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27 December 2015

Online at https://mpra.ub.uni-muenchen.de/73457/
MPRA Paper No. 73457, posted 11 October 2016 12:50 UTC
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Abstract

Poyang Lake suffers from a severe shortage of water, due to the construction of the Three Gorges Dam and other hydropower stations along the upper Yangtze River. In this paper, we propose a two-sector endogenous growth model to investigate the effects of extractive water use and impoundment activities on the optimal sustainable growth path. To offset the negative impact of the extractive use on the regenerative capacities of water resources, the impoundment activities should grow at a constant rate. We also provide a numerical example to illustrate the mechanisms of the endowment and the regenerative capacities of water resources on the long-run economic growth path. We find out some theoretical evidence to support the construction of a water conservancy project to restore regeneration capacities of Poyang Lake.

Keywords: Endogenous economic growth; Renewable water resources; Poyang Lake; Extractive use; Impoundment activities; water sustainability
1. Introduction

The Three Gorges Dam is the largest operating hydroelectric facility in the world, generating 83.7 TWh in 2013 and 98.8 TWh in 2014.1 The impoundment project of the Three Gorges Dam was separated into four phases. Beginning in November of 1997, the water level of the Yangtze River was increased to 10-75 meters. In June 2003, the second phase of the project, the water level was increased to 135 meters. In 2006, the water level was increased to 156 meters. After completion of the Three Gorges project in 2009, the water level was increased to 175 meters.

Although the extractive use of water in the Three Gorges Dam contributes to the economic growth of China by generating electricity, it causes a severe shortage of water in areas of the lower Yangtze River such as Poyang Lake. Located in Jiangxi Province, Poyang Lake is the largest freshwater lake in China. It connects to the lower Yangtze River through a channel. Figure 1 shows the average, highest and lowest monthly water levels of Poyang Lake for the last 30 years (1983-2012).2 We can see that all three indicators decreased dramatically following completion of the four impoundment phases of the Three Gorges Dam project.

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2 The raw data is drawn from the daily observations of water levels from 10 stations around Poyang Lake. We calculate the monthly water levels of all 10 stations.
The shortage of water has a negative influence on local agriculture and manufacturing industries and impedes economic growth. To deal with this problem, the local government of Jiangxi Province proposed the construction of a new water conservancy project on Poyang Lake to impound water during drought periods and restore the regeneration capacities of Poyang Lake. The plan attracted international attention, with much of the controversy focusing on the effects of the project on the ecological system of the Poyang Lake Wetland and animal protection. However, few opinions about the relationship between the long-run economic growth and the construction and maintenance costs of impoundment projects were offered.

Many scholars have contributed to study of the relationship between economic growth and environment quality and the effects of technological change on this relationship. (See the reviews of Brock and Taylor, 2005; Foray and Grübler, 1996; Löschel, 2002; Van Den Bergh and Hofkes, 1998) An important theoretical branch of this literature is the economic modeling of sustainable development using the

Source: Jiangxi Provincial Bureau of Water Resources, China.

Figure 1. Water Levels of Poyang Lake
endogenous growth model. Based on the work of Lucas (1988) and Rebelo (1991), Bovenberg and Smulders (1995) present a two-sector endogenous model in which the production of new technical knowledge in the knowledge sector will improve the effective use of the renewable resources and reduce pollution. Hofkes (1996) introduces the concept of abatement into the two-sector endogenous model, which assumes that a proportion of final output can be used as abatement to reduce the level of pollution and recover the regenerative capacities of nature resources. The literature focuses on either optimal pollution/abatement choices (Byrne, 1997; Grimaud, 1999; Michel and Rotillon, 1995; Schou, 2000; Withagen, 1995), or the endogenous technical change and the optimal environmental regulation (Acemoglu, et al., 2009; Barbier, 1999; Groth and Schou, 2007; Hart, 2004; Rosendahl, 2004; Smulders and De Nooij, 2003). Though theoretical analysis of the optimal management of groundwater resources has been provided (Esteban and Dinar, 2013; Hannouche et al., 2016; Roseta-Palma, 2002, 2003), it has not been in the framework of the endogenous growth model.

Many empirical studies have investigated the relationship between economic growth and environment quality (Brock and Taylor, 2010; De Bruyn, 2012; Du et al., 2016; Färe et al., 2005), including several on China (Choi et al., 2015; Sun et al., 2008; Wang and Song, 2014; Yu, 2015; Zhang and Cheng, 2009; Zhang et al., 2014, 2015; Zhang and Xie, 2015). Deng et al. (2011) studied economic growth and pollution in and around Poyang Lake. However, theoretical and empirical studies that incorporate water resources and impoundment activities into a sustainable economic growth model remain scarce. This study addresses this gap by applying an endogenous growth model to the optimal levels of extractive water use and impoundment activities, which has policy implications for the problem of Poyang Lake.

Based on the work of Bovenberg and Smulders (1995) and Hofkes (1996), we construct a two-sector endogenous growth model to study the problem of the long-run economic growth with restrictions on water resources. This paper contributes to the literature in the following ways. First, we construct a theoretical framework for analyzing the long-run sustainable economic growth path with extractive water use
and impoundment activities, based on a two-sector endogenous growth model. Second, our study gives a numerical analysis of the sustainable growth path and detailed policy implications for Poyang Lake, which has been neglected in previous studies.

The results show that the impoundment activities should grow at a constant rate to offset the negative impact of the extractive use on the regenerative capacities of water resources in a sustainable growth path. We then provide a numerical example that illustrates when the endowment or the regenerative capacities of water resources are low, the long-run sustainable growth cannot be achieved without sufficient impoundment activities.

The paper is organized as follows. Section 2 presents the setting of model. The feasibility and optimality conditions of the long-run sustainable growth are derived in Section 3. Section 4 provides a numerical example. Finally, Section 5 concludes.

2. Model setting

2.1. Regenerative capacities of water resources

First, we introduce the assumptions about the regenerative capacities of water resources. The aggregate stock of water resources (\(N\)) is modelled as a renewable resource and can be regenerated by itself. The regenerative capacities of water resources will decrease as the gross extractive use of water (denoted \(Q\), e.q., the water used in the generation of hydropower stations) increases. Meanwhile, the regenerative capacities will be restored with increasing intensity of impoundment activities (denoted \(I\), e.q., constructing impounding reservoirs to increase the stock of water resources). Because the construction and maintenance costs of reservoirs, which offset the negative influence of the gross extractive use of water, are funded in terms of physical capital, we introduce a variable of the net extractive use of water, \(X \equiv X(Q, I)\). It is clear that \(X_Q > 0\) and \(X_I < 0\).

The regenerative capacities of water resources are described by the following function \(E\):
\[ \dot{N} \equiv \frac{dN}{dt} = E(N, X(Q, I)). \] (1)

We assume that the function \( E \) is twice continuously differentiable and concave, and \( E_X < 0, \ E_{NX} > 0 \) and \( E_{XX} < 0 \). The regenerative capacities of water resources will decrease as the net extractive use of water increases \( (E_X < 0) \). Furthermore, we assume that the higher the stock of water resources, the smaller the negative influence of the net extractive use on regenerative capacity \( (E_{NX} > 0) \). If the stock of water resources is high, nature can easily recover from the extractive use of water and a change in the net extractive use of water has only negligible effects on the regeneration capacities. Finally, higher levels of net extractive use affect nature at an increasing rate, or the marginal reduction of the net extractive use on water resources increases \( (E_{XX} < 0) \).

According to the literature on environment quality and pollution, we assume that \( E \) is an inverted U-shape function with respect to \( N \). When the stock of water resources decreases below a certain threshold level (or the net extractive use of water resources grows beyond a certain threshold level), the degeneration process becomes irreversible. Conversely, for some high values of \( N \) the regenerative capacities of the water resources will also decrease because it becomes more and more difficult to regenerate the entire stock of water resources. For each level of net extractive use, the stock of water will stabilize at a long-run equilibrium level. At this stable level the regenerative capacities are such that the stock of water resources remains constant. This is ensured by assuming \( E_N(N, X(Q, I)) < 0 \) in a neighborhood of this stable level (as shown in Bovenberg and Smulders, 1995).

**Proposition 1.** If the stock of water resources grows at a constant (maybe zero) rate, the net extractive use should be reduced at an increasing (maybe zero) rate.

**Proof:** Following the Appendix of Hofkes (1996).³

³ Although we change the form of some functions in the two-sector endogenous growth model, and apply it to a different issue, the basic logic and conclusions of this proof are similar to those of Hofkes (1996).
constant rate if the net extractive use \((X)\) decreases at an increasing rate, which means either impoundment activities \((I)\) grow at an increasing rate or the gross extractive use of water resources \((Q)\) decreases at an increasing rate. Note that, both growth rates of \(N\) and \(X\) could equal zero in the extreme case, which implies that \(I\) and \(Q\) grow at some constant rate. Given the fact that an increasing number of hydropower stations, in addition to the Three Gorges Dam, are being constructed in the upper Yangtze River, we predict that the gross extractive use of water resources will not fall in the near future. Hence, to maintain sustainable growth, the intensity of impoundment activities should grow, minimally, at a constant rate.

2.2. The two-sector economy

The economy consists of two sectors, a production-sector producing final (consumption) goods \((Y)\) and a knowledge or R&D sector producing knowledge \((h)\) about efficient use of water resources. However, the output of the knowledge sector only accumulates technological knowledge. It does not contribute to the production of final goods. The production functions for the final goods and for knowledge are as follows:

\[
Y = Y(K_Y, Z_Y, N),
\]

\[
H = H(K_H, Z_H).
\]

We assume that both \(Y\) and \(H\) are twice continuously differentiable and concave. The variable \(K_Y\) represents the stock of physical capital allocated to the final goods sector, while \(K_H\) represents the stock of physical capital allocated to the knowledge sector. Analogously, \(Z_Y\) represents the effective input of water resources in the final goods sector, and \(Z_H\) represents the effective input of water resources in the knowledge sector. Hence, \(K = K_Y + K_H\) and \(Z = Z_Y + Z_H\), represent the total stock of physical capital and the total effective input of water resources. The effective input of water resources is a function of knowledge and the gross extractive use of water, i.e., \(Z = hQ\). The efficiency of water use increases as the stock of knowledge increases, given that the gross extractive use unchanged. Finally, the aggregate stock
of water resources \((N)\) matters to the production of final goods because a low (high) stock of water resources will increase (reduce) the cost of producing final goods.

The final good can either be consumed \((C)\), used for impoundment activities \((I)\) to restore the regenerative capacities of water resources, or invested \((\dot{K})\) to accumulate physical capital, which increases future consumption and impoundment activities. The accumulation of physical capital is given by the following equation:

\[
\dot{K} \equiv \frac{dK}{dt} = Y(K_Y, Z_Y, N) - C - I. \tag{2}
\]

The accumulation of knowledge is given by:

\[
\dot{h} \equiv \frac{dh}{dt} = H(K_H, Z_H). \tag{3}
\]

The total stock of physical capital and total effective use of water resources are allocated between two sectors. Let \(u\) and \(v\) denote the share of physical capital and the share of effective water resources used in the final goods sector, respectively, then \((1 - u)\) and \((1 - v)\) denote the share of physical capital and the share of effective water resources used in the knowledge sector, respectively:

\[
K_Y = uK, \tag{4}
\]

\[
K_H = (1 - u)K, \tag{5}
\]

\[
Z_Y = vhQ, \tag{6}
\]

\[
Z_H = (1 - v)hQ. \tag{7}
\]

Social welfare \((W)\) is described by the utility of a representative consumer, who lives infinitely. The individual’s utility \((U)\) depends upon her consumption \(c\) \((c = C/L, \text{ where } C \text{ is aggregate consumption and } L \text{ which is assumed to be constant over time})\) only, but not upon the stock of water resources \((N)\). Unlike the literature on economic growth and environment quality, the stock of water resources does not affect the utility of the individual consumer directly. One can enjoy the beautiful environment but not the water level of some lake miles away.\(^5\) By introducing the discount rate as the indicator of time preference \((\theta)\), we can construct the welfare

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\(^4\) One can also assume that the population will grow at a constant rate, which will not change the main results of our model. However, labour is not included in the production function. We use this assumption to simplify the analysis.

\(^5\) Another explanation is that the stock of water resources may affect individual utility, but the water resources elasticity of utility is very small, close to zero.
function of the representative consumer:

\[ W = \int_0^\infty e^{-\theta t} U(c(t))dt. \]

After introducing the basic assumptions of the model, we will examine the feasibility and optimality conditions of long-run sustainable growth in Section 3.

3. Sustainable growth

3.1. The feasibility conditions

We first consider the conditions under which sustainable (balanced) growth is feasible in our model. Sustainable growth is defined as a situation in which all variables grow at a constant (maybe zero) rate and in which the allocative variables, i.e. the share of consumption in final output \( C/Y \) and the share of impoundment activities in final output \( I/Y \), are constant. In any situation of sustainable growth, \( Y, C, K \) and \( I \) should grow at the same common rate. Let us denote this common rate by \( g \) and the growth rates of \( Y, C, K \) and \( I \) by \( g_Y, g_C, g_K, g_I \), respectively. Hence, the first four growth rates should be the same and equal to the common growth rate in a sustainable growth path:

\[ g = g_Y = g_C = g_K = g_I. \]  
(8)

Bovenberg and Smulders (1995) show that sustainable growth requires that the production elasticities be constant over time. Hence, let us denote the production elasticities of \( K_Y, Z_Y \) and \( N \) and the knowledge elasticities of \( K_H \) and \( Z_H \) by \( \eta_K, \eta_Z, \eta_N, \zeta_K \) and \( \zeta_Z \), respectively. By log-linearizing the production function of the final good, we can derive:

\[ g_Y = \eta_K g_K + \eta_Z (g_Q + g_h) + \eta_N g_N. \]  
(9)

By log-linearizing the production function of the knowledge, we have:

\[ g_H = \zeta_K g_K + \zeta_Z (g_Q + g_h). \]  
(10)

Furthermore, in a situation of sustainable growth:

\[ \dot{g}_h = \frac{g_H - g_h}{g} = 0. \]  
(11)
Hence, we can derive that \( g_H = g_h \). The knowledge should grow at constant rate \( g_h \). However, the growth of knowledge does not necessarily equal the common growth rate \( g \), since impoundment activities are used to increase the regenerative capacities of water resources. Finally, by log-linearizing the net extractive use of water, we can derive:

\[
g_X = \eta_Q g_Q + \eta_I g_I. \tag{12}
\]

where \( \eta_Q \) is the regeneration-elasticity of the gross extractive use of water resources \( Q \) and \( \eta_I \) is the regeneration-elasticity of the impoundment activities \( I \). Then we can derive the feasibility conditions of growth rates \( g_Q \) and \( g_h \) in the following proposition:

**Proposition 2.** In any sustainable growth path, the growth rate of the gross use of water resources and the growth rate of knowledge are as follows:

\[
g_Q = \frac{(1-\eta_K)(1-\zeta_Z)-\zeta_K \eta_Z}{\eta_Z} g, \tag{13}
\]

\[
g_h = \frac{(1-\eta_K)\zeta_Z+\zeta_K \eta_Z}{\eta_Z} g. \tag{14}
\]

Proof: see Appendix A.1.

Proposition 2 specifies the relationship between the growth rate of the gross use of water resources, the growth rate of knowledge accumulation, and the common growth rate.

Combing Eq. (13) and Eq. (A.1), and substituting \( g_A = g \), we have:

\[
\frac{\eta_A}{\eta_Q} = \zeta_K - \frac{(1-\eta_K)(1-\zeta_Z)}{\eta_Z}. \tag{15}
\]

Eq. (15) implies that given the assumptions of the regeneration function, a feasible sustainable growth path exists only for a very specific relationship between the gross extractive use of water resources and the impoundment activities. The stock of water resources, \( N \), will only be constant if the regeneration-elasticities of \( Q \) and \( I \) exactly offset each other, such that the net extractive use of water resources, \( X \), remains constant. Hence, the equality \( g_X = g_N = 0 \) holds in any feasible sustainable growth path.
3.2. The optimality conditions

After identifying the feasibility conditions of a sustainable growth path, we then analyze the society’s dynamic optimization problem, which is given by:

$$\max \int_0^\infty e^{-\theta t} U(c(t)) dt$$

s.t. \( \dot{K} = Y(K_Y, Z_Y, N) - C - I, \)

\( \dot{h} = H(K_H, Z_H), \)

\( \dot{N} = E(N, X(Q, I)). \) \tag{16}

The dynamic optimization problem (16) is designed to solve the optimal solutions of control variables, i.e., the optimal choices of the individual consumption \( c \), the optimal level of impoundment activities \( I \), the optimal gross extractive use of water resources \( Q \), and the optimal allocations of physical capital \( u \) and that of the gross extractive use of water resources \( v \). In the Poyang Lake case, local governments along the lower Yangtze River and around Poyang Lake have the incentive to construct the impoundment projects, while local governments along the upper Yangtze River benefit from building hydropower stations to use water resources extractively. Given this conflict, the dynamic optimization method may still be used to solve the problem because both the impounding and hydropower projects should be approved by the central government before construction. The central government is thus the social planner and determines the number of hydropower stations permitted along the upper Yangtze River and the number of impoundment projects permitted along the lower Yangtze River and around Poyang Lake, which are also represented by \( Q \) and \( I \), respectively. Following a standard procedure, we can solve dynamic optimization problem (16):

**Proposition 3.** The optimal choices of the individual consumption, the impoundment activities, the gross extractive use of water resources, and optimal allocations of physical capital and the gross extractive use of water resources, i.e., control variables
c, I, Q, u, and v are determined by the following five equations:

\[
\frac{\dot{c}}{c} = \frac{1}{\rho} \left( \frac{\partial Y}{\partial K_Y} - \theta \right), \quad (17)
\]
\[
\frac{\partial Y}{\partial K_Y} = \frac{\partial E}{\partial A} \frac{\partial Y}{\partial N} + \frac{\partial E}{\partial N} + \frac{\dot{\mu}}{\mu} - \frac{\dot{\lambda}}{\lambda}, \quad (18)
\]
\[
\frac{\partial Y}{\partial K_Y} = \frac{\partial H}{\partial Z_H} Q + \frac{k}{\kappa} - \frac{\lambda}{\lambda'}, \quad (19)
\]
\[
\frac{\partial Y}{\partial Z_Y} = \frac{\partial H}{\partial Z_H} \frac{\partial H}{\partial K_H}, \quad (20)
\]
\[
\frac{\partial Y}{\partial Z_Y} h = - \frac{\partial X}{\partial Q} \frac{\partial X}{\partial A}. \quad (21)
\]

Proof: see Appendix A.2.

The optimal allocation between current and future consumption is shown in Eq. (17). If the marginal return of physical capital in the production of final goods \( \left( \frac{\partial Y}{\partial K_Y} \right) \) is higher than the rate of time preference \( \theta \), consumption is increasing over time. Hence, individuals choose to increase saving, which accumulates physical capital and increases future output of consumption goods. Because the equality \( g = g_c \) holds, and the population \( L \) is a constant, in any sustainable growth path, the inequality condition \( \frac{\partial Y}{\partial K_Y} > \theta \) must hold.

Equations (18) and (19) imply that the marginal return on physical capital, the marginal return on water resources and the marginal return on knowledge should be the same. The marginal return on water resources is given by the R.H.S. of Eq. (18), the term \( \frac{\partial E}{\partial I} \frac{\partial Y}{\partial N} \) represents the increased marginal productivity with respect to water resources; the term \( \frac{\partial E}{\partial N} \) represents the regenerative capacity with respect to water resources and the changes in relative capital gains \( \left( \frac{\dot{\mu}}{\mu} - \frac{\dot{\lambda}}{\lambda} \right) \) are also included. Similarly, the marginal return on knowledge is given by the R.H.S. of Eq. (19).

We can rewrite the first three optimality conditions by substituting the shadow prices. By using Eq. (8) and assuming that \( \rho = 1 \) without loss of generality, (17), (18) and (19) can be rewritten as:

\[
g = \frac{\partial Y}{\partial K_Y} - \theta, \quad (22)
\]
\[
\frac{\partial Y}{\partial K_Y} = \frac{\partial E}{\partial I} \frac{\partial Y}{\partial N} + \frac{\partial E}{\partial N} + g, \tag{23}
\]
\[
\frac{\partial Y}{\partial K_Y} = \frac{\partial H}{\partial Z_H} Q + [1 - \zeta_K - \zeta_Z \frac{(1-\eta)}{\eta}] g. \tag{24}
\]

Note that on a sustainable growth path, the marginal return of physical capital in the production of final goods \(\frac{\partial Y}{\partial K_Y}\) should be a constant number, otherwise the growth rate \(g\) will not be constant according to Eq. (22). Hence, \(\frac{\partial Y}{\partial K_Y}\) is a constant number, determined by (22) and (23), together with the optimal sustainable growth rate \((g)\). Eq. (22) implies that the common growth rate equals the difference of the rate of capital return and intertemporal preferences, while Eq. (23) implies that the sustainable growth rate should be consistent with the optimal allocation in the long run. Together with (22) and (23), Eq. (24) determines the optimal allocation of capital and water resources among the final goods and knowledge sectors and the optimal level of impoundment activities, given the optimal sustainable growth rate \(g\).

4. A numerical example

In this section, we present a numerical example because of the complexity of model, because we are not able to collect the empirical data for all parameters, especially those of production functions. The example shows the specifications of variables, which satisfies the restrictions of dynamic optimization and regeneration elasticities of water resources in an optimal sustainable growth path. By studying the effects of modifying parameter values on the long-term steady state growth rates of those important indicators, we can also obtain clues to the mechanisms in the two-sector endogenous growth model.

Following Hofkes (1996), both the production function of final good and production function of knowledge are assumed to be Cobb-Douglas functions:

\[
Y = \Lambda_1 K_Y^\alpha Z_Y^{1-\alpha} N^\beta, \tag{25}
\]
\[
H = \Lambda_2 K_H^e Z_H^\delta, \tag{26}
\]
where \(\Lambda_1\) and \(\Lambda_2\) are technological parameters. Note that the production of final
goods exhibits constant return to scale with respect to physical capital and the effective use of water resources.\(^6\)

Taking the first-order derivative of Eq. (25) with respect to \(K_Y\), we have:

\[
\frac{\partial Y}{\partial K_Y} = \alpha \Lambda_1 K_Y^{a-1} Z_Y^{1-a} N^B = \frac{\alpha}{u} \cdot \frac{Y}{K}.
\]

Recall that the equality \(g = g_Y = g_K\) holds on any sustainable growth path. The output-to-capital ratio \(Y/K\) is a constant.

The utility function is simply given by \(U(c) = \ln(c)\), which satisfies the conditions of a constant intertemporal elasticity of substitution (i.e., \(\rho = -c \frac{\partial U}{\partial c} \frac{\partial U}{\partial c} = 1\)).

Finally, the regeneration function has to satisfy all the conditions and restrictions derived in the previous sections. Because the form of the regeneration function is similar to that of Hofkes (1996), we can adopt a similar function as follows:

\[
E(N, X(Q, I)) = -\frac{\gamma_1 X^2}{N} - \gamma_2 (N - \bar{N})^2 + \Gamma,
\]

where \(X(Q, I) = Q/1^{1-\epsilon-\delta}\) and \(\bar{N}\) and \(\Gamma\) are constants. Given function (27), we can see that \(E\) is modeled as an inverted U-shape of \(N\), such that the peak of the inverted U-shape curve exists at the stock of water resources \(N = \bar{N}\). The regenerative capacity of water resources reaches its maximum value at this level of water stock. The higher the value of \(\bar{N}\), the more abundant the water resources. We can denote the steady state stock of water resources as \(N^* > \bar{N}\), because \(E_N(N, X(Q, I)) < 0\) in the neighborhood of this stable level. The stable level \(N^*\) also increases as \(\bar{N}\) increases. Conversely, because \(\Gamma\) is the only positive term in Eq. (27), it represents the restriction of regenerative capacities of water resources. The regenerative capacities of water resources are increasing in \(\Gamma\).

In our model, there are 12 parameters that determine the optimal sustainable growth path and characterize the relationship between two sectors of production, as shown in the following Table 1:

\[^6\] The property of constant returns to scale with respect to physical capital and the effective use of water resources, together with the function form of the net extractive use of water resources \(X = Q/A^{1-\epsilon-\delta}\), make the restriction of elasticities (15) is always satisfied. Hence, using those two assumptions, we can change the values of the parameters as desired.
Table 1. Parameters of the model

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Specification</th>
<th>Benchmark values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>The weight of physical capital in the production function of final goods; 0.75</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$(1 - \alpha)$ is the weight of the effective use of water resources in the production function of final goods</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>The weight of the stock of water resources in the production function of final goods</td>
<td>0.001</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>The weight of capital in the production function of knowledge</td>
<td>0.4</td>
</tr>
<tr>
<td>$\delta$</td>
<td>The weight of the effective use of water resources in the production function of knowledge</td>
<td>0.4</td>
</tr>
<tr>
<td>$\gamma_1, \gamma_2$</td>
<td>Impact-parameters of the regeneration function</td>
<td>1, 0.5</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The rate of time preference</td>
<td>0.05</td>
</tr>
<tr>
<td>$\Lambda_1, \Lambda_2$</td>
<td>The constants of technology levels of efficient</td>
<td>0.2, 0.2</td>
</tr>
<tr>
<td></td>
<td>production in two sectors</td>
<td></td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>The level of the stock of water resources that exhibits the maximum regenerative capacity</td>
<td>1</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>The restriction of regenerative capacities of water resources</td>
<td>1</td>
</tr>
<tr>
<td>$L$</td>
<td>Total population</td>
<td>1</td>
</tr>
</tbody>
</table>

Taking the benchmark as example, we can easily obtain the values of the production elasticities with respect to capital, the effective use of water resources, and the stock of water resources in two production sectors: $\eta_K = \alpha = 0.75$, $\eta_Z = 1 - \alpha = 0.25$, $\eta_N = \beta = 0.001$, $\zeta_K = \epsilon = 0.4$ and $\zeta_Z = \delta = 0.4$.\(^7\) By log-linearizing the net extractive use function of water, $X = Q/A^{1-\epsilon-\delta}$, we can derive:

$$g_X = g_Q + (\epsilon + \delta - 1)g_A.$$  

Hence, the regeneration-elasticity of the gross extractive use of water and the regeneration-elasticity of impoundment activities are $\eta_Q = 1$, $\eta_I = \epsilon + \delta - 1 = -0.2$. Substituting those values of elasticities into (13) and (14), we have:

$$g_Q = 0.2g, \ g_h = 0.8g.$$  

Taking the first-order derivative of Eq. (25) with respect to $N$, we have:

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\(^7\) We select these values for parameters based on previous literature and the empirical data from Jiangxi Province, China. Hence, the predicted results of the numerical example should shed some lights on our understanding of the sustainable growth path.
\[
\frac{\partial Y}{\partial N} = \beta \Lambda_1 K_Y^a Z_Y^{1-a} N^{\beta - 1} = \frac{\beta Y}{N}. \tag{28}
\]

Taking the first-order derivative of Eq. (27) with respect to \( N \), we have:

\[
\frac{\partial E}{\partial N} = \frac{\gamma_1 X^2}{N^2} - 2\gamma_2(N - \bar{N}). \tag{29}
\]

Taking the first-order derivative of Eq. (27) with respect to \( I \), we have:

\[
\frac{\partial E}{\partial I} = \frac{2\gamma_1(1-\epsilon-\delta)X^2}{N}. \tag{30}
\]

Substituting (28), (29) and (30) into Eq. (23), and rearranging:

\[
\frac{\alpha}{u} \cdot \frac{Y}{K} = \frac{\gamma_1 X^2}{N^2} \left[ 1 + 2(1-\epsilon-\delta)\beta \frac{Y}{I} \right] - 2\gamma_2(N - \bar{N}) + g. \tag{31}
\]

Recall that equalities \( g_N = g_X = 0 \) and \( g = g_I \) hold in any sustainable growth path, \( N, X \) and \( Y/A \) are constants. Then the Eq. (27) becomes:

\[-\frac{\gamma_1 X^2}{N} - \gamma_2(N - \bar{N})^2 + \Gamma = 0. \tag{32}\]

Because at steady state \( g_N = 0 \), the equality \( E(N, X(Q, I)) = 0 \) always hold in the sustainable growth path, given the utility function \( U(c) = \ln(c) \), and combining (A.3) and (A.4), we can derive that:

\[2\gamma_1(1-\epsilon-\delta)X^2 \frac{C}{Y} = N \frac{I}{Y}. \tag{33}\]

Similarly, we can rewrite (22), (24), (20) and (21):

\[g = \frac{\alpha}{u} \cdot \frac{Y}{K} - \theta, \tag{34}\]

\[\frac{\alpha}{u} \cdot \frac{Y}{K} = \left[ \frac{\delta}{1-\nu} \frac{(1-\alpha)(\epsilon+\delta)}{1-\alpha} + (1-\epsilon-\delta) \right] g, \tag{35}\]

\[\frac{\alpha v}{(1-\alpha)u} = \frac{\epsilon(1-\nu)}{\delta(1-u)}, \tag{36}\]

\[\frac{A}{Y} = \frac{(1-\epsilon-\delta)(1-\alpha)}{\nu}. \tag{37}\]

Rearranging Eq. (2), we have:

\[g = (1 - \frac{C}{Y} - \frac{I}{Y}) \frac{Y}{K}. \tag{38}\]

Hence, we can use the eight equations above (31)-(38) to solve the eight unknown constants of the sustainable growth path, which are the common growth rate \( g \), the output-to-capital ratio \( Y/K \), the share of impoundment activities in final production \( I/Y \), the share of consumption in final production \( C/Y \), the net extractive use of water resources \( X \), the aggregate stock level of water resources \( N \), the share of physical
capital used in the final goods sector $u$ and the share of effective water resources used in the final goods sector $v$.\(^8\)

Table 2 below provides the long-run solutions for the combinations of some specific parameter values. The benchmark case is displayed in the first row of Table 2. In each of the following rows, the value of one parameter is changed with respect to the benchmark, while the values of other parameters remain the same as the benchmark case.

Table 2. Solutions of sustainable growth path

<table>
<thead>
<tr>
<th></th>
<th>$g$</th>
<th>$Y/K$</th>
<th>$I/Y$</th>
<th>$C/Y$</th>
<th>$X$</th>
<th>$N$</th>
<th>$u$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>5.16%</td>
<td>0.13</td>
<td>0.06</td>
<td>0.54</td>
<td>1.34</td>
<td>1.68</td>
<td>0.93</td>
<td>0.82</td>
</tr>
<tr>
<td>$\alpha = 0.7$</td>
<td>4.74%</td>
<td>0.13</td>
<td>0.07</td>
<td>0.57</td>
<td>1.37</td>
<td>1.68</td>
<td>0.92</td>
<td>0.83</td>
</tr>
<tr>
<td>$\beta = 0.005$</td>
<td>5.48%</td>
<td>0.13</td>
<td>0.06</td>
<td>0.52</td>
<td>1.35</td>
<td>1.67</td>
<td>0.93</td>
<td>0.82</td>
</tr>
<tr>
<td>$\epsilon = 0.35$</td>
<td>5.37%</td>
<td>0.13</td>
<td>0.08</td>
<td>0.51</td>
<td>1.35</td>
<td>1.67</td>
<td>0.94</td>
<td>0.84</td>
</tr>
<tr>
<td>$\gamma_1 = 2$</td>
<td>5.04%</td>
<td>0.12</td>
<td>0.07</td>
<td>0.51</td>
<td>1.31</td>
<td>1.42</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>$\Lambda_1 = 0.3$</td>
<td>9.66%</td>
<td>0.17</td>
<td>0.07</td>
<td>0.36</td>
<td>1.45</td>
<td>1.61</td>
<td>0.87</td>
<td>0.75</td>
</tr>
<tr>
<td>$\theta = 0.04$</td>
<td>5.83%</td>
<td>0.13</td>
<td>0.06</td>
<td>0.49</td>
<td>1.39</td>
<td>1.62</td>
<td>0.9</td>
<td>0.78</td>
</tr>
<tr>
<td>$N = 1.5$</td>
<td>6.84%</td>
<td>0.15</td>
<td>0.06</td>
<td>0.48</td>
<td>1.66</td>
<td>2.03</td>
<td>0.95</td>
<td>0.86</td>
</tr>
<tr>
<td>$\Gamma = 0.5$</td>
<td>4.23%</td>
<td>0.11</td>
<td>0.07</td>
<td>0.57</td>
<td>1.02</td>
<td>1.34</td>
<td>0.88</td>
<td>0.76</td>
</tr>
</tbody>
</table>

From the second row of Table 2, we see that a decrease in the weight of capital in the production function of final goods (decreasing $\alpha$), gives a lower long-run growth rate, an increasing intensity of impoundment activities, an increasing share of consumption in final production, and greater net extractive use of water resources.

An increase in the impact of water resources in the production function of final goods (increasing $\beta$), changes the values of the long-run growth rate, the share of consumption in final production, and the net extractive use of water resources slightly.

A decrease in the weight of capital in the production function of knowledge (decreasing $\epsilon$), causes an increasing intensity of impoundment activities. Meanwhile, fewer resources are allocated to the knowledge sector (increasing $u$ and $v$).

An increasing negative impact of the net extractive use on the regenerative capacities of water resources (increasing $\gamma_1$), gives an increasing intensity of

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\(^8\) Because some of equations (31)-(38) are nonlinear with respect to the unknowns, we use the strategies of guess-and-verify to derive the answer for each combination of parameters, such as the Kuhn-Tucker conditions.
impoundment activities and a lower stock of water resources. Meanwhile, more resources are allocated to the knowledge sector (decreasing \( u \) and \( v \)). In the sixth row, similar effects are found from an increase in the technology level of final goods production \( \Lambda_1 \), except the long-run growth rate increases substantially.

A deceased rate of time preference (decreasing \( \theta \)) increases the long-run growth rate, while more resources are allocated to the knowledge sector.

Finally, the two bottom rows of Table 2 show that as the water resources become more abundant (increasing \( \bar{N} \)), the long-run growth rate increases, while fewer resources are allocated to the knowledge sector. The opposite effects are observed when the regenerative capacities of water resources are decreasing (decreasing \( \Gamma \)).

Summarizing all the sensitivity analysis above, we find that the changes in the production technology do not affect the stock and use of water resources very much. However, the pace of the sustainable growth receives a major impact, which implies the effect of knowledge accumulation is a long-run effect. A change in the endowment and the regenerative capacities of water resources influences both the long-run growth rate and the stock and use of water resources. The good environment and abundant water resources are helpful to the sustainable economic growth. However, if the environment suffers some damages and the regenerative capacities of water resources become low, we can identify situations under which more impoundment activities should be carried out. From the analysis, we can see that the share of impoundment in final production \( I/Y \) never nears zero for all combinations of parameters. When the endowment or the regenerative capacities of water resources are low, long-run sustainable growth cannot be achieved without sufficient impoundment activities.

5. Policy implications

From the results of model and numerical example, we can get the policy implications for the case of Poyang Lake.

First, impoundment activities for Poyang Lake are necessary to achieve a
sustainable growth path, especially when the endowment or the regenerative capacities of water resources are low in current circumstances.

Second, impoundment activities should be used in conjunction with extractive water use, in order to restore the regenerative capacities of water resources. The government should pay special attentions on the coordination between two types of policies in practice.

Third, the government should encourage the production of knowledge, and the resources allocated to the knowledge sector depend on the technological levels and the endowment of Poyang Lake. Because it is a long-run effect of knowledge accumulation on the sustainable growth, the government should keep on financing the knowledge innovation activities even if no obvious effect can be observed in the short run.

6. Conclusions

In this paper, we construct a two-sector endogenous growth model in which the extractive use and the impoundment activities of water resources are incorporated, and apply the model to analyzing the case of Poyang Lake. The results of that model imply that on any sustainable growth path, the impoundment activities should grow at a constant rate to offset the negative impact of the extractive use of water on the regenerative capacities of water resources. Given that more and more hydropower stations are being constructed along the upper Yangtze River in addition to Three Gorges Dam, the gross extractive use of water resources will not decrease in the near future. Hence, to maintain sustainable growth, the construction of impoundment projects along the lower Yangtze River should grow at a constant rate. We also provide a numerical example showing that when the endowment or the regenerative capacities of water resources are low, the long-run sustainable growth cannot be achieved without sufficient impoundment activities. For policy implications, we find that the optimal levels of extractive water use and impoundment activities should matched, and the government should encourage the production of knowledge
based on the technological levels and the endowment of Poyang Lake.

There are several limitations of this study. 1) Because we analyze the long-run sustainable growth path in the endogenous growth model, which may not fit some empirical data perfectly and cannot predict the fluctuations around the long-run growth path; 2) We are not able to collect the empirical data for all parameters in our model, so that we have to propose a numerical example, instead of the empirical analysis.

The suggestions for future study are as follows. Because of conflicts between the local governments along the lower and upper Yangtze River, to analyze the interactive decision making of constructing impounding and hydropower projects using Game Theory, may be an interesting theoretical direction for further study. We can also apply our model to empirical analysis. By collecting more data related to the water resources of Poyang Lake and the economic development of surrounding regions, we can add proper specifications and parameter values of the regenerative function of water resources in the numerical example and test the results of our model. We can also predict the long-run economic influence of proposed impoundment projects around Poyang Lake by the local government of Jiangxi Province. Another interesting dimension of future research should be the comparative study between the case of Poyang Lake and foreign cases. For instance, Zhang and Choi (2013) and Zhang et al. (2014) provide the comparison of dynamic changes in CO2 emission performance of fossil fuel power plants in China and Korea. One can also conduct the comparison of water sustainable growth path in China and Korea.

Acknowledgements

We thank the financial support provided by the National Natural Science Foundation of China (41461118, 71473105), National Social Science Foundation of China (15ZDA054), Humanities and Social Science Fund of Jiangxi (JJ1420), and the China Postdoctoral Science Foundation (2015T80684).
Appendix

A.1. Proof of Proposition 2

Given a sustainable growth path, Proposition 1 in Section 2 shows that the stock of water resources can only increase at a constant rate if the net extractive use decreases at an increasing rate. Hence, on a sustainable growth path the net extractive use have to be constant \( g_N = g_X = 0 \), where the stock of water resources increases at a zero rate). By rearranging Eq. (12), we have:

\[
g_Q = -\frac{\eta_A}{\eta_Q} g_t. \tag{A.1}
\]

Substituting \( g_X = 0 \), \( g_H = g_h \), and Eq. (8) into (9) and (10), we can derive that:

\[
g_Q + g_h = \frac{(1-\eta_K)}{\eta_z} g,
\]

\[
\zeta_Z g_X - (1 - \zeta_Z) g_h = \zeta_K g.
\]

Solving the unknowns \( g_Q \) and \( g_h \) of two equations above, we get:

\[
g_Q = \frac{(1-\eta_K)(1-\zeta_Z) - \zeta_K \eta_Z}{\eta_z} g,
\]

\[
g_h = \frac{(1-\eta_K)\zeta_Z + \zeta_K \eta_Z}{\eta_z} g.
\]

A.2. Proof of Proposition 3

Proof: The present value Hamiltonian function of the maximization problems is:

\[
H_c \equiv He^{\theta t} = U(c) + \lambda \cdot (Y(K_Y, Z_Y, N) - C - I) + \mu \cdot E(N, X(Q, I)) + \kappa \cdot H(K_H, Z_H),
\]

(A.2)

where \( \lambda \), \( \mu \) and \( \kappa \) denote the shadow price of physical capital, of water resources and of knowledge, respectively. Using the maximum principle, we can derive the following conditions:

\[
U_c = \lambda L, \tag{A.3}
\]

\[
\lambda = \mu \frac{\partial E}{\partial I}, \tag{A.4}
\]

\[
\lambda \frac{\partial Y}{\partial Z_Y} + \kappa \frac{\partial H}{\partial Z_H} (1 - v) h = -\mu \frac{\partial E}{\partial Q} \tag{A.5}
\]
\[ \lambda \frac{\partial Y}{\partial K_Y} K = \kappa \frac{\partial H}{\partial K_H} K, \quad (A.6) \]
\[ \lambda \frac{\partial Y}{\partial Z_Y} hQ = \kappa \frac{\partial H}{\partial Z_H} hQ. \quad (A.7) \]

The five Eq.s are derived by the first order conditions with respect to \( c, I, X, u \) and \( v \). And the optimal conditions of the co-state variables \( \lambda, \mu \) and \( \kappa \) are as follows:

\[ \dot{\lambda} = \theta \lambda - \lambda \frac{\partial Y}{\partial K_Y} u - \kappa \frac{\partial H}{\partial K_H} (1 - u), \quad (A.8) \]
\[ \dot{\mu} = \theta \mu - \lambda \frac{\partial Y}{\partial N} - \mu \frac{\partial E}{\partial N}, \quad (A.9) \]
\[ \dot{\kappa} = \theta \kappa - \lambda \frac{\partial Y}{\partial Z_Y} vQ - \kappa \frac{\partial H}{\partial Z_H} (1 - v)Q. \quad (A.10) \]

Taking derivatives on two sides of Eq. (A.3), then dividing by \( U_c \) and substituting Eq. (A.3), we have:

\[ \rho \cdot \frac{\dot{c}}{c} = -\frac{\dot{\lambda}}{\lambda}, \quad (A.11) \]

where \( \rho = -c(U_{cc}/U_c) \) is the intertemporal substitution elasticity of utility.

Furthermore, substituting Eq. (A.6) into Eq. (A.8) yields:

\[ \dot{\lambda} = \theta \lambda - \lambda \frac{\partial Y}{\partial K_Y}, \quad (A.12) \]

and dividing both sides by \( \lambda \) and gives:

\[ \frac{\dot{\lambda}}{\lambda} = \theta - \frac{\partial Y}{\partial K_Y}. \quad (A.13) \]

Substituting Eq. (A.13) into Eq. (A.11) and rearranging, we get:

\[ \frac{\dot{c}}{c} = \frac{1}{\rho} \left( \frac{\partial Y}{\partial K_Y} - \theta \right). \]

Following the similar procedures, we can derive the remaining four equations:

\[ \frac{\partial Y}{\partial K_Y} = \frac{\partial E}{\partial I} \frac{\partial Y}{\partial I} + \frac{\partial E}{\partial N} + \frac{\dot{\mu}}{\mu} - \frac{\dot{\lambda}}{\lambda}, \]
\[ \frac{\partial Y}{\partial Z_H} = \frac{\partial H}{\partial K_H} Q + \frac{\dot{\kappa}}{\kappa} - \frac{\dot{\lambda}}{\lambda}, \]
\[ \frac{\partial Y}{\partial Z_H} = \frac{\partial H}{\partial K_H} \frac{\partial K_H}{\partial Z_H} \frac{\partial K_H}{\partial Z_H}, \]
\[ \frac{\partial Y}{\partial Z_Y} h = -\frac{\partial X/\partial Q}{\partial X/\partial I}. \]

Reference


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