. While deciding whether to incur costly information, jurors therefore are not deterred by the prospect of others free riding off their information. On the other hand, they do need to consider whether they are likely to be pivotal, as this crucially affects their expected benefits from incurring effort. Note that they cannot directly observe if they are pivotal, as they cannot observe the votes of other jurors.

Our results imply that the presence of bias, in the sense that inattention is more likely to induce a mistake than not, has nuanced repercussions. It can increase the case for larger jury panels on two grounds.

First, consider the case when the cost of paying attention is high. Then, an asymmetric pure strategy equilibrium holds over a greater parameter range as compared to the mixed strategy equilibria when bias is present, relative to the case where it is not (compare Figures 7 and 1). Thus, the presence of bias would entail that for a given cost of information, there is a zone in which only the APSE would obtain, but the mixed strategy equilibria would not; and as seen already, the efficiency of the APSE increases in the number of jurors. However, the ideal number of jurors would remain finite as there is an upper bound on the panel size that can be supported in an APSE, for a given c. If, on the other hand, bias is absent, so that an inattentive juror places equal probability on the two alternatives, there is a zone in which the APSE does not obtain, while the (unique) SMSE does; in this zone, efficiency is decreasing in jury size. Moreover, in the zone where the APSE and the SMSE coexist in the bias-free model, it is not clear which would be chosen. According to Mukhopadhyaya (2003), the problem of determining which jurors pay attention and which do not may be severe in an APSE, since all jurors are essentially alike, whereas a mixed strategy equilibrium has the advantage of being symmetric and therefore requiring identical behavior from every juror. If we apply this argument to our model, the case for a small jury panel increases when there is no bias.

Secondly, consider the case where the cost of paying attention is not too high. With bias, there is a zone where the APSE does not exist, but there are multiple SMSE, a high-attention one and a low-attention one. Norms can serve to solve the co-ordination problem here; if social norms favor high effort on tasks like jury duty, jurors are more likely to co-ordinate on the high-attention equilibrium, while the reverse is true if norms are tolerant of slacking. This suggests that if norms are, indeed, tolerant of slacking, there is another reason to encourage larger jury panels, as the efficiency of the low-attention equilibrium improves when the number of jurors goes up. This factor is absent when there is no bias, as the unique mixed strategy equilibrium that obtains then yields better outcomes when the number of jurors is small.

More generally, the second factor mentioned above indicates that biases introduce the need to be sensitive to social norms while deciding whether bigger or smaller jury sizes are better.

We now apply our results to a context other than jury duty – voting in elections. Identical voters incur a cost if they obtain information about the candidates. In a two-candidate election, suppose one candidate is undisputably better, and this becomes clear to any voter who incurs the cost of getting informed. However, voters may also vote without getting informed, in particular if voting is compulsory (with voluntary voting, there is a larger possibility that uninformed voters simply abstain). If uninformed voters choose either candidate with equal probability, and all voters behave symmetrically, a larger electorate may worsen electoral outcomes (the SMSE of Section 2) by lowering the probability of a voter's acquiring information, and reducing the probability that the better candidate is elected. For smaller electorates, an APSE where a bare majority of voters acquire information can also be supported. The presence of bias among uninformed voters – which increases their probability of voting incorrectly – would, however, change the results to those obtained in our Section 3. Even if all voters act symmetrically, they may either acquire information with high probability or low, depending perhaps on the general interest in politics in their environment. If voters are generally apathetic and co-ordinate on the low information equilibrium, an increase in the numbers of the electorate improves electoral outcomes, while the reverse would be true if voters co-ordinate on a high information equilibrium. Moreover, if the cost of getting informed is rather high, mixed strategy equilibria will not exist, while an APSE will, whose efficiency increases as the number of voters expands. Nonetheless, the optimal number of voters will be bounded above even in such an equilibrium.

It is interesting to compare these results with Triossi (2013). In his paper, voters are heterogeneous and can invest in acquiring information depending on their level of skill. He finds a justification for restricting the electorate to more skilled citizens, in that this can lead to better electoral outcomes. My results suggest that while, in the absence of bias, there can be a strong argument for restricting the electorate even when voters are identical, rather than heterogeneous, expanding the electorate up to a limit may be better if uninformed voters are more likely to be wrong than right, and the cost of information is high. In particular, norms, such as the general level of interest in politics, would need to be taken into account before deciding whether a large or a small electorate would lead to better electoral outcomes.

5. Conclusion

I construct a model of jury voting where individual jurors do not share information, and each juror decides whether to pay attention by incurring costly effort. An attentive juror receives perfect information. This removes one advantage to a larger jury that is often considered in the literature; the information aggregation advantage that obtains if multiple jurors all independently receive imperfect signals, and pool their information. However, here each attentive juror receives perfect information, and, moreover, information is not pooled, except through the voting rule (simple majority). On the other hand, the fact that in my model, jurors do not have to worry about other jurors free riding on their information, increases their incentives to acquire information, and ensures that, for instance, it is not sufficient for just one juror in a panel to pay attention (unlike in Mukhopadhyay 2003). I follow the literature that emphasizes the importance of strategic voting by noting that jurors' incentives to acquire information depend on their estimates of the probability that they are pivotal.

I find that, if inattentive jurors are as likely to be right as wrong, an APSE exists with a bare majority of jurors paying attention, and the correct decision being made overall, provided the number of jurors is below a ceiling. Each juror is better off, on average, in a larger panel provided this APSE obtains and provided the ceiling is respected. However, if all jurors are likely to behave identically, a SMSE obtains instead in which the probability of paying attention and the likelihood of the outcome being correct both decline in the number of jurors. Moreover, this SMSE obtains over a greater parameter range than the APSE.

If, on the other hand, inattentive jurors are more likely to be wrong than right, the results change. There are two SMSE, a high-attention one and a low-attention one, for intermediate jury sizes. Larger jury panels can be supported under SMSE than in the benchmark model. If norms dictate co-ordination on the high-attention equilibrium, outcomes improve as the number of jurors decreases; however, if norms encourage co-ordination on the low attention equilibrium, increasing the number of jurors can help. If the cost of information is high, an APSE with a bare majority of jurors paying attention now exists over a greater parameter range than the mixed strategy equilibria. In this zone, increasing the number of jurors up to a finite upper bound helps increase efficiency. Our results are also applicable to voting in elections, indeed, they apply to any context where votes are cast without deliberation, and a voluntary cost may be incurred to improve the efficiency of individual decisions.

References

Austen-Smith, D. and J.S Banks (1996): "Information Aggregation, Rationality, and the Condorcet jury theorem", *American Political Science Review* 90:34-45.

Cai, H (2009): "Costly participation and heterogeneous preferences in informational committees", *RAND Journal of Economics* 40:173-189.

Fedderson, T. and W. Pesendorfer (1998): "Convicting the Innocent: the inferiority of unanimous jury verdicts under strategic voting", *American Political Science Review* 92:23-35.

Guha, B (2011): "Preferences, prisoners and private information: was Socrates rational at his trial?" *European Journal of Law and Economics* 31:249-264.

Hansen, M.H (1991): *The Athenian democracy in the age of Demosthenes: structure, principles and ideology.* Oxford, UK and Cambridge, USA: Blackwell.

Koriyama, Y. and B. Szentes (2009): “A resurrection of the Condorcet jury theorem”, *Theoretical Economics* 4:227-252.

Leib, E.J (2007): “A comparison of criminal jury decision rules in democratic countries”, *Ohio State Journal of Criminal Law* 5:629-644.

Martinelli, C (2006): “Would rational voters acquire costly information?” *Journal of Economic Theory* 129:225-251.

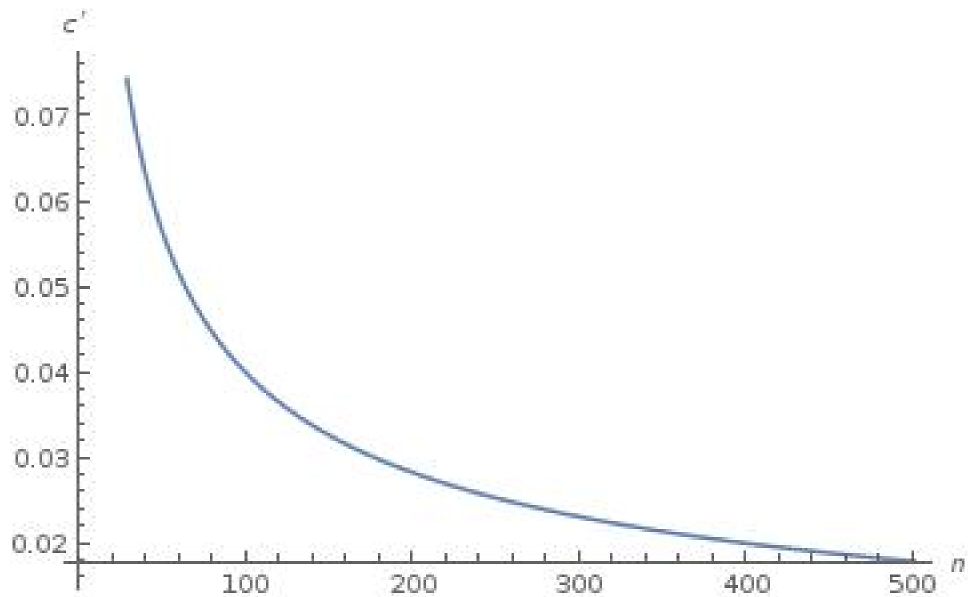
McCannon, B (2011): “Jury size in classical Athens: an application of the Condorcet jury theorem”, *Kyklos* 64:106-121.

McLean, I. and F. Hewitt (1994): *Condorcet: Foundations of social choice and political theory.* Cheltenham: Edward Elgar.

Mukhopadhyaya, K (2003): “Jury size and the free rider problem”, *Journal of Law, Economics and Organization* 19:24-44.

Persico, N (2004): “Committee design with endogenous information”, *Review of Economic Studies* 71:165-191.

Triossi, M (2013): “Costly information acquisition: is it better to toss a coin?” *Games and Economic Behavior* 82:169-191.



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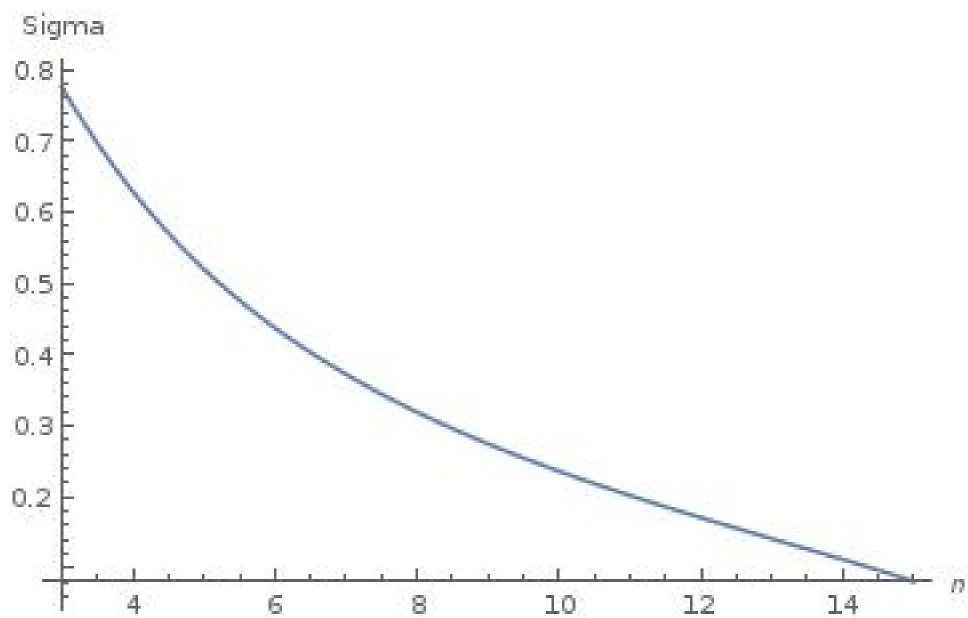

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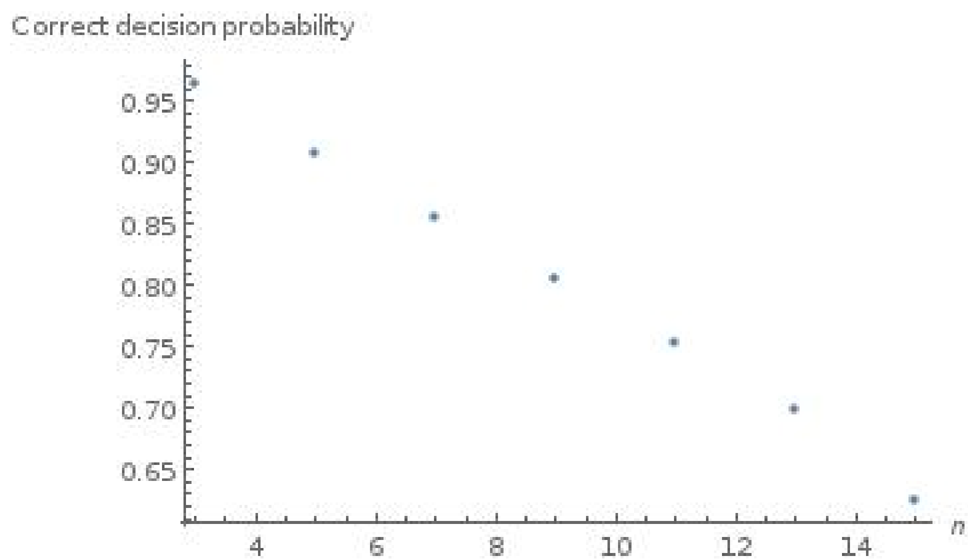
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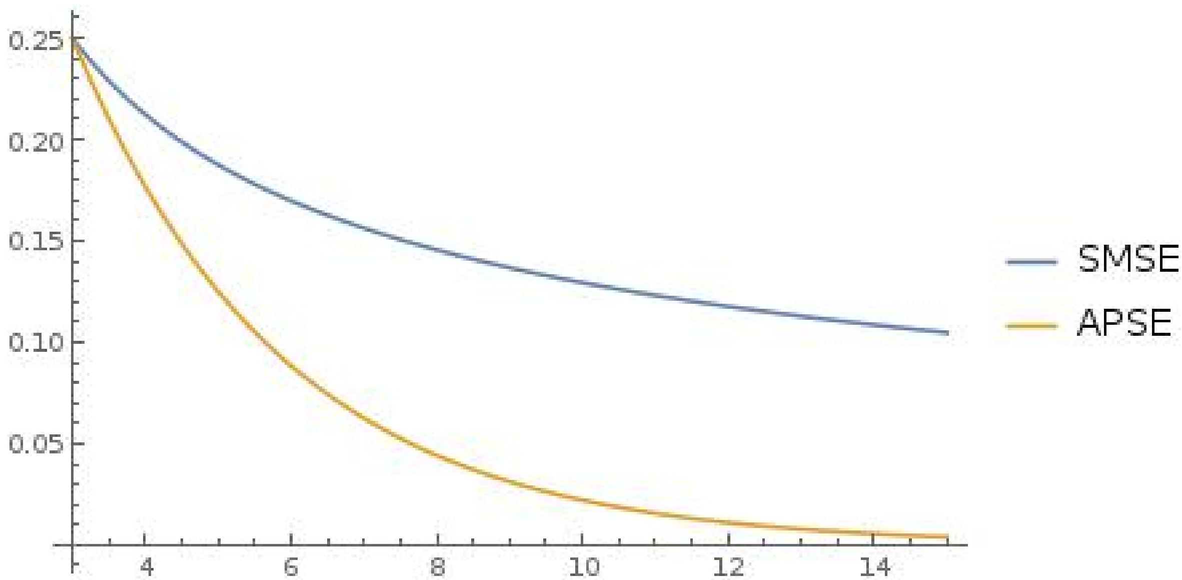
14
n



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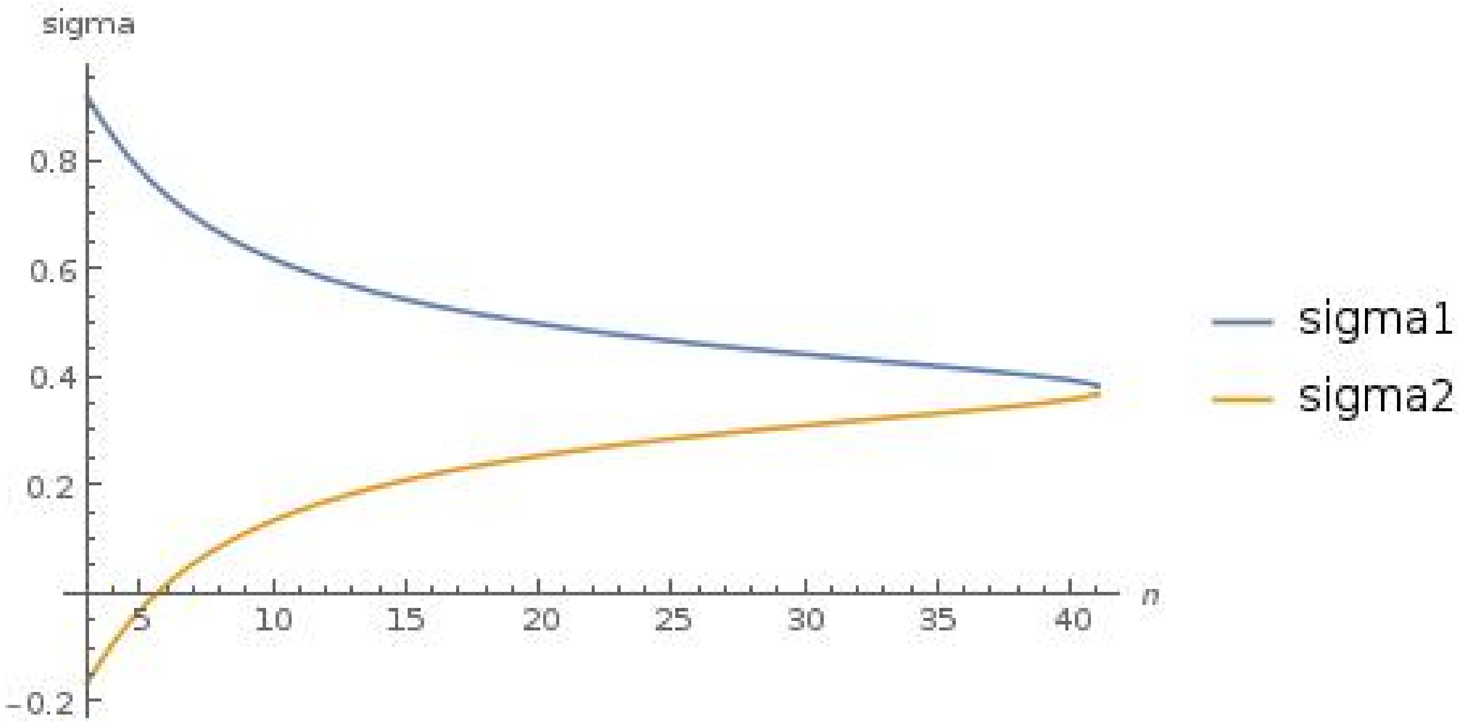
SMSE
APSE



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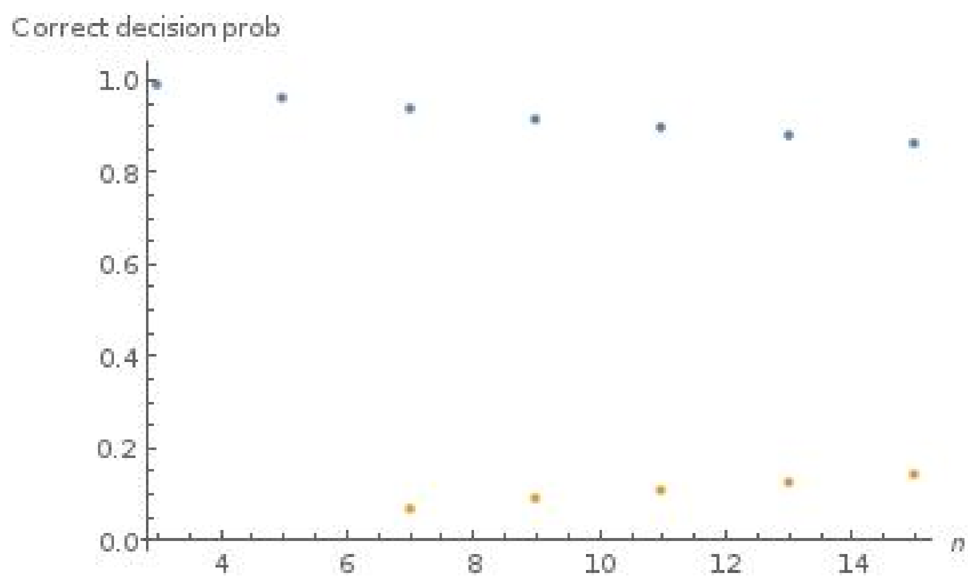
sigma1
sigma2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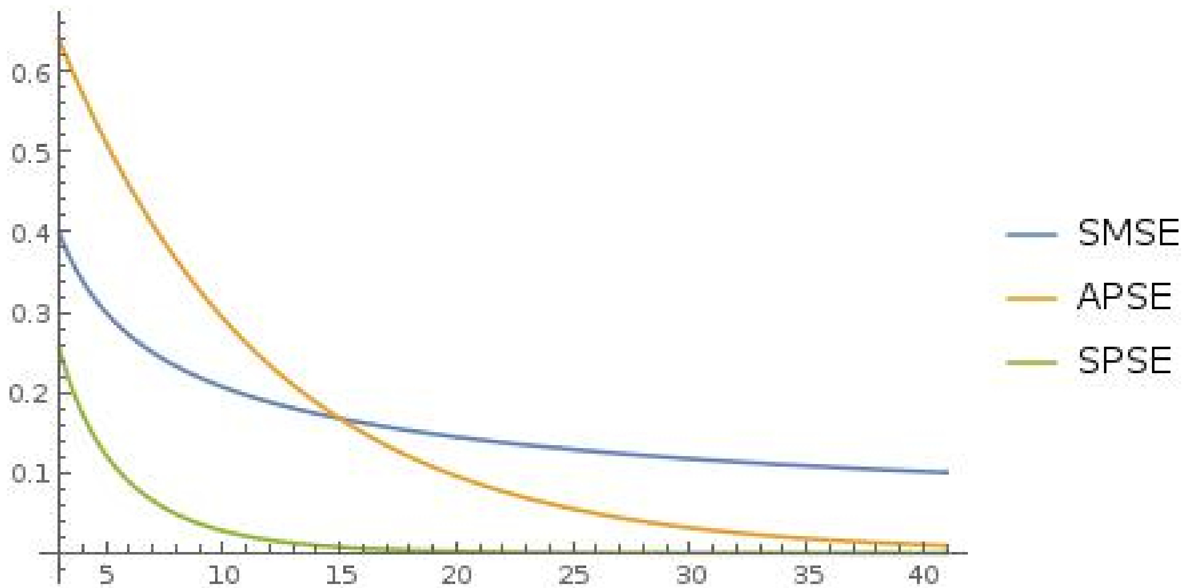

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SMSE
APSE
SPSE