Infrastructure Development vs Direct Cash Transfer: A General Equilibrium Comparison

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Abstract

This paper attempts to provide an explanation to the debate whether infrastructure development is more effective than direct cash transfer to reduce wage disparity between skilled and unskilled workers. We use a simple general equilibrium structure to argue that in the presence of symmetric productivity effects direct cash transfer meets the target when such transfer is financed by tax revenue collected from skilled wage bill. Nevertheless, in case of asymmetric productivity effects the arguments boil down to how different sectors absorb infrastructural facility to improve their productivity.

Keywords: Infrastructure, Redistribution, Personnel income tax and General Equilibrium,

JEL Classification: H54, H23, H24, D5, F1

1. **Introduction**

There has been a huge hue and cry over the issue of relevance and effectiveness of direct cash transfer to the poor. In India, it is observed that following economic reform in 1991 there is a policy of cutting down of transfer of funds to state from central level in order to reduce the fiscal deficit. This has led to a significant decline in production augmenting investment. On the other hand direct subsidies or cash transfer to the poorer and deserving segment is on a rise thereby worsening the public investment problem further[1]. These issues are highlighted in Jha (2007). Similar type of argument has also been described in a policy paper of OECD (2012). Moreover, this paper explains clear reasons of income inequality in the presence of taxes and transfers. Beside this, poverty-reducing policy implications drew much attention in the recent past since the rate of decline in poverty has been reduced in post reform period compared to 1980s. Bhattacharya, Romani and Stern (2012) is another important reference in this line. This paper focuses on the necessity of investment on infrastructure in developing countries, where transport, electricity connections etc are major components that come under infrastructure[2]. Papers by Kumo (2012) and Sahoo, Dash and Nataraj (2010) have also nicely pointed out the causality between

[1] However, Standing (2007) has shouted that cash transfer is an effective way of combating poverty and economic insecurity.

[2] In this context one can also go through the following articles, namely, Easterly and Rebelo (1993), World Bank (1994), Yoshida (2000) and Kim (2006). It is to be noted that these studies have examined the impact of infrastructural investment on economic growth. Again, Ahmed, Abbas and Ahmed (2013) have claimed that in the long run public infrastructure investments have the same direction of impact whether funded by taxation or international borrowing.

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infrastructure investment and economic growth that runs both ways in the context of developing economy[3].

Conventionally, government collects revenue from various sources to finance such welfare policies. Economists, policy makers are not always of the view that an initiative like direct cash transfer to the poor has been very helpful in raising the relative status of the target group as the alternative mechanism normally raises investment in general. Investment in infrastructural development such as transport, electricity etc must induce some changes in the productivity of all economic activities. Hence the issue becomes very interesting from any welfare state's perspective as to which alternative ends with better outcome.

This paper focuses on the debate by starting with a simple general equilibrium structure where infrastructure influences productivity symmetrically. Government finances this through taxing the relatively rich and dividing the revenue between infrastructure development and direct cash transfer to poor or unskilled labor. Here we primarily look at the wage inequality between rich and poor and try to check the effectiveness of direct cash transfer in reducing inequality. So, in a sense our paper is in line with OECD (2012). We then extend the model for asymmetric productivity effect case.

[3] These issues are also explained by Lall and Rastogi (2007). Their paper is primarily written with reference to India.

Remaining paper is arranged as follows. Section 2 develops the basic model with symmetric infrastructural benefits through productivity, per se. It has one subsection. Subsection 2.1 considers the impact of an increase in transfer share on wage inequality. The extension of the basic model has been considered in the section 3 and section 3.1 considers the impact of an increase in transfer share on wage inequality in the presence of asymmetric benefits of spending on infrastructure. The last section winds up the paper with some concluding remarks. Mathematical details are, however, relegated to the Appendix.

2. The Basic Model

We consider a small open economy comprising of two sectors, one produces a skilled labour intensive good(X) with skilled labour (S) and unskilled labour (L). The second one produces an unskilled labour intensive product (Y) with same factors of production that have been used in the first sector. We further assume the presence of welfare state in the sense that total tax revenue (g) collected from of skilled labour is used either for 'infrastructural development' (g_1) and 'direct cash transfer to the poor' (or 'allowance') (g_2) in form of redistribution of λ proportion of total tax revenue (tW_SS) only among unskilled labours or both[4]. We also assume that the portion of tax revenue that will be used for infrastructural development may improve the productivity of both sectors, which can be captured by the productivity parameter $\alpha(g_1)$. Competitive

[4] From a developing economy perspective we realistically assume that unskilled workers need not to pay income tax as their income is very low and does not fall in the taxable income bracket.

markets, CRS technology, diminishing marginal productivity and full employment of factors of production are also assumed[5].

The system of equations can be written as follows:

Competitive price conditions imply

$$W_S a_{SX} + W a_{LX} = P_X \alpha(g_1) \quad (1)$$

$$W_S a_{SY} + W a_{LY} = \alpha(g_1) \quad (2)$$

Note that $\alpha > 0$ and $\alpha' > 0$ as an improvement of infrastructure over the basic level raises productivity at a higher rate for that the system starts functioning very smoothly.

Budget is balanced in the sense that total tax revenue is entirely distributed for two different welfare policies defined earlier. Hence

$$g = g_1 + g_2 = (1-\lambda)tW_S S + \lambda t W_S S \quad (3)$$

$$g_1 = (1-\lambda)tW_S S \quad (4)$$

Full utilization of skilled and unskilled labour, respectively, imply

[5] The following is the list of notations used in the model. P_X = world price X ; P_Y = world price of Y, we assume $P_Y = 1$;L = fixed number of unskilled workers; S = stock of skilled labour; a_{ji} = quantity of the jth factor for producing one unit of output in the ith sector,j=L, S and i = X, Y; θ_{ji} = distributive share of the jth input in the ith sector; W = competitive unskilled wage rate ; W_S = competitive skilled wage rate ; g = total tax revenue; g_1 = part of total tax revenue used for infrastructural development ; g_2 = portion of total tax revenue that transferred to unskilled labour; λ = transfer share ; t = tax rate; α = productivity parameter;

$$a_{SX}X + a_{SY}Y = S \tag{5}$$

$$a_{LX}X + a_{LY}Y = L \tag{6}$$

This is a standard Heckscher-Ohlin set up *a la* Jones (1965). So, given P_X and $\alpha(g_1)$, W_S and W can be determined from equations (1) and (2). Thus g_1 and g_2 are also solved from equations (3) and (4) if we start with some given t and λ. Since all factor prices are determined, a_{ij}s are calculated from CRS assumption. Hence X and Y are solved from (5) and (6).

2.1 Increase in transfer share

In this section we want to examine the impact of an increase in λ on skilled-unskilled wage gap. In the presence of government intervention by means of redistributing tax revenue an increase in λ implies higher g_2 and lower g_1. This in turn leads to low quality of infrastructure or governance in addition to a redistribution of revenue in favour of unskilled workers. A 'hat' over a variable represents proportional change.

Let us begin with an in increase in λ. From equations (1) and (2) we get (for details see Appendix 1),

$$(\theta_{SX} - \varepsilon)\hat{W}_S + \theta_{LX}\hat{W} = \varepsilon\eta\hat{\lambda} \tag{7}$$

$$(\theta_{SY} - \varepsilon)\hat{W}_S + \theta_{LY}\hat{W} = \varepsilon\eta\hat{\lambda} \tag{8}$$

Where, $\varepsilon = (\frac{\delta\alpha}{\delta g_1}\frac{g_1}{\alpha}) > 0$ and $\eta = -\frac{\lambda}{1-\lambda} < 0.$

Note that, when government redistributes tax revenue the relevant skilled and unskilled wages become $\tilde{W}_S$ and $\tilde{W}$ respectively. Therefore, post-tax income of the skilled workers become

Where $(1-t)W_S = \tilde{W}_S$ (9)

Hence, proportionate change in pre-tax and post-tax skilled wage remain unchanged as t is given. So, equation (9) provides

$$\hat{W}_S = \hat{\tilde{W}}_S \tag{10}$$

Similarly, post transfer income of unskilled labour can be written as

$$W + \lambda \frac{tW_S S}{L} = \tilde{W} \tag{11}$$

The proportionate change in unskilled wage is represented as

$$\hat{\tilde{W}} = \theta_{W\tilde{W}}\hat{W} + \theta_{W_d\tilde{W}}(\hat{W}_S + \hat{\lambda}) \tag{12}$$

Where, $\theta_{W\tilde{W}} = \frac{W}{\tilde{W}} = 1 - \lambda\frac{tW_S S}{\tilde{W}L}$ and $\theta_{W_d\tilde{W}} = (\tilde{W} - W)/\tilde{W}$.

The analysis, however, would remain incomplete if we don't allow the effect of government intervention on g_1 which again depends on λ. When λ fraction of total tax revenue is used for direct tax transfer, $(1-\lambda)$ is left for general infrastructure development that should help increasing productivity in both X and Y. If one carefully looks at the structure, it could be easily understood that this change will have no effect on factor return as both the sectors are equally affected by $\alpha(g_1)$.

From equation (10) and (12) we get

$$\hat{\tilde{W}}_S - \hat{\tilde{W}} = -\theta_{W_d\tilde{W}}\hat{\lambda} \tag{13}$$

From the above expression it is clear that for given P_X and t, wage inequality will go down due to an increase in λ. The intuition behind the above argument is that a rise in λ implies more funds will be transferred from government revenue account to unskilled workers. $\tilde{W}$ will increase. Now a rise in λ implies an increase in g_2 and a decrease in g_1. It is to be noted that a decline in g_1 due to rise in λ implies a reduction in productivity ($\alpha(g_1)$) in both the sectors. Therefore, both of $\tilde{W}_S$ and $\tilde{W}$ will go down equi-proportionately. Thus if an increase in $\tilde{W}$ due to a rise in g_2 completely offsets the reduction in $\tilde{W}$ due to a fall in g_1, wage inequality may fall as $\tilde{W}_S$ also falls. Thus we propose that,

> **Proposition 1:** *An increase in government's direct cash transfer to unskilled workers will unambiguously lead to a reduction in wage inequality.*

3. Extended Model: Asymmetric Benefits of Infrastructure

Here we assume that the portion of government tax revenue spends on infrastructural development (g_1) in both skilled and unskilled labour intensive sectors affect both them asymmetrically unlike the basic model. For brevity of analysis we assume that change in infrastructure affects sector X and sector Y by $\alpha_1(g_1)$ and $\alpha_2(g_1)$, respectively.

Thus modified competitive price conditions can be written as

$$W_S a_{SX} + W a_{LX} = P_X \alpha_1(g_1) \quad (1')$$

$$W_S a_{SY} + W a_{LY} = \alpha_2(g_1) \quad (2')$$

Where, $\alpha_1, \alpha_2 > 0$ and $\alpha_1', \alpha_2' > 0$.

All other equations of the basic model remain unchanged. As the structure, number of endogenous variables and number independent equations are akin to the basic model, the working of the extended model would be similar to that of the basic model.

3.1 Changes in transfer share

In this section we also analyze the impact of changes in λ on post-tax wage inequality. It is to be noted that the assumption of symmetric benefits of infrastructure on both sectors may not be plausible since in reality infrastructure affects both the sectors asymmetrically. For instance if we consider the case of power and road: where infrastructural development related to power may help skilled labour intensive sector more compared to unskilled intensive sector, whereas infrastructural improvement related to road may benefit sector Y more compared to sector X. Thus infrastructure generally have asymmetric effects on different sectors.

Using (1′) and (2′) and after some algebraic calculation we obtain (see appendix 2 for detail derivation)

$$\hat{\tilde{W}}_S - \hat{\tilde{W}} = \{\theta_{W\tilde{W}} \frac{1}{|\theta_1|}(\varepsilon_1 - \varepsilon_2)\eta - \theta_{W_d\tilde{W}}\}\hat{\lambda} \tag{3'}$$

From expression (3′) it is evident that the movement of post-tax wage inequality may be positive or negative depending upon the relative value of ε_1 and ε_2.

<u>Case 1, where</u> $\underline{\varepsilon_1 > \varepsilon_2}$***<u>:</u>*** Now, a rise in λ implies more funds will be transferred from government total tax revenue to unskilled workers and hence $\tilde{W}$ will increase. Now a rise in λ implies an increase in g_2 and a decrease in g_1. Hence productivity of sector X declines more compared to sector Y, since $\varepsilon_1 > \varepsilon_2$ (i.e. $\alpha_1(g_1) > \alpha_2(g_1)$ and both falls) Thus, $\tilde{W}_S$ will go down and $\tilde{W}$ will go up. Hence wage inequality woul fall due to an increase in λ. Equation (3′) also ensures that as by definition $\theta_{W_d\tilde{W}}$ must be very low relative to $\theta_{W\tilde{W}}$.

<u>Case 2, where</u> $\underline{\varepsilon_1 < \varepsilon_2}$***<u>:</u>*** In this case responsiveness of α due to a change in g_1 is higher in unskilled labor intensive sector i.e. $\alpha_1(g_1) < \alpha_2(g_1)$. An increase in λ diverts more fund for unskilled workers and hence induces the first round increase in $\widetilde{W}$. However, productivity effect hurts unskilled labor intensive sector more compared to skilled labor intensive sector. This calls for an increase in $\widetilde{W}_S$ and a decrease in $\widetilde{W}$. Therefore, it is certain whether $\widetilde{W}$ eventually rises or falls. If falls, wage inequality unequivocally widens, but if it goes up, the subsequent effect on wage disparity remains unclear.

Proposition 2: *A rise in government's direct cash transfer to unskilled workers will unambiguously lead to a reduction in wage inequality if* $\varepsilon_1 > \varepsilon_2$, *but the effect is ambiguous if* $\varepsilon_1 < \varepsilon_2$.

4. Concluding remarks

In this paper we tried to understand the theoretical underpinnings of a much debated question: whether direct cash transfer is better option than infrastructure development to ameliorate existing wage disparity between skilled and unskilled workers. Using a general equilibrium set up, where production of any good requires some basic infrastructure, it has been explained here that an increase in direct cash transfer to unskilled workers financed by tax revenue collected from skilled workers unambiguously reduces wage disparity if both the sectors experience symmetric infrastructural productivity effects. Interestingly when productivity effects are not symmetric, the result is not so straight forward. The eventual effect on wage inequality crucially depends on which sector is more responsive to any change in infrastructure development. So, we believe that a sound welfare policy with an eye to equity should not be ad hoc, but based on the nature of factor intensity of sectors and their capacities to appropriate infrastructural facilities through improved productivity.

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Appendices

Appendix 1:

Differentiation of equations (1) and (2) gives us

$$\hat{W}_S \theta_{SX} + \hat{W}\theta_{LX} = \varepsilon \hat{g}_1 \tag{1.1}$$

$$\hat{W}_S \theta_{SY} + \hat{W}\theta_{LY} = \varepsilon \hat{g}_1 \tag{1.2}$$

Here, $\varepsilon = (\frac{\delta\alpha}{\delta g_1}\frac{g_1}{\alpha}) > 0$

From (4) one can obtain

$$\hat{g}_1 = \hat{W}_S + \eta\hat{\lambda} \tag{1.3}$$

Where, $\eta = -\frac{\lambda}{1-\lambda} < 0.$

Using (1.3) in equations (1.1) and (1.2) we can get

$$(\theta_{SX} - \varepsilon)\hat{W}_S + \theta_{LX}\hat{W} = \varepsilon\eta\hat{\lambda} \tag{1.4}$$

$$(\theta_{SY} - \varepsilon)\hat{W}_S + \theta_{LY}\hat{W} = \varepsilon\eta\hat{\lambda} \tag{1.5}$$

Arranging equations (1.4) and (1.5) we obtain

$$\begin{bmatrix} (\theta_{SX} - \varepsilon) & \theta_{LX} \\ (\theta_{SY} - \varepsilon) & \theta_{LY} \end{bmatrix} \begin{bmatrix} \hat{W}_S \\ \hat{W} \end{bmatrix} = \begin{bmatrix} \varepsilon\eta\hat{\lambda} \\ \varepsilon\eta\hat{\lambda} \end{bmatrix} \tag{1.6}$$

Solving equation (1.6) by Cramer's rule we can obtain following expressions

$$\hat{W}_S = \frac{1}{|\theta|}[\{(\theta_{LY} - \theta_{LX})\varepsilon\eta\}\hat{\lambda}]$$

$$\hat{W} = \frac{1}{|\theta|}[\{(\theta_{SX} - \theta_{SY})\varepsilon\eta\}\hat{\lambda}]$$

Here $|\theta| = (\theta_{LY} - \theta_{LX})(1-\varepsilon) > 0$ under the assumption that $0 < \varepsilon < 1$. Recall that $\theta_{LY} > \theta_{LX}$ because of factor intensity assumption.

Now, $\hat{W}_S - \hat{W} = \frac{1}{|\theta|}[\{\theta_{LY} + (\theta_{SY} - \varepsilon)\}\hat{P}_X + \{(\theta_{LY} - \theta_{LX} - \theta_{SX} + \theta_{SY})\varepsilon\eta\}\hat{\lambda}]$

Post tax income of skilled labours can be written as

$$W_S S - tW_S S = S\tilde{W}_S \tag{1.7}$$

Therefore $(1-t)W_S = \tilde{W}_S$ (1.8)

Differentiation of equation (1.8) gives us

$$\hat{W}_S = \hat{\tilde{W}}_S \tag{1.9}$$

Similarly, post transfer income of unskilled labour can be written as

$$WL + \lambda\frac{tW_S S}{L}L = L\tilde{W} \tag{1.10}$$

$$W + \lambda\frac{tW_S S}{L} = \tilde{W} \tag{1.11}$$

Differentiation of equation (1.11) gives us

$$\hat{\tilde{W}} = \frac{W}{\tilde{W}}\hat{W} + \frac{(\tilde{W} - W)}{\tilde{W}}(\hat{W}_S + \hat{\lambda})$$

$$\hat{\tilde{W}} = \theta_{W\tilde{W}}\hat{W} + \theta_{W_S\tilde{W}}(\hat{W}_S + \hat{\lambda}) \tag{1.12}$$

Where, $\theta_{W\tilde{W}} = \frac{W}{\tilde{W}} = 1 - \lambda\frac{tW_S S}{\tilde{W}L}$

From (1.12) we get

$$\hat{\tilde{W}} = \theta_{W\tilde{W}}\hat{W} + (1 - \theta_{W\tilde{W}})\hat{W}_S + \theta_{W_S\tilde{W}}\hat{\lambda}$$

$$\hat{\tilde{W}} = \theta_{W\tilde{W}}(\hat{W} - \hat{W}_S) + \hat{W}_S + \theta_{W_S\tilde{W}}\hat{\lambda}$$

Using (1.9) in the above expression and after some simplification we get

$$\hat{\tilde{W}}_S - \hat{\tilde{W}} = \frac{W}{\tilde{W}}(\hat{W}_S - \hat{W}) - \frac{(\tilde{W} - W)}{\tilde{W}}\hat{\lambda} \quad (1.13)$$

Where, $\theta_{W\tilde{W}} = \dfrac{W}{\tilde{W}} = 1 - \lambda\dfrac{tW_S S}{\tilde{W}L}$

$$\hat{\tilde{W}}_S - \hat{\tilde{W}} = -\theta_{W_S\tilde{W}}\hat{\lambda} \quad (1.14)$$

Appendix 2:

Differentiation of equations (1/) and (2/)

$$\hat{W}_S\theta_{SX} + \hat{W}\theta_{LX} = \hat{P}_X + \varepsilon_1\hat{g}_1 \quad (2.1')$$

$$\hat{W}_S\theta_{SY} + \hat{W}\theta_{LY} = \varepsilon_2\hat{g}_1 \quad (2.2')$$

Here, $\varepsilon_1 = (\dfrac{\partial\alpha_1}{\partial g_1}\dfrac{g_1}{\alpha_1}) > 0, \varepsilon_2 = (\dfrac{\partial\alpha_2}{\partial g_1}\dfrac{g_2}{\alpha_2}) > 0.$

Using (1.3) in equations (2.1′) and (2.2′)

$$(\theta_{SX} - \varepsilon_1)\hat{W}_S + \theta_{LX}\hat{W} = \varepsilon_1\eta\hat{\lambda} \quad (2.3')$$

$$(\theta_{SY} - \varepsilon_2)\hat{W}_S + \theta_{LY}\hat{W} = \varepsilon_2\eta\hat{\lambda} \quad (2.4')$$

Arranging equations (2.3′) and (2.4′) in a matrix form

$$\begin{bmatrix} (\theta_{SX} - \varepsilon_1) & \theta_{LX} \\ (\theta_{SY} - \varepsilon_2) & \theta_{LY} \end{bmatrix}\begin{bmatrix} \hat{W}_S \\ \hat{W} \end{bmatrix} = \begin{bmatrix} \varepsilon_1\eta\hat{\lambda} \\ \varepsilon_2\eta\hat{\lambda} \end{bmatrix} \quad (2.5')$$

Solving equation (2.5′) by Cramer's rule we can obtain following expressions

$$\hat{W}_S = \frac{1}{|\theta_1|}[\{(\theta_{LY}\varepsilon_1 - \theta_{LX}\varepsilon_2)\eta\}\hat{\lambda}]$$

$$\hat{W} = \frac{1}{|\theta_1|}[\{(\theta_{SX}\varepsilon_2 - \theta_{SY}\varepsilon_1)\eta\}\hat{\lambda}]$$

Where, $|\theta_1| = [(\theta_{SX}\theta_{LY} - \theta_{SY}\theta_{LX}) - (\theta_{LY}\varepsilon_1 - \theta_{LX}\varepsilon_2)] > 0$

$$\hat{W}_S - \hat{W} = \frac{1}{|\theta_1|}[\{(\varepsilon_1 - \varepsilon_2)\eta\}\hat{\lambda}] \qquad (2.6')$$

It implies $\hat{W}_S - \hat{W} > 0$ when $\hat{\lambda} > 0$.

From (1.12) and (1.9) we get

$$\hat{\tilde{W}} = \theta_{W\tilde{W}}\hat{W} + (1 - \theta_{W\tilde{W}})\hat{W}_S + \theta_{W_S\tilde{W}}\hat{\lambda}$$

$$\hat{\tilde{W}} = \theta_{W\tilde{W}}(\hat{W} - \hat{W}_S) + \hat{W}_S + \theta_{W_S\tilde{W}}\hat{\lambda}$$

After some algebraic calculations we get

$$\hat{\tilde{W}}_S - \hat{\tilde{W}} = [\{\theta_{W\tilde{W}}\frac{1}{|\theta_1|}(\varepsilon_1 - \varepsilon_2)\eta - \theta_{W_S\tilde{W}}\}\hat{\lambda}] \qquad (2.7')$$