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# Patents vs R&D Subsidies on Income Inequality

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## Abstract

This study explores the effects of patent protection and R&D subsidies on economic growth and income inequality using a Schumpeterian growth model with heterogeneity in household asset holdings. We find that although strengthening patent protection and raising R&D subsidies have the same macroeconomic effect of stimulating economic growth, they have drastically different microeconomic implications on income inequality. Specifically, strengthening patent protection increases income inequality whereas raising R&D subsidies decreases (increases) it if the quality step size is sufficiently small (large). An empirically realistic quality step size is smaller than the threshold implying a negative effect of R&D subsidies on income inequality. We also calibrate the model to provide a quantitative analysis and find that strengthening patent protection causes a moderate increase in income inequality and a negligible increase in consumption inequality whereas raising R&D subsidies causes a significant decrease in both income inequality and consumption inequality.

*JEL classification:* D30, O30, O40

*Keywords:* R&D subsidies, patents, income inequality, economic growth

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# 1 Introduction

In this study, we explore the effects of patent protection and R&D subsidies on economic growth and income inequality. We consider the Schumpeterian growth model and extend it by allowing for heterogeneity in the asset holdings of households. As Piketty (2014) argues, an unequal distribution of wealth is an important cause of income inequality. Within this growth-theoretic framework, we find that although strengthening patent protection and raising R&D subsidies have the same macroeconomic effect of stimulating economic growth, they have drastically different microeconomic implications on income inequality. Therefore, it is important to consider beyond aggregate effects and investigate distributional implications when evaluating the overall effects of a policy instrument.

When a strengthening of patent protection or a raise in R&D subsidies leads to a higher rate of economic growth, the real interest rate also rises leading to an increase in asset income, which is the cause of inequality in the model. As a result, strengthening patent protection and raising R&D subsidies both have a positive effect on income inequality via this *interest-rate* channel. Intuitively, the higher interest rate increases the income of asset-wealthy households relative to asset-poor households. Furthermore, the two policy instruments carry an *asset-value* effect that affects income inequality. By increasing monopolistic profits, strengthening patent protection increases asset value and causes an additional positive effect on income inequality. In contrast, raising R&D subsidies suppresses income inequality by reducing asset value through creative destruction<sup>1</sup> and consequently causing a decrease in asset income. As a result of the opposing interest-rate and asset-value effects, raising R&D subsidies has an overall ambiguous effect on income inequality. Specifically, if the quality step size is smaller (larger) than a threshold, then raising R&D subsidies leads to a lower (higher) degree of income inequality. An empirically realistic quality step size is smaller than the threshold implying a negative effect of R&D subsidies on income inequality. In contrast, a strengthening of patent protection causes a positive effect on income inequality. This theoretical result is consistent with the empirical finding in Adams (2008) who uses an index of patent rights constructed by Ginarte and Park (1997) and finds that strengthening patent protection has a positive and statistically significant effect on income inequality.

We also explore the effects of patent protection and R&D subsidies on consumption inequality. We find that strengthening patent protection increases consumption inequality whereas raising R&D subsidies continues to have an overall ambiguous effect on consumption inequality. Finally, we calibrate the model to investigate the quantitative effects of patent protection and R&D subsidies on growth and inequality. The policy experiments that we consider are to increase separately the rate of R&D subsidies and the level of patent protection such that the R&D share of GDP increases by one-tenth in each case. We find that the increase in patent protection causes a moderate increase in income inequality and a negligible increase in consumption inequality whereas the increase in R&D subsidies causes a quantitatively significant decrease in both income inequality and consumption inequality. These results are robust to a number of robustness checks.

This study relates to the literature on R&D and economic growth. The seminal studies

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<sup>1</sup>This creative-destruction effect on asset value is also present in the case of patent protection but is offset by its monopolistic-profit effect.

in this literature are Romer (1990), Segerstrom *et al.* (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Subsequent studies in this literature apply variants of the R&D-based growth model to explore the effects of patent protection and R&D subsidies on innovation and economic growth; see for example Peretto (1998), Li (2001), Lin (2002, 2015), Furukawa (2007), Futagami and Iwaisako (2007), Horii and Iwaisako (2007), Iwaisako and Futagami (2013), Zeng and Zhang (2007), Zeng *et al.* (2014), Chu (2009), Impullitti (2010) and Acemoglu and Akcigit (2012). These studies focus on a representative-household framework and do not consider the distributional implications of patent protection and R&D subsidies. The current study fills this gap in the literature by exploring the effects of patent protection and R&D subsidies on inequality (in addition to growth) in a Schumpeterian model with heterogeneous households.

A small number of studies in the literature also consider heterogeneous households in the R&D-based growth model. Representative studies include Chou and Talmain (1996), Li (1998), Zweimuller (2000), Foellmi and Zweimuller (2006) and Jones and Kim (2014). These studies focus on how inequality affects economic growth. In contrast, the current study explores the effects of policy instruments on income and consumption inequality. Chu (2010) also explores the effects of patent policy on income and consumption inequality; however, he does not consider R&D subsidies. Therefore, this study generalizes the analysis in Chu (2010) by providing a comparative analysis of two popular policy instruments, which appear to have similar aggregate effects but drastically different distributional implications. Furthermore, we modify the R&D specification in Chu (2010) to a lab-equipment innovation process under which R&D uses final goods (instead of labor) as input. In this case, strengthening patent protection causes the positive asset-value effect in addition to the positive interest-rate effect on income inequality.<sup>2</sup>

The rest of this study is organized as follows. Section 2 presents the model. Section 3 explores the effects of patent protection and R&D subsidies. The final section concludes.

## 2 A Schumpeterian growth model with heterogeneous households

In this section, we extend the Schumpeterian quality-ladder model in Grossman and Helpman (1991), which is a workhorse model in the literature, to allow for heterogeneous households with different asset holdings. Furthermore, we consider two policy instruments, patent protection and R&D subsidies, in order to perform a comparative policy analysis. Finally, we also modify the R&D specification by assuming a lab-equipment innovation process as in Rivera-Batiz and Romer (1991).

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<sup>2</sup>If R&D uses labor as input instead, then the positive asset-value effect of patent protection would be absent because the monopolistic-profit effect is exactly offset by the creative-destruction effect, but the negative asset-value effect of R&D subsidies would remain.

## 2.1 Households

There is a unit continuum of households with identical preferences but different levels of asset holdings. Households are indexed by  $h \in [0, 1]$ . Each household  $h$  has the following utility function:

$$u(h) = \int_0^{\infty} e^{-\rho t} \ln c_t(h) dt. \quad (1)$$

The parameter  $\rho > 0$  is the subjective discount rate. Each household  $h$  supplies one unit of labor to earn wage income and makes consumption-saving decision to maximize utility subject to the following asset-accumulation equation:

$$\dot{a}_t(h) = r_t a_t(h) + (1 - \tau_t) w_t - c_t(h). \quad (2)$$

$a_t(h)$  is the real value of financial assets owned by household  $h$ .  $r_t$  is the real interest rate.  $w_t$  is the real wage rate.  $c_t(h)$  is consumption by household  $h$ .  $\tau_t \in (0, 1)$  is the rate of a wage income tax collected by the government.<sup>3</sup> From standard dynamic optimization, the Euler equation is given by

$$\frac{\dot{c}_t(h)}{c_t(h)} = r_t - \rho, \quad (3)$$

which shows that the growth rate of consumption is the same across households such that  $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$  for all  $h \in [0, 1]$ , where  $c_t \equiv \int_0^1 c_t(h) dh$  is aggregate consumption.

## 2.2 Final goods

Competitive firms produce final good  $y_t$  using the following Cobb-Douglas aggregator over a unit continuum of differentiated intermediate goods:

$$y_t = \exp \left( \int_0^1 \ln x_t(i) di \right), \quad (4)$$

where  $x_t(i)$  denotes intermediate good  $i \in [0, 1]$ . The conditional demand function for  $x_t(i)$  is given by

$$x_t(i) = \frac{y_t}{p_t(i)}, \quad (5)$$

where  $p_t(i)$  is the price of  $x_t(i)$ .

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<sup>3</sup>Here we consider a wage income tax for two reasons. First, it is non-distortionary and does not affect aggregate equilibrium allocations. Second, if we finance R&D subsidies by a tax on asset income which is the source of inequality in the model, then raising R&D subsidies would cause an additional negative effect on inequality, which biases in favor of our finding of a negative effect of R&D subsidies on inequality.

## 2.3 Intermediate goods

There is a unit continuum of industries, which are also indexed by  $i \in [0, 1]$ , producing differentiated intermediate goods. In each industry  $i$ , there is a monopolistic industry leader, who holds a patent on the latest technology and dominates the market until the arrival of the next innovation.<sup>4</sup> The production function of the leader in industry  $i$  is

$$x_t(i) = z^{n_t(i)} l_t(i), \quad (6)$$

where the parameter  $z > 1$  is the step size of each quality improvement,  $n_t(i)$  is the number of quality improvements that have occurred in industry  $i$  as of time  $t$ , and  $l_t(i)$  is the amount of labor employed in industry  $i$ . Given the productivity level  $z^{n_t(i)}$ , the marginal cost function of the leader in industry  $i$  is  $w_t/z^{n_t(i)}$ . From Bertrand competition, the profit-maximizing price is a constant markup over the marginal cost such that

$$p_t(i) = \mu \frac{w_t}{z^{n_t(i)}}, \quad (7)$$

where the markup  $\mu \leq z$  is a policy parameter determined by the level of patent protection in the economy.<sup>5</sup> Given (7), the amount of monopolistic profit in industry  $i$  is

$$\pi_t(i) = \frac{\mu - 1}{\mu} p_t(i) x_t(i) = \frac{\mu - 1}{\mu} y_t, \quad (8)$$

and the wage payment in industry  $i$  is

$$w_t l_t(i) = \frac{1}{\mu} p_t(i) x_t(i) = \frac{1}{\mu} y_t, \quad (9)$$

where the second equality of (8) and (9) follows from (5).

## 2.4 R&D

Equation (8) shows that  $\pi_t(i) = \pi_t$  for all intermediate goods  $i \in [0, 1]$ . Therefore, the value of inventions is also the same across industries such that  $v_t(i) = v_t$  for all  $i \in [0, 1]$ .<sup>6</sup> The no-arbitrage condition that determines  $v_t$  is

$$r_t = \frac{\pi_t + \dot{v}_t - \lambda_t v_t}{v_t}, \quad (10)$$

which states that the rate of return on  $v_t$  must equal the interest rate. The rate of return on  $v_t$  is the sum of monopolistic profit  $\pi_t$ , capital gain  $\dot{v}_t$  and expected capital loss  $\lambda_t v_t$ , where  $\lambda_t$  is the rate of creative destruction.

<sup>4</sup>See Cozzi (2007) for a discussion of this Arrow replacement effect.

<sup>5</sup>The presence of monopolistic profit attracts potential imitation; therefore, stronger patent protection allows monopolistic producers to charge a higher markup without losing their markets to potential imitators. This formulation of patent breadth captures Gilbert and Shapiro's (1990) seminal insight on "breadth as the ability of the patentee to raise price".

<sup>6</sup>We follow the standard approach in the literature to focus on the symmetric equilibrium. See Cozzi *et al.* (2007) for a theoretical justification for the symmetric equilibrium to be the unique rational-expectation equilibrium in the Schumpeterian model.

Competitive entrepreneurs devote  $R_t$  units of final goods to R&D. The free-entry condition of R&D is

$$\lambda_t v_t = (1 - s)R_t, \quad (11)$$

where the policy parameter  $s \in (0, 1)$  is the rate of R&D subsidies and  $\lambda_t v_t$  is the expected return on R&D. We assume that  $\lambda_t$  is an increasing function in R&D spending  $R_t$  given by

$$\lambda_t = \frac{\varphi R_t}{Z_t}, \quad (12)$$

where  $Z_t$  is the level of technology in the economy and captures increasing R&D difficulty due to an increasing-complexity effect of technology.<sup>7</sup> Combining (11) and (12) yields

$$v_t = \frac{1 - s}{\varphi} Z_t, \quad (13)$$

which shows that invention value  $v_t$  is proportional to technology level  $Z_t$  and that the ratio  $v_t/Z_t$  is decreasing in R&D subsidy  $s$ . Intuitively, for a given  $Z_t$ , the free-entry condition implies that an increase in subsidy  $s$  makes R&D cheaper and leads to a decrease in the price of inventions. In equilibrium, this decrease in the value of inventions is caused by a higher rate of creative destruction.

## 2.5 Government

The government decides on the level  $\mu$  of patent protection in the economy. Also, it collects tax revenue to finance R&D subsidies and non-productive government expenditure  $G_t$  subject to the following balanced-budget condition:

$$\tau_t w_t = sR_t + G_t, \quad (14)$$

where  $G_t = \gamma y_t$  is assumed to be proportional to output. The parameter  $\gamma \equiv G_t/y_t \geq 0$  is the ratio of government expenditure to output.

## 3 Solving the model

In this section, we proceed to solve the model as follows. Section 3.1 defines the equilibrium. Section 3.2 shows that the aggregate economy always jumps to a unique balanced growth path and explores the effects of patent protection and R&D subsidies on the aggregate growth rate of the economy. Section 3.3 shows that the wealth distribution is stationary, and hence, it is exogenously determined by its initial condition. Sections 3.4 and 3.5 show that income and consumption distributions are also stationary, but they are endogenously determined by patent protection and R&D subsidies.

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<sup>7</sup>See Venturini (2012) who provides empirical evidence for the presence of increasing R&D difficulty.

### 3.1 Decentralized equilibrium

The equilibrium is a time path of allocations  $\{c_t(h), a_t(h), y_t, x_t(i), l_t(i), R_t\}$  and a time path of prices  $\{w_t, r_t, p_t(i), v_t\}$ . Also, at each instance of time, the following conditions hold:

- households  $h \in [0, 1]$  maximize utility taking  $\{w_t, r_t\}$  as given;
- competitive firms produce final good  $y_t$  to maximize profit taking prices as given;
- each monopolistic firm  $i$  produces intermediate good  $x_t(i)$  and chooses  $\{l_t(i), p_t(i)\}$  to maximize profit taking  $w_t$  as given;
- competitive R&D entrepreneurs choose  $R_t$  to maximize expected profit taking  $\{w_t, v_t\}$  as given;
- the market-clearing condition for labor holds such that  $\int_0^1 l_t(i) di = 1$ ;
- the market-clearing condition for final goods holds such that  $\int_0^1 c_t(h) dh + R_t + G_t = y_t$ ;
- the total value of household assets equals the value of all monopolistic firms such that  $\int_0^1 a_t(h) dh = v_t$ .

### 3.2 Aggregate economy

From (9), we see that  $l_t(i) = l_t$  for all  $i \in [0, 1]$ . Therefore, substituting (6) into (4) yields

$$y_t = Z_t l_t = Z_t, \quad (15)$$

where the second equality uses  $l_t = 1$ . Aggregate technology  $Z_t$  is defined as

$$Z_t \equiv \exp \left( \int_0^1 n_t(i) di \ln z \right) = \exp \left( \int_0^t \lambda_\omega d\omega \ln z \right), \quad (16)$$

where the last equality uses the law of large numbers. Differentiating the log of  $Z_t$  with respect to time yields the growth rate of technology given by

$$\frac{\dot{Z}_t}{Z_t} = \lambda_t \ln z, \quad (17)$$

where  $\lambda_t = \varphi R_t / Z_t$  from (12). The following proposition shows that the aggregate economy jumps to a unique balanced growth path along which aggregate variables grow at the same rate as technology.

**Proposition 1** *The aggregate economy jumps to a unique and saddle-point stable balanced growth path along which variables  $\{c_t, y_t, w_t, \pi_t, v_t\}$  grow at the same rate as technology  $Z_t$ .*



**Proof.** See the Appendix. ■

Given Proposition 1, we impose balanced growth on (10) to derive

$$v_t = \frac{\pi_t}{r - g + \lambda} = \frac{\pi_t}{\rho + \lambda}, \quad (18)$$

where  $g$  denotes the steady-state growth rate of technology. Substituting (18) into (13) yields the steady-state arrival rate of innovation given by

$$\lambda = \frac{\varphi}{1 - s} \frac{\pi_t}{Z_t} - \rho = \frac{\varphi}{1 - s} \frac{\mu - 1}{\mu} - \rho, \quad (19)$$

where the second equality uses (8) and (15). Therefore, the steady-state growth rate of technology is

$$g = \lambda \ln z = \left( \frac{\varphi}{1 - s} \frac{\mu - 1}{\mu} - \rho \right) \ln z. \quad (20)$$

The following proposition shows that a strengthening of patent protection  $\mu$  and a raise in R&D subsidy  $s$  both lead to an increase in R&D and the technology growth rate  $g$ . These aggregate effects of patent protection and R&D subsidies are quite common in the literature; see for example Peretto (1998), Li (2001) and Chu (2011).

**Proposition 2** *The steady-state equilibrium growth rate of technology is increasing in the level  $\mu$  of patent protection and the rate  $s$  of R&D subsidies.*

**Proof.** Equation (20) shows that  $g$  is increasing in  $\mu$  and  $s$ . ■

### 3.3 Wealth distribution

At time 0, the share of assets owned by household  $h$  is exogenously given by  $\theta_{a,0}(h) \equiv a_0(h)/a_0$ , which has a general distribution function  $f_a$  with a mean of one and a standard deviation of  $\sigma_a > 0$ . From (2), the aggregate value of financial assets evolves according to

$$\dot{a}_t = r_t a_t + (1 - \tau_t) w_t - c_t, \quad (21)$$

where  $a_t \equiv \int_0^1 a_t(h) dh$  is the total value of financial assets owned by all households. Combining (2) and (21) yields the law of motion for  $\theta_{a,t}(h) \equiv a_t(h)/a_t$  given by

$$\frac{\dot{\theta}_{a,t}(h)}{\theta_{a,t}(h)} = \frac{\dot{a}_t(h)}{a_t(h)} - \frac{\dot{a}_t}{a_t} = \frac{c_t - (1 - \tau_t) w_t}{a_t} - \frac{c_t(h) - (1 - \tau_t) w_t}{a_t(h)}, \quad (22)$$

which can be re-expressed as

$$\dot{\theta}_{a,t}(h) = \frac{c_t - (1 - \tau_t) w_t}{a_t} \theta_{a,t}(h) - \frac{\theta_{a,t}(h) c_t - (1 - \tau_t) w_t}{a_t}, \quad (23)$$

where consumption share  $\theta_{c,t}(h) \equiv c_t(h)/c_t$  is a stationary variable. From (3),  $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ , which in turn implies that  $\dot{\theta}_{c,t}(h)/\theta_{c,t}(h) = 0$  and that  $\theta_{c,t}(h) = \theta_{c,0}(h)$  for all  $t > 0$ . Then, recall that the aggregate economy is always on the balanced growth path along which  $c_t/a_t$  and  $w_t/a_t$  are stationary. We will also show that the steady-state equilibrium tax rate  $\tau$  is stationary. Therefore, (23) is a one-dimensional differential equation, which describes the potential evolution of  $\theta_{a,t}(h)$  given an initial  $\theta_{a,0}(h)$ . In the appendix, we show that the coefficient on  $\theta_{a,t}(h)$  in (23) is positive. Together with the fact that  $\theta_{a,t}(h)$  is a state variable, the only solution of (23) consistent with long-run stability is  $\dot{\theta}_{a,t}(h) = 0$  for all  $t$ , which is achieved by consumption share  $\theta_{c,0}(h)$  jumping to its steady-state value as shown in the appendix.

**Proposition 3** *For every household  $h$ , its asset share is constant over time and exogenously determined at time 0 such that  $\theta_{a,t}(h) = \theta_{a,0}(h)$  for all  $t$ .*

**Proof.** See the Appendix. ■

### 3.4 Income distribution

Before-tax income earned by household  $h$  is

$$I_t(h) \equiv r_t a_t(h) + w_t. \quad (24)$$

Total before-tax income earned by all households is

$$I_t = r_t a_t + w_t. \quad (25)$$

Combining these two equations yields the share of income earned by household  $h$  given by

$$\theta_{I,t}(h) \equiv \frac{I_t(h)}{I_t} = \frac{r_t a_t \theta_{a,0}(h) + w_t}{r_t a_t + w_t}. \quad (26)$$

From (3), we have  $r_t = \rho + g$ . From (13), we have  $v_t = (1 - s)Z_t/\varphi$ , and we also know that  $a_t = v_t$ . From (9) and (15), we have  $w_t = Z_t/\mu$ . Substituting these conditions into (26) yields

$$\theta_{I,t}(h) = \frac{(\rho + g)(1 - s)\mu\theta_{a,0}(h) + \varphi}{(\rho + g)(1 - s)\mu + \varphi} \quad (27)$$

for all  $t$ . Equation (27) implies that the stationary distribution function  $f_I$  of income share has a mean of one and a standard deviation of

$$\sigma_I = \sqrt{\int_0^1 [\theta_I(h) - 1]^2 dh} = \frac{(\rho + g)(1 - s)\mu}{(\rho + g)(1 - s)\mu + \varphi} \sigma_a. \quad (28)$$

Here we measure income inequality by the standard deviation  $\sigma_I$  of income share, which is equivalent to the coefficient of variation of income. Substituting the steady-state equilibrium growth rate  $g(\mu, s)$  from (20) into (28) yields

$$\sigma_I = \frac{\rho(1 - \ln z)(1 - s) + \frac{\mu-1}{\mu}\varphi \ln z}{\rho(1 - \ln z)(1 - s) + \frac{\mu-1}{\mu}\varphi \ln z + \varphi/\mu} \sigma_a. \quad (29)$$

**Proposition 4** *The degree of income inequality is always increasing in the level  $\mu$  of patent protection but decreasing (increasing) in the rate  $s$  of R&D subsidies if  $\ln z < 1$  ( $\ln z > 1$ ).*

**Proof.** Equation (29) shows that  $\sigma_I$  is increasing in  $\mu$  but decreasing (increasing) in  $s$  if  $1 - \ln z > 0$  ( $1 - \ln z < 0$ ). ■

Recall that the steady-state equilibrium growth rate  $g(\mu, s)$  is increasing in both  $\mu$  and  $s$ . Equation (28) shows that an increase in  $g$  leads to an increase in income inequality  $\sigma_I$  by increasing the real interest rate and asset income, which is the cause of inequality in the model. This is the *symmetric* interest-rate effect of patent protection  $\mu$  and R&D subsidy  $s$  on income inequality  $\sigma_I$ . However, these two policy instruments have an additional effect on income inequality captured by the term  $(1-s)\mu$  in (28), and this effect is *asymmetric* between  $\mu$  and  $s$ . To understand this asymmetric effect, one can consider the ratio  $v_t/w_t = (1-s)\mu/\varphi$  derived from (9), (13) and (15). Interestingly,  $v_t/w_t$  is decreasing in R&D subsidy  $s$  but increasing in patent protection  $\mu$ . Intuitively, an increase in patent protection reduces the share of wage income as (9) shows and raises the share of profit income as (8) shows, thereby increasing asset income relative to wage income. In contrast, an increase in R&D subsidies reduces asset income by decreasing the value of inventions as (13) shows. Therefore, while strengthening patent protection causes only positive effects on income inequality, raising R&D subsidies carries both a positive effect and a negative effect on income inequality. The positive effect via the interest-rate channel is stronger when the quality step size is larger because  $\partial g/\partial s$  is increasing in the quality step size  $z$  as (20) shows. Equation (29) shows that if  $\ln z$  is larger (smaller) than one, then a raise in R&D subsidies would have a positive (negative) effect on income inequality. The empirical value of  $z$  is often considered to be less than 1.20;<sup>8</sup> see for example Acemoglu and Akcigit (2012). Therefore, under an empirically realistic quality step size, raising R&D subsidies has a negative effect on income inequality.

### 3.5 Consumption distribution

From (2), consumption by household  $h$  is

$$c_t(h) = \left[ r_t - \frac{\dot{a}_t(h)}{a_t(h)} \right] a_t(h) + (1 - \tau)w_t = \rho a_t(h) + (1 - \tau)w_t, \quad (30)$$

where the second equality uses (3) and the balanced-growth condition  $\dot{a}_t(h)/a_t(h) = \dot{c}_t(h)/c_t(h)$ . Aggregate consumption is

$$c_t = \rho a_t + (1 - \tau)w_t. \quad (31)$$

Combining (30) and (31) yields the share of consumption by household  $h$  given by

$$\theta_{c,t}(h) \equiv \frac{c_t(h)}{c_t} = \frac{\rho a_t(h) + (1 - \tau)w_t}{\rho a_t + (1 - \tau)w_t}. \quad (32)$$

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<sup>8</sup>It is useful to note that  $\ln(1.2) \approx 0.182 < 1$ .

Following the same derivations as in the previous subsection, we obtain

$$\theta_{c,t}(h) = \frac{\rho(1-s)\mu\theta_{a,0}(h) + (1-\tau)\varphi}{\rho(1-s)\mu + (1-\tau)\varphi} \quad (33)$$

for all  $t$ . Equation (33) implies that the stationary distribution function of consumption share has a mean of one and a standard deviation of

$$\sigma_c = \sqrt{\int_0^1 [\theta_c(h) - 1]^2 dh} = \frac{\rho(1-s)\mu}{\rho(1-s)\mu + (1-\tau)\varphi} \sigma_a, \quad (34)$$

where the steady-state equilibrium tax rate  $\tau$  can be derived as follows by substituting (9), (12), (15) and (19) into (14):

$$\tau = \mu \frac{sR + G}{y} = \mu \left( \frac{s\lambda}{\varphi} + \gamma \right) = \mu s \left( \frac{1}{1-s} \frac{\mu-1}{\mu} - \frac{\rho}{\varphi} \right) + \mu\gamma. \quad (35)$$

Substituting (35) into (34) yields

$$\sigma_c = \frac{\rho(1-s)\mu}{(\rho - \gamma\varphi)\mu + \varphi(1-s\mu)/(1-s)} \sigma_a. \quad (36)$$

As before, we measure consumption inequality by the standard deviation of consumption share, which is equivalent to the coefficient of variation of consumption. Equation (34) shows that consumption inequality  $\sigma_c$  is independent of the interest rate because a higher interest rate leads to a higher saving rate such that consumption  $c_t(h)$  is always a constant fraction  $\rho$  of asset  $a_t(h)$  due to the log utility function in (1). Therefore, unlike income inequality, the interest-rate effect of patents and R&D subsidies is absent under consumption inequality. Consequently, for a given tax rate  $\tau$ , we are left with the asymmetric asset-value effect of patents and R&D subsidies captured by the term  $(1-s)\mu$  in  $v_t/w_t = (1-s)\mu/\varphi$  and in (34). As in the case of income inequality, a strengthening of patent protection has a positive asset-value effect on consumption inequality by raising asset income whereas an increase in R&D subsidies causes a negative asset-value effect. However, the tax rate  $\tau$  is also a function of  $\mu$  and  $s$ . When either  $\mu$  or  $s$  increases, R&D spending increases, which in turn leads to a higher tax rate  $\tau$  as (35) shows and worsens consumption inequality  $\sigma_c$  because the tax is levied on wage income  $w_t$  rather than asset income that is the source of inequality in the model. Therefore, strengthening patent protection increases consumption inequality due to the positive asset-income and tax-rate effects whereas raising R&D subsidies has an overall ambiguous effect on consumption inequality due to the negative asset-income effect and the positive tax-rate effect.

**Proposition 5** *The degree of consumption inequality is increasing in the level  $\mu$  of patent protection but can be decreasing or increasing in the rate  $s$  of R&D subsidies.*

**Proof.** Equation (34) shows that  $\sigma_c$  is increasing in  $\mu$  but can be decreasing or increasing in  $s$ . ■

## 4 Quantitative analysis

In this section, we calibrate the model to data in the US in order to provide a quantitative analysis on the effects of patent protection and R&D subsidies. The model features the following aggregate parameters:  $\{\rho, s, \mu, z, \varphi, \gamma\}$ . We follow Acemoglu and Akcigit (2012) to set the discount rate  $\rho$  to a conventional value of 0.05. We follow Impullitti (2010) to set the R&D subsidy rate  $s$  in the US to 0.188. As for the patent protection level  $\mu$ , we calibrate its value by setting the time between arrivals of innovation to  $1/\lambda = 3$  years as in Acemoglu and Akcigit (2012). As for the quality step size  $z$ , we calibrate its value by setting the long-run growth rate  $g$  to 2%. We calibrate the value of R&D productivity  $\varphi$  by setting the R&D share of GDP to  $R/y = 0.028$ . Finally, the ratio  $\gamma$  of government spending to GDP is set to 0.2 as in Belo *et al.* (2013). These empirical moments are representative of the US economy. Table 1 summarizes the calibrated parameter values.

$\rho$	$s$	$\mu$	$z$	$\varphi$	$\gamma$
0.050	0.188	1.027	1.062	11.905	0.200

The policy experiments that we consider are to increase separately the R&D subsidy rate  $s$  and the patent protection level  $\mu$  such that the R&D share of GDP  $R/y$  increases by one-tenth from 0.028 to 0.031 in each case. Table 2 reports the resulting implications of each of these policy changes on economic growth  $g$ , income inequality  $\sigma_I$  and consumption inequality  $\sigma_c$ . Table 2a shows that in order to increase the R&D share of GDP  $R/y$  by one-tenth, patent protection level  $\mu$  needs to increase from 1.027 to 1.029, in which case the growth rate  $g$  increases from 2.0% to 2.2%. This increase in the growth rate leads to a corresponding increase in the interest rate, which in turn drives up income inequality  $\sigma_I$ . In this case, the coefficient of variation of income increases by 3.08%.<sup>9</sup> As for consumption inequality  $\sigma_c$ , the coefficient of variation of consumption increases negligibly by 0.36% because the positive interest-rate effect is absent. Although the positive asset-value effect remains and the positive tax-rate effect appears, they are relatively minor in magnitude in the case of patent protection.

Table 2b shows that in order to increase the R&D share of GDP  $R/y$  by one-tenth, the R&D subsidy rate  $s$  needs to increase from 0.188 to 0.253. The increase in the growth rate  $g$  is the same as above and gives rise to a positive interest-rate effect on income inequality. However, the magnitude of the increase in  $s$  is large, which in turn gives rise to a strong negative asset-value effect on income inequality  $\sigma_I$ . In this case, the coefficient of variation of income decreases significantly by 5.35%. As for consumption inequality  $\sigma_c$ , the coefficient of variation of consumption decreases even more significantly by 7.67% because the positive interest-rate effect is now absent. Although the positive tax-rate effect appears, its magnitude is relatively minor because R&D spending is a small share of GDP, and hence, the negative asset-value effect remains the dominant force.

<sup>9</sup>Here the change in income inequality  $\sigma_I$  is reported as percent change (i.e.,  $\Delta\sigma_I \equiv \sigma_I^{new}/\sigma_I^{old} - 1$ ).

Table 2a: Effects of patent protection				
$\mu$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$
1.029	0.031	2.20%	3.08%	0.36%
Table 2b: Effects of R&D subsidies				
$s$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$
0.253	0.031	2.20%	-5.35%	-7.67%

## 4.1 Robustness check: technology growth rate

Starting from this section, we consider a number of robustness checks on our numerical analysis. In the previous section, we calibrate the value of the quality step size  $z$  by targeting the long-run growth rate of output per capita in the US. It is equally reasonable to calibrate the value of  $z$  by targeting the long-run growth rate of technology instead. In this case, we re-calibrate the parameter values by setting  $g = 1\%$ . Table 3 reports the re-calibrated parameter values and shows that the calibrated value of  $z$  decreases from 1.062 to 1.030 (because a lower  $g = \lambda \ln z$  implies a lower  $z$ ) whereas other parameter values remain largely the same. Under the new parameter values, the long-run growth rate  $g$  increases by the same proportion of one-tenth from 1.0% to 1.1%. Table 4 shows that the pattern of changes in inequality is the same as before except that the increase in income inequality becomes smaller at 1.90% whereas the decrease in consumption inequality becomes larger at 6.44%.

Table 3: Calibrated parameter values					
$\rho$	$s$	$\mu$	$z$	$\varphi$	$\gamma$
0.050	0.188	1.027	1.030	11.905	0.200
Table 4a: Effects of patent protection					
$\mu$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$	
1.029	0.031	1.10%	1.90%	0.36%	
Table 4b: Effects of R&D subsidies					
$s$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$	
0.253	0.031	1.10%	-6.44%	-7.67%	

## 4.2 Robustness check: innovation arrival rate

So far we have considered a value of  $1/3$  for the innovation arrival rate  $\lambda$ , which however is imprecisely estimated in the literature. Here we consider an alternative value of 0.1, which implies a time between innovation arrivals of 10 years, while keeping a technology growth rate of  $g = 1\%$  as in the previous section. Table 5 reports the re-calibrated parameter values and shows that the parameter that changes the most is R&D productivity  $\varphi$  that decreases from 11.905 to 3.571 because the innovation arrival rate  $\lambda(\varphi)$  is an increasing function of  $\varphi$ . The value of  $z$  increases quite significantly as well because a lower innovation arrival rate  $\lambda$  requires a larger quality step size  $z$  to maintain the same technology growth rate  $g = \lambda \ln z$ . In this case, Table 6 shows that the increases in income and consumption

inequality under patent protection are almost the same as before. Although the decreases in income and consumption inequality under R&D subsidies become smaller at 4.63% and 5.91% respectively, they remain quantitatively significant and larger in magnitude than the changes in inequality under patent protection.

Table 5: Calibrated parameter values					
$\rho$	$s$	$\mu$	$z$	$\varphi$	$\gamma$
0.050	0.188	1.035	1.105	3.571	0.200

  

Table 6a: Effects of patent protection				
$\mu$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$
1.038	0.031	1.10%	1.88%	0.36%

  

Table 6b: Effects of R&D subsidies				
$s$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$
0.239	0.031	1.10%	-4.63%	-5.91%

### 4.3 Robustness check: R&D share of GDP

Finally, we examine data on R&D share of GDP. As Comin (2004) argues, data on R&D expenditures reported by firms may not capture all the resources devoted to innovation-related activities. Here we consider a rough exercise by doubling the R&D share of GDP from 0.028 to 0.056. Table 7 reports the re-calibrated parameter values and shows that the markup ratio  $\mu$  increases from 1.035 to 1.073 because  $R/y$  is increasing in  $\mu$ . The re-calibrated value of R&D productivity  $\varphi$  decreases also significantly because a lower R&D productivity is required (given a higher R&D spending  $R/y$ ) in order to keep the innovation arrival rate  $\lambda$  at 0.10 as in the previous section. Table 8 shows that the increases in income and consumption inequality under patent protection become larger at 2.10% and 0.76% respectively whereas the decreases in income and consumption inequality under R&D subsidies become slightly smaller at 4.56% and 5.54% respectively. However, the qualitative pattern remains that income and consumption inequality increases under patent protection but decreases under R&D subsidies and that the magnitude of the changes under R&D subsidies is larger than under patent protection.

Table 7: Calibrated parameter values					
$\rho$	$s$	$\mu$	$z$	$\varphi$	$\gamma$
0.050	0.188	1.073	1.105	1.786	0.200

  

Table 8a: Effects of patent protection				
$\mu$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$
1.078	0.062	1.10%	2.10%	0.76%

  

Table 8b: Effects of R&D subsidies				
$s$	$R/y$	$g$	$\Delta\sigma_I$	$\Delta\sigma_c$
0.239	0.062	1.10%	-4.56%	-5.54%

## 5 Conclusion

In this study, we have explored the effects of innovation policies, such as patent protection and R&D subsidies, on economic growth and also income inequality, which is often neglected by studies in the literature. We have shown that policy instruments may have similar aggregate effects on economic growth but very different distributional effects on income inequality. We have obtained this result in a Schumpeterian model with heterogeneity in household asset holdings. Our model focuses on asset income inequality instead of wage income inequality for two reasons. First, wage inequality in the form of skill premium has received much attention in the literature, but only a relatively small number of studies have considered asset income inequality in the Schumpeterian growth model. Second, empirical studies, such as Atkinson (2000, 2003) and Piketty (2014), have shown that inequality in asset income is playing an increasingly important role.<sup>10</sup> Finally, although our model does not feature tangible assets such as physical capital, intangible assets are an important part of the modern economy.<sup>11</sup>

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<sup>10</sup>Reed and Cancian (2001) show that asset income causes about one-fourth of the increase in income inequality in the 1990s compared to one-tenth of the rise in income inequality in the 1970s.

<sup>11</sup>Nakamura (2003) documents that the market value of intangible assets in the US is about half the value of GDP.



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## Appendix

**Proof of Proposition 1.** From (8), (9), (13) and (15), we have

$$\frac{\dot{v}_t}{v_t} = \frac{\dot{Z}_t}{Z_t} = \frac{\dot{y}_t}{y_t} = \frac{\dot{w}_t}{w_t} = \frac{\dot{\pi}_t}{\pi_t}, \quad (\text{A1})$$

which shows that  $\{y_t, w_t, \pi_t, v_t\}$  grow at the same rate as technology  $Z_t$ . We also know that the value of inventions equals the value of assets such that  $v_t = a_t$ ; therefore,  $a_t$  also grows at the same rate as technology  $Z_t$ . Recall from (17) that the growth rate of  $Z_t$  is  $\dot{Z}_t/Z_t = \lambda_t \ln z$ . In the rest of this proof, we will show that  $\lambda_t$  jumps to a unique and stable steady state, so that  $\dot{Z}_t/Z_t$  also jumps to its steady-state value. From (12), we have

$$\lambda_t = \frac{\varphi R_t}{Z_t} = \varphi \left( 1 - \gamma - \frac{c_t}{Z_t} \right), \quad (\text{A2})$$

where the second equality uses  $y_t = c_t + R_t + G_t$ ,  $y_t = Z_t$  and  $G_t = \gamma y_t$ . If we define  $\chi_t \equiv c_t/Z_t$ , then  $\lambda_t = \varphi(1 - \gamma - \chi_t)$ , which shows that the dynamics of  $\lambda_t$  is solely determined by the dynamics of  $\chi_t$ . Taking the log of  $\chi_t$  and differentiating it with respect to time yield

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{Z}_t}{Z_t} = \frac{\dot{c}_t}{c_t} - \frac{\dot{v}_t}{v_t}, \quad (\text{A3})$$

where the second equality uses (A1). Recall from (3) that  $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ . Then, substituting (3) and (10) into (A3) yields

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\pi_t}{v_t} - \lambda_t - \rho = \frac{\pi_t}{v_t} - \varphi(1 - \gamma - \chi_t) - \rho, \quad (\text{A4})$$

where the second equality uses (A2). Substituting (8), (13) and (15) into (A4) yields

$$\frac{\dot{\chi}_t}{\chi_t} = \frac{\varphi}{1-s} \frac{\mu-1}{\mu} - \varphi(1 - \gamma - \chi_t) - \rho = \varphi\chi_t - \Phi, \quad (\text{A5})$$

where we define a composite parameter  $\Phi \equiv \varphi(1 - \gamma) + \rho - \frac{\varphi}{1-s} \frac{\mu-1}{\mu}$ , which is assumed to be positive by imposing parameter restrictions.<sup>12</sup> Equation (A5) shows that the dynamics of  $\chi_t$  is characterized by instability, so that  $\chi_t$  must jump to its unique and saddle-point stable steady state given by  $\chi = \Phi/\varphi$ . At the steady state,  $c_t$  and  $Z_t$  grow at the same rate given by  $g = \lambda \ln z = \varphi(1 - \gamma - \chi) \ln z = [\varphi(1 - \gamma) - \Phi] \ln z$  as in (20). ■

**Proof of Proposition 3.** Substituting (9), (13), (15) and  $a_t = v_t$  into (23) yields

$$\dot{\theta}_{a,t}(h) = \varphi \frac{\chi_t - (1 - \tau)/\mu}{1 - s} \theta_{a,t}(h) - \varphi \frac{\theta_{c,t}(h)\chi_t - (1 - \tau)/\mu}{1 - s}, \quad (\text{A6})$$

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<sup>12</sup>Otherwise,  $\chi_t$  and  $c_t$  would be negative.

which also uses  $\chi_t \equiv c_t/Z_t$ . Recall from (3) that  $\dot{c}_t(h)/c_t(h) = \dot{c}_t/c_t$ , which in turn implies  $\dot{\theta}_{c,t}(h) = 0$  for all  $t$ . Substituting  $\theta_{c,t}(h) = \theta_{c,0}(h)$  and  $\chi_t = \Phi/\varphi$  for all  $t$  into (A6) yields

$$\dot{\theta}_{a,t}(h) = \frac{\Phi - \varphi(1 - \tau)/\mu}{1 - s} \theta_{a,t}(h) - \frac{\theta_{c,0}(h)\Phi - \varphi(1 - \tau)/\mu}{1 - s}. \quad (\text{A7})$$

Therefore, the dynamic property of  $\theta_{a,t}(h)$  depends on the sign of  $[\Phi - \varphi(1 - \tau)/\mu]/(1 - s)$ . To see that  $\Phi > \varphi(1 - \tau)/\mu$ , one can use (35) to show that

$$\Phi > \frac{\varphi(1 - \tau)}{\mu} \Leftrightarrow s < 1. \quad (\text{A8})$$

Therefore, the coefficient on  $\theta_{a,t}(h)$  in (A7) is positive, which in turn implies that  $\dot{\theta}_{a,t}(h) = 0$  for all  $t$  is the only solution of (A7) consistent with long-run stability. Finally, imposing  $\dot{\theta}_{a,t}(h) = 0$  on (A7) yields the steady-state value of  $\theta_{c,t}(h)$  given by

$$\theta_{c,0}(h) = \frac{\varphi(1 - \tau)}{\mu\Phi} + \left[ 1 - \frac{\varphi(1 - \tau)}{\mu\Phi} \right] \theta_{a,0}(h). \quad (\text{A9})$$

Equation (A9) shows that  $\partial\theta_{c,0}(h)/\partial\theta_{a,0}(h) > 0$  given  $\Phi > \varphi(1 - \tau)/\mu$ . ■