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Decomposition of a Certain Cash Flow Stream: Differential Systemic Value and Net Final Value

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Decomposition of a Certain Cash Flow Stream: Differential Systemic Value and Net Final Value

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Abstract. This paper proposes a new way of decomposing net present values and net final values in periodic shares. Such a decomposition generates a new notion of residual income, radically different from the classical one available in the financial and accounting literature. While the standard residual income is formally computed as profit minus cost of capital times actual capital invested, the new paradigm introduces a fourth element: the capital invested in the so-called *shadow project*. Such a capital is the counterfactual capital that the investor would own if, at time 0, he invested his funds at the cost of capital, rather than in the project. Two important features are found: in primis, the new residual income is obtained as the sum of the standard residual incomes and the interest earned on past standard residual incomes; in secundis, the new paradigm is shown to be additive: the net final value of the project is computed as the sum of all periodic shares (residual incomes) with no capitalization process (abnormal earnings aggregation). A generalization is provided for a levered portfolio of projects, and a fourthfold decomposition is reached: (i) periodic decomposition, (ii) opportunity account decomposition, (iii) project decomposition, (iv) financing decomposition.

Note to the reader:

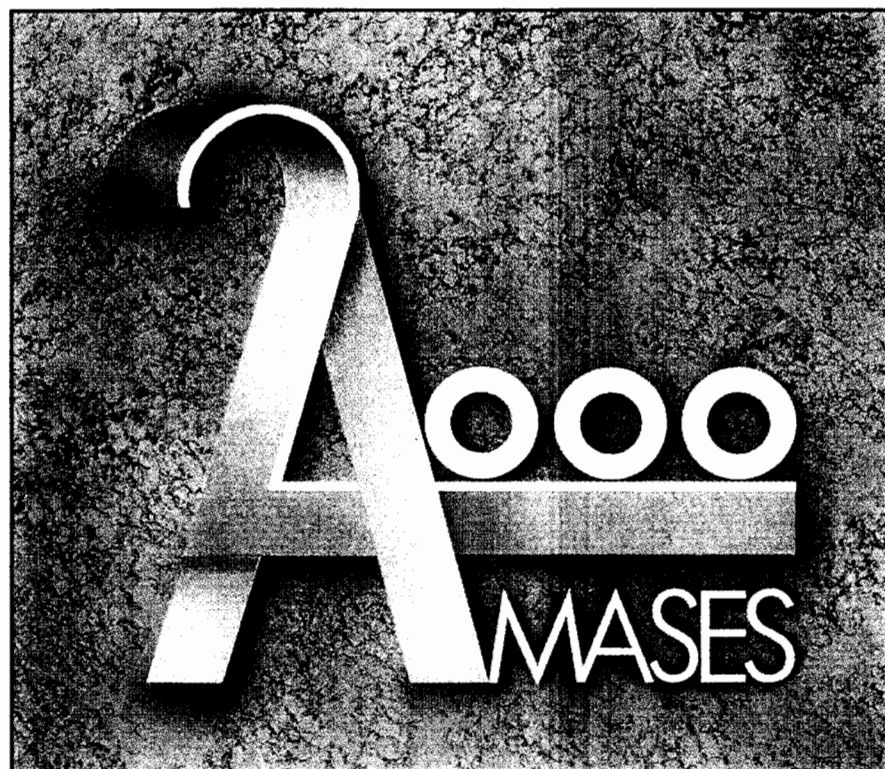
This paper, presented at the XXIV annual AMASES Conference in September 2000, presents a new notion of residual income, here called “Differential Systemic Value”. This notion has been extensively studied theoretically in later papers of mine, where it has been called “Systemic Value Added” and, more recently, “lost-capital paradigm”.

If you are interested in this topic and its implications for corporate finance, management accounting, cognitive psychology, visit my webpages at SSRN: <http://ssrn.com/author=812528> and IDEAS: <http://ideas.repec.org/f/pma506.html>.

Carlo Alberto Magni, February 21, 2008

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Università degli Studi di Bergamo
Università degli Studi di Brescia

- R. GIACOMETTI, UNIVERSITA' DI BERGAMO pag. 141
 S. ORTOBELLI LOZZA, UNIVERSITA' DELLA CALABRIA
A comparison between three risk measures for the asset allocation problem
- M. GILLI, UNIVERSITY OF GENEVA pag. 145
 E. KËLLEZI, FAME
Heuristic optimization in finance: an application to portfolio selection
- R. GRISAFI, UNIVERSITA' DI CATANIA pag. 147
 G. I. BISCHI, UNIVERSITA' DI URBINO
 S. GRECO, UNIVERSITA' DI CATANIA
L'influenza del tasso di crescita del valore azionario sulle dinamiche di mercato: un modello dinamico non lineare
- H. GZYL, USB-UCV pag. 155
Maxentropic construction of risk neutral measures: discrete market models
- C. A. MAGNI, UNIVERSITA' DI MODENA E REGGIO EMILIA pag. 163
Decomposition of a certain cash flow stream: differential systemic value and net final value
- R. MANSINI, UNIVERSITA' DI BRESCIA pag. 171
 M. G. SPERANZA, UNIVERSITA' DI BRESCIA
Mixed integer linear programming models for portfolio optimization
- L. MANTELLI, UNIVERSTA' CATTOLICA DI BRESCIA pag. 177
Comparative analysis between two options by stochastic dominance
- S. MUZZIOLI, UNIVERSITY OF BERGAMO pag. 185
 C. TORRICELLI, UNIVERSITY OF MODENA AND REGGIO EMILIA
Combining the theory of evidence with fuzzy sets for binomial option pricing
- F. M. PARIS, UNIVERSITY OF BRESCIA pag. 189
Monitoring distressed banks and the bank shareholders' incentives under risk-neutral valuation

**DECOMPOSITION OF A CERTAIN CASH FLOW STREAM:
DIFFERENTIAL SYSTEMIC VALUE AND NET FINAL VALUE
EXTENDED ABSTRACT**

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1. This paper focuses on (nondeferrable) investments under certainty. The decision maker faces the opportunity of undertaking a project and she must decide whether accepting or rejecting it. A widely accepted evaluation index is the so-called Net Present Value (NPV), or Net Final Value (NFV) if compounded, which evaluates the differential profit between the two alternatives. A periodic decomposition of NPV (NFV) is proposed in Peccati ([2], [3], [4]). Hereafter we summarize it: Assume that a decision maker currently invests funds at a rate of interest i and that she faces the opportunity of a nondeferrable investment, say P : For the sake of simplicity we can assume that the project consists of an initial outlay $-a_0$ at time 0, and equidistant cash flows $a_s \in \mathbb{R}$ will be available at time s , $s=1, \dots, n$. The evaluator's initial wealth is E_0 , with $0 < a_0 \leq E_0$, and she aims at maximizing her terminal wealth at time $T=n$. We assume that she finances her investment with a loan contract, whose cash flows are f_0 at time 0 and $-f_s \leq 0$ at time s , with $0 \leq f_0 \leq a_0$. According to Peccati's model the Net Present Value's share for period s is given by

$$g_s = (w_{s-1}(x_s - i) + D_{s-1}(i - \delta_s))(1 + i)^{-s} \quad (1)$$

where

$$w_0 = a_0 \quad w_s = w_{s-1}(1 + x_s) - a_s \quad s = 1, 2, \dots, n.$$

represents the *outstanding capital (project balance)* at time s at the periodic rate of return x_s .

$$D_0 = f_0 \quad D_s = D_{s-1}(1 + \delta_s) - f_s \quad s = 1, 2, \dots, n$$

represents the *outstanding debt (debt balance)* at time s at the periodic rate of cost δ_s . i is the (constant) opportunity cost of capital. Compounding (1) until n we find the Net Final Value's share for period s , which we denote by G_s .

2. Now we propose a different decomposition model based on an accounting-like perspective. The evaluator compares two lines of action: (i) undertaking the project; (ii) investing funds in an asset yielding the opportunity cost of capital i . Let us begin to construct prospective (cash) balance sheets for (ii). Let C be the asset yielding interest at the rate i . We call it the "opportunity" account.

As the decision maker invests her funds in C her net worth E^s at time s is given by the sum C^s , representing the value of “opportunity” account C :

$$\begin{array}{c|c} \underline{\text{Assets}} & \underline{\text{Equities}} \\ \hline C^s & E^s \end{array} \quad (2a)$$

for $s = 0, 1, 2, \dots, n$. For case (i), our investor/accountant will record two accounts in the debit side and two accounts in the credit side, expressing the fact that she holds an asset C (whose rate of return is i), an asset P whose periodic rate of return is x_s , a loan contract D whose periodic rate (of cost) is δ_s and her own net worth E : At time s we have

$$\begin{array}{c|c} \underline{\text{Assets}} & \underline{\text{Equities}} \\ \hline C_s & D_s \\ w_s & E_s \end{array} \quad (2b)$$

where C_s, w_s, D_s, E_s are the values of accounts C, w, D, E respectively. For (2a) and (2b), respectively, we state the following relations:

$$C^0 = E_0 \quad C^s = C^{s-1}(1+i) \quad (3a)$$

$$C_0 = E_0 - a_0 + f_0 \quad C_s = C_{s-1}(1+i) + a_s - f_s \quad (3b)$$

for $s \geq 1$. As for the income statement at time $s, s \geq 1$, (3a) leads to the net profit iC^{s-1} , whereas (3b) leads to the net profit $x_s w_{s-1} + iC_{s-1} - \delta_s D_{s-1}$. The sum

$$E_n - E^0 = \sum_{s=1}^n \left(x_s w_{s-1} - \delta_s D_{s-1} - i(C^{s-1} - C_{s-1}) \right) \quad (4)$$

shows the global profitability of P . We call (4) the Differential Systemic Value (DSV) as it is the difference between the two alternative financial systems. It is easy to see that the DSV coincides with the NFV, but the former's decomposition is different from Peccati's. In fact, the periodic share \overline{M}_s of P 's DSV is given by the difference between net earnings *sub* (i) and net earnings *sub* (ii), that is

$$\overline{M}_s := (E_s - E_{s-1}) - (E^s - E^{s-1}). \quad (5)$$

(5) is nothing but $x_s w_{s-1} - \delta_s D_{s-1} - i(C^{s-1} - C_{s-1})$. Letting $M_s := G_s(1+i)^{n-s}$ it is easy to show that

$$\overline{M}_s = M_s + \sum_{k=1}^{s-1} i M_k (1+i)^{s-k-1}. \quad (6)$$

Summing for s , after simple algebraic manipulations, we obtain

$$DSV = \sum_{s=1}^n \overline{M}_s = \sum_{s=1}^n M_s(1+i)^{n-s} = NFV.$$

While coinciding in overall terms, the DSV model and the NFV model give rise to different partitions. The s -th share of the NFV is the compound amount of M_s , i.e. G_s , whereas the “accounting-flavored” partition provides us with \overline{M}_s , with $\overline{M}_s \neq G_s$. The DSV model is grounded on a systemic/accounting way of reasoning which makes no use of NFVs nor compounding processes, whereas Peccati’s model rests on financial arguments, in particular on the concept of Net Final (Present) Values and on capitalization processes. It is easy to see that \overline{M}_s can be rewritten as an s -th order difference equation

$$\overline{M}_s = M_s + i \left(\sum_{h=1}^{s-1} \overline{M}_h \right). \quad (7)$$

This reformulation enables us to interpret \overline{M}_s as the sum of a direct factor M_s (generated by the capital invested w_{s-1} and by the residual debt D_{s-1}) and the periodic interest on the $(s-1)$ indirect factors \overline{M}_h : the latter represent the gain generated in period s by those shares referring to the previous periods $\overline{M}_1, \overline{M}_2, \dots, \overline{M}_{s-1}$, which yield returns at the rate i . These returns are borne in the s -th period: Therefore, each share depends on all the preceding ones, which periodically bear interest at the rate i . Such an imputation collides with the NFV-based imputation. To see why, let us assume, for the sake of convenience, $n=3$, and let us decompose both DSV and NFV. We have the following decomposition table:

$$\begin{array}{lll} G_1 = M_1 + (iM_1) + (iM_1 + i^2M_1) & G_2 = M_2 + (iM_2) & G_3 = M_3 \\ \overline{M}_1 = M_1 & \overline{M}_2 = M_2 + (iM_1) & \overline{M}_3 = M_3 + (iM_1 + i^2M_1) \\ & & + (iM_2) \end{array} \quad (8)$$

where the first row decomposes the DSV, the second one decomposes the NFV. As we can see, the NFV decomposition accomplishes a two-step evaluation. The idea is the following: M_1, M_2, M_3 are the three shares for period 1, 2, 3 respectively. As this is money referred to the dates 1, 2, 3, respectively, the basic principles of financial calculus force the evaluator to compound (or discount) flows to take time into consideration. After capitalization (and only after) the evaluator may sum the three shares. Conversely, in the light of our

systemic perspective the decision maker can construct, in a gradual way, the three shares of the DSV. The first share is M_1 , which exactly represents the difference between what the investor receives in the first period and what she would receive should she decide to forego the project opportunity and invest her funds at the opportunity cost of capital i . In the second period the difference between what she receives and what she would have received must take into account that, in addition to M_2 , the first share does not disappear, but yields interest equal to iM_1 . This is natural, for if in the first period the investor gains, say, a differential 100\$, in order to calculate the second differential gain, we must necessarily consider that 100\$ produce, just in the second period (not in the first one) a differential return of $100i$. Iterating the argument, the third share must consider the return on the two first shares \overline{M}_1 and \overline{M}_2 , which is produced just in the third period, and is equal to $100i$ and $(100i+100i^2)$ respectively. You can see that this line of argument is not obeyed by the “financial-flavored” decomposition. G_1 embodies the term iM_1 which, as we have seen, is to be ascribed to the second share, since it is generated in the second period. In addition, it comprehends the term $iM_1+i^2M_1$ which in turn is related to the third period. At the same time G_2 includes iM_2 , which is pertinent to the third period, but lacks the term iM_1 (previously embodied in G_1). Finally, the third share G_3 forgets the return on previous periods’ shares.

3. Now we look at the implicit assumptions of the two decomposition models. For the sake of convenience, we will assume $D_s=0$ for every s . Both the NFV and the DSV decomposition aim at answering the following question: “*What is the differential gain of (i) over (ii) that we are to ascribe to the s-th period?*”(*). The answer to (*) is the difference $(E_s - E_{s-1}) - (E^s - E^{s-1})$, which is reached under the DSV outlook by directly drawing up two sequences of (cash) balance sheets for alternative (i) and (ii) respectively (we call it the *systemic* argument). According to the NFV approach (the *financial* argument) the decision maker should evaluate a (fictitious) uniperiodic sub-project whose initial outlay is the project balance at time $(s-1)$ and the receipt is given by the sum of a_s and the project balance at time s . Let $\text{NFV}(s)$ denote this sub-project’s Net Final Value, calculated at time s (or, which is the same, its Net Present Value compounded until time s): we have

$$\text{NFV}(s) = -w_{s-1}(1 + i) + w_s + a_s = w_{s-1}(x_s - i). \quad (9)$$

If this is to be the answer to (*), then we must have

$$\text{NFV}(s) = (E_s - E_{s-1}) - (E^s - E^{s-1}). \quad (10)$$

But we know that a Net Final Value represents the difference between alternative final net worths. We have must therefore have $\text{NFV}(s) = E_s - E^s$. The

latter and (10) yield

$$E_{s-1} = E^{s-1} \quad (11)$$

for every s . This tells us that if project P is undertaken the net worth at time $(s-1)$ (left-hand side) coincides with the net worth produced if the project is not undertaken (right-hand side). We distinguish two exhaustive cases: (a) $x_s \neq i$ for at least one s ; (b) $x_s = i$ for all s . If (a) holds, we have two kinds of contradiction: the *mathematical* and the *factual* contradiction. As for the *mathematical* contradiction, let s^* be an index such that $x_{s^*} \neq i$. As (10) and (11) must hold, $\text{NFV}(s^*)$ is zero. This implies $x_{s^*} = i$, due to (9), but this contradicts the assumption. As for the *factual* contradiction, it is due to a vitiated interpretation of facts. In fact, the *financial* argument accomplishes the decomposition by calculating the $\text{NFV}(s)$ for period s , which presupposes that the following assumption is made: *at time 0 the investor invests her net worth E_0 in asset C at the opportunity cost of capital until time $(s-1)$. At time $(s-1)$ the sum w_{s-1} is withdrawn by account C and invested in a uniperiodic project with rate of return x_s . At time s , the investor holds the final amount w_s alongside the value of account C .* As the latter is assumed to hold for every period $s=1, 2, \dots, n$, then it boils down to a set of n incompatible assumptions. As for (b), it causes the decision process to be an idle issue, as alternative (i) coincides, from a *mathematical-financial* point of view, with alternative (ii): There is no difference, financially speaking, in investing at the opportunity cost of capital the whole net worth or only a part of it, if the remainder is invested in a project whose rate of return is the opportunity cost of capital itself. In this case the two arguments lead to the same obvious (and uninteresting) result. In this situation (i) and (ii) are different only under a *factual* perspective, for they, though financial equivalent, represent distinct courses of action. Further, the *factual* contradiction persists, as the aforementioned assumption holds regardless of (a) and (b). No such contradictions, mathematical or factual, arise in the *systemic* argument, which presupposes the following hypothesis: *at time 0 the investor invests the sum $(C^0 - w_0)$ in asset C at the opportunity cost of capital and the sum w_0 in project P .* While the *factual* contradiction cannot be repaired, we might ask if we are able to heal the mathematical contradiction inherent in the *financial* argument. In other terms, can the *systemic* perspective be incorporated in the *financial* argument so as to provide a correct partition of the NFV on the basis of the *financial* argument? Let \bar{w}_s be the value of w_s obtained by replacing each x_s with i :

$$\bar{w}_s := a_0(1+i)^s - a_1(1+i)^{s-1} - \dots - a_s \quad s = 1, 2, \dots, n.$$

Note that the following then hold:

$$\bar{w}_s = C^s - C_s \quad \text{and} \quad \bar{w}_s = \bar{w}_{s-1}(1+i) - a_s.$$

The *systemic* C_s can be then rewritten as

$$C_0 = C^0 - \bar{w}_0, \quad C_s = C_{s-1}(1+i) + a_s = (C^{s-1} - \bar{w}_{s-1})(1+i) + a_s, \quad s = 1, \dots, n \quad (12)$$

where $\bar{w}_0 := a_0$. Now let

$$\bar{a}_0 := a_0 \quad \text{and} \quad \bar{a}_s := x_s w_{s-1} - i \bar{w}_{s-1} + a_s \quad s = 1, \dots, n. \quad (13)$$

Suppose that the investor undertakes a project \bar{P} consisting of the cash flow stream $(-\bar{a}_0, \bar{a}_1, \dots, \bar{a}_n)$. It easy to demonstrate that we can correctly decompose P by applying Peccati's decomposition to the *shadow* project \bar{P} provided we avoid compounding the Net Final Valus so obtained. From (13) we obtain

$$\bar{w}_0 = \bar{a}_0, \quad \bar{w}_s = \bar{w}_{s-1}(1 + \bar{x}_s) - \bar{a}_s \quad s = 1, \dots, n \quad (14)$$

where $\bar{x}_s := x_s w_{s-1} / \bar{w}_{s-1}$. We can then interpret \bar{w}_s as the *project balance* of \bar{P} at the rate \bar{x}_s , and the \bar{a}_s as withdrawals from (if positive) or investments in (if negative) an account yielding interest at the periodic rate \bar{x}_s , $s=1, 2, \dots, n$.

Let us decompose the *shadow* project \bar{P} by using the *financial* argument: the investor invests \bar{w}_{s-1} at the beginning of the s -th period and receives the sum $\bar{w}_s + \bar{a}_s$ at the end of the period: At time s the value of the project is

$$\begin{aligned} \overline{\text{NFV}}(s) &= -\bar{w}_{s-1}(1+i) + \bar{w}_s + \bar{a}_s \\ &= -\bar{w}_{s-1}(1+i) + (\bar{w}_{s-1}(1 + \bar{x}_s) - \bar{a}_s) + \bar{a}_s \\ &= \bar{w}_{s-1}(\bar{x}_s - i). \end{aligned} \quad (15)$$

Denoting with \bar{E}_s and \bar{E}^s the net worths at time s for (i) and (ii) respectively, we have

$$\overline{\text{NFV}}(s) = E_s - E^s = [\text{for (11)}] = (E_s - E_{s-1}) - (E^s - E^{s-1}) \quad (16)$$

(16) tells us that $\overline{\text{NFV}}(s)$ is the s -th share of the Net Final Value of P and, as we expect, it coincides with \bar{M}_s , previously found by following a *systemic* argument:

$$\overline{\text{NFV}}(s) = \bar{w}_{s-1}(\bar{x}_s - i) = x_s w_{s-1} - i (C^{s-1} - C_{s-1}) = \bar{M}_s.$$

Hence,

$$\text{DSV} = \sum_{s=1}^n \bar{w}_{s-1}(\bar{x}_s - i) = \sum_{s=1}^n \overline{\text{NFV}}(s) = \text{NFV} \quad (17)$$

and the two models are, to a certain extent, reconciled with no need of compounding.

4. We now generalize the systemic model by allowing a portfolio of q projects, p “opportunity” accounts and m loan contracts, with respective periodic rates i_s^j , δ_s^l , x_s^r , $j = 1, \dots, p$, $l = 1, \dots, m$, $r = 1, \dots, q$.¹ The latter generalization forces the evaluator to select one or more “opportunity” accounts K^j to be activated for withdrawals and reinvestment of the cash flows released by the projects w^r and the debts D^l . Referring to time s , denote with a_s^{rj} the quota of project r 's cash flow invested in (if positive) or withdrawn from (if negative) account K^j . Likewise, denote with f_s^{lj} the quota of debt l 's cash flow withdrawn from account K^j , $j=1,2,\dots,p$. We must obviously have

$$\sum_{r=1}^q \sum_{j=1}^p a_s^{rj} = a_s \quad \text{and} \quad \sum_{l=1}^m \sum_{j=1}^p f_s^{lj} = f_s.$$

Let us give the following notation:

$$\begin{aligned} \bar{w}_0^{rj} &:= w_0^{rj} := a_0^{rj} & \bar{D}_0^{lj} &:= D_0^{lj} := f_0^{lj} \\ w_s^{rj} &:= w_{s-1}^{rj}(1 + x_s^r) - a_s^{rj} & \bar{w}_s^{rj} &:= \bar{w}_{s-1}^{rj}(1 + i_s^j) - a_s^{rj} \\ D_s^{lj} &:= D_{s-1}^{lj}(1 + \delta_s^l) - f_s^{lj} & \bar{D}_s^{lj} &:= \bar{D}_{s-1}^{lj}(1 + i_s^j) - f_s^{lj} \end{aligned}$$

The value of K^j is

$$K_s^j = \left(K_0^j \prod_{k=1}^{s-1} (1 + i_k) - \sum_{r=1}^q \bar{w}_{s-1}^{rj} + \sum_{l=1}^m \bar{D}_{s-1}^{lj} \right) (1 + i_s^j) + \sum_{r=1}^q a_s^{rj} - \sum_{l=1}^m f_s^{lj}. \quad (18)$$

Let us focus on a generic account K^j . We have for it a portfolio of q *shadow* projects with initial outlays \bar{w}_0^{rj} . It is easy to see that this portfolio's periodic Net Final Value is

$$\sum_{r=1}^q \bar{w}_{s-1}^{rj} (\bar{x}_s^r - i_s^j) - \sum_{l=1}^m \bar{D}_{s-1}^{lj} (\bar{\delta}_s^l - i_s^j) \quad (19)$$

¹The financial system's structure is now articulated as

<u>Assets</u>	<u>Equities</u>
K_s^1	D_s^1
K_s^2	D_s^2
...	...
K_s^p	...
w_s^1	...
w_s^2	...
...	D_s^m
w_s^q	E_s .

where $\bar{x}_s^r = x_s w_{s-1}^r / \bar{w}_{s-1}^r$ and $\bar{\delta}_s^r = \delta_s D_{s-1}^l / \bar{D}_{s-1}^l$. (19) is therefore the share of the q projects' portfolio generated in period s by account j . We can rearrange (19) so as to decompose the share according to the source of funds used. Let $\alpha_s^{rj} := \bar{w}_{s-1}^{rj} / (\sum_{r=1}^q \bar{w}_{s-1}^{rj})$; then $\alpha_s^{rj} \bar{D}_{s-1}^{lj}$ is that part borrowed from creditor l financing the initial outlay \bar{w}_{s-1}^{rj} . Rearranging terms and manipulating we obtain

$$\sum_{r=1}^q \left[\left(\sum_{l=1}^m \alpha_s^{rj} \bar{D}_{s-1}^{lj} (\bar{x}_s^r - \bar{\delta}_s^l) \right) + \left((\bar{w}_{s-1}^{rj} - \sum_{l=1}^m \alpha_s^{rj} \bar{D}_{s-1}^{lj}) (\bar{x}_s^r - i_s^j) \right) \right] \quad (20)$$

Let $A_s^{lrj} := \alpha_s^{rj} \bar{D}_{s-1}^{lj} (\bar{x}_s^r - \bar{\delta}_s^l)$, $l=1, \dots, m$ and denote with $A_s^{m+1,rj}$ the last addend in (20). Summing for j and s we obtain the portfolio's Net Final Value

$$\text{NFV} = \sum_{s=1}^n \sum_{j=1}^p \sum_{r=1}^q \sum_{l=1}^{m+1} A_s^{lrj} \quad (21)$$

A_s^{lrj} is the quota of the portfolio's NFV to be ascribed to source l , to project r , to account j , to period s . The evaluation we have arrived to provides us with four types of decomposition: (I) periodic decomposition (the share of portfolio's NFV generated in period s), obtained by summing A_s^{lrj} for all variables except s ; (II) opportunity account decomposition (the share of portfolio's NFV generated by the use of account K^j), obtained by summing A_s^{lrj} for all variables except j ; (III) project decomposition (the share of portfolio's NFV generated by project r), obtained by summing A_s^{lrj} for all variables except r ; (IV) financing decomposition (the share of portfolio's NFV generated by the use source l), obtained by summing A_s^{lrj} for all variables except l . We further point out that the portfolio's NFV in (21) coincides, as we expect, with the portfolio's DSV we would have obtained by directly using the *systemic* argument, that is

$$\sum_{s=1}^n \left(\sum_{r=1}^q x_s^r w_{s-1}^r - \sum_{l=1}^m \delta_s^l D_{s-1}^l - \sum_{j=1}^p i_s^j (K_0^j \prod_{k=1}^{s-1} (1 + i_k) - K_{s-1}^j) \right)$$

REFERENCES

- [1] LUCIANO, E. and PECCATI, L., A Decomposition of Random Net Present Values, in *Recent Research in Financial Modelling* (E.J. Stokking, L. Zambruno, eds.), Physical Verlag, Heidelberg, pp. 17-23, 1987.
- [2] PECCATI, L., DCF e risultati di periodo, Atti XI Convegno A.M.A.S.E.S., Berlin, 469-83, suppl. al n.39, 1987.
- [3] PECCATI, L., Valutazione analitica e sintetica di attività finanziarie, Rivista Milanese di Economia, serie Quaderni n.21, Cariplo-Laterza, 1991.
- [4] PECCATI, L., Multiperiod Analysis of a Levered Portfolio, in *Modelling for Financial Decisions* (J. Spronk, B. Matarazzo, eds.), Springer-Verlag, Berlin, 1992.
- [5] PRESSACCO, F. and STUCCHI, P., Su un estensione bidimensionale del teorema di scomposizione di Peccati, Rivista di Matematica per le Scienze Economiche e Sociali **20**, 169-85, 1997.