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Accounting for Needs in Cost Sharing[*]

Étienne Billette de Villemeur
Université de Lille and Chaires Universitaires Toussaint Louverture

Justin Leroux
HEC Montréal, CIRANO and CRÉ

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Abstract

We introduce basic needs in cost-sharing problems so that agents with higher needs are not penalized, all the while holding them responsible for their consumption. We characterize axiomatically two families of cost-sharing rules, each favoring one aspect—compensation or responsibility—over the other. We also identify specific variants of those rules that protect small users from the cost externality imposed by larger users. Lastly, we show how one can implement these schemes with realistic informational assumptions; i.e., without making explicit interpersonal comparisons of needs and consumption.

Keywords: Cost Sharing; Needs; Responsibility; Liberal Egalitarianism.
JEL code: D63.

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1 Introduction

Most allocation mechanisms rely only on demand, that is to say on what biologists would call *wants*. We argue in favor of introducing objective characteristics to complement this subjective input. Specifically, we study pricing formulas that do not penalize agents with higher *needs*.

Some public utilities, like water and wastewater services, are essential to achieving a decent standard of living. In a society where households differ in terms of their basic needs for utility services, these should be taken into account when setting utility rates. In practice, commendable efforts have been made in this regard, with rate schedules typically taking the form of multi-part tariffs (block pricing), including discounts given to households with higher needs (for the case of water supply in the US, see AWWA, 2012). These discounts can take the form of a rebate to low-income households, which is subsidized by a higher overall rate structure. Alternatively, increasing-block rate schedules subsidize the lowest block through rate premiums for large users, hence affording all households a low rate to meet basic needs. In the case of water services, this also addresses the issue of resource conservation.[1] Nevertheless, while these practices recognize the fact that some households should be subsidized, the design of such subsidies, both in shape and in magnitude, is largely left to rule-of-thumb considerations.[2]

Our aim is to design a pricing scheme that does not penalize agents for having higher needs. As we shall see below, social transfers to finance the basic needs of the poor actually cannot achieve this. This is because what will remain to be paid will necessarily depend on their needs. Alternatively, adjusting prices to reflect differences in income may be unfeasible, either because it is too informationally costly or simply illegal. In fact, our objective is not quite the same as making essential consumption 'affordable'; we merely require that agents with higher needs are not at a disadvantage in their ability to achieve a given welfare level.

We develop a framework to formally take matters of partial responsibility into account when devising rates for utility services, which we will assume to be water

[1]The recent move towards "water budget-based rates" or, more accurately, to "sustainable" rate design in some U.S. municipalities reflects these concerns (Barr and Ash, 2015; Barraqué and Montginoul, 2015; Dinar and Ash, 2015)

[2]For example, the M1 Manual of the American Water Works Association, a highly regarded reference by North American water utilities, gives surprisingly little guidance on how to determine rate blocks: "Generally, rate blocks should be set at logical break points." (AWWA, 2012, p.107)

services to fix ideas.[3] Each agent is summarized by her water consumption and her basic water needs, which may differ from one agent to the next. For instance, one can think of agents as being households of possibly different sizes. We take the view that agents should not be penalized for their needs, but are fully responsible for their consumption beyond those needs.

Our approach builds on the axiomatic framework of liberal egalitarianism, which aims at compensating differences in "non-responsibility" characteristics while rewarding differences in characteristics under the agents' control. Classically, agents are deemed responsible for their effort but have no control over their talents. Here, agents have no control over their basic water needs—say, 50 liters of clean water per day (Gleick, 1996)—but are responsible for their consumption beyond that amount. Thus, water consumption is a 'hybrid' characteristic of sorts: the portion required to meet basic needs falls into the non-responsibility category, whereas the remainder falls into the sphere of responsibility.

A general theme of that literature is that the two desiderata of compensation and reward are incompatible (Bossert, 1995; Bossert and Fleurbaey, 1996; Cappelen and Tungodden, 2006). Accordingly, one must set less ambitious goals for redistributive policies. This is typically done by giving priority to one ideal—compensation or reward—while limiting the scope of the other (Fleurbaey 2008, and references therein), leading to the *Egalitarian Equivalent* and *Conditional Equality* solutions, respectively. Likewise, we characterize two polar families of solutions: *Conditional Equality* solutions emphasize responsibility for excessive usage (Theorem 1) while *Egalitarian Equivalent* solutions stress compensation for differences in needs (Theorem 4).

Contrasting with previous results, the solutions we obtain are not unique because they depend on two additional dimensions that the literature is currently not equipped to handle: how to account for 'hybrid' characteristics and how to account for cost externalities. The latter is embodied by the nonlinearity of the cost function, which links the agents through the requirement of balancing the budget. Regarding the former, each family of solutions will produce different solutions whether one measures responsibility in terms of consumption (q) beyond needs ($\bar{q}$), formally $q-\bar{q}$, or in terms of its fraction relative to one's own needs, $(q-\bar{q})/\bar{q}$, for example. We call these views *absolute responsibility* and *relative responsibility*, respectively. When welfare can be

[3] Our analysis applies to all utilities necessary for a decent standard of living, including electricity services.

evaluated by means of a (common) utility function—i.e., when agents differ only in their needs—and when the responsibility measure is chosen so as to reflect the actual welfare of the agents—a more sophisticated exercise—Conditional Equality solutions are actually compatible with a much stronger compensation requirement than when responsibility is computed arbitrarily (Theorem 3). This implies that, when differences in needs summarize the relevant differences across agents, sufficient knowledge of the utility function can afford greater compatibility between the desiderata of compensation and reward, a sharp contrast with existing results in the literature on liberal egalitarianism.

Even with a specific view on responsibility, much freedom remains regarding how to account for cost externalities within each family of solutions. Indeed, the partial responsibility approach determines what portion of the total cost is devoted to meeting basic needs. How to split the remainder—for which agents are deemed responsible—falls into the realm of cost-sharing theory. In principle, any cost-sharing rule can be associated with each family of solutions and with each responsibility view. However, given the nature of the service at hand, when costs are convex we posit an axiom that protects parsimonious users from the cost externality caused by wasteful users. When costs are concave, so that there are economies of scale, we ask 'small users' to fully benefit from a further reduction in their consumption. This characterizes unique solutions: the *serial* (Moulin and Shenker, 1992) and *decreasing-serial* (de Frutos, 1998) cost-sharing variant of each family of solutions, respectively when costs are convex or concave (Propositions 2-5).

Lastly, we show how one can implement the above schemes with realistic informational assumptions; i.e., without making explicit interpersonal comparisons of needs and consumption, which would prove very difficult and possibly counterproductive for all but very small populations. In particular, we use household size as a proxy for needs and denote by $\bar{q}_s$ the needs of a household of size s. Using aggregate information to summarize distributional aspects, we design rate schedules that otherwise explicitly depend on the sole individual characteristics of households.

For instance, consider affine costs of the form $C(Q) = F + cQ$, with $F, c > 0$, where Q is the aggregate demand of the population.[4] When responsibility is measured by

[4] Such a cost structure is typical of water services, which exhibit high fixed costs (infrastructure) and low marginal costs (electricity for pumping and chemicals for treatment).

absolute responsibility, $q - \bar{q}_s$, the *decreasing serial conditional equality* solution[5] yields the following rate schedule for households of size s:

$$\frac{F + c\bar{Q}}{N} + c\left(q - \bar{q}_s\right), \tag{1}$$

where $\bar{Q}$ is the quantity needed to cover the needs of the entire population, and N is the total number of households. In addition to splitting the fixed cost equally, this rate schedule shares the cost of the population's needs equally before pricing consumption at marginal cost (minus a rebate equal to the cost of meeting one's own needs).

The rate schedule changes significantly under the relative responsibility view. Assuming responsibility is identically distributed across types, we obtain the following rate schedule for households of size s:

$$\frac{F}{N} + \frac{c}{\bar{q}_s / \left(\bar{Q}/N\right)} q \tag{2}$$

The result is still a two-part tariff but one where only the fixed cost is split equally. No rebate is granted, and consumption is priced at a rate that is inversely proportional to one's needs.

By contrast, the family of egalitarian equivalent solutions is based on utility comparisons with households having a hypothetical reference level of needs, $\bar{q}_0$, chosen by the planner. Under the *decreasing serial egalitarian equivalent* solution, which emphasizes compensating differences in needs over responsibility, the rate schedule for households of size s is as follows:

$$\frac{F}{N} + cq + \left[u_s\left(q, \bar{q}_s\right) - u_s\left(q, \bar{q}_0\right)\right] - \frac{1}{N} \sum_t \int_{z=0}^{\infty} \left[u_t\left(z, \bar{q}_t\right) - u_t\left(z, \bar{q}_0\right)\right] n_t\left(z\right) dz,$$

where $u_s\left(q, \bar{q}_s\right)$ is the utility of a representative household of size s and where $n_s\left(q\right)$ is the density of households that are consuming q units in the distribution of size-s households. The cost-sharing portion of the schedule, $\frac{F}{N} + cq$, splits the fixed cost equally and prices consumption at marginal cost. Needs are completely absent from that component. However, they enter in the remaining, redistributive portion to

[5] As mentioned, the decreasing serial cost-sharing rule is the more appropriate for concave costs.

ensure that heterogeneity in needs does not drive differences in welfare.

The remainder is organized as follows. The next section offers a brief discussion of the related literature. Section 3 presents the formal model. In Section 4, we take the cost-sharing rule as given in order to focus on our contribution; namely, the introduction of essential needs in cost-sharing problems. We then introduce a specific property of the rate function, which aims at protecting small users while still holding them accountable, and show how doing so calls for adopting a specific underlying cost-sharing rule depending on the convexity of the externality (Section 5). Finally, we show in Section 6 how these abstract formulas actually boil down to specific two-part tariffs for which we provide an explicit and complete determination using only coarse information on characteristics of the population.

2 Related Literature

Liberal egalitarianism. Our work expands the literature on liberal egalitarianism in two ways. First, we extend the theory to settings with externalities. To our knowledge, the only other effort in this direction is Billette de Villemeur and Leroux (2011), which tackles the issue of global climate change and the design of transfer schemes between countries to account for their responsibility in current emissions and, possibly, their non-responsibility in past emissions. Externalities are introduced through a (nonlinear) damage function, but basic needs are absent from their setting.

Our second contribution has to do with our consideration of a characteristic—here, consumption—for which one is both partly responsible and partly non-responsible. Ooghe and Peichl (2014) and Ooghe (2015) very recently introduced the notion of 'partial control' over some characteristics to handle different degrees of responsibility in any given characteristic. According to this 'soft cut', an agent may be responsible for, say, only 30% of his intellectual skills, the remainder being attributable to inborn abilities or environmental factors. Our view of consumption as a hybrid characteristic differs from theirs in that we deem households fully non-responsible for the portion aimed at satisfying their needs, but fully responsible for the remaining portion, viewed as discretionary.

Needs. Economists have been aware for quite some time that the welfare interpretation of income inequality measures is problematic (see among others Garvy, 1954;

David, 1959; Morgan, 1962). How to account for differences in ability and needs is still the topic of lively discussions in public economics, in particular in the literature on taxation, but not only (e.g., Mayshar and Yitshaki, 1996, Trannoy, 2003, Duclos et al. 2005, Duclos and Araar 2007). Ebert (1997) adopts an axiomatic approach to discuss the comparison of income distributions when the population consists of heterogeneous households. Observing that economic growth had done very little for the poorer half of the third world population, some economists at the World Bank have pointed out the importance of looking at basic needs (Streeten and Burki, 1978; Streeten, 1979; Hicks and Streeten, 1979). Similarly, rather than being concerned with the 'affordability' of services to low-income households, as do most approaches to rate setting, we focus on the material—as opposed to the financial—needs of households.

Fair division. Despite mounting empirical evidence suggesting that needs are a relevant ingredient of fairness (Konow, 2001; Traub et al, 2005; Schwettman, 2012), the literature on fair division has only recently considered basic needs in a formal fashion. Specifically, although in a setting different from ours, Bergantiños et al. (2012) and Manjunath (2012) modify the classical rationing problem—where a fixed social endowment must be divided among several recipients—to account for a minimal requirement. There, agents are indifferent between receiving less than this minimal share and receiving nothing.

Because we ask for full cost recovery, the relevant strand of the fair division literature is that of cost sharing. Yet, this literature does not explicitly address the issue of basic needs. The closest work in that direction lead to sharing rules that protect small users when costs are convex (Moulin and Shenker, 1992) or guarantee that small users will indeed be rewarded from reducing their consumption to the tune of their effort (de Frutos, 1998). We build upon these two sharing rules to complement our approach (Section 5).

3 Accounting for Needs

The Model. Let $N = \{1, ..., n\}$ be the set of agents. Agent i consumes a quantity $q_i \geq 0$ of water. Serving all of the agents' demands $Q = \sum_{i=1}^{n} q_i$ costs $C(Q) \geq 0$, where C is an increasing cost function.[6]

[6]We use the following convention: by 'increasing' we mean 'strictly increasing'. We use the term 'non-decreasing' when the monotonicity is not strict. Similarly, by 'positive' we mean 'strictly

Full cost recovery is essential to the sustainability of the infrastructure.[7] Thus, we require that the agents' water bills, x_i's, cover the total cost:

$$\sum_{i=1}^{n} x_i \geq C(Q). \tag{3}$$

We denote by Γ the class of cost functions. Our aim is to define appropriate formulas to compute the agents' bills. We restrict ourselves to the case where no profits are made, owing to the public nature of the service, so that the budget constraint (3) is binding.

The needs of agent i, in terms of water use, are denoted $\bar{q}_i \geq 0$. We adopt a quasi-linear setup, where agent i's utility level is defined by:

$$U_i(q_i, \bar{q}_i, x) = u_i(q_i, \bar{q}_i) - x_i.$$

The utility function u_i, which is possibly agent specific, is defined on $\mathbb{D} \equiv \left\{(x, y) \in \mathbb{R}_+^2 | x \geq y\right\}$.[8] It is assumed to be increasing in q_i and decreasing in $\bar{q}_i$. We denote by Υ the class of utility functions. When agents consume exactly their needs, they share a common utility level $\underline{u}$ that, without loss of generality, we can set to zero. Formally,

$$u_i(\bar{q}_i, \bar{q}_i) \equiv 0, \qquad \forall \bar{q}_i \geq 0, \forall i \in N.$$

Defining responsibility. Our aim is to design a pricing rule that does not penalize agents with higher needs while taking individual responsibilities into account. In order to do so, we must define the sphere of responsibility of the agents. We consider that agents are not responsible for their essential needs, $\bar{q}_i$, but are responsible for any consumption beyond those needs. The extent of their responsibility can be measured in many different ways. For the sake of generality, we define a real-valued function,

positive', and use 'nonnegative' when zero is not excluded.

[7]For example, while it remains an empirical matter whether pricing water actually leads to economic efficiency in practice, it is widely recognized that full cost recovery is essential to the sustainability of the infrastructure (Massarutto, 2007; AWWA, 2012; Canadian Water and Wastewater Association, 2015) and is "a key preoccupation" of many OECD countries (OECD, 2010). Still in the context of water services, Massarutto (2007) identifies three important benefits of recovering costs through the pricing structure: to "ensure the viability of water management systems", to "maintain asset value over time", and to "guarantee the remuneration of inputs".

[8]Because we consider $\bar{q}_i$ to represent agent i's essential needs, it is a lower bound to her consumption.

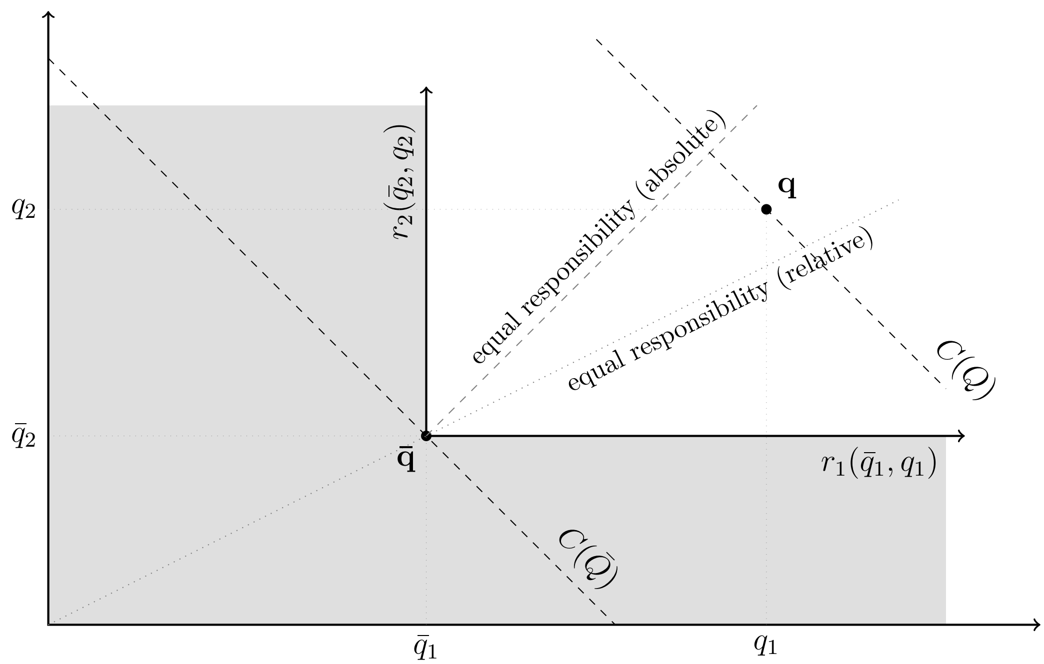

Figure 1: Responsibility is measured from $\bar{\mathbf{q}}$. Given the position of $\mathbf{q}$ relative to $\bar{\mathbf{q}}$ in this figure, if responsibility is defined as $q_i - \bar{q}_i$ (absolute responsibility) agent 1 is considered to bear more responsibility than agent 2 in her discretionary consumption. If it is defined as $(q_i - \bar{q}_i)/\bar{q}_i$ (relative responsibility), the reverse holds.

$r(q_i, \bar{q}_i)$, defined on $\mathbb{D}$, which is increasing in water consumption q_i, non-increasing in needs $\bar{q}_i$, and normalized to zero when $q_i = \bar{q}_i$. When no confusion is possible, we abuse notations slightly by denoting $r_i = r(q_i, \bar{q}_i)$. We denote by R the class of responsibility functions.

A *consumption-needs profile* (or simply *a profile*) is a list of n consumption-needs pairs that we shall denote $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$, abusing notations slightly.[9]

Rate functions and cost-sharing rules. Having defined the notion of responsibility, we can now share the total cost according to the responsibility profile, $\mathbf{r} \equiv (r_1, r_2, ..., r_n)$. In doing so, *cost-sharing rules* (ξ) will allow us to highlight the distinction between the handling of the production externality—governed by the shape of the cost function—and the redistribution problem that follows from taking essen-

[9] We shall adopt the convention that boldface type refers to the vector of the relevant variables. E.g., $\mathbf{q} = (q_1, ..., q_n)$ and $\bar{\mathbf{q}} = (\bar{q}_1, ..., \bar{q}_n)$.

tial needs into account. We ultimately provide pricing formulas, that we shall refer to as *rate functions* (x).

Formally, let $\mathcal{C}(\mathbf{q}, \bar{\mathbf{q}})$ stand for the portion of the cost for which the population is considered to be responsible, once needs are accounted for. The principles of liberal reward and compensation will guide us in defining $\mathcal{C}(\mathbf{q}, \bar{\mathbf{q}})$. A cost-sharing rule is a mapping that splits this portion of the cost across users: $\xi : \mathbb{R}^n \times \Gamma \to \mathbb{R}^n$, such that $\sum_i \xi_i(\mathbf{r}, \mathcal{C}) = \mathcal{C}(\mathbf{q}, \bar{\mathbf{q}})$. By contrast, a rate function takes all the information in the economy into account and is a mapping $x : \mathbb{D}^n \times R \times \Upsilon \times \Gamma \to \mathbb{R}^n$ such that $\sum_{i \in N} x_i(\mathbf{q}, \bar{\mathbf{q}}, r, u, C) = C(Q)$ where $C(Q)$ is the total cost to be covered.

Section 6 will be devoted to obtaining explicit formulas based on illustrative examples. Until then, fix the cost function, C, the common utility function, u, and the responsibility function, r. As a result, we abuse notations slightly and write $x(\mathbf{q}, \bar{\mathbf{q}})$ instead of the more cumbersome $x(\mathbf{q}, \bar{\mathbf{q}}, r, u, C)$.

4 Fair Treatment

4.1 Interdependence and Anonymity

A natural and seemingly minimal fairness requirement is that two agents with identical needs face the same pricing schedule:

Axiom. ***(Equal Rate Schedule for Equal Needs, ERSEN)***
The functions $q_i \mapsto x_i(\mathbf{q}, \bar{\mathbf{q}})$ *and* $q_j \mapsto x_j(\mathbf{q}, \bar{\mathbf{q}})$ *must be identical whenever* $\bar{q}_i = \bar{q}_j$.

As it turns out, however, **ERSEN** is unfeasible:

Theorem 1. *No rate function satisfies **ERSEN** unless the cost function is linear.*

Proof. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ such that $\bar{q}_i = \bar{q}_j$ for some $i \neq j$. By budget balance, the rate schedule of agent 1, $f : q_i' \mapsto x_i((q_i', \mathbf{q_{-i}}), \bar{\mathbf{q}})$, writes as follows:

$$f(q_i') - f(q_i) = C(Q - q_i + q_i') - C(Q) \qquad \forall q_i' \in [\bar{q}_i, +\infty). \tag{4}$$

By **ERSEN**, the function f cannot depend on q_j, so that :

$$f(q_i') - f(q_i) = C\left(Q - q_i + q_i' - q_j + q_j'\right) - C\left(Q - q_j + q_j'\right), \tag{5}$$

for all $\left(q_i', q_j'\right) \in [\bar{q}_i, +\infty) \times [\bar{q}_j, +\infty)$. Taken together, Expressions (4) and (5) yield:

$$C\left(Q - q_i + q_i'\right) - C\left(Q\right) = C\left(Q - q_i + q_i' - q_j + q_j'\right) - C\left(Q - q_j + q_j'\right), \qquad (6)$$

for all $\left(q_i', q_j'\right) \in [\bar{q}_i, +\infty) \times [\bar{q}_j, +\infty)$.

Already, Expression (6) suggests that C increases at a constant rate. We prove this formally by rewriting the expression as a Cauchy functional equation. Let $h > 0$ and consider $q_i' = q_i + h$ and $q_j' = q_j + h$. Expression (6) becomes:

$$C\left(Q + h\right) - C\left(Q\right) = C\left(Q + 2h\right) - C\left(Q + h\right) \qquad \forall h \geq 0. \qquad (7)$$

Rearranging and defining $g : h \mapsto C\left(Q + h\right)$ on $\mathbb{R}_+$ yields:

$$g\left(2h\right) + g\left(0\right) = 2g\left(h\right) \qquad \forall h \geq 0. \qquad (8)$$

Expression (8) must hold for all h and thus defines a functional equation in g. This is a well-known Cauchy equation (Aczél, 1967), which requires g—and therefore C—to be linear in its argument. Having started from an arbitrary profile $(\mathbf{q}, \bar{\mathbf{q}})$, linearity follows on the full domain of C. □

ERSEN effectively requires that the rate schedule an agent faces depends only on the profile of needs, but not on the consumption vector. However, this ignores the interdependence that exists between agents through the cost function. Theorem 1 makes it clear that, if this interdependence is not accounted for, rate schedules cannot be determined *ex ante*, on the sole basis of needs.[10] It follows that we must depart from the simplistic view according to which agents can ignore the impact they have on others, as is assumed to be the case under perfect competition, for instance. We therefore adopt a more comprehensive view in which bills depend explicitly on the entire profile of consumption and needs.

Moreover, just as individuals cannot be considered in isolation, essential needs cannot be handled separately from consumption beyond them. Financing the provision of essential needs ($\bar{Q}$) through, say, the income tax—and having agents pay for $Q - \bar{Q}$ through a pricing scheme that depends on the sole q_i's, ignoring the $\bar{q}_i$'s—would not solve our problem. In fact, agents' bills would be required to finance $C\left(Q\right) - C\left(\bar{Q}\right)$,

[10] For a general proof of the incompatibility between budget balance and equal treatment of equals, albeit when needs are absent, see Billette de Villemeur and Leroux (2016).

which depends explicitly on $\bar{Q}$. This is in contradiction with the fact that once essential needs have been financed, they can be ignored in pricing the remaining consumption.

Even worse, if agents' bills were to depend solely on the q_i's, an agent whose needs happened to increase but whose consumption remained unchanged would end up paying the same amount despite a lower responsibility in consumption. This can lead to situations where one agent ends up paying more than another despite having both lower responsibility and higher needs. Indeed, consider any i such that $q_i > q_j > \bar{q}_j$ for some $j \neq i$. Nevertheless, her needs can increase to some level $\bar{q}_i \in (\bar{q}_j, q_i)$ such that $r_i < r_j$ even though she ends up paying more than j (because $q_i > q_j$).

In addition, budget balance would necessarily be violated. Suppose the needs of the population happen to decrease, while again consumption remains unchanged. Then, the portion of costs that must be financed through pricing—$C(Q) - C(\bar{Q})$—increases and revenue requirements are no longer met.

We shall thus stick to our encompassing approach, which aims at financing the total cost, $C(Q)$, by accounting jointly for the q_i's and the $\bar{q}_i$'s.

The fairness requirement we shall adopt is that the rate function satisfies *anonymity*. Formally, we shall require that, for any permutation of the agents $\pi : N \to N$:

$$x_{\pi(i)}(\mathbf{q}_\pi; \bar{\mathbf{q}}_\pi) = x_i(\mathbf{q}; \bar{\mathbf{q}}) \qquad \text{for all } i \in N,$$

where $\mathbf{q}_\pi$ (resp. $\bar{\mathbf{q}}_\pi$) is the vector of consumption (resp. needs) after permutation of the agents along π.

Remark 1. Anonymity implies the equal treatment of equals: $(q_i, \bar{q}_i) = (q_j, \bar{q}_j) \implies x_i(\mathbf{q}; \bar{\mathbf{q}}) = x_j(\mathbf{q}; \bar{\mathbf{q}})$. Two users with identical needs and identical consumption must pay the same bill.

4.2 The Reward Principle: Responsibility Axioms

The general idea behind the reward principle is that conservative users should be rewarded in the form a lower bill. Of course, if needs are accounted for, whether consumption is moderate or not is not measured by considering only actual consumption, but on the basis of $r(q_i, \bar{q}_i)$.

A minimal requirement in terms of responsibility is that the portion of costs resulting from consumption above and beyond the needs of the population, $C(Q) - C(\bar{Q})$, be distributed to users according to their contribution to this cost. This leads us to introducing a cost-sharing rule, ξ, to split $C(Q) - C(\bar{Q})$ according to the responsibility profile, $\mathbf{r}$. Keeping with the desideratum of anonymity, we shall consider only *symmetric* cost-sharing rules:

$$\xi\left(\mathbf{r}, C - C(\bar{Q})\right) \text{ is a symmetric function of the variables } r_i,\ i \in N.$$

The function ξ embodies how we want to hold agents accountable for their consumption.[11] Given ξ, the following axioms specify how responsibility is assigned, and are presented in decreasing order of stringency.

Axiom. ***(Shared Responsibility, SR)***

$$x_k\left(\mathbf{q}, \bar{\mathbf{q}}\right) - x_k\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) = \xi_k(\mathbf{r}, C - C(\bar{Q})) \qquad \forall k \in N$$

A less demanding axiom consists in sharing $C(Q) - C(\bar{Q})$ according to ξ only when all agents have equal needs.

Axiom. ***(Shared Responsibility for Uniform Needs, SRUN)***

$$[\bar{q}_i = \bar{q}_j, \forall i, j \in N] \implies \left[x_k(\mathbf{q}, \bar{\mathbf{q}}) - x_k(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_k(\mathbf{r}, C - C(\bar{Q})), \forall k \in N\right]$$

Finally, an even less demanding axiom consists in sharing costs according to ξ only when the needs of all are identical and equal to a reference level, $\bar{q}_0 \in \mathbb{R}_+$.

Axiom. ***(Shared Responsibility for Reference Needs, SRRN)***
For some reference level of needs, $\bar{q}_0 \in \mathbb{R}_+$:

$$[\bar{q}_i = \bar{q}_0, \forall i \in N] \implies [x_k(\mathbf{q}, \bar{\mathbf{q}}_0) - x_k(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_k(\mathbf{r}_\mathbf{0}, C - C(n\bar{q}_0)), \forall k \in N]$$

where $\bar{\mathbf{q}}_\mathbf{0} = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0)$ and $r_{0,i} = r\left(q_i, \bar{q}_0\right)$ for all $i \in N$.

[11] If needs were not an issue, we would be back to the classical cost-sharing framework where $\xi\left(\mathbf{q}, C\right)$ alone defines the shares to be paid (see Moulin, 2002, for a thorough survey).

4.3 The Compensation Principle: No Responsibility for One's Needs

Throughout, we take the view that agents are not responsible for their needs. Ideally, difference in needs should not drive differences in welfare:

Axiom. ***(Group Solidarity, GS)***
For any $i \in N$ and any two profiles $(\mathbf{q}, \overline{\mathbf{q}})$ and $(\mathbf{q}, \overline{\mathbf{q}}')$ such that $\overline{q}'_i \neq \overline{q}_i$ and $\overline{q}'_j = \overline{q}_j$ for all $j \in N \backslash \{i\}$, then

$$[u_i(q_i, \bar{q}'_i) - x'_i] - [u_i(q_i, \bar{q}_i) - x_i] = \left[u_j\left(q_j, \bar{q}'_j\right) - x'_j\right] - [u_j(q_j, \bar{q}_j) - x_j],$$

for all $j \in N$, where $x = x(\mathbf{q}, \overline{\mathbf{q}})$ and $x' = x(\mathbf{q}, \overline{\mathbf{q}}')$.

Another, weaker approach consists in requiring that when two agents bear an equal responsibility, their welfare should be equal:

Axiom. ***(Equal Welfare for Equal Responsibility, EWER)***

$$r_i = r_j \quad \Longrightarrow \quad u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j$$

We shall also consider a weaker axiom, which consists in requiring equality of welfare only if all agents bear an equal responsibility:

Axiom. ***(Uniform Welfare for Uniform Responsibility, UWUR)***

$$[r_i = r_j, \forall i, j \in N] \implies [u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j, \forall i, j \in N]$$

An even weaker axiom consists in having the same requirement only if this common level of responsibility is equal to a reference level:

Axiom. ***(Uniform Welfare for Reference Responsibility, UWRR)***
For some reference responsibility level, $r_0 \in \mathbb{R}_+$:

$$[r(q_i, \bar{q}_i) = r_0, \forall i \in N] \implies [u_i(q_i, \bar{q}_i) - x_i = u_j(q_j, \bar{q}_j) - x_j, \forall i, j \in N]$$

Finally, we shall say that a rate function satisfies *Uniform Welfare for Minimal Consumption (**UWMC**)* if it satisfies **UWRR** with reference responsibility level $r_0 = 0$.

4.4 Pricing Mechanisms

We now turn to the design of pricing mechanisms. The principles of responsibility and compensation will determine how to allocate the cost of meeting the needs of the population, $C\left(\bar{Q}\right)$, but not only. As we shall see, these principles will also interact with how the cost $C\left(Q\right) - C\left(\bar{Q}\right)$, is to be split. The two portions of the cost cannot be considered in isolation.

Conditional Equality: SR+UWRR

Turning first to rate functions that prioritize holding agents responsible for their consumption, we identify the strongest compensation axioms compatible with **SR.** We find that **UWRR** and **SR** jointly characterize a family of rate functions, which we call *Conditional Equality solutions*,[12] that is parametrized by the choice of a reference responsibility level, r^0:

Theorem 2. *A rate function x satisfies **SR** and **UWRR** if and only if $x = x^{CE}$ where, for some reference level $r^0 > 0$,*

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right),$$

for all $i \in N$, where q_i^0 is defined by $r\left(q_i^0, \bar{q}_i\right) = r^0$.

Proof. In Appendix A.1. □

A special variant of the Conditional Equality solutions consists in choosing zero responsibility as a reference: $\mathbf{q_0} = \bar{\mathbf{q}}$. This implies charging households the same fee to meet their own needs, whatever those needs may be. Should they choose to consume more, they would bear the consequences according to the cost-sharing rule in effect.

Corollary 1. *The unique rate function satisfying **SR** and **UWMC** is the following:*

$$x_i^{CE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i(\mathbf{r}, C - C(\bar{Q})) \qquad \text{for all } i \in N.$$

[12] The name reflects the fact that this family of solutions is reminiscent of the conditional equality solution in Fleurbaey (1995) in a different context.

A limit of x^{CE0} is that compensation for needs is established on the basis of a single scenario which is unlikely to ever arise. However, it possesses the advantage of not requiring knowledge of the utility function.

Theorem 2 is generically tight because x^{CE} generically does not satisfy the stronger compensation axiom **UWUR**. The only exception is when the agents share a common utility function and the responsibility function, r, reflects the utility derived by the agents:

Proposition 1. *x^{CE} does not satisfy **UWUR** unless the following two assertions are true:*
(1) all agents share a common utility function; i.e., $u_i = u \in \Upsilon$, for all $i \in N$
(2) the responsibility function co-varies with agents utility; i.e., $r = \rho \circ u$, for some increasing function $\rho : \mathbb{R} \to \mathbb{R}_+$.

Proof. In Appendix A.2. □

In fact, when the conditions of Proposition 1 are true, **SR** is even compatible with the stronger compensation axiom **EWER**. Together, they characterize a unique solution:

Theorem 3. *If $u_i = u \in \Upsilon$, for all $i \in N$ and if $r = \rho \circ u$, for some increasing function $\rho : \mathbb{R} \to \mathbb{R}_+$, a rate function x satisfies **EWER** and **SR** if and only if*

$$x \equiv x^{CE0}$$

Proof. In Appendix A.3. □

The above result applies only to specific circumstances: agents differ only in their needs, but not in their preferences. A remarkable feature of the above characterization is that it does not require specifying a reference responsibility level, although it obviously requires knowledge of the (common) utility function.[13]

Theorem 3 is a tight characterization because **SR** is incompatible with the strongest solidarity axiom, **GS**, as Theorem 4 below implies.

[13]Knowledge of the common utility function u is merely required to check whether the theorem applies, not to compute cost shares. In particular, cardinal information about preferences is not needed.

Egalitarian Equivalence: GS+SRRN

We now turn to rate functions that prioritize negating the impact of differences in needs on welfare. Axiom **GS** embodies this desideratum. We show that **GS** together with **SRRN** determine a family of rate functions, which we call the *Egalitarian Equivalent solutions*,[14] that is parametrized by a reference level of needs, $\bar{q}_0$:

Theorem 4. *A rate function x satisfies* ***GS*** *and* ***SRRN*** *if and only if $x = x^{EE}$ where, for a given reference level of needs, $\bar{q}_0 > 0$,*

$$\begin{aligned} x_i^{EE}(\mathbf{q},\bar{\mathbf{q}}) &= \frac{C(n\bar{q}_0)}{n} + \xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right) \\ &\quad + \left[u_i\left(q_i,\bar{q}_i\right) - u_i\left(q_i,\bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k\left(q_k,\bar{q}_k\right) - u_k\left(q_k,\bar{q}_0\right)\right], \end{aligned}$$

for all $i \in N$, where $\mathbf{r}_0 = \left(r\left(q_1,\bar{q}_0\right), r\left(q_2,\bar{q}_0\right), ..., r\left(q_n,\bar{q}_0\right)\right)$.[15]

Proof. In Appendix A.4. □

x^{EE} measures responsibility relative to the common reference level, $\bar{q}_0$: $r_{i,0} = r\left(q_i,\bar{q}_0\right)$ and splits costs accordingly. Differences between actual needs and the reference level are compensated for so as to preserve the relative welfare distribution.

The characterization is tight, in the sense that the Egalitarian Equivalent solution does not satisfy stronger responsibility axioms. This can be shown by considering a profile $(\mathbf{q},(\bar{q}_1,\bar{q}_1,...,\bar{q}_1)) \in \mathbb{D}^n$ such that $\bar{q}_1 \neq \bar{q}_0$ to obtain that **SRUN** is not satisfied. The formal proof can be found in Appendix A.5.

Remark 2. The cost-sharing portion of the transfer, $(1/n)\, C(n\bar{q}_0)+\xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right)$, is driven by the consumption profile of the agents and by the cost structure, but is actually independent of individual needs. By contrast, the redistributive component of the bill, $\left[u_i\left(q_i,\bar{q}_i\right) - u_i\left(q_i,\bar{q}_0\right)\right] - (1/n)\sum_{k=1}^{n}\left[u_k\left(q_k,\bar{q}_k\right) - u_k\left(q_k,\bar{q}_0\right)\right]$, is based on the benefits the agents derive from consumption and is independent of costs.

Remark 3. Whenever needs summarize all relevant differences across agents so that they share a common utility function u, whatever the value of $\bar{q}_0$, the Egalitarian

[14] The name reflects the fact that this family of solutions is reminiscent of the egalitarian equivalent allocations in the seminal contribution by Pazner and Schmeidler (1978).

[15] Given the domain of definition of the utility functions, x^{EE} is well defined only on the set $\left\{(\mathbf{q},\bar{\mathbf{q}}) \in \mathbb{D}^n \middle| \min_i q_i \geq \bar{q}_0\right\}$.

Equivalent solution fully addresses the issue of differences in needs whenever consumption is uniform. Formally, if $q_1 = q_2 = ... = q_n$, then $u_i(q_i, \bar{q}_i) - x_i^{EE}(\mathbf{q}, \bar{\mathbf{q}}) = u_j(q_j, \bar{q}_j) - x_j^{EE}(\mathbf{q}, \bar{\mathbf{q}})$ for all $i, j \in N$. In other words, under x^{EE}, any differences in utility levels are attributable to differences in consumption.

Remark 4. The fact that the Conditional Equality solutions satisfy weaker compensation axioms does not mean that the Egalitarian Equivalent solutions are more redistributive. Indeed, for the latter, the parameter $\bar{q}_0$ dictates both the portion of the cost to be shared in an egalitarian fashion and how differences in needs are accounted for. In particular, when $\bar{q}_0 = 0$, the portion of costs to be split equally under x^{EE} is nil—$C(n\bar{q}_0)/n = 0$—and users are held responsible for their whole consumption. By contrast, x^{CE} always splits equally the portion of costs corresponding to the needs of the population: $C(\bar{Q})$.

5 Protecting small users while holding them responsible

5.1 Convex Costs

We introduce an axiom that aims to protect parsimonious users from the cost externality caused by wasteful users: An agent who increases her responsibility level cannot lead consumers with lower responsibility to pay a higher amount.

Axiom. ***(Independence of Higher Responsibility, IHR)*** *For all* $(\mathbf{q}, \bar{\mathbf{q}})$ *and* $(\mathbf{q}', \bar{\mathbf{q}}')$ *such that* $\bar{\mathbf{q}}' = \bar{\mathbf{q}}$ *and* $\mathbf{r}' \geq \mathbf{r}$*. For all* $i \in N$*, define* $L(i) = \{j \in N \text{ s.t. } r_j \leq r_i\}$ *the set of users with lower responsibility than* i*. Then,*

$$\begin{aligned} & \left\{r'_j = r_j \text{ for all } j \in L(i)\right\} \\ \Longrightarrow \quad & \left\{\xi_j\left(\mathbf{r}', C - C\left(\bar{Q}'\right)\right) = \xi_j\left(\mathbf{r}, C - C\left(\bar{Q}\right)\right) \text{ for all } j \in L(i)\right\}. \end{aligned}$$

Remark 5. Note that for a given profile $(\mathbf{q}, \bar{\mathbf{q}})$, such that $q_i > q_j$ and $\bar{q}_i > \bar{q}_j$ for some i and j, then one can find two functional forms $\tilde{r}$ and $\hat{r}$ such that

$$\tilde{r}(q_i, \bar{q}_i) \geq \tilde{r}(q_j, \bar{q}_j) \quad \text{and} \quad \hat{r}(q_i, \bar{q}_i) < \hat{r}(q_j, \bar{q}_j).$$

Hence, the identity of consumers with a smaller responsibility depends on how responsibility is measured; i.e., upon the specific functional form for r (Figure 1).

Serial Conditional Equality

Recall that $r(\cdot, \bar{q}_i)$ maps an agent's consumption to her responsibility level, given her needs. Define the inverse of this function, $g_i(\cdot) = (r)^{-1}(\cdot, \bar{q}_i)$, which maps a responsibility level to the corresponding consumption level given the needs of the agent.

Proposition 2. *The unique rate function satisfying* ***UWMC****,* ***SR*** *and* ***IHR*** *is the following:*

$$x_i^{SCE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\hat{Q}^i\right)}{(n-i+1)} - \sum_{k=1}^{i-1} \frac{C\left(\hat{Q}^k\right)}{(n-k)(n-k+1)} \qquad \text{for all } i \in N,$$

where, for all $k \in N$,

$$\hat{Q}^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i(r_k),$$

where the set of agents is ordered so as to have $r_1 \leq r_2 \leq ... \leq r_n$.

Proof. In Appendix B.1. □

Remark 6. x^{SCE0} amounts to applying the serial cost-sharing rule to responsibility levels. In fact,

$$x_i^{SCE0}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C(\bar{Q}) + \sum_{k=1}^{i} \frac{1}{n-k+1} \left[C\left(\hat{Q}^k\right) - C\left(\hat{Q}^{k-1}\right)\right],$$

with $\hat{Q}^0 = \bar{Q}$. This is of notable interest because the serial cost-sharing rule is known for its strong incentives properties (Moulin and Shenker, 1992).

Notice that a higher responsibility level leads to a higher bill: $r_i \geq r_j$ implies $x_i^{SCE0}(\mathbf{q}, \bar{\mathbf{q}}) \geq x_j^{SCE0}(\mathbf{q}, \bar{\mathbf{q}})$ because

$$\hat{Q}^{k+1} - \hat{Q}^k = \sum_{i=k+1}^{n} \left[g_i(r_{k+1}) - g_i(r_k)\right] \geq 0.$$

Serial Egalitarian Equivalence

Proposition 3. *A rate function x satisfies* ***GS****,* ***SRRN*** *and* ***IHR*** *if and only if $x = x^{SEE}$ where, for a given reference level of needs $\bar{q}_0 > 0$,*

$$\begin{aligned} x_i^{SEE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{C\left(\tilde{Q}^i\right)}{(n-i+1)} - \sum_{k=1}^{i-1} \frac{C\left(\tilde{Q}^k\right)}{(n-k)(n-k+1)} \\ &\quad + \left[u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)\right] \end{aligned}$$

for all $i \in N$, where $\tilde{Q}^k = \sum_{l=1}^{k} q_l + (n-k)\, q_k$ with the set of agents ordered so as to have $q_1 \leq q_2 \leq ... \leq q_n$.[16]

Proof. In Appendix B.2. □

Remark 7. The expression for x^{SEE} is independent of the form of responsibility.

Remark 8. x^{SEE} amounts to applying the serial cost-sharing rule directly to consumption, along with transfers to compensate for differences in needs. In fact,

$$\begin{aligned} x_i^{SEE}(\mathbf{q}, \bar{\mathbf{q}}) &= \frac{1}{n} C\left(n \inf_j q_j\right) + \sum_{k=1}^{i-1} \frac{1}{n-k}\left[C\left(\tilde{Q}^{k+1}\right) - C\left(\tilde{Q}^k\right)\right] \\ &\quad + \left[u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)\right]. \end{aligned}$$

Note that the compensation terms may affect the well-known incentives properties of the serial cost-sharing rule.

At first blush, the expressions of x^{SCE0} and x^{SEE} may seem similar, with x^{SEE} having an additional compensation term. However, note that agents are ordered according to their consumption under x^{SEE} but are ordered according to their responsibility level under x^{SCE0}. Also, the Q^k's that enter in the cost-sharing portion stand for different aggregate consumption levels. In particular, x^{SEE} applies the serial cost-sharing rule directly on consumption levels, with needs appearing only in the compensation portion. By contrast, x^{SCE0} applies the serial cost-sharing rule to responsibility levels which, by design, take individual needs into account.

[16] x^{SEE} is well defined only on the subdomain $\{(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n | \min_i q_i \geq \bar{q}_0\}$.

5.2 Concave Costs

With increasing marginal cost, we wished to protect users with smaller responsibility levels from bearing a high marginal cost due to the presence of 'large users'. By contrast, when the technology exhibits increasing returns to scale, we want 'small users' to fully benefit from a further reduction in their consumption. It follows that larger users never benefit from the effort of smaller users in reducing their consumption.

Axiom. ***(Independence of Lower Responsibility, ILR)*** *For all* $(\mathbf{q},\bar{\mathbf{q}})$ *and* $(\mathbf{q}',\bar{\mathbf{q}}')$ *such that* $\bar{\mathbf{q}}'=\bar{\mathbf{q}}$ *and* $\mathbf{r}'\leq\mathbf{r}$*. For all* $i\in N$*, define* $H(i)=\{j\in N \text{ s.t. } r_j\geq r_i\}$ *the set of users with higher responsibility level than* i*. Then,*

$$\begin{aligned} & \left\{r_j'=r_j \text{ for all } j\in H(i)\right\} \\ \Longrightarrow\quad & \left\{\xi_j\left(\mathbf{r}',C-C\left(\bar{Q}'\right)\right)=\xi_j\left(\mathbf{r},C-C\left(\bar{Q}\right)\right) \text{ for all } j\in H(i)\right\}. \end{aligned}$$

Decreasing Serial Conditional Equality

Proposition 4. *The unique rate function satisfying* ***UWMC***, ***SR*** *and* ***ILR*** *is the following:*

$$x_i^{DSCE0}(\mathbf{q},\bar{\mathbf{q}})=\frac{C\left(\check{Q}^i\right)}{i}-\sum_{k=i+1}^{n}\frac{C\left(\check{Q}^k\right)}{k(k-1)}\qquad \text{for all } i\in N,$$

where, for all $k\in N$*,*

$$\check{Q}^k=\sum_{l=1}^{k}g_l(r_k)+\sum_{l=k+1}^{n}q_l,$$

where the set of agents is ordered so as to have $r_1\leq r_2\leq\ldots\leq r_n$*.*

Proof. In Appendix B.3. □

Remark 9. x^{DSCE0} amounts to applying the decreasing serial cost-sharing rule to responsibility levels in order to split the associated costs. In fact,

$$x_i^{DSCE0}(\mathbf{q},\bar{\mathbf{q}})=\frac{1}{n}C\left(\check{Q}^n\right)-\sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(\check{Q}^{k+1}\right)-C\left(\check{Q}^k\right)\right]$$

with $\check{Q}^1=Q$. Like the serial rule, the decreasing serial cost-sharing rule is also known for its strong incentives properties (de Frutos, 1998).

Note that a higher responsibility level indeed leads to a higher bill: $q_i^r \geq q_j^r$ implies $x_i^{DSCE0}(\mathbf{q}, \bar{\mathbf{q}}) \geq x_j^{DSCE0}(\mathbf{q}, \bar{\mathbf{q}})$ because

$$\breve{Q}^{k+1} - \breve{Q}^k = \sum_{l=1}^{k} [g_l(r_{k+1}) - g_l(r_k)] \geq 0.$$

Decreasing Serial Egalitarian Equivalence

Proposition 5. *A rate function x satisfies **GS**, **SRRN** and **ILR** if and only if $x = x^{SEE}$ where, for a given reference level of needs $\bar{q}_0 > 0$,*

$$x_i^{DSEE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\breve{Q}^i\right)}{i} - \sum_{k=i+1}^{n-1} \frac{C\left(\breve{Q}^k\right)}{k(k-1)} + [u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n} [u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)]$$

for all $i \in N$, where $\breve{Q}^k = kq_k + \sum_{l=k+1}^{n} q_l$ for all $k = 1, ..., n$, with the set of agents ordered so as to have $q_1 \leq q_2 \leq ... \leq q_n$.[17]

Proof. In Appendix B.4. □

Remark 10. x^{DSEE} amounts to applying the decreasing serial cost-sharing rule directly to consumption, along with transfers to compensate for differences in needs. In fact,

$$x_i^{DSEE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n} C\left(n \sup_j q_j\right) - \sum_{k=i}^{n-1} \frac{1}{k}\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right] + [u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n} [u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)]$$

6 Accounting for responsibility in practice

In practice, making explicit interpersonal comparisons of needs and consumption would be very difficult and possibly counterproductive. Nevertheless, we show how one can implement the above schemes with realistic informational assumptions.[18]

[17] x^{DSEE} is well defined only on the subdomain $\{(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n | \min_i q_i \geq \bar{q}_0\}$.
[18] Computations can be found in Appendix C

6.1 Pricing using aggregate distributions

We now represent the population by a distribution. Assume that there is a finite number of types in the needs dimension due to, say, household size, and let $\bar{q}_s$ denote the needs of a household of size $s \in S$. The planner does not know each individual's utility function, but has enough information to infer, u_s, the typical utility function of a household of type $s \in S$. Let $n_s(q)$ be the density of type-s households with consumption level q and let $N_s(q)$ be the associated cumulative distribution: $N_s(q) = \int_{z=0}^{q} n_s(z)\,dz$. Define $n(q) = \sum_{s \in S} n_s(q)$ and $N(q) = \sum_{s \in S} N_s(q)$. We slightly abuse notation and write $r(q, s)$ instead of $r(q, \bar{q}_s)$ whenever it is unambiguous. Given the responsibility function r, define $n_s^r(\rho)$ the density of type-s households with responsibility level ρ. Let $N_s^r(\rho)$ be the associated cumulative distribution: $N_s^r(\rho) = \int_{z=0}^{\rho} n_s^r(z)\,dz$ and define $N^r(\rho) = \sum_{s \in S} N_s^r(\rho)$. We now define the following continuous counterparts to the quantities $\hat{Q}, \tilde{Q}, \check{Q}$ and $\breve{Q}$, respectively corresponding to the SCE0, SEE, DSCE0 and DSEE schemes:

$$\text{SCE0} \quad : \qquad \hat{Q}(\rho) = \sum_{s \in S} \left[\int_0^{+\infty} g_s(\inf\{\rho, z\})\, n_s^r(z)\,dz \right] \tag{9}$$

$$\text{SEE} \quad : \qquad \tilde{Q}(q) = \int_0^{\infty} \inf\{q, z\} n(z)\,dz \tag{10}$$

$$\text{DSCE0} \quad : \qquad \check{Q}(\rho) = \int_{z=0}^{\infty} \sum_{s \in S} g_s(\sup\{\rho, z\})\, n_s^r(z)\,dz \tag{11}$$

$$\text{DSEE} \quad : \qquad \breve{Q}(q) = \int_{z=0}^{\infty} \sup\{q, z\} n(z)\,dz \tag{12}$$

with $g_s(\cdot) \equiv r^{-1}(\cdot, \bar{q}_s)$.

With this notation, the expressions for x^{SCE0}, x^{SEE}, x^{DSCE0}, and x^{DSEE} take the following forms:

$$x^{SCE0}(\rho) = \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right) \frac{d\hat{Q}(z)}{d\rho} dz \tag{13}$$

$$x^{SEE}(q,s) = \frac{C(N \inf \mathbf{q})}{N} + \int_{z=0}^{q} C'\left(\tilde{Q}(z)\right) dz + \left[u_s(q,\bar{q}_s) - u(q_s,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u_t(z,\bar{q}_t) - u_t(z,\bar{q}_0)\right] n_t(z)\, dz \tag{14}$$

$$x^{DSCE0}(\rho) = \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup\vec{\rho}} \frac{1}{N^r(z)} C'\left(\check{Q}(z)\right) \frac{d\check{Q}(z)}{dz} dz \tag{15}$$

$$x^{DSEE}(q,s) = \frac{1}{N}\; C(N \sup \mathbf{q}) - \int_{z=q}^{\sup \mathbf{q}} C'\left(\check{Q}(z)\right) dz + \left[u_s(q,\bar{q}_s) - u_s(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u_t(z,\bar{q}_t) - u_t(z,\bar{q}_0)\right] n_t(z)\, dz \tag{16}$$

where $\check{Q}_{\sup} = \check{Q}(\sup\vec{\rho})$ with $\sup\vec{\rho}$ the largest responsibility level in the population and where N still denotes the total number of households.

$$x_i^{SCE0}(\mathbf{q},\bar{\mathbf{q}}) = \frac{C\left(\hat{Q}^i\right)}{(n-i+1)} - \sum_{k=1}^{i-1} \frac{C\left(\hat{Q}^k\right)}{(n-k)(n-k+1)} \qquad \text{for all } i \in N, where, for all k \in N, \hat{Q}^k = \sum_{i=1}^{k-1} q_i +$$

6.2 Illustrative Examples

To illustrate, we now consider two specific forms for r. In the *absolute responsibility view*, $r(q,s) = q - \bar{q}_s$, whereas in the *relative responsibility view*, $r(q,s) = (q - \bar{q}_s)/\bar{q}_s$. If s indeed denotes household size, the former holds households equally responsible for consumption above needs regardless of their size. By contrast, the latter view holds larger households less responsible than smaller households for an identical consumption level above needs. In other words, needs also impact the way consumption beyond them is considered.

Decreasing Returns to Scale : Quadratic Costs

Assume that costs are given by the following quadratic function: $C(Q) = cQ^2/2$. Under absolute responsibility, the serial conditional equality rule with zero responsibility as a reference yields:

$$x^{SCE0}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(q - \bar{q}_s - \frac{Q-\bar{Q}}{N}\right). \tag{17}$$

In words, users share the total cost equally and are rewarded or penalized for deviation from the average responsibility level. These deviations are valued at marginal cost.

Under relative responsibility, however, marginal consumption is not priced equally across household types. When responsibility is equally distributed across types, we obtain the following expression:

$$x^{SCE0}(q,s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\frac{\bar{Q}}{N}\left(\frac{q-\bar{q}_s}{\bar{q}_s} - \frac{Q-\bar{Q}}{\bar{Q}}\right). \tag{18}$$

Again, x^{SCE0} charges everyone the average cost and prices deviations from the average responsibility, but this time at the marginal cost of responsibility if needs were equal to $\bar{Q}/N$. Observe that if $\bar{q}_s > \bar{Q}/N$ consumption is priced at less than the marginal cost while the consumption of households with lower-than-average needs ($\bar{q}_s < \bar{Q}/N$) is priced above marginal cost.[19]

The serial egalitarian equivalent solution takes on the following form:

$$\begin{aligned} x^{SEE}(q,s) = {} & cQ\left(q - \frac{Q}{2N}\right) \\ & + \left[u_s(q,\bar{q}_s) - u_s(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u_t(z,\bar{q}_t) - u_t(z,\bar{q}_0)\right]n_t(z)\,dz. \end{aligned} \tag{19}$$

Recall that the expression for x^{SEE} is independent of the responsibility view (e.g., absolute or relative responsibility). However, payments now depend upon the utility function. This calls for an observation. Suppose that a household's type is simply its size and that $\bar{q}_s = \tilde{q} \times s$ for some reference per-person level of needs, $\tilde{q}$. Given a consumption level, q, it seems natural for the total bill to be lower for larger households. For this to be the case, it must be that $u_s(q, \tilde{q} \times s)$ is decreasing in s, according to Expression (19). This implies that household utility cannot be written as a simple sum of the utility of its members, $s \times v_{\tilde{q}}(q/s)$, where $v_{\tilde{q}}$ is some increasing and concave

[19]This is unlike the case of absolute responsibility above, where the marginal cost of responsibility was identical across households and equal to the marginal cost.

function. Indeed, we would have:

$$\frac{d}{ds}\left[s \times v_{\tilde{q}}\left(q/s\right)\right] = v_{\tilde{q}}\left(\frac{q}{s}\right) - \frac{q}{s}v_{\tilde{q}}'\left(\frac{q}{s}\right) \geq 0, \tag{20}$$

by the concavity of $v_{\tilde{q}}$. Thus, one must refrain from modeling households as a sum of individual utility functions.[20]

Increasing Returns to Scale: Affine Costs

Assume costs are of the form $C(Q) = F + cQ$, with $F, c \in \mathbb{R}_+$. When responsibility is measured by absolute responsibility, the decreasing serial conditional equality rule yields:

$$x^{DSCE0}\left(q, s\right) = \frac{F + c\bar{Q}}{N} + c\left(q - \bar{q}_s\right). \tag{21}$$

In addition to splitting the fixed cost equally, x^{DSCE0} also splits the cost of the population's needs equally before charging users at marginal cost with a rebate equal to the cost of meeting their needs.

Under the relative responsibility view, and if responsibility is identically distributed across types, we obtain:

$$x^{DSCE0}\left(q, s\right) = \frac{F}{N} + c\frac{1}{\bar{q}_s/\left(\bar{Q}/N\right)}q. \tag{22}$$

As with absolute responsibility, x^{DSCE0} splits the fixed cost equally. No rebate is granted, however, but consumption is priced at a rate that is inversely proportional to one's needs.

We now turn to x^{DSCEE}:

[20] This is reminiscent of the Repugnant Conclusion in population ethics (Blackorby et al., 2005). The latter is a consequence of the pure utilitarian criterion, which deems any population always worse off than a larger one sharing the same resources, even if the population size is such that individuals have barely enough to survive (see also Fleurbaey et al., 2014).

$$x^{DSEE}(q,s) = \frac{F}{N} + cq \tag{23}$$
$$+ \left[u_s(q,\bar{q}_s) - u_s(q,\bar{q}_0)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u_t(z,\bar{q}_t) - u_t(z,\bar{q}_0)\right] n_t(z)\, dz$$

The cost-sharing portion of x^{DSEE} splits the fixed cost equally and prices consumption at marginal cost. Needs are completely absent from that component. However, the redistributive portion of x^{DSEE} ensures that heterogeneity in needs does not drive differences in welfare.

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A Appendix: Section 4 Proofs

A.1 Proof of Theorem 2

Let $r^0 \in \mathbb{R}_+$ be a reference responsibility level and $(\mathbf{q^0}, \bar{\mathbf{q}}) \in \mathcal{P}$ be such that,

$$r\left(q_i^0, \bar{q}_i\right) = r^0, \qquad \text{for all } i \in N. \tag{24}$$

By **UWRR**,

$$u_i\left(q_i^0, \bar{q}_i\right) - x_i\left(\mathbf{q^0}, \bar{\mathbf{q}}\right) = u_j\left(q_j^0, \bar{q}_j\right) - x_j\left(\mathbf{q^0}, \bar{\mathbf{q}}\right), \qquad \text{for all } i, j \in N. \tag{25}$$

Hence, for all $i \in N$,

$$x_i\left(\mathbf{q^0}, \bar{\mathbf{q}}\right) = u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N}\left[u_j\left(q_j^0, \bar{q}_j\right) - x_j\left(\mathbf{q^0}, \bar{\mathbf{q}}\right)\right], \tag{26}$$

$$= \frac{C\left(Q^0\right)}{n} + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \tag{27}$$

where $Q^0 \equiv \sum_{j\in N} q_j^0$.

Applying **SR** between profiles $(\mathbf{q^0}, \bar{\mathbf{q}})$ and $(\bar{\mathbf{q}}, \bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q^0}, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) \;=\; \xi_i\left(\mathbf{r}^0, C - C(\bar{Q})\right). \tag{28}$$

Hence, by symmetry of ξ,

$$x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = x_i\left(\mathbf{q^0}, \bar{\mathbf{q}}\right) - \frac{C\left(Q^0\right) - C\left(\bar{Q}\right)}{n}. \tag{29}$$

Applying **SR** between profiles $(\bar{\mathbf{q}}, \bar{\mathbf{q}})$ and $(\mathbf{q}, \bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q}, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) \;=\; \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right). \tag{30}$$

Thus,

$$x_i(\mathbf{q}, \bar{\mathbf{q}}) = \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) \tag{31}$$

$$= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + x_i\left(\mathbf{q^0}, \bar{\mathbf{q}}\right) - \frac{C\left(Q^0\right) - C\left(\bar{Q}\right)}{n} \tag{32}$$

$$= \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + \frac{C\left(\bar{Q}\right)}{n} + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right). \tag{33}$$

A.2 Proof of Proposition 1

Let $(\mathbf{q}^0, \bar{\mathbf{q}}) \in \mathcal{P}$ and $(\mathbf{q}^1, \bar{\mathbf{q}}) \in \mathcal{P}$ be two profiles associated respectively with the uniform responsibility profiles $\mathbf{r^0} = (r^0, r^0, ..., r^0)$ and $\mathbf{r^1} = (r^1, r^1, ..., r^1)$ with $r^1 \neq r^0$. Suppose that x satisfies **UWUR** so that it satisfies in particular **UWRR** for the reference responsibility level r^0. If it does also satisfy **SR,** it must be written as

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N. \tag{34}$$

This says in particular that when $\mathbf{q} = \mathbf{q^1}$, we have:

$$x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}^1, C - C(\bar{Q})\right) + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N. \tag{35}$$

By symmetry of ξ, we have $\xi_i\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \left[C\left(Q^1\right) - C\left(\bar{Q}\right)\right]/n$, for all $i \in N$ so that

$$x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = \frac{C\left(Q^1\right)}{n} + u_i\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i \in N. \tag{36}$$

If $x\left(\mathbf{q}, \bar{\mathbf{q}}\right)$ satisfies **UWRR** for the reference responsibility level r^1 (to which $\mathbf{q^1}$ is associated), it must be the case that

$$u_i\left(q_i^1, \bar{q}_i\right) - x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right) = u_j\left(q_j^1, \bar{q}_j\right) - x_j\left(\mathbf{q^1}, \bar{\mathbf{q}}\right), \qquad \text{for all } i, j \in N. \tag{37}$$

From the expression of $x_i\left(\mathbf{q^1}, \bar{\mathbf{q}}\right)$ established above, we must have

$$u_i\left(q_i^1, \bar{q}_i\right) - u_i\left(q_i^0, \bar{q}_i\right) = u_j\left(q_j^1, \bar{q}_j\right) - u_j\left(q_j^0, \bar{q}_j\right), \qquad \text{for all } i, j \in N. \tag{38}$$

This implies in turn that

$$u_i\left(q_i^1, \bar{q}_i\right) - u_i\left(q_i^0, \bar{q}_i\right) = \frac{1}{n}\sum_{j\in N}\left[u_j\left(q_j^1, \bar{q}_j\right) - u_j\left(q_j^0, \bar{q}_j\right)\right], \qquad \text{for all } i \in N. \tag{39}$$

This must be true for any responsibility level r^0 and r^1 and the associated profiles $(\mathbf{q}^0, \bar{\mathbf{q}}) \in \mathcal{P}$ and $(\mathbf{q}^1, \bar{\mathbf{q}}) \in \mathcal{P}$. Thus, by setting $r^1 = 0$ and considering the associated profile $(\bar{\mathbf{q}}, \bar{\mathbf{q}}) \in \mathcal{P}$, we obtain that, for **SR** and **UWUR** to be compatible, the utility function must be such that

$$u_i\left(q_i^0, \bar{q}_i\right) = \frac{1}{n}\sum_{j\in N} u_j\left(q_j^0, \bar{q}_j\right) \tag{40}$$

for all $i \in N$ and for all profiles $(\mathbf{q}^0, \bar{\mathbf{q}}) \in \mathcal{P}$ such that

$$r\left(q_i^0, \bar{q}_i\right) = r^0, \qquad \text{for all } i \in N. \tag{41}$$

Fix r^0 and $\bar{\mathbf{q}}$ and define, for all $i \in N$, $q\left(r^0, \bar{q}_i\right) = \left\{q \in \mathbb{R}_+ | r\left(q, \bar{q}_i\right) = r^0\right\}$. By continuity and strict monotonicity of r, $q\left(r^0, \bar{q}_i\right)$ is a singleton and $\left(r^0, \bar{q}_i\right) \mapsto q\left(r^0, \bar{q}_i\right)$ defines a continuous function that is increasing in its first argument. Also, define $u^0 = \frac{1}{n}\sum_{j\in N} u_j\left(q\left(r^0, \bar{q}_j\right), \bar{q}_j\right)$. It follows from Expression (40) that we must have $u_i\left(q\left(r^0, \bar{q}_i\right), \bar{q}_i\right) = u^0$ for all i and all $\bar{q}_i$. Because u^0 depends neither upon i, nor upon$\bar{q}_i$, it must be that $\left(\bar{q}_i, r^0\right) \mapsto u_i\left(q\left(r^0, \bar{q}_i\right), \bar{q}_i\right)$ is a function of r^0 only. Therefore, for all r^0, all i and all $\bar{q}_i$,

$$u_i\left(q\left(r^0, \bar{q}_i\right), \bar{q}_i\right) = v\left(r^0\right) \tag{42}$$

for some function v on $\mathbb{R}$. Because u_i and q are both continuous and increasing in their first argument, v is also a continuously increasing function.

Finally, let $\left(q_i, \bar{q}_i\right) \in \mathbb{D}$, evaluating the above expression at $r^0 = r\left(q_i, \bar{q}_i\right)$, and noticing that

$$q\left(r\left(q_i, \bar{q}_i\right), \bar{q}_i\right) = q_i \tag{43}$$

yields:

$$u_i\left(q_i, \bar{q}_i\right) = v\left(r\left(q_i, \bar{q}_i\right)\right). \tag{44}$$

This in turn implies that the utility must be a transformation of the responsibility function:

$$u_i = u \equiv v \circ r. \tag{45}$$

Because v is a continuous and increasing function of $\mathbb{R}$, we can write:

$$r = \rho \circ u, \tag{46}$$

with $\rho = v^{-1}$, so that r is a transformation of the common utility function$u$, as was to be shown.

A.3 Proof of Theorem 3

Only if. Let x satisfy **EWER** and **SR**. Because **EWER** is more demanding than **UWUR**, x must also satisfy **UWUR** . By Proposition 1, this can only occur if $u_i = u$ for some utility function u and $r = \rho \circ u$ for some continuous and increasing function ρ. Because **UWUR** is more demanding than **UWRR**, x must also satisfy **UWRR**. By Theorem 2, x must be a Conditional Equivalent solution:

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) + u\left(q_i^0, \bar{q}_i\right) - \frac{1}{n}\sum_{j \in N} u\left(q_j^0, \bar{q}_j\right), \tag{47}$$

where u is the common utility function and $\mathbf{q^0}$ is such that, for all $i \in N$, $r\left(q_i^0, \bar{q}_i\right) = r^0$ for some reference responsibility level, r^0. Moreover, it follows from $r = \rho \circ u$ that $u\left(q_i^0, \bar{q}_i\right) = \rho^{-1}\left(r^0\right)$ for all $i \in N$. Hence,

$$x_i^{CE}(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} + \xi_i\left(\mathbf{r}, C - C(\bar{Q})\right), \qquad \text{for all } i \in N. \tag{48}$$

If. We already know from Theorem 1 that x^{CE0} satisfies **SR**. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ such that $r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right)$for some $i, j \in N$. It follows from the symmetry of ξ that

$$\xi_i\left(\mathbf{r}, C - C(\bar{Q})\right) = \xi_j\left(\mathbf{r}, C - C(\bar{Q})\right). \tag{49}$$

As a result,

$$x_i^{CE0}(\mathbf{q}, \bar{\mathbf{q}}) = x_j^{CE0}(\mathbf{q}, \bar{\mathbf{q}}). \tag{50}$$

Moreover, because $r = \rho \circ u$ for some continuous and increasing function ρ, we can write $u = \rho^{-1} \circ r$. Thus,

$$r\left(q_i, \bar{q}_i\right) = r\left(q_j, \bar{q}_j\right) \implies u\left(q_i, \bar{q}_i\right) = u\left(q_j, \bar{q}_j\right), \tag{51}$$

and $u_i = u_j = u$ yields

$$u_i\left(q_i, \bar{q}_i\right) - x_i^{CE0}(\mathbf{q}, \bar{\mathbf{q}}) = u_j\left(q_j, \bar{q}_j\right) - x_j^{CE0}(\mathbf{q}, \bar{\mathbf{q}}). \tag{52}$$

Hence, x^{CE0} satisfies **EWER**.

A.4 Proof of Theorem 4

Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}_+^n$ the associated reference vector. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ such that $\min_i q_i \geq \bar{q}_0$. By budget balance and anonymity,

$$x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n}. \tag{53}$$

By **SRRN**,

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0, C - C\left(n\bar{q}_0\right)\right) \qquad \text{for all } i \in N, \tag{54}$$

where $r_{0,i} = r\left(q_i, \bar{q}_0\right)$ for all i.

Define $\bar{\mathbf{q}}_0^1 = (\bar{q}_1, \bar{q}_0, ..., \bar{q}_0)$. Applying **GS** between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ yields, for all $j \neq 1$:

$$u_1\left(q_1, \bar{q}_1\right) - x_1^1 - u_1\left(q_1, \bar{q}_0\right) + x_1^0 \quad = \quad u_j\left(q_j, \bar{q}_0\right) - x_j^1 - u_j\left(q_j, \bar{q}_0\right) + x_j^0 \tag{55}$$

where $x_j^0 = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0\right)$ and $x_j^1 = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^1\right)$ for all $j \in N$. This yields:

$$x_j^0 - x_j^1 = u_1\left(q_1, \bar{q}_1\right) - u_1\left(q_1, \bar{q}_0\right) + x_1^0 - x_1^1, \tag{56}$$

hence, by budget balance:

$$\begin{cases} x_1^1 - x_1^0 & = \frac{n-1}{n}\left[u_1\left(q_1, \bar{q}_1\right) - u_1\left(q_1, \bar{q}_0\right)\right] \\ x_j^1 - x_j^0 & = -\frac{1}{n}\left[u_1\left(q_1, \bar{q}_1\right) - u_1\left(q_1, \bar{q}_0\right)\right] \qquad \forall j \neq 1. \end{cases} \tag{57}$$

Applying **GS** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq$

$j \leq n$:

$$
\begin{aligned}
& u_i(q_i, \bar{q}_i) - x_i^k - u_i(q_i, \bar{q}_i) + x_i^{k-1} & (58) \\
= \; & u_k(q_k, \bar{q}_k) - x_k^k - u_k(q_k, \bar{q}_0) + x_k^{k-1} & (59) \\
= \; & u_j(q_j, \bar{q}_0) - x_j^k - u_j(q_j, \bar{q}_0) + x_j^{k-1} & (60)
\end{aligned}
$$

Hence, for all $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$
\begin{aligned}
& x_i^{k-1} - x_i^k & (61) \\
= \; & u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0) + x_k^{k-1} - x_k^k & (62) \\
= \; & x_j^{k-1} - x_j^k & (63)
\end{aligned}
$$

By budget balance, $\sum_j \left(x_j^k - x_j^{k-1}\right) = 0$, yielding:

$$
\begin{aligned}
x_k^k - x_k^{k-1} &= \frac{n-1}{n}\left[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)\right] & (64) \\
x_j^k - x_j^{k-1} &= -\frac{1}{n}\left[u_k(q_k, \bar{q}_k) - u_k(q_k, \bar{q}_0)\right] \qquad \text{for all } j \neq k. & (65)
\end{aligned}
$$

Summing up over all iterations k yields the following:

$$
\begin{aligned}
x_1^n - x_1^0 &= \sum_{k>1}^{n} \left(x_1^k - x_1^{k-1}\right) + x_1^1 - x_1^0 & (66) \\
&= -\frac{1}{n}\sum_{j>1}^{n}\left[u_j(q_j, q_j) - u_j(q_j, \bar{q}_0)\right] + \left(1 - \frac{1}{n}\right)\left[u_1(q_1; \bar{q}_1) - u_1(q_1; \bar{q}_0)\right] & (67) \\
&= \left[u_1(q_1, \bar{q}_1) - u_1(q_1, \bar{q}_0)\right] - \frac{1}{n}\sum_{j=1}^{n}\left[u_j(q_j, \bar{q}_j) - u_j(q_j; \bar{q}_0)\right] & (68)
\end{aligned}
$$

Likewise, for all $i \in N$:

$$
x_i^n - x_i^0 = \left[u_i(q_i, \bar{q}_i) - u_i(q_i, \bar{q}_0)\right] - \frac{1}{n}\sum_{j=1}^{n}\left[u_j(q_j, \bar{q}_j) - u_j(q_j, \bar{q}_0)\right] \qquad (69)
$$

Finally, upon noticing that $x_i^n = x(\mathbf{q}, \bar{\mathbf{q}})$ and $x_i^0 = x_i(\mathbf{q}, \bar{\mathbf{q}}_0)$, Expression (54) yields:

$$x_i(\mathbf{q},\bar{\mathbf{q}}) = \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) + x_i(\bar{\mathbf{q}}_0,\bar{\mathbf{q}}_0) \tag{70}$$

$$+ [u_i(q_i,\bar{q}_i) - u_i(q_i,\bar{q}_0)] - \frac{1}{n}\sum_{j=1}^{n}[u_j(q_j,\bar{q}_j) - u_j(q_j,\bar{q}_0)]. \tag{71}$$

Expression (53) yields the result.

A.5 Proof of tightness of the characterization of EE by SRRN and GS

Let x^{EE} be the egalitarian equivalent solution defined relative to reference needs level $\bar{q}_0 \geq 0$ and consider a profile $(\mathbf{q},\bar{\mathbf{q}}_\mathbf{1}) \in \mathbb{D}^n$ such that $\bar{\mathbf{q}}_\mathbf{1} = (\bar{q}_1, \bar{q}_1, ..., \bar{q}_1)$ with $\bar{q}_1 > \bar{q}_0$. Then:

$$\begin{aligned} x_i^{EE}(\mathbf{q},\bar{\mathbf{q}}_\mathbf{1}) - x_i^{EE}(\bar{\mathbf{q}}_\mathbf{1},\bar{\mathbf{q}}_\mathbf{1}) = & \frac{C(n\bar{q}_0)}{n} + \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) \\ & + [u_i(q_i,\bar{q}_1) - u_i(q_i,\bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(q_k,\bar{q}_1) - u_k(q_k,\bar{q}_0)] \\ & - \left(\frac{C(n\bar{q}_0)}{n} + \xi_i(\bar{\mathbf{r}}_0, C - C(n\bar{q}_0)) \ldots \right. \\ & \left. \ldots + [u_i(\bar{q}_1,\bar{q}_1) - u_i(\bar{q}_1,\bar{q}_0)] - \frac{1}{n}\sum_{k=1}^{n}[u_k(\bar{q}_1,\bar{q}_1) - u_k(\bar{q}_1,\bar{q}_0)]\right) \end{aligned} \tag{72}$$

where $\bar{\mathbf{r}}_0 \equiv (r(\bar{q}_1,\bar{q}_0), r(\bar{q}_1,\bar{q}_0), ..., r(\bar{q}_1,\bar{q}_0)) \in \mathbb{R}_+^n$. Hence, upon noticing that $\xi_i(\bar{\mathbf{r}}_0, C - C(n\bar{q}_0)) = \frac{1}{n}(C(n\bar{q}_1) - C(n\bar{q}_0))$, Expression (72) simplifies into:

$$\begin{aligned} x_i^{EE}(\mathbf{q},\bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1,\bar{\mathbf{q}}_1) = & \xi_i(\mathbf{r}_0, C - C(n\bar{q}_0)) - \frac{1}{n}(C(n\bar{q}_1) - C(n\bar{q}_0)) \\ & + (u_i(q_i,\bar{q}_1) - u_i(\bar{q}_1,\bar{q}_1) - [u_i(q_i,\bar{q}_0) - u_i(\bar{q}_1,\bar{q}_0)]) \\ & - \frac{1}{n}\sum_{k=1}^{n}(u_k(q_k,\bar{q}_1) - u_k(q_k,\bar{q}_0) - [u_k(\bar{q}_1,\bar{q}_1) - u_k(\bar{q}_1,\bar{q}_0)]) \end{aligned} \tag{73}$$

The above expression reveals that $x_i^{EE}(\mathbf{q},\bar{\mathbf{q}}_1) - x_i^{EE}(\bar{\mathbf{q}}_1,\bar{\mathbf{q}}_1)$ depends on u_i, hence cannot be driven only by the cost sharing function ξ. In other words, it cannot be the case that:

$$x_i^{EE}\left(\mathbf{q},\bar{\mathbf{q}}_1\right)-x_i^{EE}\left(\bar{\mathbf{q}}_1,\bar{\mathbf{q}}_1\right)=\xi_i\left(\mathbf{r}_1,C-C\left(n\bar{q}_1\right)\right),$$

as required by **SRUN**.

B Section 5 Proofs

B.1 Proof of Proposition 2

Let $(\mathbf{q}\bar{\mathbf{q}})\in\mathbb{D}_+^n$. By **UWMC**,

$$\begin{aligned} x_i(\bar{\mathbf{q}},\bar{\mathbf{q}}) \quad &= \quad x_j(\bar{\mathbf{q}},\bar{\mathbf{q}}) \qquad \text{for all } i,j\in N && (74)\\ &\Longrightarrow \quad x_i(\bar{\mathbf{q}},\bar{\mathbf{q}})=\frac{C\left(\bar{Q}\right)}{n} \qquad \text{for all } i\in N && (75)\end{aligned}$$

by budget balance. Without any loss of generality, assume that $r_1\leq r_2\leq ...\leq r_n$. Let $f_i:w\mapsto r\left(w,\bar{q}_i\right)$ map consumption to individual responsibility for agent i. By construction, f_i is monotonic and strictly increasing. Its inverse, $g_i:v\mapsto f_i^{-1}\left(v\right)$ is well defined and is also monotonic and strictly increasing. Note that $g_i\left(r_i\right)=q_i$ for all $i\in N$.

Define the following profile:

$$\mathbf{q}^1=\left(q_1,g_2\left(r_1\right),...,g_i\left(r_1\right),...,g_n\left(r_1\right)\right). \tag{76}$$

Note that, by construction $(\mathbf{q}^1,\bar{\mathbf{q}})$ is such that $r_i^1=r_1$ for all $i\in N$. Applying **SR** with profile $(\mathbf{q}^1,\bar{\mathbf{q}})$ yields:

$$x_i\left(\mathbf{q}^1,\bar{\mathbf{q}}\right)-x_i\left(\bar{\mathbf{q}},\bar{\mathbf{q}}\right)=\xi_i\left(\mathbf{r}^1,C-C\left(\bar{Q}\right)\right), \tag{77}$$

By symmetry of ξ and because all r_i^1 are identical, we have:

$$\xi_i\left(\mathbf{r}^1,C-C\left(\bar{Q}\right)\right)=\frac{1}{n}\left[C\left(Q^1\right)-C\left(\bar{Q}\right)\right], \tag{78}$$

where

$$\hat{Q}^1=\sum_{i=1}^n q_i^1=\sum_{i=1}^n g_i\left(r_1\right). \tag{79}$$

Similarly, let

$$\mathbf{q}^2 = \left(q_1, q_2, g_3\left(r_2\right), ..., g_i\left(r_2\right), ..., g_n\left(r_2\right)\right). \tag{80}$$

Again by construction $(\mathbf{q}^2, \bar{\mathbf{q}})$ is such that $r_i^2 = r_2$ for all $i = 2, ..., n$. Applying now **SR** with profile $(\mathbf{q}^2, \bar{\mathbf{q}})$ yields:

$$x_i\left(\mathbf{q}^2, \bar{\mathbf{q}}\right) - x_i\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) = \xi_i\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right). \tag{81}$$

As before, the symmetry of ξ yields:

$$\xi_i\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right) = \xi_j\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right), \tag{82}$$

for all $i, j \geq 2$. Moreover, because $r_1 \leq r_2$, applying **IHR** between profiles $(\mathbf{q}^1, \bar{\mathbf{q}})$ and $(\mathbf{q}^2, \bar{\mathbf{q}})$ yields that agent 1's contribution is the same under both profiles:

$$\xi_1\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \xi_1\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right) = \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C\left(\bar{Q}\right)\right], \tag{83}$$

Thus, agents $2, ..., n$ share the remaining cost equally:

$$\begin{aligned} \xi_i\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right) &= \frac{1}{n-1}\left[C\left(\hat{Q}^2\right) - C\left(\bar{Q}\right) - \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C(\bar{Q})\right]\right] && (84) \\ &= \frac{1}{n-1}\left[C\left(\hat{Q}^2\right) - C\left(\hat{Q}^1\right)\right] + \frac{1}{n}\left[C\left(\hat{Q}^1\right) - C(\bar{Q})\right] && (85) \end{aligned}$$

for all $i \geq 2$, where

$$\hat{Q}^2 = \sum_{i=1}^{n} q_i^2 = q_1 + \sum_{i=2}^{n} g_i\left(r_2\right) \geq \hat{Q}^1. \tag{86}$$

Alternatively,

$$\xi_i\left(\mathbf{r}^2, C - C\left(\bar{Q}\right)\right) - \xi_i\left(\mathbf{r}^1, C - C\left(\bar{Q}\right)\right) = \frac{1}{n-1}\left[C\left(\hat{Q}^2\right) - C\left(\hat{Q}^1\right)\right]$$

all $i \geq 2$.

Similarly, for all $k \geq 2$, we define

$$\mathbf{q}^k = \left(q_1, q_2, ..., q_k, g_{k+1}\left(r_k\right), ..., g_n\left(r_k\right)\right), \tag{87}$$

and obtain by **SR** that

$$x_i\left(\mathbf{q}^k,\bar{\mathbf{q}}\right)-x_i\left(\bar{\mathbf{q}},\bar{\mathbf{q}}\right)=\xi_i\left(\mathbf{r}^k,C-C\left(\bar{Q}\right)\right), \tag{88}$$

for all $i\in N$. It follows that

$$\begin{cases}\xi_i\left(\mathbf{r}^k,C-C\left(\bar{Q}\right)\right)-\xi_i\left(\mathbf{r}^{k-1},C-C\left(\bar{Q}\right)\right)=0 & \text{for all } i<k, \text{ and}\\ \xi_i\left(\mathbf{r}^k,C-C\left(\bar{Q}\right)\right)-\xi_i\left(\mathbf{r}^{k-1},C-C\left(\bar{Q}\right)\right)=\frac{1}{n-k+1}\left[C\left(\hat{Q}^k\right)-C\left(\hat{Q}^{k-1}\right)\right] & \text{for all } i\geq k,\end{cases} \tag{89}$$

with

$$\hat{Q}^k=\sum_{i=1}^n q_i^k=\sum_{i=1}^{k-1}q_i+\sum_{i=k}^n g_i\left(r_k\right). \tag{90}$$

Observe that

$$\hat{Q}^{k+1}-\hat{Q}^k=\sum_{i=k+1}^n\left[g_i\left(r_{k+1}\right)-g_i\left(r_k\right)\right]\geq 0 \tag{91}$$

by monotonicity of the g_i's. It follows that $x_{i+1}\left(\mathbf{q}^k,\bar{\mathbf{q}}\right)\geq x_i\left(\mathbf{q}^k,\bar{\mathbf{q}}\right)$, for all $i\in N$, so that agents with a higher r_i pay a higher bill for all k.

To sum up, upon observing that $\mathbf{r}^n=\mathbf{r}$ (as associated to profile $(\mathbf{q},\bar{\mathbf{q}})$), we obtain

$$x_k\left(\mathbf{q},\bar{\mathbf{q}}\right)-x_k\left(\bar{\mathbf{q}},\bar{\mathbf{q}}\right)=\sum_{i=1}^k\frac{1}{n-i+1}\left[C\left(\hat{Q}^i\right)-C\left(\hat{Q}^{i-1}\right)\right] \tag{92}$$

where $Q^0=\bar{Q}$. Finally,

$$\begin{aligned}x_k\left(\mathbf{q},\bar{\mathbf{q}}\right) &= \frac{1}{n}C\left(\bar{Q}\right)+\sum_{i=1}^k\frac{1}{n-i+1}\left[C\left(\hat{Q}^i\right)-C\left(\hat{Q}^{i-1}\right)\right] && (93)\\ &= \left[\frac{1}{n}-\frac{1}{n}\right]C\left(\hat{Q}^0\right)+\left[\frac{1}{n}-\frac{1}{n-1}\right]C\left(\hat{Q}^1\right)+\left[\frac{1}{n-1}-\frac{1}{n-2}\right]C\left(\hat{Q}^2\right) && (94)\\ &\quad +...+\left[\frac{1}{n-i+1}-\frac{1}{n-i}\right]C\left(\hat{Q}^i\right)+...+\frac{1}{n-k+1}C\left(\hat{Q}^k\right)\\ x_k\left(\mathbf{q},\bar{\mathbf{q}}\right) &= \frac{C\left(\hat{Q}^k\right)}{(n-k+1)}-\sum_{i=1}^{k-1}\frac{C\left(\hat{Q}^i\right)}{(n-i)\,(n-i+1)}=\frac{C\left(\hat{Q}^k\right)}{n-k}-\sum_{i=1}^k\frac{C\left(\hat{Q}^i\right)}{(n-i)\,(n-i+1)} && (95)\end{aligned}$$

with

$$\hat{Q}^k = \sum_{i=1}^{k-1} q_i + \sum_{i=k}^{n} g_i\left(r_k\right). \tag{96}$$

B.2 Proof of Proposition 3

Let $(\mathbf{q}\bar{\mathbf{q}}) \in \mathbb{D}_+^n$. Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}_+^n$ the associated reference vector. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ such that $\min_i q_i \geq \bar{q}_0$. By budget balance and anonymity,

$$x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n} \qquad \text{for all } i \in N. \tag{97}$$

Without loss of generality, assume that $q_1 \leq q_2 \leq ... \leq q_n$, so that $r_{0,1} \leq r_{0,2} \leq ... \leq r_{0,n}$, where $r_{0,i} = r\left(q_i, \bar{q}_0\right)$ for all $i \in N$.
For all $k \in N$, define

$$\mathbf{q}^k = \left(q_1, q_2, ..., q_{k-1}, q_k, ..., q_k\right). \tag{98}$$

Notice that $\mathbf{q}^1 = (q_1, q_1, ..., q_1)$; hence, by anonymity,

$$x_i\left(\mathbf{q}^1, \bar{\mathbf{q}}_0\right) = \frac{C\left(nq_1\right)}{n} \tag{99}$$

and

$$x_i\left(\mathbf{q}^1, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \frac{1}{n}\left[C\left(nq_1\right) - C\left(n\bar{q}_0\right)\right] \tag{100}$$

for all $i \in N$.

Similarly, for $k \geq 2$, **SRRN** yields

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C\left(n\bar{q}_0\right)\right) \tag{101}$$

and

$$x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^{k-1}, C - C\left(n\bar{q}_0\right)\right) \tag{102}$$

for all $i \in N$, with $r_{0,i}^k = r\left(q_i^k, \bar{q}_0\right)$ and $r_{0,i}^{k-1} = r\left(q_i^{k-1}, \bar{q}_0\right)$. Therefore, by subtraction,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C\left(n\bar{q}_0\right)\right) - \xi_i\left(\mathbf{r}_0^{k-1}, C - C\left(n\bar{q}_0\right)\right) \tag{103}$$

for all $i \in N$. Summing up over all agents, we find:

$$\sum_{i=1}^{n} \left[x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right)\right] = C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right), \tag{104}$$

where $\tilde{Q}^{k-1} = \sum_{l=1}^{n} q_l^{k-1} = \sum_{l=1}^{k-1} q_l + (n-k+1)\, q_{k-1}$ and $\tilde{Q}^k = \sum_{l=1}^{n} q_l^k = \sum_{l=1}^{k} q_l + (n-k)\, q_k$.

Observe that if $i < j$ then $r_{0,i}^{k-1} \leq r_{0,j}^{k-1}$ and $r_{0,i}^k \leq r_{0,j}^k$. Moreover for all $1 \leq i \leq k-1$, $\mathbf{q}_i^{k-1} = \mathbf{q}_i^k = q_i$, and $\mathbf{r}_{0,i}^{k-1} = \mathbf{r}_{0,i}^k = r\left(q_i, \bar{q}_0\right)$. Therefore, by **IHR**,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = 0, \tag{105}$$

for all $1 \leq i \leq k-1$. It follows that the previous summation can truncated from below:

$$\sum_{i=k}^{n} \left[x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right)\right] = C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right). \tag{106}$$

Moreover, for all $i, j \geq k$, we have $q_i^{k-1} = q_j^{k-1} = q_{k-1}$ and $q_i^k = q_j^k = q_k$. Therefore, by anonymity,

$$x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) \qquad \text{and} \qquad x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) \tag{107}$$

for all $i, j \geq k$.

Hence,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right] \tag{108}$$

for all $i \geq k$, with the convention that $\tilde{Q}^0 = n\bar{q}_0$.
Finally, upon observing that $\mathbf{q}^n = \mathbf{q}$, it follows by summation that

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right]; \tag{109}$$

i.e., substituting according to Expression (97):

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0\right) = \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right] \tag{110}$$

We now work along the needs dimension. Define $\bar{\mathbf{q}}_0^1 = (\bar{q}_1, \bar{q}_0, ..., \bar{q}_0)$. Applying **GS** between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ yields, for all $j \neq 1$:

$$u(q_1, \bar{q}_1) - x_1^1 - u(q_1, \bar{q}_0) + x_1^0 = u(q_j, \bar{q}_0) - x_j^1 - u(q_j, \bar{q}_0) + x_j^0, \tag{111}$$

where $x_j^0 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $x_j^1 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ for all $j \in N$. This yields

$$x_j^0 - x_j^1 = u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0) + x_1^0 - x_1^1. \tag{112}$$

Since total consumption is unchanged, we have, by budget balance

$$x_1^1 - x_1^0 = \frac{n-1}{n}\left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right], \text{ and} \tag{113}$$

$$x_j^1 - x_j^0 = -\frac{1}{n}\left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right]. \tag{114}$$

for all $j \neq 1$.

Iterating and applying **GS** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all $1 \leq i \leq k \leq j \leq n$:

$$u(q_i, \bar{q}_i) - x_i^k - u(q_i, \bar{q}_i) + x_i^{k-1} = u(q_k, \bar{q}_k) - x_k^k - u(q_k, \bar{q}_0) + x_k^{k-1} \tag{115}$$

$$= u(q_j, \bar{q}_0) - x_j^k - u(q_j, \bar{q}_0) + x_j^{k-1} \tag{116}$$

where $x_j^{k-1} = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1}\right)$ and $x_j^k = x_j\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$. Hence, for all $k = 1, ..., n$, and all $1 \leq i \leq k \leq j \leq n$:

$$x_i^{k-1} - x_i^k = u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) + x_k^{k-1} - x_k^k \tag{117}$$

$$= x_j^{k-1} - x_j^k \tag{118}$$

Since total consumption does not change from $\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1}\right)$ to $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$, but only needs, budget balance implies $\sum_j \left(x_j^k - x_j^{k-1}\right) = 0$. Therefore,

$$x_j^k - x_j^{k-1} = -\frac{1}{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \qquad \text{for all } j \neq k, \text{ and} \tag{119}$$

$$x_k^k - x_k^{k-1} = \frac{n-1}{n}\left[u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0)\right] \tag{120}$$

Summing up over all iterations $k = 1, ..., n$ yields the following for agent 1:

$$x_1^n - x_1^0 = \sum_{k>1}^{n} \left(x_1^k - x_1^{k-1}\right) + x_1^1 - x_1^0 \tag{121}$$

$$= -\frac{1}{n}\sum_{k>1}^{n} \left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] + \frac{n-1}{n}\left[u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right)\right] \tag{122}$$

$$= \left[u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n} \left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{123}$$

Similarly, for all $i > 1$:

$$x_i^n - x_i^0 = \left[u\left(q_i, \bar{q}_i\right) - u\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n} \left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{124}$$

Finally, observing that $\bar{\mathbf{q}}_0^n = \bar{\mathbf{q}}$, Expressions (110) and (124) yield the following :

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) = x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0^n\right) = \frac{C(n\bar{q}_0)}{n} + \sum_{k=1}^{i} \frac{1}{n-k+1}\left[C\left(\tilde{Q}^k\right) - C\left(\tilde{Q}^{k-1}\right)\right] \tag{125}$$

$$+ \left[u\left(q_i, \bar{q}_i\right) - u\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n} \left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right]$$

for all $i \in N$, where $\tilde{Q}^k = \sum_{l=1}^{k} q_l + (n-k)\, q_k$ for all $k = 1, ..., n$. Rearranging, we get the desired result:

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) = \frac{C\left(\tilde{Q}^i\right)}{n-i+1} - \sum_{k=1}^{i-1} \frac{C\left(\tilde{Q}^k\right)}{(n-k)(n-k+1)} \tag{126}$$

$$+ \left[u\left(q_i, \bar{q}_i\right) - u\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n} \left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right].$$

B.3 Proof of Proposition 4

The proof follows the same blueprint as that of Proposition 2. Let $(\mathbf{q}\bar{\mathbf{q}}) \in \mathbb{D}_+^n$. By **UWMC**,

$$x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = x_j(\bar{\mathbf{q}}, \bar{\mathbf{q}}) \quad \text{for all } i, j \in N \tag{127}$$

$$\Longrightarrow \quad x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \frac{C\left(\bar{Q}\right)}{n} \tag{128}$$

Without loss of generality, assume that $r_1 \leq r_2 \leq ... \leq r_n$. Let $f_i : w \mapsto r\left(w, \bar{q}_i\right)$ map consumption to the individual responsibility for agent i. By construction, f_i is monotonic and strictly increasing. Its inverse, $g_i : v \mapsto f_i^{-1}\left(v\right)$ is well defined, monotonic and strictly increasing. Note that $g_i\left(r_i\right) = q_i$ for all $i \in N$.

Define the following profile:

$$\mathbf{q}^n = \left(g_1\left(r_n\right), ..., g_i\left(r_n\right), ..., g_{n-1}\left(r_n\right), q_n\right). \tag{129}$$

Note that, by construction $(\mathbf{q}^n, \bar{\mathbf{q}})$ is such that

$$r_i^n = r_n, \tag{130}$$

for all $i \in N$.

Applying **SR** to profile $(\mathbf{q}^n, \bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q}^n, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^n, C - C(\bar{Q})), \tag{131}$$

where $\xi_i(\mathbf{r}, C - C(\bar{Q}))$ is symmetric in $\mathbf{r}$. Since all r_i^n's are identical, we have

$$\xi_i(\mathbf{r}^n, C - C(\bar{Q})) = \frac{1}{n}\left[C\left(\check{Q}^n\right) - C(\bar{Q})\right], \tag{132}$$

where

$$\check{Q}^n = \sum_{i=1}^{n} q_i^n = \sum_{i=1}^{n} g_i\left(r_n\right). \tag{133}$$

This gives

$$x_i(\mathbf{q}^n, \bar{\mathbf{q}}) = \frac{1}{n} C\left(\check{Q}^n\right). \tag{134}$$

Similarly, let

$$\mathbf{q}^{n-1}= \left(g_1\left(r_{n-1}\right), ..., g_i\left(r_{n-1}\right), ..., g_{n-2}\left(r_{n-1}\right), q_{n-1}, q_n\right). \tag{135}$$

Again, by construction $(\mathbf{q}^{n-1}, \bar{\mathbf{q}})$ is such that

$$r_i^{n-1} = r_{n-1}, \tag{136}$$

for all $i = 1, ..., n-1$.
Applying now **SR** to profile $(\mathbf{q}^{n-1}, \bar{\mathbf{q}})$ yields:

$$x_i(\mathbf{q}^{n-1}, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^{n-1}, C - C(\bar{Q})), \tag{137}$$

where $\xi_i(\mathbf{r}, C - C(\bar{Q}))$ is symmetric in $\mathbf{r}$, therefore

$$\xi_i(r^{n-1}, C - C(\bar{Q})) = \xi_j(\mathbf{r}^{n-1}, C - C(\bar{Q})) \tag{138}$$

for all $i, j \leq n-1$. In words, agents $1, ..., n-1$ are assigned the same cost share. Moreover, by assumption $r_n \geq r_{n-1}$. Thus, applying **ILR** between profiles $(\mathbf{q}^n, \bar{\mathbf{q}})$ and $(\mathbf{q}^{n-1}, \bar{\mathbf{q}})$ yields that agent n's contribution is the same under both profiles:

$$\xi_n(\mathbf{r}^{n-1}, C - C(\bar{Q})) = \xi_n(\mathbf{r}^n, C - C(\bar{Q})) = \frac{1}{n}\left[C\left(\check{Q}^n\right) - C(\bar{Q})\right], \tag{139}$$

Thus, agents $1, ..., n-1$ share the remaining cost equally:

$$\begin{aligned} \xi_i(\mathbf{r}^{n-1}, C - C(\bar{Q})) &= \frac{1}{n-1}\left[C\left(\check{Q}^{n-1}\right) - C\left(\bar{Q}\right) - \frac{1}{n}\left[C\left(\check{Q}^n\right) - C(\bar{Q})\right]\right] && (140) \\ &= \frac{1}{n}\left[C\left(\check{Q}^n\right) - C(\bar{Q})\right] - \frac{1}{n-1}\left[C\left(\check{Q}^n\right) - C\left(\check{Q}^{n-1}\right)\right] && (141) \end{aligned}$$

for all $i \leq n-1$, where

$$\check{Q}^{n-1} = \sum_{i=1}^{n} q_i^{n-1} = \sum_{i=1}^{n-1} g_i\left(r_{n-1}\right) + q_n \leq \check{Q}^n. \tag{142}$$

Alternatively,

$$\xi_i(\mathbf{r}^{n-1}, C - C(\bar{Q})) - \xi_i(\mathbf{r}^n, C - C(\bar{Q})) = -\frac{1}{n-1}\left[C\left(\check{Q}^n\right) - C\left(\check{Q}^{n-1}\right)\right] \tag{143}$$

all $i \leq n-1$.

Similarly, for all $k \leq n-1$, we define

$$\mathbf{q}^k = \left(g_1\left(r_k\right), ..., g_{k-1}\left(r_k\right), q_k, ..., q_{n-1}, q_n\right) \tag{144}$$

and obtain by **SR** that

$$x_i(\mathbf{q}^k, \bar{\mathbf{q}}) - x_i(\bar{\mathbf{q}}, \bar{\mathbf{q}}) = \xi_i(\mathbf{r}^k, C - C(\bar{Q})), \tag{145}$$

for all $i \in N$. It follows that

$$\xi_i(\mathbf{r}^k, C - C(\bar{Q})) - \xi_i(\mathbf{r}^{k+1}, C - C(\bar{Q})) = 0 \qquad \text{for all } i > k\text{, and} \tag{146}$$

$$\xi_i(\mathbf{r}^k, C - C(\bar{Q})) - \xi_i(\mathbf{r}^{k+1}, C - C(\bar{Q})) = -\frac{1}{k}\left[C\left(\check{Q}^{k+1}\right) - C\left(\check{Q}^k\right)\right] \qquad \text{for all } i \leq k, \tag{147}$$

with

$$\check{Q}^k = \sum_{i=1}^{n} q_i^k = \sum_{i=1}^{k} g_i\left(r_k\right) + \sum_{i=k+1}^{n} q_i. \tag{148}$$

Observe that

$$\check{Q}^{k+1} - \check{Q}^k = \sum_{i=1}^{k}\left[g_i\left(r_{k+1}\right) - g_i\left(r_k\right)\right] \geq 0 \tag{149}$$

by monotonicity of the g_i's. It follows that $x_{i+1}\left(\mathbf{q}^k, \bar{\mathbf{q}}\right) \geq x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}\right)$, all $i \in N$. It follows from our initial ordering of the agents that agents with a higher r_i pay a higher bill for all k.

To sum up, upon observing that $\mathbf{r}^1 = \mathbf{r}$ (as associated to $(\mathbf{q}, \bar{\mathbf{q}})$), we obtain

$$x_k\left(\mathbf{q}, \bar{\mathbf{q}}\right) - x_k\left(\bar{\mathbf{q}}, \bar{\mathbf{q}}\right) = \frac{1}{n}\left[C\left(\check{Q}^n\right) - C(\bar{Q})\right] - \sum_{i=k}^{n-1}\frac{1}{i}\left[C\left(\check{Q}^{i+1}\right) - C\left(Q^i\right)\right] \tag{150}$$

where $Q^1 = Q$. Finally,

$$x_k(\mathbf{q}, \bar{\mathbf{q}}) = \frac{1}{n}C(Q^n) - \sum_{i=k}^{n-1}\frac{1}{i}\left[C\left(\check{Q}^{i+1}\right) - C\left(\check{Q}^{i}\right)\right] \tag{151}$$

$$= \left[\frac{1}{n} - \frac{1}{n-1}\right]C\left(\check{Q}^n\right) + \left[\frac{1}{n-1} - \frac{1}{n-2}\right]C\left(\check{Q}^{n-1}\right) + \left[\frac{1}{n-2} - \frac{1}{n-3}\right]C\left(\check{Q}^{n-2}\right) \tag{152}$$

$$+ ... + \left[\frac{1}{i} - \frac{1}{i-1}\right]C\left(\check{Q}^i\right) + ... + \frac{1}{k}C\left(\check{Q}^k\right)$$

$$x_k(\mathbf{q}, \bar{\mathbf{q}}) = \frac{C\left(\check{Q}^k\right)}{k-1} - \sum_{i=k}^{n}\frac{C\left(\check{Q}^i\right)}{i(i-1)} = \frac{C\left(\check{Q}^k\right)}{k} - \sum_{i=k+1}^{n}\frac{C\left(\check{Q}^i\right)}{i(i-1)} \tag{153}$$

with

$$\check{Q}^k = \sum_{i=1}^{k} g_i(r_k) + \sum_{i=k+1}^{n} q_i. \tag{154}$$

B.4 Proof of Proposition 5

The proof follows the same blueprint as that of Proposition 3. Let $\bar{q}_0 \in \mathbb{R}_+$ be a reference level of needs and denote by $\bar{\mathbf{q}}_0 = (\bar{q}_0, \bar{q}_0, ..., \bar{q}_0) \in \mathbb{R}_+^n$ the associated reference vector. Let $(\mathbf{q}, \bar{\mathbf{q}}) \in \mathbb{D}^n$ such that $\min_i q_i \geq \bar{q}_0$. By budget balance and anonymity,

$$x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \frac{C(n\bar{q}_0)}{n}. \tag{155}$$

Without loss of generality, assume that $q_1 \leq q_2 \leq ... \leq q_n$, so that $r_{0,1} \leq r_{0,2} \leq ... \leq r_{0,n}$, where $r_{0,i} = r(q_i, \bar{q}_0)$ for all $i \in N$.

Define

$$\mathbf{q}^k = (q_k, ..., q_k, q_{k+1}, ..., q_{n-1}, q_n) \tag{156}$$

for all $k = 1...n$.

Notice that $\mathbf{q}^n = (q_n, q_n, ..., q_n)$; hence, by anonymity, $x_i(\mathbf{q}^n, \bar{\mathbf{q}}_0) = C(nq_n)/n$ so that

$$x_i(\mathbf{q}^n, \bar{\mathbf{q}}_0) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \frac{1}{n}\left[C(nq_n) - C(n\bar{q}_0)\right] \tag{157}$$

for all $i \in N$.

Similarly, for $k \leq n-1$, **SRRN** yields

$$x_i\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \xi_i\left(\mathbf{r}_0^{k+1}, C - C(n\bar{q}_0)\right) \tag{158}$$

and

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C\left(n\bar{q}_0\right)\right) \tag{159}$$

with $r_{0,i}^{k+1} = r\left(q_i^{k+1}, \bar{q}_0\right)$ and $r_{0,i}^k = r\left(q_i^k, \bar{q}_0\right)$. Therefore, by subtraction,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right) = \xi_i\left(\mathbf{r}_0^k, C - C\left(n\bar{q}_0\right)\right) - \xi_i\left(\mathbf{r}_0^{k+1}, C - C\left(n\bar{q}_0\right)\right) \tag{160}$$

for all $i \in N$ and summing up over all agents, we find

$$\sum_{i=1}^{n}\left[x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right)\right] = -\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right], \tag{161}$$

where $\breve{Q}^k = \sum_{l=1}^n q_l^k = kq_k + \sum_{l=k+1}^n q_l$ and $\breve{Q}^{k+1} = \sum_{l=1}^n q_l^{k+1} = (k-1)\, q_{k+1} + \sum_{l=k+1}^n q_l$.

Observe that if $i < j$ then $r_{0,i}^k \leq r_{0,j}^k$ and $r_{0,i}^{k+1} \leq r_{0,j}^{k+1}$. Moreover for all $k+1 \leq i \leq n$, $\mathbf{q}_i^k = \mathbf{q}_i^{k+1} = q_i$, and $\mathbf{r}_{0,i}^k = \mathbf{r}_{0,i}^{k+1} = r\left(q_i, \bar{q}_0\right)$. Therefore, by **ILR**,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right) = 0, \tag{162}$$

for all $k+1 \leq i \leq n$. It follows that the previous summation can truncated from above:

$$\sum_{i=1}^{k}\left[x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right)\right] = -\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right], \tag{163}$$

where $1 \leq k \leq n-1$.

Moreover, for all $i, j \leq k$, we have $q_i^{k+1} = q_j^{k+1} = q_{k+1}$ and $q_i^k = q_j^k = q_k$. Therefore, by anonymity,

$$x_i\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^{k+1}, \bar{\mathbf{q}}_0\right) \tag{164}$$

all $i, j \leq k$, and

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) = x_j\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right). \tag{165}$$

Hence,

$$x_i\left(\mathbf{q}^k, \bar{\mathbf{q}}_0\right) - x_i\left(\mathbf{q}^{k-1}, \bar{\mathbf{q}}_0\right) = -\frac{1}{k}\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right] \tag{166}$$

all $i \leq k$.
Finally, upon observing that $\mathbf{q}^1 = \mathbf{q}$, it follows by summation that

$$x_i(\mathbf{q}, \bar{\mathbf{q}}_0) - x_i(\bar{\mathbf{q}}_0, \bar{\mathbf{q}}_0) = \frac{1}{n}\left[C(nq_n) - C(n\bar{q}_0)\right] - \sum_{k=i}^{n-1} \frac{1}{k}\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right], \quad (167)$$

so that

$$x_i(\mathbf{q}, \bar{\mathbf{q}}_0) = \frac{1}{n} C(nq_n) - \sum_{k=i}^{n-1} \frac{1}{k}\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right]. \quad (168)$$

Define $\bar{\mathbf{q}}_0^1 = (\bar{q}_1, \bar{q}_0, ..., \bar{q}_0)$. Applying **GS** between $(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ yields, for all $j \neq 1$:

$$u(q_1, \bar{q}_1) - x_1^1 - u(q_1, \bar{q}_0) + x_1^0 = u(q_j, \bar{q}_0) - x_j^1 - u(q_j, \bar{q}_0) + x_j^0$$

where $x_j^0 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0)$ and $x_j^1 = x_j(\mathbf{q}, \bar{\mathbf{q}}_0^1)$ for all $j \in N$. This yields:

$$x_j^0 - x_j^1 = u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0) + x_1^0 - x_1^1. \quad (169)$$

Becuase total consumption is unchanged, we have by budget balance:

$$x_1^1 - x_1^0 = \frac{n-1}{n}\left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right] \quad (170)$$

$$x_j^1 - x_j^0 = -\frac{1}{n}\left[u(q_1, \bar{q}_1) - u(q_1, \bar{q}_0)\right] \quad (171)$$

all $j \neq 1$.
Iterating and applying **GS** to profiles $\left(\mathbf{q}, \bar{\mathbf{q}}_0^k\right)$ where $\bar{\mathbf{q}}_0^k = (\bar{q}_1, \bar{q}_2, ..., \bar{q}_k, \bar{q}_0, ..., \bar{q}_0)$, successively leads to the following expression, for all iterations, $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$u(q_i, \bar{q}_i) - x_i^k - u(q_i, \bar{q}_i) + x_i^{k-1} = u(q_k, \bar{q}_k) - x_k^k - u(q_k, \bar{q}_0) + x_k^{k-1} \quad (172)$$

$$= u(q_j, \bar{q}_0) - x_j^k - u(q_j, \bar{q}_0) + x_j^{k-1} \quad (173)$$

Hence, for all $k = 1, ..., n$, and all agents $1 \leq i \leq k \leq j \leq n$:

$$x_i^{k-1} - x_i^k = u(q_k, \bar{q}_k) - u(q_k, \bar{q}_0) + x_k^{k-1} - x_k^k \quad (174)$$

$$= x_j^{k-1} - x_j^k \quad (175)$$

Because total consumption does not change from $\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k-1}\right)$ to $\left(\mathbf{q}, \bar{\mathbf{q}}_0^{k}\right)$, but only needs, budget balance implies $\sum_j \left(x_j^k - x_j^{k-1}\right) = 0$. Therefore,

$$x_j^k - x_j^{k-1} = -\frac{1}{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \qquad \text{for all } j \neq k \tag{176}$$

$$x_k^k - x_k^{k-1} = \frac{n-1}{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{177}$$

Summing up over all iterations $k = 1, ..., n$ yields the following for agent 1:

$$x_1^n - x_1^0 = \sum_{k>1}^{n}\left(x_1^k - x_1^{k-1}\right) + x_1^1 - x_1^0 \tag{178}$$

$$= -\frac{1}{n}\sum_{k>1}^{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] + \frac{n-1}{n}\left[u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right)\right] \tag{179}$$

$$= \left[u\left(q_1, \bar{q}_1\right) - u\left(q_1, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{180}$$

Similarly, for all $i > 1$:

$$x_i^n - x_i^0 = \left[u\left(q_i, \bar{q}_i\right) - u\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{181}$$

Finally, observing that $\bar{\mathbf{q}}_0^n = \bar{\mathbf{q}}$ yields the following:

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) = x_i\left(\mathbf{q}, \bar{\mathbf{q}}_0^n\right) = \frac{1}{n}C\left(\breve{Q}^n\right) - \sum_{k=i}^{n-1}\frac{1}{k}\left[C\left(\breve{Q}^{k+1}\right) - C\left(\breve{Q}^k\right)\right] + \left[u\left(q_i, \bar{q}_i\right) - u\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{182}$$

Rearranging, we obtain the desired result:

$$x_i\left(\mathbf{q}, \bar{\mathbf{q}}\right) = \frac{C\left(\breve{Q}^i\right)}{i} - \sum_{k=i+1}^{n-1}\frac{C\left(\breve{Q}^k\right)}{k\left(k-1\right)} + \left[u\left(q_i, \bar{q}_i\right) - u\left(q_i, \bar{q}_0\right)\right] - \frac{1}{n}\sum_{k=1}^{n}\left[u\left(q_k, \bar{q}_k\right) - u\left(q_k, \bar{q}_0\right)\right] \tag{183}$$

where $\breve{Q}^k = kq_k + \sum_{l=k+1}^{n} q_l$ for all $k = 1, ..., n$.

C Supplementary material: Calculations not intended for publication

For the upcoming calculations, it will be convenient to introduce the $N_s = \int_{z=0}^{\infty} n_s(z)\,dz$ is the total number of type-s households.

C.1 Decreasing Returns to Scale: Quadratic Costs

SCE0 with absolute responsibility

Recall that

$$x^{SCE0}(\rho) = \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right) \frac{d\hat{Q}(z)}{d\rho} dz, \tag{184}$$

where

$$\hat{Q}(\rho) = \sum_{s \in S} \left[\int_0^{+\infty} \inf\{g_s(z), g_s(\rho)\} n_s^r(z)\,dz \right]. \tag{185}$$

Under the absolute responsibility view,

$$\frac{d\hat{Q}(\rho)}{d\rho} = \sum_{s \in S} (N_s - N_s^r(\rho))\, g_s'(\rho) = N - N^r(\rho), \tag{186}$$

with the second equality following from the fact that $g_s(\rho) \equiv \bar{q}_s + \rho$. Hence,

$$x^{SCE0}(\rho) = \frac{C(\bar{Q})}{N} + \int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz, \tag{187}$$

with

$$\begin{aligned}
\hat{Q}_s(\rho) &= \int_0^{\rho} (\bar{q}_s + z)\, n_s^r(z)\,dz + (N_s - N_s^r(\rho))(\bar{q}_s + \rho) && (188)\\
&= \bar{q}_s N_s + \int_0^{\rho} z n_s^r(z)\,dz + (N_s - N_s^r(\rho))\,\rho && (189)\\
&= \bar{q}_s N_s + \int_0^{+\infty} \min\{z, \rho\} n_s^r(z)\,dz, && (190)
\end{aligned}$$

so that

$$\hat{Q}(\rho) = \bar{Q} + \int_0^{+\infty} \min\{z, \rho\} n^r(z)\, dz \tag{191}$$

$$= \bar{Q} + \int_0^{r} z n^r(z)\, dz + (N - N^r(\rho))\,\rho \tag{192}$$

Consider the case where $C(Q) = \frac{c}{2}Q^2$. It follows that $C'(Q) = cQ$, so that

$$x^{SCE0}(\rho) = \frac{c\bar{Q}^2}{2N} + c\int_{z=0}^{\rho} \hat{Q}(z)\, dz \tag{193}$$

$$= \frac{c\bar{Q}^2}{2N} + c\int_{z=0}^{\rho} \left[\bar{Q} + \int_{y=0}^{+\infty} \min\{y, z\} n^r(y)\, dy\right] dz \tag{194}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \int_{z=0}^{\rho} \min\{y, z\} dz dy \tag{195}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[\int_{z=0}^{y} z dz + \int_{z=y}^{\rho} y dz\right] dy \tag{196}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[\frac{y^2}{2} + y(\rho - y)\right] dy, \tag{197}$$

$$= \frac{c\bar{Q}^2}{2N} + c\bar{Q}\rho + c\int_{y=0}^{+\infty} n^r(y) \left[y\rho - \frac{y^2}{2}\right] dy. \tag{198}$$

Upon noticing that $\bar{Q} + \int_{y=0}^{+\infty} n^r(y)\, y dy = Q$ under absolute responsibility, the above expression rewrites as follows:

$$x^{SCE0}(\rho) = \frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy + cQ\rho. \tag{199}$$

By budget balance,

$$c\frac{Q^2}{2} = \int_{z=0}^{+\infty} x^{SCE0}(z)\, n^r(z)\, dz \tag{200}$$

$$= N\left[\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy\right] + cQ\int_{z=0}^{+\infty} z n^r(z) dz \tag{201}$$

$$= N\left[\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y) \frac{y^2}{2} dy\right] + cQ\left(Q - \bar{Q}\right). \tag{202}$$

Thus,

$$\frac{c\bar{Q}^2}{2N} - c\int_{y=0}^{+\infty} n^r(y)\,\frac{y^2}{2}dy = \frac{1}{N}\left(\frac{cQ^2}{2} - cQ\left(Q - \bar{Q}\right)\right). \tag{203}$$

Finally, it follows that

$$x^{SCE0}(\rho) = \frac{1}{N}\left[cQ\left(\bar{Q} - \frac{Q}{2}\right)\right] + cQ\rho \tag{204}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + cQ\left(\rho - \frac{Q - \bar{Q}}{N}\right) \tag{205}$$

Upon recalling that $\rho = q - \bar{q}_s$ under absolute responsibility, we obtain the result:

$$x^{SCE0}(q, s) = \frac{1}{N}\frac{cQ^2}{2} + cQ\left(q - \bar{q}_s - \frac{Q - \bar{Q}}{N}\right). \tag{206}$$

SCE0 with relative responsibility

Recall that

$$x^{SCE}(\rho) = \frac{C\left(\bar{Q}\right)}{N} + \int_{z=0}^{\rho} \frac{1}{N - N^r(z)} C'\left(\hat{Q}(z)\right)\frac{d\hat{Q}(z)}{dz}dz, \tag{207}$$

where, from Expressions (191) and (192), we can write:

$$\hat{Q}(\rho) = \sum_{s\in S}\left[\int_0^{+\infty} g_s\left(\inf\{z, \rho\}\right) n_s^r(z)\,dz\right] \tag{208}$$

$$= \sum_{s\in S}\left[\int_0^{\rho} g_s(z)\, n_s^r(z)\,dz + \left(N_s - N_s^r(\rho)\right) g_s(\rho)\right] \tag{209}$$

Under relative responsibility, $\rho = (q - \bar{q}_s)/\bar{q}_s$ so that $g_s(\rho) = \bar{q}_s(1 + \rho)$. It follows that $g_s'(\rho) = \bar{q}_s$ and

$$\frac{d\hat{Q}_s(\rho)}{d\rho} = \left(N_s - N_s^r(\rho)\right) g_s'(\rho) = \left(N_s - N_s^r(\rho)\right)\bar{q}_s. \tag{210}$$

We now make an additional assumption. Namely, we posit that responsibility is evenly spread across types, so that its distribution is independent of needs, $\bar{q}_s$:

$$N_s^r(\rho) = \alpha(\rho)\, N_s \qquad \forall s \in S, \tag{211}$$

for some increasing function $\alpha : \mathbb{R}_+ \to [0,1]$ which we take to be differentiable. This yields:

$$\frac{d\hat{Q}(\rho)}{d\rho} = (1-\alpha(\rho))\,\bar{Q}. \tag{212}$$

Also, because $N - N^r(\rho) = (1-\alpha(\rho))\,N$, we have

$$\frac{1}{N - N^r(\rho)} \frac{d\hat{Q}(\rho)}{d\rho} = \frac{\bar{Q}}{N}, \tag{213}$$

so that $x^{SCE0}(\rho)$ simplifies to

$$x^{SCE0}(\rho) = \frac{C(\bar{Q})}{N} + \frac{\bar{Q}}{N} \int_{z=0}^{\rho} C'\left(\hat{Q}(z)\right) dz. \tag{214}$$

Upon noticing that $n_s^r(\rho) = \alpha'(\rho)\,N_s$ we get

$$\begin{aligned}
\hat{Q}(\rho) &= \int_0^{+\infty} \sum_{s \in S} \inf\{g_s(z), g_s(\rho)\} N_s \alpha'(z)\,dz && (215)\\
&= \int_0^{+\infty} \inf\{\sum_{s \in S} N_s g_s(z), \sum_{s \in S} N_s g_s(\rho)\} \alpha'(z)\,dz && (216)
\end{aligned}$$

where the summation sign enters the minimum operator because, for any $s \in S$, $g_s(z) \leq g_s(\rho)$ if and only if $z \leq \rho$. Therefore,

$$\begin{aligned}
\hat{Q}(\rho) &= \int_0^{+\infty} \inf\{\sum_{s \in S} N_s \bar{q}_s(1+z), \sum_{s \in S} N_s \bar{q}_s(1+\rho)\} \alpha'(z)\,dz && (217)\\
&= \bar{Q}\left[1 + \int_0^{+\infty} \inf\{z, \rho\} \alpha'(z)\,dz\right]. && (218)
\end{aligned}$$

Assuming $C(Q) = \frac{1}{2}cQ^2$,

$$\begin{aligned}
x^{SCE0}(\rho) &= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}c}{N}\int_{z=0}^{\rho} \hat{Q}(z)\,dz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}c}{N}\int_{z=0}^{\rho} \bar{Q}\left[1 + \int_{y=0}^{+\infty} \inf\{y,z\}\alpha'(y)\,dy\right] dz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty}\int_{z=0}^{r} \inf\{y,z\}\alpha'(y)\,dydz \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty} \alpha'(y)\left[\int_{z=0}^{y} zdz + y\int_{z=y}^{\rho} dz\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + \frac{\bar{Q}^2 c\rho}{N} + \frac{\bar{Q}^2 c}{N}\int_{y=0}^{+\infty} \frac{n^r(y)}{N}\left[\frac{y^2}{2} + y(\rho - y)\right] dy \\
&= \frac{c\bar{Q}^2}{2N} + \frac{c\bar{Q}^2}{N}\rho + \frac{c\bar{Q}^2}{N^2}\int_{y=0}^{+\infty}\left[\left(\rho - \frac{y}{2}\right) yn^r(y)\right] dy \\
&= \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\rho.
\end{aligned}$$

For households of type s this writes:

$$\begin{aligned}
x^{SCE0}(q,s) &= \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\left(\frac{q - \bar{q}_s}{\bar{q}_s}\right) \\
&= \left\{\frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2}\int_{y=0}^{+\infty} y^2 n^r(y)\,dy - \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\right\} \\
&\quad + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\int_{y=0}^{+\infty} yn^r(y)\,dy\right]\frac{q}{\bar{q}_s}.
\end{aligned}$$

Also, by budget balance,

$$\frac{cQ^2}{2} = \sum_s \int_{z=0}^{+\infty} x^{SCE0}(z) n_s^r(z) dz \tag{219}$$

$$= \sum_s \int_{z=0}^{+\infty} \left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} n_s^r(z) dz \tag{220}$$

$$+ \sum_s \int_z \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \frac{q}{\bar{q}_s} n_s^r(z) dz$$

$$= \left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} \sum_s \int_{z=0}^{+\infty} n_s^r(z) dz \tag{221}$$

$$+ \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \sum_s \int_z (z+1)\, n_s^r(z) dz$$

$$= \left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} N \tag{222}$$

$$+ \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left[\sum_s \frac{Q_s}{\bar{q}_s} \right].$$

because $z+1 = g_s(z)/\bar{q}_s$ and $\int_z (z+1)\, n_s^r(z) dz = \int_z [g_s(z)/\bar{q}_s]\, n_s^r(z) dz = \int_q (q/\bar{q}_s)\, n_s(q) dq = Q_s/\bar{q}_s$.

Therefore,

$$\left\{ \frac{c\bar{Q}^2}{2N} - \frac{c\bar{Q}^2}{2N^2} \int_{y=0}^{+\infty} y^2 n^r(y)\, dy - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \right\} = \frac{1}{N} \left\{ \frac{cQ^2}{2} \right. \tag{223}$$

$$- \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left[\sum_s \right.$$

Hence,

$$x^{SCE0}(q,s) = \frac{1}{N} \left\{ \frac{cQ^2}{2} - \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left[\sum_s \frac{Q_s}{\bar{q}_s} \right] \right\} + \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \frac{q}{\bar{q}_s} \tag{224}$$

$$= \frac{1}{N} \frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N} \left[1 + \frac{1}{N} \int_{y=0}^{+\infty} y n^r(y)\, dy \right] \left(\frac{q}{\bar{q}_s} - \frac{1}{N} \sum_s \frac{Q_s}{\bar{q}_s} \right). \tag{225}$$

Observing that $N_s(q) = N_s^r\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right)$ implies $n_s(q)dq = \frac{1}{\bar{q}_s}n_s^r\left(\frac{q-\bar{q}_s}{\bar{q}_s}\right)dq = n_s^r\left(y\right)dy$. Hence,

$$x^{SCE0}\left(q,s\right) = \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\sum_s \int_{q=\bar{q}_s}^{+\infty} \frac{q-\bar{q}_s}{\bar{q}_s} n_s\left(q\right)dq\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right) \tag{226}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[1 + \frac{1}{N}\sum_s \left(\frac{Q_s}{\bar{q}_s} - N_s\right)\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right) \tag{227}$$

$$= \frac{1}{N}\frac{cQ^2}{2} + \frac{c\bar{Q}^2}{N}\left[\frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right]\left(\frac{q}{\bar{q}_s} - \frac{1}{N}\sum_s \frac{Q_s}{\bar{q}_s}\right). \tag{228}$$

Moreover, the distributional assumption that $N_s^r\left(r\right)/N_s = \alpha(\rho)$ for all s implies that:

$$Q_s = \int_{\bar{q}_s}^{+\infty} q n_s\left(q\right)dq \tag{229}$$

$$= \int_0^{+\infty} \bar{q}_s\left(1+y\right) n_s\left(y\right)dy \tag{230}$$

$$= \bar{q}_s \int_0^{+\infty} \left(1+y\right)\alpha'\left(y\right) N_s dy \tag{231}$$

$$= \bar{Q}_s \int_0^{+\infty} \left(1+y\right)\alpha'\left(y\right)dy \tag{232}$$

This says that $Q_s/\bar{Q}_s = \int_0^{+\infty}\left(1+y\right)\alpha'\left(y\right)dy$ is independent of s. Hence, for all s,

$$Q_s/\bar{Q}_s = Q/\bar{Q}. \tag{233}$$

Finally,

$$x^{SCE0}\left(q,s\right)=\frac{1}{N}\frac{cQ^{2}}{2}+\frac{c\bar{Q}^{2}}{N}\left[\frac{1}{N}\frac{Q}{\bar{Q}}\sum_{s}\frac{\bar{Q}_{s}}{\bar{q}_{s}}\right]\left(\frac{q}{\bar{q}_{s}}-\frac{1}{N}\frac{Q}{\bar{Q}}\sum_{s}\frac{\bar{Q}_{s}}{\bar{q}_{s}}\right) \tag{234}$$

$$=\frac{1}{N}\frac{cQ^{2}}{2}+\frac{c\bar{Q}^{2}}{N}\left[\frac{Q}{\bar{Q}}\frac{1}{N}\sum_{s}N_{s}\right]\left(\frac{q}{\bar{q}_{s}}-\frac{Q}{\bar{Q}}\frac{1}{N}\sum_{s}N_{s}\right) \tag{235}$$

$$=\frac{1}{N}\frac{cQ^{2}}{2}+\frac{c\bar{Q}^{2}}{N}\left[\frac{Q}{\bar{Q}}\right]\left(\frac{q}{\bar{q}_{s}}-\frac{Q}{\bar{Q}}\right) \tag{236}$$

$$=\frac{1}{N}\frac{cQ^{2}}{2}+cQ\frac{\bar{Q}}{N}\left(\frac{q}{\bar{q}_{s}}-\frac{Q}{\bar{Q}}\right) \tag{237}$$

$$=\frac{1}{N}\frac{cQ^{2}}{2}+cQ\frac{\bar{Q}}{N}\left(\frac{q-\bar{q}_{s}}{\bar{q}_{s}}-\frac{Q-\bar{Q}}{\bar{Q}}\right). \tag{238}$$

SEE

Recall that:

$$x^{SEE}\left(q,s\right)=\frac{C(N\bar{q}_{0})}{N}+\int_{z=0}^{q}\frac{1}{N-N\left(z\right)}C'\left(\tilde{Q}\left(z\right)\right)\frac{d\tilde{Q}\left(z\right)}{dq}dz \tag{239}$$

$$+\left[u_{s}\left(q,\bar{q}_{s}\right)-u_{s}\left(q,\bar{q}_{0}\right)\right]-\frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u_{t}\left(z,\bar{q}_{t}\right)-u_{t}\left(z,\bar{q}_{0}\right)\right]n_{t}\left(z\right)dz,$$

where

$$\tilde{Q}\left(q\right) = \int_{0}^{q}zn\left(z\right)dz+\left(N-N\left(q\right)\right)q \tag{240}$$

$$= \int_{0}^{\infty}\inf\{z,q\}n\left(z\right)dz. \tag{241}$$

We have

$$\frac{d\tilde{Q}}{dq} = qn\left(q\right)+\left(N-N\left(q\right)\right)-qn\left(q\right) \tag{242}$$

$$= N-N\left(q\right), \tag{243}$$

so that

$$\int_{z=0}^{q} \frac{1}{N - N(z)} C'\left(\tilde{Q}(z)\right) \frac{d\tilde{Q}(z)}{dq} dz = \int_{z=0}^{q} C'\left(\tilde{Q}(z)\right) dz \tag{244}$$

$$= \int_{z=0}^{q} c\tilde{Q}(z) dz \tag{245}$$

$$= c \int_{z=0}^{q} \left(\int_{y=0}^{\infty} \inf\{y, z\} n(y) dy \right) dz \tag{246}$$

$$= c \int_{y=0}^{\infty} \left(\int_{z=0}^{q} \inf\{y, z\} dz \right) n(y) dy \tag{247}$$

$$= c \int_{y=0}^{\infty} \left(\int_{z=0}^{y} z dz + \int_{z=y}^{q} y dz \right) n(y) dy \tag{248}$$

$$= c \int_{y=0}^{\infty} \left(\frac{y^2}{2} + y(q - y) \right) n(y) dy \tag{249}$$

$$= c \int_{y=0}^{\infty} \left(yq - \frac{y^2}{2} \right) n(y) dy \tag{250}$$

$$= cq \int_{y=0}^{\infty} y n(y) dy - c \int_{y=0}^{\infty} \frac{y^2}{2} n(y) dy \tag{251}$$

$$= cQq - c \int_{y=0}^{\infty} \frac{y^2}{2} n(y) dy. \tag{252}$$

Hence,

$$x^{SEE}(q, s) = \frac{C(N\bar{q}_0)}{N} - c \int_{y=0}^{\infty} \frac{y^2}{2} n(y) dy + cQq \tag{253}$$

$$+ [u_s(q, \bar{q}_s) - u_s(q, \bar{q}_0)] - \frac{1}{N} \sum_{t \in S} \int_{z=0}^{\infty} [u_t(z, \bar{q}_t) - u_t(z, \bar{q}_0)] n_t(z) dz.$$

By budget balance

$$\sum_s \int_0^{\infty} x^{SEE}(z, s) n_s(z) dz = \frac{cQ^2}{2} \tag{254}$$

so that

$$N \left[\frac{C(N\bar{q}_0)}{N} - c \int_{y=0}^{\infty} \frac{y^2}{2} n(y) dy \right] + cQ^2 = \frac{cQ^2}{2}; \tag{255}$$

hence,

$$\frac{C(N\bar{q}_0)}{N} - c \int_{y=0}^{\infty} \frac{y^2}{2} n(y) dy = -\frac{1}{N} \frac{cQ^2}{2}. \tag{256}$$

Therefore, the cost-sharing component of x^{SEE} writes

$$\frac{C(N\bar{q}_0)}{N} - c\int_{y=0}^{\infty} \frac{y^2}{2} n\left(y\right) dy + cQq = -\frac{1}{N}\frac{cQ^2}{2} + cQq \tag{257}$$

$$= cQ\left(q - \frac{Q}{2N}\right). \tag{258}$$

To sum up,

$$x^{SEE}\left(q,s\right) = cQ\left(q - \frac{Q}{2N}\right) \tag{259}$$

$$+ \left[u_s\left(q,\bar{q}_s\right) - u_s\left(q,\bar{q}_0\right)\right] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}\left[u_t\left(z,\bar{q}_t\right) - u_t\left(z,\bar{q}_0\right)\right] n_t\left(z\right) dz.$$

C.2 Increasing Returns to Scale: Affine Costs

DSCE0 with absolute responsibility

Recall the expression for x^{DSCE0} :

$$x^{DSCE0}\left(\rho\right) = \frac{1}{N}C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup\vec{\rho}} \frac{1}{N^r\left(z\right)} C'\left(\check{Q}\left(z\right)\right) \frac{d\check{Q}\left(z\right)}{dz} dz \tag{260}$$

where,

$$\check{Q}\left(\rho\right) = \int_{z=0}^{\infty}\sum_{s\in S} g_s\left(\sup\left\{\rho, z\right\}\right) n_s^r\left(z\right) dz, \tag{261}$$

and where $\sup\vec{\rho}$ is the largest responsibility level in the population with the associated virtual consumption level that brings all users to that same level of responsibility:

$$\check{Q}_{\sup} \equiv \check{Q}\left(\sup\vec{\rho}\right) \tag{262}$$

$$= \int_{z=0}^{\infty}\sum_{s\in S} g_s\left(\sup\left\{\sup\vec{\rho}, z\right\}\right) n_s^r\left(z\right) dz \tag{263}$$

$$= \sum_{s\in S} g_s\left(\sup\vec{\rho}\right)\int_{z=0}^{\infty} n_s^r\left(z\right) dz \tag{264}$$

$$= \sum_{s\in S} N_s g_s\left(\sup\vec{\rho}\right). \tag{265}$$

Moreover,

$$\begin{aligned}
\frac{d\check{Q}(z)}{dz} &= \frac{d}{dz}\left[\sum_{s\in S} N_s^r(z)\, g_s(z) + \int_{y=z}^{\infty}\sum_{s\in S} g_s(y)\, n_s^r(y)\, dy\right] && (266)\\
&= \sum_{s\in S} n_s^r(z)\, g_s(z) + \sum_{s\in S} N_s^r(z)\, g_s'(z) - \sum_{s\in S} g_s(z)\, n_s^r(z) && (267)\\
&= \sum_{s\in S} N_s^r(z)\, g_s'(z)\,. && (268)
\end{aligned}$$

With absolute responsibility, $\rho = q - \bar{q}_s$ so that $g_s(\rho) = \rho + \bar{q}_s$. Hence, $g_s'(\rho) = 1$ for all $s \in S$. It follows that:

$$\begin{aligned}
\check{Q}(\rho) &= \int_{z=0}^{\infty}\sum_{s\in S} g_s(\sup\{\rho, z\})\, n_s^r(z)\, dz && (269)\\
&= \int_{z=0}^{\infty}\sum_{s\in S} [\sup\{\rho, z\} + \bar{q}_s]\, n_s^r(z)\, dz && (270)\\
&= \int_{z=0}^{\infty} \sup\{\rho, z\}\, n^r(z)\, dz + \bar{Q}. && (271)
\end{aligned}$$

Moreover, Expression (268) becomes:

$$\frac{d\check{Q}(\rho)}{d\rho} = \sum_{s\in S} N_s^r(\rho) = N^r(\rho)\,, \qquad (272)$$

and,

$$\begin{aligned}
\check{Q}_{\sup} &= \sum_{s\in S} N_s g_s(\sup\vec{\rho}) && (273)\\
&= \sum_{s\in S} N_s [\bar{q}_s + \sup\vec{\rho}] && (274)\\
&= \bar{Q} + N\sup\vec{\rho}. && (275)
\end{aligned}$$

Hence,

$$x^{DSCE0}(\rho) = \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup\vec{\rho}} C'\left(\check{Q}(z)\right) dz. \qquad (276)$$

Suppose that $C(Q) = F + cQ$ with $F, c > 0$. We obtain

$$x^{DSCE0}(\rho) = \frac{1}{N}\left(F + c\check{Q}_{\sup}\right) - c\left[\sup\vec{\rho} - \rho\right] \tag{277}$$

$$= \frac{F + c\bar{Q}}{N} + c\sup\vec{\rho} - c\sup\vec{\rho} + c\rho \tag{278}$$

$$= \frac{F + c\bar{Q}}{N} + c\rho, \tag{279}$$

which, expressed in terms of q and s, yields the result:

$$x^{DSCE0}(\rho) = \frac{F + c\bar{Q}}{N} + c\left(q - \bar{q}_s\right) \tag{280}$$

DSCE0 with relative responsibility

When responsibility is measured by relative responsibility, $\rho = \left(q - \bar{q}_s\right)/\bar{q}_s$ so that $g_s(\rho) = \bar{q}_s(1 + \rho)$. Hence, $g_s'(\rho) = \bar{q}_s$ for any $s \in S$. It follows that:

$$\check{Q}(\rho) = \int_{z=0}^{\infty}\sum_{s\in S} g_s\left(\sup\{\rho, z\}\right) n_s^r(z)\, dz \tag{281}$$

$$= \int_{z=0}^{\infty}\sum_{s\in S}\left[1 + \sup\{\rho, z\}\right]\bar{q}_s n_s^r(z)\, dz \tag{282}$$

$$= \bar{Q} + \int_{z=0}^{\infty}\sup\{\rho, z\}\sum_{s\in S}\bar{q}_s n_s^r(z)\, dz. \tag{283}$$

Moreover, Expression (268) becomes:

$$\frac{d\check{Q}(\rho)}{d\rho} = \sum_{s\in S}\bar{q}_s N_s^r(\rho), \tag{284}$$

and,

$$\check{Q}_{\sup} = \sum_{s\in S} N_s g_s \left(\sup \vec{\rho}\right) \tag{285}$$

$$= \sum_{s\in S} N_s \bar{q}_s \left[1 + \sup \vec{\rho}\right] \tag{286}$$

$$= \left[1 + \sup \vec{\rho}\right] \bar{Q}. \tag{287}$$

Therefore, taking $C(Q) = F + cQ$ yields:

$$x^{DSCE0}(\rho) = \frac{1}{N} C\left(\check{Q}_{\sup}\right) - \int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} C'\left(\check{Q}(z)\right) \frac{d\check{Q}(z)}{dz} dz \tag{288}$$

$$= \frac{1}{N}\left(F + c\check{Q}_{\sup}\right) - c\int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} \frac{d\check{Q}(z)}{dz} dz \tag{289}$$

$$= \frac{F + c\bar{Q}}{N} + \frac{1}{N} c\bar{Q} \sup \vec{\rho} - c\int_{z=\rho}^{\sup \vec{\rho}} \frac{1}{N^r(z)} \sum_{s\in S} \bar{q}_s N_s^r(z)\, dz \tag{290}$$

Assuming that responsibility is evenly spread across types, the distribution of responsibility is independent of needs:

$$N_s^r(\rho) = \alpha(\rho) N_s, \tag{291}$$

for some increasing function $\alpha : \mathbb{R}_+ \to [0, 1]$ which we take to be differentiable. This yields:

$$\sum_{s\in S} \bar{q}_s N_s^r(\rho) = \sum_{s\in S} \bar{q}_s \alpha(\rho) N_s = \alpha(\rho) \bar{Q}, \tag{292}$$

and,

$$N^r(z) = \sum_{s\in S} N_s^r(z) = \alpha(\rho) \sum_{s\in S} N_s = \alpha(\rho) N. \tag{293}$$

Finally, it follows that:

$$x^{DSCE0}(\rho) = \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\sup\vec{\rho} - c\int_{z=\rho}^{\sup\vec{\rho}} \frac{1}{\alpha(\rho)N}\alpha(\rho)\bar{Q}dz \qquad (294)$$

$$= \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\sup\vec{\rho} - c\frac{\bar{Q}}{N}(\sup\vec{\rho}-\rho) \qquad (295)$$

$$= \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\rho, \qquad (296)$$

which, expressed in terms of q and s, yields the result:

$$x^{DSCE0}(\rho) = \frac{F+c\bar{Q}}{N} + c\frac{\bar{Q}}{N}\frac{q-\bar{q}_s}{\bar{q}_s}$$
$$= \frac{F}{N} + c\frac{1}{\bar{q}_s/\left(\bar{Q}/N\right)}q.$$

DSEE

Recall that

$$x^{DSEE}(q,s) = \frac{1}{N}C(N\sup\mathbf{q}) - \int_{z=q}^{\sup\mathbf{q}} C'\left(\breve{Q}(z)\right)dz + [u_s(q,\bar{q}_s) - u_s(q,\bar{q}_0)] - \frac{1}{N}\sum_{t\in S}\int_{z=0}^{\infty}[u_t(z,\bar{q}_t) - u_t($$

Notice that

$$\breve{Q}(q) = \int_{z=0}^{\infty}\sup\{q,z\}n(z)\,dz \qquad (297)$$

$$= N(q)\,q + \int_{z=q}^{\infty} zn(z)\,dz \qquad (298)$$

Hence,

$$\frac{d\breve{Q}(q)}{dq} = N(q), \qquad (299)$$

and

$$\breve{Q}(\sup\mathbf{q}) = N\sup\mathbf{q}. \qquad (300)$$

With $C(Q) = F + cQ$, the cost-sharing component of $x^{DSEE}(q, s)$ becomes:

$$\frac{1}{N} C\left(\breve{Q}(\sup \mathbf{q})\right) - \int_{z=q}^{\sup \mathbf{q}} C'\left(\breve{Q}(z)\right) dz = \frac{F}{N} + c \sup \mathbf{q} - c \int_{z=q}^{\sup \mathbf{q}} dz \quad (301)$$

$$= \frac{F}{N} + cq, \quad (302)$$

as was to be shown.