A Hedonic Output Index based Approach to Modeling Polluting Technologies

Malikov, Emir and Bokusheva, Raushan and Kumbhakar, Subal C.

Auburn Univeristy, Swiss Federal Institute of Technolog, State University of New York at Binghamton

2016

A Hedonic Output Index based Approach to Modeling Polluting Technologies*

Emir Malikov[1] Raushan Bokusheva[2,3] Subal C. Kumbhakar[4]

[1]Department of Agricultural Economics & Rur. Soc., Auburn University, Auburn, AL
[2]Agricultural Economics, Swiss Federal Institute of Technology (ETH Zurich), Switzerland
[3]Trade and Agriculture Directorate, OECD, Paris, France
[4]Department of Economics, State University of New York at Binghamton, Binghamton, NY

This Draft: June 20, 2016

Abstract

Despite some recent criticisms, the conventional radial distance function, which treats undesirable by-products as either frontier shifters or inputs, remains a popular go-to formulation of polluting production processes among practitioners. This unfading popularity is arguably driven by the ability of radial distance functions, unlike alternative directional distance functions, to allow for unit-free multiplicative changes in arguments as well as, by implicitly postulating the radial direction, to free researchers from the dilemma of having to explicitly choose the directional vector. In this paper, we offer a generalization of the standard radial distance function to polluting technologies that can accommodate undesirable by-products in a more economically meaningful way. Specifically, we propose modeling undesirable outputs via a hedonic output index, which is meant to ensure that pollutants are treated as outputs, as opposed to inputs or theoretically unregulated frontier shifters, while also recognizing their undesirable nature. By using a radial input distance function generalized to encompass an (unobservable) hedonic output index of desirable and undesirable outputs, we are able to meaningfully describe relationships between different products (including the complementarity of desirable and undesirable outputs) within producible output sets as well as to represent technically feasible polluting production possibilities given inputs. An empirical application of our methodology to the case of Dutch dairy farms in 2001–2009 demonstrates the complexity of interactions between outputs, thereby attesting to the value of more elaborate representations of production possibilities.

Keywords: bad output, dairy production, input distance function, livestock, nitrogen pollution, shadow price

JEL Classification: D24, D62, Q12

* *Email:* emalikov@auburn.edu (Malikov), raushan.bokusheva@oecd.org (Bokusheva), kkar@binghamton.edu (Kumbhakar).

The data used in the present paper come from the Dutch FADN system as collected by the Dutch Agricultural Economics Research Institute (LEI). The Centre of Economic Information (CEI) has provided access to these data. The reported results are and remain entirely the responsibility of the authors; they neither represent the views of LEI/IEC nor do they constitute official statistics. Bokusheva also acknowledges financial support by the Swiss National Science Foundation (Research Grant No. 100014_128967).

1 Introduction

The by-production of undesirable, or so-called "bad", outputs is an inherent attribute of many production processes. In agriculture, some examples of such processes include the use of pesticides in farming which, while increasing crop yields (good outputs), also results in an increase in undesirable and oftentimes non-marketed environmental risks associated with leaching and runoff, or dairy farming where the production of desirable dairy products like milk is accompanied by the natural but undesirable generation of nitrogen surpluses in the form of manure. Moreover, the by-production of undesirable outputs is not confined to agricultural production only. Undesirable by-products are also relevant in carbon-based electric power generation that is accompanied by the emission of pollutant gases or even in the provision of banking services which suffers from the by-production of undesirable outputs like non-performing loans due to banks' exposure to credit uncertainty. These examples highlight the importance of accounting for undesirable outputs in the estimation of polluting production technologies.

The modeling of polluting production technologies is however not a clear-cut issue. The assortment of existing approaches to the formulation of multiple-output multiple-input production processes in the presence of undesirable by-products include *(i)* hyperbolic distance functions that allow equiproportional expansion of desirable and contraction of undesirable outputs (e.g., Färe et al., 1989; Cuesta et al., 2009), *(ii)* directional distance functions that allow additive expansion of desirable and contraction of undesirable outputs in a pre-specified direction (e.g., Chung et al., 1997; Färe et al., 2005) or *(iii)* by-production systems of separable technologies for desirable production and undesirable pollution generation (e.g., Fernández et al., 2002, 2005; Murty et al., 2012; Malikov et al., 2015a).

Despite this apparent abundance of modeling frameworks, the conventional radial distance function of Shephard (1953, 1970), which measures equiproportional changes in inputs or desirable outputs, however persistently remains a popular go-to formulation of polluting production processes among practitioners. The conventional radial distance function is usually augmented to incorporate undesirable outputs in two ways. The function is either conditioned on the quantity of undesirable by-products treated as (theoretically unregulated) technology shifters (e.g., Atkinson and Dorfman, 2005; Assaf et al., 2013) or expanded to effectively incorporate undesirable by-products in the role of inputs (e.g., Reinhard et al., 1999, 2000; Hailu and Veeman, 2000, 2001). Arguably, these approaches remain popular among applied economists in spite of their criticism for the implied strong disposability of undesirable outputs (Färe et al., 2005) because, unlike alternative directional distance functions, radial functions permit unit-free multiplicative changes in arguments as well as, by implicitly postulating the radial direction, free researchers from the dilemma of having to explicitly choose the directional vector.

In this paper, we offer a generalization of the standard radial distance function to polluting technologies that can accommodate undesirable by-products in a more economically meaningful way. Specifically, we propose modeling undesirable outputs via a hedonic output index, which is meant to ensure that pollutants are treated as outputs, as opposed to inputs or theoretically unregulated controls, while also recognizing their undesirable nature. By using a radial input distance function (IDF) generalized to encompass an (unobservable) hedonic output index of desirable and undesirable outputs, we are able to meaningfully describe relationships between different products (including the complementarity of desirable and undesirable outputs) within producible output sets as well as to represent technically feasible polluting production possibilities given inputs.

We apply our methodology to study the production technology of Dutch dairy farms which, in addition to desirable outputs such as milk, other livestock products and crops, also generate environmentally detrimental nitrogen surpluses. This nitrogen pollution mostly arises from an application of manure produced by livestock to soil well in excess of what crops need for growth (Reinhard et al., 1999). The relationships among the outputs are rather complex in this setting. For instance, while a nitrogen surplus has a tendency to increase with higher levels of dairy and livestock production, the surplus might decrease (per unit) for farms with more crop production, since the latter absorbs manure generated by livestock. Furthermore, although the crops produced for feeding the farm livestock are complements to dairy products, crop production for sale is certainly a substitute for dairy production. Accordingly, a more elaborate approach is required to consider technological relations between desirable and undesirable outputs in this context. Following our hedonic-output-index-based approach, we study the links between two desirable outputs, namely dairy and crop products for sale, and an undesirable nitrogen surplus.

We estimate our proposed hedonic-output-index-based model subject to theoretical regularity conditions using the Augmented Lagrangian Adaptive Barrier Minimization Algorithm (Lange, 2004) that is capable of handling nonlinear inequality constraints. We impose monotonicity and curvature regularity restrictions (at every data point) in order to ensure that our results are economically meaningful, as emphasized by Barnett et al. (1991) and Barnett (2002). Among other things, our findings confirm significant complementarity between nitrogen surpluses and dairy farms' desirable outputs. Our estimates also suggest the median price of non-marketed nitrogen pollutants at €893 per ton (in 2005 prices). In addition to our proposed model, we also estimate two auxiliary models based on the traditional IDF, as customarily done in the literature. Our findings exemplify the need in a more elaborate approach to modeling undesirable outputs in the setup of radial distance functions. Specifically, we find that modeling undesirable outputs as technology shifters yields rather counter-intuitive results suggesting that nitrogen surpluses may not only be substitutable with other desirable outputs but also desirable. On the other hand, when treating nitrogen surplus as an input, the radial IDF produces unrealistically high estimates of the shadow price of this non-marketable pollutant as well as stark evidence of the dramatic technological regress in the Dutch dairy sector in 2001–2009. In contrast to these popular specifications of radial distance functions, our hedonic-output-index-based model yields more reasonable and intuitive results.

The rest of the paper proceeds as follows. Section 2 describes the (radial) distance function formulation of the production process in the presence of undesirable outputs where the latter are incorporated via a hedonic output index. We present our econometric model in Section 3. Data on Dutch dairy farms are discussed in Section 4. Section 5 presents the empirical results, and Section 6 concludes.

2 Input Distance Function with a Hedonic Output Index

We start by introducing the (radial) distance function formulation of the production process in the presence of undesirable outputs. Consider the pollution-generating production process in which J inputs $\mathbf{x} \in \Re^J_+$, which include both "good" and pollution-generating inputs, are being transformed into M desirable ("good") outputs $\mathbf{y} \in \Re^M_+$ and P undesirable by-products $\mathbf{b} \in \Re^P_+$ ("bad" outputs) such as pollution. The production technology is given by

$$\mathbb{T} \stackrel{\text{def}}{=} \{(\mathbf{x}, \mathbf{y}, \mathbf{b}) : \ \mathbf{x} \text{ can produce } (\mathbf{y}, \mathbf{b})\}, \tag{2.1}$$

subject to the usual axioms (see Chambers et al., 1998; Färe et al., 2005):

(T.1) closedness of $\mathbb{T}$;

(T.2) no free lunch: if $(\mathbf{x}, \mathbf{y}, \mathbf{b}) \in \mathbb{T}$ and $\mathbf{x} = \mathbf{0}$, then $(\mathbf{y}, \mathbf{b}) = (\mathbf{0}, \mathbf{0})$;

(T.3) null-jointness of the output production: if $(\mathbf{x}, \mathbf{y}, \mathbf{b}) \in \mathbb{T}$ and $\mathbf{b} = \mathbf{0}$, then $\mathbf{y} = \mathbf{0}$;

(T.4) free input disposability: if $(\mathbf{x}, \mathbf{y}, \mathbf{b}) \in \mathbb{T}$ and $\mathbf{x}' \geq \mathbf{x}$, then $(\mathbf{x}', \mathbf{y}, \mathbf{b}) \in \mathbb{T}$;

(T.5) weak *joint* disposability of desirable and undesirable outputs: if $(\mathbf{x}, \mathbf{y}, \mathbf{b}) \in \mathbb{T}$ and $0 \leq \kappa \leq 1$, then $(\mathbf{x}, \kappa\mathbf{y}, \kappa\mathbf{b}) \in \mathbb{T}$;

(T.6) free disposability of desirable outputs: if $(\mathbf{x}, \mathbf{y}, \mathbf{b}) \in \mathbb{T}$ and $\mathbf{y}' \leq \mathbf{y}$, then $(\mathbf{x}, \mathbf{y}', \mathbf{b}) \in \mathbb{T}$;

(T.7) feasibility of inaction: $(\mathbf{0}, \mathbf{0}, \mathbf{0}) \in \mathbb{T}$;

(T.8) convexity of $\mathbb{T}$.

To allow for a meaningful relationship between desirable and undesirable outputs as well as to ensure that the above theoretical properties of production technology are maintained, we propose modeling the pollution-generating nature of $\mathbb{T}$ using a hedonic output (index) function. Specifically, we formulate technology $\mathbb{T}$ in the form of the radial input distance function $D_i : \Re^J_+ \times \Re_+ \to [1; +\infty)$ defined as

$$D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big) \stackrel{\text{def}}{=} \sup_{\beta} \left\{\beta \geq 1 : \ \big(\mathbf{x}/\beta, h(\mathbf{y}, \mathbf{b})\big) \in \mathbb{T}\right\}, \tag{2.2}$$

where $h(\mathbf{y}, \mathbf{b}) : \Re^M_+ \times \Re^P_+ \to \Re_+$ is a scalar hedonic function which aggregates both desirable and undesirable outputs into an output index. The hedonic-output-index-based IDF in (2.2) measures the (input-)radial distance between the observed $(\mathbf{x}, \mathbf{y}, \mathbf{b})$ and the boundary of technology $\mathbb{T}$. It seeks the maximal proportionate contraction of inputs $\mathbf{x}$ while still preserving the feasibility of outputs $(\mathbf{y}, \mathbf{b})$. The function $D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big)$ satisfies the following usual theoretical properties extended to accommodate a single output index:

(D.1) $D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big) \geq 1 \iff (\mathbf{x}, \mathbf{y}, \mathbf{b}) \in \mathbb{T}$;

(D.2) linear homogeneity in inputs: $D_i\big(\alpha\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big) = \alpha D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big)$ for $\alpha > 0$;

(D.3) positive monotonicity in inputs: if $\mathbf{x}' \geq \mathbf{x}$, then $D_i\big(\mathbf{x}', h(\mathbf{y}, \mathbf{b})\big) \geq D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big)$;

(D.4) strict negative monotonicity in the hedonic output index: if $h'(\mathbf{y}, \mathbf{b}) \leq h(\mathbf{y}, \mathbf{b})$, then $D_i\big(\mathbf{x}, h'(\mathbf{y}, \mathbf{b})\big) > D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big)$;

(D.5) concavity of $D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big)$ in $\mathbf{x}$.

The formulation in (2.2) differs from a standard textbook definition of the IDF (e.g., Färe and Primont, 1995; Kumbhakar and Lovell, 2000) in that it imposes a structure on both desirable and undesirable outputs by having the latter enter the distance function via a hedonic output index $h(\cdot)$. Such a specification is meant to ensure that undesirable by-products are explicitly treated as outputs rather than as "controls" or, worse, inputs. Function $h(\cdot)$ is an equivalent representation of the output vector $(\mathbf{y}, \mathbf{b})$ and satisfies the following properties:

(H.1) linear homogeneity: $h(\alpha\mathbf{y}, \alpha\mathbf{b}) = \alpha h(\mathbf{y}, \mathbf{b})$ for $\alpha > 0$;

(H.2) positive monotonicity in desirable outputs: if $\mathbf{y}' \geq \mathbf{y}$, then $h(\mathbf{y}', \mathbf{b}) \geq h(\mathbf{y}, \mathbf{b})$;

(H.3) strict negative monotonicity in undesirable outputs: if $\mathbf{b}' \leq \mathbf{b}$, then $h(\mathbf{y}, \mathbf{b}') > h(\mathbf{y}, \mathbf{b})$.

Properties (H.1)–(H.3) together suggest costly disposability of undesirable outputs $\mathbf{b}$ as well as complementarity between the two types of outputs (on the technological frontier), i.e., $\partial b_p/\partial y_m \geq 0 \ \forall \ m = 1, \ldots, M; \ p = 1, \ldots, P$. The latter is especially important because, given the polluting nature of $\mathbb{T}$, the quantity of desirable outputs $\mathbf{y}$ cannot be increased without also increasing the quantity of its by-products $\mathbf{b}$ for a given input vector $\mathbf{x}$ (in the absence of inefficiency and abatement). However, we emphasize that $h(\cdot)$ is unobserved in practice.

3 Econometric Model

In this paper, we consider a stochastic production process, where for simplicity we assume that all firms are technically efficient, i.e., operate on the technological frontier. The stochastic IDF with a hedonic output index is then given by

$$1 = D_i\big(\mathbf{x}, h(\mathbf{y},\mathbf{b})\big)\exp\{\epsilon\}, \tag{3.1}$$

where ϵ is an i.i.d. zero-mean finite-variance random disturbance. We impose the linear homogeneity property onto the IDF by making use of (D.2) and setting $\alpha = 1/x_1$, which yields the following equation in logs:

$$-\ln x_1 = \ln D_i\big(\mathbf{x}/x_1, h(\mathbf{y},\mathbf{b})\big) + \epsilon. \tag{3.2}$$

In a similar fashion, we use property (H.1) to impose linear homogeneity onto the hedonic output index (by setting $\alpha = 1/b_1$):

$$\ln h(\mathbf{y},\mathbf{b}) = \ln b_1 + \ln h\big(\mathbf{y}/b_1, \mathbf{b}/b_1)\big), \tag{3.3}$$

where we remind the reader that $h(\mathbf{y},\mathbf{b})$ is unobserved.

We assume that both the IDF and the hedonic output index take flexible translog functional forms as well as allow for temporal shifts in the technological frontier via the time trend t:

$$\begin{aligned}
-\ln x_1 = \alpha_0 + \alpha_h \ln h(\mathbf{y},\mathbf{b}) + \frac{1}{2}\alpha_{hh}\Big(\ln h(\mathbf{y},\mathbf{b})\Big)^2 + \sum_{j\neq 1}\alpha_{hj}\ln h(\mathbf{y},\mathbf{b})\ln\left(\frac{x_j}{x_1}\right) + \\
\sum_{j\neq 1}\beta_j \ln\left(\frac{x_j}{x_1}\right) + \frac{1}{2}\sum_{j\neq 1}\sum_{j'\neq 1}\beta_{jj'}\ln\left(\frac{x_j}{x_1}\right)\ln\left(\frac{x_{j'}}{x_1}\right) + \\
\gamma_t t + \frac{1}{2}\gamma_{tt}t^2 + \gamma_{th}\; t\ln h(\mathbf{y},\mathbf{b}) + \sum_{j\neq 1}\gamma_{tj}\; t\ln\left(\frac{x_j}{x_1}\right) + \epsilon,
\end{aligned} \tag{3.4}$$

where

$$\begin{aligned}
\ln h(\mathbf{y},\mathbf{b}) = \ln b_1 + \sum_m \omega_m \ln\left(\frac{y_m}{b_1}\right) + \frac{1}{2}\sum_m\sum_{m'}\omega_{mm'}\ln\left(\frac{y_m}{b_1}\right)\ln\left(\frac{y_{m'}}{b_1}\right) + \\
\sum_{p\neq 1}\delta_p \ln\left(\frac{b_p}{b_1}\right) + \frac{1}{2}\sum_{p\neq 1}\sum_{p'\neq 1}\delta_{pp'}\ln\left(\frac{b_p}{b_1}\right)\ln\left(\frac{b_{p'}}{b_1}\right) + \\
\sum_m\sum_{p\neq 1}\theta_{mp}\ln\left(\frac{y_m}{b_1}\right)\ln\left(\frac{b_p}{b_1}\right).
\end{aligned} \tag{3.5}$$

The model in (3.4)–(3.5) is estimated by substituting the equation for the hedonic output index in (3.5) into the IDF given in (3.4), which produces a highly nonlinear (in parameters) specification of the production technology.[1]

We estimate the hedonic-output-index-based IDF in (3.4)–(3.5) subject to the Slutsky symmetry as well as the theoretical monotonicity and curvature restrictions in order to ensure that our results are economically meaningful, as emphasized by Barnett et al. (1991), Barnett (2002) and Malikov

[1]Note that the constant term in the translog specification of the hedonic output index is normalized to zero for identification purposes.

et al. (2015b).[2] Specifically, the monotonicity of $D_i\big(\mathbf{x}, h(\mathbf{y}, \mathbf{b})\big)$ in inputs and hedonic output index, respectively, requires that the log-derivatives satisfy the following conditions:

$$\frac{\partial \ln D_i(\cdot)}{\partial \ln x_j} = \alpha_{hj} \ln h(\mathbf{y}, \mathbf{b}) + \beta_j + \sum_{j'} \beta_{jj'} \ln x_{j'} + \gamma_{tj} t \geq 0 \quad \forall \quad j = 1, \ldots, J \tag{3.6a}$$

$$\frac{\partial \ln D_i(\cdot)}{\partial \ln h(\cdot)} = \alpha_h + \alpha_{hh} \ln h(\mathbf{y}, \mathbf{b}) + \sum_{j} \alpha_{hj} \ln x_j + \gamma_{th} t < 0, \tag{3.6b}$$

whereas monotonicity of the hedonic output index $h(\mathbf{y}, \mathbf{b})$ in desirable and undesirable outputs, respectively, requires that

$$\frac{\partial \ln h(\cdot)}{\partial \ln y_m} = \omega_m + \sum_{m'} \omega_{mm'} \ln y_{m'} + \sum_{p} \theta_{mp} \ln b_p \geq 0 \quad \forall \quad m = 1, \ldots, M \tag{3.7a}$$

$$\frac{\partial \ln h(\cdot)}{\partial \ln b_p} = \delta_p + \sum_{p'} \delta_{pp'} \ln b_{p'} + \sum_{m} \theta_{mp} \ln y_m < 0 \quad \forall \quad p = 1, \ldots, P. \tag{3.7b}$$

The concavity of the IDF requires that its Hessian matrix

$$\mathbb{H}_i = \begin{bmatrix} \beta_{11} & \cdots & \beta_{1J} \\ \vdots & \ddots & \vdots \\ \beta_{J1} & \cdots & \beta_{JJ} \end{bmatrix} \tag{3.8}$$

is a negative semi-definite. We guarantee the latter by restricting odd-numbered (even-numbered) principal minors of $\mathbb{H}_i$ to be non-positive (non-negative).

Few remarks are warranted here. Both the monotonicity and concavity constraints are formulated in the level form, i.e., *prior to* the "linear homogeneity" normalization. To recover parameters for the normalizing input x_1 and output b_1, we use the following restrictions implicitly built into the normalized IDF (3.4) and hedonic output index (3.5):

$$\sum_j \beta_j = 1, \qquad \sum_j \beta_{jj'} = 0 \; \forall \; j', \qquad \sum_j \alpha_{hj} = 0, \qquad \sum_j \gamma_{tj} = 0;$$
$$\sum_m \omega_m + \sum_p \delta_p = 1, \qquad \sum_m \omega_{mm'} + \sum_p \theta_{pm'} = 0 \; \forall \; m', \qquad \sum_p \delta_{pp'} + \sum_m \theta_{mp'} = 0 \; \forall \; p'.$$

The monotonicity restrictions are observation-specific and imposed at every data point. In total, we have $(J+M+P+1)NT+J$ theoretical regularity conditions, where N and T are the numbers of firms and time periods, respectively.

4 Data

The data for a sample of Dutch dairy farms come from the Farm Data Accountancy Network (FADN) which represents 99% of the dairy production in the Netherlands. Our data sample is an unbalanced panel of 348 farms observed over the period from 2001 to 2009. The sample consists of 1,866 observations on dairy farms whose revenues from sales of milk and livestock products account for at least 80% of their total revenues.

[2] Recall that the linear homogeneity properties are already imposed by construction.

Table 1. Summary Statistics for Dutch Dairy Production, 2001–2009

Variable	Description and Units	Mean	Median	SD
x_1	Materials & Feed, *Real* €1,000	84.91	72.73	55.11
x_2	Land, *Hectares*	51.47	43.99	30.40
x_3	Labor, *Man-Years*	1.71	1.59	0.66
x_4	Capital, *Real* €1,000	48.81	39.57	34.40
x_5	Livestock, *Livestock Std. Units*	114.24	98.00	66.66
b	Nitrogen Surplus, *Tons*	9.10	7.44	6.47
y_1	Dairy & Livestock Products, *Real* €1,000	227.12	188.81	154.03
y_2	Crop Products, *Real* €1,000	13.34	7.93	15.78
s_1	Share of y_1 in Total Output, *%*	94.60	96.09	4.56

NOTE: Zero values appear due to rounding.

We distinguish between two desirable and one undesirable outputs. The good outputs are y_1, defined as the real revenue from sales of milk and livestock products plus changes in the valuation of the livestock, and y_2, defined as the real revenue from sales of crops and other agricultural products. The production of both desirable outputs by farms in the livestock sector is accompanied by the by-production of an undesirable, environmentally detrimental output – "nitrogen surplus" (b). This nitrogen pollution mostly arises from an application of manure produced by livestock to soil well in excess of what crops need for growth (Reinhard et al., 1999). To document such nitrogen pollution, Dutch dairy farms are required to keep nutrient balance sheets, from which we can quantify a nitrogen surplus as the difference between the quantity of nitrogen applied on the farm and the quantity of nitrogen in the farms' desirable outputs (see Reinhard et al., 1999, 2000).

The farm's five inputs are materials (x_1) which also include purchased feed, land (x_2), labor (x_3), capital (x_4) and livestock (x_5). Materials are defined as farm variable costs including those on purchased feed and concentrated feeding stuff. Land is defined as the farm total agricultural land measured in hectares. The number of agricultural work units is used to measure the farm labor force. Capital is defined in terms of the value of the machines and building depreciation. Livestock is measured as the number of standardized livestock units. All monetary values were deflated to the 2005 price level by using the price indices for each category as reported by Eurostat (2012). All variables are normalized by their respective geometric means. Table 1 reports the summary statistics for our variables.

On average, nitrogen application per farm is 14.87 tons per year over the course of the sample period. The intensity of nitrogen use during the 2001–2009 period is 0.29 tons per hectare of agricultural land and 0.13 tons as measured per livestock unit. The surplus nitrogen during the same period amounts to an average of 9.10 tons per farm-year, which results in intensity values of 0.18 tons per hectare of land and 79.9 kilograms per livestock unit. These figures indicate that the use of nitrogen and the accompanying (by-)generation of a nitrogen surplus has decreased substantially in the last decade. For instance, the average nutrient input by Dutch dairy farms from a comparable FADN data sample used by Reinhard et al. (2000) was significantly higher in 1991–1994 amounting to 17.75 tons per farm-year. The dairy farms in Reinhard et al.'s (2000) sample also generated considerably more nitrogen surpluses: an average of 14.63 tons per farm-year. Furthermore, dairy farms in 1991–1994 also appear to have had higher ratios of total revenue to nitrogen surpluses than farms in our 2001–2009 sample: an average of €26.4 vs. €17.3 of revenue per kilogram of nitrogen surplus (in 2005 prices).

Table 2. Summary of the Estimated Models

Model	Formulation of the IDF	Treatment of $\mathbf{b}$
(I)	$D_i\big(\mathbf{x}, h(\mathbf{y},\mathbf{b})\big) = \sup_\beta \{\beta \geq 1 : \ \big(\mathbf{x}/\beta, h(\mathbf{y},\mathbf{b})\big) \in \mathbb{T}\}$	Undesirable output
(II)	$D_i(\mathbf{x},\mathbf{y},\mathbf{b}) = \sup_\beta \{\beta \geq 1 : \ (\mathbf{x}/\beta, \mathbf{y}, \mathbf{b}) \in \mathbb{T}\}$	(Theoretically unregulated) technology shifter
(III)	$D_i(\mathbf{x},\mathbf{y},\mathbf{b}) = \sup_\beta \{\beta \geq 1 : \ (\mathbf{x}/\beta, \mathbf{y}, \mathbf{b}/\beta) \in \mathbb{T}\}$	Input

5 Results

This section reports estimates of the hedonic-output-index-based IDF that we propose in this paper for Dutch dairy farms in 2001–2009. We estimate the nonlinear model in (3.4)–(3.5) subject to $(J + M + P + 1)NT + J$ restrictions in (3.6)–(3.8) via nonlinear least squares.[3] The theoretical restrictions that we seek to impose (to ensure an economically meaningful analysis) are observation-specific and nonlinear, which makes their computational implementation rather difficult. For instance, the standard constrained optimization routines subject to the so-called "box" constraints on parameters are of no use here. Instead, we implement the constrained optimization of the nonlinear least squares objective function corresponding to our model in (3.4)–(3.5) using the Augmented Lagrangian Adaptive Barrier Minimization Algorithm (Lange, 2004) that is capable of handling nonlinear inequality constraints. The algorithm searches for a solution by replacing the original constrained problem with a sequence of unconstrained problems adding a penalty and Lagrange-multiplier-type terms per constraint to the objective function.

In addition to our preferred hedonic-output-index-based IDF model [hereafter referred to as Model (I)], we also estimate two auxiliary models based on the more conventional IDF formulations of the production processes augmented to incorporate undesirable by-products. The two models are as follows. First, as is oftentimes done in the literature (e.g., Atkinson and Dorfman, 2005; Assaf et al., 2013), we estimate the standard IDF that treats undesirable by-products as (theoretically unregulated) technology shifters [Model (II)]. As usual, here the IDF is defined as $D_i\big(\mathbf{x},\mathbf{y},\mathbf{b}\big) = \sup_\beta \{\beta \geq 1 : \ (\mathbf{x}/\beta, \mathbf{y}, \mathbf{b}) \in \mathbb{T}\}$ which is assumed to be linearly homogeneous, positively monotone and concave in inputs $\mathbf{x}$ and negatively monotone in desirable outputs $\mathbf{y}$. No explicit theoretical regularity assumption is made about undesirable outputs $\mathbf{b}$ that are treated as mere "control variables" shifting the frontier. The other auxiliary model [Model (III)] is also based on the standard IDF with the sole difference from Model (II) in that it treats undesirable by-products effectively as inputs necessary for the production of desirable outputs, i.e., $D_i\big(\mathbf{x},\mathbf{y},\mathbf{b}\big) = \sup_\beta \{\beta \geq 1 : \ (\mathbf{x}/\beta, \mathbf{y}, \mathbf{b}/\beta) \in \mathbb{T}\}$ (e.g., Reinhard et al., 1999, 2000; Hailu and Veeman, 2000). In this case, the IDF is assumed to be linearly homogeneous, positively monotone and concave in both $\mathbf{x}$ and $\mathbf{b}$ and negatively monotone in desirable outputs $\mathbf{y}$. Both auxiliary models are estimated subject to the corresponding regularity conditions.

For the reader's convenience, all estimated models are summarized in Table 2. By estimating these models, we are able to empirically assess the sensitivity of the estimates of a polluting production technology to the treatment of undesirable by-products.

5.1 Elasticities

The estimates of the IDF elasticities obtained from the three models are summarized in Table 3. The last column reports the fraction of observations for which the elasticity estimates would have

[3] Quasi maximum likelihood is another feasible alternative.

Table 3. Elasticities of the Input Distance Function, 2001–2009

Elast.	Mean	1st Qu.	Median	3rd Qu.	Violations
			Model (I)		
x_1	0.2657	0.2231	0.2672	0.3109	0.21%
x_2	0.0708	0.0541	0.0705	0.0869	22.24%
x_3	0.3150	0.2310	0.3117	0.3953	0.91%
x_4	0.0646	0.0484	0.0643	0.0808	0.32%
x_5	0.2837	0.2155	0.2859	0.3517	0.00%
y_1	–0.7972	–0.8799	–0.7980	–0.7169	0.00%
y_2	–0.0968	–0.1084	–0.0973	–0.0847	0.16%
b	0.0083	0.0050	0.0081	0.0111	96.62%
			Model (II)		
x_1	0.2669	0.2128	0.2677	0.3236	0.16%
x_2	0.0551	0.0438	0.0551	0.0658	19.34%
x_3	0.2839	0.2144	0.2797	0.3573	1.87%
x_4	0.1531	0.1135	0.1534	0.1925	0.53%
x_5	0.2410	0.1870	0.2415	0.2918	0.00%
y_1	–0.7657	–0.8573	–0.7698	–0.6700	0.00%
y_2	–0.0368	–0.0446	–0.0366	–0.0284	3.00%
b	–0.2692	–0.3510	–0.2716	–0.1928	—
			Model (III)		
x_1	0.3344	0.2891	0.3351	0.3837	1.33%
x_2	0.0533	0.0411	0.0533	0.0648	42.65%
x_3	0.2222	0.1648	0.2176	0.2710	0.37%
x_4	0.1250	0.1060	0.1253	0.1450	0.80%
x_5	0.2321	0.1810	0.2322	0.2842	0.37%
y_1	–0.8793	–1.0950	–0.8919	–0.6562	0.00%
y_2	–0.0490	–0.0603	–0.0482	–0.0378	4.55%
b	0.0329	0.0258	0.0329	0.0399	53.00%

violated theoretical monotonicity properties had we not imposed the latter during the estimation. The two elasticities that stand out here are those with respect to land (x_2) and nitrogen surplus (b). Unconstrained models produce the most violations of the monotonicity conditions for these two elasticities. For instance, if we estimate Models (I) and (III) without imposing regularity conditions, the elasticities of IDF with respect to the undesirable nitrogen surplus are wrongly negative for 96.6% and 53.0% farm-years, respectively, which would counter-intuitively suggest that a nitrogen surplus is a desirable output.[4] This finding exemplifies the importance of the estimation of production technologies subject to theoretically warranted regularity conditions, as advocated by Barnett et al. (1991) and Barnett (2002). Thus, in what follows, we focus solely on the results obtained from the constrained estimation.

Despite rather large quantitative differences in elasticity estimates across the models, all three seem to agree about the following. All models suggest that labor, livestock and materials (including feed) have the highest values of elasticity among inputs. This expectedly implies (by duality) that the cost of dairy farms is the most sensitive to the quantities of these variable inputs that are

[4] Note that no "violations" are computed for the same elasticity in the case of Model (II) since this model does not theoretically regulate the sign of monotonicity of the IDF with respect to undesirable by-products.

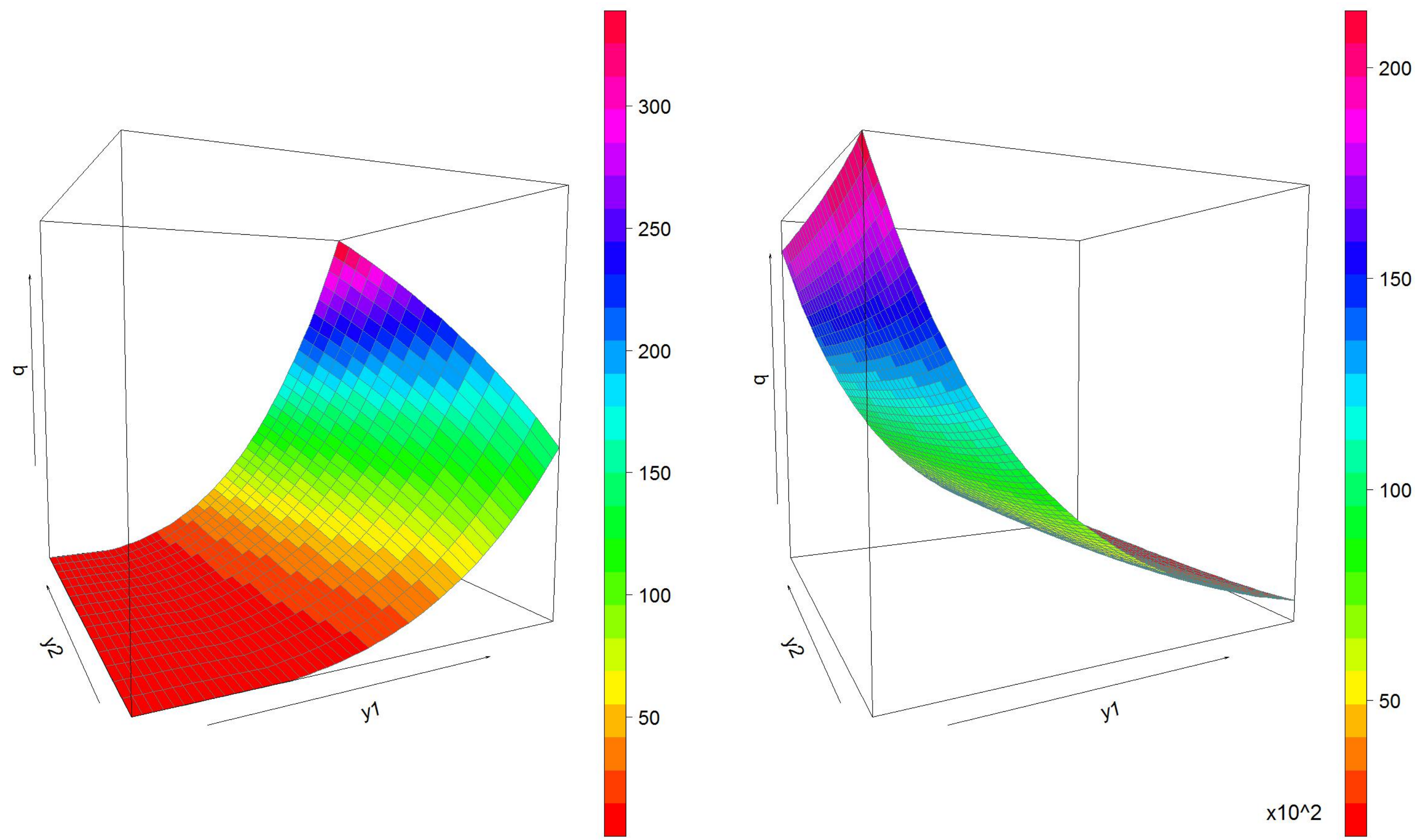

Figure 1. Fitted (b, y_1, y_2) Relation based on Model (I) on the left and Model (II) on the right

Table 4. Returns to Scale, 2001–2009

	Point Estimates				*Statistical Significance*		
Model	Mean	1st Qu.	Median	3rd Qu.	< 1	$= 1$	> 1
(I)	1.1496	1.0140	1.1163	1.2492	5.25%	43.67%	51.07%
(II)	1.3017	1.1179	1.2416	1.4202	0.69%	31.72%	67.57%
(III)	1.2251	0.8727	1.0645	1.4184	27.11%	38.58%	34.29%

NOTE: Percentage points may not sum up to a hundred due to rounding.

directly used in the production of milk and other dairy products. Similarly, all models indicate that the farms' cost is far more sensitive to the volume of diary production (y_1) than to the quantity of produced crop products (y_2). Specifically, across the models, the implied (by duality) average cost elasticity of dairy products ranged between 0.77 and 0.88, whereas the average cost elasticity of crop products falls between 0.04 and 0.10.

We document stark differences in the estimates of the IDF elasticity with respect to the undesirable nitrogen surplus across the fitted models. Contrasting the results from the two models that theoretically regulate undesirable byproducts [Models (I) and (III)], we find that our preferred hedonic-output-index-based Model (I) produces the IDF elasticity with respect to this undesirable output of considerably smaller magnitudes than does Model (III) which treats bads as inputs: 0.008 versus 0.033, on average. Even more starkly, we find that when one models undesirable by-products as theoretically unregulated controls as in Model (II), the results imply that, contrary to common sense, a nitrogen surplus is a *desirable* output. To further exemplify the consequences of modeling undesirable by-products as technology shifters, consider Figure 1 which plots the fitted three-dimensional relationships between dairy farms' desirable and undesirable outputs based on Models (I) and (II) at median values of inputs. The left subfigure, which is produced using the estimated hedonic-output-index-based IDF, depicts an expected complementary relation between the nitrogen surplus b and two desirable outputs y_1 and y_2. In contrast, the estimates from Model (II) point to the existence of the trade-off (substitution) between b and desirable outputs y_1 and y_2 (see the right subfigure in Figure 1), which is rather counter-intuitive given the complementary by-production nature of a nitrogen surplus. These findings lend support to our preferred Model (I) which explicitly treats b as an undesirable output.

5.2 Returns to Scale

Table 4 presents the summary statistics of the point estimates of returns to scale based on all three models over the entire sample period. The returns to scale are computed as the negative of the inverse of the sum of *desirable* output elasticities (Färe and Primont, 1995), i.e.,[5]

$$\text{RTS} = -\left[\sum_m \frac{\partial \ln D_i(\cdot)}{\partial \ln y_m}\right]^{-1}. \tag{5.1}$$

Our preferred Model (I) produces an average estimate of the returns to scale at about 1.15 suggesting the presence of economies of scale among Dutch dairy farms during our sample period. We also find that Model (II) tends to over-estimate whereas Model (III) tends to under-estimate the magnitude of returns to scale. These differences across the models are vividly illustrated in

[5]Following Malikov et al. (2015b), the concept of returns to scale is defined over desirable outputs only.

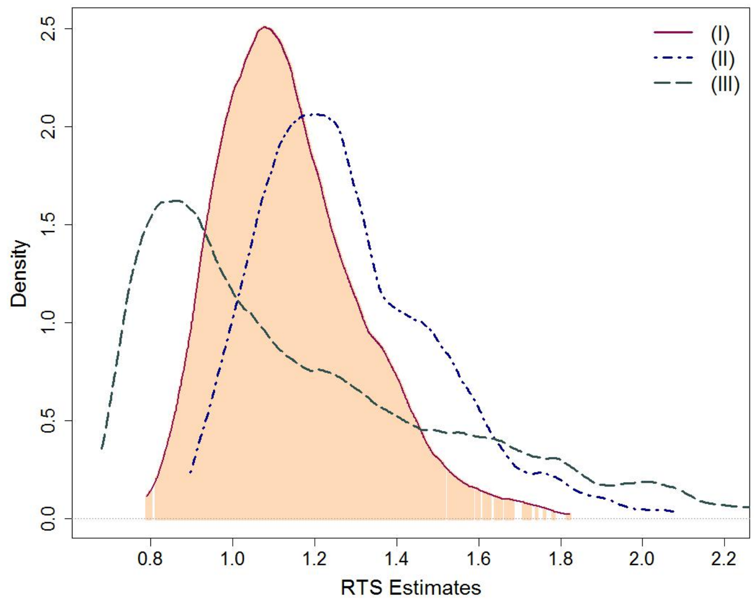

Figure 2. Returns to Scale across Models (I) through (III)

Figure 2 which plots kernel densities of RTS estimated using a second-order Epanechnikov kernel with cross-validated bandwidths.

The right panel of Table 4 also reports the groupings of dairy farms by the returns to scale categories: decreasing, constant and increasing returns to scale. We classify a dairy farm as exhibiting decreasing/constant/increasing returns to scale if the point estimate of its returns to scale is found to be statistically less than/equal to/greater than one at the 5% significance level.[6] As expected, the empirical evidence suggests that Model (III) that treats a nitrogen surplus as an input misleadingly predicts that about a quarter of dairy farms in our sample operate at decreasing returns to scale, whereas our preferred Model (I) suggests that the number of such farms hardly exceeds 5% of the sample. Overall, when modeling b as an undesirable output [as in Model (I)], we find that about a half of farms in our sample (51%) exhibit significant scale economies indicating the potential for the cost reduction as a result of expansion of the scale of operation. The other 44% of farms are found to be scale-efficient, i.e., they operate at unitary constant returns to scale. Our findings are generally consistent with earlier results reported in Brümmer et al. (2002) and Emvalomatis et al. (2011) who also document the evidence of increasing returns to scale among Dutch dairy farms, although no previous study reports farm-level estimates of scale economies providing only average estimates thereof.

5.3 Technical Change Decomposition

We next proceed to the analysis of the technical change experienced by dairy farms in the Netherlands during the 2001–2009 period. The annual rate of technical change is defined as a *ceteris paribus* outward shift in the technological frontier over time and can be measured as the time

[6]Standard errors for the returns to scale estimates are constructed using the delta method. In turn, the standard errors for the IDF parameter estimates are computed using the robust outer-product sandwich variance-covariance matrix.

Table 5. Technical Change, 2001–2009

Model	*Point Estimates*				*Statistical Significance*		
	Mean	1st Qu.	Median	3rd Qu.	<0	$=0$	>0
(I)	0.0078	–0.0055	0.0077	0.0212	8.36%	44.10%	47.53%
(II)	0.0057	–0.0033	0.0076	0.0172	10.61%	50.85%	38.53%
(III)	–0.0185	–0.0568	–0.0159	0.0175	0.00%	100.00%	0.00%

NOTES: Percentage points may not sum up to a hundred due to rounding.

semi-elasticity of the IDF, i.e.,

$$\mathrm{TC} \stackrel{\mathrm{def}}{=} \frac{\partial \ln D_i(\cdot)}{\partial t}. \tag{5.2}$$

The summary of the technical change estimates is reported in Table 5. The table also reports the break-down of dairy farms by the statistical significance (at the 5% significance level) of these estimates. Our preferred Model (I) estimates the rate of technical change of the highest magnitude among all three models: the average annual rate of 0.78% with estimates being significantly positive for 48% of the sample. Our average estimate of technical change falls in the middle between those reported for Dutch dairy farms by earlier studies that do not account for the by-production of undesirable outputs. For instance, Brümmer et al. (2002) report the average annual rate of technical progress of 0.53% in 1991–1994, whereas Emvalomatis et al. (2011) report the rate of 1.25% per annum in 1995–2005.[7]

We find that Model (II) reports slightly lower estimates of technical advancement than does our preferred hedonic-output-index-based model. However, the TC estimates are the most surprising in the case of Model (III) which suggests a persistent technological *regress* at the average rate of –1.85% per annum, although none of the estimates are statistically significant at conventional levels. The stark distortions in the technical change estimates produced by Model (II) which treats a nitrogen surplus as an input can be clearly assessed from Figure 3. The plotted are the technical change indices implied by all three models. The indices are normalized to 100 in the year 2000 and are constructed using the total-revenue-weighted average annual technical change rates computed as $\mathrm{TC}_t = \sum_i w_{it}\mathrm{TC}_{it}$, where $w_{it} = TotalRevenue_{it}/\sum_j TotalRevenue_{jt}\ \forall\ t$. From the figure, modeling the production technology in which undesirable outputs are treated as inputs produces a hard-to-believe estimate of the cumulative ten-year decline in dairy farming technology of 40.5%. In contrast, our preferred Model (I) suggests a modest but more reasonable 9.7% cumulative ten-year improvement in technology.

Changes in the weighted estimates of the annual rate of technical change TC_t over years can be attributed to two primary sources: a secular shift in the technological frontier across farms and the reallocation of fixed factors towards more technologically advanced farms that operate on "higher" frontiers. To differentiate between these two sources, we decompose the weighted measure of technical change TC_t into two components à la Olley and Pakes (1996):

$$\begin{aligned}\mathrm{TC}_t &= \sum_i w_{it}\mathrm{TC}_{it} \\ &= \sum_i \left[\overline{w}_t + (w_{it} - \overline{w}_t)\right]\left[\overline{\mathrm{TC}}_t + \left(\mathrm{TC}_{it} - \overline{\mathrm{TC}}_t\right)\right]\end{aligned}$$

[7]It is also noteworthy that the two papers use different formulations of the production technology. Brümmer et al. (2002) estimate a conventional input distance function, whereas Emvalomatis et al. (2011) specify an output distance function.

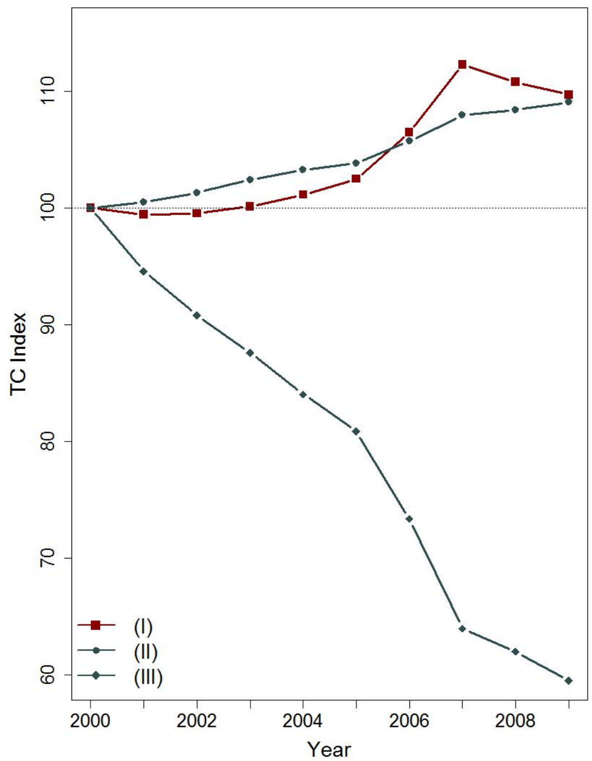

Figure 3. Technical Change Indices across Models (I) through (III)

$$= \overline{\mathrm{TC}}_t + \sum_i (w_{it} - \overline{w}_t) \left(\mathrm{TC}_{it} - \overline{\mathrm{TC}}_t\right) \ \forall \ t,$$

where $\overline{\mathrm{TC}}_t = 1/N_t \sum_i \mathrm{TC}_{it}$ and $\overline{w}_t = 1/N_t$ are the unweighted average technical change and unweighted revenue share (a uniform weight), respectively. According to the above decomposition, the (aggregate) weighted average rate of technical change TC_t is a sum of the *un*weighted average of farm-level technical change $\overline{\mathrm{TC}}_t$ and a sample covariance between the farm-level (total) revenue and technical change $\mathrm{TC}_t - \overline{\mathrm{TC}}_t = \sum_i (w_{it} - \overline{w}_t) \left(\mathrm{TC}_{it} - \overline{\mathrm{TC}}_t\right)$. While changes in the first component represent a secular trend in technical change, yearly changes in the covariance term capture the reallocation of economic activity from farms that exhibit slow technological growth to those exhibiting faster technological advancement. The larger the covariance term, the larger the (total) revenue share of farms with faster technological improvement in the dairy sector.

The decomposition results are presented in Table 6. Both Models (I) and (II) suggest that there is a steady decline in the covariance between farms' total revue and the rate of technical change in the first five years of our sample period followed by a dramatic increase in 2006–2007 which seems to fade out by 2009. The latter findings indicate that the acceleration in the rate of technical change observed during the 2001–2007 period is primarily driven by a secular improvement in the pace of technological advancement except in 2006–2007 when the reallocation of resources towards more technologically advanced farms has played a bigger role. Interestingly, Model (III) produces a contrary result according to which in 2006–2007 resources seem to have reallocated towards *less* technologically advanced farms. These differences yet again highlight the importance of modeling a nitrogen surplus as an undesirable output as opposed to an input.

Table 6. Technical Change Decomposition, in %

	Model (I)			Model (II)			Model (III)		
Year	TC_t	$\overline{TC}_t$	Realloc.	TC_t	$\overline{TC}_t$	Realloc.	TC_t	$\overline{TC}_t$	Realloc.
2001	–0.54	–0.44	–0.09	0.49	0.20	0.28	–5.41	–1.95	–3.46
2002	0.08	0.16	–0.08	0.81	0.46	0.35	–3.98	–1.28	–2.70
2003	0.59	0.76	–0.16	1.09	0.91	0.18	–3.54	–1.39	–2.15
2004	1.02	1.34	–0.32	0.81	0.71	0.09	–4.04	–1.73	–2.31
2005	1.32	1.94	–0.62	0.58	0.49	0.08	–3.76	–2.13	–1.63
2006	3.91	2.51	1.40	1.80	0.72	1.08	–9.29	–2.13	–7.15
2007	5.43	3.11	2.31	2.09	0.82	1.26	–12.8	–3.32	–9.48
2008	–1.32	–1.62	0.29	0.44	0.29	0.15	–3.09	–1.36	–1.72
2009	–0.98	–1.04	0.05	0.58	0.52	0.06	–3.99	–1.24	–2.74

NOTES: TC_t is the farm-revenue-share-weighted average estimate of annual farm-level technical change; $\overline{TC}_t$ is the simple average of annual farm-level technical change; "Realloc." equals the sample covariance between $\overline{TC}_t$ and farm revenue.

5.4 Shadow Price of the Undesirable By-Product

Like most environmentally detrimental pollutants, a nitrogen surplus is a non-marketable output with no observable market price. However, the derivation of shadow prices for undesirable by-products is of particular relevance for designing and targeting environmental policy instruments. We can obtain the estimate of unobserved shadow prices for a nitrogen surplus using estimates of a distance function under some behavioral assumptions. Specifically, the shadow prices for b can be obtained under the proposition that the ratio of the shadow prices for two outputs ought to be equal to the marginal rate of transformation between the two, where the latter equals the ratio of derivatives of the distance function with respect to outputs (Färe et al., 1993).

Following Färe et al. (2005), we derive the shadow price of a non-marketed nitrogen surplus using our preferred hedonic-output-index-based IDF in the revenue maximization framework. Specifically, for dairy farms that seek to maximize revenues, the revenue function is given by

$$\mathcal{R}(\mathbf{x},\mathbf{y},b) \overset{\text{def}}{=} \max_{\mathbf{y},b} \left\{ \boldsymbol{p}'\mathbf{y} - rb : \; D_i\big(\mathbf{x}, h(\mathbf{y},b)\big) \geq 1 \right\} \tag{5.3}$$

with the corresponding first-order conditions given by

$$\frac{p_m}{r} = -\left[\frac{\partial D_i\big(\mathbf{x}, h(\mathbf{y},b)\big)}{\partial b}\right]^{-1} \frac{\partial D_i\big(\mathbf{x}, h(\mathbf{y},b)\big)}{\partial y_m} \quad \forall \quad m = 1,\dots,M, \tag{5.4}$$

where $\boldsymbol{p} = (p_1,\dots,p_m,\dots,p_M)'$ is an $M \times 1$ vector of market prices of desirable outputs, and r is the shadow price of a (scalar) undesirable output.

Summing over M first-order conditions yields a unique shadow price of the nitrogen surplus b:

$$\begin{aligned} r &= -\frac{1}{M}\frac{\partial D_i\big(\mathbf{x}, h(\mathbf{y},b)\big)}{\partial b} \sum_m p_m \left[\frac{\partial D_i\big(\mathbf{x}, h(\mathbf{y},b)\big)}{\partial y_m}\right]^{-1} \\ &= -\frac{1}{Mb}\left[\frac{\partial \ln D_i\big(\mathbf{x}, h(\mathbf{y},b)\big)}{\partial \ln b}\right] \sum_m p_m y_m \left[\frac{\partial \ln D_i\big(\mathbf{x}, h(\mathbf{y},b)\big)}{\partial \ln y_m}\right]^{-1}, \end{aligned} \tag{5.5}$$

which can be computed using the estimates of the IDF elasticities.

Table 7. Shadow Price of Nitrogen Surplus, in € per ton

Model	Mean	1st Qu.	Median	3rd Qu.
(I)	1,282.0	482.7	892.6	1,551.5
(II)	–58,999.4	–74,475.17	–47,088.2	–30,627.5
(III)	479,711.5	253,344.5	387,703.1	606,189.1

The summary statistics for the shadow price estimates are reported in Table 7. Here, we also report the shadow price estimates based on the two auxiliary models. In the case of Models (II) and (III), we use the same formula given in (5.5) where we replace $D_i\big(\mathbf{x}, h(\mathbf{y}, b)\big)$ with the conventional IDF $D_i(\mathbf{x}, \mathbf{y}, b)$.[8] According to our preferred Model (I), the average shadow price of a nitrogen surplus is estimated at €1,282 per ton during the 2001–2009 period, which is significantly lower than the estimate of €1,960 per ton (in 2005 prices)[9] reported by Reinhard et al. (1999) for 1991–1994. The latter suggests a substantial decline in the price of an undesirable by-product like a nitrogen surplus over the course of years, indicating that it might have become less costly for Dutch dairy farms to cut their nitrogen surpluses.

The remaining two models yield rather odd results. Model (II), which imposes no theoretical regularity onto b and treats it as a frontier shifter, produces negative values for the shadow price of a nitrogen surplus, yet again indicating that this pollutant is desirable. In the instance of Model (III), while we generally obtain positive values for the r estimates, they are all however of unreasonably high magnitudes in excess of €100,000 per ton, likely pointing to the misspecification of the model due to the modeling of a nitrogen surplus as an input.

5.5 Elasticity of Substitution between Desirable and Undesirable Outputs

We conclude our analysis of the production technology in Dutch dairy farming by looking at the complementarity of desirable and undesirable outputs. To do so, we compute the the Morishima output elasticity of substitution (Blackorby and Russell, 1981) between a desirable output y_m and undesirable output b defined as $MES_m \equiv \partial \ln(r/p_m)/\partial \ln(y_m/b)$. This elasticity measures how the pollution intensity (an inverse of y_m/b) influences the undesirable-desirable shadow price ratio. We compute the Morishima output elasticity only for our preferred Model (I), since this is the only model that treats a nitrogen surplus as an output.[10] To estimate the Morishima elasticity using the fitted hedonic-output-index-based IDF, we rewrite the revenue-maximizing first-order condition (5.4) as follows

$$\begin{aligned} \frac{r}{p_m} &= -\frac{\partial \ln D_i\big(\mathbf{x}, h(\mathbf{y}, b)\big)}{\partial \ln b} \left[\frac{\partial \ln D_i\big(\mathbf{x}, h(\mathbf{y}, b)\big)}{\partial \ln y_m}\right]^{-1} \times \frac{y_m}{b} \\ &= -\frac{\partial \ln h(\mathbf{y}, b)}{\partial \ln b} \left[\frac{\partial \ln h(\mathbf{y}, b)}{\partial \ln y_m}\right]^{-1} \times \frac{y_m}{b}. \end{aligned} \tag{5.6}$$

[8] It might appear that one cannot use the same methodology to compute the shadow price of b for Model (III) because it treats b as an input. However, the formula for r even when b is an input stays unchanged if one replaces the revenue maximization framework with the profit maximization in which b is treated as one of the inputs.

[9] To be able to compare our shadow price estimates with those reported by Reinhard et al. (1999), we convert their average shadow price estimate measured in 1991 guilders into euros of 2005.

[10] Morishima elasticity computed using the IDF estimates from the other two models will lack a proper economic interpretation.

Table 8. Morishima Elasticities of Substitution based on Model (I)

	Point Estimates				*Statistical Significance*		
Elasticity	Mean	1st Qu.	Median	3rd Qu.	< 0	$= 0$	> 0
MES: b vs. y_1	1.7882	1.3944	1.5638	1.9387	0.00%	2.24%	97.75%
MES: b vs. y_2	0.2895	0.1612	0.4812	0.6251	9.91%	40.72%	49.35%

NOTES: Percentage points may not sum up to a hundred due to rounding.

Log-differentiating both sides with respect to $\ln(y_m/b)$, we get

$$MES_m \stackrel{\text{def}}{=} \frac{\partial \ln\left(\frac{r}{p_m}\right)}{\partial \ln\left(\frac{y_m}{b}\right)} = 1 + \frac{\partial \ln\left[-\frac{\partial \ln h(\mathbf{y},b)}{\partial \ln b}\right]}{\partial \ln\left(\frac{y_m}{b}\right)} - \frac{\partial \ln\left[\frac{\partial \ln h(\mathbf{y},b)}{\partial \ln y_m}\right]}{\partial \ln\left(\frac{y_m}{b}\right)}. \tag{5.7}$$

Obtaining the closed-form expression for MES_m is quite tedious given that our specifications of the IDF and the hedonic output index are in the log form. We therefore compute these elasticities of substitution using numerical derivatives.

Table 8 presents the summary of the MES_m estimates based on our Model (I). In the (y_1, b) dimension, our statistically non-negative estimates of MES_1 expectedly suggest that milk production and a nitrogen surplus are complements, given that most nitrogen surpluses originate in the form of unused or over-utilized manure. The sample average estimate of MES_1 is 1.78 with 97% of the estimates being statistically above zero at the 5% level. In the (y_2, b) dimension, the estimates of Morishima elasticity are still mostly positive pointing to complementarity between crop production and nitrogen surplus generation. However, the MES_2 estimates are somewhat lower in magnitude than are the MES_1 estimates, suggesting that an increase in the y_1/b ratio leads to a much bigger increase in the shadow price ratio r/p_1 than a commensurate increase in the y_2/b ratio evokes in the r/p_2 ratio. The average value of MES_2 is estimated at 0.29 with 49% (41%) of the estimates being statistically above zero (statistically insignificant).

Lastly, we examine the relationship (if any) between the degree of complementarity between desirable and undesirable outputs and a farm's extent of specialization in one of desirable outputs. We assess this relationship not just at the mean but distribution-wise. To accomplish the latter, we estimate bivariate kernel densities[11] of farm-level estimates of Morishima elasticities of substitution and (nominal) dairy output share in the total revenue defined as $p_1y_1/\sum_m p_my_m$. Figure 4 depicts contour plots for these densities. The left sub-figure suggests that the degree of complementarity between the dairy output and nitrogen surplus (MES_1) appears to decrease as the dairy output share in the farm's total agricultural output goes up. This result suggests that a higher degree of specialization in dairy production allows farms to decrease the marginal rate of substitution between y_1 and b given a change in their quantity ratio y_1/b, i.e., highly specialized farms forgo less revenue per unit of y_1 due to the generation of a nitrogen surplus. In contrast, the right subfigure in Figure 4 indicates a different picture, according to which the extent of specialization in dairy production is positively related with the Morishima elasticity of substitution between crop products y_2 and nitrogen surplus b.

[11] We employ an axis-aligned bivariate Gaussian kernel, evaluated on a square grid using the normal reference bandwidth.

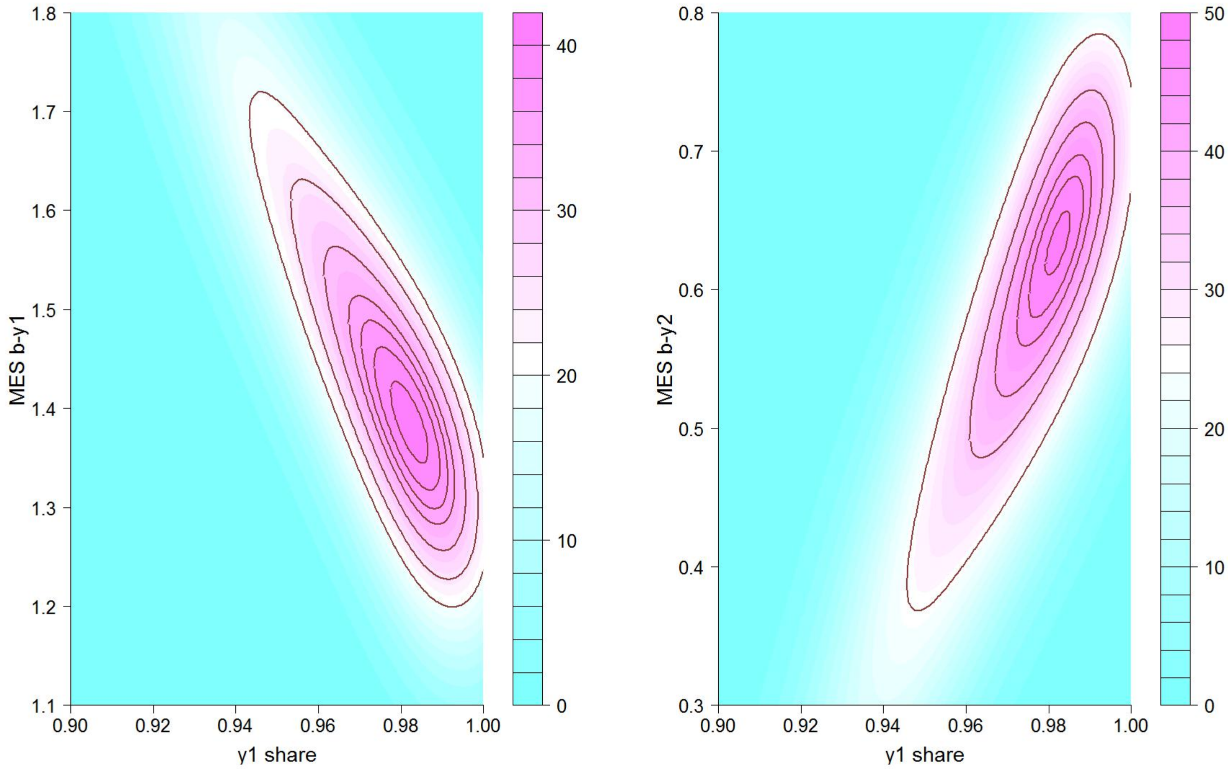

Figure 4. Morishima Output Elasticity of Substitution vs. Dairy Output Share based on Model (I)

6 Conclusion

The by-production of undesirable, or so-called "bad", outputs is an inherent attribute of many production processes. It is therefore imperative to account for such undesirable outputs in the estimation of polluting production technologies. Despite the apparent abundance of modeling frameworks which include hyperbolic and directional distance functions, the conventional radial distance function of Shephard (1953, 1970) however persistently remains a popular go-to formulation of polluting production processes among practitioners. The unfading popularity of radial distance functions in spite of their recent criticisms is arguably driven by their ability, unlike alternative directional distance functions, to allow for unit-free multiplicative changes in arguments as well as, by implicitly postulating the radial direction, to free researchers from the dilemma of having to explicitly choose the directional vector. The conventional radial distance function is usually augmented to incorporate undesirable outputs in two ways. The function is either conditioned on the quantity of undesirable by-products treated as (theoretically unregulated) technology shifters or expanded to effectively incorporate undesirable by-products in the role of inputs.

In this paper, we offer a generalization of the standard radial distance function to polluting technologies that can accommodate undesirable by-products in a more economically meaningful way. Specifically, we propose modeling undesirable outputs via a hedonic output index, which is meant to ensure that pollutants are treated as outputs, as opposed to inputs or theoretically unregulated frontier shifters, while also recognizing their undesirable nature. By using a radial input distance function generalized to encompass an (unobservable) hedonic output index of desirable and undesirable outputs, we are able to meaningfully describe relationships between different products (including the complementarity of desirable and undesirable outputs) within producible output sets as well as to represent technically feasible polluting production possibilities given inputs. An empirical application of our methodology to the case of Dutch dairy farms in 2001–2009 demonstrates the complexity of interactions between outputs, thereby attesting to the value of more elaborate representations of production possibilities.

References

Assaf, A. G., Matousek, R., and Tsionas, E. G. (2013). Turkish bank efficiency: Bayesian estimation with undesirable outputs. *Journal of Banking & Finance*, 37:506–517.

Atkinson, S. E. and Dorfman, J. H. (2005). Bayesian measurement of productivity and efficiency in the presence of undesirable outputs: Crediting electric utilities for reducing air pollution. *Journal of Econometrics*, 126:445–468.

Barnett, W. A. (2002). Tastes and technology: Curvature is not sufficient for regularity. *Journal of Econometrics*, 108:199–202.

Barnett, W. A., Geweke, J., and Wolfe, M. (1991). Seminonparametric Bayesian estimation of the Asymptotically Ideal Production Model. *Journal of Econometrics*, 49:5–50.

Blackorby, C. and Russell, R. (1981). The Morishima elasticity of substitution: Symmetry, constancy, separability, and its relationship with the Hicks and Allen elasticities. *Review of Economic Studies*, 48:147–158.

Brümmer, B., Glauben, T., and Thijssen, G. (2002). Decomposition of productivity growth using distance functions: The case of dairy farms in three European countries. *American Journal of Agricultural Economics*, 84:628–644.

Chambers, R. G., Chung, Y., and Färe, R. (1998). Profit, directional distance functions, and Nerlovian efficiency. *Journal of Optimization Theory and Applications*, 98:351–364.

Chung, Y., Färe, R., and Grosskopf, S. (1997). Productivity and undesirable outputs: A directional distance function approach. *Journal of Environmental Management*, 51:229–240.

Cuesta, R. A., Lovell, C. A. K., and Zofio, J. L. (2009). Environmental efficiency measurement with translog distance functions: A parametric approach. *Ecological Economics*, 68:2232–2242.

Emvalomatis, G., Stefanou, S. E., and Oude Lansink, A. (2011). A reduced-form model for dynamic efficiency measurement: Application to dairy farms in Germany and the Netherlands. *American Journal of Agricultural Economics*, 93:161–174.

Färe, R., Grosskopf, S., Lovell, C. A. K., and Pasurka, C. (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. *Review of Economics and Statistics*, 71:90–98.

Färe, R., Grosskopf, S., Lovell, C. A. K., and Yaisawarng, S. (1993). Derivation of shadow prices for undesirable outputs: A distance function approach. *Review of Economics and Statistics*, 75:374–380.

Färe, R., Grosskopf, S., Noh, D.-W., and Weber, W. (2005). Characteristics of a polluting technology: Theory and practice. *Journal of Econometrics*, 126:469–492.

Färe, R. and Primont, D. (1995). *Multi-Output Production and Duality: Theory and Applications*. Kluwer Academic Publishers, Boston, MA.

Fernández, C., Koop, G., and Steel, M. F. J. (2002). Multiple-output production with undesirable outputs: An application to nitrogen surplus in agriculture. *Journal of the American Statistical Association*, 97(458):432–442.

Fernández, C., Koop, G., and Steel, M. F. J. (2005). Alternative efficiency measures for multiple-output production. *Journal of Econometrics*, 126:411–444.

Hailu, A. and Veeman, T. S. (2000). Environmentally sensitive productivity analysis of the Canadian pulp and paper industry, 1959–1994: An input distance function approach. *Journal of Environmental Economics and Management*, 40:251–274.

Hailu, A. and Veeman, T. S. (2001). Non-parametric productivity analysis with undesirable outputs: An application to the Canadian pulp and paper industry. *American Journal of Agricultural Economics*, 83:605–616.

Kumbhakar, S. C. and Lovell, C. A. K. (2000). *Stochastic Frontier Analysis*. Cambridge University Press, Cambridge.

Lange, K. (2004). *Optimization (Springer Texts in Statistics)*. Springer, Verlag, NY.

Malikov, E., Kumbhakar, S. C., and Tsionas, E. G. (2015a). Bayesian approach to disentangling technical and environmental productivity. *Econometrics*, 3:443–465.

Malikov, E., Kumbhakar, S. C., and Tsionas, E. G. (2015b). A cost system approach to the stochastic directional technology distance function with undersirable outputs: The case of U.S. banks in 2001-2010. *Journal of Applied Econometrics*. forthcoming.

Murty, S., Russell, R. R., and Levkoff, S. B. (2012). On modeling pollution-generating technologies. *Journal of Environmental Economics and Management*, 64:117–135.

Olley, G. S. and Pakes, A. (1996). The dynamics of productivity in the telecommunications equipment industry. *Econometrica*, 64:1263–1297.

Reinhard, S., Lovell, C. A. K., and Thijssen, G. (1999). Econometric estimation of technical and environmental efficiency: An application to Dutch dairy farms. *American Journal of Agricultural Economics*, 81:44–60.

Reinhard, S., Lovell, C. A. K., and Thijssen, G. (2000). Environmental efficiency with multiple environmentally detrimental variables; estimated with SFA and DEA. *European Journal of Operational Research*, 121:287–303.

Shephard, R. W. (1953). *Cost and Production Functions*. Princeton University Press, Princeton, NJ.

Shephard, R. W. (1970). *Theory of Cost and Production Functions*. Princeton University Press, Princeton, NJ.