Technology Shocks, the Service Sector and Economic Growth

Mitra Thakur, Gogol

Jawaharlal Nehru University, New Delhi, India

21 July 2016

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Gogol Mitra Thakur*†

Abstract

Advances in ICTs as well as financial developments have greatly increased the scope for joint utilisation of various industrial goods and services. For example, consumption of many durable goods like telecommunication equipment (e.g. mobile sets), various electronic products, computer hardware and automobiles leads to joint purchases of services such as telecommunications, software services, insurance and other financial services. In this paper, we propose a specification for demand interlinkage between industry and the service sector, indicative of such developments, wherein final demand for service not only depends on industrial output but also on the relative price of service. This specification implies that a labour productivity increase in the service sector, say due to adoption of ICTs, can generate enough demand to increase both the growth rate in the economy and the relative size of the service sector if demand for service per unit industrial output is sufficiently elastic with respect to its relative price.
JEL Classification: O11; O14; O41
Keywords: Service sector; Information and Communication Technologies (ICTs); Demand-led growth; Two sector growth models

1. Introduction

The world economy today is a predominantly service economy. Services contributed 70 percent of the world GDP in the year 2013. In case of high

*Ph.D. Candidate, Centre for Economic Studies and Planning, Jwaharlal Nehru University, New Delhi. Email: gogol86@gmail.com
†This paper forms a chapter of my Ph.D. thesis. I am grateful to my Ph.D. supervisor Dr. Subrata Guha for his comments and suggestions. The usual disclaimer applies.

income economies, the average share of services in GDP was 74 percent in the same year and the share of services in male and female employment for the period 2009-2012 were 64 percent and 86 percent respectively.[1] Traditionally, the rise of the service sector has been perceived as a matter of concern, in contrast to the importance attached to the role of industrialization in growth and development. This view, formalized by the two-sector 'unbalanced growth' model of Baumol (1967), posits that the inherent nature of services is such that labour productivity improvements are rare phenomena. Therefore, if output share of the service sector does not decline then resources continuously shift away from more productive sectors to the service sector, causing stagnation in the economy. Advances in information and communication technologies (ICTs) and their rapid adoption in many services, however, have ensured that this traditional view regarding expansion of services has few takers today.[2] Particularly important in this regard is the fact that ICT using services such software and IT, telecommunications, banking & finance have emerged as important sectors not only in advanced economies but also in developing economies such as India. According to Eichengreen and Gupta (2013), ICT using services are driving the expansion in the output share of the service sector at much lower levels of per capita income after 1990 than before. This suggests that the service sector can also be a source of productivity increase and growth in developing economies.

Naturally these developments have generated a renewed theoretical interest in exploring relationships between the expansion of the service sector and economic growth. Most of the recent theoretical literature attempts to

[1] Source: Table 2.3 and Table 4.2 of *World Development Indicators* (2015) for employment shares and GDP share respectively.

[2] Adoption of ICTs in services not just limited to advanced economies. For example, Qiang et al. (2006), using data from Investment Climate Surveys (ICS) conducted by the World Bank between 1999 and 2003 covering 20,000 firms from 26 sectors in 56 low- and middle-income countries, find that 55 and 50 percent of service firms use e-mails and websites, respectively, to interact with clients and in both use of websites and percentage of employees using computers, service sector firms are much ahead of manufacturing firms. Among the various services are telecommunications and IT services, real estate and hotels & restaurants were the heaviest users of e-mails and websites whereas percentage of employees using computers was highest (67 percent) in case of accounting and finance sector.

reverse the traditional view of a negative relationship between the expansion of the service sector and economic growth in Baumol (1967) by focusing on the importance of various services for endogenous productivity and output growth in the economy.[3] Our focus in this paper, however, is on new kinds of demand interlinkages between the service sector and industry or manufacturing ushered in by development of ICTs as well as financial developments. In the modern world, thanks to rapid development and diffusion of ICTs, purchase of various services go hand in hand with purchase of many industrial goods. For example, consumption of many durable goods like telecommunication equipment (e.g. mobile sets), various electronic products and computer hardware make sense only if purchased with telecommunication and software services. Similarly, because of financial developments, purchase of durable goods like automobiles give rise to purchase of insurance and other financial services. Even in case of investment demand, firms might employ financial and business consultancy services in order to arrange financing for their investments. We argue that if exogenous improvements in labour productivity of the service sector, say due to adoption of modern ICTs, lower prices of services and increase the quantity purchased of services along with purchases of industrial goods then both the growth rate of the economy and the size of the service sector can increase.

The importance of demand interlinkages between sectors for economic growth was stressed upon by Kaldor, who using a two-sector model consisting of agriculture and industry argued:

> ...industrial growth is dependent on the exogenous component of demand for industry- that part of the demand which comes from "outside" the industrial sector: growth of its exports...growth of purchasing power of the primary sector...will determine the growth rates of both...[Kaldor (1989b, pp. 431-432)]

A crucial assumption that allows Kaldor to arrive at the above conclusion is

[3]Section 2 contains a short review of this literature.

the availability of an unlimited supply of labour to industry at a subsistence wage rate fixed in terms of food. On the contrary, as pointed out by Dutt (1992), in the model by Baumol (1967) the only factor of production in both sectors, labour, is always fully employed in the economy and, therefore, expansion of the service sector necessarily shift resources away from the more productive sector. Dutt (1992) considers a two-sector growth model consisting of an industrial and a service sector, where both the sectors produce under conditions of excess capacity and labour is not fully employed. He shows that industry and the service sector can grow in a balanced manner because of demand interlinkages between the two. In this paper, we first review both Baumol's and Dutt's models and show that in Dutt's model, unlike the case of Baumol's model, exogenous increase in labour productivity of the industrial sector increases both its relative size and the growth rate in the economy. We also show an exogenous increase in the labour productivity of the service sector, on the other hand, decreases the growth rate of the economy in Dutt's model.

The negative association of exogenous increase in labour productivity of the service sector and the growth rate in Dutt's model is slightly disappointing. This because the widespread adoption ICTs in various service activities can be expected to have a positive impact on the labour productivity of the service sector. However, we also show that in models such as that of Dutt, where resources are not fully utilised and both sectors generate demand for each other's output, implications of sector-specific technology shocks for growth and structural change depend upon the specification of demand interlinkages. This leads to our main argument in this essay, which is in contemporary times advances in ICTs as well as financial developments have greatly increased the scope for joint utilisation various industrial goods and services. Consumption of many durable goods like telecommunication equipment, various electronic products, computer hardware and automobiles leads to joint purchases of services such as telecommunications, software services, insurance and other financial services. We propose a specification for demand interlinkage between industry and the service sector, indicative of such developments, wherein final demand for service not only depends on the industrial output but also on the relative price of service. We show that if demand for service per unit industrial output is sufficiently elastic with respect to its relative price then an exogenous increase in the labour productivity of the service sector, say due to adoption of ICTs, generates enough demand to not only increase the growth rate in the economy but also the relative size of the service sector.

The next section in the paper presents the 'unbalanced growth' model of Baumol (1967) and also provides a brief review of recent theoretical contributions which counter Baumol's argument and that emphasise the contribution of various services towards endogenous productivity and output growth. Section 3 presents the two-sector demand constrained growth model of Dutt (1992) which emphasises balanced growth between industry and the service sector as a result of demand interlinkages between the two. We examine the effects of sector-specific technology shocks on the balanced growth rate of the model. Further, using two simple variants of this model, we show that the effects of sector specific technology shocks on both the growth rate and the structure of the economy are sensitive to specifications of demand interlinkages between the two sectors. In section 4 we present a two sector demand constrained growth model similar to Dutt (1992), where the demand for a service generated per unit of industrial output is negatively related to the relative price of the service. In this model we show improvements in labour productivity of the service sector can increase both the growth rate of the economy as well as the relative size of the service sector. Finally, section 5 concludes the paper.

2. Service Sector and Stagnation *à la* Baumol (1967)

Baumol (1967) argues that labour productivity increase in services is at best sporadic compared to industry, where it rises in a cumulative fashion. As a result, if the ratio of outputs of the service sector and industry is not allowed to decline then resources shift towards service sector away from 'technologically progressive' industry causing stagnation in the economy. In the 'unbalanced growth' model of Baumol (1967) there are two sectors - the industrial sector, which produces a single good, and the service sector, which produces a single service. Production technologies of the two sectors are specified as $X_j = x_j L_j$ where $j \in \{i, s\}$ with i and s denoting the industrial sector and the service sector respectively.[4] X_j and L_j represents output and employment in sector j. Labour is the only factor of production in both the sectors and is fully employed in the economy, i.e. $L_i + L_s = L$, which is the total labour supply in the economy. x_j is labour productivity in sector j. Baumol assumes that x_i grows exponentially at a constant rate, say $\eta > 0$, whereas x_s is a constant. He also assumes that the the labour market is such that the money wage rate is same for the two sectors, say W, and grows at the rate η. Given this simple structure Baumol makes four strong predictions.

First, cost per unit output in service, $\frac{W}{x_s}$, grows in an unbounded fashion whereas the same in industry, $\frac{W}{x_i}$, remains constant. Second, if the ratio of total costs in the two sectors remains constant, which is same as the employment ratio of the two sectors remaining constant, then the ratio of output of the service sector output to the same of the industrial sector will become zero. To see, this notice that the ratio of outputs of the two sectors is

$$\frac{X_i}{X_s} = \frac{L_i x_i}{L_s x_s} = \frac{L_i x e^{\eta t}}{L_s x_s} \tag{1}$$

where $x > 0$ is the industrial labour productivity at time $t = 0$. Thus if $\frac{L_i}{L_s}$ is constant then $\frac{X_i}{X_s}$ approaches infinity as x_i grows at the constant rate η and x_s is constant. Third, if, instead, the ratio of outputs of the two sectors remains constant then employment share of the the service sector approaches one. This follows from (1) because $\frac{X_i}{X_s}$ can remain constant only if $\frac{L_i}{L_s}$ approaches zero given that $\frac{x_i}{x_s}$ is growing at a constant rate.

Finally, if both the ratio of output of the two sector and labour supply remain constant then the aggregate growth rate in the economy approaches

[4] Throughout this paper, notations with subscript i refer to the industry sector and notations with subscript s refer to the service sector.

zero. This follows largely from the previous result. With labour supply in the economy being constant, growth rate of aggregate output is equal to the growth rate of aggregate labour productivity. Aggregate labour productivity is weighted average of labour productivities of the two sectors with weights being their respective employment shares. Since employment share of service sector approaches one, aggregate labour productivity approaches the labour productivity of the service sector which is a constant by assumption. Moreover, if the demand for service increases faster than that for industrial goods, as is expected at high levels of per capita income, then the third and the fourth predictions of Baumol become even more pronounced. Since it is generally agreed that services have greater income elasticity of demand than industrial goods, particularly at higher levels of per capita income,[5] the 'unbalanced growth' model of Baumol (1967) predicts that as economies develop more and more resources will shift to the provisioning of technologically stagnant services causing stagnation. This association of the service sector with stagnation led Rowthorn and Ramaswamy (1997, p. 22) to argue that "...growth of living standards in the advanced economies is likely to be increasingly influenced by productivity developments in the service sector". However, note that merely allowing labour productivity growth in the service sector does not prevent a decline in the growth rate in Baumol's model. Growth rates of labour productivity in the two sectors have to be exactly equal.

In Baumol's model both sectors produce only final output. Oulton (2001), therefore, argues that the 'unbalanced growth' model of Baumol (1967) is not suitable for explaining implications of expansion of services such as business services, that are primarily required as intermediate inputs, for growth. In a two-sector model where the single service is required just as an intermediate input in industry, Oulton (2001) shows that under the assumption of perfect competition, a slower rate of labour productivity growth in service sector does not necessarily imply increase in the service sector's share of primary input

[5]See for example Kaldor (1989a); Kaldor (1967); Rowthorn and Ramaswamy (1999); Ray (1998)

usage. Further, if the elasticity of substitution between the service input and the primary input in industry is greater than one and the growth rate of labour productivity in the service sector is positive then the service sector's share of primary input usage asymptotically increases to approach one and the growth rate of total factor productivity (TFP) increases to approach the sum of the labour productivity growth rates of the two sectors. However, Sasaki (2007) using a CES production function for industry shows that once final demand for service is included in Oulton's model, a slower growth rate of labour productivity in the service sector ultimately causes a decline in the growth rate of TFP if the consumption ratio of the service and the industrial good is held constant. This result of Sasaki (2007) suggests that just highlighting the role of services as an intermediate input is not enough to counter the gloomy predictions of Baumol (1967).

Quite a few of the other contributions in the literature, which deal with this issue, resort to endogenous growth theory. For example Pugno (2006) extends Baumol's model by including human capital stock of the economy in the production functions of both sectors. Pugno (2006) argues that consumption of services like health, education and cultural services contributes towards human capital formation. Using a linear human capital production function, Pugno shows that expansion of these services need not necessarily lead to a decline in growth rate of the economy so long as their contribution towards human capital formation is substantial. Similarly, Vincenti (2007), in a model based on two hypotheses - service sector produces a positive externality on industry, via R & D and general human capital improvements, and 'learning by doing' in both sectors - shows that the share of service employment can be positively related to the growth rate of the economy. Sasaki (2012) combines 'learning by doing' in industry along with the hypothesis of Pugno (2006) that consumption of services leads to human capital formation and generates a U-shaped relationship between the growth rate of the economy and the employment share of the service sector. De (2014) argues that services such as finance, insurance, software and various other business services that use ICTs are part of the 'new economy' and contribute towards creation of 'intangible capital'. Extending the Uzawa-Lucas model by including 'intangible capital' as a separate non-rival but excludable factor in the production of the final good and a separate sector for its production, De (2014) shows that accumulation of 'intangible capital' can result in sustained growth in the economy.

Although these contributions highlight the importance of various services for endogenous technological progress and growth, it is important to realise that the negative relation between the expansion of the service sector and economic growth as implied by Baumol (1967) is to a large extent determined by the macroeconomic structure of the model. Particularly consequential is the assumption of full employment of labour because of which any expansion in the service sector necessarily shifts resources away from the more productive industry sector. This point is made by Dutt (1992), who also shows that if resources are not fully employed there can be balanced growth between industrial and service sectors, with each sector generating demand for the other. In the next section we discuss the demand constrained two-sector model of Dutt (1992).

3. Dutt (1992) and Balanced Growth between Industry & Service

Unlike Baumol (1967), Dutt (1992) assumes that production in both the sectors require both capital and labour as inputs. The industrial sector produces a tangible good which is used both as consumption good and as capital good. The service sector produces an intangible service which is required as input in the industrial sector.[6] Further, Dutt (1992) assumes that the industrial sector requires the service as an overhead input in a constant proportion, say $\lambda > 0$, to its capital stock. Thus, the total service input required by the industrial sector is

$$N_s = \lambda K_i \tag{2}$$

[6] In Dutt (1992) the two sectors are referred to as 'productive' and 'unproductive' sectors, where the latter is meant to represent overlapping sets of service, nonmarket and unproductive activities.

where K_i is the capital stock of the industrial sector. There is no technological progress. This implies that labour productivity and full capacity output-capital ratios of both the sectors can be treated as constants. Let $x_j > 0$ and $\bar{u}_j > 0$ respectively denote labour productivity and full capacity output-capital ratio of sector $j \in \{i, s\}$.

The two sectors are both assumed to be characterized by the presence of excess capacity and imperfect competition. Excess capacity means output capital ratios in both the sectors can not be greater than their respective full capacity output-capital ratios. Output capital ratios represent degree or rate of capacity utilization in the two sectors and it must be that $\frac{X_j}{K_j} \leq \bar{u}_j$ for $j \in \{i, s\}$. Imperfect competition in the product market implies that firms are not price takers, but instead enjoy a degree of monopoly power in both the sectors. Price in both the sectors is determined by applying a fixed mark-up on unit prime cost. Let price of the industrial good be

$$P_i = (1 + z_i)\frac{W}{x_i} \tag{3}$$

and that of the service be

$$P_s = (1 + z_s)\frac{W}{x_s} \tag{4}$$

Nominal wage W is exogenously given and is assumed to be the same in both the sectors. $z_j > 0$ is the constant price mark-up in the jth sector with $j \in \{i, s\}$, which following Kalecki (1971) is taken to be exogenously determined by the 'degree of monopoly' prevailing in the sector. These assumptions imply that the relative price of the service in terms of the industrial good, p, is a constant as shown below.

$$p = \frac{P_s}{P_i} = \frac{(1 + z_s)x_i}{(1 + z_i)x_s} \tag{5}$$

Real wage in terms of the industrial good, $\frac{W}{P_i}$, is a positive constant following assumptions regarding W and P_i. Both sectors can employ as much labour as they require at this real wage. Therefore, levels of employment in the two sectors are determined by their output levels, i.e. $E_i = \frac{X_i}{x_i}$ and $E_s = \frac{X_s}{x_s}$ where E_j is the employment in sector $j \in \{i, s\}$.

There is capital accumulation in both the sectors. Dutt (1992) assumes that rates of investment of the two sectors are increasing linear functions of their respective rates of capacity utilization. Let X_j, K_j and I_j be output, capital stock and investment of sector $j \in \{i, s\}$. Then rates of capacity utilization in the industrial service sectors are $\frac{X_i}{K_i}$ and $\frac{X_s}{K_s}$ respectively. The rate of investment of the industrial sector is given by

$$\frac{I_i}{K_i} = \alpha_i + \beta_i \frac{X_i}{K_i} \tag{6}$$

where $\frac{X_i}{K_i}$ is the rate of capacity utilization in the industrial sector. Similarly for the service sector, the rate of investment is given by

$$\frac{I_s}{K_s} = \alpha_s + \beta_s \frac{X_s}{K_s} \tag{7}$$

where $\frac{X_s}{K_s}$ is the rate of capacity utilization in the service sector. α_j and β_j, for all $j \in \{i, s\}$, are positive constants. There is no depreciation of capital.[7]

Finally regarding savings behaviour in the model, Dutt (1992) assumes that all wages and a fraction of profits in the economy are used for consumption. Thus, using (3) and (4), the total consumption expenditure incurred on the industrial good is

$$\begin{aligned} C_i &= \frac{P_i X_i}{1+z_i} + \frac{P_s X_s}{1+z_s} + (1-s)(\frac{z_i P_i X_i}{1+z_i} + \frac{z_s P_s X_s}{1+z_s} - P_s X_s) \\ &= (P_i X_i - \frac{s z_i P_i X_i}{1+z_i}) + (\frac{s P_s X_s}{1+z_s}) \end{aligned} \tag{8}$$

where $s \in (0, 1)$ is a constant. Now let us consider the short-run and long-run dynamics of this model.

[7]The assumption of no depreciation of capital stocks is merely a simplifying assumption. Constant rates of depreciation in both the sectors can be easily accommodated in such models without any significant effect on the conclusions. For simplicity of exposition, through out this paper, we are going to assume that there is no depreciation of capital.

In the short run, the capital stock of both the sectors -K_i and K_s- are given. Since prices are fixed any mis match between demand and supply in the two sectors is corrected via adjustments of output of respective sectors. The short-run dynamics can be represented in the following manner. For $j \in \{i, s\}$,

$$\dot{X}_j = \psi_j[d_j - X_j] \tag{9}$$

where ψ_i and ψ_s are positive constants; $\dot{X}_i$ and $\dot{X}_s$ are time derivatives or rates of change of X_i and X_s respectively; and $d_i = \frac{C_i}{P_i} + I_i + I_s$ and $d_s = N_s$ are real demands for the industrial good and the service respectively. Substituting for d_i and d_s, using (2), (6), (7) and (8), in (9) reduces the short run dynamics of the model to

$$\begin{aligned} \dot{X}_i &= \psi_i[-(\frac{sz_i}{1+z_i} - \beta_i)X_i + (\frac{sp}{1+z_s} + \beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[-X_s + \lambda K_i] \end{aligned} \tag{10}$$

Short-run equilibrium requires $X_i > 0$ and $X_s > 0$ such that $\dot{X}_i = \dot{X}_s = 0$. Let

$$X_i^* = \frac{\alpha_i K_i + \alpha_s K_s + \lambda K_i(\frac{sp}{1+z_s} + \beta_s)}{\Omega} \tag{11}$$

$$X_s^* = \lambda K_i \tag{12}$$

where $\Omega = \frac{sz_i}{1+z_i} - \beta_i$.[8]

Proposition 1. *If $\Omega > 0$ then (X_i^*, X_s^*) is a unique and asymptotically stable short-run equilibrium of* (10)*.*

Proof. Suppose $\Omega > 0$. Setting the right hand sides of the two equations in (10) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(\frac{sz_i}{1+z_i} - \beta_i) & (\frac{sp}{1+z_s} + \beta_s) \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ -\lambda K_i \end{bmatrix} \tag{13}$$

[8] $\frac{1}{\Omega}$ is the expenditure multiplier for industrial output.

Ω is the determinant of 2×2 matrix in (13). Since $\Omega \neq 0$, *Cramer's rule* yields the unique solution of (13) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s + \lambda K_i\{sp/(1+z_s)+\beta_s\}}{\Omega} = X_i^*$ and $X_s = \lambda K_i = X_s^*$. $\Omega > 0$ implies $X_i^* > 0$ and $X_s^* > 0$ in (11) and (12) respectively as α_i, α_s, K_i, K_s, λ, s, p, z_s and β_s are all positive. For stability, notice that the Jacobian matrix for (10) is

$$\begin{bmatrix} -\psi_i(\frac{sz_i}{1+z_i} - \beta_i) & \psi_i(\frac{sp}{1+z_s} + \beta_s) \\ 0 & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega > 0$ and trace $-\psi_i\Omega - \psi_s < 0$ when $\Omega > 0$, as ψ_i and ψ_s are both positive.[9] □

The long-run dynamics in Dutt (1992) considers capital accumulation in the two sectors as a result of investments carried out in the short equilibrium described above. For this analysis it is assumed that $\Omega > 0$ and the economy is always in a short-run equilibrium given by X_i^* and X_s^*.[10] In the absence of depreciation, growth rate of capital stock of sector j, say g_j, is equal to its rate of investment $\frac{I_j}{K_j}$ where $j \in \{i, s\}$. Substituting the short-run equilibrium outputs of the two sectors from (11) and (12) respectively in (6) and (7) yields the growth rate of their capital stocks as

$$g_i = \alpha_i + \frac{\beta_i}{\Omega}\{\alpha_i + \frac{\alpha_s}{k} + \lambda(\frac{sp}{1+z_s} + \beta_s)\} \tag{14}$$

and

$$g_s = \alpha_s + \beta_s\lambda k \tag{15}$$

where $k = \frac{K_i}{K_s}$ is the relative capital stock of industry sector vis-a-vis the service sector, hence forth referred to as the relative capital stock of the industrial sector. Thus in the long run growth rate of capital stock of the industry sector is a strictly function of its relative capital stock. Intuitively,

[9]For a discussion of stability of equilibrium in linear systems of two differential equations in two variables, see, for example, Hirsch et al. (2004, pp. 61-64).

[10]It is assumed that full capacity output-capital ratios of the two sectors- $\bar{u}_i$ and $\bar{u}_s$- are such that, given K_i and K_s, X_i^* and X_s^* allow for excess capacity in both the sectors. We make similar assumption regarding $\bar{u}_i$ and $\bar{u}_s$ throughout this paper.

a higher value of either K_i or K_s means, from (11), a higher level of output in the industrial sector. This implies a *ceteris paribus* increase in K_s increases degree of capacity utilization in the industrial sector at the short-run equilibrium $\frac{X_i^*}{K_i}$, which increases the growth rate of capital stock of the sector in the long run. On the other hand, while a *ceteris paribus* in K_i increases X_i^* it not enough to counter the negative effect of K_i on $\frac{X_i^*}{K_i}$ as, from (11), $\frac{\partial(X_i^*/K_i)}{\partial K_i} = -\frac{\alpha K_s}{\Omega K_i^2}$. Since a *ceteris paribus* in K_i decreases $\frac{X_i^*}{K_i}$, it also decreases g_i in the long run. Thus, g_i is negatively related to k and, for similar reasons, g_s is positively related to k.

The long-run dynamics of the model is captured by changes in the relative capital stock of the industrial sector k because of different rates of growth of capital stocks of the two sectors. For all $k > 0$, the rate of change in k is

$$\dot{k} = k[g_i - g_s] \tag{16}$$

Substituting for g_i from (14) and g_s from (15) in (16), we obtain, for all $k > 0$,

$$\dot{k} = k[\alpha_i + \frac{\beta_i}{\Omega}\{\alpha_i + \frac{\alpha_s}{k} + \lambda(\frac{sp}{1+z_s} + \beta_s)\} - \alpha_s - \beta_s\lambda k] \tag{17}$$

Existence of steady state in the long run requires $\dot{k} = 0$ in (17) for some $k > 0$. Let

$$k^* = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{18}$$

where $a = -\beta_s\lambda\Omega$, $b = (\alpha_i - \alpha_s)\Omega + \alpha_i\beta_i + \beta_i\lambda(\frac{sp}{1+z_s} + \beta_s)$ and $c = \alpha_s\beta_i$.

Proposition 2. *Given $\Omega > 0$, k^* is a unique and asymptotically stable steady state of (17) in $\mathbb{R}_{++}$.*

Proof. We can rearrange (17) as $\dot{k} = \frac{ak^2+bk+c}{\Omega}$ where $a = -\beta_s\lambda\Omega$, $b = (\alpha_i - \alpha_s)\Omega + \alpha_i\beta_i + \beta_i\lambda(\frac{sp}{1+z_s} + \beta_s)$ and $c = \alpha_s\beta_i$. In the steady state $ak^2+bk+c = 0$ as $\Omega > 0$. Now $a < 0$ and $c > 0$ as α_s, β_i, β_s, λ and Ω are all positive. $a < 0$ and $c > 0$ imply $b^2 - 4ac > 0$. Therefore $ak^2+bk+c = 0$ has two distinct real roots, $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$. Since $a < 0$, the steady state value of k is $k^* = \frac{-b-\sqrt{b^2-4ac}}{2a}$. For stability, define a function $V : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V(k) = (g_i - g_s)^2$,

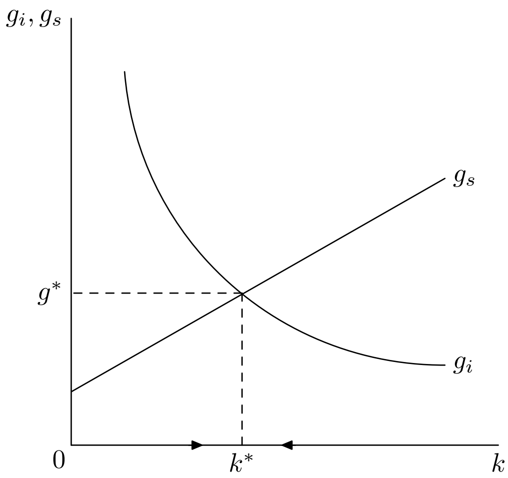

Figure 1: Balanced growth of industry and service in Dutt (1992)

where g_i and g_s are given by (14) and (15) respectively. Notice that, by definition, $V(k^*) = 0$ and $V(k \neq k^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, using (14), (15) and (16), $\dot{V} = -2k(g_i - g_s)^2(\frac{\beta_i \alpha_s}{\Omega k^2} + \beta_s \lambda) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k^*$ as Ω, α_s, β_i, β_s and λ are all positive. Thus, V is a *strict Liapunov function*[11] for k^*. $\square$

In Figure 1, we show the relative capital stock of the industrial sector k on the x-axis and the growth rates of capital stocks of the two sectors g_i and g_s on the y-axis. The downward sloping curve g_i represents (14) and the upward sloping line g_s represents (15). These two curves intersect at k^*, which is the long-run steady state. At k^* capital stocks of both the sectors grow at the same rate g^*. g^* is a positive constant, as can be checked by substituting k^* for k in either (14) or (15). Moreover, given that labour productivity in both sectors is a constant, it can be easily deduced from (11) and (12) that outputs as well as employment levels of the two sectors grow at the rate g^* at this steady state. In Figure 1, at any $k < k^*$, the industrial sector accumulates at a higher rate than the service sector so, from (19), k increases and continues to increase so long as it is below k^*. Similarly, at any $k > k^*$ the service sector accumulates at a faster rate than the industrial

[11] For the statement of Liapunov stability theorem see, for example, Hirsch et al. (2004, pp. 194-195).

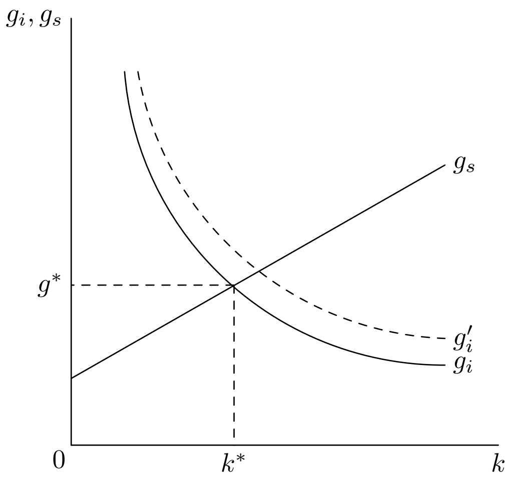

Figure 2: Effect of increase in x_i in Dutt (1992)

sector causing a decreases in k, which continues to decrease so long it is above k^*.

Dutt (1992) offers two conclusions from this model. First, an increase in λ increases the growth rate of the economy. This is fairly obvious as, from (14) and (15), an increase in λ will shift both g_i and g_s curves upwards in Figure 1. Second, there is no inverse relation between expansion of the service sector and the growth rate of the economy if the latter is measured by g_i. This conclusion is rather peculiar, because outside the steady state, the rate of accumulation of the industrial sector can not be taken as the growth rate of the economy in the model. Moreover since the steady state is globally stable, it is more natural to consider the rate at which both sectors grow at the steady state as the growth rate of the economy. Changes in the steady state growth rate depend on the nature of shifts in g_i and g_s curves. For example, if for some reason g_i curve shifts downwards in Figure 1 and the g_s curve remains unaffected then there is a relative expansion of the service sector and a decline in the growth rate.

Analyzing the implications of an exogenous labour productivity increase in the industrial sector on the steady state appears to provide a more interesting comparison of this demand constrained model of Dutt (1992) with

Baumol (1967). An increase in x_i increases the relative price of service, as p in (5) is an increasing function of x_i. This in turn increases the growth rate of capital stock of the industrial sector for all k but, given k, has no effect on the growth rate of capital stock of the service sector, from (15). We show this in Figure 2 where increase in the labour productivity of service sector causes the schedule for growth rate of capital stock of the industrial sector to shift upwards from g_i to g_i'. while there is no change in the schedule for growth rate of capital stock of the service sector. The new steady state is the intersection point of the dashed curve g_i' and the upward sloping line g_s in Figure 2. At the new steady state, both relative capital stock of the industrial sector and the growth rate are greater. Thus, contrary to the conclusions of Baumol (1967), in the demand constrained model of Dutt (1992) an exogenous labour productivity increase in the industrial sector increases both the relative size of the sector (measured in terms of the intersectoral ratio of capital stocks) and the growth rate of the economy. On the other hand, an increase in the labour productivity of the service sector has the opposite effect on the steady state because in this case p decreases as it is a decreasing function of x_s in (5). Proposition 3 proves this formally.

Proposition 3. *Given that $\Omega > 0$. Let $g^* = \alpha_i + \frac{\beta_i}{\Omega}\{\alpha_i + \frac{\alpha_s}{k^*} + \lambda(\frac{sp}{1+z_s} + \beta_s)\} = \alpha_s + \beta_s \lambda k^*$, where k^* is given by (18). Then $\frac{\partial g^*}{\partial p} > 0$.*

Proof. Suppose $\Omega > 0$. By the definition of g^*, $\frac{\partial g^*}{\partial p} = \beta_s \lambda \frac{\partial k^*}{\partial p}$. Thus $\frac{\partial g^*}{\partial p} > 0$ if and only if $\frac{\partial k^*}{\partial p} > 0$ as β_s and λ are positive. Now, from (18),

$$\frac{\partial k^*}{\partial p} = -\frac{1}{2a}\{1 + \frac{b}{\sqrt{b^2 - 4ac}}\}\frac{\partial b}{\partial p}$$

Since $b = (\alpha_i - \alpha_s)\Omega + \alpha_i\beta_i + \beta_i\lambda(\frac{sp}{1+z_s} + \beta_s)$, $\frac{\partial b}{\partial p} = \frac{\beta_i \lambda s}{1+z_s} > 0$ as β_i, λ and z_s are positive. Also $a = -\beta_s\lambda\Omega < 0$ and $c = \alpha_s\beta_i > 0$ as Ω, β_s, λ, α_s and β_i are positive. $a < 0$ and $c > 0$ imply $b^2 - 4ac > b^2$. Then, it must be that $-1 < \frac{b}{\sqrt{b^2-4ac}} < 1$ or $\{1 + \frac{b}{\sqrt{b^2-4ac}}\} > 0$. Thus, it follows that $\frac{\partial k^*}{\partial p} > 0$. □

3.1 Alternative Specifications for Demand Interlinkages

However the implications of sector specific technology shocks in the demand constrained model of Dutt (1992), given by Proposition 3, are not so much driven by the fact that resources are not fully utilised - existence of surplus labour and excess capacity - but by the specification as well as the functional forms of demand interlinkages between the two sectors. To bring this out, let us separately consider two variants of this model. First, instead of assuming that production in the industrial sector requires the service as an overhead input, let us assume that it requires the service as an intermediate input. Specifically let

$$N_s = \lambda_1 X_i \tag{19}$$

where λ_1 is a positive constant. (19) implies that the industrial sector's demand for service input is in constant proportion to its output instead of its capital stock, as is the case in (2). As a consequence price of the industrial product is now $P_i = (1 + z_i)(\frac{W}{x_i} + P_s\lambda_1)$, combining this with (4) yield the following expression for relative price of service

$$p = \frac{1 + z_s}{(1 + z_i)\{\frac{x_s}{x_i} + (1 + z_s)\lambda_1\}} \tag{20}$$

This change in the the expression for the relative price of service is not of much interest here because an increase in x_i decreases p and an increase in x_s increases p in (20), same as before, when p was given by (5). The more consequential change is in the short-run equilibrium. Substituting for d_i and d_s in (9), using (6), (7), (8) and (19), we can represent the short run dynamics in this case as the following system of differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(\frac{sz_i}{1 + z_i} - \beta_i)X_i + (\frac{sp}{1 + z_s} + \beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\lambda_1 X_i - X_s] \end{aligned} \tag{21}$$

Short-run equilibrium now requires that there exists $X_i > 0$ and $X_s > 0$ such that $\dot{X}_i = \dot{X}_s = 0$ in (21). Let

$$X_{i1}^* = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_1} \tag{22}$$

$$X_{s1}^* = \frac{\lambda(\alpha_i K_i + \alpha_s K_s)}{\Omega_1} \tag{23}$$

where $\Omega_1 = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp}{1+z_s} + \beta_s)$.[12]

Proposition 4. *If $\Omega_1 > 0$ then (X_{i1}^*, X_{s1}^*) is a unique and asymptotically stable short-run equilibrium of (21).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

Assuming $\Omega_1 > 0$ and substituting (X_{i1}^*, X_{s1}^*) for (X_i, X_s) in (6) and (7) yields the growth rate of capital stock of the two sectors as

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_1} \tag{24}$$

$$g_s = \alpha_s + \frac{\beta_s \lambda(\alpha_i k + \alpha_s)}{\Omega_1} \tag{25}$$

The long-run dynamics is now obtained by substituting for g_i and g_s from (24) and (25) respectively in (16):

$$\dot{k} = k[\alpha_i + \frac{\beta(\alpha_i + \frac{\alpha_s}{k})}{\Omega_1} - \alpha_s - \frac{\beta_s \lambda(\alpha_i k + \alpha_s)}{\Omega_1}] \tag{26}$$

for all $k > 0$. Like in the previous model, there exists a stable long run steady state with a constant relative capital stock of industrial sector,

$$k_1^* = \frac{-b_1 - \sqrt{b_1^2 - 4a_1 c_1}}{2a_1} \tag{27}$$

where $a_1 = -\alpha_i \beta_s \lambda_1$, $b_1 = (\alpha_i - \alpha_s)\Omega_1 + \alpha_i \beta_i - \lambda_1 \alpha_s \beta_s$ and $c_1 = \alpha_s \beta_i$. Outputs, capital stocks and employment levels in both the sector grow at

[12] $\frac{1}{\Omega_1}$ is the expenditure multiplier for industrial goods in this case.

the same constant rate at k^*.

Proposition 5. *Given* $\Omega_1 > 0$, k_1^* *is a unique and asymptotically stable steady state of* (26) *in* $\mathbb{R}_{++}$.

Proof. Similar to the proof of Proposition 2, see appendix A. □

Despite these similarities between the two models, implications of sector-specific technology shocks on the steady state are not exactly same. Like the previous model, in this model too, a *ceteris paribus* increase in x_i unambiguously increases g_1^*, however, unlike the previous model, its effect on k_1^* is ambiguous. From (20) it follows that an exogenous increase in x_i increases the relative price service p, like before. Changes in p in this model affects the expenditure multiplier as $\Omega_1 = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp}{1+z_s} + \beta_s)$. Thus increase in x_i increases the expenditure multiplier $\frac{1}{\Omega_1}$ because $\frac{\partial \Omega_1}{\partial p} = -\frac{\lambda_1 s}{1+z_s} < 0$. This means that, for any arbitrary combination of capital stocks of the two sectors, there is a greater short-run equilibrium output of the industry sector, as can be shown from (22). Since the service is now demanded as an intermediate input by the industrial sector, a larger short-run equilibrium level of industrial output means a larger short-run equilibrium output of the service sector. Further, since the short-run equilibrium output in both the sectors increase irrespective of their capital stocks, in the long run growth rates of their capital stocks increase for all values of k. In Figure 3, we show this upward shifts in the schedules for growth rates of capital stocks of the two sectors from g_i to g_i' and g_s to g_s'. The new steady state is given by the intersection point of the two dashed curves g_i' and g_s'. Clearly, growth rate at the new steady state must be greater. We formally prove this in Proposition 6.

Proposition 6. *Let* $g_1^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_1^*)\}}{\Omega_1} = \alpha_s + \frac{\beta_s \lambda_1(\alpha_i k_1^* + \alpha_s)}{\Omega_1}$ *where* k_1^* *is given by* (27). *Then* $\frac{\partial g_1^*}{\partial p} > 0$.

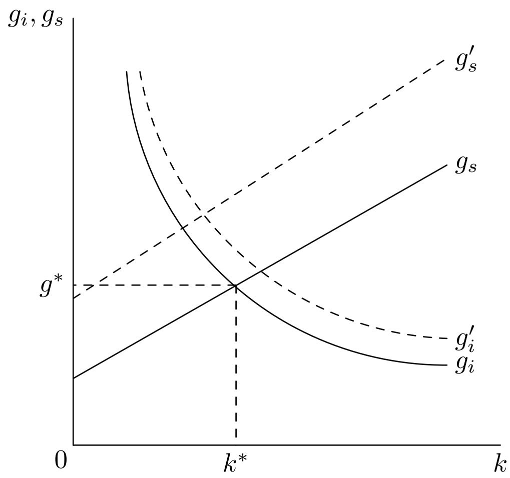

Figure 3: Effect of increase in x_i when N_s is described by (19)

Proof. From $g_1^* = \alpha_s + \frac{\beta_s \lambda_1 (\alpha_i k_1^* + \alpha_s)}{\Omega_1}$,

$$\frac{\partial g_1^*}{\partial p} = \frac{\beta_s \lambda_1}{\Omega_1^2} \{\Omega_1 \alpha_i \frac{\partial k_1^*}{\partial p} - (\alpha_i k_1^* + \alpha_s) \frac{\partial \Omega_1}{\partial p}\} \tag{28}$$

Now from the definition of Ω_1, $\frac{\partial \Omega_1}{\partial p} = -\frac{\lambda_1 s}{1+z_s} < 0$ as λ_1, s and z_s are positive. Next from (27),

$$\frac{\partial k_1^*}{\partial p} = -\frac{1}{2a_1} \frac{\partial b_1}{\partial p} \{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1 c_1}}\}$$

Since $b_1 = (\alpha_i - \alpha_s)\Omega_1 + \alpha_i \beta_i - \lambda_1 \alpha_s \beta_s$ we have $\frac{\partial b_1}{\partial p} = (\alpha_i - \alpha_s) \frac{\partial \Omega_1}{\partial p}$. Substituting for $\frac{\partial b_1}{\partial p}$ in above expression yields

$$\frac{\partial k_1^*}{\partial p} = -\frac{(\alpha_i - \alpha_s)}{2a_1} \frac{\partial \Omega_1}{\partial p} \{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1 c_1}}\} \tag{29}$$

Now $a_1 = -\alpha_i \beta_s \lambda_1 < 0$ and $c_1 = \alpha_s \beta_i > 0$ as α_i, α_s, β_i, β_s and λ_1 are all positive. $a_1 < 0$ and $c_1 > 0$ imply $b_1^2 - 4a_1 c_1 > b_1^2$. Therefore, $-1 < \frac{b_1}{\sqrt{b_1^2 - 4a_1 c_1}} < 1$ or $\{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1 c_1}}\} > 0$. Thus, from (29), it follows that $\frac{\partial k_1^*}{\partial p} < 0$ if and only if $\alpha_i - \alpha_s > 0$ as $\frac{\partial \Omega_1}{\partial p} < 0$ and $a_1 < 0$. And, from (28), if $\frac{\partial k_1^*}{\partial p} \geq 0$ then $\frac{\partial g^*}{\partial p} > 0$ as $\frac{\partial \Omega_1}{\partial p} < 0$ and all other factors in the right hand side of (28) are positive. To complete the proof, we need to show that $\frac{\partial g_1^*}{\partial p} > 0$ when

$\frac{\partial k_1^*}{\partial p} < 0$. Suppose at $p_1 > 0$, $\frac{\partial k_1^*}{\partial p} < 0$ and $\frac{\partial g_1^*}{\partial p} \leq 0$. Let k_{11}^* be the steady state of (26) when relative price of service is p_1. Also, let $g_{11}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{11}^*)\}}{\Omega_1(p_1)} = \alpha_s + \frac{\beta_s\lambda_1(\alpha_i k_{11}^* + \alpha_s)}{\Omega_1(p_1)}$, where $\Omega_1(p_1) = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp_1}{1+z_s} + \beta_s)$. Since $\frac{\partial k_1^*}{\partial p} < 0$ and $\frac{\partial g_1^*}{\partial p} \leq 0$ at p_1, there exists a $p_2 > p_1$ such that $k_{12}^* < k_{11}^*$ and $g_{12}^* \leq g_{11}^*$, where k_{12}^* is the steady state of (26) when relative price of service is p_2 and $g_{12}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{12}^*)\}}{\Omega_1(p_2)} = \alpha_s + \frac{\beta_s\lambda_1(\alpha_i k_{12}^* + \alpha_s)}{\Omega_1(p_2)}$ with $\Omega_1(p_2) = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp_2}{1+z_s} + \beta_s)$. Now $g_{12}^* \leq g_{11}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{12}^*)\}}{\Omega_1(p_2)} \leq \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{11}^*)\}}{\Omega_1(p_1)}$. This in turn implies $\Omega_1(p_1) < \Omega_1(p_2)$ since $\alpha_s > 0$ and $k_{11}^* > k_{12}^*$ imply $\frac{\alpha_s}{k_{11}^*} < \frac{\alpha_s}{k_{12}^*}$. However this is a contradiction as it must be that $\Omega_1(p_1) > \Omega_1(p_2)$ since $\frac{\partial \Omega_1}{\partial p} < 0$ for all p and $p_1 < p_2$. □

In Figure 3, we also show that the steady state relative capital stock of the industrial sector decreases, but, this is not necessarily true. From the proof of Proposition 6, $\frac{\partial k^*}{\partial p} = -\frac{(\alpha_i - \alpha_s)}{2a_1}\frac{\partial \Omega_1}{\partial p}\{1 + \frac{b_1}{\sqrt{b_1^2 - 4a_1c_1}}\} < 0$ if and only if $\alpha_i - \alpha_s > 0$. Similarly, an increase in x_s decreases the steady state growth rate but has ambiguous effect on the steady state relative capital stock of the industrial sector.

Next, consider another simple change in the model of Dutt (1992). Instead of the classical savings function, let us assume that consumption expenditure incurred on the industrial good is a constant fraction of the the value added. Since there is no final demand for service output, total value added in the model is P_iX_i. So consumption expenditure incurred on the industrial good now is

$$C_i = cP_iX_i \tag{30}$$

where $c \in (0, 1)$ is a constant. Also we revert back to (2), that is the service input in the industrial sector is an overhead input rather than an intermediate input. There is no other change from the model of Dutt (1992) except that we replace (8) with (30). Using (2), (6), (7) and (30), to substitute for d_i and d_s in (9) we can represent the short run dynamics of this model as the following system of differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(1-c-\beta_i)X_i + \beta_s X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi[-X_s + \lambda K_i] \end{aligned} \tag{31}$$

Let

$$X_{i2}^* = \frac{\alpha_1 K_i + \alpha_s K_s + \beta_s \lambda K_i}{\Omega_2} \tag{32}$$

$$X_{s2}^* = \lambda K_i \tag{33}$$

where $\Omega_2 = 1 - c - \beta_i > 0$.

Proposition 7. *If $\Omega_2 > 0$ then (X_{i2}^*, X_{s2}^*) is a unique and asymptotically stable short-run equilibrium of (31).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

For the long-run, once again assuming that $\Omega_2 > 0$ and the economy is always in a short-run equilibrium, we obtain growth rates of capital stocks of the two sectors by substituting (X_{i2}^*, X_{s2}^*) for (X_i, X_s) in (6) and (7),

$$g_i = \alpha_i + \frac{\beta_i}{\Omega_2}(\alpha_i + \frac{\alpha_s}{k} + \beta_s \lambda) \tag{34}$$

$$g_s = \alpha_s + \beta_s \lambda k \tag{35}$$

The long run dynamics of this model is then obtained by substituting for g_i and g_s respectively from (34) and (35) in (16).

$$\dot{k} = k[\alpha_i + \frac{\beta_i}{\Omega_2}(\alpha_i + \frac{\alpha_s}{k} + \beta_s \lambda) - \alpha_s - \beta_s \lambda k] \tag{36}$$

There exists a stable steady state of (36) with a constant relative capital stock of industrial sector,

$$k_2^* = \frac{-b_2 - \sqrt{b_2^2 - 4a_2 c_2}}{2a_2} \tag{37}$$

where $a_2 = -\beta_s\lambda$, $b_2 = (\alpha_i - \alpha_s)\Omega_2 + \alpha_i\beta_i + \beta_i\beta_s\lambda$ and $c_2 = \alpha_s\beta_i$.

Proposition 8. *Given $\Omega_2 > 0$, k_2^* in (37) is a unique and asymptotically stable steady state of (36) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. □

However, in this case exogenous increase in labour productivity in either of the sectors has no effect on the steady state, as shown in Proposition 9. This is because growth rate of capital stocks of both the sectors are independent of the relative price of the service.

Proposition 9. *Let $g_2^* = \alpha_i + \frac{\beta_i}{\Omega_2}(\alpha_i + \frac{\alpha_s}{k_2^*} + \beta_s\lambda) = \alpha_s + \beta_s\lambda k_2^*$ where k_2^* is given by (37). Then $\frac{\partial g_2^*}{\partial p} = 0$.*

Proof. From (37) and the definition of g_2^*, it follows that $\frac{\partial g_2^*}{\partial p} = 0$. □

To sum up this section, the demand constrained model of Dutt (1992) shows that in presence of excess capacity in both the industrial sector and the service sector and perfectly elastic supply of labour, balanced growth of the two sectors is feasible if they generate demand for each others product. We showed that in this particular model exogenous increase in labour productivity of the industrial sector, contrary to the predictions of Baumol (1967), increases both growth rate of the economy and the relative size of the industrial sector measured in terms of k^*. Using two variants of this model, we also showed that implications of sector-specific technology shocks in such models is likely to vary with variations in the specification as well as the functional form of demand interlinkages between the two sectors. In none of the models considered in this section, however, increase in labour productivity of the service sector have any positive implication for growth. This is somewhat perplexing considering application of modern information and communication technologies (ICTs). In the next section we present a similar model in which increase in labour productivity in the service sector has a positive effect on the growth rate using a different specification of demand for service generated by the industrial sector.

4. ICTs and Service-Led Growth

Pessimistic views about technological progress in services, like that of Baumol (1967), may no longer hold for all kinds of services with continuing advances in and diffusion of information and communication technology (ICTs). There has also been continuous advances in the application of ICTs in many services such as telecommunications, banking & finance and various business services like software and IT services. Application of modern ICTs in these services have coincided with growing importance of the service sector not only in developed economies but also in many developing economies.[13] For example, in the Indian economy these services have been amongst the fastest growing sectors from the late 1990s. In the models presented in the previous section, there is no growth-boosting effect of labour productivity increases in the service sector, as there is for the industrial sector. In those models sector specific technology shocks affected the growth rate via their effect on the relative price of the service on the demand for the industrial good, d_i. On the other hand, the demand for the service, d_s, was completely determined by either the capital stock or the output of the industrial sector. In this section we argue that if d_s depends on both p and X_i then an increase in labour productivity in the service sector can not only increase the relative size of the service sector but also the growth rate of the economy.

Due to tremendous advances in ICTs and in electronics, many services are today required to complement the use of various industrial products. For example purchase of computer hardware without software and Internet services is not very useful. Similar is the case for mobile telephony and other electronic goods in general. This is not only true for consumption of industrial products but can also be true for investment demand. For example it is possible that a firm can raise more funds for investment if it employs

[13] See for example Eichengreen and Gupta (2013)

the services of a financial firm to underwrite its shares. Moreover there is no reason why the joint utilization of industrial goods and services needs to be a perfectly complementary one. With lower prices of various services, more services can be purchased along various industrial goods. In a two-sector model with industry and service demand interactions like the ones discussed in the previous section, this aspect can be incorporated by stipulating that industrial output can be utilized for consumption or used as investment good only if it is purchased along with service output. Formally let, $P_s d_s = \theta P_i X_i$ or,

$$d_s = \frac{\theta X_i}{p} \tag{38}$$

where θ is a positive constant. Thus we assume that demand for service d_s is now positively related to output of the industrial sector X_i and negatively related to the relative price of service $p = \frac{P_s}{P_i}$. For the sake of simplicity, we do not consider demand for services as inputs in the industrial sector in this section.[14] Prices of the industrial good and the service are given by (3) and (4) respectively and therefore relative price of service is given by (5) where $p = \frac{(1+z_s)x_i}{(1+z_i)x_s}$.

Like in the models of the previous section, the industrial good is demanded for consumption and as capital good by both the sectors. Therefore demand for industrial output once again is $d_i = \frac{C_i}{P_i} + I_i + I_s$. Investment demands of the two sectors, I_i and I_s, are described by (6) and (7) and once again we assume that there is no depreciation of capital in both the sectors. As far as C_i is concerned, in this section we are going to assume that consumption expenditure incurred on the industrial sector is a constant fraction of total value added in the economy like in (30). However since there is no input demand for the service, total value added in the economy now is equal to $P_i X_i + P_s X_s$. Therefore total consumption expenditure is

$$C_i = c(P_i X_i + P_s X_s) \tag{39}$$

[14] We can easily include a price sensitive term for intermediate input demand for the service from the industrial sector, such as $N_s = \frac{\lambda_2 X_i}{p}$ where λ_2 is a positive constant , without significantly effecting any result.

where $c \in (0,1)$.[15] Using (6), (7), (38) and (39), to substitute for d_i and d_s in (9) we obtain the short-run dynamics of this model as the following system of two differential equations.

$$\begin{aligned}\dot{X}_i &= \psi_i[-(1-c-\beta_i)X_i + (cp+\beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\frac{\theta}{p}X_i - X_s]\end{aligned} \tag{40}$$

Let

$$X_{i3}^* = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_3} \tag{41}$$

and

$$X_{s3}^* = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_3} \tag{42}$$

where $\Omega_3 = 1 - c(1+\theta) - \beta_i - \frac{\theta\beta_s}{p}$.

Proposition 10. *If $\Omega_3 > 0$ then (X_{i3}^*, X_{s3}^*) is a unique and asymptotically stable short-run equilibrium of (40).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

For the long-run analysis, we once again assume that $\Omega_3 > 0$ and the economy is always in a short run equilibrium and capital stocks of both the sectors grow because of investment carried out in the short run. Using (6), 7, (41) and (42), we get the growth rate of capital stock of the industrial sector as

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_3} \tag{43}$$

and the growth rate of capital stock of the service sector as

$$g_s = \alpha_s + \frac{\beta_s\theta(\alpha_i k + \alpha_s)}{p\Omega_3} \tag{44}$$

[15] It can be verified that, if, instead of (38) and (39), we assume that the service is used only for consumption and derive consumption demands for the two sectors as constant fractions of the total consumption expenditure, obtained using the classical savings function, then implications of sector-specific technology shocks are no different from what is discussed in this section (with the exception of subsection 4.2, where there can be some differences).

The long run dynamics of this model can be studied by analysing the following differential equation derived from (16), (43) and (44).

$$\dot{k} = k[\alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_3} - \alpha_s - \frac{\beta_s\theta(\alpha_i k + \alpha_s)}{p\Omega_3}] \tag{45}$$

for all $k > 0$. Let

$$k_3^* = \frac{-b_3 - \sqrt{b_3^2 - 4a_3c_3}}{2a_3} \tag{46}$$

where $a_3 = -\alpha_i\beta_s\theta$, $b_3 = (\alpha_i - \alpha_s)p\Omega_3 + \alpha_i\beta_i p - \alpha_s\beta_s\theta$ and $c_3 = \alpha_s\beta_i p$.

Proposition 11. *Given $\Omega_3 > 0$, k_3^* is a unique and asymptotically stable steady state of (45) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. □

Thus in this model too, outputs, capital stocks and employment levels of both sectors grow at the same rate as the economy converges to k_3^* in (46) in the long run.

Now let us examine the implication of an increase in labour productivity of the service sector in this model. We know from (5), that an increase in x_s decreases p. A fall in p, however, has completely different effect on the steady state in this model compared to the models in the previous sections. Here, a lower relative price of service means more service demand per unit industrial output as from (38), $\frac{\partial d_s}{\partial p} = -\frac{\theta X_i}{p^2} < 0$. Further, since greater service demand means greater service output, there is an increase in consumption and investment demand for the industrial good generated by the service sector because of which the industrial output increases. This effect is reflected in an increase in the expenditure multiplier for the industrial output, $\frac{1}{\Omega_3}$, as $\frac{\partial \Omega_3}{\partial p} = \frac{\theta\beta_s}{p^2} > 0$. As a consequence, the short-run equilibrium output of the industrial sector in (41) increases, which in turn combines with increase in service demand per unit industrial output to increase the short-run equilibrium output of the service sector in (42). Since short-run equilibrium output of both sectors increase because of a rise in x_s irrespective of their capital stocks, growth

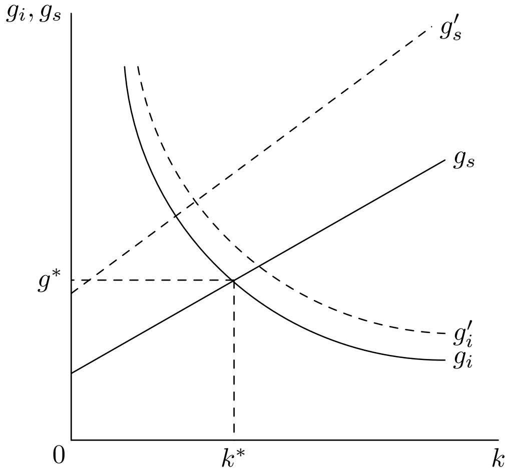

Figure 4: Effect of an increase in x_s when d_s is described by (38)

rates of capital stock of both sectors increase for all $k > 0$. We show this in Figure 4, where schedules for the growth rate of capital stock of both sectors shift upwards from g_i to g'_i and g_s g'_s because of the increase in x_s. The new steady state is given by the intersection point of the curves labeled g'_i and g'_s in Figure 4. Clearly in this case increase in labour productivity of the service sector increases the steady state growth rate.

Proposition 12. *Let* $g_3^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_3^*)\}}{\Omega_3} = \alpha_s + \frac{\beta_s\theta(\alpha_i k_3^* + \alpha_s)}{p\Omega_3}$ *where* k_3^* *is given by* (46)*. Then* $\frac{\partial g_3^*}{\partial p} < 0$.

Proof. From $g_3^* = \alpha_s + \frac{\beta_s\theta(\alpha_i k_3^* + \alpha_s)}{p\Omega_3}$,

$$\frac{\partial g_3^*}{\partial p} = \frac{\beta_s\theta}{p^2\Omega_3^2}[p\Omega_3\alpha_i\frac{\partial k_3^*}{\partial p} - (\alpha_i k_3^* + \alpha_s)\{\Omega_3 + p\frac{\partial \Omega_3}{\partial p}\}] \tag{47}$$

From the definition of Ω_3, $\Omega_3 + p\frac{\partial\Omega_3}{\partial p} = 1 - c(1+\theta) - \beta_i > 0$ since $\Omega_3 > 0$. Therefore sign of $\frac{\partial g_3^*}{\partial p}$ in (47) depends on the sign of $\frac{\partial k_3^*}{\partial p}$ as p, Ω_3, p, α_s, k_3^*,

β_s and θ are all positive. From (46),

$$\frac{\partial k_3^*}{\partial p} = -\frac{1}{2a_3}\{\frac{\partial b_3}{\partial p} + \frac{1}{2\sqrt{b_3^2 - 4a_3c_3}}(2b_3\frac{\partial b_3}{\partial p} - 4a_3\frac{\partial c_3}{\partial p})\}$$
$$= -\frac{1}{2a_3}\frac{\partial b_3}{\partial p}\{1 + \frac{b_3}{\sqrt{b_3^2 - 4a_3c_3}}\} + \frac{\partial c_3}{\partial p} \tag{48}$$

Now $a_3 < 0$, $c_3 > 0$ and $\frac{\partial c_3}{\partial p} = \alpha_s\beta_i > 0$ as α_i, α_s, β_i, β_s, p and θ are all positive. Also, $b_3^2 - 4a_3c_3 > b_3^2$ as $a_3 < 0$ and $c_3 > 0$. Therefore $-1 < \frac{b_3}{\sqrt{b_3^2-4a_3c_3}} < 1$ or $\{1+\frac{b_3}{\sqrt{b_3^2-4a_3c_3}}\} > 0$. However, sign of $\frac{\partial b_3}{\partial p} = \{(\alpha_i - \alpha_s)(\Omega_3 + p\frac{\partial \Omega_3}{\partial p}) + \alpha_i\beta_i\}$ is ambiguous because of which sign of $\frac{\partial k_3^*}{\partial p}$ is also ambiguous. Then it follows from (47) that $\frac{\partial g_3^*}{\partial p} < 0$ if $\frac{\partial k_3^*}{\partial p} \leq 0$. To complete the proof, we need to show that $\frac{\partial g_3^*}{\partial p} < 0$ when $\frac{\partial k_3^*}{\partial p} > 0$. Suppose, on the contrary, that at some arbitrary value of $p = p_1$, $\frac{\partial k_3^*}{\partial p} > 0$ and $\frac{\partial g_3^*}{\partial p} \geq 0$. Let k_{31}^* be the steady state of (45) when relative price of service is p_1. Also, let $g_{31}^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{31}^*)\}}{\Omega_3(p_1)} = \alpha_s + \frac{\beta_s\theta(\alpha_i k_{31}^*+\alpha_s)}{p_1\Omega_3(p_1)}$, where $\Omega_3(p_1) = 1 - c(1+\theta) - \beta_i - \frac{\theta\beta_s}{p_1}$. Since $\frac{\partial k_3^*}{\partial p} > 0$ and $\frac{\partial g_3^*}{\partial p} \geq 0$ at p_1, there exists a $p_2 > p_1$ such that $k_{32}^* > k_{31}^*$ and $g_{32}^* \geq g_{31}^*$, where k_{32}^* is the steady state of (45) when relative price of service is p_2 and $g_{32}^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{32}^*)\}}{\Omega_3(p_2)} = \alpha_s + \frac{\beta_s\theta(\alpha_i k_{32}^*+\alpha_s)}{p_2\Omega_3(p_2)}$ with $\Omega_3(p_2) = 1 - c(1+\theta) - \beta_i - \frac{\theta\beta_s}{p_2}$. Now $g_{32}^* \geq g_{31}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{32}^*)\}}{\Omega_3(p_2)} \geq \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{31}^*)\}}{\Omega_3(p_1)}$. This, in turn, implies $\Omega_3(p_1) > \Omega_3(p_2)$ since $k_{32}^* > k_{31}^*$ and $\alpha_s > 0$. However, this is a contradiction because $p_1 < p_2$ and $\frac{\partial \Omega_3}{\partial p} = \frac{\beta_s\theta}{p^2} > 0$ for all p imply $\Omega_3(p_1) < \Omega_3(p_2)$. $\square$

Although in Figure 4, we show that steady state relative capital stock of the industry sector decreases, the effect on the steady state relative capital stock of the industrial sector depends on which of the two schedules shifts more and is, therefore, ambiguous. From (48) in the proof Proposition 12, $\frac{\partial k^*}{\partial p} > 0$ if $\frac{\partial b_3}{\partial p} = \{(\alpha_i - \alpha_s)(\Omega_3 + p\frac{\partial \Omega_3}{\partial p}) + \alpha_i\beta_i\} = \{(\alpha_i - \alpha_s)(1 - c(1+\theta) - \beta_i) + \alpha_i\beta_i\} > 0$. Thus this model predicts if application of ICTs causes an increase in labour productivity of the service sector, then both the growth rate of economy and the relative size of the service sector can increase. Rise in labour productivity in the industrial sector, on the other hand, decreases growth rate in this model. However, unlike Baumol (1967), where the decline in growth rate is because of a shift in resources to a stagnant service sector, here it is due to an increase in the relative price of the service. As a result, outputs and growth rates of capital stocks of both sectors decline in the short run and the long run respectively, which causes a decrease in the steady state growth rate of the model. Such an unambiguous conclusion regarding effect of sector specific technology shocks on the growth rate, however, does not follow if we adopt a slightly more general specification for demand for the service.

4.1 A More General Formulation of d_s

Instead of (38), let demand for the service be given by

$$d_s = \theta(p)X_i \tag{49}$$

where θ now is a function $\theta : \mathbb{R}_{++} \mapsto \mathbb{R}_{++}$ with derivative $\theta'(p) < 0$ for all $p \in \mathbb{R}_{++}$. Using (6), (7), (39) and (49), to substitute for d_i and d_s in (9) we obtain the short run dynamics of this model as the following system of two differential equations.

$$\begin{aligned} \dot{X}_i &= \psi_i[-(1-c-\beta_i)X_i + (cp+\beta_s)X_s + \alpha_i K_i + \alpha_s K_s] \\ \dot{X}_s &= \psi_s[\theta(p)X_i - X_s] \end{aligned} \tag{50}$$

Let

$$X_{i4}^* = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_4} \tag{51}$$

$$X_{s4}^* = \frac{\theta(p)(\alpha_i K_i + \alpha_s K_s)}{\Omega_4} \tag{52}$$

where $\Omega_4 = 1 - c - \beta_i - \theta(p)(cp + \beta_s)$.

Proposition 13. *If $\Omega_4 > 0$ then (X_{i4}^*, X_{s4}^*) is a unique and asymptotically stable short-run equilibrium of (50).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

As usual, substituting for (X^*_{i4}, X^*_{s4}) from (51) and (52) in (6) and (7) yields the growth rate of capital stocks of the two sectors in the long run as the following.

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_4} \tag{53}$$

$$g_s = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k + \alpha_s)}{\Omega_4} \tag{54}$$

The long run dynamics of this model can be obtained by substituting for g_i and g_s from (53) and (54) respectively in (16).

$$\dot{k} = k[\alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_4} - \alpha_s - \frac{\beta_s\theta(p)(\alpha_i k + \alpha_s)}{\Omega_4}] \tag{55}$$

for all $k > 0$. Let

$$k^* = \frac{-b_4 - \sqrt{b_4^2 - 4a_4c_4}}{2a_4} \tag{56}$$

where $a_4 = -\alpha_i\beta_s\theta(p)$, $b_4 = (\alpha_i - \alpha_s)\Omega_4 + \alpha_i\beta_i - \alpha_s\beta_s\theta(p)$ and $c_4 = \alpha_s\beta_i$.

Proposition 14. *Given $\Omega_4 > 0$, k^*_4 is a unique and asymptotically stable steady state of* (55) *in* $\mathbb{R}_{++}$.

Proof. Similar to the proof of Proposition 2, see appendix A. □

Like in the previous model, a decrease in p increases demand for service per unit industrial output $\theta(p)$, which tends to increase the industrial output. However in this case, the total indirect effect on the industrial output per unit increase in the industrial output, $(cp + \beta_s)\theta(p)$, may not be positive. As a result increase in p now has an ambiguous effect on the expenditure multiplier $\frac{1}{\Omega_4}$ and, therefore, on the steady state growth rate too. Nonetheless in Proposition 15 we show if $\theta(p)$ is sufficiently elastic then a decrease in p increases the steady state growth rate.

Proposition 15. *Let* $g_4^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_4^*)\}}{\Omega_4} = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_4^* + \alpha_s)}{\Omega_4}$ *where* k_4^* *is given by* (56). *Then* $\frac{\partial g_4^*}{\partial p} < 0$ *if* $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ *for all* $p > 0$.

Proof. Suppose $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$. From the definition Ω_4, $\frac{\partial \Omega_4}{\partial p} = -(cp+\beta_s)\theta'(p) - c\,\theta(p)$. Now $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ implies $-(cp+\beta_s)\theta'(p) - c\,\theta(p) > 0$ as p, $\theta(p)$, c and β_s are all positive. Thus $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$ implies $\frac{\partial \Omega_4}{\partial p} > 0$ for all p. Next, from $g_4^* = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_4^* + \alpha_s)}{\Omega_4}$,

$$\frac{\partial g_4^*}{\partial p} = \frac{\beta_s}{\Omega_4^2}[\Omega_4\{\theta'(p)(\alpha_i k_4^* + \alpha_s) + \theta(p)\alpha_i \frac{\partial k_4^*}{\partial p}\} - \theta(p)(\alpha_i k_4^* + \alpha_s)\frac{\partial \Omega_4}{\partial p}] \quad (57)$$

Since $\theta'(p) < 0$ and $\frac{\partial \Omega_4}{\partial p} > 0$ for all p, it follows from (57) that $\frac{\partial g_4^*}{\partial p} < 0$ if $\frac{\partial k_4^*}{\partial p} \leq 0$ as α_i, α_s, β_s, $\theta(p)$, k_4^* and Ω_4 are all positive. To complete the proof, we need to show that $\frac{\partial g_4^*}{\partial p} < 0$ when $\frac{\partial k_4^*}{\partial p} > 0$. Suppose, on the contrary that at some arbitrary value of $p = p_1 > 0$, $\frac{\partial k_4^*}{\partial p} > 0$ and $\frac{\partial g_4^*}{\partial p} \geq 0$. Let k_{41}^* be the steady state of (55) when relative price of service is p_1. Also, let $g_{41}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{41}^*)\}}{\Omega_4(p_1)} = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_{41}^* + \alpha_s)}{\Omega_4(p_1)}$, where $\Omega_4(p_1) = 1 - c - \beta_i - \theta(p_1)(cp_1 + \beta_s)$. Since $\frac{\partial k_4^*}{\partial p} > 0$ and $\frac{\partial g_4^*}{\partial p} \geq 0$ at p_1, there exists a $p_2 > p_1$ such that $k_{42}^* > k_{41}^*$ and $g_{42}^* \geq g_{41}^*$, where k_{42}^* is the steady state of (55) when relative price of service is p_2 and $g_{42}^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{42}^*)\}}{\Omega_4(p_2)} = \alpha_s + \frac{\beta_s\theta(p)(\alpha_i k_{42}^* + \alpha_s)}{\Omega_4(p_2)}$ with $\Omega_4(p_2) = 1 - c - \beta_i - \theta(p_2)(cp_2 + \beta_s)$. Now, $g_{42}^* \geq g_{41}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{42}^*)\}}{\Omega_4(p_2)} \geq \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_{42}^*)\}}{\Omega_4(p_2)}$. This, in turn, implies $\Omega_4(p_1) > \Omega_4(p_2)$ because $k_{41}^* < k_{42}^*$ and $\alpha_s > 0$. However this is a contradiction since $\frac{p\theta'(p)}{\theta(p)} < -\frac{cp}{cp+\beta_s}$ for all $p > 0$ implies $\frac{\partial \Omega_4}{\partial p} > 0$ for all p and, therefore, $p_1 < p_2$ implies $\Omega_4(p_1) < \Omega_4(p_2)$. $\square$

4.2 Sensitiveness of z_s to Changes in x_s

The analysis in this section so far has been carried out on the assumption that application of modern ICTs in the service sector affects its labour productivity, x_s, but not the 'degree of monopoly', z_s. Introduction of new technology can have effects on the concentration of market power in the economy or in particular sectors. For example if new technology in a sector is introduced by new entrants then the degree of concentration in the sector might decline whereas, in case new technology is introduced by an incumbent then it might increase as the new technology can act as a barrier to entry. Even when new technology is introduced by a new entrant, market power can become more concentrated if either an incumbent acquires or merges with the entrant firm or the new entrant drives out incumbent firms from the market.

Let the mark-up in the service sector be a differentiable function of its labour productivity. That is, let $z_s = z_s(x_s)$. The derivative $z_s'(x_s) > 0$ would mean that an increase in labour productivity of the service sector is accompanied by an increase in the 'degree of monopoly' of the sector and $z_s'(x_s) < 0$ would mean that an increase in labour productivity of the service sector is accompanied by a decrease in the 'degree of monopoly' of the sector. In order to consider the implication for either $z_s'(x_s) > 0$ or $z_s'(x_s) < 0$ in our analysis we need to include the rate of profit of the service sector in its investment function. By definition, the rate of profit of the service sector is

$$r_s = ph_s\frac{X_s}{K_s} \tag{58}$$

where h_s is the profit share in the service sector, which, from (4), is positively related to z_s, i.e $h_s = \frac{z_s}{1+z_s}$. Let the investment function of the service sector be

$$\frac{I_s}{K_s} = \alpha_s + (\beta_s + \gamma_s ph_s)\frac{X_s}{K_s} \tag{59}$$

For simplicity, let the demand for service be given by (38). Using (6), (38), (39) and (59) to substitute for d_i and d_s in (9), the short run dynamics is now given by the following system of differential equations.

$$\begin{aligned}\dot{X}_i &= \psi_i[-(1-c-\beta_i)X_i + (cp+\beta_s+\gamma_s ph_s)X_s + \alpha_i K_i + \alpha_s K_s]\\ \dot{X}_s &= \psi_s[\frac{\theta}{p}X_i - X_s]\end{aligned} \tag{60}$$

Let

$$X_{i5}^* = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_5} \tag{61}$$

$$X_{s5}^* = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_5} \tag{62}$$

where $\Omega_5 = 1 - c - \beta_i - \theta c - \frac{\theta \beta_s}{p} - \gamma_s h_s$.

Proposition 16. *If $\Omega_5 > 0$ then (X_{i5}^*, X_{s5}^*) is a unique and asymptotically stable short run equilibrium of (60).*

Proof. Similar to the proof of Proposition 1, see appendix A. □

Substituting for (X_{i5}^*, X_{s5}^*) from (61) and (62) in (6) and (59) yields the growth rate of capital stocks of both sectors in the long run as the following.

$$g_i = \alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_5} \tag{63}$$

$$g_s = \alpha_s + \frac{(\beta_s + \gamma_s p h_s)\theta(\alpha_i k + \alpha_s)}{p\Omega_5} \tag{64}$$

The long run dynamics is now obtained by substituting for g_i and g_s from (63) and (64) respectively in (16) as the following differential equation.

$$\dot{k} = k[\alpha_i + \frac{\beta_i(\alpha_i + \frac{\alpha_s}{k})}{\Omega_5} - \alpha_s - \frac{(\beta_s + \gamma_s p h_s)\theta(\alpha_i k + \alpha_s)}{p\Omega_5}] \tag{65}$$

for all $k > 0$. Let

$$k_5^* = \frac{-b_5 - \sqrt{b_5^2 - 4a_5 c_5}}{2a_5} \tag{66}$$

where $a_5 = -\alpha_i \theta(\beta_s + \gamma_s p h_s)$, $b_5 = (\alpha_i - \alpha_s)p\Omega_5 + \alpha_i \beta_i p - \alpha_s \theta(\beta_s + \gamma_s p h_s)$ and $c_5 = \alpha_s \beta_i p$.

Proposition 17. *Given $\Omega_5 > 0$, k_5^* is a unique and asymptotically stable steady state of (65) in $\mathbb{R}_{++}$.*

Proof. Similar to the proof of Proposition 2, see appendix A. □

A rise in labour productivity of the service sector affects the steady state of this model in a much more complicated manner. First of all, with the price mark-up in service sector z_s being a function of x_s, an increase in x_s no longer necessarily decreases p. Since $z_s = z_s(x_s)$, from (5) we obtain

$$\frac{\partial p}{\partial x_s} = p(\frac{z_s'(x_s)}{1+z_s} - \frac{1}{x_s}) \tag{67}$$

From (67), if a rise in labour productivity of the service sector is associated with a sufficiently large increase in the degree of monopoly of the sector then the relative price of the service increases instead of decreasing. Second, a rise in x_s effects the multiplier for industrial output $\frac{1}{\Omega_5}$ not only by changing the relative price of service but also through a change in profit share of the service sector. The net effect can be ambiguous. To see this, note that the derivative of Ω_5 with respect to x_s is,

$$\frac{\partial \Omega_5}{\partial x_s} = \frac{\beta_s \theta}{p^2}\frac{\partial p}{\partial x_s} - \gamma_s \frac{\partial h_s}{\partial x_s} \tag{68}$$

And finally, the third source of complication is that increase in x_s now has another potentially ambiguous effect on investment of the service sector through the term ph_s in (59) in addition to its effect on the same through the short-run equilibrium service output. We end this section with Proposition 18 which provides a sufficient condition for an increase in x_s to increase the steady state growth rate when $z_s'(x_s) > 0$.

Proposition 18. *Let* $g_5^* = \alpha_i + \frac{\beta_i\{\alpha_i + (\alpha_s/k_5^*)\}}{\Omega_5} = \alpha_s + \frac{(\beta_s+\gamma_s ph_s)\theta(\alpha_i k_5^* + \alpha_s)}{p\Omega_5}$ *where* k_5^* *is given by* (66). *Then* $\frac{\partial g_5^*}{\partial x_s} > 0$ *if* $\frac{z_s}{x_s} < z'(x_s) < \frac{1+z_s}{x_s}$.

Proof. Suppose $\frac{z_s}{x_s} < z'(x_s) < \frac{1+z_s}{x_s}$. From $g_5^* = \alpha_s + \frac{(\beta_s+\gamma_s ph_s)\theta(\alpha_i k_5^* + \alpha_s)}{p\Omega_5}$,

$$\begin{aligned}\frac{\partial g_5^*}{\partial x_s} = {} & [p\Omega_5\{\gamma_s\theta(\alpha_i k_5^* + \alpha_s)(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) + (\beta_s + \gamma_s ph_s)\theta\alpha_i\frac{\partial k_5^*}{\partial x_s}\} \\ & - (\beta_s + \gamma_s ph_s)\theta(\alpha_i k_5^* + \alpha_s)(\Omega_5\frac{\partial p}{\partial x_s} + p\frac{\partial \Omega_5}{\partial x_s})] \times \frac{1}{p^2\Omega_5^2}\end{aligned} \tag{69}$$

Now $\frac{z_s}{x_s} < z'(x_s)$ implies $z_s'(x_s) > 0$ as z_s and x_s are positive. Therefore,

$\frac{\partial h_s}{\partial x_s} = \frac{z_s'(x_s)}{(1+z_s)} > 0$. Also from (67), $z'(x_s) < \frac{1+z_s}{x_s}$ implies $\frac{\partial p}{\partial x_s} < 0$ as $p > 0$. Further, from (67) and $h_s = \frac{z_s}{1+z_s}$, $(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) = \frac{p}{1+z_s} \times (z_s'(x_s) - \frac{z_s}{x_s})$. Thus $\frac{z_s}{x_s} < z'(x_s)$ implies $(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) > 0$. Next, from (68) note that, $\frac{\partial p}{\partial x_s} < 0$ and $\frac{\partial h_s}{\partial x_s} > 0$ imply $\frac{\partial \Omega_5}{\partial x_s} < 0$ as β_s, θ and γ_s are positive. Then, from (69), $(h_s\frac{\partial p}{\partial x_s} + p\frac{\partial h_s}{\partial x_s}) > 0$, $\frac{\partial p}{\partial x_s} < 0$ and $\frac{\partial \Omega_5}{\partial x_s} < 0$ imply $\frac{\partial g_5^*}{\partial x_s} > 0$ if $\frac{\partial k_5^*}{\partial x_s} \geq 0$. Finally, in order to complete the proof, we need to show $\frac{\partial g_5^*}{\partial x_s} > 0$ when $\frac{\partial k_5^*}{\partial x_s} < 0$. Suppose, on the contrary, $\frac{\partial k_5^*}{\partial x_s} < 0$ and $\frac{\partial g_5^*}{\partial x_s} \leq 0$ at some arbitrary value of $x_s = x_{s1}$. Let k_{51}^*, p_1, h_{s1} and $\frac{1}{\Omega_5(x_{s1})}$ be the steady state of (65), the relative price of the service, profit share in service sector and the multiplier respectively when labour productivity of the service is x_{s1}. Also, let $g_{51}^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{51}^*)\}}{\Omega_5(x_{s1})} = \alpha_s + \frac{(\beta_s+\gamma_s p_1 h_{s1})\theta(\alpha_i k_{51}^*+\alpha_s)}{p_1\Omega_5(x_{s_1})}$. Since $\frac{\partial k_5^*}{\partial x_s} < 0$ and $\frac{\partial g_5^*}{\partial x_s} \leq 0$ at $x_s = x_{s1}$, there exist a $x_{s2} > x_{s1}$ such that $k_{51}^* > k_{52}^*$ and $g_{51}^* \geq g_{52}^*$, where k_{52}^* is the the steady state of (65) when $x_s = x_{s2}$, $g_{52}^* = \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{52}^*)\}}{\Omega_5(x_{s2})} = \alpha_s + \frac{(\beta_s+\gamma_s p_2 h_{s2})\theta(\alpha_i k_{52}^*+\alpha_s)}{p_2\Omega_5(x_{s_2})}$ with p_2 the relative price of the service, h_{s2} profit share in service sector and $\frac{1}{\Omega_5(x_{s2})}$ the multiplier respectively when $x_s = x_{s2}$. Now, $g_{51}^* \geq g_{52}^*$ implies $\alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{51}^*)\}}{\Omega_5(x_{s1})} \geq \alpha_i + \frac{\beta_i\{\alpha_i+(\alpha_s/k_{52}^*)\}}{\Omega_5(x_{s2})}$. This implies $\Omega_5(x_{s1}) < \Omega_5(x_{s2})$ because $k_{51}^* > k_{52}^*$ and $\alpha_s > 0$. However this is a contradiction because $\frac{\partial \Omega_5}{\partial x_s} < 0$ and $x_{s1} < x_{s2}$ imply $\Omega_5(x_{s1}) > \Omega_5(x_{s2})$. $\square$

5. Conclusion

We sum up this paper with the following comments. First, the result of Baumol (1967) that the inherent technologically stagnant nature of the service sector implies that expansion of the service sector inevitably leads to stagnation in the economy is no longer apt considering the widespread application of modern ICTs in various services. Further, the negative relationship between growth and the expansion of service sector in Baumol (1967) is driven largely by the assumption of full employment of resources, as pointed out by Dutt (1992). Second, a *ceteris paribus* increase in labour productivity of industry increases both the relative size of the industrial sector and the growth rate of the economy in the demand-constrained two-sector model of Dutt (1992). On the other hand, a *ceteris paribus* increase in the labour productivity of the service sector increases the relative size of the service sector but decreases the growth rate of the economy. Nonetheless the implications of sector specific technology shocks in models such as that of Dutt (1992) are sensitive to specification of demand interlinkages between the two sectors and the form of the demand function for industrial products. In a model similar to the demand constrained model of Dutt (1992) it can be shown that if demand for services per unit of industrial output increases with a fall in the relative price of services, then a *ceteris paribus* increase in the labour productivity of the service sector can increase both the growth rate of the economy and the relative size of the service sector. It can be argued that in modern times not only have many services have been extremely receptive towards adoption of ICTs but more and more of such services are being jointly purchased along with various industrial goods. So, if demand for services is sufficiently elastic with respect to its relative price then improvements in labour productivity of the service sector can provide sufficient boost to demand for both sectors and increase the growth rate of economy. In this model the growth rate of the economy can increase even if application of ICTs to the service sector not only increases the labour productivity of the sector but also its 'degree of monopoly'.

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Appendix A

Proof of Proposition 4. Suppose $\Omega_1 > 0$. Setting the right hand sides of the two equations in (21) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(\frac{sz_i}{1+z_i} - \beta_i) & (\frac{sp}{1+z_s} + \beta_s) \\ \lambda_1 & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \tag{70}$$

Now, $\Omega_1 = \frac{sz_i}{1+z_i} - \beta_i - \lambda_1(\frac{sp}{1+z_s} + \beta_s)$ is the determinant of 2×2 matrix in (70). Since $\Omega_1 \neq 0$, *Cramer's rule* yields the unique solution of (70) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_1} = X_{i1}^*$ and $X_s = \frac{\lambda_1(\alpha_i K_i + \alpha_s K_s)}{\Omega_1} = X_{s1}^*$. $\Omega_1 > 0$ implies $X_{i1}^* > 0$ and $X_{s1}^* > 0$ as α_i, α_s, K_i, K_s and λ_1 are all positive. For stability, notice that the Jacobian matrix for (21) is

$$\begin{bmatrix} -\psi_i(\frac{sz_i}{1+z_i} - \beta_i) & \psi_i(\frac{sp}{1+z_s} + \beta_s) \\ \lambda_1 & -\psi_s \end{bmatrix}$$

with determinant $\psi_i \psi_s \Omega_1 > 0$ and trace $-\psi_i \Omega_1 + \lambda_1(\frac{sp}{1+z_s} + \beta_s) - \psi_s < 0$ when $\Omega_1 > 0$, as ψ_i, ψ_s, λ_1, s, p, z_s and β_s are all positive. □

Proof of Proposition 5. We can re-arrange (26) as $\dot{k} = \dfrac{a_1 k^2 + b_1 k + c_1}{\Omega_1}$ where $a_1 = -\alpha_i \beta_s \lambda_1$, $b_1 = (\alpha_i - \alpha_s)\Omega_1 + \alpha_i \beta_i + \lambda_1 \alpha_s \beta_s$ and $c_1 = \alpha_s \beta_i$. In the steady state $a_1 k^2 + b_1 k + c_1 = 0$ as $\Omega_1 > 0$. Now $a_1 < 0$ and $c_1 > 0$ as α_i, α_s, β_i, β_s and λ_1 are all positive. $a_1 < 0$ and $c_1 > 0$ imply $b_1^2 - 4a_1 c_1 > 0$. Thus $a_1 k^2 + b_1 k + c_1 = 0$ has two distinct real roots, $\frac{-b_1 \pm \sqrt{b_1^2 - 4a_1 c_1}}{2a_1}$. Since $a_1 < 0$, the steady state value of k is $k_1^* = \frac{-b_1 - \sqrt{b_1^2 - 4a_1 c_1}}{2a_1}$. For stability, define a function $V_1 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_1(k) = (g_i - g_s)^2$, where g_i and g_s are given by (24) and (25) respectively. Note that, by definition, $V_1(k^*) = 0$ and $V_1(k \neq k^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, using (16), (24) and (25), $\dot{V}_1 = -2k(g_i - g_s)^2(\frac{\beta_1 \alpha_s}{\Omega_1 k^2} + \frac{\beta_s \lambda_1 \alpha_i}{\Omega_1}) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k_1^*$ as α_i, α_s, β_i, β_s, λ_1 and Ω_1 are all positive. Thus, V_1 is a *strict Liapunov function* for k_1^*. □

Proof of Proposition 7. Suppose $\Omega_2 > 0$. Setting the right hand sides of the two equations in (31) equal to zero yields the following system of linear equations.

$$\begin{bmatrix} -(1-c-\beta_i) & \beta_s \\ 0 & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ -\lambda K_i \end{bmatrix} \tag{71}$$

$\Omega_2 = 1 - c - \beta_i$ is the determinant of 2×2 matrix in (71). Since $\Omega_2 \neq 0$, *Cramer's rule* yields the unique solution of (71) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s + \beta_s \lambda K_i}{\Omega_2} = X_{i2}^*$ and $X_s = \lambda K_i = X_{s2}^*$. $\Omega_2 > 0$ implies $X_{i2}^* > 0$ and $X_{s2}^* > 0$ as α_i, α_s, K_i, K_s and λ are all positive. For stability, notice that the Jacobian matrix for (31) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & \beta_s \\ 0 & -\psi_s \end{bmatrix}$$

with determinant $\psi_i \psi_s \Omega_2 > 0$ and trace $-\psi_i \Omega_2 - \psi_s < 0$ when $\Omega_2 > 0$, as ψ_i and ψ_s are both positive. □

Proof of Proposition 8. We can re-arrange (36) as $\dot{k} = \dfrac{a_2 k^2 + b_2 k + c_2}{\Omega_2}$ where $a_2 = -\beta_s \lambda$, $b_2 = \{(\alpha_i - \alpha_s)\Omega_2 + \alpha_i \beta_i + \beta_i \beta_s \lambda\}$ and $c_2 = \alpha_s \beta_i$. In the steady state $a_2 k^2 + b_2 k + c_2 = 0$ as $\Omega_2 > 0$. Now $a_2 < 0$ and $c_2 > 0$ as α_s, β_i, β_s an λ are all positive. $a_2 < 0$ and $c_2 > 0$ imply $b_2^2 - 4a_2 c_2 > 0$. This means that $a_2 k^2 + b_2 k + c_2 = 0$ has two distinct real roots, $\frac{-b_2 \pm \sqrt{b_2^2 - 4a_2 c_2}}{2a_2}$. Since $a_2 < 0$, the steady state value of k is $k_2^* = \frac{-b_2 - \sqrt{b_2^2 - 4a_2 c_2}}{2a_2}$. For stability, define a function $V_2 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_2(k) = (g_i - g_s)^2$, where g_i and g_s are given by (34) and (35) respectively. Note that, by definition, $V_2(k_2^*) = 0$ and $V_2(k \neq k_2^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, using (16), (34) and (35), $\dot{V}_2 = -2k(g_i - g_s)^2 (\frac{\beta_i \alpha_s}{\Omega_2 k^2} + \beta_s \lambda) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k_2^*$ as α_s, β_i, β_s, λ and Ω_2 are all positive. Thus, V_2 is a *strict Liapunov function* for k_2^*. □

Proof of Proposition 10. Suppose $\Omega_3 > 0$. Setting the right hand sides of the two equations in (40) equal to zero yields the following system of linear

equations.

$$\begin{bmatrix} -(1-c-\beta_i) & (cp+\beta_s) \\ \frac{\theta}{p} & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \tag{72}$$

$\Omega_3 = 1 - c(1+\theta) - \beta_i - \frac{\theta\beta_s}{p}$ is the determinant of 2×2 matrix in (72). Since $\Omega_3 \neq 0$, *Cramer's rule* yields the unique solution of (72) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_3} = X^*_{i3}$ and $X_s = \frac{\theta(\alpha_i K_i + \alpha_s K_s)}{p\Omega_3} = X^*_{s3}$. $\Omega_3 > 0$ implies $X^*_{i3} > 0$ and $X^*_{s3} > 0$ as α_i, α_s, K_i, K_s, p, and θ are all positive. For stability, notice that the Jacobian matrix for (40) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & (cp+\beta_s) \\ \frac{\theta}{p} & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_3 > 0$ and trace $-\psi_i(\Omega_3 + \theta c + \frac{\theta\beta_s}{p}) - \psi_s < 0$ when $\Omega_3 > 0$, as ψ_i, ψ_s, θ, c, p and β_s are all positive. □

Proof of Proposition 11. We can re-arrange (45) as $\dot{k} = \frac{a_3k^2+b_3k+c_3}{p\Omega_3}$ where $a_3 = -\alpha_i\beta_s\theta$, $b_3 = (\alpha_i - \alpha_s)p\Omega_3 + \alpha_i\beta_i p - \alpha_s\beta_s\theta$ and $c_3 = \alpha_s\beta_i p$. In the steady state $a_2k^2 + b_2k + c_2 = 0$ as $p > 0$ and $\Omega_3 > 0$. Now, $a_3 < 0$ and $c_3 > 0$ as α_i, α_s, β_i, β_s, θ and p are all positive. This means that $a_3k^2 + b_3k + c_3 = 0$ has two distinct real roots since $a_3 < 0$ and $c_3 > 0$ imply $b_3^2 - 4a_3c_3 > 0$. These are $\frac{-b_3 \pm \sqrt{b_3^2 - 4a_3c_3}}{2a_3}$. Since $a_3 < 0$, the steady state value of k is $k^*_3 = \frac{-b_3 - \sqrt{b_3^2 - 4a_3c_3}}{2a_3}$. For stability, define a function $V_3 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_3(k) = (g_i - g_s)^2$, where g_i and g_s are given by (43) and (44) respectively. Note that, by definition, $V_3(k^*_3) = 0$ and $V_3(k \neq k^*_3) > 0$ for all $k \in \mathbb{R}_{++}$. Also, from (16),(43) and (44), $\dot{V}_3 = -2k(g_i - g_s)^2(\frac{\beta_i\alpha_s}{\Omega_3 k^2} + \frac{\beta_s\theta\alpha_i}{p\Omega_3}) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k^*_3$ as α_i, α_s, β_i, β_s, θ, p and Ω_3 are all positive. Thus, V_3 is a *strict Liapunov function* for k^*_3. □

Proof of Proposition 13. Suppose $\Omega_4 > 0$. Setting the right hand sides of the two equations in (50) equal to zero yields the following system of linear

equations.

$$\begin{bmatrix} -(1-c-\beta_i) & (cp+\beta_s) \\ \theta(p) & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \tag{73}$$

$\Omega_4 = 1-c-\beta_i-\theta(p)(cp+\beta_s)$ is the determinant of 2×2 matrix in (73). Since $\Omega_4 \neq 0$, *Cramer's rule* yields the unique solution of (73) as $X_i = \frac{\alpha_i K_i + \alpha_s K_s}{\Omega_4} = X_{i4}^*$ and $X_s = \frac{\theta(p)(\alpha_i K_i + \alpha_s K_s)}{\Omega_4} = X_{s4}^*$. $\Omega_4 > 0$ implies $X_{i3}^* > 0$ and $X_{s3}^* > 0$ as α_i, α_s, K_i, K_s and $\theta(p)$ are all positive. For stability, notice that the Jacobian matrix for (50) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & (cp+\beta_s) \\ \theta(p) & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_4 > 0$ and trace $-\psi_i(\Omega_4 + \theta(p)(cp+\beta_s) - \psi_s < 0$ when $\Omega_4 > 0$, as ψ_i, ψ_s, $\theta(p)$, c, p and β_s are all positive. $\square$

Proof of Proposition 14. We can re-arrange (55) as $\dot{k} = \frac{a_4k^2+b_4k+c_4}{\Omega_4}$ where $a_4 = -\alpha_i\beta_s\theta(p)$, $b_4 = (\alpha_i - \alpha_s)\Omega_4 + \alpha_i\beta_i - \alpha_s\beta_s\theta(p)$ and $c_4 = \alpha_s\beta_i$. In the steady state $a_4k^2 + b_4k + c_4 = 0$ as $\Omega_4 > 0$. Now, $a_4 < 0$ and $c_4 > 0$ as α_i, α_s, β_i, β_s and $\theta(p)$ are all positive. This means that $a_4k^2 + b_4k + c_4 = 0$ has two distinct real roots since $a_4 < 0$ and $c_4 > 0$ imply $b_4^2 - 4a_4c_4 > 0$. These are $\frac{-b_4 \pm \sqrt{b_4^2 - 4a_4c_4}}{2a_4}$. Since $a_4 < 0$, the steady state value of k is $k_4^* = \frac{-b_4 - \sqrt{b_4^2-4a_4c_4}}{2a_4}$. For stability, define a function $V_4 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_4(k) = (g_i - g_s)^2$, where g_i and g_s are given by (53) and (54) respectively. Note that, by definition, $V_4(k_4^*) = 0$ and $V_4(k \neq k_4^*) > 0$ for all $k \in \mathbb{R}_{++}$. Also, from (16), (53) and (54), we have $\dot{V}_4 = -2k(g_i - g_s)^2(\frac{\beta_i\alpha_s}{\Omega_4 k^2} + \frac{\beta_s\theta(p)\alpha_i}{\Omega_4}) < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq k_4^*$ as α_i, α_s, β_i, β_s and $\theta(p)$ are all positive. Thus, V_4 is a *strict Liapunov function* for k_4^*. $\square$

Proof of Proposition 16. Suppose $\Omega_5 > 0$. Setting the right hand sides of the two equations in (60) equal to zero yields the following system of linear

equations.

$$\begin{bmatrix} -(1-c-\beta_i) & (cp+\beta_s+\gamma_s p h_s) \\ \frac{\theta}{p} & -1 \end{bmatrix} \begin{bmatrix} X_i \\ X_s \end{bmatrix} = \begin{bmatrix} -(\alpha_i K_i + \alpha_s K_s) \\ 0 \end{bmatrix} \quad (74)$$

$\Omega_5 = 1-c-\beta_i-\theta c-\frac{\theta\beta_s}{p}-\gamma_s h_s$ is the determinant of 2×2 matrix in (74). Since $\Omega_5 \neq 0$, *Cramer's rule* yields the unique solution of (74) as $X_i = \frac{\alpha_i K_i+\alpha_s K_s}{\Omega_5} = X^*_{i5}$ and $X_s = \frac{\theta(\alpha_i K_i+\alpha_s K_s)}{p\Omega_5} = X^*_{s5}$. $\Omega_5 > 0$ implies $X^*_{i5} > 0$ and $X^*_{s5} > 0$ asα_i, α_s, K_i, K_s, p, and θ are all positive. For stability, notice that the Jacobian matrix for (60) is

$$\begin{bmatrix} -\psi_i(1-c-\beta_i) & (cp+\beta_s) \\ \theta(p) & -\psi_s \end{bmatrix}$$

with determinant $\psi_i\psi_s\Omega_5 > 0$ and trace $-\psi_i(\Omega_4+\theta c+\frac{\theta\beta_s}{p}+\gamma_s h_s - \psi_s < 0$ when $\Omega_4 > 0$, as ψ_i, ψ_s, θ, c, p, β_s, h_s and γ_s are all positive. □

Proof of Proposition 17. We can re-arrange (65) as $\dot{k} = \frac{a_5k^2+b_5k+c_5}{p\Omega_5}$ where $a_5 = -\alpha_i\theta(\beta_s+\gamma_s p h_s)$, $b_5 = (\alpha_i-\alpha_s)p\Omega_5+\alpha_i\beta_i p-\alpha_s\theta(\beta_s+\gamma_s p h_s)$ and $c_5 = \alpha_s\beta_i p$. Since $p > 0$ and Ω_5, $a_5k^2+b_5k+c_4 = 0$ in the steady state. Now, since α_i, α_s, β_i, β_s, θ, γ_s, p and h_s are all positive, $a_5 < 0$ and c_4, which imply $b_5^2-4a_5c_5 > 0$. Thus, $a_5k^2+b_5k+c_5 = 0$ has two distinct real roots, $\frac{-b_5\pm\sqrt{b_5^2-4a_5c_5}}{2a_5}$. Since $a_5 < 0$, the steady state value of k is $k^*_5 = \frac{-b_5-\sqrt{b_5^2-4a_5c_5}}{2a_5}$. For stability, define a function $V_5 : \mathbb{R}_{++} \mapsto \mathbb{R}$ such that $V_5(k) = (g_i-g_s)^2$, where g_i and g_s are given by (63) and (64) respectively. Note that, by definition, $V_5(k^*_5) = 0$ and $V_5(k \neq k^*_5) > 0$ for all $k \in \mathbb{R}_{++}$. Also, from 16, (63) and (64), we have $\dot{V}_5 = -2k(g_i-g_s)^2\{\frac{\beta_i\alpha_s}{\Omega_5 k^2}+\frac{(\beta_s+\gamma_s p h_s)\theta\alpha_i}{p\Omega_5}\} < 0$ for all $k \in \mathbb{R}_{++}$ and $k \neq K^*_5$ as α_i, α_s, β_i, β_s, γ_s, p, h_s, θ and Ω_5 are all positive. Thus, V_5 is a *strict Liapunov function* for k^*_5. □